Dynamic Competition in Electricity Markets: Hydropower and Thermal Generation

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Abstract

We study dynamic duopolistic competition between hydro and thermal generators under demand uncertainty. Producers compete in quantities and each is constrained: the thermal generator by capacity and the hydro generator by water availability. Two versions of the model are analysed: an infinite-horizon game with a fixed thermal capacity and a finite-horizon game in which the thermal generator can make investments in capacity.

In the infinite-horizon game, we find the Feedback equilibrium by using collocation methods to approximate the hydro generator's value function. The thermal generator's equilibrium strategy is decreasing in the water level, hence there is a strategic withholding of water by the hydro generator. Comparing the Feedback equilibrium to the efficient one, we find that the thermal capacity and water availability constraints bind less frequently under the duopoly than is efficient. However, for a large range of possible thermal production capacities and water flow levels, the outcome under duopoly is near the efficient outcome in terms of the level of prices.

We analyse the choice of capacity by the thermal producer using a two-period variant of the model. We characterize both the closed-loop and S-adapted open-loop equilibria for this game. We find that investment is higher under closed-loop information.

Keywords: Electricity markets; Dynamic game; Duopoly

1 Introduction

It is common to find alternative electricity generation technologies coexisting in a market. In many jurisdictions, electricity is generated from a mix of thermal (coal, oil, gas), nuclear, and hydro generation plants. Of these technologies, one special characteristic of hydroelectric power generation is that it is constrained by the availability of water, which varies over time. Consequently, an interesting situation from the point of view of the dynamics of competition is when a hydroelectric generator coexists with a thermal generator.[1] Hydroelectric generation can be characterized by low marginal cost when operating, but subject to the availability of water to drive the turbines. In contrast, thermal generation units have more flexibility in the sense that their inputs (gas, coal, etc.) are not subject to the same constraints as water in a reservoir, however the marginal cost of generation is higher.

Another common feature of deregulated electricity markets is price volatility. One reason for the relatively high volatility of electricity prices is the inability to store electricity at a scale that would enable speculation to smooth prices. However, the ability to store water behind a hydro dam does allow for some degree of price smoothing. A hydro operator may benefit from withholding water in periods with low prices in order to have more available for use in periods with high prices. In a perfectly competitive market, it is likely that the hydro operators would choose their water release in an efficient way. However, in most jurisdictions, hydroelectric generators tend to be rather large producers, in which case there is no guarantee that water will be released efficiently. We investigate this issue by analysing the equilibrium of a dynamic game between a hydro producer and a thermal producer. Comparing this equilibrium to the efficient outcome then allows us to discuss the potential for inefficient water use in an imperfectly competitive market for electricity.

A second issue that we address in this paper is that of investment in new generation capacity in the presence of a large hydro competitor. In many jurisdictions, following deregulation and in combination with demand growth, we expect to see investment in new generation capacity. Given regulatory and environmental hurdles and substantial fixed costs, new hydro development is often not an option. In this case, new capacity is commonly

[1] In some jurisdictions, hydroelectric power generation is the dominant source of electricity. It accounts for 80% of generation in New Zealand, 97% in Brazil, 90% in Quebec, and 98% in Norway (Crampes and Moreaux (2001)). In other jurisdictions, such as Ontario and Western US, it is a significant source of electricity, but not as dominant.

provided by thermal plants. Since the incentive to invest in new capacity depends on the expected distribution of future prices, the extent to which incumbent hydro generation affects the distribution of prices will have an effect on investment in thermal capacity.

Although there has been much recent interest in models of electricity markets, there has not been much analysis of the implications for market performance when one of the producers has significant hydroelectric generation capability. Exceptions include Crampes and Moreaux [3], Bushnell [1], Scott and Read [14].

The paper closest to what we do here is Crampes and Moreaux [3] who model a Cournot duopoly in which a hydro producer uses a fixed stock of water over two periods facing competition from a thermal producer and a known, deterministic demand. They address the question of how the available water is used over the two periods under a variety of market structures, finding that under duopoly, hydro production is tilted towards the second period relative to what is efficient. I.e., there is a strategic incentive to withhold water in the first period. We find a similar effect in the infinite horizon version of our model. However, we also examine the possibility of a binding constraint on thermal production. This allows us to discuss implications of the model for investment in thermal generation capacity.

Bushnell [1] examines a Cournot oligopoly with fringe producers in which each producer controls both hydro and thermal generation facilities. Both hydro and thermal units face capacity constraints and the producers must decide how to allocate the available water over a number of periods. He solves the model with parameters calibrated to the western United States electricity market and finds that the dynamic allocation of water under imperfectly competitive conditions is not the efficient one. In particular, firms tend to allocate more water to off-peak periods than is efficient.

Scott and Read [14] develop a Cournot model of mixed hydro/thermal generation that is calibrated to the New Zealand wholesale electricity market, which is dominated by hydro generation. They focus on the effect of forward contracting, concluding that high levels of contracting are necessary to approach an efficient outcome. In addition, they do not find that hydro producers strategically withhold water in any significant way.

There is a recent literature examining capacity investment in electricity markets including Garcia and Shen [5], Garcia and Stacchetti [6], Bushnell and Ishii [2], Murphy and Smeers [12].

We turn next to a description of the basic aspects of the model. We then examine the Feedback equilibrium for the infinite horizon dynamic game between the hydroelectric and thermal generators holding the capacity of the thermal generator fixed. We solve this game using collocation methods to find numerical solutions to the game which we then analyse using simulations of the equilibrium outcomes. Finally, we turn our attention to the investment choices of the thermal generator using an alternative two-period version of the model. We are able to find closed-form solutions for both the S-adapted open-loop and closed-loop equilibria.

2 The model

Consumers of electricity have no control over price, with their behaviour in any period $t = 0, 1, 2, ..., T$ summarized by the following inverse demand function:

$$P_t = D_t - \beta(h_t + q_t), \qquad \beta > 0. \tag{1}$$

The demand intercept, D_t, is stochastic, which provides an incentive to use water in a manner that smooths price fluctuations. The particular distribution of D_t will be specified as we discuss each particular version of the model.

There are two types of technologies in the industry: a hydroelectric generator owns generation units that use water held behind dams to spin the electric generators and a thermal electric generator owns thermal units that burn fossil fuel. Thermal generation costs are given by $C(q) = c_1 q + (c_2/2)q^2$.

The marginal cost of production for hydro units is zero. The hydro producer is not involved in capacity investments, and doesn't expand the production capacity by any means.[2] The hydro producer's electricity generation, h_t, is determined by at one-to-one relation with the amount of water it releases from its reservoir. Its output is constrained by the amount of water available for release, W_t. The transition equation governing the level of water in the reservoir is

$$W_{t+1} = (1 - \gamma)(W_t - h_t) + \omega, \tag{2}$$

where W_t is the level of the reservoir at the beginning of period t, γ is a parameter that determines the rate of evaporation in the reservoir over an interval of time, and ω is the rate of inflow into the reservoir over an interval

[2] Construction of a new river dam, depending on the production capacity, to produce electricity can take up to fifteen years. e.g., the Three Gorges Dam in China, the largest dam on earth, was completed in about 14 years. Also because of environmental and several other reasons (e.g., irrigation, recreational), it may not be possible to expand the available production capacity of the hydro player. Marginal cost of production is generally assumed to be zero, since the water turning the turbines is commonly free.

of time. For and infinite time horizon, this specification implies a "natural" steady-state water level in the absence of hydroelectric generation of ω/γ.

Producers choose their outputs simultaneously in each period and both producers discount future payoffs at a common rate, δ, which is also the social rate of discount.

3 Infinite horizon

In this section we present an infinite horizon version of the model in order to analyse the stationary distribution of hydro and thermal production. We assume here that the stochastic component of demand in (1) is normally distributed, i.e., $D_t \sim N(\mu, \sigma^2)$, with a variance small enough to render the probability of non-positive demand very small.

We next describe the game played by the duopoly, after which we describe the efficient solution. Following that, we analyse the differences in the two market structures by way of numerical solutions.

3.1 Duopoly

Each player, the hydro producer and the thermal producer, are assumed to maximize the discounted present value of profits, where each discounts the future using the common discount factor $\delta \in (0,1)$. We focus on the case in which producers use Feedback strategies, which are functions of the current state (W_t, D_t) only.[3] Denote the strategies of the two producers by $\sigma^H(D_t, W_t)$ and $\sigma^T(D_t, W_t)$. We assume that both producers observe W_t and D_t before making decisions in period t.The Feedback equilibrium is a Nash equilibrium in Feedback strategies.

Given the hydro producer's strategy, $\sigma^H(D_t, W_t)$, the problem for the thermal producer is then

$$\max_{\{q_t\}} E \sum_{t=0}^{\infty} \delta^t \left[(D_t - \beta(\sigma^H(D_t, W_t) + q_t))q_t - c_1 q_t - (c_2/2){q_t}^2 \right] \qquad (3)$$

subject to

$$0 \leq q_t \leq K,$$

The thermal producer's problem is simplified by the fact that the thermal producer does not influence the future state through its actions. Since its

[3]This rules out other equilibria, typically collusive ones, which emerge when strategies are allowed to be functions of all previous play.

production decision does not affect its continuation payoff, thermal production is governed by its "static" best response function for an interior solution. Incorporating the capacity and non-negativity constraints, we have

$$\sigma^T(D_t, W_t) = \max\left[0, \min\left[\frac{D_t - c_1 - \beta\sigma^H(D_t, W_t)}{2\beta + c_2}, K\right]\right] \tag{4}$$

Given the thermal producer's strategy, $\sigma^T(D_t, W_t)$, the problem faced by the hydro producer is to

$$\max_{\{h_t\}} E \sum_{t=0}^{\infty} \delta^t \left[(D_t - \beta(h_t + \sigma^T(D_t, W_t)))h_t\right] \tag{5}$$

subject to

$$0 \leq h_t \leq W_t$$

and

$$W_{t+1} = (1 - \gamma)(W_t - h_t) + \omega,$$

The hydro producer's best response is determined by the solution to a dynamic optimization problem. The Bellman equation for the hydro producer's problem is

$$V(D_t, W_t) = \max_{h_t \in [0, W_t]} \left\{(D_t - \beta(h_t + \sigma^T(D_t, W_t)))h_t + \delta E_t V(D_{t+1}, W_{t+1})\right\} \tag{6}$$

subject to (2). The value of h_t that solves the maximization problem in (6) is the hydro producer's best response to $q_t = \sigma^T(D_t, W_t)$ which yields $\sigma^H(D_t, W_t)$.

Define $\psi(h_t)$ as the derivative of the payoff in the maximization problem in (6), i.e.,

$$\psi(h_t) = D_t - 2\beta h_t - \beta\sigma^T(D_t, W_t) - \delta(1-\gamma)E_t V_W(D_{t+1}, (1-\gamma)(W_t - h_t) + w). \tag{7}$$

Let b_{0t} and b_{Wt} be the Lagrange multipliers on the non-negativity and water availability constraints for the maximization problem in (6). The necessary conditions for optimal hydro output are

$$\psi(h_t) + b_{0t} - b_{Wt} = 0 \tag{8}$$

$$b_{Wt}(W_t - h_t) = 0, \quad b_{Wt} \geq 0, \quad (W_t - h_t) \geq 0 \tag{9}$$

and

$$b_{0t}h_t = 0, \quad b_{0t} \geq 0, \quad h_t \geq 0. \tag{10}$$

We can illustrate the strategic effect by expanding the $E_t V_W(D_{t+1}, W_{t+1})$ term in (7). Evaluating (6) at $t+1$ and differentiating with respect to W_{t+1} yields

$$\begin{aligned} E_t V_W(D_{t+1}, W_{t+1}) = E_t\Big[&(\psi(h_{t+1}) + b_{0t+1} - b_{Wt+1})\sigma_W^H(D_{t+1}, W_{t+1}) \\ &- \beta\sigma_W^t(D_{t+1}, W_{t+1})\sigma^H(D_{t+1}, W_{t+1}) \\ &+ b_{Wt+1} + \delta(1-\gamma)E_{t+1}V_W(D_{t+2}, W_{t+2})\Big] \end{aligned} \quad (11)$$

Using (8) we have

$$\begin{aligned} E_t V_W(D_{t+1}, W_{t+1}) = E_t\Big[&-\beta\sigma_W^t(D_{t+1}, W_{t+1})\sigma^H(D_{t+1}, W_{t+1}) \\ &+ b_{Wt+1} + \delta(1-\gamma)E_{t+1}V_W(D_{t+2}, W_{t+2})\Big] \end{aligned} \quad (12)$$

Applying the same process for $V_W(D_{T+2}, W_{t+2})$, $V_W(D_{T+3}, W_{t+3})$, ... yields

$$\begin{aligned} E_t V_W(D_{t+1}, W_{t+1}) = E_t\Big[&-\beta\sum_{i=0}^{\infty}\delta^i(1-\gamma)^i h_{t+1+i}\sigma_W^T(D_{t+1+i}, W_{t+1+i}) \\ &+ \sum_{i=0}^{\infty}\delta^i(1-\gamma)^i b_{Wt+1+i}\Big]. \end{aligned} \quad (13)$$

The strategic effect works through the influence of hydro output on future thermal output via future water availability. Since $\sigma_W^T(D, W) < 0$ (which occurs in all cases we examine below) the difference between the hydro producer's output in the Feedback equilibrium versus the open loop equilibrium is negative: the hydro producer reduces output relative to the open loop equilibrium strategy. This results in more water available future periods and hence lower thermal output.

In order to describe the Feedback equilibrium strategies more carefully, we need to find the value function for the hydro producer, which we do using numerical approximation techniques. Rather than approximate the value function itself, we solve the problem by approximating $E_t V(D_{t+1}, W_{t+1})$, which has the benefit of allowing us to approximate a function of one state variable only (W_{t+1}) since the future demand shock is integrated out.[4]

[4]This is a consequence of the assumption that the demand states are i.i.d. If we were to allow any serial correlation in this process, the expected value function would be a function of two state variables as well.

3.1.1 Numerical algorithm: duopoly

We approximate the hydro producer's expected value function using the collocation method[5]. In particular,

$$E_t V(D_{t+1}, W_{t+1}) \approx \sum_{i=1}^{n} d_i \phi_i(W_{t+1}) \equiv \tilde{V}(W_{t+1}) \tag{14}$$

where the ϕ_i are known basis functions. Collocation proceeds by determining the d_i, $i = 1, ...n$, in order for the approximation to hold exactly at n collocation nodes, $W_+^1, W_+^2, ..., W_+^n$. The algorithm we use to solve the problem is described as follows:

0. Choose a starting approximation of $\tilde{V}^0(W_{t+1})$, i.e., starting values d_i^0, $i = 1, 2, ..., n$.

1. Given the current approximation, $\tilde{V}^k(W_{t+1})$ compute the value function at the collocation nodes, $W_+^1, W_+^2, ..., W_+^n$. In order to do this, we determine the Nash Equilibrium quantities for each producer. At every node i, conditional on the demand state, D_+:

   a) Use a root-finding algorithm to solve $\psi(h, R^T(h)) = 0$ if a root exists in $(0, W_+^i)$. If not, determine whether $h = 0$ or $h = W_+^i$ is appropriate. Here $R^T(h)$ is the thermal producers best-response to h, i.e., equation (4) using h in place of σ^H.

   b) Given the value found for h, compute q from (4).

   Use these quantities to compute $V^k(D_+, W_+^i)$. This step yields the value in the next period as a function of the demand state for each W_+^i.

2. Integrate the new value function numerically over demand states to update $\tilde{V}(W_t^{k+1})$, i.e. find new values d_i^1, $i = 1, ..., n$.

3. If convergence achieved, stop. Else, return to step 1.

3.2 Efficient solution

We wish to compare the outcome under duopoly to what is efficient. To this end, we solve the problem faced by a social planner choosing thermal

[5] See Judd [11]

and hydro generation with the objective of maximizing the expected present value of the stream of consumer surplus less generation costs:

$$\max_{\{h_t,q_t\}} \sum_{t=0}^{\infty} \delta^t \left(D_t(h_t+q_t) - \frac{\beta}{2}(h_t+q_t)^2 - c_1 q_t - \frac{c_2}{2} {q_t}^2 \right) \tag{15}$$

subject to

$$\begin{aligned} W_{t+1} &= (1-\gamma)(W_t - h_t) + \omega, \\ 0 \le h_t &\le W_t, \\ 0 \le q_t &\le K, \end{aligned}$$

and

$$D_{t+1} \sim N(\mu, \sigma^2).$$

The planner's value function then satisfies the Bellman equation:

$$V^P(D_t, W_t) = \max_{h_t,q_t} D_t(h_t+q_t) - \frac{\beta}{2}(h_t+q_t)^2 - c_1 q_t - \frac{c_2}{2} {q_t}^2 + \delta E_t V^P(D_{t+1}, W_{t+1}) \tag{16}$$

subject to the constraints.

The necessary conditions for the maximization problem are

$$D_t - \beta(h_t+q_t) - \delta(1-\gamma)\frac{\partial[E_t V^P(D_{t+1}, W_{t+1})]}{\partial W_{t+1}} - b_W + b_0 = 0 \tag{17}$$

and

$$D_t - \beta(h_t+q_t) - c_1 - c_2 q_t - a_K + a_0 = 0 \tag{18}$$

where b_W and b_0 are the Lagrange multipliers on hydro production's capacity and non-negativity constraints and a_K and a_0 are the multipliers on thermal production's capacity and non-negativity constraints. Equations (17) and (18) imply

$$a_K - a_0 + c_1 + c_2 q_t = \delta(1-\gamma)\frac{\partial[E_t V^P(D_{t+1}, W_{t+1})]}{\partial W_{t+1}} + b_W - b_0 \tag{19}$$

which for an interior solution simplifies to

$$\delta(1-\gamma)\frac{\partial[E_t V^P(D_{t+1}, W_{t+1})]}{\partial W_{t+1}} = c_1 + c_2 q_t, \tag{20}$$

the marginal value of water is equalized with the marginal cost of thermal production.

The numerical algorithm used to solve the planner's problem is similar to that described for the duopoly, using collocation to approximate the planner's expected value function.

3.3 The effects of water flow

We now present solutions for some particular cases of the model's parameters. For all examples below, we maintain some of the parameter values at specific values: $c_1 = 0, c_2 = 1.0,\ \delta = 0.9,\ \gamma = 0.3,\ \mu = 10.0,\ \beta = 1.0,\ \sigma = 1.0$. We present results for alternative values of ω, each of which has "large" thermal capacity ($K = 4.0$).

A useful benchmark to keep in mind is what the equilibrium of an unconstrained situation would be. For a static Cournot game with production costs as given for these two producers and a demand intercept of 10, the hydro producer would produce 4 units and the thermal producer 2 units.

In all the examples below, Chebyshev polynomials are used for the ϕ_i functions and n varies by example and market structure.[6]

3.3.1 Example: Low inflow

We start with $\omega = 1.0$. This level of water inflow is "low" in the sense that 1.0 units of water per period is substantially less than the hydro producer would choose to produce if it were unconstrained. Hence, in this example, we expect hydro production to be frequently constrained by water availability.

We present the solution as a series of plots over a range of D_t and W_t values. We choose a range for the plots with the range of D_t given by $\pm$ 2.5 standard deviations around the mean, while the range of W_t is between zero and the "natural" steady state water level, ω/γ, which is equal to 10/3 in this case. Production of hydro and thermal producers under each market structure are presented in Figure 1. For low water levels, h_t increases one for one with the available water and does not vary with the demand state. In this region, the constraint that $h_t \leq W_t$ binds and the only water available in the next period is due to the inflow, ω. We see that water availability is less constraining in the duopoly than in the efficient solution. The hydro producer restricts its output relative to the efficient level, and hence the water availability constraint is not binding for a larger area of the state space.

Figure 1 demonstrates that there are three "regimes" characterizing the optimal thermal output. For low water levels and high demand states, thermal output is constrained by capacity. For somewhat higher water levels, the capacity constraint no longer binds, but hydro output is still constrained by W_t, so thermal output falls quickly as water levels increase. Finally for values

[6] The computations are done with C++ and make use of routines for Chebyshev approximation, numerical integration, and root finding from the Gnu Scientific Library. [4]

	Low inflow		Medium inflow		High Inflow	
	Duopoly	Efficient	Duopoly	Efficient	Duopoly	Efficient
$E(h)$	1.00	1.00	3.78	4.00	4.00	5.00
$E(q)$	3.00	3.96	2.07	2.99	2.00	2.50
$E(p)$	6.00	5.04	4.14	3.00	4.00	2.50
st.dev.(p)	0.67	0.92	0.44	0.51	0.40	0.49
skew(p)	0.02	0.34	0.23	0.25	0.00	0.13
% $h = W$	0.994	0.997	0.039	0.988	0.000	0.969
% $q = K$	0.001	0.840	0.000	0.022	0.000	0.001
$E\Pi^T$	137.0	122.1	65.3	46.6	60.7	32.7
$E\Pi^H$	60.1	50.5	158.6	120.5	162.0	125.4
E(Welfare)	277.8	295.7	397.4	413.3	405.0	440.9

Table 1: Simulated Descriptive Statistics

of the state in which hydro production is not constrained, optimal thermal output decreases less quickly with water levels. Except for the points where the thermal strategy exhibits a kink, $\sigma_W^T < 0$, so the strategic effect works to decrease hydro production as discussed above. Comparing the duopoly with the efficient case, thermal output is lower in equilibrium than is efficient for all values of the state. This is the consequence of market power. For this lower water inflow case, since hydro production is frequently constrained, it is efficient to use more thermal production to take up the slack. However, the thermal producer uses its market power to restrict output, keeping price higher.

The first column of Table 1 provides some descriptive statistics for the values of the outcome when equilibrium strategies are played. They are created by generating 100 simulations of the model over 1,000 periods each.[7] The values in Table 1 are averages over the 100 runs. For this low water inflow case, the hydro producer is almost always producing at capacity (the reservoir is drained each period) which is also what is efficient. The thermal producer produces less than what is efficient, resulting in an inefficiently high average price. The thermal producer almost never produces at capacity (it does so less than 1% of the time), whereas the planner would have the thermal producer at capacity 84% of the time.

[7] An initial run of 100 periods precedes the 5,000 period sample to minimize any effects of starting values. We use 1/2 of the natural steady state water level as a starting value.

3.3.2 Example: Medium inflow

In order to relax the constraint on hydro production we now present another example in which all parameter values except the inflow of water, ω, are as above. We now set $\omega = 4.0$ which is the hydro producer's Cournot equilibrium output in a "static" version of this game with demand at the mean level of the demand disturbance. This guarantees that there is enough water in any period for the hydro producer to produce the same output as it would in the "static" game with no uncertainty.

Now the hydro producer chooses output that drains its reservoir only 4% of the time, whereas the planner would have it do so 99% of the time. This reduction of output by the hydro producer, along with the thermal producer's inefficiently low output now results in a larger price gap between the duopoly and efficient outcomes. The duopoly price falls compared to the low inflow case, but not by as much as is efficient.[8]

3.3.3 Example: High inflow

A third possibility that we examine is the case where the flow of water is sufficient that it is rarely constraining. For this scenario, we choose an inflow of water that is equal to 2.5 standard deviations above the mean hydro production in the "static" game discussed above. For these parameter values, this translates into a value of $\omega = 5.0$.

Not surprisingly, the duopoly equilibrium now results in outcomes that are what occurs in the static Cournot game. Neither producer operates at capacity, so we just have an interior solution that replicates the Cournot outcome. This is not efficient, since the planner would like to use more of the low cost output, having the hydro producer at capacity in almost all periods.

3.4 Effects of Thermal Capacity

In order to analyse how thermal generation capacity, K, affects the equilibrium outcomes under the two market structures we examine the "medium inflow" case of section 3.3.2 allowing for different levels of K. We solve the model for 20 different capacities ranging between zero and four units. For each solution, we simulate the model over 5,000 periods as we did above. We plot some of the resulting statistics in Figures 2 and 3.

[8]The duopoly price is 19% higher than the efficient one in the low inflow case and 38% higher in the medium inflow case.

The top row of Figure 2 plots the average outputs of each producer by market structure. At low levels of thermal capacity, the thermal producer is essentially always operating at capacity, which is efficient. At large levels of capacity (roughly speaking, larger than 2 — the average Cournot output), the thermal producer reduces output below capacity more frequently, resulting in inefficiently low output at higher capacities. In contrast, the hydro producer's average output is below capacity for any level K, although the size of the difference to the efficient level is not large.

The bottom left graph in Figure 2 demonstrates that price is very close to the efficient level until thermal capacity reaches approximately 2.0. After this point, price levels off and slowly falls to the Cournot price of 4.0, whereas the planner has price falling until thermal capacity is beyond 3.0. The implications for price volatility are demonstrated in the bottom right graph in Figure 2. The duopoly results in prices that are less volatile than is efficient. While this result is as expected[9] it is notable that the gap between price volatility is largest for intermediate values of K.

We plot payoffs in Figure 3: profits for each producer and social welfare generated. Each payoff is very close to the efficient one for thermal capacities less than 2.0. From the above discussion we know that this is because the thermal constraint frequently binds under both market structures and the hydro producer does not reduce output greatly under duopoly.

An interesting question to now address is what thermal capacity would be chosen? Consider allowing the thermal producer to make a one time investment in capacity before time 0. Looking at the thermal payoff (solid line) graph in Figure 3, unless the marginal cost of capacity is very low, the thermal producer would choose a capacity below 2.0. As this is in the region where the equilibrium is near efficiency, we can suggest that *conditional on the level of capacity chosen*, the equilibrium in the dynamic duopoly game is "close" to the efficient one. Of course the planner may wish to choose a higher level of capacity, so an interesting extension of this game would be to examine optimal versus actual capacity choices. We do not do that with this infinite horizon game, but rather discuss this issue in detail using a finite-horizon version of the model in the next section.

[9] Thille [15] demonstrates that firms with market power will "over-smooth" demand fluctuations when demand is linear.

4 A Two-Period Model with Capacity Choice

In this section, we examine a two-period version of the model described in section 2. In period 0, inverse demand is known to be $P_0(Q) = D - Q$, with D a constant. Inverse demand in period 1 is random:

$$P_1(Q) = \begin{cases} D + \delta - Q & \text{with probability } u \\ D - \delta - Q & \text{with probability } d \end{cases} \tag{21}$$

with $u + d = 1$.

There is a constant inflow of water into the reservoir in each period, which we denote ω. We assume no evaporation so the amount of water available in period 1 is $W_1 = W_0 + \omega - h_0$ with W_0 the initial level of water in the reservoir. We assume that W_0 is large enough so that the hydro producer is unconstrained in either of the period 1 scenarios and low enough that the efficient solution is constrained. The hydro producer must choose three actions in this game: period 0 production and period 1 production in each of the two demand states. We denote a vector of hydro producer actions as (h_0, h_{1u}, h_{1d}).

Production costs for the thermal producer are as above: $c_1 q_t + c_2/2 q_t{}^2$. We now allow for the thermal producer to invest in capacity in period 0. An investment of I_0 units of capacity costs the thermal producer $e_1/2 I_0^2$. Investment is irreversible: $I_0 \geq 0$ and capacity does not depreciate. The thermal producer begins the game with K_0 units of capacity, so in period one has $K_1 = K_0 + I_0$ units of capacity available. Actions taken by the thermal producer consist of investment and production in period 0 and production in each of the period 1 demand states: $(I_0, q_0, q_{1u}, q_{1d})$. We are interested in a case where K_0 is large enough so that the capacity constraint does not always bind, but low enough that the thermal producer has an incentive to invest in increasing capacity. In particular, we present equilibria that have $q_{1u} = K_1$, and $q_{1d} < K_1$. The specific parameter values that satisfy this assumption will be presented below in Assumptions 1 and 2.

It will be useful in the presentation of the results to let q_u^c and q_d^c denote the Cournot equilibrium thermal outputs in the first period game when no constraints bind and q_0^c the corresponding quantity in period 0, i.e.,

$$q_0^c \equiv \frac{D - 2c_1}{3 + 2c_2}, \qquad q_d^c \equiv \frac{D - \delta - 2c_1}{3 + 2c_2}, \qquad q_u^c \equiv \frac{D + \delta - 2c_1}{3 + 2c_2}. \tag{22}$$

The timing of the game is as follows. In the first period players choose production quantities simultaneously and independently to maximize their own profits. At the same period, thermal player who makes capacity investment under uncertainty understands that there are two possible demand states at the next stage and investment in this period will become available for production in the following period. In the second period, players make their optimal production decisions conditional on the demand state that reveals, and no investments will take place since it is the final stage.

We analyse this game by computing three outcomes. For the non-cooperative game between producers we compute both the open-loop and closed-loop equilibria. This allows us to demonstrate the strategic effects of thermal capacity choice. In addition, we compute the efficient solution in order to examine the degree of inefficiency under the duopoly market structure.

4.1 S-adapted open-loop equilibrium

In this subsection, we wish to compute the equilibrium outcome when the thermal producer does not choose its investment level strategically. If there were no uncertainty, the appropriate equilibrium concept would be the open-loop Nash equilibrium. However, we want the producers to be able to respond to the future demand state, in which case the appropriate solution concept is the S-adapted open-loop equilibrium. Here we assume that players have S-adapted information. This equilibrium concept first introduced by Zaccour [16] and Haurie, Zaccour and Smeers [9]. It is extended and employed for large-scale oligopolies by Haurie and Moresino [10], Genc, Reynolds and Sen [7], and Genc and Sen [8]. In this equilibrium, players condition their decisions on time period, demand state and initial capacity levels. Players choose their profit maximizing strategies given the rivals' strategies. This equilibrium concept is a half way between closed loop and open loop equilibrium paradigms (see, e.g., Genc, Reynolds and Sen [7] and Pineau and Murto [13]).

In terms of our model, strategies can depend explicitly on the demand state, but not on the level of thermal capacity. An S-adapted strategy for the hydro producer is $\sigma^H = (h_0, h_{1u}, h_{1d})$, where h_{1u} is period one production in the high demand state and h_{1d} is period one production in the low demand state. The thermal producer's strategy is $\sigma^T = (I_0, q_0, q_{1u}, q_{1d})$.

The hydro producer chooses its strategy to solve

$$\max_{\sigma^H} E_0 \sum_{t\in\{0,1u,1d\}} (D_t - (h_t + q_t))h_t \tag{23}$$

subject to

$$0 \leq h_t \leq W_t.$$

E_0 denotes the expectation taken with respect to information available at time 0. As mentioned above, we assume that W_0 is sufficiently large that the constraint will not be binding.

The thermal producer faces the problem:

$$\max_{\sigma^T} E_0 \sum_{t \in \{0,1u,1d\}} \left[(D_t - (h_t + q_t))q_t - c_1 q_t - \frac{c_2}{2} {q_t}^2 - \frac{e_1}{2} I_0^2 \right] \tag{24}$$

subject to

$$0 \leq q_t \leq K_t,$$

$$K_1 = K_0 + I_0.$$

For the equilibrium to involve a binding thermal capacity constraint in the high-demand period 1 state only, we make the following assumption regarding the initial level of capacity:

Assumption 1.

$$K_0 \in \left[q_d^c - \frac{2u\delta}{2e_1 + u(3 + 2c_2)}, q_u^c \right].$$

The following proposition summarizes the equilibrium strategies for this game:

Proposition 1. *For W_0 sufficiently large that the hydro producer is not constrained, and under Assumption 1, the S-adapted open-loop Nash equilibrium strategies are:*

$$I_0 = \frac{u(D + \delta - 2c_1 - K_0(3 + 2c_2))}{2e_1 + u(3 + 2c_2)} \tag{25}$$

$$(q_0, h_0) = \begin{cases} \left(q_0^c, \frac{D(1+c_2)+c_1}{3+2c_2} \right) & \textit{if } q_0^c < K_0 \\ \left(K_0, \frac{D-K_0}{2} \right) & \textit{otherwise.} \end{cases} \tag{26}$$

$$(q_{1u}, h_{1u}) = \left(K_0 + I_0, \frac{D + \delta - K_0 - I_0}{2} \right) \tag{27}$$

$$(q_{1d}, h_{1d}) = \left(q_d^c, \frac{(D - \delta)(1 + c_2) + c_1}{3 + 2c_2} \right) \tag{28}$$

Proof. Since there is enough water available that the hydro constraints do not bind, the hydro producer plays its "static" best response in each period. Specifically $h_0 = \frac{D-q_0}{2}$, $h_{1u} = \frac{D+\delta-q_{1u}}{2}$, and $h_{1d} = \frac{D-\delta-q_{1d}}{2}$.

The period 0 thermal production choice has no bearing on the payoffs of any of other thermal actions, so which case obtains in (26) is of no consequence to the equilibrium investment and period 1 outputs. The Lagrangian function for the thermal producer's problem is

$$L^T = E_0 \sum_{t\in\{0,1u,1d\}} \left[(D_t - (h_t + q_t))q_t - c_1 q_t - \frac{c_2}{2} {q_t}^2\right] - \frac{e_1}{2} I_0^2 + \sum_{t\in\{0,1u,1d\}} [a_t(K_t - q_t)] \tag{29}$$

where $a_t \geq 0$ are the Lagrange multipliers on the capacity constraints[10]. The KKT conditions for the thermal producer's problem are then

$$\frac{\partial L^T}{\partial q_t} q_t = 0, \quad \frac{\partial L^T}{\partial a_t} a_t = 0 \quad \text{and} \quad \frac{\partial L^T}{\partial I_0} I_0 = 0,$$

for $t = 0, 1u, 1d$.

Given the assumptions regarding K_0, it clear that $a_{1u} > a_{1d} = 0$. It follows that $q_{1u} = K_0 + I_0$, and $q_{1d} = \frac{D-\delta-h_{1d}-c_1}{2+c_2}$. Also at $t = 0$, since we assume interior solution, $a_0 = 0$ holds, hence $q_0 = \frac{D-h_0-c_1}{2+c_2}$.

Next we solve the best response functions for the equilibrium points. By substituting one player's response functions into other's functions we obtain that $q_{1d} = \frac{(D-\delta)-2c_1}{3+2c_2}$ and $h_{1d} = \frac{(D-\delta)(1+c_2)+c_1}{3+2c_2}$. Since $q_{1u} = K_0 + I_0$, $h_{1u} = \frac{D+\delta-K_0-I_0}{2}$. At time 0, either $q_0 = \frac{D-2c_1}{3+2c_2}$ or $q_0 = K_0$ and the hydro producer plays its best response.

For optimal investment outcomes we note that the period one capacity constraints only bind when demand is high, so investment only has an impact in that state. We then obtain $a_{1u} = u[D + \delta - 2q_{1u} - h_{1u} - c_2 q_{1u} - c_1]$. Using the equilibrium q_{1u} and h_{1u} from above and noting that the optimal investment choice satisfies $I_0 = \frac{a_{1u}}{e_1}$, we get the equilibrium I_0 of the proposition.

Comparing the equilibrium thermal quantities with the assumed pattern of binding constraints gives the bounds in Assumption 1. □

We will discuss the equilibrium investment of Proposition 1 more once we have computed the investment in the closed-loop equilibrium.

[10]The non-negativity constraints do not bind for the situations we are interested in, so we suppress their multipliers to simplify the presentation.

4.2 Closed-loop equilibrium

In the S-adapted open-loop equilibrium, the thermal producer does not take into account the influence that its investment choice has on the hydro producer's output choice in period one. This was a consequence of the S-adapted information structure. We now allow for a closed-loop information structure.

For the equilibrium to involve a binding thermal capacity constraint in the high-demand period 1 state only, we make the following assumption regarding the initial level of capacity:

Assumption 2.

$$K_0 \in \left[q_d^c\left(1-\frac{u}{2e_1}\right)-\frac{u\delta}{e_1}, q_u^c\left(1-\frac{u}{2e_1}\right)\right]$$

Solving for the equilibrium quantities we get

Proposition 2. *For W_0 sufficiently large that the hydro producer is not constrained, and under Assumption 2, the closed-loop Nash equilibrium strategies are:*

$$I_0^M = \frac{u[D+\delta-K_0(2+2c_2)-2c_1]}{2(e_1+u(1+c_2))} \tag{30}$$

$$(q_0, h_0) = \begin{cases} \left(q_0^c, \frac{D(1+c_2)+c_1}{3+2c_2}\right) & \textit{if } q_0^c < K_0 \\ \left(K_0, \frac{D-K_0}{2}\right) & \textit{otherwise.} \end{cases} \tag{31}$$

$$(q_{1u}, h_{1u}) = \left(K_0 + I_0^M, \frac{D+\delta-K_0-I_0^M}{2}\right) \tag{32}$$

$$(q_{1d}, h_{1d}) = \left(\frac{D-\delta-2c_1}{3+2c_2}, \frac{(D-\delta)(1+c_2)+c_1}{3+2c_2}\right) \tag{33}$$

Proof. The only difference in the proof of this proposition and that of Proposition 1 is in the determination of investment. The best responses by both players in period one are the same as they are in the S-adapted open-loop game. Hence, conditional on K_1, outputs in period one are the same. However, investment in capacity by the thermal producer, and hence K_1, may differ.

Under Assumption 2, investment only provides benefits in stage $1u$. Let $\pi_{1u}^T(K_1)$ be the profit to the thermal investor in period $1u$ when it has capacity of $K_1 = K_0 + I_0$. Optimal investment must satisfy

$$-e_1 I_0 + u\frac{\partial \pi_{1u}^T}{\partial K_1}\frac{\partial K_1}{\partial I_0} = 0, \tag{34}$$

or

$$-e_1 I_0 + u\left[D + \delta - h_{1u}(K_1) - 2K_1 - h'_{1u}(K_1) - c_1 - c_2 K_1\right] = 0. \qquad (35)$$

When $q_{1u} = K_1$, we know that $h_{1u}(K_1) = \frac{D+\delta-K_1}{2}$ is the hydro producers best response. Substituting this for $h_{1u}(K_1)$ and $K_1 = K_0 + I_0$ and simplifying we have

$$I_0^M = \frac{u[D + \delta - K_0(2 + 2c_2) - 2c_1]}{2(e_1 + u(1 + c_2))} \qquad (36)$$

Comparing the equilibrium thermal quantities with the assumed pattern of binding constraints gives the bounds in Assumption 2. □

The only difference in behaviour between the open- and closed-loop games is that under closed-loop information, the thermal producer considers the influence of its investment choice on h_{1u}. From Proposition 2, we know that $\partial h_{1u}/\partial I_0^M = -1/2$ in the closed-loop game, which leads to higher equilibrium investment, i.e., the strategic effect associated with investment in thermal capacity results in "aggressive" behaviour by the thermal producer. We summarize this result in:

Corollary 1. *For K_0 satisfying both Assumptions 1 and 2, the closed-loop equilibrium investment is larger than the S-adapted open-loop equilibrium investment.*

Since total output in period $1u$ is increasing in I_0, we have price lower in period $1u$ in the closed-loop equilibrium than in the open-loop equilibrium. Prices in periods 0 and $1d$ are the same under both equilibria. It is important to note that even though prices are lower, this does not mean that the closed-loop equilibrium is more efficient. The increased output comes about through inefficient investment. Since the hydro producer is not operating at capacity, clearly it would be efficient to increase output by increasing hydro production. In the closed-loop equilibrium however, the increased output comes about through increased thermal production that is made possible through a costly investment.

Whether or not the thermal producer chooses a capacity that is greater or less than the efficient level investment depends on two conflicting forces. First, since the hydro producer is restricting output, increasing thermal capacity can be efficient in that it reduces the loss due to the exercise of market power. Second, since the hydro producer has lower production costs and is not constrained, if output is to be increased, it is efficient for it to be done by the hydro producer, not by the thermal producer increasing capacity. Either of these forces might dominate, as seen by the following proposition:

Proposition 3. *Equilibrium capacity investment by the thermal producer may be either higher or lower than is efficient.*

Proof. By way of proof, we simply look at two limiting cases $W_0 \to 0$ and $W_0 \to \infty$. As $W_0 \to 0$, hydro production goes to zero, and the thermal producer has a monopoly. Thermal monopoly output and investment is clearly lower than what would be efficient. At the other extreme, as $W_0 \to \infty$, equilibrium investment under duopoly is as we have described in Propositions 1 and 2, i.e., positive. In this case, the efficient level of investment is clearly zero since hydro production can meet all contingencies, hence we have over-investment relative to the efficient level. □

5 Conclusion

We have studied dynamic competition between thermal and hydroelectric producers under uncertainty. In an infinite horizon game between the two producers, we have demonstrated that the hydro producer has a strategic incentive to withhold water. However, we demonstrate that when capacities are "tight", the duopoly outcome is not far from the efficient one. Examination of the payoffs to the thermal producer at various capacity levels suggests if capacity were to be chosen by the thermal player, it would not choose a capacity so that it is rarely constrained. This results in a welfare loss lower than that under unconstrained Cournot duopoly. We investigate the investment question further using a two-period version of the model. We show that the thermal also has a strategic motive when choosing to invest in increased capacity: over-investing in the closed-loop versus the open-loop equilibrium.

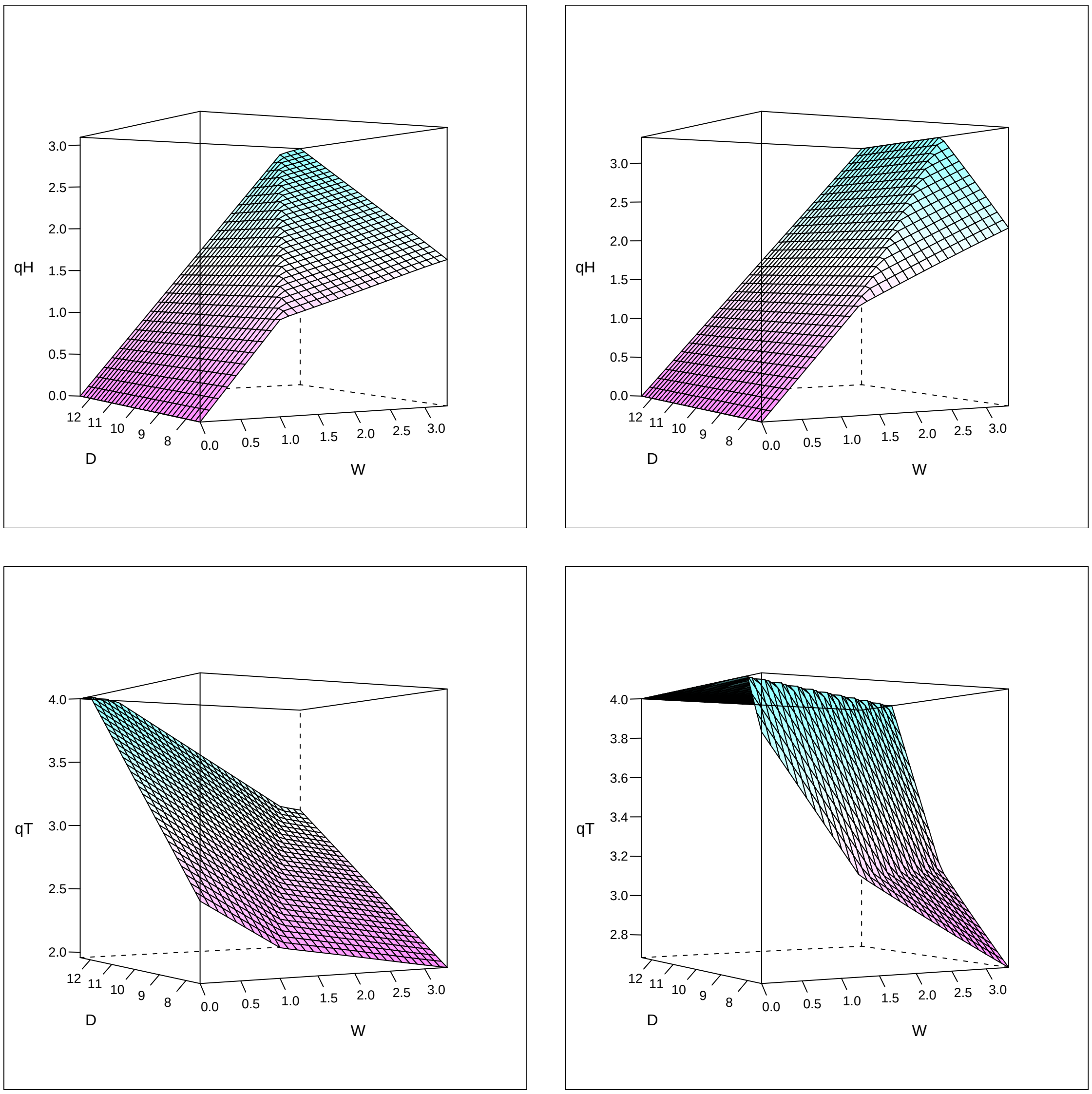

Figure 1: Production: Left column - duopoly, Right - planner

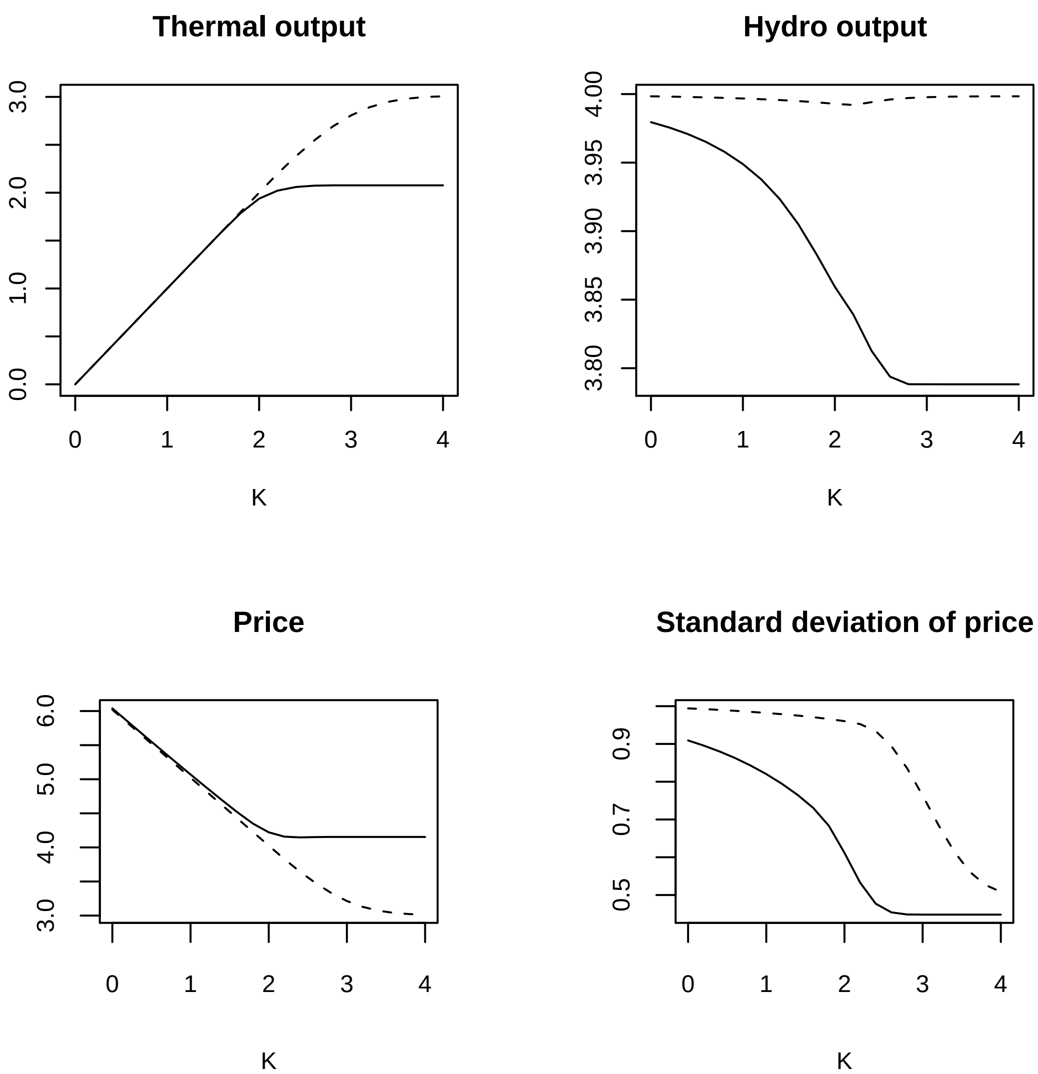

Figure 2: Average model values for alternative thermal capacities: Duopoly (solid line) and Efficient (dashed line)

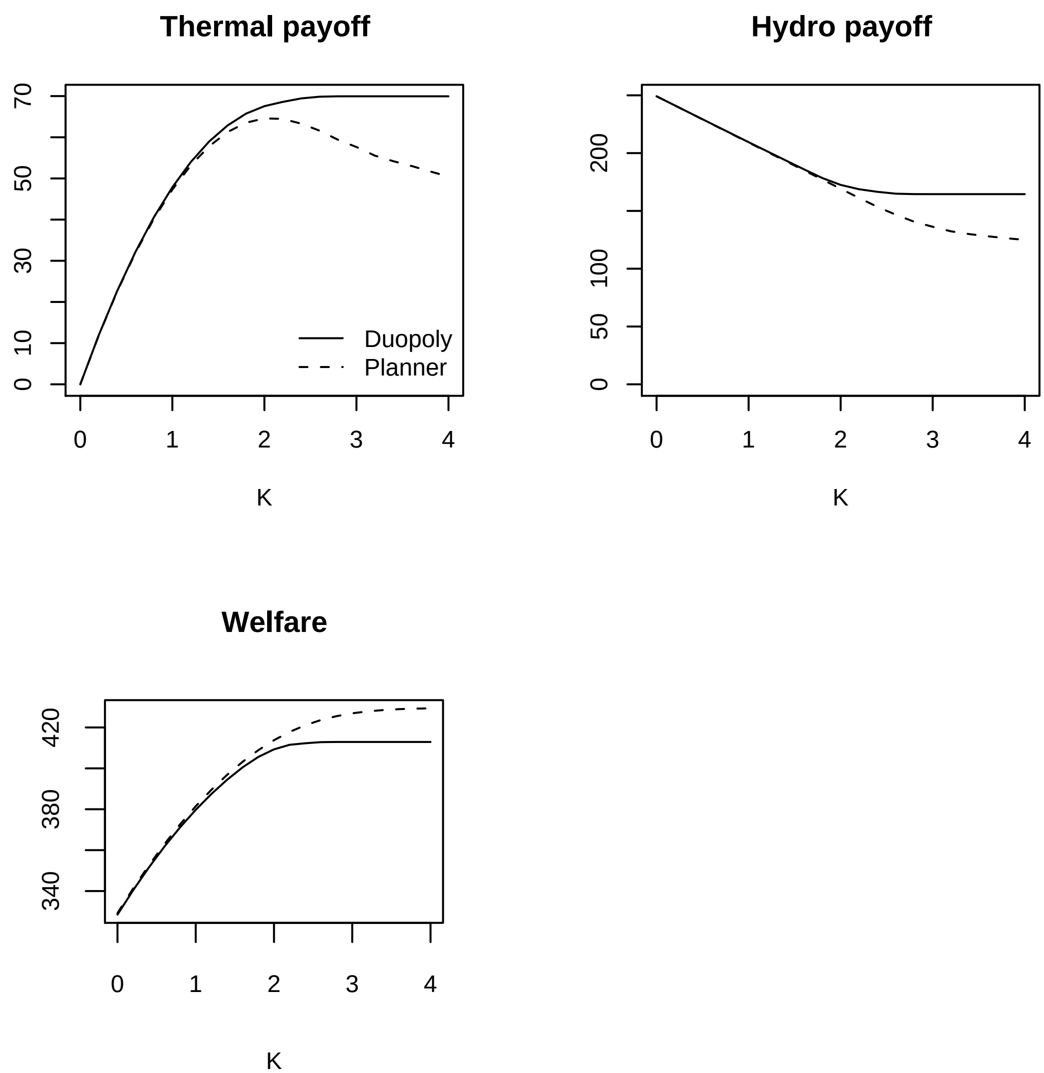

Figure 3: Payoffs for alternative thermal capacities: Duopoly (solid line) and Efficient (dashed line)

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