Investment Incentives and Auction Design in Electricity Markets*

Natalia Fabra, Nils-Henrik M von der Fehr, and María-Ángeles de Frutos

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Abstract

Motivated by the regulatory debate in electricity markets, we seek to understand how market design affects market performance through its impact on investment incentives. For this purpose, we study a two-stage game in which firms choose their capacities under demand uncertainty prior to bidding into the spot market. We analyse a number of different market design elements, including (i) two commonly used auction formats, the uniform-price and discriminatory auctions, (ii) price-caps and (iii) bid duration. We find that, although the discriminatory auction tends to lower prices, this does not imply that investment incentives at the margin are poorer; indeed, under reasonable assumptions on the shape of the demand distribution, the discriminatory auction induces (weakly) stronger investment incentives than the uniform-price format.

Keywords: Market design, uniform-price and discriminatory auctions, investment, electricity, regulatory reform.

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1 Introduction

In recent years, electricity prices have been going up almost everywhere. It is beyond dispute that sharp increases in fuel costs coupled with a tightening of supply and demand conditions have contributed to these price increases. However, there is controversy regarding the responsibility of other issues related to the design and structure of these markets. While some observers have blamed the pricing problem on wholesale market design, which has allegedly facilitated the exercise of market power,[1] others have argued that changing the design could potentially exacerbate the problem. A lack of adequate investments in generating capacity, as well as a high degree of concentration, have also been pointed to as contributing factors. These wide-spread concerns about the functioning of electricity markets have opened up the debate about how to improve their overall performance. The purpose of this paper is to contribute to this debate by analysing how market design affects the performance of electricity markets.

Previous analyses on this issue have tended to concentrate on price formation in the short run, i.e., for given capacities (e.g. Federico and Rahman, 2003; Fabra, von der Fehr and Harbord, 2006). These analyses have at least two important limitations. Firstly, while short-run price formation is certainly of interest - especially in markets characterised by imperfect competition - in the longer run market performance depends first and foremost on the availability of capacity. Secondly, there may be a trade off between achieving low prices and attracting adequate investment.

We attempt to overcome these limitations by incorporating capacity decisions into the analysis. Our analysis is based on a simple model of investment and price formation that reflects essential features of deregulated electricity markets.[2] The modelling approach extends the one developed in Fabra, von der Fehr and Harbord (2006) by allowing for endogenous capacities. In the two-stage game, it is assumed that firms make investment decisions under demand uncertainty, prior to competing in the spot market. We analyse and compare a number of different market design elements, including (i) the two commonly considered pricing formats associated with the uniform-price and discriminatory (or pay-as-bid) auction, respectively; (ii) the introduction of a price cap, and (iii) bid duration. Furthermore, we explore the effects of long-run price-responsiveness of demand on investment incentives.

Our analysis complements and extends the earlier, albeit rather scant literature on investment incentives in electricity markets. A central topic in this literature is whether market-based

[1] Complaints about the manipulation of wholesale markets led the European Commission to open an inquiry into the functioning of the European energy markets (European Comission, 2007). Some national competition authorities have also undertaken inquiries into the performance of these markets.

[2] Consciously, we abstract from the existence of several non-market mechanisms imposed on wholesale electricity markets- notably for the purposes of this paper, capacity payments or capacity markets (Joskow and Tirole (2007)). Our aim is not to derive the actual level of investment, but rather to point at fundamental characteristics of the workings of "energy-only" markets.

price signals provide sufficient incentives for new investment in markets characterised by imperfect competition.[3] One of the first analyses of this topic was provided by von der Fehr and Harbord (1998), who analyse a model closely related to ours but limited to the case in which firms bid in a uniform-price auction under demand certainty. They found that capacity may fall below or exceed the first best, depending upon parameter values. In order to investigate how firms adjust their capacities in response to demand growth, García and Stachetti (2006) introduce dynamics in a simplified version of the von der Fehr-Harbord model. They showed that there exist equilibria that involve negligible or no excess capacity along the outcome path, suggesting that additional incentives may be required for the market to deliver adequate investments. Within a dynamic model based on Cournot competition in the spot market, Bushnell and Ishii (2007) found that asymmetries between firms, demand uncertainty and contractual obligations impact on investment incentives. We confirm von der Fehr and Harbord (1998)'s result that overinvestment is a theoretical possibility, but point out (in what appears to be the empirically most relevant formulation) that underinvestment is more likely, at least if the price cap is set below consumers' willingness to pay for new capacity. We also demonstrate that investment incentives in electricity markets depend on market design, an issue that was not considered in the analyses mentioned above.

Some recent papers have also compared investment incentives in uniform-price and discriminatory auctions. Within a model very similar to Fabra, von der Fehr and Harbord (2006)'s, Úbeda (2007) finds that both auction formats result in firms choosing capacities equal to the Cournot outputs, leading to pay-off equivalence across auctions. However, this conclusion relies heavily on demand being fixed and certain at the investment stage, an assumption which is at odds with some of the features of electricity markets. Indeed, as acknowledged by the author, his model is more about short-run availability decisions for existing capacity rather than about long-run capacity investments.[4]

Cramton and Stoft (2006) provide an informal discussion of the long-run effects of choosing, respectively, a uniform-price and a discriminatory (or pay-as-bid) format for the wholesale market. Based on the premise that prices are typically lower with a discriminatory format, they argue that incentives to invest are weaker with this format and hence that in the longer run the discriminatory format does not perform as well as the uniform-price format. We demonstrate that matters are not that simple. While it is true that, on average, returns to investment are lower with a discriminatory format, at the margin investment incentives are not necessarily weaker. Capacity additions affect price formation in the wholesale market differently under the two formats, depending upon the realisation of demand. Whether the market-price ef-

[3] The adequacy of investment incentives may also depend on how demand is rationed in cases in which price cannot be adjusted to clear the market in all contingencies; we abstract from this issue here.

[4] Le Coq (2002) and Crampes and Creti (2005) analyse a similar issue but restrict attention to the uniform-price auction with inelastic demand.

fect is stronger or weaker with the discriminatory format therefore depends on how demand is distributed. For what appears to be the empirically most relevant demand distribution (i.e., uniform or concave), we find that investment incentives are stronger with the discriminatory format. The fact that the discriminatory format may provide more adequate investment incentives improves the relative supremacy of this format, at least from consumers' point of view, since the discriminatory format also consistently provides lower prices than the uniform-price format.[5]

Concerning price caps, we show that the wide-spread conjecture that eliminating them would suffice for the market to deliver efficient outcomes is flawed as a general principle.[6] While this is true under the discriminatory format, it is not under the uniform-price format; indeed, the price cap may mitigate a tendency towards inefficient over-investment. Moreover, from a consumer point-of-view, there is a trade-off between prices and capacity availability: a larger capacity, which allows for greater consumption, comes at the cost of higher prices. Hence, consumers prefer an effective price cap even though it might result in demand rationing.

On the issue of whether bids in the wholesale market should have long or short duration, our analysis also leads to clear results. When bids are long-lived (i.e., remain fixed over a period in which demand varies considerably) prices tend to be higher, while investment incentives are unaffected, compared to when bids are short-lived. Given that under long-lived bids the discriminatory auction also outperforms the uniform-price format, we can conclude that in our setting a combination of short-lived bids and the discriminatory format produces the most favourable outcome from consumers' point of view.

The question of whether price-responsiveness tends to stimulate or discourage investments depends on whether capacity expansions translate into lower or higher prices, and hence more or less demand. Such a link is not homogenous across auction formats: while capacity expansions tend to reduce prices under the discriminatory auction, the opposite is true under the uniform-price auction. Thus, the discriminatory auction delivers an aggregate capacity level that is closer to the first best as compared to the uniform-price auction.

Lastly, our paper can also be framed within the emerging auction literature that analyses the impact of market design on the longer-term choices of market participants. For instance, Arozamena and Cantillon (2004) compare first-price and second-price single-unit sealed-bid auctions in a model in which one of the bidders has the opportunity to invest in cost-reducing activities prior to the auction.[7] Even though our model differs from theirs in important aspects, both

[5]The overall assessment of the two pricing formats may depend also on concerns that are not represented in our model; in particular, on productive inefficiencies. For a discussion, see Fabra *et al.* (2006).

[6]See Joskow (2006) for a discussion of how price caps create the so-called "missing money" problem in electricity markets. More generally, Earle, Schmedders and Tatur (2007) show that an increase in the price cap might be welfare improving when firms compete *à la* Cournot under demand uncertainty.

[7]See also Tan (1992) and Piccione and Tan (1996). Unlike ours, all of these authors assume incomplete cost information, single-unit auctions, investment in cost reducing activities, and they do not allow for demand

analyses highlight the importance of strategic effects generated by investment decisions. Athey, Levin and Seira (2004) endogenise entry decisions by heterogenous bidders in sealed-bid and open auctions. Both theoretically end empirically they find that sealed bidding promotes entry of weaker bidders, and discourages entry of stronger bidders, in ways that may reverse the revenue comparison obtained in models with a fixed set of bidders.

The structure of the paper is the following. In Section 2, we first describe our basic model, in which we assume that demand is uniformly distributed, and then characterise and compare equilibrium behaviour and outcomes across auction formats. In Section 3, we extend and modify the basic model in several directions: we consider alternative equilibrium selection criteria, we modify the assumptions on timing of bids and realisation of demand uncertainty, we explore the effects of allowing for price-elastic demand functions in the long-run and we introduce a general demand distribution function. The last section contains our conclusions.

2 The Basic Model

The modelling framework builds on that developed in Fabra, von der Fehr and Harbord (2006). We consider a market in which two firms - Firm 1 and Firm 2 - offer a homogenous product. The supply of firm i, q_i, is constrained by installed capacity k_i, i.e., $0 \leq q_i \leq k_i$, $i = 1, 2$. Firm i's marginal cost of production equals zero for production levels below capacity, while production above capacity is impossible (i.e. is infinitely costly). Demand θ is a random variable, independent of price, which is distributed according to a uniform distribution function on the unit interval.

The timing of the game is as follows. At the beginning of the game, firms make investment decisions simultaneously. The unit cost of capacity is c. Once investment decisions have been made, information about capacities become public knowledge. Next, demand is realised and publicly observed and, subsequently, firms compete in prices. Each firm simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply the whole of its capacity.[8] We let $\mathbf{b} \equiv (b_1, b_2)$ denote the bid profile, where $b_i \leq P$ is the bid of firm i, $i = 1, 2$, and $P > c$ is the 'market reserve price', possibly determined by regulation. On the basis of the bid profile an auctioneer calls firms into operation. If firms submit different bids, the lower-bidding firm's capacity is despatched first. If this capacity is not sufficient to satisfy demand, the higher-bidding firm's capacity is despatched to serve residual or remaining demand. If firms submit identical bids demand is split equally between them. Formally, the

uncertainty.

[8]Fabra *et al.* (2006) allow firms to submit upward-sloping step offer-price functions. They show that (pure-strategy) *equilibrium outcomes* - but not the equilibrium *pricing strategies* - are essentially independent of the number of admissible steps in each firm's bid function (and whether the 'step sizes' are choice variables for suppliers). This implies that constraining firms to submit a single bid for their entire capacity is without loss of generality in this setting.

output allocated to firm i, $i = 1, 2$, is given by

$$q_i(\theta; \mathbf{b}) = \begin{cases} \min\{\theta, k_i\} & \text{if} \quad b_i < b_j \\ \frac{1}{2}\min\{\theta, k_i\} + \frac{1}{2}\max\{0, \theta - k_j\} & \text{if} \quad b_i = b_j \\ \max\{0, \theta - k_j\} & \text{if} \quad b_i > b_j. \end{cases} \tag{1}$$

Note that supplies q_i, $i = 1, 2$, are solely functions of demand and the bid profile. Payments made to firms do depend upon the auction format, however. In the *uniform-price auction*, the price received is equal to the highest accepted bid. In the *discriminatory auction*, the price received by firm i is equal to its own offer price whenever its bid is wholly or partly accepted.

Both firms are assumed to be risk neutral and maximise expected profits.

For comparison purposes, we characterise the first-best capacity level, defined as the level that maximises the sum of consumer and producer surplus. Let v denote consumers' gross utility per unit consumed - or total willingness to pay - and let $K = k_1 + k_2$ denote aggregate capacity. Given that demand is completely inelastic prices are a pure transfer between consumers and producers. Hence, total welfare is a function of K only:

$$W = v\left[\int_0^K \theta d\theta + \int_K^1 K d\theta\right] - cK. \tag{2}$$

Maximisation of (2) with respect to K gives the optimal capacity:

$$K^{FB} = 1 - \frac{c}{v}\cdot$$

2.1 Uniform-Price Auction

We first consider the uniform-price auction, in which the price received by firms equals the highest accepted bid. We start by characterising equilibrium bidding behaviour for every possible demand realisation and then move to analysing the investment stage.

Let $k^- = \min(k_1, k_2) \leq k^+ = \max(k_1, k_2)$. We call the firm with capacity k^- 'the small firm' and the other 'the large firm'. The following result then follows directly from the argument in the proof of Proposition 2 in Fabra, von der Fehr and Harbord (2006):

Proposition 1 *In the uniform-price auction, for given capacities and a given demand realisation, equilibrium bidding behaviour and equilibrium outcomes are characterised as follows:*

(i) (Low Demand) If $\theta \leq k^-$, there exists a unique pure-strategy equilibrium in which both firms bid at marginal cost and make zero profits.

(ii) (High Demand) If $\theta \geq k^-$, there exist multiple pure-strategy equilibria in all of which the highest accepted price is P. Equilibrium bidding behaviour and equilibrium outcomes depend on to which of the following regions θ belongs:

(Region I) If $k^- \leq \theta \leq k^+$, the large firm bids at P and the small firm bids a sufficiently low price to make undercutting unprofitable. The small firm serves all capacity k^- at P, whereas the large firm serves residual demand $\theta - k^-$ at P.

(Region II) If $k^+ \leq \theta < K$, either one of the two firms bids at P and the other firm bids a sufficiently low price to make undercutting unprofitable. The low-bidding firm sells all capacity at P, whereas the high-bidding firm serves residual demand at P.

(iii) (Very High Demand) If $\theta \geq K$, there exist multiple pure-strategy equilibria in all of which at least one firm bids at P and both firms sell all capacity at P.

Equilibrium bidding behaviour depends on the relationship between firms' capacities and demand. For Low Demand realisations, both firms have enough capacity to serve total demand; hence, competition drives prices down to marginal cost, and firms make zero profits. When demand exceeds that level, so that at least one firm is unable to serve all of demand, marginal-cost bidding is no longer an equilibrium. For High Demand realisations in Region I, only the small firm's capacity is below demand. Over this range, at the unique equilibrium outcome, the small firm sells its capacity while the large firm maximises its profits by serving residual demand at the market reserve price P. The bid submitted by the small firm has to be low enough to discourage the large firm from undercutting, but it is otherwise irrelevant, as the market price is set by the high bid. For High Demand realisations in Region II, the capacity of both firms is needed to cover demand. Hence, there also exists equilibria in which the small firm bids high and therefore sells below capacity if there is excess capacity overall. Last, for Very High Demand realisations, demand exceeds aggregate capacity, so that both firms sell at capacity at P.

Note that, for a given demand realisation, equilibrium outcomes are unique, except for High Demand realisations in Region II. For the analysis of investment decisions, we need to know which equilibrium will be played in this case. We assume that, when there exist multiple equilibrium outcomes at the price-competition stage, either equilibrium is played with equal probability; that is, the equilibrium in which firm i bids high is played with probability $\frac{1}{2}$. As we point out in Section 3.1 below, where we discuss alternative equilibria, the qualitative nature of our results do not depend on equilibrium selection.

Figure 1 summarises profit realisations; the upper expressions represent profits of the small firm, while the lower expressions represent profits of the large firm. When demand is Low, profits are zero for both firms. When demand is High or Very High, all output is paid the market reserve price; in High Demand Region I, the small firm produces at full capacity, while the large firm serves residual demand; in High Demand Region II, each firm is equally likely to produce at full capacity and to serve residual demand only; when demand is Very High, both firms produce at full capacity.

At the capacity-investment stage, demand is uncertain. Since equilibrium outcomes at the price-competition stage depend on the relative size of firms' capacities, expected profits are given

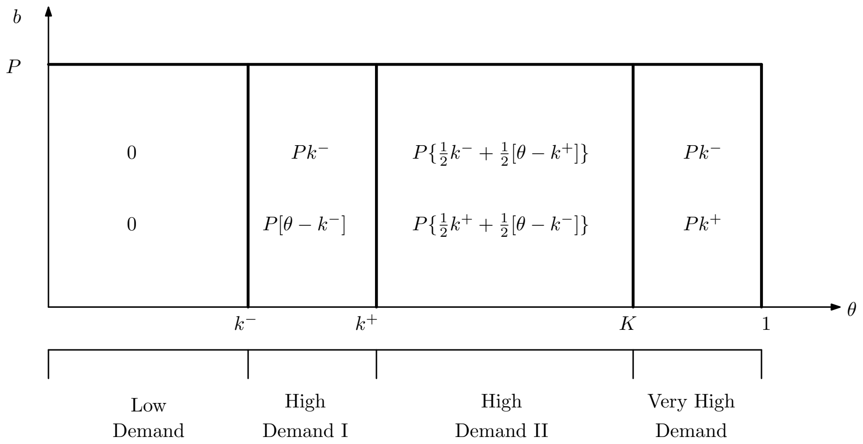

Figure 1: Profits in the uniform-price auction

as follows, for $i = 1, 2$, $i \neq j$,

$$\pi_i^u(k_i, k_j) = \begin{cases} \pi_i^{u-} & \text{if} \quad k_i \leq k_j \\ \pi_i^{u+} & \text{if} \quad k_i \geq k_j \end{cases}$$

where

$$\begin{aligned} \pi_i^{u-} &= P\left[\int_{k^-}^{k^+} k^- d\theta + \int_{k^+}^{K} \left\{\tfrac{1}{2}k^- + \tfrac{1}{2}\left[\theta - k^+\right]\right\} d\theta + \int_{K}^{1} k^- d\theta\right] - ck^- \\ \pi_i^{u+} &= P\left[\int_{k^-}^{k^+} \left[\theta - k^-\right] d\theta + \int_{k^+}^{K} \left\{\tfrac{1}{2}k^+ + \tfrac{1}{2}\left[\theta - k^-\right]\right\} d\theta + \int_{K}^{1} k^+ d\theta\right] - ck^+ \end{aligned} \tag{3}$$

The three elements in the main square brackets in (3) correspond to the equilibrium outcomes for High and Very High Demand realisations, as defined in Proposition 1 and illustrated in Figure 1 above. Note that, since $\pi_i^{u-} = \pi_i^{u+}$ at $k^- = k^+$, the expected-profit function is everywhere continuous.

To gain insight into investment incentives, we decompose the impact of an increase in a firm's capacity on its profits. The impact on the small firm's profits of an increase in its capacity of $\triangle$ is illustrated in Figure 2, which builds on Figure 1. An increase in capacity has two distinct effects on the firm's profit: (i) it leads to an increase in output whenever the firm is capacity constrained; and (ii) it affects the type of equilibrium being played. The first effect is present both for Very High and High Demand realisations, although in Region II it occurs with probability $\frac{1}{2}$ only (i.e., the probability that an equilibrium is played in which the firm bids low). The second effect is present only in the region where the state of demand is shifted from High to Low, where now the realised price becomes equal to marginal costs rather than to the market reserve price (there is a second-order effect in the region where the state of demand is shifted from Very High to High, but this becomes negligible for marginal increases in capacity).

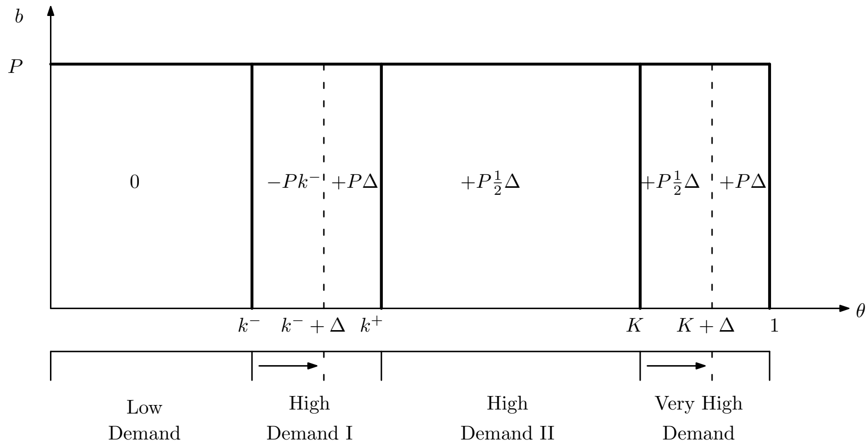

Figure 2: Effect on the small firm's profits of a capacity increase in the uniform-price auction

Figure 3 shows the corresponding impact on the large firm's profit of an increase in its capacity. Here the effect of an increase in output whenever the firm is capacity constrained is present only for High Demand Region II and Very High Demand realisations, whereas the effect of a change in equilibrium is present where the state of demand is shifted from Region I to Region II in High Demand and the large firm, rather than producing at full capacity with probability $\frac{1}{2}$, serves residual demand only (again there is a second-order effect in the region where the state of demand is shifted from Very High to High, which becomes negligible for marginal increases in capacity).

Formally, the effect of a marginal increase in capacity becomes

$$\frac{\partial \pi_i^{u-}}{\partial k_i} = P[k_j - k_i] - \frac{1}{2}Pk_i + P[1-K] - c \tag{4}$$

$$\frac{\partial \pi_i^{u+}}{\partial k_i} = P[1-K] - c \tag{5}$$

We see that the effect of a change in capacity differs between the small and the large firm. The reason is twofold. First, only the small firm earns additional profits from a larger capacity in High Demand Region I. Second, there are different effects on states of demand; an increase in the small firm's capacity shifts the border between Low and High Demand, whereas an increase in the capacity of the large firm shifts the border between High Demand Regions I and II. When the state of demand is shifted from High to Low, the small firm loses all profits from producing at full capacity and selling at the market reserve price; when the state of demand is shifted from High Demand Region I to Region II, the large firm loses profits corresponding to serving residual demand rather than producing at full capacity.

The difference in marginal returns to investment for a small and a large firm translate into a kink in firms' profit functions. In particular, the partial derivative of the profit function of firm

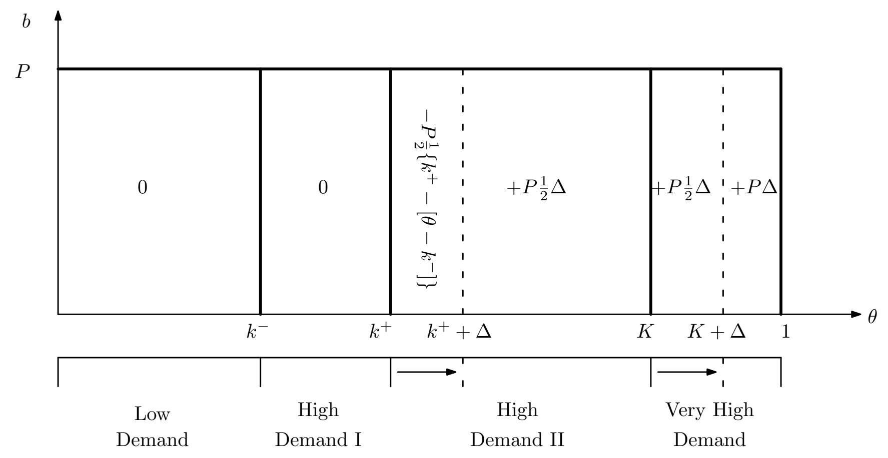

Figure 3: Effect on the large firm's profits of a capacity increase in the uniform-price auction

i with respect to its own capacity is discontinuous at $k_i = k_j$; that is,

$$\lim_{k_i \downarrow k} \frac{\partial \pi_i^{u+}(k_i, k)}{\partial k_i} - \lim_{k_i \uparrow k} \frac{\partial \pi_i^{u-}(k_i, k)}{\partial k_i} = \frac{1}{2} k > 0,$$

In other words, the gain in profit from (marginally) increasing capacity 'jumps up' at the point where capacities are identical.

Second-order and cross derivatives are given by

$$\begin{aligned} \frac{\partial^2 \pi_i^{u-}}{\partial k_i^2} &= -\frac{5}{2} P < \frac{\partial^2 \pi_i^{u-}}{\partial k_i \partial k_j} = 0 \\ \frac{\partial^2 \pi_i^{u+}}{\partial k_i^2} &= \frac{\partial^2 \pi_i^{u+}}{\partial k_i \partial k_j} = -P < 0. \end{aligned}$$

Since second-order derivatives are negative, the expected-profit function is piece-wise concave; that is, for given k_j, π_i is concave as a function of k_i both to the left and to the right of the point at which capacities are identical (i.e., $k_i = k_j$). Furthermore, the sign of the cross derivatives implies that marginal return to capacity is (weakly) decreasing in the rival's capacity.

Figure 4 shows, for two different values of k_j, firm i's expected profits as a function of its own capacity for an example in which $P = 2$ and $c = 1$. We have drawn π_i^{u-} and π_i^{u+} for all $k_i \geq 0$ for which the two functions attain positive values. The solid lines represent π_i^u.

The unorthodox nature of profit functions implies that equilibrium cannot be determined by the standard first-order approach. Moreover, reaction functions are discontinuous, and at most one reaction-function crosses the 45°-line. It follows that there cannot exist a symmetric equilibrium.[9]

[9] We refer to the Appendix for details and proof of this and subsequent results.

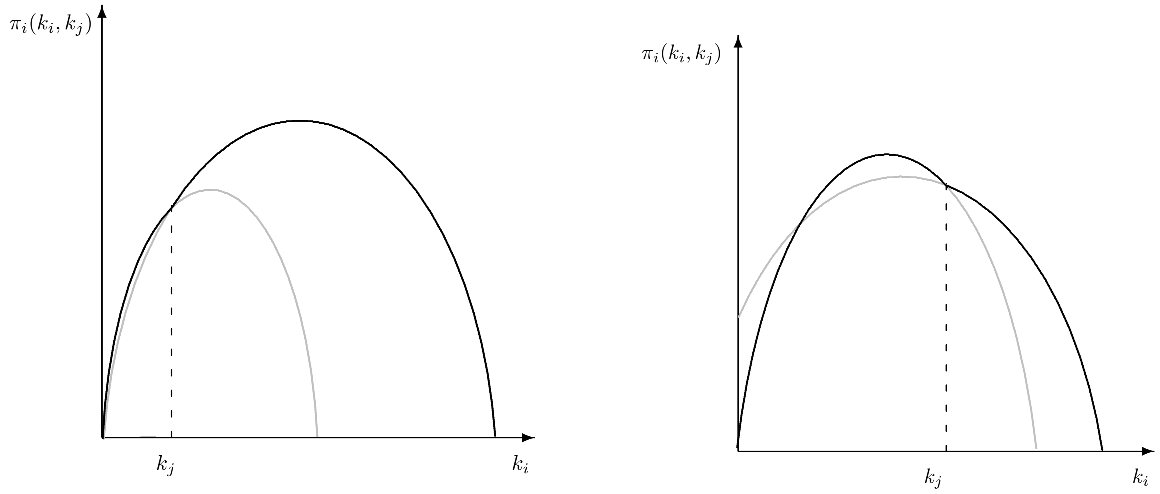

Figure 4: Firm i's profits in the uniform auction for $k_j = 0.15$ (left) and $k_j = 0.3$ (right)

Proposition 2 *In the uniform-price auction,*

(i) there are exactly two pure-strategy equilibria in capacity choices, one with $(k_1, k_2) = (k^{u+}, k^{u-})$ *and the other with* $(k_1, k_2) = (k^{u-}, k^{u+})$*, where*

$$k^{u+} = \frac{3}{5}\left[1 - \frac{c}{P}\right] > k^{u-} = \frac{2}{5}\left[1 - \frac{c}{P}\right].$$

(ii) Aggregate equilibrium capacity is $K^u = 1 - \frac{c}{P}$*.*

Proposition 2 says that there exists two asymmetric pure-strategy Nash equilibria. In any one of these equilibria, one firm invests more than its rival. Note that equilibria are asymmetric even though firms are fully symmetric *ex ante*.[10] Since equilibria differ only by the identity of the small and the large firm, they are outcome equivalent; in particular, aggregate capacity and expected prices are the same independently of which equilibrium is played. Capacities are decreasing in capacity costs and increasing in the market reserve price.

Equilibrium profits of the small and the large firm are given as

$$\begin{array}{l} \pi_i^{u-} = \frac{20}{100} P \left[1 - \frac{c}{P}\right]^2 \\ \pi_i^{u+} = \frac{22}{100} P \left[1 - \frac{c}{P}\right]^2 \end{array}$$

Note that while the large firm earns higher profits than the small firm, the difference is relative smaller than the difference in capacities; the reason is that the small firm tends to be despatched at full capacity more often than the large firm. Nevertheless, if given a choice, either firm would

[10] A symmetric equilibrium in pure strategies also fails to exist in a two-stage duopoly model of capacity investment and pricing in a homogeneous product market when market demand is uncertain, see Hviid (1991) and Reynolds and Wilson (2000).

prefer to be large. Profits are decreasing in capacity costs c and increasing in the market reserve price P.

Since total capacity is increasing in the market reserve price, so is total welfare. An increase in P affects consumer surplus in two different and opposing ways; on the one hand, consumers benefit from a larger capacity (and hence a lower probability of rationing), but, on the other hand, they lose from higher payments. Since the former effect dominates for low values of P, while the latter dominates for high P values, consumers are better off with an effective price cap.[11]

2.2 Discriminatory Auction

In the discriminatory auction, the price received by firm i for its output is equal to its own offer price whenever its bid is wholly or partly accepted. We concentrate our attention on the points where the analysis for the discriminatory auction differs from that of the uniform-price auction. The following result follows from the argument in the proof of Proposition 2 in Fabra, von der Fehr and Harbord (2006):

Proposition 3 *In the discriminatory auction, for given capacities and a given demand realisation, equilibrium bidding behaviour and equilibrium outcomes are characterised as follows:*

(i) (Low Demand) If $\theta \leq k^-$, there exists a unique pure-strategy equilibrium in which both firms bid at marginal cost and make zero profits.

(ii) (High demand) If $\theta \geq k^-$, a pure-strategy equilibrium does not exist. In the unique mixed-strategy equilibrium expected prices exceed marginal costs. Equilibrium bidding behaviour and equilibrium outcomes depend on to which of the following regions θ belongs:

(Region I) If $k^- \leq \theta \leq k^+$, the small firm makes expected profits $P\frac{k^-}{\theta}\left[\theta - k^-\right]$ whereas the large makes expected profits $P\left[\theta - k^-\right]$.

(Region II) If $k^+ \leq \theta < K$, the small firm makes expected profits $P\frac{k^-}{k^+}\left[\theta - k^-\right]$ whereas the large makes expected profits $P\left[\theta - k^-\right]$.

(iii) (Very High Demand) If $\theta \geq K$, both firms sell all of their capacity at P.

Equilibrium outcomes are the same as in the uniform-price auction when demand is either below the capacity of the small firm or above aggregate capacity. However, for other demand realisations, competition is more vigorous in the discriminatory auction. In particular, for High Demand realisations an equilibrium in which all production is paid at P cannot exist, given that both firms would have incentives to undercut P in order to increase their production with only a slight reduction in price. Indeed, only mixed-strategy equilibria exist for these realisations. These are such that the two firms mix over a common support - that lies above marginal costs

[11] In particular, we find that consumer surplus is concave in P, increasing in P at $P = c$, and decreasing in P at $P = v$. It follows that consumer surplus is maximized at some $\widetilde{P} \in (c, v)$.

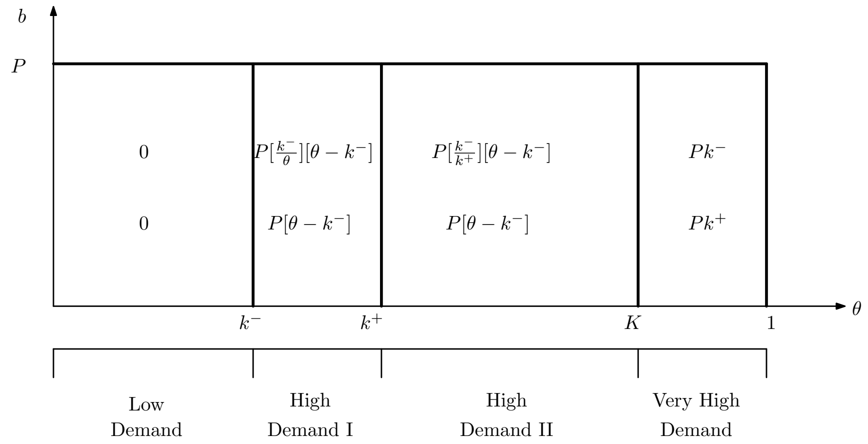

Figure 5: Profits in the discriminatory auction

and includes the market reserve price - but according to different probability distributions; in particular, only the large firm plays the upper bound, P, with positive probability. Since, with probability 1, the small firm bids below that level, profits of the large firm are the same as if it offered to sell residual demand (when the capacity of the small firm is fully despatched) at P. Figure 5 summarises profit realisations.

Expected profits of the small and the large firms, respectively, are

$$\begin{aligned}
\pi_i^{d-} &= P\left[\int_{k^-}^{k^+} \frac{k^-}{\theta}\left[\theta - k^-\right] d\theta + \int_{k^+}^{K} \frac{k^-}{k^+}\left[\theta - k^-\right] d\theta + \int_{K}^{1} k^- d\theta\right] - ck^- \\
\pi_i^{d+} &= P\left[\int_{k^-}^{k^+} \left[\theta - k^-\right] \underset{1}{d\theta} + \int_{k^+}^{K} \left[\theta - k^-\right] d\theta + \int_{K}^{1} k^+ d\theta\right] - ck^+
\end{aligned}$$

The effect of a marginal increase in capacity becomes,

$$\frac{\partial \pi_i^{d-}}{\partial k_i} = P\int_{k_i}^{K}\left[\frac{\theta - 2k_i}{\min\{\theta, k_j\}}\right] d\theta + P\left[1-K\right] - c \tag{6}$$

$$\frac{\partial \pi_i^{d+}}{\partial k_i} = P\left[1-K\right] - c \tag{7}$$

Changes in capacities affect the intensity of competition for a given demand realisation, thus affecting profits. Changes in capacities also alter the probability of playing the different types of equilibria; however, unlike in the uniform-price auction, since profits are everywhere continuous in the demand parameter θ, these changes do not affect expected profits. Accordingly, as can be seen from (7), the large firm benefits from marginal increases in its capacity with probability

$1-K$, since it only sells at capacity when demand exceeds aggregate capacity. An increase in the small firm's capacity has additional effects on its profits whenever demand is below aggregate capacity: (i) it allows the small firm to expand output when it prices below the rival; (ii) however, as this also makes the large firm more aggressive, the probability that the small firm sells at capacity is reduced. The overall additional effect on the small firm's profits from an increase in its capacity may be positive or negative, depending on the relative strength of these two effects.

The following result characterises equilibrium investment behaviour in the discriminatory auction.

Proposition 4 *In the discriminatory auction,*

(i) there are exactly two pure-strategy equilibria in capacity choices, one with $(k_1, k_2) = \left(k^{d+}, k^{d-}\right)$ *and the other with* $(k_1, k_2) = \left(k^{d-}, k^{d+}\right)$*, where* $k^{d+} = \alpha k^{d-}$ *with* $\alpha > 2$ *given implicitly by the equation*

$$\alpha^2 - 2\alpha \ln(\alpha) = \frac{3}{2}. \tag{8}$$

(ii) Aggregate equilibrium capacity is $K^d = 1 - \frac{c}{P}$*.*

As in the uniform-price auction, we find that, even though firms are symmetric *ex ante,* equilibrium behaviour is asymmetric; in particular, one firm chooses a capacity that is more than twice the size of that of its rival. Furthermore, the two equilibria are outcome equivalent, with the same aggregate capacity and expected prices.

Equilibrium profits become

$$\pi_i^{d-} = 0.11P\left[1 - \tfrac{c}{P}\right]^2$$
$$\pi_i^{d+} = 0.25P\left[1 - \tfrac{c}{P}\right]^2$$

Also in this auction, the larger firm is more profitable; indeed, here the larger firm's profit is more than twice that of its smaller rival. As in the uniform-price auction, we find that profits, capacities and total welfare are increasing in the market reserve price, P, while consumer surplus reaches its maximum for some intermediary value $\widetilde{P} \in (c, v)$.

2.3 Uniform-Price versus Discriminatory

Having characterised equilibria for the two auction formats, we turn to a comparison of equilibrium outcomes.

Proposition 5

(i) Aggregate capacity is $1 - \frac{c}{P}$ *in both auctions. Hence, unless* $P = v$*, both auction formats result in under-investment relative to the first best.*

(ii) The discriminatory auction results in more capacity asymmetry than the uniform-price auction.

(iii) Expected prices are higher in the uniform-price auction than in the discriminatory auction.

The two auction formats result in the same level of aggregate investment. Unless the market reserve price P is set equal to consumers' per-unit gross utility v, the marginal return to extra capacity is below the social benefit; hence, there is under-investment relative to the first best.

The distribution of aggregate capacity across firms differs between auction formats. While in the uniform-price auction the capacity of the large firm is 1.5 times that of the small firm, in the discriminatory auction the large firm has more than double the capacity of the small firm.

Since the small firm is larger in the uniform-price auction, the incidence of Low Demand realisations is higher. Taken in isolation, this effect implies that prices tend to be lower in the uniform-price auction than in the discriminatory auction. However, for High Demand realisations competition is fiercer and price lower in the discriminatory action than in the uniform-price auction. The relative importance of these two different and opposing effects depends on the relative incidence of Low Demand and High Demand realisations. With a uniform demand distribution, the latter effect outweighs the former and hence expected prices are lower in the discriminatory auction than in the uniform-price auction.

Lastly, given that total welfare is a function of aggregate capacity only, both auction formats result in the same level of total welfare. However, in expectation consumer surplus is higher in the discriminatory auction since it leads to lower prices. Since the difference in profits is increasing in the market reserve price, the gain to consumers from moving from the uniform-price to the discriminatory format is greater the higher is P.

3 Extensions and Variations

In the previous section we have compared equilibria of the uniform-price and discriminatory auctions under a number of simplifying assumptions. In this section we extend the analysis by considering various alternative formulations, thereby not only casting light on the importance of the assumptions underlying the basic model, but also allowing for a discussion of how the comparison across auction formats depends on market characteristics.

3.1 Equilibrium Selection

The above analysis of the uniform-price auction was limited to pure-strategy equilibria, and, moreover, based on the assumption that, for demand realisations where multiple pure-strategy equilibria exist at the price-competition stage, firms are equally likely to play either of these. In this section we characterise and discuss alternative equilibria of the uniform-price auction.

The assumption that firms are equally likely to play either of the two possible pure-strategy equilibria at the price-competition stage was chosen to maximise the *ex ante* symmetry of firms, thereby highlighting the underlying mechanism that drives the asymmetric capacity outcome. This equilibrium may be justified by assuming that firms adopt the following coordination mechanism: they toss a coin and, if heads come up, they play the equilibrium in which Firm 1 bids at P, whereas if tails come up they play the equilibrium in which Firm 2 bids at P.[12] It turns out that the analysis may be straightforwardly extended to the case in which, at the outset of the second stage of the game, firms observe the outcome of a public signal, $\tilde{\rho}$, uniformly distributed on $[0,1]$, which allows them to coordinate on either of the two price equilibria.[13] Specifically, in High Demand Region II, an equilibrium in which firm i bids high is played whenever $\tilde{\rho} \leq \rho$, where ρ is a constant independent of installed capacity levels. Without loss of generality, we set $i = 1$ for all $\rho \leq \frac{1}{2}$, so that the probability with which Firm 2 bids high is $1 - \rho$. Note that setting $\rho = 0$ corresponds to assuming that, wherever relevant, firms play the equilibrium at which Firm 2 bids at P. The following result generalises Proposition 2:

Proposition 6 *In the uniform-price auction,*

(i) there exists $\hat{\rho} \in \left(0, \frac{1}{2}\right)$, such that if $\rho \in [0, \hat{\rho})$, there is a unique pure-strategy Nash equilibrium in capacity choices; it has the form $\left(k_1^{u+}, k_2^{u-}\right)$. Otherwise, if $\rho \in \left[\hat{\rho}, \frac{1}{2}\right]$, there are exactly two pure-strategy equilibria in capacity choices, one with $\left(k_1^{u+}, k_2^{u-}\right)$ and the other with $\left(k_1^{u-}, k_2^{u+}\right)$. In either case,

$$
\begin{aligned}
k_1^{u+} &= \frac{2-\rho}{3-\rho}\left[1 - \frac{c}{P}\right] > k_2^{u-} = \frac{1}{3-\rho}\left[1 - \frac{c}{P}\right] \\
k_1^{u-} &= \frac{1}{2+\rho}\left[1 - \frac{c}{P}\right] < k_2^{u+} = \frac{1+\rho}{2+\rho}\left[1 - \frac{c}{P}\right]
\end{aligned}
$$

(ii) Aggregate equilibrium capacity is $K^u = 1 - \frac{c}{P}$.

There always exists a pure-strategy Nash equilibrium in which one firm - here called Firm 1 - invests more than its rival. For a range of values of the parameter ρ, this equilibrium is unique. For other parameter values, there exists another pure-strategy equilibrium in which the other firm - Firm 2 - invests more.

In any equilibrium, independently of the value of ρ, aggregate capacity equals $1 - \frac{c}{P}$. The degree of capacity asymmetry however depends on which equilibrium is played, as well as on the value of ρ. For $\rho = \frac{1}{2}$, the two equilibrium outcomes mirror each other, i.e., $k_1^{u+} = k_2^{u+} > k_1^{u-} = k_2^{u-}$. For smaller ρ, Firm 1 is less likely to bid high and hence has a larger probability of being despatched at full capacity; therefore its incentive to expand capacity is larger. Consequently, as ρ is reduced, Firm 1 becomes larger, and Firm 2 correspondingly smaller, leading to more asymmetry if $\left(k_1^{u+}, k_2^{u-}\right)$ is played, but less asymmetry if $\left(k_1^{u-}, k_2^{u+}\right)$ is played. When $\rho = 0$ -

[12] Since both strategy profiles constitute a Nash equilibrium, such a random coordination on each of them is also an equilibrium. This is the idea that underlies the concept of correlated equilibrium proposed by Aumann (1974).

[13] A public randomizing device in the second stage expands the set of Nash equilibria, so that any convex combination of the Nash equilibria in the second stage is an equilibrium. This allows firms to achieve any point in the convex hull of the set of continuation payoffs. Furthermore, firms play a continuation game with payoffs on the Pareto frontier of the convex hull.

that is, when firms coordinate on the equilibrium in which Firm 1 never plays high - capacity asymmetry is at its maximum; here Firm 1 is twice as large as Firm 2.

Expected price depends on the size of the small firm only; the larger it is, the higher is the probability that price equals marginal costs rather than the market reserve price P. If the equilibrium $\left(k_1^{u+}, k_2^{u-}\right)$ is played, expected price is decreasing in ρ, whereas if $\left(k_1^{u-}, k_2^{u+}\right)$ is played it is increasing in ρ. Since, for $\rho = \frac{1}{2}$, expected prices are the same in both equilibria, it follows that when $\rho < \frac{1}{2}$ the $\left(k_1^{u+}, k_2^{u-}\right)$-equilibrium results in a higher expected price than the $\left(k_1^{u-}, k_2^{u+}\right)$-equilibrium. In other words, prices tend to be higher when it is more likely that an equilibrium is played in which the small firm prices high, because this decreases the incentive of the small firm to expand its capacity, thereby reducing the range of demand realisations at which the price is competed down to marginal cost.

We conclude that aggregate investment, and therefore total welfare, do not depend on whether firms coordinate on one of the pure-strategy equilibria or on whether they play both with positive probability. Market concentration is lower when firms play a correlated equilibrium because such an equilibrium involves weaker incentives for the large firm to expand capacity. The increase in the relative size of the small firm, which implies greater incidence of Low Demand realisations, tends to reduce prices; in particular, prices are at their lowest when both firms are equally likely to bid high, i.e., when $\rho = \frac{1}{2}$. Since even in this case - as expressed in Proposition 5 above - the uniform-price auction results in higher prices and lower consumer surplus than the discriminatory auction, it follows that this result holds independently of which equilibrium is considered.

So far the analysis of the uniform-price auction has been made under two restrictions: first, we have concentrated on pure-strategy equilibria at which the pricing strategies do not depend on the first-stage capacity choices; second, we have restricted attention to pure-strategy price equilibria. Regarding the first restriction, we could instead have considered a case in which the large firm always bids high for demand realisations where multiple pure-strategy equilibria exist at the price-competition stage. However, this assumption introduces a discontinuity in firms' profit function at symmetric capacity pairs, with $\pi_i^{u-}(k,k) - \pi_i^{u+}(k,k) = \frac{P}{2}k^2 > 0$. Such a discontinuity results in the non-existence of a pure-strategy equilibrium in capacity choices, as either firm would always prefer to invest slightly less than its rival. Nevertheless, at any mixed-strategy equilibria in capacity choices, it would still be true that aggregate capacity in the uniform-price auction is $1 - \frac{c}{P}$.[14]

Regarding the second restriction, if firms play a mixed-strategy equilibrium at the price-competition stage for demand realisations in High Demand Region II, the qualitative nature of the equilibrium of the overall game is essentially the same as when we consider pure-strategies only. This last result on equilibrium selection is summarised in the following Proposition.

[14] Conditionally on being the large firm, the first- order condition under this equilibrium selection criterion is the same as (5).

Proposition 7 *In the uniform-price auction, when demand is high, there also exist mixed-strategy equilibria at the price-competition stage. When firms play a mixed-strategy equilibrium for High Demand realisations in Region II, equilibria in the overall game have the same qualitative characteristics as when firms play (correlated) pure strategies. In particular,*

(i) equilibrium capacities are asymmetric;

(ii) aggregate equilibrium capacity is $K^u = 1 - \frac{c}{P}$*;*

(iii) for a given capacity configuration, industry profits and prices are lower than when firms play (correlated) pure strategies;

(iv) industry profits and prices are nevertheless higher than in the discriminatory auction.

There turns out to be a very close connection between the set of correlated equilibria and the set of mixed-strategy equilibria. In particular, as far as capacity configurations are concerned, each outcome in a correlated equilibrium corresponds to an outcome in a mixed-strategy equilibrium, and *vice versa*. In other words, whether firms randomise over which firm should bid high and which firm should bid low, or whether each firm individually randomises over its choice of bid, is immaterial as far as investment incentives are concerned.[15]

3.2 Long-Lived Bids

In the basic model, it was assumed that price competition took place after demand was realised and observed. The assumption that firms know demand when they set prices is a reasonable approximation for markets in which prices are set for short periods of time, say for hourly or half -hourly periods. Given the relatively high persistence of demand and the very high accuracy with which demand can be forecasted, market players will in effect be able to foresee the level of demand when they prepare their bids. However, in markets in which prices are set for longer periods of time - say, for a whole day - demand will typically vary considerably over the pricing period; the assumption that demand is fixed and known is then not appropriate.

In this subsection we consider instead a variant of the model in which bids are made before demand is realised. We term this bidding format 'long-lived bids'.[16] Equilibrium at the pricing stage may then be characterised as follows:

Proposition 8 *Suppose bids are made before demand is realised. Under both auction formats, for given capacities* $k_i \leq k_j \leq 1$*, there does not exist a pure-strategy equilibrium. In the unique*

[15] The fact that the mixed-strategy equilibria generate lower profits but the same aggregate capacity as the pure or correlated equilibria provides an additional example of how investment incentives depend on marginal profits rather than profits *per se*.

[16] Note that this amounts to assuming that the variation in demand over the pricing period corresponds to that over the lifetime of investment. In practice, demand may vary over the day (a typical pricing period), as well as between seasons and years. Taking account of different types of demand variation would require a multi-period set up.

mixed-strategy equilibrium, firms choose prices from a common support, with a lower bound strictly above (zero) marginal costs and an upper bound equal to P. If $k_i \neq k_j$, the firm with the larger capacity bids less aggressively (i.e., plays prices below any threshold with lower probability) than the firm with the smaller capacity; in particular, the firm with the larger capacity plays P with positive probability.

Equilibrium differs substantially from the case in which demand is known with certainty before prices are set. In particular, two forces destroy any candidate pure-strategy equilibrium: on the one hand, a higher price translates into higher profits if demand turns out to exceed aggregate capacity; on the other hand, pricing high reduces a firm's expected sales (Fabra, von der Fehr and Harbord, 2006). Therefore, the only equilibrium is in mixed strategies.

At the unique equilibrium, the large firm plays a mass point at P, so that it receives the same profits as if it served residual demand at P with probability one (these profits are the same as in the discriminatory auction with short-lived bids). The small firm's profits differ substantially from the profits it makes with short-lived bids. Nevertheless, it preserves features that account for the non-existence of a symmetric equilibrium in capacity choices; in particular, the small firm's returns to investment are lower than those of the large firm, since the small firm takes into account that an increase in its capacity would affect the aggressiveness of the pricing behaviour of its rival.

The following proposition characterises equilibrium capacity choices in the uniform-price and discriminatory auctions with long-lived bids.

Proposition 9 *Under each auction format, there exist exactly two pure-strategy equilibria in capacity choices, one with $(k_1, k_2) = (k^{a+}, k^{a-})$ and the other with $(k_1, k_2) = (k^{a-}, k^{a+})$, where $k^{a-} < k^{a+}$, and $a = d, u$ denotes discriminatory and uniform-price format, respectively. At equilibrium, aggregate capacity is $K^a = 1 - \frac{c}{P}$, $a = d, u$.*

The next proposition compares equilibrium outcomes across pricing formats.

Proposition 10 *When bids are made before demand is realised, the discriminatory auction generates the same aggregate capacity, it induces a more skewed capacity distribution and it results in lower expected prices than the uniform-price auction.*

The comparison across auction formats therefore corresponds to that with short-lived bids, (Proposition 5). However, the fact that the discriminatory auction performs better than the uniform-price auction contrasts with results for the case in which capacities are taken as given. With ex-ante symmetric firms, fixed capacities and long-lived bids, the uniform-price and the discriminatory auctions yield equal expected prices (Fabra, von der Fehr and Harbord, 2006); when capacities are endogenous, this is no longer the case.

We end by comparing equilibrium outcomes across bid formats.

Proposition 11 *Comparing equilibrium outcomes when bids are made, respectively, before and after demand is realised, we find:*

(i) *Aggregate capacity is the same and equals* $1-\frac{c}{P}$ *in all cases.*

(ii) *With the uniform-price format, the distribution of capacity is more skewed when bids are made before demand is realised iff* $\frac{P}{c}>1.22$ *(or* $\frac{c}{P}\leq 0.82$*). With the discriminatory format, the distribution of capacity is more skewed when bids are made before demand is realised iff* $\frac{P}{c}>12.71$ *(or* $\frac{c}{P}<0.08$*).*

(iii) *Under both auction formats, aggregate profits - and hence expected prices - are higher when bids are made before demand is realised.*

Bid duration does not have an impact on overall capacity, and hence on total welfare. Nevertheless, it alters the way in which total capacity is distributed among firms, thereby affecting market concentration. In perhaps the most relevant case, when the market reserve price is close, but not very close, to capacity costs (specifically, $1.22<\frac{P}{c}<12.71$), moving from short-lived to long-lived bids tends to reduce the difference between the two auction formats; the capacity distribution becomes less concentrated under the discriminatory format - where concentration tends to be higher in any case - and more concentrated under the uniform-price format - where concentration is less.

A move from short-lived to long-lived bids does however increase profits - and hence consumer prices - under both auction formats. It follows that the combination of a reasonable price cap, short-lived bids and the discriminatory format produces the most favourable outcome from consumers' point of view.

3.3 Price-Responsive Demand

So far we have restricted attention to the case in which demand is completely price inelastic, both in the short and in the long run. In this subsection we extend the basic model by introducing a long-term demand function that depends on retail price.[17] For analytical convenience, we assume aggregate demand has the multiplicative form $\theta D(p)$, where D is a deterministic function, decreasing in consumer or retail price, p, and (as before) θ is a stochastic parameter uniformly distributed on the unit interval.

While with a price-inelastic formulation it is not essential to specify the exact form of consumer payments, here we need to be explicit about the determination of retail price. We assume that retail price is set so that the market clears in average or expected terms, i.e., payments by consumers exactly cover payments to producers:

$$p\left[\int_{0}^{K/D(p)}\theta D(p)\,d\theta+\int_{K/D(p)}^{1}K d\theta\right]=\pi_1+\pi_2+cK. \tag{9}$$

[17] The analytical details are included in the Appendix.

Clearly, for given capacities, an auction format that leads to lower payments to producers will result in lower retail prices, more demand and hence higher welfare.

To understand how price-responsiveness of demand may affect investment incentives, consider first the impact on the profit of the large firm from a marginal increase in its capacity under the discriminatory format:

$$\frac{\partial \pi_i^{d+}}{\partial k^+} = P \frac{\partial p}{\partial k^+} \int_{k^-/D(p)}^{K/D(p)} \theta D'(p) \, d\theta + P \left[1 - \frac{K}{D(p)} \right] - c.$$

Comparing with the case in which demand is price-inelastic (cf. (7) above), there are now additional effects. Firstly, as captured by the second element on the right-hand side in the above expression, the probability that the marginal unit of capacity is despatched depends on the level of demand and hence on the market price; the lower is the price, the higher is the probability that capacity will be fully utilised.

Secondly, as captured by the first element in the above expression, there is what we may term a 'market-size' effect; that is, capacity affects retail price and hence demand. Whether this effect tends to stimulate or to depress investment incentives depends on whether increases in the large firm's capacity tend to reduce or increase retail price; that is, it depends on the sign of $\frac{\partial p}{\partial k^+}$. The link between capacities and retail price is complex; an increase in capacities allow for an expansion of consumption (cf. the left-hand side of (9)), but also raises total costs and affect producers' profits (cf. the right-hand side of (9)); the overall impact on the retail price depends on which of these effects dominates. We have not been able to derive general results on the relation between capacities and retail price, but, as we explain below, numerical simulations for a linear specification suggests that an increase in capacities is associated with a fall in retail price. If so, the market-size effect tends to enhance investment incentives.

In the uniform-price auction, an increase in the large firm's capacity unambiguously raises retail price, implying that overall capacity will be smaller when demand is price-elastic than when it is not. Specifically, the impact on the profit of the large firm of a marginal increase in its capacity may be written

$$\frac{\partial \pi_i^{u+}}{\partial k^+} = P \frac{dp}{dk^+} \frac{1}{2} \left[\frac{k^- k^+}{[D(p)]^2} D'(p) + \int_{k^+/D(p)}^{K/D(p)} \theta D'(p) \, d\theta \right] + P \left[1 - \frac{K}{D(p)} \right] - c.$$

In addition to the effects identified above for the discriminatory auction, for the uniform-price auction we find that price-responsiveness of demand also affects delineation of different spot-market outcomes. In particular, as price increases and demand falls following an expansion of the large firm's capacity, the relative incidence of demand realisations in High Demand Region I is increased while that of High Demand Region II is reduced. Since the large firm earns a higher profit in Region II (where it may be despatched at full capacity) than in Region I (where it serves residual demand only), this effect tends to discourage investments even further.

To gain some further insight into the possible direction and magnitude of the effects involved, we have analysed a linear specification of the demand function; in particular, we let $D(p) = 1 - \gamma p$ (note that $\gamma = 0$ corresponds to the case with inelastic demand). We have performed a series of numerical simulations which show that, with this specification, an increase in the large firm's capacity induces a reduction in the retail price in the discriminatory auction. Hence, whereas the market-size effect tends to discourage investments in the uniform-price auction, it has the opposite effect in the discriminatory auction. Investment incentives are further strengthened in the discriminatory auction as compared to the uniform-price auction by the fact that under the former auction format retail prices are lower. These two results lead to both higher consumer surplus and overall welfare in the discriminatory auction than in the uniform-price auction.

The figures below depict aggregate capacity and retail prices as a function of the slope of the demand function for the case in which $c/P = 0.1$. As can be seen, the discriminatory auction induces more aggregate investment and lower retail prices than the uniform-price auction, and the differences between the two become larger the more elastic is the demand function.

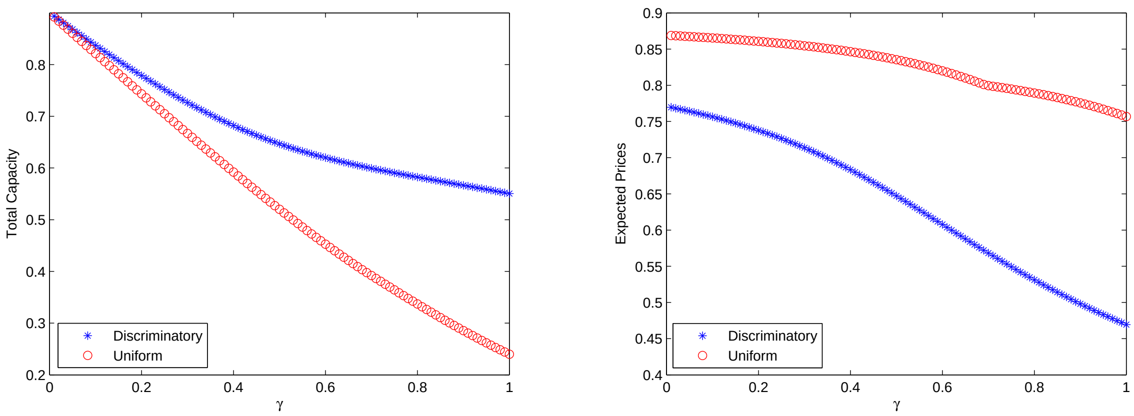

Figure 6: Aggregate capacity (left) and expected prices (right) as a function of γ

3.4 Distribution of Demand

In the basic model, it was assumed that demand is uniformly distributed. In this section, we relax this assumption. Since price competition takes place after demand has been realised and observed, characterisation of equilibrium bidding behaviour remains as in Propositions 1 and 3 for the uniform-price and discriminatory auctions, respectively. We may therefore concentrate our attention on the stage where capacities are chosen.

In the analysis of the basic model we saw how the marginal impact of investment on profits could be decomposed into a number of distinct and partly off-setting effects. First, investment allows for an increase in output whenever capacity acts as a constraint. Second, investment affects the type of equilibrium being played, by moving the borders between different regions of demand. In the discriminatory auction, there is a third effect also; investment affect the intensity of price competition when demand is High. The overall impact of investment depends on the relative importance of these effects, which further depends on the distribution of demand; the distribution determines the likelihood with which demand falls into different regions and hence the relative weight on each type of effect.

While equilibrium analysis is essentially the same with a general demand distribution as with the uniform distribution, it does become rather involved; in particular, existence of pure-strategy equilibrium is not guaranteed.[18] This may be seen as being due to the fact that reaction functions are not well behaved; not only are they not continuous (as with the uniform demand distribution), but they may also slope in different directions. We refer to the Appendix for details on equilibrium characterisation; here we concentrate attention on the total capacities that result at equilibrium.

The following proposition compares aggregate equilibrium capacities across auction formats, as well as with the first-best:

Proposition 12 *Suppose demand is distributed according to the function G on $[0,1]$. Then, when a pure-strategy equilibrium in capacities exists, the following is true:*

(i) $K^d \leq K^{FB}$, where the inequality is strict for $P < v$.

(ii) If G is strictly concave, $K^u < K^d$.

(iii) If G is strictly convex, $K^d < K^u$. Moreover, there exists $\widehat{P} \in (c, v)$, such that if $P < \widehat{P}$, $K^u < K^{FB}$, and if $P > \widehat{P}$, $K^{FB} < K^u$.

The relative size of installed capacities in the uniform-price and discriminatory auctions depends on whether the demand distribution function is concave or convex; when the distribution function is concave, the uniform-price auction induces lower investment than the discriminatory auction, and *vice versa*.

To understand this result, consider the effect on the profit of the large firm from a marginal increase in its capacity for the uniform-price and discriminatory formats, respectively:

$$\frac{\partial \pi^{u+}}{\partial k^+} = P\left[1 - G(K)\right] - c + \frac{P}{2}\left[G(K) - G\left(k^+\right) - G'\left(k^+\right)k^-\right] \tag{10}$$

$$\frac{\partial \pi^{d+}}{\partial k^+} = P\left[1 - G(K)\right] - c \tag{11}$$

In both cases, a marginal increase in the large firm's capacity allows it to sell more output at P whenever demand exceeds aggregate capacity. As may be seen from (11), in the discriminatory auction the firm balances this effect, $P\left[1 - G(K)\right]$, against the unit cost of capacity, c, and this determines aggregate investment.

[18] As shown in the Appendix, the following are sufficient conditions for equilibrium existence: either G is convex, or G is concave and G' is convex. These properties are satisfied by a large family of distribution functions.

As seen from (10), in the uniform-price auction there are additional effects. Firstly, when the large firm increases its capacity it enlarges the range of demand realisations over which, with probability $\frac{1}{2}$, it is bidding high and is despatched with only part of its capacity; the corresponding loss of profit is captured by the term $-G'(k^+)k^-$. Secondly, in the event that demand is in the range $[k^+, K]$, the firm is despatched at full capacity and hence a marginal increase in its capacity induces a gain in profit equal to $G(K) - G(k^+)$. Whether investment incentives are weaker or stronger in the uniform-price auction as compared to in the discriminatory auction depends on the relative importance of these two effects; in particular, it depends on the relative frequency of demand realisations on the interval $[k^+, K]$, or, more precisely, on the sign of $G(K) - G(k^+) - G'(k^+)\, k^-$, which is determined by the shape of the demand distribution function G. For example, when G is concave, the total gain from being able to increase output by one unit for each realisation of demand in the range $[k^+, K]$ - which has length k^- - is smaller than the loss from having to reduce output by k^- in the event that demand equals k^+; therefore, investment incentives are weaker in the uniform-price auction than in the discriminatory auction. The reverse is true when G is convex.

This discussion should also help understanding the comparison with first best. Independently of how demand is distributed, the discriminatory auction results in underinvestment because the large firm does not capture the entire social gain of its investment unless the price at which the extra capacity is sold equals consumers' willingness to pay, i.e., $P = v$.

In the uniform-price auction, market performance relative to the First Best depends on how demand is distributed also. For $P = v$, so that market price reflects the social value of capacity, underinvestment results if G is concave, whereas overinvestment results if G is convex. If $P < v$, investment incentives are lower in either case. If G is concave, a reduction in P strengthens the underinvestment result; if G is convex, overinvestment is mitigated, and ultimately eliminated as P is reduced to $\widehat{P}$. For $P < \widehat{P}$, the uniform-price auction results in underinvestment also for a convex distribution function.

Proposition 12 has immediate implications for the welfare ranking of the two auction formats:

Corollary 1 *At equilibrium, the comparison of total welfare is as follows:*
(i) If G is strictly concave, $W^u < W^d$.
(ii) If G is strictly convex, there exists $\overline{P} \in \left(\widehat{P}, v\right)$ such that if $P \leq \overline{P}$, $W^d \leq W^u$ and $W^u < W^d$ otherwise.

If the demand distribution function is concave, the discriminatory auction induces a more efficient outcome than the uniform-price auction, independently of the value of P. Furthermore, with a discriminatory auction first best may be attained by setting $P = v$, whereas with a uniform-price auction under-investment cannot not be avoided.

If the demand distribution is convex, the welfare ranking depends on the level of the market reserve price P. If $P \leq \widehat{P}$, the uniform-price auction induces a more efficient outcome in the sense that aggregate capacity is closer to the first-best level. As P is raised above $\widehat{P}$, welfare decreases in the uniform-price auction since over-investment results, but increases in the discriminatory as the degree of under-investment is mitigated. Hence there exists some price level $\overline{P}$ such that for $P > \overline{P}$ it is less costly is terms of welfare losses to ration demand than to finance the over-investment that results in the uniform-price auction. Both auction formats would result in an efficient level of installed capacity if the market reserve price is set at the right level: this level is $P = \overline{P}$ in the uniform-price auction and $P = v$ in the discriminatory auction. Note that while the market reserve price that induces first-best investment is lower in the uniform-price than in the discriminatory action, prices may still be higher in the uniform-price auction.

Given the importance of the shape of the demand distribution function, it is relevant to ask what real distribution functions look like. The figure below depicts demand distribution functions for the Spanish electricity industry for each of the years 2002 to 2005.[19] As can be seen from Figure 7, the demand distribution function is convex for low demand values, approximately uniform for intermediate demand values and concave for high demand values. Our theoretical model indicates that, as far as overall capacity is concerned, the relevant range is intermediate and high demand values. If so, these data suggest that the distribution of demand is uniform or concave.[20] The implication is that both auction formats lead to underinvestment, but that performance is better with the discriminatory format, both with regard to investment and average prices. Of course, given the highly stylised nature of our model, this result should be taken with the necessary caution.

4 Conclusions

Fabra, von der Fehr and Harbord (2006) demonstrated that, in a model which captures essential features of price setting in deregulated electricity markets, the discriminatory price format consistently leads to lower prices than the uniform-price format. This analysis, which was based on the assumption that installed capacities were given, suggested three sets of issues for future research. First, given that market prices depend on the pricing format, and given that price signals influence investment incentives, how does the choice of auction format affect capacity investment? Second, does allowing for endogenous capacities affect the relative performance of the two market designs? And, third, how do price caps - which mitigate market power but also reduce the profitability of investment - affect overall market performance once the effects on investment incentives is accounted for; in particular, is it still true that consumers are better off

[19]This demand distribution function has been constructed using the $35,064$ hourly demand values registered in the Spanish electricity market from 2002 to 2005. These data are provided by the Market Operator, OMEL. The precise shape of this function may change from year to year depending on factors such as weather, economic activity, etc.

[20]As far as we know, the distribution of demand in other markets tend to have similar shapes.

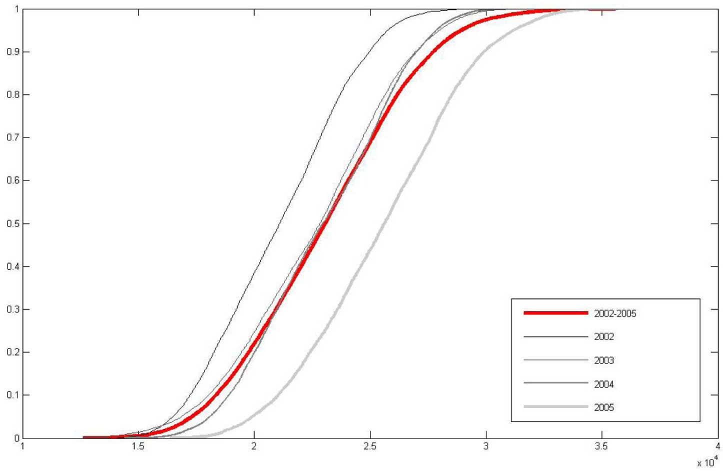

Figure 7: Demand Distribution Function in the Spanish Electricity Market, 2002-2005

the more stringent is the price cap?

In this paper, we have attempted to cast some light on these issues. Firstly, we have demonstrated that, although the discriminatory auction generally leads to more competitive behaviour and lower prices than the uniform-price auction, it is not necessarily the case that incentives to invest are weaker; indeed, investment incentives may be greater with the discriminatory format. Moreover, even under conditions of imperfect competition, market-based incentives do not necessarily lead to under-investment; in fact, with the uniform-price format over-investment may well occur. Nevertheless, aggregate investment is not necessarily affected by the choice of either pricing rule or bid format; so long as demand is uniformly distributed on the relevant range (which appears to be the empirically relevant case), total investment remains the same. The intuition for this result follows from two observations: (i) at the margin, capacity is always determined by a firm that in effect acts as a monopolist with respect to residual demand; and (ii) the marginal unit is despatched when capacity is fully utilised, in which case it receives the market reserve price under both auction formats.

On the second set of issues, relating to market performance, the relative supremacy of the discriminatory auction as far as prices are concerned tends to be true even when we allow for endogenous capacities. The dominant effect is the one identified by Fabra, von der Fehr and Harbord (2006), that, when demand is high, competition is fiercer in the discriminatory auction than in the uniform-price auction. Allowing for endogenous capacity choice does however introduce two new effects that may modify the conclusion. Firstly, a larger total capacity generally reduces prices, especially when demand is close to full capacity utilisation; when the discriminatory auction leads to higher investment, this effect enhances the supremacy of that format, and *vice versa*. Secondly, more asymmetry between firms (which, for a given aggregate capacity, means higher market concentration) tends to raise prices; when the discriminatory auction leads to more asymmetry, this effect reduces the supremacy of that format, and *vice versa*. The determinants of these additional effects are complex and the comparison across formats depend on underlying market characteristics. However, with uniformly distributed demand, while total capacity is the same and the asymmetry is greater with the discriminatory auction, prices are nevertheless lower with that format.

On the third set of issues, the relationship between price caps and market performance gets richer and probably more realistic once capacities are endogenised. Price caps have two countervailing effects on consumers' welfare. On the one hand, for given capacities, lowering the price cap reduces equilibrium prices, thereby benefitting consumers; however, a lower price cap also decreases firms' incentives to expand capacity, leading to a greater likelihood of demand rationing. It turns out that consumers may be made better off with a price cap than with no price cap at all. It may be noted that the choice of price cap also affects the relative performance of the two auction formats.

Admittedly, our model is highly stylised and we would not want to over-emphasise the empirical relevance of the theoretical results. Nevertheless, some of the insights appear quite robust and seem to point to more fundamental characteristics of the workings of deregulated electricity markets. One of these has already been pointed out: although the discriminatory auction format tends to lower prices, this does not imply that investment incentives are poorer; profit-maximising firms are not concerned with profit levels *per se*, but how the level of profit is affected by capacity choices.

A second apparently robust result is the asymmetry of investment incentives. By choosing a smaller capacity than its rival, a firm can ensure a higher frequency of market outcomes at which not only is price competition softer, but it itself is despatched at full capacity. At the same time, a firm facing a relatively small rival has incentive to expand capacity so as to take advantage of higher prices in periods of high demand. It may be noted that such asymmetric investment incentives imply that incumbency size advantages may be maintained also after market-based competition has taken effect. A natural question to analyse next is to which extent the different auction formats favour investment by the current market leader, thereby leading to increasing asymmetries in the long-run.[21]

[21] Athey and Schmutzler (2001) analyse a model of oligopolistic competition with ongoing investment and derive conditions under which the leading firms invest more, thereby reinforcing their initial market dominance.

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Appendix

A The Basic Model

Proof of Propositions 2 and 6

As Proposition 2 is a special case of Proposition 6, we provide a general argument that covers both cases.

Let ρ_i be the probability with which, when there exist multiple equilibrium outcomes at the price-competition stage, an equilibrium in which firm i bids high is played. Recall that ρ_i is a constant, independent of installed capacity levels, such that $\rho_i = \rho \leq \frac{1}{2}$ if $i = 1$ and $\rho_i = 1 - \rho$ if $i = 2$.

Expected profits are given as follows, for $i = 1, 2$, $i \neq j$,

$$\pi_i^u(k_i, k_j) = \begin{cases} \pi_i^{u-} & \text{if} \quad k_i \leq k_j \\ \pi_i^{u+} & \text{if} \quad k_i \geq k_j, \end{cases}$$

where

$$\pi_i^{u-} = P\left[\int_{k^-}^{k^+} k^- d\theta + \int_{k^+}^{K} \{[1-\rho_i]\, k^- + \rho_i\,[\theta - k^+]\}\, d\theta + \int_{K}^{1} k^- d\theta\right] - ck^-$$
$$\pi_i^{u+} = P\left[\int_{k^-}^{k^+} [\theta - k^-]\, d\theta + \int_{k^+}^{K} \{[1-\rho_i]\, k^+ + \rho_i\,[\theta - k^-]\}\, d\theta + \int_{K}^{1} k^+ d\theta\right] - ck^+. \tag{12}$$

Let $k_i^-(k_j)$ and $k_i^+(k_j)$ be (implicitly) defined as the solutions to the first-order conditions $\frac{\partial \pi_i^{u-}\left(k_i^-(k_j),k_j\right)}{\partial k_i} = 0$ and $\frac{\partial \pi_i^{u+}\left(k_i^+(k_j),k_j\right)}{\partial k_i} = 0$, respectively. Simple algebra shows that

$$k_i^-(k_j) = \frac{1}{2+\rho_i}\left[1 - \frac{c}{P}\right], \tag{13}$$

$$k_i^+(k_j) = 1 - \frac{c}{P} - k_j. \tag{14}$$

Note that since both $\pi_i^{u-}(k_i, k_j)$ and $\pi_i^{u-}(k_i, k_j)$ are concave in k_i and since π_i^u is continuous, it follows that $k_i^-(k_j)$ and $k_i^+(k_j)$ are local profit maximisers only if they are interior, i.e., if $k_i^-(k_j) \leq k_j$ and $k_i^+(k_j) \geq k_j$, respectively. There are three cases to consider: (a) $k_j < k_i^-(k_j) \leq k_i^+(k_j)$, so that only $k_i^+(k_j)$ supports a maximum; (b) $k_i^-(k_j) \leq k_j \leq k_i^+(k_j)$, so that both $k_i^-(k_j)$ an $k_i^+(k_j)$ are local maximisers; and (c) $k_i^-(k_j) \leq k_i^+(k_j) < k_j$, so that only $k_i^-(k_j)$ supports a maximum. Figure 4 depicts cases (a) and (c).

In case (b), a simple monotonicity argument - based on the observations that while $\pi_i^{u-}\left(k_i^-(k_j), k_j\right)$ is a constant, independent of k_j, $\pi_i^{u+}(k_i, k_j)$ is strictly decreasing in k_j - implies that there exists some value $\widehat{k}_j$ such that $\pi_i^{u-}\left(k_i^-(k_j), k_j\right) \geq \pi_i^{u+}\left(k_i^+(k_j), k_j\right)$ if and only if $k_j \geq \widehat{k}_j$. From the expressions

$$\pi_i^{u-}\left(k_i^-(k_j), k_j\right) = \frac{1}{2}\frac{1}{2+\rho_i} P \left[1 - \frac{c}{P}\right]^2$$
$$\pi_i^{u+}\left(k_i^+(k_j), k_j\right) = \frac{1}{2} P \left[1 - \frac{c}{P}\right]^2 - [P - c]\, k_j + \frac{P}{2}\,[2-\rho_i]\, k_j^2$$

we find[22]

$$\widehat{k}_j = \frac{2 + \rho_i - \rho_i\sqrt{2+\rho_i}}{2-\rho_i}\frac{1}{2+\rho_i}\left[1 - \frac{c}{P}\right].$$

Therefore, using (13) and (14), the best-reply function of firm i becomes

$$k_i^u(k_j) = \begin{cases} 1 - \frac{c}{P} - k_j & \text{if} \quad k_j \leq \widehat{k}_j \\ \frac{1}{2+\rho_i}\left[1 - \frac{c}{P}\right] & \text{if} \quad k_j \geq \widehat{k}_j \end{cases} \tag{15}$$

Note that k_i^u is a continuous, non-increasing function everywhere on $[0, 1]$, except at $\widehat{k}_j$, where $k_i^-\left(\widehat{k}_j\right) \leq \hat{k}_j \leq k_i^+\left(\widehat{k}_j\right)$ (the inequalities are strict unless $\rho_i = 0$); in particular, for $k_j \geq \widehat{k}_j$ the reaction function is flat, whereas for $k_j \leq \widehat{k}_j$ it is strictly decreasing in the rival's

[22]The second root in the equation $\pi_i^{u-}\left(k_i^-(k_j), k_j\right) = \pi_i^{u+}\left(k_i^+(k_j), k_j\right)$ is ruled out by the condition $\hat{k}_j \leq \frac{1}{2}\frac{P-c}{P}$.

capacity. For $\rho > 0$, the discontinuity in the reaction function implies that it never crosses the 45°-line.

Using the same parameter values as in the example above, Figure 8 depicts firms' reaction functions for $\rho = \frac{1}{2}$ and $\rho = 0$.

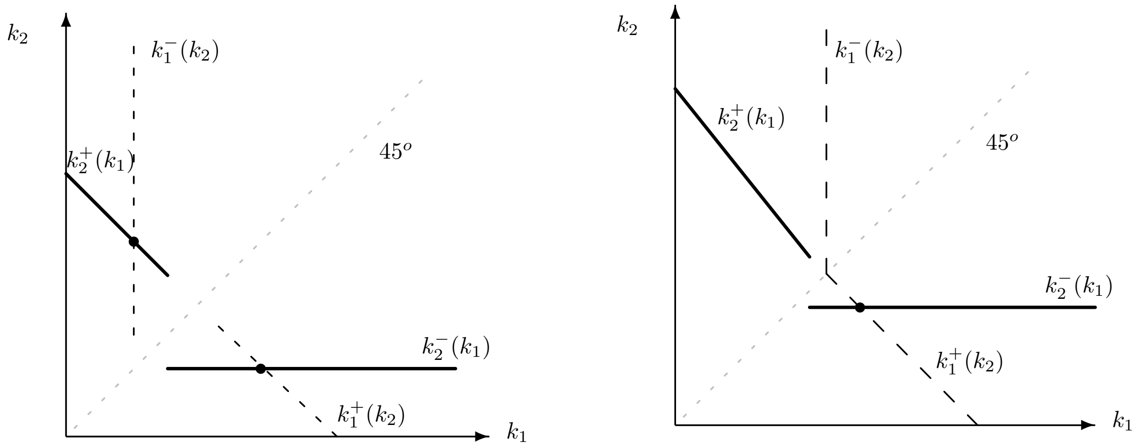

Figure 8: Best reply functions in the uniform auction for $\rho = \frac{1}{2}$ (left) and $\rho = 0$ (right)

Since, for a given value of ρ, at most one reaction function crosses the 45°-line a symmetric pure-strategy equilibrium cannot exist. Hence, at equilibrium, firms choose asymmetric capacities. For a pure-strategy equilibrium with $k_i > k_j$ to exist it must be the case that the flat part of firm j's reaction function, $k_j^-(k_i)$, crosses the decreasing part of firm i's reaction function, $k_i^+(k_j)$, to the left of the discontinuity point $\widehat{k}_j$; that is, at such an equilibrium, $k_j^-\left(\widehat{k}_j\right) < \widehat{k}_j$.

Consider first the case $\rho = \frac{1}{2}$. Since, in this case, firms' payoff functions are identical, their best-reply functions are symmetric, with discontinuity at some $\widehat{k}_1 = \widehat{k}_2 = \widehat{k}$. Consequently, the two reaction functions must cross at two points, (k^+, k^-) and (k^+, k^-), with $k^- < \widehat{k} < k^+$.

As ρ is reduced below $\frac{1}{2}$, so that Firm 1 sells at capacity more frequently (and Firm 2 correspondingly less often), $k_1^-(k_2)$ shifts up, $k_2^-(k_1)$ shifts down, while both $k_1^+(k_2)$ and $k_2^+(k_1)$ remain unaltered. It follows that, in (k_1, k_2)-space, the two crossing points move towards South-East, implying that, at equilibrium, the capacity of Firm 1 increases, while that of Firm 2 decreases, as ρ is reduced. Note, however, that whereas the equilibrium with $k_1 > k_2$ exists for all $\rho \leq \frac{1}{2}$, the equilibrium with $k_1 < k_2$ fails to exist when ρ falls below a critical level, $\widehat{\rho}$. We must have $\widehat{\rho} > 0$, since, for $\rho = 0$, firms' reaction functions cannot cross at any point $k_1 < k_2$.

1 1

Proof of Proposition 4

In reduced form, we have

$$\begin{aligned} \pi^{d-} &= [P-c]\,k^{-} - P\,[k^{-}]^2 \left[1 + \tfrac{1}{2}\tfrac{k^{-}}{k^{+}} - \ln\left(\tfrac{k^{-}}{k^{+}}\right)\right] \\ \pi^{d+} &= [P-c]\,k^{+} - P\left\{\tfrac{1}{2}\,[k^{+}]^2 + k^{-}k^{+}\right\} \end{aligned} \tag{16}$$

Note that $\pi^{d-} = \pi^{d+}$ at $k^{-} = k^{+}$. Moreover,

$$\begin{aligned} \frac{\partial \pi^{d-}}{\partial k^{-}} &= P - c - Pk^{-}\left[1 - 2\ln\left(\frac{k^{-}}{k^{+}}\right) + \frac{3}{2}\frac{k^{-}}{k^{+}}\right] \\ \frac{\partial \pi^{d+}}{\partial k^{+}} &= P - c - P\left[k^{+} + k^{-}\right] \end{aligned}$$

and

$$\begin{aligned} \frac{\partial^2 \pi^{d-}}{\partial k^{-2}} &= P\left[1 + 2\ln\left(\frac{k^{-}}{k^{+}}\right) - 3\frac{k^{-}}{k^{+}}\right] \\ \frac{\partial^2 \pi^{d+}}{\partial k^{+2}} &= -P \end{aligned}$$

Since $\frac{\partial^2 \pi^{d-}}{\partial k^{-2}}$ is increasing in k^{-} for $k^{-} \in \left(0, \frac{2}{3}k^{+}\right)$ and decreasing in k^{-} for $k^{-} > \frac{2}{3}k^{+}$, while $\frac{\partial^2 \pi^{d-}}{\partial k^{-2}} < 0$ at $k^{-} = \frac{2}{3}k^{+}$, π^{d-} is strictly concave in k^{-}. Also π^{d+} is concave in k^{+}. It follows that firm i's profit function is a piecewise concave function, continuous everywhere.

Let $k_i^{-}(k_j)$ and $k_i^{+}(k_j)$ be (implicitly) defined as the solutions to the first-order conditions $\frac{\partial \pi^{d-}\left(k_i^{-}(k_j), k_j\right)}{\partial k_i} = 0$ and $\frac{\partial \pi^{d+}\left(k_i^{+}(k_j), k_j\right)}{\partial k_i} = 0$, respectively. Simple algebra shows that $k_i^{-}(k_j)$ solves

$$1 - \frac{c}{P} = k_i\left[1 - 2\ln\left(\frac{k_i}{k_j}\right) + \frac{3}{2}\frac{k_i}{k_j}\right], \tag{17}$$

while $k_i^{+}(k_j)$ is given by

$$k_i^{+}(k_j) = 1 - \frac{c}{P} - k_j. \tag{18}$$

Since both $\pi^{d-}(k_i, k_j)$ and $\pi^{d+}(k_i, k_j)$ are concave in k_i and since π_i^d is continuous, it follows that $k_i^{-}(k_j)$ and $k_i^{+}(k_j)$ are local profit maximisers if they are interior, i.e., if $k_i^{-}(k_j) \le k_j$ and $k_i^{+}(k_j) \ge k_j$, respectively. We first establish conditions under which $k_i^{-}(k_j)$ and $k_i^{+}(k_j)$ can be interior. First, for $k_i^{+}(k_j)$ to be interior, we require $k_j \le \frac{1}{2}\left[1 - \frac{c}{P}\right]$. To see this, note that $k_i^{+}(k_j)$ is downward sloping. Furthermore, if $k_i^{+}(k_j) = k_j$, (18) implies $k_i = k_j = \frac{1}{2}\left[1 - \frac{c}{P}\right]$. It follows that $k_i^{+}(k_j) \ge k_j$ iff $k_j \le \frac{1}{2}\left[1 - \frac{c}{P}\right]$.

Next, for $k_i^{-}(k_j)$ to be interior, we require $k_j \ge \frac{2}{5}\left[1 - \frac{c}{P}\right]$. To see this note that $k_i^{-}(k_j)$ is downward sloping, or

$$\frac{\partial k_i^{-}(k_j)}{\partial k_j} = \frac{2\frac{k_i}{k_j} - \frac{3}{2}\left[\frac{k_i}{k_j}\right]^2}{1 + 2\ln\left(\frac{k_i}{k_j}\right) - 3\frac{k_i}{k_j}} < 0,$$

since the numerator is positive, given that $k_i \leq k_j$, and the denominator, which has the same sign as $\frac{\partial^2 \pi^{d-}}{\partial k^{-2}}$, is negative. Furthermore, if $k_i^-(k_j) = k_j$, (17) implies $k_i = k_j = \frac{2}{5}\left[1 - \frac{c}{P}\right]$. It follows that $k_i^-(k_j) \leq k_j$ iff $k_j \geq \frac{2}{5}\left[1 - \frac{c}{P}\right]$.

There are three cases to consider: (a) $k_j < k_i^-(k_j) \leq k_i^+(k_j)$, so that only $k_i^+(k_j)$ supports a maximum; (b) $k_i^-(k_j) \leq k_j \leq k_i^+(k_j)$, so that both $k_i^-(k_j)$ an $k_i^+(k_j)$ are local maximisers; and (c) $k_i^-(k_j) \leq k_i^+(k_j) < k_j$, so that only $k_i^-(k_j)$ supports a maximum. The three alternative cases may therefore be delineated as follows: (a)$k_j < \frac{2}{5}\left[1 - \frac{c}{P}\right]$; (b)$\frac{2}{5}1 - \frac{c}{P} \leq k_j \leq \frac{1}{2}\left[1 - \frac{c}{P}\right]$; and (c) $k_j > \frac{1}{2}\left[1 - \frac{c}{P}\right]$. Clearly, in regions (a) and (b), given the continuity of the profit function, the global maxima are the interior solutions $k_i^+(k_j)$ in (a) and $k_i^-(k_j)$ in (c).

In region (b), where both maxima are interior, we compare profits at the two candidate solutions. Since

$$\begin{aligned}
\pi^{d-}\left(k_i^-\left(\frac{2}{5}\left[1-\frac{c}{P}\right]\right), \frac{2}{5}\left[1-\frac{c}{P}\right]\right) - \pi^{d+}\left(k_i^+\left(\frac{2}{5}\left[1-\frac{c}{P}\right]\right), \frac{2}{5}\left[1-\frac{c}{P}\right]\right) &\leq 0 \text{ and} \\
\pi^{d-}\left(k_i^-\left(\frac{1}{2}\left[1-\frac{c}{P}\right]\right), \frac{1}{2}\left[1-\frac{c}{P}\right]\right) - \pi^{d+}\left(k_i^+\left(\frac{1}{2}\left[1-\frac{c}{P}\right]\right), \frac{1}{2}\left[1-\frac{c}{P}\right]\right) &\geq 0,
\end{aligned}$$

and by the continuity of the profit functions, there exists $\hat{k}_j$ such that

$$\pi^{d-}\left(k_i^-\left(\hat{k}_j\right), \hat{k}_j\right) - \pi_i^{d+}\left(k_i^+\left(\hat{k}_j\right), \hat{k}_j\right) = 0.$$

At $k_j = \hat{k}_j$ both k_i^- and k_i^+ are global maximisers and hence best replies. For values $k_j < \hat{k}_j$ the best reply is k_i^+, whereas for $k_j > \hat{k}_j$ the best reply is k_i^-. The uniqueness of $\hat{k}_j$ is guaranteed by the fact that the difference in profits is a strictly increasing function in k_j:

$$\begin{aligned}
\frac{d\pi^{d-}\left(k_i^-(k_j), k_j\right)}{dk_j} - \frac{d\pi^{d+}\left(k_i^+(k_j), k_j\right)}{dk_j} &= -\frac{P}{2}\left\{\left[\frac{k_i^-(k_j)}{k_j}\right]^2\left[2k_j - k_i^-(k_j)\right] - 2k_i^+(k_j)\right\} \\
&\geq -\frac{P}{2}\left\{\left[2k_j - k_i^-(k_j)\right] - 2k_j\right\} > 0
\end{aligned}$$

where the first inequality from the fact that in region (b) $k_i^-(k_j) \leq k_j \leq k_i^+(k_j)$.

In summary, the best-response function of firm $i = 1, 2$, $i \neq j$, is

$$k_i^d(k_j) = \begin{cases} k_i^+(k_j) & \text{if} \quad k_j \leq \hat{k} \\ k_i^-(k_j) & \text{if} \quad k_j \geq \hat{k} \end{cases}$$

Note that $k_i^d(k_j)$ is discontinuous at $k_j = \hat{k}$, where it jumps down from $k_i^+\left(\hat{k}\right) > \hat{k}$ to $k_i^-\left(\hat{k}\right) < \hat{k}$.

To establish the character of equilibria we use a geometric argument. We have that $k_2^d(0) = 1 - \frac{c}{P}$ and $\lim_{k_1 \uparrow \widehat{k}} k_2^d(k_1) > \widehat{k}$. Furthermore, if we let $k_1^{d\ inv}$ denote the inverse of k_1^d, we have $k_1^{d\ inv}(A) = 1 - \frac{c}{P}$, where $A = k_1^d\left(1 - \frac{c}{P}\right) > 0$, and $k_1^{d\ inv}(B) = \widehat{k}$, where $B = \lim_{k_2 \downarrow \widehat{k}} k_1^d(k_2) < \widehat{k}$. Since reaction functions are everywhere decreasing, it follows that k_2^d and $k_1^{d\ inv}$ must cross once on the interval $[A, B]$ and do not cross on either $[0, A]$ or $\left[B, \widehat{k}\right]$. Therefore, there exists exactly

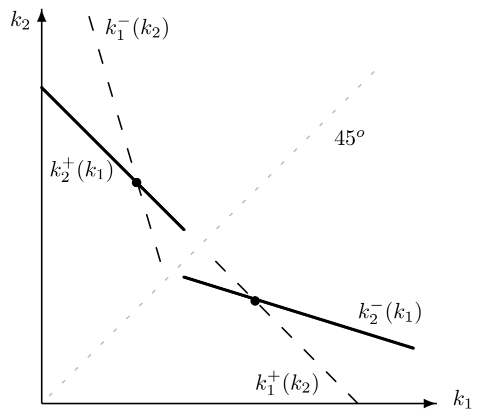

Figure 9: Best reply functions in the discriminatory auction

one equilibrium in $\left[0, \widehat{k}\right] \times \left[\widehat{k}, 1-\frac{c}{P}\right]$. A corresponding argument establishes that there exists one equilibrium in $\left[\widehat{k}, 1-\frac{c}{P}\right] \times \left[0, \widehat{k}\right]$.

To finally characterise equilibria, let $k^+ = \alpha k^-$, with $\alpha > 1$. From the first-order condition (18), we find

$$k^- = \frac{1}{1+\alpha}\left[1-\frac{c}{P}\right]. \tag{19}$$

Furthermore, from this result and the first-order condition (17), we find $\alpha^2 - 2\alpha \ln(\alpha) - \frac{3}{2} = 0$. The left-hand side of this equation is negative for $\alpha = 1$ and increasing in α for any $\alpha > 1$. It follows that it has a unique solution, which is $\alpha \approx 2.34$.

Proof of Proposition 5

(i) The fact that aggregate equilibrium capacities coincide under the two auction formats derives from the fact that the first-order conditions for the large firm's capacity choice are the same, equations (5) and (7) above.

(ii) From Proposition 2 it follows that $k^{u+} = \frac{3}{2}k^{u-}$, while from Proposition 4 it follows that $k^{d+} > 2k^{d-}$. Since aggregate capacities are the same it follows that $k^{d-} < k^{u-} < k^{u+} < k^{d+}$.

(iii) In the uniform-price auction, equilibrium capacities are

$$k^{u-} = \frac{2}{5}\left[1-\frac{c}{P}\right] \text{ and } k^{u+} = \frac{3}{5}\left[1-\frac{c}{P}\right]$$

and so, by inserting these expressions into (20), we find that profits are

$$\pi^{u-} = \frac{10}{50}P\left[1-\frac{c}{P}\right]^2 \text{ and } \pi^{u+} = \frac{11}{50}P\left[1-\frac{c}{P}\right]^2$$

implying that aggregate profits are

$$\pi^{u-} + \pi^{u+} = \frac{21}{50}P\left[1-\frac{c}{P}\right]^2. \tag{20}$$

In the discriminatory auction, equilibrium capacities are

$$k^{d-}=\frac{1}{1+\alpha}\left[1-\frac{c}{P}\right] \text{ and } k^{d+}=\frac{\alpha}{1+\alpha}\left[1-\frac{c}{P}\right]$$

where α is given by (8), and, so by inserting these expressions into (16), we find that profits are

$$\begin{aligned}\pi^{d-} &= \left\{\tfrac{1}{1+\alpha}-\tfrac{1}{[1+\alpha]^2}\left[1+\tfrac{1}{2\alpha}+\ln(\alpha)\right]\right\}P\left[1-\tfrac{c}{P}\right]^2\\ \pi^{d+} &= \left\{\tfrac{\alpha}{1+\alpha}-\tfrac{1}{[1+\alpha]^2}\left[\tfrac{\alpha^2}{2}+\alpha\right]\right\}P\left[1-\tfrac{c}{P}\right]^2\end{aligned}$$

implying that aggregate profits are

$$\pi^{d-}+\pi^{d+}=\left\{1-\frac{1}{1+\alpha}-\frac{1}{[1+\alpha]^2}\left[\frac{1}{2\alpha}+\ln\alpha+\frac{\alpha^2}{2}\right]\right\}P\left[1-\frac{c}{P}\right]^2. \tag{21}$$

Since the bracketed term in (21) is smaller than $\frac{21}{50}$, it follows that aggregate profits are lower in the discriminatory auction than in the uniform-price auction.

Consumer payments equal total revenues of firms, which equals the sum of capacity costs and profits. Since aggregate capacity is the same in both auctions, so are capacity costs. It follows that consumer payments are higher in the uniform-price than in the discriminatory action. Moreover, if we define the average price p as total payments divided by expected consumption, that is,

$$p\left[\int_0^K \theta d\theta+\int_K^1 K d\theta\right]=cK+\pi_1+\pi_2,$$

it follows that it is lower in the discriminatory auction than in the uniform-price auction.

B Equilibrium Selection

Proof of Proposition 7

Fabra *et al.* (2006) demonstrate that a mixed-strategy equilibrium at the price-competition stage when demand is in High Demand Region II has the form

$$\begin{aligned}F_1^u(b) &= \begin{cases} A_1\left[\frac{b}{P}\right]^{\frac{\theta-k_1}{k_1+k_2-\theta}} & \text{for } \quad 0<b<P\\ 1 & \text{for } \quad b=P\end{cases}\\ F_2^u(b) &= \begin{cases} A_2\left[\frac{b}{P}\right]^{\frac{\theta-k_2}{k_1+k_2-\theta}} & \text{for } \quad 0<b<P\\ 1 & \text{for } \quad b=P\end{cases}\end{aligned}$$

where $F_i^u(b)=\Pr\{b_i\leq b\}$ denotes the mixed strategy of firm i, $i=1,2$, and either (i) $A_1=1$ and $0<A_2\leq 1$ or (ii) $0<A_1\leq 1$ and $A_2=1$.

Furthermore, for a given demand realisation in High demand-Region II, equilibrium profits of firm i, $i=1,2$, are given by

$$P\left\{\Pr(b_j=P)\,k_i+[1-\Pr(b_j=P)]\,[\theta-k_j]\right\}-ck_i,$$

or, equivalently,

$$P\left\{(1-A_j)\,k_i + A_j\left[\theta - k_j\right]\right\} - ck_i.$$

Note that for demand realisations in this region, within the class of equilibria in which $\lim_{b\to P} F_1^u(b) = 1$ (implying $\Pr(b_1 = P) = 0$ or $A_1 = 1$) total industry profits are maximised in the limiting case $\Pr(b_2 = P) = 1$ (which corresponds to $A_2 = 0$), where profits are $[P-c]\,k_1$ and $P[\theta - k_1] - ck_2$,[23] and they are minimised in the case $\Pr(b_2 = P) = 0$ (which corresponds to $A_2 = 1$), where profits are $P[\theta - k_2] - ck_1$ and $P[\theta - k_1] - ck_2$. Corresponding results hold for the other class of mixed-strategy equilibria.

Assuming that firms play one particular mixed-strategy equilibrium at the price-competition stage, we find that, for a given capacity configuration, expected profits are, for $i = 1, 2$, $i \neq j$,

$$\pi_i^u(k_i, k_j) = \begin{cases} \pi_i^{u-} & \text{if} \quad k_i \leq k_j \\ \pi_i^{u+} & \text{if} \quad k_i \geq k_j \end{cases}$$

where

$$\pi_i^{u-} = P\left[\int_{k^-}^{k^+} k^- d\theta + \int_{k^+}^{K} \left\{[1-\phi_i]\,k^- + \phi_i\left[\theta - k^+\right]\right\} d\theta + \int_{K}^{1} k^- d\theta\right] - ck^-$$

$$\pi_i^{u+} = P\left[\int_{k^-}^{k^+} \left[\theta - k^-\right] d\theta + \int_{k^+}^{K} \left\{[1-\phi_i]\,k^+ + \phi_i\left[\theta - k^-\right]\right\} d\theta + \int_{K}^{1} k^+ d\theta\right] - ck^+$$

and $\phi_i = 1 - \Pr(b_j = P) = A_j$, $i, j = 1, 2$, $i \neq j$.

Comparing with (12) above, we see that here ϕ_i is analytically equivalent to ρ_i. Consequently, best reply functions and equilibrium characterisations can be derived by appealing to the proof of Proposition 6, leading us to the two solutions $\left(k_1^{u+}, k_2^{u-}\right)$ and $\left(k_1^{u-}, k_2^{u+}\right)$, where

$$\begin{aligned} k_1^{u+} &= \frac{1+\phi_2}{2+\phi_2}\left[1 - \frac{c}{P}\right] > k_2^{u-} = \frac{1}{2+\phi_2}\left[1 - \frac{c}{P}\right] \\ k_1^{u-} &= \frac{1}{2+\phi_1}\left[1 - \frac{c}{P}\right] < k_2^{u+} = \frac{1+\phi_1}{2+\phi_1}\left[1 - \frac{c}{P}\right] \end{aligned}$$

and either $\phi_1 = 1$ and $0 < \phi_2 \leq 1$ or $0 < \phi_1 \leq 1$ and $\phi_2 = 1$. As explained in the proof of Proposition 6, for certain parameter values only one of these solutions constitute an equilibrium; in particular, an equilibrium with $k_1 < k_2$ ($k_2 < k_1$) fails to exist when ϕ_1 (ϕ_2) is sufficiently small.

Consequently, in all equilibria, aggregate capacity equals $1 - \frac{c}{P}$. Moreover, capacity asymmetry is larger when firms' strategies are more symmetric. Consider for example the equilibrium in which Firm 2 bids the reserve price with positive probability and Firm 1 does not; that is, $\Pr(b_1 = P) = 0$ and $\Pr(b_2 = P) \geq 0$, or $\phi_1 \leq 1$ and $\phi_2 = 1$. Then the capacity of Firm 1 (the small firm) is decreasing, and that of Firm 2 (the large firm) is increasing, as the probability

[23] This is the same as in the corresponding pure-strategy equilibrium in which Firm 2 is always bidding high, implying that profits in this pure-strategy equilibrium dominate those in any mixed-strategy equilibrium.

that Firm 2 plays the reserve price goes down (i.e., ϕ_1 goes up). The difference in capacities is maximised in the limiting case when $\Pr(b_2 = P) = 0$, which corresponds to $\phi_1 = 1$, when the capacity of Firm 2 is double that of Firm 1.

At the equilibrium $\left(k_1^{u+}, k_2^{u-}\right)$, profits become

$$\begin{aligned}\pi_1^u &= \frac{1}{2}\frac{2+2\phi_2+\phi_2^2-\phi_1}{[2+\phi_2]^2}\frac{[P-c]^2}{P} \\ \pi_2^u &= \frac{1}{2}\frac{1}{2+\phi_2}\frac{[P-c]^2}{P}\end{aligned}$$

implying that industry profits are

$$\pi_1^u + \pi_2^u = \frac{1}{2}\frac{4+3\phi_2+\phi_2^2-\phi_1}{[2+\phi_2]^2}\frac{[P-c]^2}{P}. \tag{22}$$

With $0 < \phi_1 \leq 1$ and $\phi_2 = 1$, (22) is minimised at the upper bound $\phi_1 = 1$, where

$$\pi_1^u + \pi_2^u = \frac{7}{18}\frac{[P-c]^2}{P}.$$

With $\phi_1 = 1$ and $0 < \phi_2 \leq 1$, (22) is minimised in the limit $\phi_2 = 0$, where

$$\pi_1^u + \pi_2^u = \frac{3}{8}\frac{[P-c]^2}{P}$$

Since the bracketed term in (21) is smaller than $\frac{3}{8}$, it follows that also in these mixed-strategy equilibria industry profits exceed those obtained in the discriminatory auction.

C Long-Lived Bids

Proof of Propositions 8 and 9

Uniform-Price Auction

Let $F_i^u(b) = \Pr\{b_i \leq b\}$ denote the equilibrium mixed-strategy of firm i, $i = 1, 2$, in the uniform-price auction, with $f_i^u(b) = F_i^{u\prime}(b)$, and let S_i^u be the support of F_i^u. Standard arguments imply that $S_1^u \cap S_2^u = [\underline{b}, P)$, $\underline{b}^u \geq c$, and that F_1^u and F_2^u do not have mass points on $(\underline{b}^u, P)$.

We concentrate our attention on the case in which $k_1 + k_2 \leq 1$.[24] Firm i's profit, when bidding b, may then be written

$$\pi_i^u(b) = F_j^u(b) b \int_{k_j}^1 \min\{\theta - k_j, k_i\} d\theta + \int_b^P \left[\int_0^{k_i} b\theta d\theta + \int_{k_i}^1 \upsilon k_i d\theta\right] dF_j^u(\upsilon) - ck_i.$$

The first term on the right-hand side represents firm i's profits in the event that the rival bids below b, in which case firm i produces a positive quantity (limited by its installed capacity) only when demand is above the capacity of the rival. The second term represents firm i's profits

[24]It is easy to show that $k_1 + k_2 > 1$ will never be part of a subgame perfect equilibrium.

in the event that the rival bids above b. As given by the first element of this term, firm i will then serve all demand at its own price when capacity is sufficient to satisfy all of demand. Furthermore, as given by the second element, firm i will produce at full capacity and receive a price determined by the rival's bid in the event that demand exceeds firm i's capacity.

On $(\underline{b}^u, P)$, strategies must satisfy the following differential equations:

$$
\begin{aligned}
& F_j^u(b) \int_{k_j}^{1} \min\{k_i, \theta - k_j\}\, d\theta + \left[1 - F_j^u(b)\right] \int_0^{k_i} \theta d\theta \\
-b f_j^u(b) & \left\{ \int_0^{k_i} \theta d\theta + \int_{k_i}^{1} k_i d\theta - \int_{k_j}^{1} \min\{k_i, \theta - k_j\}\, d\theta \right\} = 0
\end{aligned}
$$

On the interior of the support of the mixed strategies the net gain from raising the bid marginally must be zero. The first element on the left-hand side represents the gain to a firm from the resulting increase in price received in the event that demand exceeds the capacity of the rival and the rival submits the lowest bid. The second element represents the gain to a firm from the resulting increase in price in the event that demand is lower than its capacity and the rival submits the highest bid. Lastly, the third term represents the loss from being despatched with a smaller output: in case demand falls below the firm's capacity, the loss of output equals total demand; in case demand exceeds the firm's capacity but remains below aggregate capacity, the loss equals the difference between being despatched at full capacity and serving residual demand only (i.e., $k_i - \min\{k_i, \theta - k_j\}$).

The above expressions may alternatively be written

$$
f_j^u(b) - \frac{\lambda_j}{b} F_j^u(b) = \frac{\beta_j}{b},
$$

where

$$
\begin{aligned}
\lambda_j &= \frac{1 - k_i - k_j}{k_j} \\
\beta_j &= \frac{1}{2} \frac{k_i}{k_j}
\end{aligned}
$$

which have solutions

$$
F_j^u(b) = \Omega^j \left[b\right]^{\lambda_j} - \frac{\beta_j}{\lambda_j},
$$

where Ω^j, $j = 1, 2$, are constants of integration. Note that, if $k_i \leq k_j$, $\beta_i \geq \beta_j$, $\lambda_i \geq \lambda_j$ and $\frac{\beta_i}{\lambda_i} \geq \frac{\beta_j}{\lambda_j}$. Furthermore, $\beta_1 = \beta_2$ and $\lambda_1 = \lambda_2$ when $k_1 = k_2$.

Given the boundary condition $F_j^u(\underline{b}^u) = 0$, these equations yield the mixed-strategy distribution functions for $b \in [\underline{b}, P)$:

$$
F_j^u(b) = \frac{\beta_j}{\lambda_j} \left\{ \left[\frac{b}{\underline{b}}\right]^{\lambda_j} - 1 \right\}.
$$

Let k^- and k^+ denote the capacities of the smaller and larger firm, respectively, and define β^-, β^+, λ^-, λ^+, F^- and F^+ correspondingly. Then $\lambda^- \geq \lambda^+$ and $\frac{\beta^-}{\lambda^-} \geq \frac{\beta^+}{\lambda^+}$ and therefore

$$\begin{aligned}
\lim_{b \uparrow P} F_j^{u-}(b) &= \frac{\beta^-}{\lambda^-}\left\{\left[\frac{P}{\underline{b}^u}\right]^{\lambda^-} - 1\right\} \\
&\geq \frac{\beta^+}{\lambda^+}\left\{\left[\frac{P}{\underline{b}^u}\right]^{\lambda^+} - 1\right\} \\
&= \lim_{b \uparrow P} F^{u+}(b).
\end{aligned}$$

Since at most one player can bid P with positive probability, it follows that we must have $\lim_{b\uparrow P} F_j^{u+}(b) \leq \lim_{b\uparrow P} F_j^{u-}(b) = 1$. Then it is straightforward to verify that $\underline{b}^u$ is given uniquely by

$$\underline{b}^u = P\left[\frac{\beta^-}{\lambda^- + \beta^-}\right]^{\frac{1}{\lambda^-}} = P\left[\frac{k^+}{2 - 2k^- - k^+}\right]^{\frac{k^-}{1-k^--k^+}}.$$

Substituting for $\underline{b}^u$, we find

$$\begin{aligned}
F_j^{u-}(b) &= \frac{\beta^-}{\lambda^-}\left\{\frac{\lambda^- + \beta^-}{\beta^-}\left[\frac{b}{P}\right]^{\lambda^-} - 1\right\} \\
&= \frac{1}{2}\frac{k^+}{1-k^--k^+}\left\{\frac{2-k^+-2k^-}{k^+}\left[\frac{b}{P}\right]^{\frac{1-k^--k^+}{k^-}} - 1\right\},
\end{aligned}$$

while $F_j^{u+}(P) = 1$ and, for $b \in [\underline{b}, P)$,

$$\begin{aligned}
F_j^{u+}(b) &= \frac{\beta^+}{\lambda^+}\left\{\left[\frac{\lambda^- + \beta^-}{\beta^-}\right]^{\frac{\lambda^+}{\lambda^-}}\left[\frac{b}{P}\right]^{\lambda^+} - 1\right\} \\
&= \frac{1}{2}\frac{k^-}{1-k^--k^+}\left\{\left[\frac{2-k^+-2k^-}{k^+}\right]^{\frac{k^-}{k^+}}\left[\frac{b}{P}\right]^{\frac{1-k^--k^+}{k^+}} - 1\right\}.
\end{aligned}$$

Equilibrium profits become

$$\begin{aligned}
\pi_i^{u-} &= P\left[\Pr(b_j < P)\int_{k_j}^1 \min\{\theta - k_j, k_i\}\,d\theta + \Pr(b_j = P)\int_0^1 \min\{\theta, k_i\}\,d\theta\right] - ck_i \\
\pi_i^{u+} &= P\int_{k_j}^1 \min\{k_i, \theta - k_j\}\,d\theta - ck_i
\end{aligned}$$

where

$$\Pr(b_j < P) = \lim_{b\uparrow P} F_j^{u+}(b).$$

Profits can also be expressed in terms of k^- and k^+as follows:

$$\begin{aligned}
\pi_i^{u-} &= Pk^-\left[1 - \frac{1}{2}k^- - \frac{1}{2}\frac{k^-k^+}{1-k^--k^+}\left\{\left[\frac{2-2k^--k^+}{k^+}\right]^{\frac{k^-}{k^+}} - 1\right\}\right] - ck^- \\
\pi_i^{u+} &= Pk^+\left[1 - k^- - \frac{1}{2}k^+\right] - ck^+
\end{aligned}$$

We find

$$
\begin{aligned}
\frac{\partial \pi_i^{u-}}{\partial k_i} &= P\left[1-k^- - \frac{1}{2}\frac{k^-k^+\left[2-k^--2k^+\right]}{\left[1-k^--k^+\right]^2}\left(\left[\frac{2-2k^--k^+}{k^+}\right]^{\frac{k^-}{k^+}}-1\right)\right. \qquad (23)\\
&\quad \left. -\frac{1}{2}\frac{\left[k^-\right]^2}{1-k^--k^+}\left[\frac{2-2k^--k^+}{k^+}\right]^{\frac{k^-}{k^+}}\left[\ln\left(\frac{2-2k^--k^+}{k^+}\right)-\frac{2k^-}{2-2k^--k^+}\right]\right]-c \\
\frac{\partial \pi_i^{u+}}{\partial k_i} &= P\left[1-k^--k^+\right]-c
\end{aligned}
$$

The latter expression implies that at equilibrium aggregate capacity is the same as with short-lived bids. Furthermore,

$$
\lim_{k_i \downarrow k}\frac{\partial \pi_i^{u+}(k_i,k)}{\partial k_i}-\lim_{k_i \uparrow k}\frac{\partial \pi_i^{u-}(k_i,k)}{\partial k_i}=Pk\left[1+\frac{1}{2}\frac{2-3k}{1-2k}\ln\left(\frac{2-3k}{k}\right)\right]>0,
$$

when $0<2k<1$, which rules out existence of symmetric equilibria.

Given the complexity of $\frac{\partial \pi_i^{u-}}{\partial k^-}$, we have not been able to derive closed-form solutions for equilibrium capacities. However, the problem may be solved by numerical methods. From the condition $\frac{\partial \pi^{u+}}{\partial k^{u+}}=0$, we have $k^+=1-\frac{c}{P}-k^-$. We define $g\left(k^-\right)=\frac{1}{P}\frac{\partial \pi^{u-}\left(k^-,1-\frac{c}{P}-k^-\right)}{\partial k^-}$. From (23) we have that

$$
\begin{aligned}
g\left(k^-\right) &= 1-k^--\frac{1}{2}\frac{k^-\left[1-\frac{c}{P}-k^-\right]\left[2\frac{c}{P}+k^-\right]}{\left[\frac{c}{P}\right]^2}\left\{\left[\frac{1+\frac{c}{P}-k^-}{1-\frac{c}{P}-k^-}\right]^{\frac{k^-}{1-\frac{c}{P}-k^-}}-1\right\} \qquad (24)\\
&\quad -\frac{1}{2}\frac{\left[k^-\right]^2}{\frac{c}{P}}\left[\frac{1+\frac{c}{P}-k^-}{1-\frac{c}{P}-k^-}\right]^{\frac{k^-}{1-\frac{c}{P}-k^-}}\left[\ln\left(\frac{1+\frac{c}{P}-k^-}{1-\frac{c}{P}-k^-}\right)-\frac{2k^-}{1+\frac{c}{P}-k^-}\right]-\frac{c}{P}.
\end{aligned}
$$

A necessary (albeit not sufficient) condition for an equilibrium to exist is that g is downward-sloping and crosses the horizontal axis for some $0\le k^-\le\frac{1}{2}$. Figure 10, which shows plots of g for different values of $\frac{c}{P}$ (lines corresponding to higher values of $\frac{c}{P}$ lie below those corresponding to lower values), demonstrates that this is indeed the case.

Discriminatory Auction

Firm i's profit, when pricing at b, may be written

$$
\pi_i^d(b)=b\left\{F_j^d(b)\int_{k_j}^1\min\left[\theta-k_j,k_i\right]d\theta+\left[1-F_j^d(b)\right]\int_0^1\min\left[\theta,k_i\right]d\theta\right\}-ck_i.
$$

A necessary condition for firm i to be indifferent between any price in the support S^d is that, for all $b\in S^d$, $\pi_i^d(b)=\overline{\pi}_i$, implying

$$
F_j^d(b)=\frac{\frac{\overline{\pi}_i+ck_i}{b}-\int_0^1\min\left[\theta,k_i\right]d\theta}{\int_{k_j}^1\min\left[\theta-k_j,k_i\right]d\theta-\int_0^1\min\left[\theta,k_i\right]d\theta}.
$$

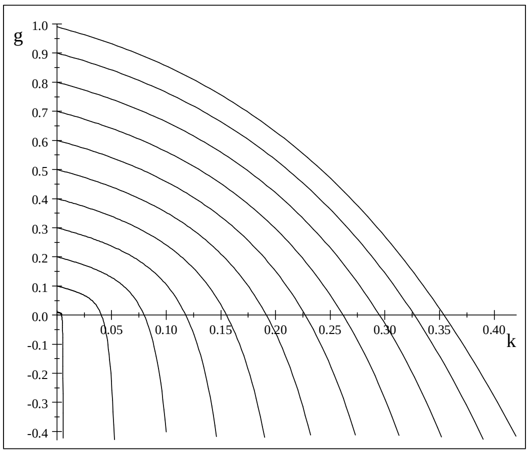

Figure 10: $g(k)$ for $\frac{c}{P} \in \{.01, .1, .2, .3, .4, .5, .6., .7, .8, .9, .99\}$

The boundary condition $F_j^d(\underline{b}^d) = 0$ implies

$$\overline{\pi}_i + ck_i = \underline{b}^d \int_0^1 \min[\theta, k_i]\, d\theta,$$

and so

$$\begin{aligned} F_j^d(b) &= \frac{\int_0^1 \min[\theta, k_i]\, d\theta}{\int_0^1 \min[\theta, k_i]\, d\theta - \int_{k_j}^1 \min[\theta - k_j, k_i]\, d\theta} \frac{b - \underline{b}^d}{b} \\ &= \frac{1}{2} \frac{2 - k_i}{k_j} \frac{b - \underline{b}^d}{b}. \end{aligned}$$

Let k^- and k^+ denote the capacities of the smaller and larger firm, respectively and define F^{d-} and F^{d+} correspondingly. Then $F^{d-}(b) \geq F^{d+}(b)$. It follows that we cannot have $\lim_{b \uparrow P} F^{d+}(b) = 1$, since this would imply $\lim_{b \uparrow P} F^{d-}(b) \geq 1$, with strict inequality for $k^- < k^+$. Consequently, we have the boundary condition $\lim_{b \uparrow P} F^{d-}(b) = 1$, from which it follows that

$$\underline{b}^d = P \frac{2 - 2k^- - k^+}{2 - k^+}.$$

Equilibrium profits become

$$\begin{aligned} \pi^{d-} &= \frac{1}{2} P k^- \frac{2 - k^-}{2 - k^+} \left[2 - 2k^- - k^+\right] - ck^- \\ \pi^{d+} &= \frac{1}{2} P k^+ \left[2 - 2k^- - k^+\right] - ck^+ \end{aligned}$$

It follows that

$$\begin{aligned} \frac{\partial \pi^{d-}}{\partial k^-} &= P \frac{\left[1 - k^-\right]\left[2 - 2k^- - k^+\right] - k^- \left[2 - k^-\right]}{2 - k^+} - c \qquad (25) \\ \frac{\partial \pi^{d+}}{\partial k^+} &= P\left[1 - k^- - k^+\right] - c \end{aligned}$$

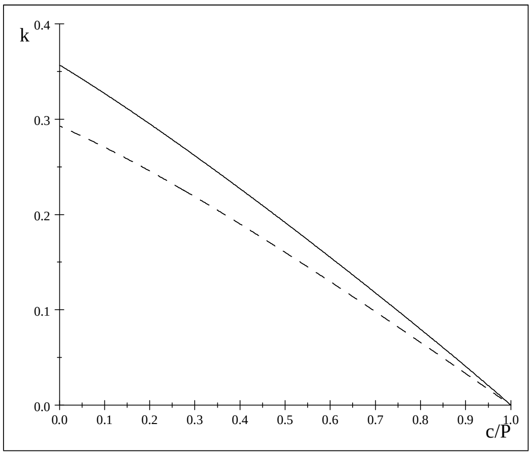

Figure 11: The small firm's capacity in the uniform-price auction (solid line) and in the discriminatory auction (dashed line) with long-lived bids

Note that

$$\lim_{k_i \downarrow k} \frac{\partial \pi_i^{d+}(k_i,k)}{\partial k_i} - \lim_{k_i \uparrow k} \frac{\partial \pi_i^{d-}(k_i,k)}{\partial k_i} = 2Pk\frac{1-k}{2-k} > 0,$$

which rules out symmetric solutions. Now since best replies are everywhere decreasing functions if an equilibrium with $k_i < k_j$ exists, it will be unique. Indeed, such an equilibrium exists and closed-form solutions for equilibrium capacities are given by,

$$k^{d-} = \frac{1}{2}\left[2+\frac{c}{P}-\sqrt{2+4\frac{c}{P}+3\left[\frac{c}{P}\right]^2}\right] \tag{26}$$

$$k^{d+} = \frac{1}{2}\left[-3\frac{c}{P}+\sqrt{2+4\frac{c}{P}+3\left[\frac{c}{P}\right]^2}\right] \tag{27}$$

Proof of Proposition 10

The proof above shows that equilibrium aggregate capacity is the same under both auction formats, and that it equals $1-\frac{c}{P}$ in both cases. We next want to compare equilibrium capacities and profits (and hence expected price) between the two auction formats. Figure 11, which plots k^{u-} (solid line) and k^{d-} (dashed line), demonstrates that $k^{d-} < k^{u-}$.

We next want to compare equilibrium profits under the two formats. Total profits in the discriminatory auction are

$$\Pi^d\left(k^-,k^+\right) = \pi^{d-}+\pi^{d+} = \frac{1}{2}P\left[2-2k^- - k^+\right]\frac{\left[2-k^-\right]k^- + \left[2-k^+\right]k^+}{2-k^+} - c\left[k^- + k^+\right]$$

Substituting for k^- and k^+ from (26) and (27) above, we find equilibrium profits in reduced

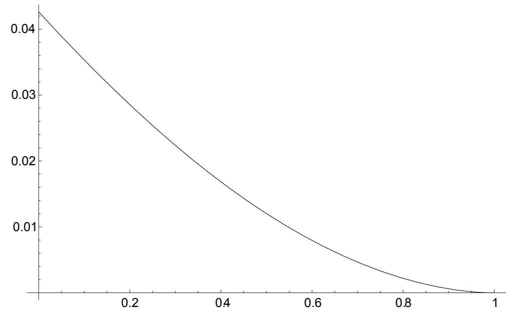

Figure 12: The difference between profits in the uniform-auction and the discriminatory auction under long-lived bids

form:

$$\Pi^d = P\left\{\frac{1}{4}\left[\frac{c}{P}+\sqrt{2+4\frac{c}{P}+3\left[\frac{c}{P}\right]^2}\right]\frac{\left[1+2\frac{c}{P}\right]\sqrt{2+4\frac{c}{P}+3\left[\frac{c}{P}\right]^2}-\frac{c}{P}\left[5+4\frac{c}{P}\right]}{2+\frac{1}{2}\left[3\frac{c}{P}-\sqrt{2+4\frac{c}{P}+3\left[\frac{c}{P}\right]^2}\right]^2} -\frac{c}{P}\left[1-\frac{c}{P}\right]\right\} \tag{28}$$

Total profits in the uniform-price auction are

$$\Pi^u = P\left[\left[k^-+k^+\right]\left\{1-\frac{1}{2}\left[k^-+k^+\right]\right\}-\frac{1}{2}\frac{\left[k^-\right]^2k^+}{1-k^--k^+}\left\{\left[\frac{2-2k^--k^+}{k^+}\right]^{\frac{k^-}{k^+}}-1\right\}\right] -c\left[k^-+k^+\right]$$

Using the fact that, at equilibrium, $k^- + k^+ = \frac{P-c}{c}$ and $k^+ = 1 - \frac{c}{P} - k^-$, we may write total profits as a function of k^- alone:

$$\Pi^u\left(k^-\right) = P\left[\frac{1}{2}\left\{1-\left[\frac{c}{P}\right]^2\right\} -\frac{1}{2}\frac{\left[k^-\right]^2\left[1-\frac{c}{P}-k^-\right]}{\frac{c}{P}}\left\{\left[\frac{1+\frac{c}{P}-k^-}{1-\frac{c}{P}-k^-}\right]^{\frac{k^-}{1-\frac{c}{P}-k^-}}-1\right\}-\frac{c}{P}\left[1-\frac{c}{P}\right]\right] \tag{29}$$

Figure 12, which shows the difference between profits under the uniform-price and the discriminatory format, demonstrates that profits - and hence prices - are higher with the uniform-price format.

Proof of Proposition 11

We next turn to a comparison of equilibrium capacity choices and profits when bids are made, respectively, before and after demand is realised. We start from the observation that aggregate capacities are the same in both cases.

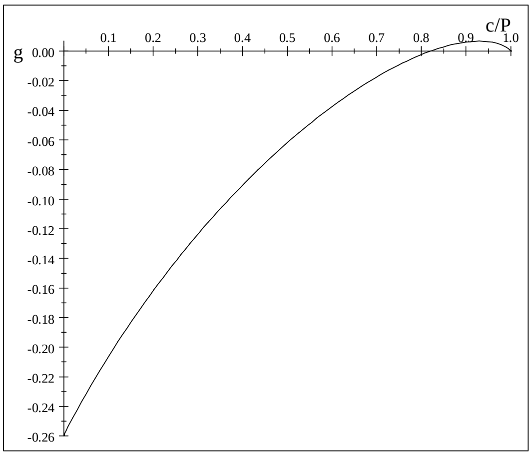

Figure 13: The function g evaluated at $\frac{2}{5}\left[1-\frac{c}{P}\right]$

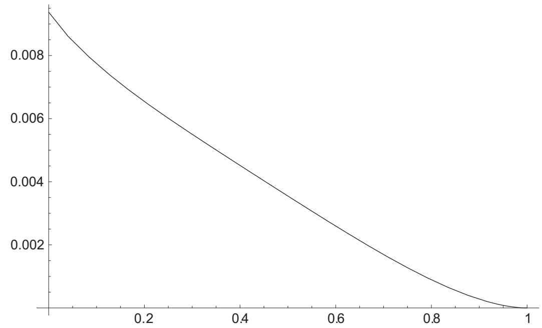

Figure 14: The difference between profits with long-lived and short-lived bids in the uniform-price auction

In the uniform-price auction, we rely on numerical methods. With short-lived bids, equilibrium capacity of the small firm is given by $\frac{2}{5}\left[1-\frac{c}{P}\right]$, whereas with long-lived bids the corresponding capacity is given implicitly by the equation $g(k)=0$, where g is defined in (24) above. Figure 13, which provides a plot of $g\left(\frac{2}{5}\left[1-\frac{c}{P}\right]\right)$, shows that $g\left(\frac{2}{5}\left[1-\frac{c}{P}\right]\right)$ is negative if and only if $\frac{c}{P}\leq 0.82078$. This - together with the fact that g is decreasing in k - implies that the equilibrium capacity choice of the small firm is smaller with long-lived bids than the corresponding choice with short-lived bids when $\frac{c}{P}\leq 0.82078$, and *vice versa*.

Aggregate equilibrium profits with short-lived bids are given by $\frac{21}{50}P\left[1-\frac{c}{P}\right]^2$, whereas with long-lived bids, equilibrium profits are given by (29). Figure 14, which shows the difference between profits with long-lived and short-lived bids, demonstrates that profits - and hence average prices - are higher with long-lived bids.

For the discriminatory format, we can compare closed-form solutions for equilibrium capacity choices. With short-lived bids, equilibrium capacity of the small firm is given by $\frac{1}{1+\alpha}\left[1-\frac{c}{P}\right]$,

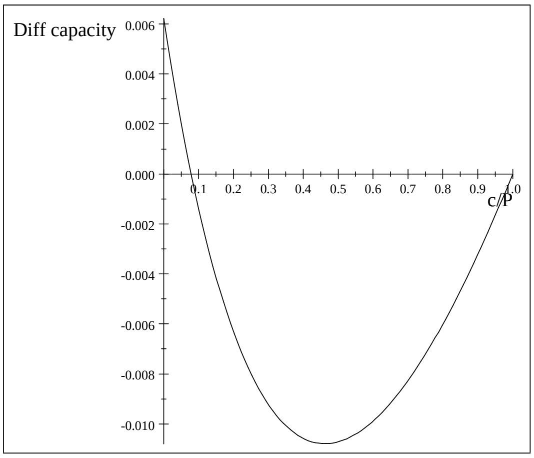

Figure 15: The difference between the capacity of the small firm with short-lived and long-lived bids in the discriminatory auction

where α is the solution to the equation $\alpha^2 - 2\alpha\ln(\alpha) = \frac{3}{2}$, or $\alpha \approx 2.343164$. With long-lived bids, equilibrium capacity of the small firm is given by $\frac{1}{2}\left[2 + \frac{c}{P} - \sqrt{2 + 4\frac{c}{P} + 3\left[\frac{c}{P}\right]^2}\right]$. Figure 15 plots the difference between the capacity of the small firm with short-lived and long-lived bids as a function of $\frac{c}{P}$. As can be seen, small-firm capacity with short-lived bids exceeds that with long-lived bids when $\frac{c}{P} < 0.07866$, and *vice versa*.

Aggregate profits are given by (21), or approximately $0.359\,987P\left[1 - \frac{c}{P}\right]^2$, when bids are short-lived, whereas in the case of long-lived bids, aggregate profits are given by (28). Figure 16, which shows a plot of $\frac{1}{P}\left[\Pi^{ds} - \Pi^{dl}\right]$, demonstrates that profits - and hence prices - are always lower with short-lived bids.

D Price-Responsive Demand

This section provides details on the equilibrium characterisation when demand is price-responsive.

Uniform-Price Auction

Profits for the small and the large firm are, respectively,

$$\pi_i^{u-} = P\left[\int_{\frac{k^-}{D(p)}}^{\frac{k^+}{D(p)}} k^- d\theta + \int_{\frac{k^+}{D(p)}}^{\frac{K}{D(p)}} \left\{\tfrac{1}{2}k^- + \tfrac{1}{2}\left[\theta D(p) - k^+\right]\right\} d\theta + \int_{\frac{K}{D(p)}}^{1} k^- d\theta\right] - ck^-$$

$$\pi_i^{u+} = P\left[\int_{\frac{k^-}{D(p)}}^{\frac{k^+}{D(p)}} \left[\theta D(p) - k^-\right] d\theta + \int_{\frac{k^+}{D(p)}}^{\frac{K}{D(p)}} \left\{\tfrac{1}{2}k^+ + \tfrac{1}{2}\left[\theta D(p) - k^-\right]\right\} d\theta + \int_{\frac{K}{D(p)}}^{1} k^+ d\theta\right] - ck^+$$

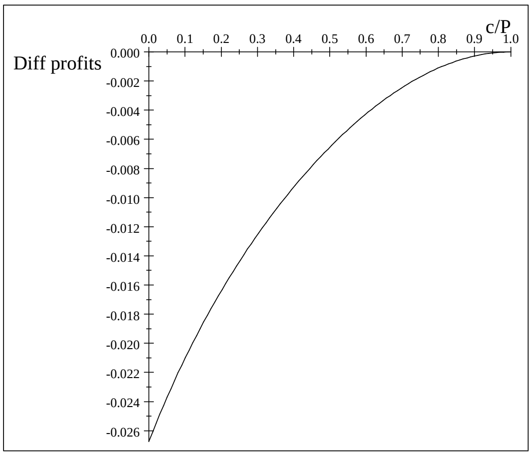

Figure 16: $\frac{1}{P}\left[\Pi^{ds} - \Pi^{dl}\right]$

The break-even constraint that determines retail prices is

$$p\left[\int_{0}^{\frac{K}{D(p)}} \theta D(p)\, d\theta + \int_{\frac{K}{D(p)}}^{1} K d\theta\right] = P\left[\int_{\frac{k^-}{D(p)}}^{\frac{K}{D(p)}} \theta D(p)\, d\theta + \int_{\frac{K}{D(p)}}^{1} K d\theta\right].$$

This may alternatively be written

$$[P-p]\,K\left[D(p) - \frac{1}{2}K\right] = \frac{1}{2}P\left[k^-\right]^2,$$

from which it follows that

$$\begin{aligned}
\frac{dp}{dk^-} &= \frac{[P-p]\,[D(p)-K] - Pk^-}{K\left[D(p)-\frac{1}{2}K\right] - [P-p]\,KD'(p)} \\
\frac{dp}{dk^+} &= \frac{[P-p]\,[D(p)-K]}{K\left[D(p)-\frac{1}{2}K\right] - [P-p]\,KD'(p)}
\end{aligned}$$

In the relevant range, $p < P$ and $D(p) > K$, and so $\frac{dp}{dk^+} > 0$. Also, $\frac{dp}{dk^-} > 0$ if k^- is sufficiently small.

Setting $D(p) = 1 - \gamma p$, the above profit expressions may be re-written as:

$$\begin{aligned}
\pi_i^{u-} &= [P-c]\,k^- - \frac{P}{1-\gamma p}\frac{5}{4}\left[k^-\right]^2 \\
\pi_i^{u+} &= [P-c]\,k^+ - \frac{P}{1-\gamma p}\left\{\frac{1}{2}\left[k^+\right]^2 + k^-k^+ - \frac{1}{4}\left[k^-\right]^2\right\}
\end{aligned}$$

The first-order derivatives of the profit functions are,

$$\begin{aligned}
\frac{\partial \pi_i^{u-}}{\partial k_i} &= P - c - \frac{5k^-}{4}\frac{P}{1-\gamma p}\left[2 + \frac{\gamma k^-}{1-p\gamma}\frac{\partial p}{\partial k^-}\right] \\
\frac{\partial \pi_i^{u+}}{\partial k_i} &= P - c - \frac{P}{1-\gamma p}\left\{K + \frac{\gamma}{1-\gamma p}\left[\frac{1}{2}\left[k^+\right]^2 + k^-k^+ - \frac{1}{4}\left[k^-\right]^2\right]\frac{\partial p}{\partial k^+}\right\}
\end{aligned}$$

With $D(p) = 1 - \gamma p$, the break-even constraint for the retail price may be written as:

$$\gamma p^2 + \left[\frac{K}{2} - 1 - P\gamma\right] p + P\left[1 - \frac{[k^-]^2}{2K} - \frac{K}{2}\right] = 0$$

which has the solutions

$$\begin{aligned} p^- &= \frac{1}{2\gamma}\left[-X - \sqrt{X^2 - 4\gamma Y}\right] \\ p^+ &= \frac{1}{2\gamma}\left[-X + \sqrt{X^2 - 4\gamma Y}\right] \end{aligned}$$

where

$$X = \frac{K}{2} - 1 - P\gamma < 0 \text{ and } Y = P\left[1 - \frac{[k^-]^2}{2K} - \frac{K}{2}\right] > 0$$

We can show that p^+ is inadmissible. Note first that $X^2 - 4\gamma Y = \left[1 - \frac{K}{2} - \gamma P\right]^2 + 2\gamma P\frac{[k^-]^2}{K} > \left[1 - \frac{K}{2} - \gamma P\right]^2$. Furthermore, if $1-\gamma P - \frac{K}{2} < 0$, $\frac{1}{2\gamma}\left[-X + \sqrt{\left[1 - \frac{K}{2} - \gamma P\right]^2}\right] = P$, while if $1 - \gamma P - \frac{K}{2} > 0$, $\frac{1}{2\gamma}\left[-X + \sqrt{\left[1 - \frac{K}{2} - \gamma P\right]^2}\right] = \frac{1}{\gamma}\left[1 - \frac{K}{2}\right] > P$. Thus, it follows that $p^+ > \frac{1}{2\gamma}\left[-X + \sqrt{\left[1 - \gamma P - \frac{K}{2}\right]^2}\right] \geq P$ (with strict inequality if $\frac{1}{2}K < 1 - \gamma P$), which cannot be the case (at least at equilibrium). Consequently, the only admissible solution is p^-.

Since $\frac{dX}{dk^-} = \frac{dX}{dk^+} = \frac{1}{2}$, the relationships between price and capacities become

$$\begin{aligned} \frac{dp}{dk^-} &= -\frac{1}{4\gamma}\left[1 + \frac{X - 4\gamma\frac{dY}{dk^-}}{\sqrt{X^2 - 4\gamma Y}}\right] \\ \frac{dp}{dk^+} &= -\frac{1}{4\gamma}\left[1 + \frac{X - 4\gamma\frac{dY}{dk^+}}{\sqrt{X^2 - 4\gamma Y}}\right] \end{aligned}$$

where

$$\begin{aligned} \frac{dY}{dk^-} &= -\frac{P}{2}\left\{1 - \left[\frac{k^-}{K}\right]^2 + 2\frac{k^-}{K}\right\} < 0 \\ \frac{dY}{dk^+} &= -\frac{P}{2}\left\{1 - \left[\frac{k^-}{K}\right]^2\right\} < 0 \end{aligned}$$

Discriminatory Auction

In the discriminatory auction, the corresponding profit expressions are

$$\pi_i^{d-} = P\left\{\int_{\frac{k^-}{D(p)}}^{\frac{k^+}{D(p)}} \frac{k^-}{\theta D(p)}\left[\theta D(p) - k^-\right] d\theta + \int_{\frac{k^+}{D(p)}}^{\frac{K}{D(p)}} \frac{k^-}{k^+}\left[\theta D(p) - k^-\right] d\theta + \int_{\frac{K}{D(p)}}^{1} k^- d\theta\right\} - ck^-$$

$$\pi_i^{d+} = P\left\{\int_{\frac{k^-}{D(p)}}^{\frac{K}{D(p)}} \left[\theta D(p) - k^-\right] d\theta + \int_{\frac{K}{D(p)}}^{1} k^+ d\theta\right\} - ck^+$$

The above equations may be written

$$\pi_i^{d-} = [P-c]\,k^- - \tfrac{P}{1-p\gamma}\left\{\tfrac{[k^-]^3}{2k^+} + [k^-]^2 + [k^-]^2 \ln\tfrac{k^+}{k^-}\right\}$$
$$\pi_i^{d+} = [P-c]\,k^+ - \tfrac{P}{1-p\gamma}\tfrac{[2k^-+k^+]k^+}{2}$$

Taking derivatives:

$$\tfrac{\partial\pi_i^{d-}}{\partial k_i} = P-c-\tfrac{P}{1-p\gamma}\left\{\tfrac{3[k^-]^2}{2k^+} + k^- + 2k^-\ln\tfrac{k^+}{k^-} + \tfrac{\gamma}{1-p\gamma}\tfrac{\partial p}{\partial k^-}[k^-]^2\left[1+\tfrac{k^-}{2k^+}+\ln\tfrac{k^+}{k^-}\right]\right\}$$
$$\tfrac{\partial\pi_i^{d+}}{\partial k_i} = P-c-\tfrac{P}{1-p\gamma}\left[K+\tfrac{\gamma}{1-p\gamma}\tfrac{\partial p}{\partial k^+}\tfrac{2k^-+k^+}{2}k^+\right]$$

The break-even constraint may be written

$$\begin{aligned} p\left[\int_0^{\frac{K}{D(p)}}\theta D(p)\,d\theta + \int_{\frac{K}{D(p)}}^1 K d\theta\right] &= P\int_{\frac{k^-}{D(p)}}^{\frac{k^+}{D(p)}}\left[\theta D(p)-k^-\right]\left[1+\frac{k^-}{\theta D(p)}\right]d\theta \\ &+P\left\{\int_{\frac{k^+}{D(p)}}^{\frac{K}{D(p)}}\left[\theta D(p)-k^-\right]\left[1+\frac{k^-}{k^+}\right]d\theta + \int_{\frac{K}{D(p)}}^1 K d\theta\right\}\end{aligned}$$

Under the linear demand specification, it becomes:

$$-2\gamma p^2 + [2P\gamma+2-K]\,p + P\left\{K+\frac{[k^-]^2}{k^+}-2+\frac{2\,[k^-]^2}{K}\ln\frac{k^+}{k^-}\right\} = 0 \tag{30}$$

which has the solutions

$$\begin{aligned} p^- &= \frac{1}{4\gamma}\left[X-\sqrt{X^2+8\gamma Y}\right] \\ p^+ &= \frac{1}{4\gamma}\left[X+\sqrt{X^2+8\gamma Y}\right]\end{aligned}$$

where

$$X = 2P\gamma+2-K>0 \text{ and } Y = P\left\{K+\frac{[k^-]^2}{k^+}-2+2\frac{[k^-]^2}{K}\ln\frac{k^+}{k^-}\right\}.$$

Again, as with the uniform-price auction, we can show that $p^+ > \frac{1}{2\gamma}\left[-X+\sqrt{[2\gamma P-2+K]^2}\right] \geq P$, and so the only admissible solution is p^-. Note that $Y<0$ (it is increasing in k^- and it attains a negative value at $k^- = k^+$) which guarantees that $p^- > 0$.

We find

$$\begin{aligned} \frac{dp}{dk^-} &= -\frac{1}{4\gamma}\left[1-\frac{X-4\gamma\frac{dY}{dk^-}}{\sqrt{X^2+8Y\gamma}}\right] = \frac{1}{\sqrt{X^2+8\gamma Y}}\left[p-\frac{dY}{dk^-}\right] < 0 \\ \frac{dp}{dk^+} &= -\frac{1}{4\gamma}\left[1-\frac{X-4\gamma\frac{dY}{dk^+}}{\sqrt{X^2+8Y\gamma}}\right] = \frac{1}{2\sqrt{X^2+8\gamma Y}}\left[p-\frac{dY}{dk^+}\right]\end{aligned}$$

where

$$\frac{dY}{dk^-} = P\left\{1+2\frac{[k^-]^2}{Kk^+}+2k^-\frac{k^-+2k^+}{K^2}\ln\frac{k^+}{k^-}\right\} > P$$
$$\frac{dY}{dk^+} = P\left\{1-\left[\frac{k^-}{k^+}\right]^2+2\frac{k^+}{K}\left[\frac{k^-}{k^+}\right]^2-2\left[\frac{k^-}{K}\right]^2\ln\frac{k^+}{k^-}\right\}$$

The sign of $-p+\frac{\partial Y}{\partial k^+}$ depends on the values of k^- and k^+; a sufficient condition for $\frac{dY}{dk^+}>P$ is $k^+<ek^-$, in which case $\frac{dp}{dk^+}<0$ also.

E Distribution of Demand

The analysis of existence of equilibrium and comparison across auction formats with a general demand distribution function rely on the properties of firms' profit functions. Therefore, we first discuss these properties. We next analyse conditions that must be satisfied for the existence of equilibrium, before we end the section by comparing outcomes across auction formats.

Expected Profits: Uniform-Price Auction

Suppose demand is distributed according to the function G on $[0,1]$ and that the density function G' is positive everywhere on $(0,1)$. Assuming that, when multiple equilibria exist at the price-competition stage, each is played with equal probability, expected profits for firm i, $i=1,2$, $i\neq j$, are given as

$$\pi_i^u(k_i,k_j)=\begin{cases}\pi_i^{u-} & \text{if} \quad k_i\leq k_j\\ \pi_i^{u+} & \text{if} \quad k_i\geq k_j\end{cases}$$

where

$$\pi_i^{u-}=P\left[\int_{k^-}^{k^+}k^-dG(\theta)+\int_{k^+}^{K}\left\{\tfrac{1}{2}k^-+\tfrac{1}{2}[\theta-k^+]\right\}dG(\theta)+\int_{K}^{1}k^-dG(\theta)\right]-ck^-$$
$$\pi_i^{u+}=P\left[\int_{k^-}^{k^+}[\theta-k^-]\,dG(\theta)+\int_{k^+}^{K}\left\{\tfrac{1}{2}k^++\tfrac{1}{2}[\theta-k^-]\right\}dG(\theta)+\int_{K}^{1}k^+dG(\theta)\right]-ck^+$$

The first-order derivatives are

$$\frac{\partial\pi_i^{u-}}{\partial k_i} = P\left[1-G(k_i)-\frac{1}{2}[G(k_i+k_j)-G(k_j)]-G'(k_i)\,k_i\right]-c \tag{31}$$
$$\frac{\partial\pi_i^{u+}}{\partial k_i} = P\left[1-G(k_i+k_j)+\frac{1}{2}\left[G(k_i+k_j)-G(k_i)-G'(k_i)\,k_j\right]\right]-c \tag{32}$$

while the second-order derivatives are

$$\frac{\partial^2 \pi_i^{u-}}{\partial k_i^2} = -P\left[2G'(k_i) + \frac{1}{2}G'(k_i+k_j) + G''(k_i)\,k_i\right] \tag{33}$$

$$\frac{\partial^2 \pi_i^{u+}}{\partial k_i^2} = -\frac{P}{2}\left[G'(k_i+k_j) + G'(k_i) + G''(k_i)\,k_j\right] \tag{34}$$

$$\frac{\partial^2 \pi_i^{u-}}{\partial k_i \partial k_j} = -\frac{P}{2}\left[G'(k_i+k_j) - G'(k_j)\right]$$

$$\frac{\partial^2 \pi_i^{u+}}{\partial k_i \partial k_j} = -\frac{P}{2}\left[G'(k_i+k_j) + G'(k_i)\right]$$

A sufficient (but not necessary) condition for the second-order derivatives to be negative is that G is convex. If G is concave, the direct second-order derivatives will be negative if the density function G' is concave also (because then $G'(k_i)+G''(k_i)\,k_j > G'(k_i+k_j)$). The cross derivative for the small firm's profit function is positive (negative) if G is concave (convex); it is always negative for the large firm.

Expected Profits: Discriminatory Auction

With the discriminatory format, the corresponding profits are

$$\pi_i^d(k_i,k_j) = \begin{cases} \pi_i^{d-} & \text{if} \quad k_i \leq k_j \\ \pi_i^{d+} & \text{if} \quad k_i \geq k_j \end{cases}$$

where

$$\pi^{d-} = P\left[\int_{k^-}^{k^+} \frac{k^-}{\theta}\left[\theta - k^-\right] dG(\theta) + \int_{k^+}^{K} \frac{k^-}{k^+}\left[\theta - k^-\right] dG(\theta) + \int_{K}^{1} k^- dG(\theta)\right] - ck^-$$

$$\pi^{d+} = P\left[\int_{k^-}^{k^+} \left[\theta - k^-\right] dG(\theta) + \int_{k^+}^{K} \left[\theta - k^-\right] dG(\theta) + \int_{K}^{1} k^+ dG(\theta)\right] - ck^+$$

The first-order derivatives are

$$\frac{\partial \pi_i^{d-}}{\partial k_i} = P\left[\int_{k_i}^{k_i+k_j} \left[\frac{\theta - 2k_i}{\min\{\theta, k_j\}}\right] G'(\theta)\,d\theta + 1 - G(k_i+k_j)\right] - c \tag{35}$$

$$\frac{\partial \pi_i^{d+}}{\partial k_i} = P\left[1 - G(k_i+k_j)\right] - c \tag{36}$$

while the second-order derivatives are

$$\begin{aligned}
\frac{\partial^2 \pi_i^{d-}}{\partial k_i^2} &= -P\left[2\int_{k_i}^{k_j}\frac{dG(\theta)}{\theta} + 2\int_{k_j}^{k_i+k_j}\frac{dG(\theta)}{k_j} + G'(k_i+k_j)\frac{k_i}{k_j} - G'(k_i)\right] \qquad (37)\\
\frac{\partial^2 \pi_i^{d+}}{\partial k_i^2} &= -PG'(k_i+k_j)\\
\frac{\partial^2 \pi_i^{d-}}{\partial k_i \partial k_j} &= -\frac{P}{k_j}\left[\int_{k_j}^{k_i+k_j}\frac{\theta-2k_i}{k_j}G'(\theta)\,d\theta + G'(k_i+k_j)\,k_i\right]\\
\frac{\partial^2 \pi_i^{d+}}{\partial k_i \partial k_j} &= -PG'(k_i+k_j)
\end{aligned}$$

For the large firm's profit function, both the direct and cross second-order derivatives are always negative. For the small firm's profit function, the direct second-order derivative is negative when G is convex. If G is concave and the density function G' is convex, the result also holds. To see this note that a sufficient condition for the direct second-order derivative to be negative is

$$2\int_{k^-}^{k^-+k^+}\frac{dG(\theta)}{k^+} + G'\left(k^-+k^+\right)\frac{k^-}{k^+} - G'\left(k^-\right) > 0.$$

In what follows we show that this inequality holds for a convex pdf. To do so we use an auxiliary result whose statement and proof follows. If G' is convex then

$$2\int_{k^-}^{k^-+k^+}\frac{dG(\theta)}{k^+} - G'\left(k^-\right) \geq G'\left(k^-\right) + \left[k^+-k^-\right]G''\left(k^-\right). \qquad (38)$$

Let g be a convex function. Since a convex function is locally Lipschitzian, integration by parts implies

$$\int_x^b [b-t]\,g'(t)dt - \int_a^x [t-a]\,g'(t)dt = \int_a^b g(t)dt - [b-a]\,g(x)$$

Since $g'(t) \geq g'_+(x)$ for all $t \in [x,b]$, if we multiply by $[b-t] \geq 0$, $t \in [x,b]$ and we integrate on $[x,b]$ we get,

$$\int_x^b [b-t]\,g'(t)dt \geq \frac{1}{2}[b-x]^2\,g'_+(x). \qquad (39)$$

Similarly, since $g'(t) \leq g'_-(x)$ for all $t \in [a,x]$, multiplying both sides by $[t-a] \geq 0$, $t \in [a,x]$ and integrating on $[a,x]$ we get,

$$\int_a^x [t-a]\,g'(t)dt \leq \frac{1}{2}[x-a]^2\,g'_-(x). \qquad (40)$$

Extracting (40) from (39), we deduce

$$\int_a^b g(t)dt - [b-a]\,g(x) \geq \frac{1}{2}\left[[b-x]^2\,g'_+(x) - [x-a]^2\,g'_-(x)\right]$$

If x is a point of differentiability for g, then $g'_+(x) = g'_-(x) = g'(x)$ and the inequality above simplifies to

$$\frac{1}{b-a}\int_a^b g(t)dt - g(x) \geq \left[\frac{a+b}{2} - x\right]g'(x)$$

Taking $a = k^-$, $b = k^- + k^+$, $x = k^-$, and $g = G'$, we have

$$\begin{aligned} \int_{k^-}^{k^-+k^+} \frac{dG(\theta)}{k^+} - G'\left(k^-\right) &\geq \left[\frac{k^+ - k^-}{2}\right] G''\left(k^-\right) \text{ and} \\ \int_{k^-}^{k^-+k^+} \frac{dG(\theta)}{k^+} &\geq G'\left(k^-\right) + \left[\frac{k^+ - k^-}{2}\right] G''\left(k^-\right) \end{aligned}$$

Adding up the two inequalities above, the result (38) is derived. Using the derived inequality, the second-order derivative is negative if

$$G'\left(k^-\right) + \left[k^+ - k^-\right] G''\left(k^-\right) + G'\left(k^- + k^+\right) \frac{k^-}{k^+} > 0,$$

which holds trivially as $G'\left(k^-\right) + \left[k^+ - k^-\right] G''\left(k^-\right)$ is the linear approximation (the tangent line $y(x) = G'\left(k^-\right) + (x - k^-)G''(k^-)$) to G'' at argument k^- passing by $x = k^+$, and it hence satisfies

$$G'\left(k^-\right) + \left[k^+ - k^-\right] G''\left(k^-\right) > G'\left(k^- + k^+\right) > 0$$

as G' is convex.

Last, the second-order cross derivative of the small firm's profit function is always negative. This is easy to see for the case $k_j \geq 2k_i$, as then $\int_{k_j}^{k_i+k_j} \frac{[\theta - 2k_i]}{k_j} G'(\theta)\, d\theta > 0$. We therefore concentrate on the case $k_i \leq k_j < 2k_i$.

If G is convex, we find, using integration by parts,

$$\begin{aligned} & \int_{k_j}^{k_i+k_j} \frac{\theta - 2k_i}{k_j} G'(\theta)\ d\theta + G'(k_i + k_j)\, k_i \\ = \quad & \frac{k_j - k_i}{k_j} G(k_i + k_j) + \frac{2k_i - k_j}{k_j} G(k_j) - \int_{k_j}^{k_i+k_j} \frac{G(\theta)}{k_j}\, d\theta + G'(k_i + k_j)\, k_i \\ \geq \quad & \frac{k_j - k_i}{k_j} G(k_i + k_j) + \frac{2k_i - k_j}{k_j} G(k_j) - \frac{k_i}{k_j} G(k_i + k_j) + G'(k_i + k_j)\, k_i \\ = \quad & \frac{k_j - k_i}{k_j} \left[G(k_i + k_j) - G(k_j)\right] + \frac{k_i}{k_j} \left[G(k_i) + G'(k_i + k_j)\, k_j - G(k_i + k_j)\right] \\ \geq \quad & \frac{k_j - k_i}{k_j} \left[G(k_i + k_j) - G(k_j)\right] \geq 0 \end{aligned}$$

The first and last inequalities follow from the fact that G is a non-decreasing function, while the second inequality follows from the assumption that G is convex. Therefore $\frac{\partial^2 \pi_i^{d-}}{\partial k_i \partial k_j} \leq 0$, where the inequality is strict if $k_i > 0$.

When G is concave, we have

$$\begin{aligned} \int_{k_j}^{k_i+k_j} \frac{\theta - 2k_i}{k_j} G'(\theta)\ d\theta + G'(k_i + k_j)\, k_i &\geq \int_{k_j}^{k_i+k_j} \frac{\theta - 2k_i}{k_j} G'(k_i + k_j)\ d\theta + G'(k_i + k_j)\, k_i \\ &= \frac{k_i}{2k_j} \left[4k_j - 3k_i\right] G'(k_i + k_j) > 0, \end{aligned}$$

from which the result follows.

Note that, if G is convex over some intervals and concave over others, we may combine the above arguments to prove that $\frac{\partial^2 \pi_i^{d-}}{\partial k_i \partial k_j} \leq 0$ for any G.

Equilibrium Existence: Uniform-Price Auction

In this and the next subsection we show that, provided second-order conditions are satisfied, (i) there exist exactly two pure-strategy equilibria in capacity choices, one with $k_1 < k_2$ and the other with $k_1 > k_2$, and (ii) aggregate equilibrium capacity in the discriminatory auction is given by $G^{-1}\left(1-\frac{c}{P}\right)$ whereas aggregate capacity in the uniform-price auction is bounded below (above) by $G^{-1}\left(1-\frac{c}{P}\right)$ if G is convex (concave). We start with the uniform-price auction in this section and consider the discriminatory auction in the next.

If the second-order derivatives are negative, the profit function is piecewise concave and continuous everywhere, in particular at $k_i = k_j$. Choose an arbitrary but fixed value for k_j. Then, the payoff functions $\pi_i^{u-}(\cdot, k_j)$, $\pi_i^{u+}(\cdot, k_j)$ are single-peaked on the interval $[0,1]$, with unconstrained maxima at $k_i^-(k_j)$, and $k_i^+(k_j)$. Since, along the diagonal,

$$\lim_{k_i \downarrow k} \frac{\partial \pi_i^{u+}(k_i,k)}{\partial k_i} - \lim_{k_i \uparrow k} \frac{\partial \pi_i^{u-}(k_i,k)}{\partial k_i} = G'(k)\,\frac{k}{2} > 0, \tag{41}$$

$k_i^+(k_j)$ is interior if and only if $\lim_{k_i \downarrow k} \frac{\partial \pi_i^{u+}(k_i,k)}{\partial k_i} \geq 0$, whereas $k_i^-(k_j)$ is interior if and only if $\lim_{k_i \uparrow k} \frac{\partial \pi_i^{u-}(k_i,k)}{\partial k_i} \leq 0$. Thus we only need to distinguish between three cases: (a) If $\lim_{k_i \downarrow k} \frac{\partial \pi_i^{u+}(k_i,k)}{\partial k_i} \geq \lim_{k_i \uparrow k} \frac{\partial \pi_i^{u-}(k_i,k)}{\partial k_i} \geq 0$, the global maximum is $k_i^+(k_j)$; (b) If $\lim_{k_i \uparrow k} \frac{\partial \pi_i^{u-}(k_i,k)}{\partial k_i} \leq \lim_{k_i \downarrow k} \frac{\partial \pi_i^{u+}(k_i,k)}{\partial k_i} < 0$, the global maximum is $k_i^-(k_j)$; (c) If $\lim_{k_i \downarrow k} \frac{\partial \pi_i^{u+}(k_i,k)}{\partial k_i} \geq 0$ and $\lim_{k_i \uparrow k} \frac{\partial \pi_i^{u-}(k_i,k)}{\partial k_i} \leq 0$, both $k_i^-(k_j)$ and $k_i^+(k_j)$ are interior. To determine the global maximum, we compare profits at the two local maxima. To do so, let us first implicitly define k^* and k^{**} as,

$$\begin{aligned}
\lim_{k_i \uparrow k^*} \frac{\partial \pi_i^{u-}(k_i,k^*)}{\partial k_i} &= 0, \text{ and} \\
\lim_{k_i \downarrow k^{**}} \frac{\partial \pi_i^{u+}(k_i,k^{**})}{\partial k_i} &= 0.
\end{aligned}$$

Given that (41) implies $\lim_{k_i \downarrow k^*} \frac{\partial \pi_i^{u+}(k_i,k^*)}{\partial k_i} > 0$, it follows from the concavity of π_i^{u+} that $k^{**} > k^*$. Using the above definitions, note that (41) also implies

$$\begin{aligned}
\pi_i^{u-}\left(k_i^-\left(k_j^*\right), k_j^*\right) - \pi_i^{u+}\left(k_i^+\left(k_j^*\right), k_j^*\right) &< 0 \\
\pi_i^{u-}\left(k_i^-\left(k_j^{**}\right), k_j^{**}\right) - \pi_i^{u+}\left(k_i^+\left(k_j^{**}\right), k_j^{**}\right) &> 0.
\end{aligned} \tag{42}$$

Furthermore, the difference in profits is a strictly increasing function in $k_j \in [k^*, k^{**}]$,

$$\frac{d\pi_i^{u-}\left(k_i^-(k_j), k_j\right)}{dk_j} - \frac{d\pi_i^{u+}\left(k_i^+(k_j), k_j\right)}{dk_j} > 0. \tag{43}$$

It is straightforward to check this result if G is concave, given the signs of the first-order cross derivatives. If G is convex, both derivatives are negative, so we compute the difference explicitly. It is given by

$$\frac{P}{2}\left[G'(k_j)\,k_i^- + G\left(k_i^+ + k_j\right) - G\left(k_i^- + k_j\right) + G\left(k_i^+\right) - G(k_j)\right] > 0,$$

where the inequality follows from the fact that both k_i^+ and k_i^- are interior within this region so that $k_i^- < k_j < k_i^+$ implies $G\left(k_i^+ + k_j\right) > G\left(k_i^- + k_j\right)$ and $G\left(k_i^+\right) > G\left(k_j\right)$. Consequently, (42) and (43) ensure that there exists a unique $\hat{k}_j \in [k^*, k^{**}]$ such that

$$\pi_i^{u-}\left(k_i^-\left(\hat{k}_j\right), \hat{k}_j\right) - \pi_i^{u+}\left(k_i^+\left(\hat{k}_j\right), \hat{k}_j\right) = 0.$$

At $k_j = \hat{k}_j$ both k_i^- and k_i^+ are a best reply. For values $k_j < \hat{k}_j$ the best reply is k_i^+, whereas for $k_j > \hat{k}_j$ the best reply is k_i^-. Note that $k^* < \hat{k}_j < k^{**}$.

In summary, the best-response functions in the uniform-price auction for firm i, $i = 1, 2$, $i \neq j$, are discontinuous at (at most) one point and are given by:

$$k_i^u(k_j) = \begin{cases} k_i^-(k_j) & \text{if} \quad k_j \geq \hat{k}_j \\ k_i^+(k_j) & \text{if} \quad k_j \leq \hat{k}_j, \end{cases}$$

where $k_i^-(k_j)$ is strictly decreasing (increasing) for any convex (concave) demand function and $k_i^+(k_j)$ is always strictly decreasing. Finally note that firms' best-reply functions are equal as their payoff functions are identical.

If G is convex, then the best response by either firm is a decreasing function, so that any crossing of the two best replies must take place outside the discontinuity region. Furthermore, since $k_1^* = k_2^* = k^* < k_1^{**} = k_2^{**} = k^{**}$ they cross twice as $k_1^+(0) = G^{-1}\left(\frac{P-c}{P}\right) < 1$ and $k_2^- > 0$ at $k_1 = 1$, which ensures that two asymmetric equilibria exist.

If G is concave, existence of a candidate equilibrium of the form $\left(k_1^+, k_2^-\right)$ (a solution to the system of FOCs) is trivially guaranteed as $k_2^-(k_1)$ is a strictly increasing function whereas $k_1^+(k_2)$ is strictly decreasing and hence bounded above by $k_1^+(0) = G^{-1}\left(\frac{P-c}{P}\right)$. Hence, all we need to show is that the solution to the system of FOCs satisfies $k^-\left(k^+\right) \leq \hat{k}$. Assume, for contradiction, that $k^-(k^+) > \hat{k}$. If best replies cross in the discontinuity region, then

$$\pi_i^{u+}\left(k^+\left(\hat{k}\right), \hat{k}\right) = \pi_i^{u-}\left(k^-\left(\hat{k}\right), \hat{k}\right) < \pi_i^{u-}\left(k^-\left(\hat{k}\right), k^+\left(\hat{k}\right)\right) < \pi_i^{u-}\left(\hat{k}, k^+\left(\hat{k}\right)\right). \tag{44}$$

The first equality follows for the definition of $\hat{k}$. The second inequality follows from the fact that the first cross derivative of the small firm is positive, which shows that the small firm's profit is increasing in the capacity choice of its rival. Lastly, since best replies cross in the discontinuity region, $\pi_i^-\left(\cdot, k^+\left(\hat{k}\right)\right)$ must attain its maximum at some $k_i \geq \hat{k}$. This implies that the small firm's profit function is increasing in its own capacity for any capacity below $\hat{k}$, which explains the third inequality.

Let us use the following notation. For an arbitrary pair (k^-, k^+), the difference in the profits made by the large and the small firm is given by $\Delta\pi_i^u\left(k^-, k^+\right) = \pi_i^{u+}\left(k^+, k^-\right) - \pi_i^{u-}\left(k^-, k^+\right)$:

$$\Delta\pi_i^u\left(k^-, k^+\right) = k^+\left[P\int_{k^+}^{1} G'(\theta)\, d\theta - c\right] - k^-\left[P\int_{k^-}^{1} G'(\theta)\, d\theta - c\right] + P\int_{k^-}^{k^+}\left[\theta - k^-\right] G'(\theta)\, d\theta.$$

Taking the derivative of the above expression with respect to k^-, and dividing it by P, we get

$$\frac{1}{P}\frac{\partial\Delta\pi_i^u\left(k^-,k^+\right)}{\partial k^-}=-\left[1-\frac{c}{P}\right]+G'\left(k^-\right)+k^-G'\left(k^-\right)-\left[G(k^+)-G\left(k^-\right)\right]$$

Take any pair (k^-,k^+) at which the small firm's FOC is increasing in its own capacity. Rearranging (31), we get

$$-\left[1-\frac{c}{P}\right]+G\left(k^-\right)+G'\left(k^-\right)k^-<-\frac{1}{2}\left[G\left(k^-+k^+\right)-G\left(k^+\right)\right]$$

Hence, at such a point (k^-,k^+),

$$\frac{\partial\Delta\pi_i^u\left(k^-,k^+\right)}{\partial k^-}<-\frac{P}{2}\left[G\left(k^-+k^+\right)-G\left(k^+\right)\right]-P\left[G(k^+)-G\left(k^-\right)\right]<0.$$

The above result and the fact that the profit function is everywhere continuous, in particular at symmetric capacity pairs, i.e., $\Delta\pi_i^u\left(k^+,k^+\right)=0$, imply $\pi_i^{u+}\left(k^+,k^-\right)>\pi_i^{u-}\left(k^-,k^+\right)$ for pairs (k^-,k^+) at which the small firm's FOC is increasing in its own capacity.

At the capacity pair $\left(\hat{k},k^+\left(\hat{k}\right)\right)$ the small firm's FOC is increasing in its own capacity since, by the fact that best replies cross in the discontinuity region, $k^-\left(k^+\left(\hat{k}\right)\right)>\hat{k}$. Hence, applying the result immediately above, we get $\pi_i^{u+}\left(k^+\left(\hat{k}\right),\hat{k}\right)>\pi_i^{u-}\left(\hat{k},k^+\left(\hat{k}\right)\right)$, which contradicts (44). The contradiction shows our claim, i.e., $k^-(k^+)\leq\hat{k}$ which is sufficient to ensure that any crossing point between the best reply functions takes place outside the discontinuity region and to ultimately guarantee equilibrium existence.

Finally, since an equilibrium pair $\left(k_1^+,k_2^-\right)$ satisfies (32), aggregate capacity is bounded below (above) by $G^{-1}\left(\frac{P-c}{P}\right)$ if G is convex (concave) as it implies $G\left(k_1^++k_2^-\right)-G\left(k_1^+\right)-G'\left(k_1^+\right)k_2^->(<)\,0$.

Equilibrium Existence: Discriminatory Auction

The proof follows the same steps as the one for the uniform-price auction. Nevertheless, we need to add the following pieces of information, which are specific to the discriminatory auction. Along the diagonal,

$$\lim_{k_i\downarrow k}\frac{\partial\pi_i^{d+}(k_i,k)}{\partial k_i}-\lim_{k_i\uparrow k}\frac{\partial\pi_i^{d-}(k_i,k)}{\partial k_i}=P\int_k^{2k}\frac{2k-\theta}{k}G'\left(\theta\right)\,d\theta>0$$

For k^* and k^{**} implicitly defined similarly as before, we have

$$\begin{aligned}\pi_i^{d-}\left(k_i^-\left(k^*\right),k^*\right)-\pi_i^{d+}\left(k_i^+\left(k^*\right),k^*\right)&<0\\ \pi_i^{d-}\left(k_i^-\left(k^{**}\right),k^{**}\right)-\pi_i^{d+}\left(k_i^+\left(k^{**}\right),k^{**}\right)&>0.\end{aligned}$$

Furthermore, the difference in profits is a strictly increasing function of k_j for any $k_j \in [k^*, k^{**}]$:

$$\begin{aligned} & \frac{d\pi_i^{d-}\left(k_i^-\left(k_j\right), k_j\right)}{dk_j} - \frac{d\pi_i^{d+}\left(k_i^+\left(k_j\right), k_j\right)}{dk_j} \\ = & -\int_{k_j}^{k_i^- + k_j} \frac{k_i^-}{k_j^2}\left[\theta - k_i^-\right] G'(\theta)\, d\theta + G\left(k_i^+ + k_j\right) - G\left(k_j\right) \\ = & \int_{k_j}^{k_i^- + k_j} \left\{1 - \frac{k_i^-}{k_j^2}\left[\theta - k_i^-\right]\right\} G'(\theta)\, d\theta + G\left(k_i^+ + k_j\right) - G\left(k_i^- + k_j\right) \\ = & \int_{k_j}^{k_i^- + k_j} \frac{1}{k_j^2}\left[k_j^2 + k_i^{-2} - k_i^-\theta\right] G'(\theta)\, d\theta + G\left(k_i^+ + k_j\right) - G\left(k_i^- + k_j\right) > 0 \end{aligned}$$

since $k_i^+ > k_i^-$ and $k_j^2 + \left(k_i^-\right)^2 - k_i^-\theta > k_j^2 + \left[k_i^-\right]^2 - k^-\left[k_i^- + k_j\right] > 0$. Therefore, there exists a unique $k_j = \hat{k} \in (k^*, k^{**})$ such that $\pi_i^-\left(k_i^-\left(\hat{k}\right), \hat{k}\right) - \pi_i^+\left(k_i^+\left(\hat{k}\right), \hat{k}\right) = 0$.
At $k_j = \hat{k}$, both k_i^- and k_i^+ are a best reply. For values $k_j < \hat{k}$, the best reply is k_i^+, whereas for $k_j > \hat{k}$ the best reply is k_i^-.In summary, the best-response functions in the discriminatory auction for firm $i = 1, 2$, $i \neq j$, are as follows:

$$k_i^d(k_j) = \begin{cases} k_i^-(k_j) & \text{if} \quad k_j \geq \hat{k} \\ k_i^+(k_j) & \text{if} \quad k_j \leq \hat{k} \end{cases}$$

Notice that $k_i^d(k_j)$ is discontinuous at $k_j = \hat{k}$. If G is convex, both $k_i^-(k_j)$ and $k_i^+(k_j)$ are decreasing functions. If G is concave and G' is convex, $k_i^+(k_j)$ and $k_i^-(k_j)$ are also decreasing functions.

In the discriminatory auction, since $k_i^-(\hat{k}) < k_i^+(\hat{k})$ and the best replies are decreasing functions with $k_i^+(0) = G^{-1}\left(\frac{P-c}{P}\right) < 1$ and $k_i^-(1) > 0$, they must cross outside the discontinuity region, which guarantees that there is a Nash equilibrium of the form $\left(k_1^-, k_2^+\right)$. Finally, since the best replies are identical for both players $\left(k_2^+, k_1^-\right)$ is also an equilibrium.

Finally, an equilibrium satisfies $P\left[1 - G\left(k_i^- + k_j^+\right)\right] - c = 0$. Consequently aggregate capacity, $K = k_i^- + k_j^+$, equals $G^{-1}(\frac{P-c}{P})$.

Proof of Proposition 12

Since $K^{FB} = G^{-1}\left(1 - \frac{c}{v}\right)$ and $K^d = G^{-1}\left(1 - \frac{c}{P}\right)$, we have $K^d \leq K^{FB}$, independently of the shape of G. Furthermore, from the characterisation of equilibrium, we know that $K^u \leq K^d$ if G is concave and $K^u \geq K^d$ if G is convex.

Suppose G is concave. Since $K^u \leq K^d$ and $K^d \leq K^{FB}$, it follows directly that $K^u \leq K^d \leq K^{FB}$, where the first inequality is strict if G is strictly concave.

Finally, suppose G is convex. Since both K^u and K^d are monotonically decreasing in P, and since $K^{FB} = K^d \leq K^u$ at $P = v$ while $K^{FB} > K^d = K^u = 0$ at $P = c$, there must exist some value $\widehat{P} \in (c, v)$ such that $K^d \leq K^u \leq K^{FB}$ if $P \leq \widehat{P}$ and $K^d \leq K^{FB} < K^u$ otherwise.