VERY PRELIMINARY

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Strategic Supply Function Competition with Private Information

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Abstract

The paper presents a partial equilibrium model of supply function competition when firms have private information about their uncertain costs. A linear Bayesian equilibrium is characterized and comparative static results are derived. As the market grows large the equilibrium becomes competitive and we obtain an approximation to how many competitors are needed to have a certain degree of competitiveness. Results are compared with the outcome of Cournot competition. It is found that with supply function competition, and in contrast to Cournot competition, competitiveness is affected by the parameters of the information structure. In particular, supply functions are steeper with more noise in the private signals or more correlation among the costs parameters. Furthermore, competition in supply functions aggregates the dispersed information of firms while Cournot competition does not. The implication is that with the former the only source of deadweight loss is market power while with the latter we have to add private information.

1. Introduction

Competition in supply functions has been used to model several markets, in particular the spot market for electricity but also management consulting or airline reservation systems. The models considered typically do not allow for private information (exceptions are Hortaçsu and Puller (2006) and Kühn and Machado (2004)). In this paper we study supply function competition when firms have private information about costs and compare it with Cournot competition, a leading contender. Private information on costs is a relevant situation. In many instances it is not realistic to assume that there is common knowledge on costs. Instead each firm has an estimate of its own costs and uses it, together with whatever public information is available, to make inferences about the costs of rivals.

Our modeling strategy is to consider a linear-quadratic model coupled with an affine information structure that yields a linear Bayesian Supply Function Equilibrium. The characterization of a linear equilibrium with supply function competition when there is market power and private information needs some careful analysis in order to model the capacity of a firm to influence the market price at the same time that the firm learns from the price. Kyle (1989) pioneered this type of analysis in a financial market context.

It is found that there is a unique linear Bayesian Supply Function Equilibrium. This equilibrium is privately revealing. That is, the private information of a firm and the price provide a sufficient statistic of the joint information in the market. This means in particular that the incentives to acquire information are preserved despite the fact that the price aggregates information. We do not examine possible nonlinear equilibria. Linear equilibria are tractable, in particular in the presence of private information, and have desirable properties like simplicity.

In the linear equilibrium supply functions are upward sloping provided that the informative role of price does not overwhelm its traditional capacity as index of scarcity. This happens when costs shocks are not very correlated and information precision not too low. In this case an increase in the correlation of cost parameters or in the noise in private signals makes supply functions steeper. The market looks less competitive in those circumstances as reflected in increased price-cost margins. Ignoring private cost information with supply function competition may therefore overestimate the slope of supply. This is not the case with Cournot competition, where the margin is not affected by the information parameters.

The welfare evaluation of the LBSFE is in marked contrast with the Cournot equilibrium in the presence of private information. The reason is that the LBSFE aggregates information and therefore there is only a deadweight loss due to market power but not due to private information. The result is that in a large market with supply function competition there is no efficiency loss (in the limit) and the order of magnitude of the deadweight loss is $1/n^2$ where n is the number of firms (and the size of the market as well). With Cournot competition we have to add a deadweight loss due to private information (on top of a larger deadweight loss due to market power). A large Cournot market does not aggregate information (i.e. a large Cournot market does not approach a full information competitive outcome) and in the limit there is a welfare loss due to private information.

We characterize also the limit competitive economy with competition in supply functions and à la Cournot and the convergence rate to the limit as the economy grows large. We find that LBSFE prices converge in mean square to the full information competitive limit at the rate of $1/\sqrt{n}$. Furthermore, the asymptotic variance is increasing in prior uncertainty, the noise in the signals, and decreasing with the correlation of cost parameters. With Cournot competition, the Bayesian Cournot price converges (in mean square) to the price-taking limit (which is not a full information equilibrium) also at the rate of $1/\sqrt{n}$. In this case the asymptotic variance is also increasing in prior uncertainty, and decreasing with the correlation of cost parameters but the influence of the noise in the signals is ambiguous.

A potential application of the model is to competition in the electricity spot market. In quite a few spot markets firms submit supply schedules in a day-ahead pool market which is organized as a uniform price multiunit auction. In the British Pool, the first liberalized wholesale market, generators had to submit a single supply schedule for the entire day. The schedules are increasing since the Pool's rules rank plants in order of increasing bids. Other wholesale markets have different rules (and the British Pool was replaced by NETA in 2001).[1] In our modeling the supply functions are smooth (the old English pool was modeled like this by Green and Newbery (1992) and Green (1996, 1999)) while typically supplies are discrete. However the modeling of the auction with discrete supplies leads to existence problems of equilibrium in pure strategies (see von der Fher and Harbord (1993)). The linear supply function model has been widely used in electricity markets and new developments include cost asymmetries, capacity constraints, piecewise affine supply functions and non-negativity generation constraints (see Baldick, Grant, and Kahn (2004) and Rudkevich (2005)).

There is a lively debate about the best way of modeling competition in the wholesale electricity market. The Cournot framework has been used in a variety of studies.[2] The advantage of the Cournot model is that it is a robust model in which capacity constraints and fringe suppliers are easily incorporated. The Cournot model also provides the least competitive equilibria of all possible equilibria with supply function competition. A drawback is that the Cournot model tends to predict prices that are too high given realistic estimates of the demand elasticity. However, including vertical relations and

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[1] In the day-ahead market in the Spanish pool generators submit supply functions which have to be nondecreasing and can include up to 25 price-quantity pairs for each production unit, as well as some other ancillary conditions. The demand side can bid in a similar way and the market operator constructs a merit order dispatch by ordering in the natural way supply and demand bids. The intersection of the demand and supply schedules determines the (uniform) price. Once the market closes the system operator solves congestion problems and market participants may adjust their positions in a sequence of intra-day markets, which have similar clearing procedures as in the day-ahead market. (See Crampes and Fabra (2005)).

[2] See, for example, Borenstein, and Bushnell (1999) for the US; Alba et al. (1999), Ramos et a. (1998), and Ocaña and Romero (1998) for Spain; and Andersson and Bergman (1995) for Scandinavia.

contracts in a Cournot setting provides good estimates (see Bushnell, Mansur, and Saravia (2005)). The supply function approach is more realistic but less robust. There is either non-existence of equilibrium in pure strategies if discrete supplies are taken into account or a plethora of equilibria in smooth models. Under some circumstances a unique equilibrium can be pinned down (Klemperer and Meyer (1989) and Green and Newbery (1992)).[3] Baldick and Hogan (2006) justify to concentrate attention on linear supply function equilibria in a linear-quadratic model because other equilibria (in the range between the least competitive Cournot one and the most competitive) are unstable. Another potential advantage of the supply function approach, over either the Cournot or the pure auction approaches, is that it implies that firms bid in a consistent way over an extended time horizon.

Hortaçsu and Puller (2006) study the Texas balancing market (the day-ahead market is resolved with bilateral contracts) and argue that there the relevant private information is not about costs but the contract positions of firms. (The authors also argue that to take a linear approximation to marginal costs in the Texas electricity market is reasonable.) Information on costs would be available by purchase to firms selling information and also because the balancing market takes place very close to the generation moment. However, private cost information related to plant availability may be relevant when there is a day-ahead market organized as a pool where firms submit hourly or daily supply schedules. Even if there was a market for information on costs the solution of the model with private information would yield the value of information. Kühn and Machado (2004) introduce private information on retail sales by vertically integrated firms in the Spanish pool.

The plan of the paper is as follows. Section 2 presents the supply function model with n strategic firms and characterizes linear Bayesian Supply Function equilibria. Section 3 looks at the competitive limit of the market and the convergence to this limit as the

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[3] In supply function models with uncertainty with unbounded support, and no private information, it is possible to show the existence of a unique equilibrium (in the linear-quadratic model this is a linear equilibrium, see Klemperer and Meyer (1989)).

market grows large. Section 4 considers the Cournot case. Some proofs are gathered in the appendix.

2. A strategic supply function model

Consider a market for a homogenous product with n consumers, each with quasilinear preferences and having the net benefit function

$$U(x) - px \text{ with } U(x) = \alpha x - \beta x^2/2,$$

where α and β are positive parameters and x the consumption level. This gives rise to the inverse demand $P_n(X) = \alpha - \beta X/n$ where X is total output. In the electricity market the demand intercept α is a continuous function of time (load-duration characteristic) that yields the variation of demand over the time horizon considered. At any time there is a fixed α and the market clears.

There are also n firms in the market. We are considering an n-replica market and X/n is the average or per capita output. We will denote the average of a variable by a tilde (for example, $\tilde{x}_n = X/n$). Firm i produces according to a quadratic cost function

$$C(x_i;\theta_i) = \theta_i x_i + \frac{\lambda}{2} x_i^2$$

where θ_i is a random parameter and $\lambda > 0$. Total surplus is therefore given by TS = n U(X/n) $-\Sigma_i C(x_i;\theta_i)$ and per capita surplus by TS/n = U(X/n) $-(\Sigma_i C(x_i;\theta_i))/n$.

This replica market can also be interpreted as a market parameterized by the number of consumers and where firms can enter freely paying a positive fixed entry cost. Then the free entry number of firms is of the order of the number of consumers. A large market then is a market with a large number of consumers. We will consider in the paper the reduced-form replica market version instead of the free-entry version.

We assume that θ_i is normally distributed (with mean $\bar{\theta} > 0$). The parameters θ_i are correlated with correlation coefficient $\rho \in [0,1]$ Firm i receives a signal $s_i = \theta_i + \varepsilon_i$ and signals are of the same precision $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$. Error terms in the signals are uncorrelated among themselves and with the θ_i parameters. All random variables are thus normally distributed. In the electricity example the random cost shock may be linked to plant availability because of technical issues or transport problems. The common component in the shock may be related to the prices of energy in international markets to which the supply contracts of firms are linked.

Ex-ante, before uncertainty is realized, all firms face the same prospects. The vector of random variables $(\theta_1, ..., \theta_n)$ is jointly normally distributed with $E\theta_i = \bar{\theta}$, $\text{Var}\ \theta_i = \sigma_\theta^2$, and $\text{Cov}\ (\theta_i, \theta_j) = \rho\sigma_\theta^2$, for $j \neq i$, $0 \leq \rho \leq 1$. It follows that the average parameter $\tilde{\theta}_n \equiv (\sum_{i=1}^n \theta_i) / n$ is normally distributed with mean $\bar{\theta}$, $\text{Var}\ \tilde{\theta}_n = (1 + (n-1)\rho)\sigma_\theta^2/n$, and $\text{cov}\ (\tilde{\theta}_n, \theta_i) = \text{Var}\,\tilde{\theta}_n$.

Our information structure encompasses the cases of "common value" and of "private values." For $\rho = 1$ the θ parameters are perfectly correlated and we are in a *common value* model. When signals are perfect, $\sigma_{\varepsilon_i}^2 = 0$ for all i, and $0 < \rho < 1$, we will say we are in a *private values* model. Agents receive idiosyncratic shocks, which are imperfectly correlated, and each agent observes his shock with no measurement error. When $\rho = 0$, the parameters are independent, and we are in an *independent values* model.

It is not difficult to see that

$$E(\theta_i|s_i) = \xi s_i + (1-\xi)\bar{\theta} \text{ and } E(s_j|s_i) = E(\theta_j|s_i) = \xi \rho s_i + (1-\xi\rho)\bar{\theta}.$$

When signals are perfect, $\xi = 1$ and $E(\theta_i|s_i) = s_i$, and $E(\theta_j|s_i) = \rho s_i + (1-\square)\bar{\theta}$. When they are not informative, $\xi = 0$ and $E(\theta_i|s_i) = E(\theta_j|s_i) = \bar{\theta}$.

Under the normality assumption conditional expectations are affine. There are other families of conjugate prior and likelihood that also yield affine conditional expectations and allow for bounded supports of the distributions. (See Vives (1999)).

Firms compete in supply functions. We will restrict attention to Linear Bayesian Supply Function Equilibria (LBSFE). As stated before, the characterization of linear equilibria with supply function competition when there is market power and private information needs some careful analysis in order to model the capacity of a firm to influence the market price at the same time that the firm learns from the price.

The strategy for firm i is a price contingent schedule $X_i(s_i, .)$. This is a map from the signal space to the space of supply functions. Given the strategies of firms $X_j(s_j, .)$, $j = 1, \ldots, n$, for given realizations of signals market clearing implies that

$$p = P_n\left(\sum_{j=1}^{n} X_j(s_j, p)\right).$$

Let us assume that there is a unique market clearing price $\hat{p}\left(X_1(s_1,\cdot),\ldots,X_n(s_n,\cdot)\right)$ for any realizations of the signals.[4] Then profits for firm i, for any given realization of the signals, are given by

$$\pi_i\left(X_1(s_1,\cdot),\ldots,X_n(s_n,\cdot)\right) = pX_i(s_i,p) - C\left(X_i(s_i,p)\right)$$

where

$$p = \hat{p}\left(X_1(s_1,\cdot),\ldots,X_n(s_n,\cdot)\right).$$

4 If there is no market clearing price assume the market shuts down and if there are many then the one that maximizes volume is chosen.

This defines a game in supply functions and we want to characterize a Bayesian symmetric linear supply function equilibrium. Let us posit a candidate symmetric equilibrium for the game with n firms:

$$X_n(s_i, p) = b_n - a_n s_i + c_n p .$$

Average output is given by $\tilde{x}_n = b_n - a_n \tilde{s}_n + c_n p$, where $\tilde{s}_n = \left(\sum_i s_i\right)/n = \tilde{\theta}_n + \left(\sum_i \varepsilon_i\right)/n$, and substituting in the inverse demand we obtain $p = \alpha - \beta\tilde{x}_n = \alpha - \beta b_n + \beta a_n \tilde{s}_n - \beta c_n p$ and therefore

$$p = (1+\beta c_n)^{-1}(\alpha - \beta b_n + \beta a_n \tilde{s}_n)$$

where we posit that $1+\beta c_n > 0$.

Given the strategies of rivals $X_n(s_j, .)$, $j \neq i$, firm i faces a residual inverse demand

$$p = \alpha - \frac{\beta}{n}\sum_{j\neq i} X_n(s_j, p) - \frac{\beta}{n}x_i = \alpha - \frac{\beta}{n}(n-1)(b_n + c_n p_n) + \frac{\beta}{n}a_n \sum_{j\neq i} s_j - \frac{\beta}{n}x_i .$$

It follows that

$$p = I_i - \frac{\beta}{n}\left(1+\beta\frac{n-1}{n}c_n\right)^{-1} x_i$$

where

$$I_i = \left(1+\beta\frac{n-1}{n}c_n\right)^{-1}\left(\alpha - \frac{\beta}{n}(n-1)b_n + \frac{\beta}{n}a_n\sum_{j\neq i} s_j\right).$$

Note that all the information provided by the price to firm i about the signals of others is subsumed in the intercept of residual demand I_i. The information available to firm i is therefore $\{s_i, p\}$ or, equivalently, $\{s_i, I_i\}$. Firm i chooses x_i to maximize

$$E\left(\pi_i \mid s_i, p\right) = x_i\left(p - E\left(\theta_i \mid s_i, p\right)\right) - \frac{\lambda}{2}x_i^2 = x_i\left(I_i - \frac{\beta}{n}\left(1 + \beta\frac{n-1}{n}c_n\right)^{-1} x - E\left(\theta_i \mid s_i, p\right)\right) - \frac{\lambda}{2}x_i^2$$

The F.O.C. is

$$I_i - E\left(\theta_i \mid s_i, I_i\right) - 2\frac{\beta}{n}\left(1 + \beta\frac{n-1}{n}c_n\right)^{-1} x_i - \lambda x_i = 0$$

or

$$p - E\left(\theta_i \mid s_i,\ p\right) = \left(\frac{\beta}{n + \beta(n-1)c_n} + \lambda\right)x_i .$$

The second order sufficient condition for a maximum is $\left(\frac{2\beta}{n + \beta(n-1)c_n} + \lambda\right) > 0$. An equilibrium must fulfill also $1 + \beta c_n > 0$.

The following proposition characterizes the linear equilibrium when $\rho < 1$.

<u>Proposition 1.</u> In the n-firm market with $\rho < 1$ there is a unique symmetric Bayesian linear supply function equilibrium. It is given by $X_n\left(s_i, p\right) = b_n - a_n s_i + c_n p$, where

$$a_n = \frac{(1-\rho)\sigma_\theta^2}{\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)}\left(\frac{\beta}{n + \beta(n-1)c_n} + \lambda\right)^{-1}$$

$$b_n = \left(1 + \frac{\sigma_\varepsilon^2\rho}{(1-\rho)K}\right)^{-1}\left(\frac{\sigma_\varepsilon^2\rho\alpha}{\beta(1-\rho)K} - \frac{\sigma_\varepsilon^2\overline{\theta}}{nK}\left(\frac{\beta}{n + \beta(n-1)c_n} + \lambda\right)^{-1}\right)$$

and c_n is the largest solution to the quadratic equation

$$\lambda\beta(n-1)\left(1 + \frac{\rho\sigma_\varepsilon^2}{K(1-\rho)}\right)c_n^2 + \left((\beta + \lambda n)\left(1 + \frac{\rho\sigma_\varepsilon^2}{K(1-\rho)}\right) + \frac{\lambda(n-1)\rho\sigma_\varepsilon^2}{K(1-\rho)} - \beta(n-1)\right)c_n +$$

$$+ (\beta + \lambda n)\frac{\rho\sigma_\varepsilon^2}{\beta K(1-\rho)} - n = 0$$

where $K = \frac{\left(\sigma_\varepsilon^2 + \left(1+(n-1)\rho\right)\sigma_\theta^2\right)}{n}$. In equilibrium we have that $1+\beta c_n > 0$.

<u>Proof:</u> The price equation

$$p = \left(1+\beta c_n\right)^{-1}\left(\alpha - \beta b_n + \beta a_n \tilde{s}_n\right)$$

can be rearranged to define

$$h_i \equiv \frac{p\left(1+\beta c_n\right) - \alpha + \beta b_n}{\beta a_n} n - s_i = \sum_{j \neq i} s_j .$$

The pair (s_i, p) is informationally equivalent to the pair (s_i, h_i), hence

$$E\left(\theta_i \middle| s_i, p\right) = E\left(\theta_i \middle| s_i, h_i\right).$$

Because of the assumed information structure we have

$$\begin{pmatrix} \theta_i \\ s_i \\ h_i \end{pmatrix} \sim N\left[\begin{pmatrix} \bar{\theta} \\ \bar{\theta} \\ (n-1)\bar{\theta} \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & (n-1)\rho\sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_\varepsilon^2 & (n-1)\rho\sigma_\theta^2 \\ (n-1)\rho\sigma_\theta^2 & (n-1)\rho\sigma_\theta^2 & \Lambda \end{pmatrix}\right],$$

where $\Lambda = (n-1)\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right) + (n-1)(n-2)\rho\sigma_\theta^2$.

We obtain

$$E\left[\theta_i \middle| s_i, h_i\right] = E\left[\theta_i \middle| s_i, \frac{h_i}{n-1}\right] = \frac{\sigma_\varepsilon^2}{\sigma_\theta^2\left(1+(n-1)\rho\right)+\sigma_\varepsilon^2}\bar{\theta} +$$

$$\frac{\sigma_\theta^2\left[\sigma_\theta^2(1-\rho)\left(1+(n-1)\rho\right)+\sigma_\varepsilon^2\right]}{\left[\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2\right]\left[\sigma_\theta^2\left(1+(n-1)\rho\right)+\sigma_\varepsilon^2\right]} s_i + \frac{\sigma_\theta^2\sigma_\varepsilon^2\rho}{\left[\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2\right]\left[\sigma_\theta^2\left(1+(n-1)\rho\right)+\sigma_\varepsilon^2\right]} h_i.$$

We are looking strategies of the form $X_n(s_i, p) = b_n - a_n s_i + c_n p$. Using the F.O.C. and the expression for h_i we obtain the following

$$-\frac{\sigma_\varepsilon^2\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)\overline{\theta}+\left(\sigma_\theta^2\sigma_\varepsilon^2 n\rho\left(\beta b_n-\alpha\right)/\beta a_n\right)}{\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)\left(\sigma_\varepsilon^2+\left(1+(n-1)\rho\right)\sigma_\theta^2\right)}-\frac{(1-\rho)\sigma_\theta^2\left(\sigma_\varepsilon^2+\left(1+(n-1)\rho\right)\sigma_\theta^2\right)}{\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)\left(\sigma_\varepsilon^2+\left(1+(n-1)\rho\right)\sigma_\theta^2\right)}s_i$$

$$+\left(1-\frac{\sigma_\theta^2\sigma_\varepsilon^2 n\rho\left((1+\beta c_n)/\beta a_n\right)}{\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)\left(\sigma_\varepsilon^2+\left(1+(n-1)\rho\right)\sigma_\theta^2\right)}\right)p=\left(\frac{\beta}{n+\beta(n-1)c_n}+\lambda\right)\left(b_n-a_n s_i+c_n p\right)$$

We can use the method of undetermined coefficients and find a_n, b_n, c_n by solving the following system of equations

$$\begin{cases} \dfrac{(1-\rho)\sigma_\theta^2}{\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)}=\left(\dfrac{\beta}{n+\beta(n-1)c_n}+\lambda\right)a_n \\ -\dfrac{\sigma_\varepsilon^2\overline{\theta}}{nK}-\dfrac{\sigma_\varepsilon^2\rho\left(\beta b_n-\alpha\right)}{\beta(1-\rho)K}\left(\dfrac{\beta}{n+\beta(n-1)c_n}+\lambda\right)=\left(\dfrac{\beta}{n+\beta(n-1)c_n}+\lambda\right)b_n \\ \left(1-\dfrac{\sigma_\varepsilon^2\rho\left(1+\beta c_n\right)}{\beta(1-\rho)K}\left(\dfrac{\beta}{n+\beta(n-1)c_n}+\lambda\right)\right)=\left(\dfrac{\beta}{n+\beta(n-1)c_n}+\lambda\right)c_n \end{cases}$$

This characterizes linear equilibria. It can be checked that the largest root of the quadratic equation defining c is the only one compatible with the second order condition. ♦

The price p_n reveals the aggregate information $\tilde{s}_n$. The equilibrium is *privately revealing* (i.e. for firm i (s_i, p) or $(s_i, \tilde{s}_n)$ is a sufficient statistic of the joint information in the market, see Allen (1981)). The incentives to collect information are preserved because for firm i the signal s_i still helps in estimating θ_i even though p_n reveals $\tilde{s}_n$. Note that with private values (perfect signals with $\sigma_\varepsilon^2 = 0$) the price reveals $\tilde{\theta}_n$ and therefore a Bayesian price-taking supply function equilibrium would coincide with the usual complete information competitive solution.

It is worth noting that the slope of supply c may be negative if costs shocks are correlated ($\rho > 0$) and signals not perfect ($\sigma_\varepsilon^2 > 0$). The price serves a dual role as index of scarcity and as conveyor of information. Indeed, a high price has a direct effect to increase the competitive supply of a firm, but also conveys news that costs are high. If $\rho = 0$ (or $\sigma_\varepsilon^2 = 0$) then the price conveys no information on costs and $c > 0$. As $\rho\sigma_\varepsilon^2$ increases then the slope c decreases because of the informational component of the price (it is easily checked that c decreases in $\rho\sigma_\varepsilon^2$ and increases in σ_θ^2; this follows from the fact that the largest root of the quadratic equation determining c decreases with $M \equiv \frac{\rho\sigma_\varepsilon^2 n}{(1-\rho)\left(\sigma_\varepsilon^2 + \left(1+(n-1)\rho\right)\sigma_\theta^2\right)}$). As ρ tends to 1, c becomes negative. There are particular parameter combinations for which the scarcity and informational effects balance and firms set a zero weight ($c = 0$) on public information. In this case firms do not condition on the price and the model reduces to the Cournot model where firms compete in quantities. However, in this particular case, when supply functions are allowed, not reacting to the price (public information) is optimal. However, for reasonable parameter values in the electricity example (i.e. low correlation of plants outages and/or good private precision on them) supply will be upward sloping, the scarcity effect dominating the information effect.

It is interesting to note that increasing the noise in the private signal σ_ε^2 or the correlation of the random cost parameters ρ makes the slope of supply steeper (decreases c). This result may help explain the fact that in the Texas balancing market small firms use steeper supply functions than those predicted by theory (Hortaçsu and Puller (2006)). Indeed, smaller firms may have signals of worse quality because of economies of scale in information gathering while private cost information has not been taken into account in the estimation.

A conjecture to be checked is that the slope of supply becomes steeper also when decreasing the number of firms n (i.e. c increases with n).

A consequence is that the margin over expected marginal cost $\left(E\left(\theta_i \left| s_i, p\right.\right)+\lambda x_i\right)$ will tend to be increasing in σ_ε^2, ρ and decreasing in n. Indeed, from the F.O.C. we have that

$$p-\left(E\left(\theta_i \left| s_i,\ p\right.\right)+\lambda x_i\right)=\left(\frac{1}{n\beta^{-1}+\left(n-1\right)c_n}\right)x_i,$$

where the slope of residual demand is $n\beta^{-1}+\left(n-1\right)c_n$. A similar relation holds for the margin over average expected marginal cost $E\left[MC_n\right]=\frac{1}{n}\sum_{i=1}^{n}\left(E\left(\theta_i \left| s_i, p\right.\right)+\lambda x_i\right)=\frac{1}{n}\sum_{i=1}^{n}E\left(\theta_i \left| s_i, p\right.\right)+\lambda\tilde{x}_n$:

$$\frac{p-E\left[MC_n\right]}{p}=\frac{1}{\left(n+\beta\left(n-1\right)c_n\right)\eta_n}$$

where $\eta_n=p/\left(\beta\tilde{x}_n\right)$ is the elasticity of demand.

When ρ = 1 a fully revealing REE is not implementable. Indeed, if ρ = 1 and $\sigma_\varepsilon^2<\infty$ (common value) there is no linear equilibrium. The reason should be well understood: if the price reveals the common value then no firm has an incentive to put any weight on its signal (and the incentives to acquire information disappear as well). But if firms put no weight on their signals then the price can not contain any information on the costs parameters. However, if in addition the signals are pure noise (i.e. $\sigma_\varepsilon^2=\infty$) then there is always a linear equilibrium. The equilibrium is given by

$$X_n\left(p\right)=c_n\left(p-\theta\right)$$

where c_n is given implicitly by the positive root of

$$\left(\frac{\beta}{n+\beta\left(n-1\right)c_n}+\lambda\right)c_n=1.$$

To see this note that as $\sigma_\varepsilon^2 \to \infty$, $E(\theta_i | s_i, I_i) \to \bar{\theta}$, $a_n \to 0$ and

$$b_n = -\bar{\theta}\left(\frac{\beta}{n+\beta(n-1)c_n} + \lambda\right)^{-1} = -c_n\bar{\theta}.$$

Remark: The replica market considered can be the outcome of free entry in a market parameterized by size. Consider a market with m consumers (the size of the market) and inverse demand $P_m(X) = \alpha - \beta_m X$ where $\beta_m = \beta/m$. Suppose now that at a first stage firms decide whether to enter the market or not. If a firm decides to enter it pays a fixed cost F > 0. At a second stage each active firm i, upon observing its signal s_i, sets an output level. Given that n firms have entered, a Bayesian Supply Function equilibrium is realized. Given our assumptions, for any n there is a unique, and symmetric, equilibrium yielding expected profits $E\pi_n = (\frac{\lambda}{2} + \frac{\beta}{n})\, E\,(X_n(s_i, p))^2$ for each firm. A free entry equilibrium is a subgame-perfect equilibrium of the two-stage game. A subgame-perfect equilibrium requires that for any entry decisions at the first stage, a Bayesian-Nash equilibrium in supply functions obtains at the second stage. Given a market of size m, the free entry number of firms $n^*(m)$ is approximated by the solution to $E\pi_n = F$ (provided F is not so large to prevent any entry). It can be checked that $n^*(m)$ is of the same order as m (similarly as in Vives (2002)). This means that the ratio of consumers to firms is bounded away from zero and infinity for any market size. We can reinterpret, therefore, the replica market as a free entry market parameterized by market size.

Price-taking equilibrium

In order to assess the welfare loss due to strategic behavior we characterize price-taking equilibria. Full (shared) information competitive equilibria are Pareto optimal and characterized by the equality of price and expected marginal cost (with full information):

$$p = E(\theta_i | s_i, \tilde{s}_n) + \lambda x_i .$$

This allocation, provided that $\rho < 1$, is implemented by a price-taking LBSFE (denoted by a hat on the coefficients) which yields as F.O.C.:

$$p = E\left(\theta_i \left| s_i, p\right.\right) + \lambda x_i$$

and where $p = \left(1+\beta\hat{c}_n\right)^{-1}\left(\alpha - \beta\hat{b}_n + \beta\hat{a}_n\tilde{s}_n\right)$ and the coefficients given by the system of equations

$$\begin{cases} \dfrac{\left(1-\rho\right)\sigma_\theta^2}{\left(\sigma_\varepsilon^2+\left(1-\rho\right)\sigma_\theta^2\right)} = \lambda a_n \\ -\dfrac{\sigma_\varepsilon^2\overline{\theta}}{nK} - \dfrac{\sigma_\varepsilon^2\rho\left(\beta b_n - \alpha\right)}{\beta\left(1-\rho\right)K}\lambda = \lambda b_n \\ \left(1-\dfrac{\sigma_\varepsilon^2\rho\left(1+\beta c_n\right)}{\beta\left(1-\rho\right)K}\lambda\right) = \lambda c_n \end{cases}$$

It follows that

$$\hat{a}_n = \frac{\left(1-\rho\right)\sigma_\theta^2}{\lambda\left(\sigma_\varepsilon^2+\left(1-\rho\right)\sigma_\theta^2\right)}$$

$$\hat{b}_n = \left(1+\frac{\sigma_\varepsilon^2\rho}{\left(1-\rho\right)K}\right)^{-1}\left(\frac{\sigma_\varepsilon^2\rho\alpha}{\beta\left(1-\rho\right)K} - \frac{\sigma_\varepsilon^2\overline{\theta}}{nK}\lambda^{-1}\right)$$

$$\hat{c}_n = \frac{\lambda^{-1}\left(1-\rho\right)K - \beta^{-1}\rho\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2\rho+\left(1-\rho\right)K\right)}$$

where $K = \dfrac{\left(\sigma_\varepsilon^2+\left(1+\left(n-1\right)\rho\right)\sigma_\theta^2\right)}{n}$.

3. The competitive limit and convergence properties

3.1 The competitive limit

The continuum economy counterpart of the finite markets considered in Section 2 is given by the inverse demand $p = \alpha - \beta\tilde{x}$ where $\tilde{x}$ is average output. Firms are indexed in the unit interval (endowed with the Lebesgue measure).

We can derive the relationship of θ_i, s_i, and the average parameter $\tilde{\theta} = \int \theta_j dj$. The average parameter $\tilde{\theta} = \int \theta_j dj$ is normally distributed with mean $\bar{\theta}$ and variance $\rho\sigma_\theta^2$. Indeed, $E(\theta_i|\tilde{\theta}) = \tilde{\theta}$, $E(\tilde{\theta}|\theta_i) = E(\theta_j|\theta_i) = \rho\theta_i + (1-\rho)\bar{\theta}$, $E(\tilde{\theta}|s_i) = E(\theta_j|s_i)$, and

$$E(\theta_i|\tilde{\theta}, s_i) = (1- d)\tilde{\theta} + ds_i,$$

where $d = [\sigma_\theta^2(1-\rho)]/[\sigma_\theta^2(1-\rho) + \sigma_\varepsilon^2]$. If signals are perfect, then $d = 1$ and $E(\theta_i|\tilde{\theta}, s_i) = s_i$. If signals are useless or correlation perfect ($\rho = 1$), then $d = 0$ and $E(\theta_i|\tilde{\theta}, s_i) = \tilde{\theta}$. If both signals and correlation are perfect, then $E(\theta_i|\tilde{\theta}, s_i) = \tilde{\theta} = s_i$ (a.s.).

Observe that $\tilde{s} = \int \theta_i di + \int \varepsilon_i di = \tilde{\theta}$, since $\int \varepsilon_i di = 0$ since we make the convention that the average of i.i.d. random variables with mean zero is zero.

The following proposition characterizes the equilibrium.

<u>Proposition 2</u>. Let $\rho \in [0,1)$. In the continuum economy with inverse demand $p = \alpha - \beta\tilde{x}$, there is a unique LBSFE. It is given by

$$X(s_i, p) = b - as_i + cp,$$

where

$$a = \frac{(1-\rho)\sigma_\theta^2}{\lambda\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)} = \frac{1}{\lambda\left(1 + \left(\sigma_\varepsilon^2 / \left((1-\rho)\sigma_\theta^2\right)\right)\right)}$$

$$b = \frac{\alpha}{\beta}\frac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)} = \frac{\alpha}{\beta}(1-\lambda a)$$

$$c = \frac{\lambda^{-1}(1-\rho)\sigma_\theta^2 - \beta^{-1}\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)} = \frac{1}{\beta}\left(a(\beta+\lambda) - 1\right)$$

Moreover, the equilibrium price is given by $p = (1+\beta c)^{-1}\left(\alpha - \beta b + \beta a\tilde{\theta}\right) = \dfrac{\lambda\alpha + \beta\tilde{\theta}}{\lambda + \beta}$.

Proof:

In the continuum economy the F.O.C. is given by

$$p = E(\theta_i | s_i,\ p) + \lambda x_i .$$

Assuming linear strategies $X(s_i, p) = b - as_i + cp$ and using the inverse demand function $p = \alpha - \beta\tilde{x}$ and our convention $\int s_i di = \tilde{\theta}$ we obtain an expression for the price

$$p = (1+\beta c)^{-1}(\alpha - \beta b + \beta a\tilde{\theta}).$$

Given joint normality of the stochastic variables we obtain

$$\begin{pmatrix} \theta_i \\ s_i \\ p \end{pmatrix} \sim N\left[\begin{pmatrix} \bar{\theta} \\ \bar{\theta} \\ C + D\bar{\theta} \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & D\rho\sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 + \sigma_\varepsilon^2 & D\rho\sigma_\theta^2 \\ D\rho\sigma_\theta^2 & D\rho\sigma_\theta^2 & D^2\rho\sigma_\theta^2 \end{pmatrix}\right]$$

Where $C = (1+\beta c)^{-1}(\alpha - \beta b)$ and $D = (1+\beta c)^{-1}(\beta a)$. Using the projection theorem for normal random variables we obtain

$$E(\theta_i | s_i, p) = -\frac{C\sigma_\varepsilon^2}{D\left((1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2\right)} + \frac{(1-\rho)\sigma_\theta^2}{(1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2} s_i + \frac{\sigma_\varepsilon^2}{D\left((1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2\right)} p .$$

Recall that $E(\theta_i | s_i, \tilde{\theta}) = ds_i + (1-d)\tilde{\theta}$ where $d = \dfrac{(1-\rho)\sigma_\theta^2}{(1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2}$.

Plugging in the F.O.C. of the limit economy we obtain

$$-\frac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)} \frac{\beta b - \alpha}{\beta a} - \frac{(1-\rho)\sigma_\theta^2}{\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)} s_i + \left[1 - \frac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)} \frac{(1+\beta c)}{\beta a}\right] p =$$
$$= \lambda(b - as_i + cp)$$

and using the method of undetermined coefficients, we have the following system of equations

$$\begin{cases} a = \dfrac{(1-\rho)\sigma_\theta^2}{\lambda\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)} \\ -\dfrac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)} \dfrac{\beta b - \alpha}{\beta a} = \lambda b \\ 1 - \dfrac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)} \dfrac{(1+\beta c)}{\beta a} = \lambda c \end{cases}$$

The solution to the above system gives the result.

As before the equilibrium supply function can be upward or downward sloping. It will be downward sloping when the reaction to private information is small (i.e. when we are close to the common value case, when prior uncertainty is low or noise in the signals is high). We have $c \le a \le 1/\lambda$, $c = a - \beta^{-1}(1-\lambda a) = (1+\beta^{-1}\lambda)\left(a - (\beta+\lambda)^{-1}\right)$ and $1 + \beta c = a(\beta + \lambda) > 0$ provided that $\rho < 1$. Furthermore, $E[p] = \dfrac{\alpha\lambda + \beta\bar{\theta}}{\beta + \lambda}$ and $E[\pi_i] = \dfrac{\lambda}{2} E\left[x_i^2\right]$.

When signals are perfect ($\sigma_\varepsilon^2 = 0$), we have that a = c = λ^{-1}, b = 0, $x_i = \lambda^{-1}(p - \theta_i)$ and $p = \dfrac{\alpha + \beta\lambda^{-1}\tilde{\theta}}{1+\beta\lambda^{-1}} = \dfrac{\alpha\lambda + \beta\tilde{\theta}}{\lambda + \beta}$. The equilibrium is just the usual complete information competitive equilibrium (note it is independent of ρ and therefore it is Pareto optimal. The equilibrium is also efficient when $\sigma_\varepsilon^2 > 0$: it is price-taking and firms act with a sufficient statistic for the shared information in the economy.

If $\rho = 0$ or $\sigma_\varepsilon^2 = 0$ then the price conveys no information on costs and $c = 1/\lambda$, the slope of competitive supply $X(s_i, p) = \lambda^{-1}[p - \theta_i]$. As $\sigma_\varepsilon^2 / \left((1-\rho)\sigma_\theta^2\right)$ increases the slope c decreases because of the informational component of price, when $\beta / \lambda = \sigma_\varepsilon^2 / \left((1-\rho) \sigma_\theta^2 \right)$ we have that c = 0, and for larger values of $\sigma_\varepsilon^2 / \left((1-\rho) \sigma_\theta^2 \right)$ it becomes negative.

Firms are more cautious while responding to their private signals when they have market power. From the S.O.C. $\left(\lambda + 2\beta \big/ \left(n + \beta(n-1) \right) c_n \right) > 0$ with market power we have that:

$$a_n = \frac{\tau_\varepsilon}{\tau_\varepsilon + \tau_\theta / (1-\rho)} \frac{1}{\left(\lambda + \beta \big/ \left(n + \beta(n-1) \right) c_n \right)} < \frac{\tau_\varepsilon}{\left(\tau_\varepsilon + \tau_\theta / (1-\rho) \right)} \frac{1}{\lambda} = a \, .$$

This is because of the usual effect of market power: A firm takes into account the price impact coming from his production. Note that in principle a firm with market power would also be cautious because of the informational leakage from his action, but here the equilibrium is revealing.

3.2 Convergence to price-taking behavior

We examine next the convergence properties of the finite market as the economy grows. Before stating the convergence results we will recall measures of speed of convergence. We say that the sequence (of real numbers) b_n is of the *order* n^{υ}, with υ a real number,. whenever $n^{-\upsilon} b_n \xrightarrow[n]{} k$ for some nonzero constant k. We say that the sequence of random variables $\{y_n\}$ converges in *mean square* to zero at the rate $1/\sqrt{n^r}$ (or that y_n is of the order $1/\sqrt{n^r}$) if $E\left[(y_n)^2 \right]$ converges to zero at the rate $1/n^r$ (i.e. $E\left[(y_n)^2 \right]$ is of the order $1/n^r$). Given that $E\left[(y_n)^2 \right] = \left(E[y_n] \right)^2 + \operatorname{var}[y_n]$, a sequence $\{y_n\}$ such that $E[y_n] = 0$ and $\operatorname{var}[y_n]$ is of the order of 1/n, converges to zero at the rate $1/\sqrt{n}$.

For example, if the random parameters $(\theta_1, ..., \theta_n)$ are i.i.d. with finite variance and mean $\bar{\theta}$, and we let $\tilde{\theta}_n \equiv (\sum_{i=1}^{n} \theta_i) / n$, then $\tilde{\theta}_n$ - $\bar{\theta}$ converges (in mean square) to 0 at the rate of $1/\sqrt{n}$ because $E\left[\tilde{\theta}_n - \bar{\theta}\right] = 0$ and $\text{var}\left[\tilde{\theta}_n\right] = \sigma_\theta^2/n$. In our case $\tilde{\theta}_n$ is normally distributed with mean $\bar{\theta}$ and Var $\tilde{\theta}_n = (1 + (n-1)\rho)\sigma_\theta^2/n$. We have therefore that $\tilde{\theta}_n \to \tilde{\theta}$ in mean square at the rate $1/\sqrt{n}$ where $\tilde{\theta}$ is normally distributed with mean $\bar{\theta}$ and variance $\rho\sigma_\theta^2$.

A more refined measure of convergence speed for a given convergence rate is provided by the *asymptotic variance*. Suppose that $\{y_n\}$ is such that $E[y_n] = 0$ and $E\left[(y_n)^2\right] = \text{var}[y_n]$ converges to 0 at the rate $1/n^r$ for some r > 0. Then the asymptotic variance is given by the constant $AV = \lim_{n\to\infty} n^r \text{var}(y_n)$. A higher asymptotic variance means that the speed of convergence is slower. It is worth noting that if the sequence $\{y_n\}$ is normally distributed then $\sqrt{n^r}(y_n)$ converges in distribution to N(0, AV). Indeed, a normal random variable is characterized by mean and variance and we have that $\text{var}\left[\sqrt{n^r}(y_n)\right] = n^r \text{var}[y_n]$ tends to AV as n tends to infinity.

The equilibria of the finite markets tend to the equilibrium of the continuum economy as the market grows large. This justifies the use of the continuum model as an approximation to the large market with supply function competition. We characterize also the speed at which this convergence occurs. We consider, in turn, convergence to price taking and convergence to the continuum model as the economy is replicated.

The following proposition characterizes the convergence of the LBSFE to a price-taking equilibrium as the market grows.

<u>Proposition 3</u>. As the market grows large the market price p_n (at the LBSFE) converges in mean square to the price-taking Bayesian price p_n^c at the rate of 1/n. (That is, $E(p_n - p_n^c)^2$ tends to 0 at the rate of $1/n^2$.) The difference between (per capita) expected

deadweight loss at the LBSFE and at the Bayesian price-taking equilibrium (ETS_n^c - ETS_n)/n is of the order of $1/n^2$.

Sketch of proof: The results follow because $\hat{a}_n - a_n, \hat{b}_n - b_n$, and $\hat{c}_n - c_n$ are of the order of 1/n (from the expressions for a_n and c_n in Proposition 1, and c in Proposition 2 we obtain that $a_n \to a$ and $c_n \to c$ at the rate 1/n) and both p_n and p_n^c depend on $\tilde{s}_n$. Basically, the departure from price taking (marginal cost) is of the order of 1/n and the deadweight loss is of the order of the square of it. (A complete proof of a similar result for Cournot competition –Proposition 6 - can be found in the Appendix.)

Proposition 4. Let $\rho \in [0,1)$. As n tends to infinity the symmetric LBSFE of the n-replica market converges to the limit equilibrium:

(i) $p_n - p \underset{n}{\to} 0$ in mean square at the rate of $1/\sqrt{n}$;

(ii) $\sqrt{n}(p_n - p)$ converges in distribution to $N\left(0, \left(\frac{\beta}{\beta+\lambda}\right)^2 \left((1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2\right)\right)$.

Proof:

We know that $a_n \to a$ and $c_n \to c$ at the rate 1/n and that $\tilde{\theta}_n \to \tilde{\theta}$ in mean square at the rate $1/\sqrt{n}$. We have also that

$$\mathrm{Var}(p_n - p) = \mathrm{Var}\left(\frac{\beta a_n}{1+\beta c_n}\tilde{s}_n - \frac{\beta a}{1+\beta c}\tilde{\theta}\right) =$$

$$= \beta^2 \mathrm{Var}\left(\frac{a_n}{1+\beta c_n}\tilde{\theta}_n - \frac{a}{1+\beta c}\tilde{\theta} + \frac{a_n}{1+\beta c_n}\tilde{\varepsilon}_n\right) =$$

$$= \beta^2\left(\mathrm{Var}\left(\frac{a_n}{1+\beta c_n}\tilde{\theta}_n - \frac{a}{1+\beta c}\tilde{\theta}\right) + \left(\frac{a_n}{1+\beta c_n}\right)^2 \frac{\sigma_\varepsilon^2}{n}\right) =$$

$$= \beta^2\left(\left(\frac{a_n}{1+\beta c_n}\right)^2\left(\frac{1+(n-1)\rho}{n}\right)\sigma_\theta^2 - 2\left(\frac{a_n}{1+\beta c_n}\right)\left(\frac{a}{1+\beta c}\right)\rho\sigma_\theta^2 + \left(\frac{a}{1+\beta c}\right)^2 \rho\sigma_\theta^2\right) +$$

$$+\beta^2\left(\left(\frac{a_n}{1+\beta c_n}\right)^2 \frac{\sigma_\varepsilon^2}{n}\right).$$

It follows then that $\lim_{n\to\infty} \mathrm{Var}(p_n - p) = 0$, i.e. p_n converges to p in mean square.

Furthermore,

$$\lim_{n\to\infty} \mathrm{Var}\left(\sqrt{n}(p_n - p)\right) = \beta^2\left(\left(\frac{a}{1+\beta c}\right)^2 \sigma_\theta^2 - \left(\frac{a}{1+\beta c}\right)^2 \rho\sigma_\theta^2 + \left(\frac{a}{1+\beta c}\right)^2 \sigma_\varepsilon^2\right) =$$

$$= \beta^2\left(\frac{a}{1+\beta c}\right)^2 \left((1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2\right).$$

Given that $\lim_{n\to\infty} E\left(\sqrt{n}(p_n - p)\right) = 0$ we obtain

$$\sqrt{n}(p_n - p) \xrightarrow{d} N\left(0, \beta^2\left(\frac{a}{1+\beta c}\right)^2 \left((1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2\right)\right).$$

The term $\frac{\beta a}{1+\beta c}$ included in the variance of the limiting distribution can be simplified further if we use the results of Proposition 2 where we obtained that

$$c = a - \frac{1}{\beta}(1 - \lambda a).$$

This implies that

$$\frac{\beta a}{1+\beta c} = \frac{\beta a}{\beta a + \lambda a} = \frac{\beta}{\beta + \lambda}.$$

This proves the proposition.

Convergence to the equilibrium of the continuum economy happens at the rate $1/\sqrt{n}$ at which the average error in the signals of the agents $\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i$ tends to zero. Convergence is faster when we are closer to a common value environment, with better signals, or closer to the case with less prior uncertainty.

4. Cournot competition

Consider the market exactly as before but now firm i sets a quantity contingent on its information $\{s_i\}$.[5] The firm has no other source of information and, in particular, does not condition on the price. The expected profits of firm i conditional on receiving signal s_i and assuming firm j, $j \neq i$, uses strategy $X_j(\cdot)$, are

$$E\left(\pi_i \left| s_i\right.\right) = x_i\left(P_n\left(\sum_{j\neq i} X_j(s_j) + x_i\right) - E\left(\theta_i \left| s_i\right.\right)\right) - \frac{\lambda}{2}x_i^2 .$$

From the F.O.C. of the optimization of a firm we obtain

$$p - \left(E\left(\theta_i \left| s_i\right.\right) + \lambda x_i\right) = \left(\frac{\beta}{n}\right)x_i .$$

A similar relation holds for the margin over average expected marginal cost $E\left[MC_n\right] = \frac{1}{n}\sum_{i=1}^{n}\left(E\left(\theta_i \left| s_i\right.\right) + \lambda x_i\right) = \frac{1}{n}\sum_{i=1}^{n} E\left(\theta_i \left| s_i\right.\right) + \lambda\tilde{x}_n$:

$$\frac{p - E\left[MC_n\right]}{p} = \frac{1}{n\eta_n}$$

5 See Vives (2002) for related results when cost parameters are i.i.d.

where $\eta_n = p/(\beta\tilde{x}_n)$ is the elasticity of demand. The margins are augmented from the supply function equilibrium case since they correspond to the case of zero slope of supply ($c_n = 0$).

The following proposition characterizes the Bayesian Cournot equilibrium and the Bayesian price-taking equilibrium (denoted by a superscript c for "competitive"). The proof is standard and is presented in the Appendix.

Proposition 5. In the linear-normal model there is a unique Bayesian Cournot equilibrium and a unique Bayesian price-taking equilibrium. They are symmetric, and affine in the signals. Letting $\xi \equiv \tau_\varepsilon/(\tau_\theta + \tau_\varepsilon)$ the strategies of the firms are given (respectively) by:

$$X_n(s_i) = b_n(\alpha - \bar{\theta}) - a_n(s_i - \bar{\theta}), \text{ where } a_n = \frac{\xi}{\frac{2\beta}{n} + \lambda + \beta\frac{n-1}{n}\rho\xi}, \text{ and } b_n = \frac{1}{\lambda + \beta\left(\frac{1+n}{n}\right)};$$

$$X_n^c(s_i) = b_n^c(\alpha - \bar{\theta}) - a_n^c(s_i - \bar{\theta}), \text{ where } a_n^c = \frac{\xi}{\frac{\beta}{n} + \lambda + \beta\frac{n-1}{n}\rho\xi}, \text{and } b_n^c = \frac{1}{\lambda + \beta}.$$

Remark: From the F.O.C. of profit maximization it is immediate that in equilibrium expected profits for firm i are given by $E\pi_n = (\frac{\lambda}{2} + \frac{\beta}{n})\, E\,(X_n(s_i))^2$.

Remark: In the case of independent values (i.e. $\rho = 0$ and $\tau_\varepsilon = \infty$) the formulae are valid for a general distribution of the uncertainty.

When $\rho = 0$ we can see easily that in Cournot competition firms are more cautious when responding to their private information: $a_n^{\text{Cournot}} < a_n^{\text{SF}}$ whenever supply functions are upward sloping.

We show here that Bayesian Cournot equilibria converge to (Bayesian) price-taking equilibria as n tends to infinity. This justifies the use of the continuum model as an approximation to the large Cournot market. We characterize also the speed at which this convergence occurs.

We consider, in turn, convergence to price taking and convergence to the continuum model as the economy is replicated. The following proposition characterizes the convergence of the Bayesian Cournot equilibrium to a price-taking equilibrium. It is worth noting that the price-taking equilibrium, either in the finite or limit economy, does not aggregate information except in the independent values case (see Vives (2002)). In any case as the market grows large there is no convergence to a full information equilibrium. The proof is in the Appendix.

Proposition 6. As the market grows large the market price p_n (at the Bayesian Cournot equilibrium) converges in mean square to the price-taking Bayesian price p_n^c at the rate of 1/n. (That is, $E(p_n - p_n^c)^2$ tends to 0 at the rate of $1/n^2$.) The difference between (per capita) expected deadweight loss at the market outcome and at the Bayesian price-taking equilibrium $(ETS_n^c - ETS_n)/n$ is of the order of $1/n^2$.

As the market grows large market power (in terms of the margin over marginal cost) dissipates at the rate of 1/n and the welfare loss with respect to the price-taking equilibrium at the rate of $1/n^2$. These are the same rates of convergence as in the Cournot oligopoly with no uncertainty.

The following proposition characterizes convergence of the Bayesian Cournot equilibria to the price-taking equilibrium of the continuum economy as the market grows large. (The proof is in the Appendix.)

Proposition 7. As the economy is replicated the Bayesian Cournot equilibrium converges to the equilibrium in the continuum limit economy: $X(s_i) = b(\alpha - \bar{\theta}) - a(s_i - \bar{\theta})$, where a =

$\xi/(\lambda+\beta\rho\xi)$ and $b = 1/(\lambda+\beta)$. The Bayesian Cournot price p_n converges (in mean square) to $p = \alpha - \beta\,(b(\alpha - \bar{\theta}) - a(\tilde{\theta} - \bar{\theta}))$ at the rate of $1/\sqrt{n}$ and $\sqrt{n}(p_n - p)$ converges in distribution to $N(0, \beta^2 a^2((1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2))$.

Convergence is slower, according to the asymptotic variance $\beta^2 a^2((1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2)$, with larger σ_θ^2 or β and faster with larger ρ or λ. A larger ρ means that we are closer to a common value environment. As with supply function competition convergence is faster when we are closer to a common value environment or with less prior uncertainty.

However, the effect of an increase in σ_ε^2 is ambiguous: the direct effect is to slow down convergence but the indirect effect is to lower the response to information, which has the opposite effect. In the supply function equilibrium only the first effect applies because changes in a are neutralized by changes in the slope of supply c.

In summary, we have checked that the equilibria obtained in the continuum markets, be it with supply function or Cournot competition, are not an artifact but the limit of equilibria in finite economies. Furthermore, the convergence rate to the limit equilibrium is in both cases $1/\sqrt{n}$ for prices, where n is the "size" of the market, and convergence is slower for higher prior uncertainty and faster when closer to a common value environment. Finally, convergence to price-taking is faster also in both cases, at the rate of $1/n$ for prices and $1/n^2$ for the welfare loss (i.e. the deadweight loss with respect to price-taking).

Appendix

Proof of Proposition 5: Drop the subscript n labeling the replica market and let $\beta = 1$. We consider first the Bayesian Cournot equilibrium. We check that the candidate strategies form an equilibrium. The expected profits of firm i conditional on receiving signal s_i and assuming firm j, $j \neq i$, uses strategy $X_j(\cdot)$, are

$$E(\pi_i | s_i) = x_i \left(\alpha - E(\theta_i | s_i) - \frac{1}{n} \sum_{j \neq i} E(X_j(s_j) | s_i) - \left(\frac{1}{n} + \frac{\lambda}{2} \right) x_i \right).$$

Then first order conditions (F.O.C.) yield

$$2\left(\frac{1}{n} + \frac{\lambda}{2} \right) x_i(s_i) = \alpha - E(\theta_i | s_i) - \frac{1}{n} \sum_{j \neq i} E(X_j(s_j) | s_i), \quad \text{for } i = 1, \ldots, n.$$

Plugging in the candidate equilibrium strategy and using the formulae for the conditional expectations for $E(\theta_i | s_i)$ and $E(s_j | s_i)$,

$$E(\theta_i | s_i) = \xi s_i + (1 - \xi) \bar{\theta} \text{ and } E(s_j | s_i) = E(\theta_j | s_i) = \xi \rho s_i + (1 - \xi \rho) \bar{\theta},$$

it is easily checked that they satisfy the F.O.C. (which are also sufficient in our model). To prove uniqueness (1) we show that the Bayesian Cournot equilibria of our game are in one-to-one correspondence with person-by -person optimization of an appropriately defined concave quadratic team function; (2) we note that person-by-person optimization is equivalent in our context to the global optimization of the team function (since the random term does not affect the coefficients of the quadratic terms and the team function is concave in actions, Radner (1962, Theorem 4)); and (3) we invoke the result by Radner, which implies that in our linear-quadratic model with the type of uncertainty considered and jointly normal random variables, the components of the unique Bayesian team decision function of the equivalent team problem are affine (Radner (1962, Theorem 5)). Based on the above three observations we conclude that the affine Bayesian Cournot equilibrium is the unique equilibrium.[6]

Let us show (1) by displaying an appropriate team function G. A team decision rule $(X_1(s_1), ..., X_n(s_n))$ is person-by-person optimal if it can not be improved by changing only one component $X_i(\cdot)$. (This just means that each agent maximizes the team objective conditional on his information and taking as given the strategies of the other agents.) Let $G(x) = \pi_i(x) + f_i(x_{-i})$ where

$$f_i(x_{-i}) = \sum_{j \neq i} (\alpha - \theta_j) x_j - \left(\frac{1}{n} + \frac{\lambda}{2} \right) \sum_{j \neq i} x_j^2 - \frac{1}{2n} \sum_{\substack{k \neq j \\ k, j \neq i}} x_k x_j \text{ .}$$

This yields

$$G(x) = \sum_j (\alpha - \theta_j) x_j - \left(\frac{1}{n} + \frac{\lambda}{2} \right) \sum_j x_j^2 - \frac{1}{2n} \sum_{i \neq j} x_i x_j \text{ .}$$

We obtain the same outcome by solving either $\max_{x_i} E(\pi_i | s_i)$ or $\max_{x_i} E(G | s_i)$ since $f_i(x_{-i})$ does not involve x_i.

A similar argument establishes the result for the Bayesian price-taking equilibrium. Then the F.O.C. for firm i is given by

$$\lambda x_i(s_i) = \alpha - E(\theta_i | s_i) - \frac{1}{n} \sum_j E(X_j(s_j) | s_i) \text{ ,}$$

and the solution is a (person-by-person) maximum of a team problem with an objective function which is precisely the ETS. ♦

In order to perform welfare comparisons we will need the following Lemma.

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6 This method of showing uniqueness of Bayesian Cournot equilibrium in linear-quadratic models with normal distributions (or more in general, with affine information structures) has been used by Basar and Ho (1974) and Vives (1988).

Lemma. The difference in (per capita) ETS between a price-taking regime R and another regime with strategies based on less information (that is, on a weakly coarser information partition) is given by $(ETS^R - ETS)/n = (\beta E(\tilde{x}_n - \tilde{x}_n^R)^2 + \lambda (\Sigma_i E(x_{in} - x_{in}^R)^2/n)/2$.

The result follows considering a Taylor series expansion of TS (stopping at the second term due to the quadratic nature of the payoff) around price-taking equilibria. The key to simplify the computations is to notice that at price-taking equilibria total surplus is maximized. Note that if the strategies and the information structure are symmetric then $E(x_{in} - x_{in}^R)^2$ is independent of i and therefore $\Sigma_i E(x_{in} - x_{in}^R)^2/n = E(x_{in} - x_{in}^R)^2$.

Proof of Proposition 6: Let $y_n = p_n - p_n^c = \tilde{x}_n^c - \tilde{x}_n = (b_n^c - b_n)(\alpha - \bar{\theta}) + (a_n - a_n^c)(\tilde{s}_n - \bar{\theta})$. Recall that $E\left[(y_n)^2\right] = \left(E[y_n]\right)^2 + \mathrm{var}[y_n]$. We have that $E[y_n] = (b_n^c - b_n)(\alpha - \bar{\theta})$ because $E\tilde{s}_n = \bar{\theta}$. It is easily seen that $(b_n^c - b_n)$ is of order 1/n (indeed, $n(b_n^c - b_n)$ tends to $1/(1+\lambda)^2$ as n tends to infinity). Therefore $\left(E[y_n]\right)^2$ is of order $1/n^2$. Furthermore, $\mathrm{var}[y_n] = (a_n - a_n^c)^2 \ \mathrm{var}\ \tilde{s}_n$. We have that $\mathrm{var}\ \tilde{s}_n = ((1+(n-1)\rho)\sigma_\theta^2 + \sigma_\varepsilon^2)/n$, which is of the order of a constant for $\rho > 0$ (or 1/n for $\rho = 0$), and that $(a_n - a_n^c)$ is of order 1/n (because $n(a_n - a_n^c)$ tends to $((\rho\xi+\lambda)(2\rho\xi+\lambda))^{-1}$ as n tends to infinity). Therefore the order of $\mathrm{var}[y_n]$ is $1/n^2$ for $\rho > 0$ (or $1/n^3$ for $\rho = 0$). We conclude that in any case the order of $y_n = p_n - p_n^c$ is 1/n. Consider $(ETS_n^c - ETS_n)/n$ now. According to the Lemma above and given that equilibria are symmetric we have that $(ETS_n^c - ETS_n)/n = (\beta E(\tilde{x}_n - \tilde{x}_n^c)^2 + \lambda (E(x_{in} - x_{in}^c)^2)/2$. We have just shown $E(\tilde{x}_n - \tilde{x}_n^c)^2$ to be of order $1/n^2$. We have that $E(x_{in} - x_{in}^c)^2 = (E(x_{in} - x_{in}^c))^2 + \mathrm{var}\,(x_{in} - x_{in}^c)$. Now, $E(x_{in} - x_{in}^c)$ is of the same order as $E(\tilde{x}_n - \tilde{x}_n^c)$, 1/n, and $\mathrm{Var}\,(x_{in} - x_{in}^c) = (a_n^c - a_n)^2 (\sigma_\theta^2 + \sigma_\varepsilon^2)$, is of order $1/n^2$ because $(a_n^c - a_n)$ is of order 1/n. Therefore, $E(x_{in} - x_{in}^c)^2$ is of order $1/n^2$. We conclude that $(ETS_n^c - ETS_n)/n$ is of the order of $1/n^2$. ♦

Proof of Proposition 7: For the first part, from Proposition 4 we have that

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\xi}{\frac{2\beta}{n} + \lambda + \beta \frac{n-1}{n}\rho\xi} = \frac{\xi}{\lambda + \beta\rho\xi} = a$$

and

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\lambda + \beta \frac{1+n}{n}} = \frac{1}{\lambda + \beta} = b\,.$$

Hence,

$$\lim_{n\to\infty} x_n\left(s_i\right) = \lim_{n\to\infty}\left[b_n\left(\alpha - \bar{\theta}\right) - a_n\left(s_i - \bar{\theta}\right)\right] = b\left(\alpha - \bar{\theta}\right) - a\left(s_i - \bar{\theta}\right) = x\left(s_i\right).$$

For the second part of the proposition note that $\tilde{x}_n = b_n(\alpha - \bar{\theta}) - a_n(\tilde{s}_n - \bar{\theta})$. We have that

$$p_n = \alpha - \beta\left[b_n\left(\alpha - \bar{\theta}\right) - a_n\left(\tilde{s}_n - \bar{\theta}\right)\right]$$
$$p = \alpha - \beta\left[b\left(\alpha - \bar{\theta}\right) - a\left(\tilde{\theta} - \bar{\theta}\right)\right]$$

and $E(p_n - p) = \beta(b - b_n)(\alpha - \bar{\theta})$ since $E\tilde{s}_n = E\tilde{\theta} = \bar{\theta}$. Note that $(b - b_n)$, and therefore $E(p_n - p)$, tends to 0 as n tends to infinity. Since $\text{var}\left(p_n - p\right) = E\left(p_n - p\right)^2 - \left[E\left(p_n - p\right)\right]^2$ to conclude that $\lim_{n\to\infty} E\left(p_n - p\right)^2 = 0$ it is sufficient to show that $\lim_{n\to\infty} \text{var}\left[\sqrt{n}\left(p_n - p\right)\right] = \beta^2 a^2 ((1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2)$. From which it follows that $\sqrt{n}(p_n - p)$ converges in distribution to $N(0, \beta^2 a^2((1-\rho)\sigma_\theta^2 + \sigma_\varepsilon^2))$ because $(p_n - p)$ is normally distributed. We have that

$$\text{var}\left[\sqrt{n}\left(p_n - p\right)\right] = n\beta^2 \text{var}\left[a_n\tilde{s}_n - a\tilde{\theta}\right] = \beta^2 \frac{\left\{a_n^2\left[\frac{\left(1+\rho\left(n-1\right)\right)\sigma_\theta^2}{n} + \frac{\sigma_\varepsilon^2}{n}\right] - 2a_n a\rho\sigma_\theta^2 + a^2\rho\sigma_\theta^2\right\}}{1/n}.$$

Using L'Hopital's rule we obtain $\lim_{n\to\infty} \text{var}\left[\sqrt{n}\left(p_n - p\right)\right] =$

$$\beta^2 \lim_{n\to\infty} \frac{\left\{2a_n \frac{\partial a_n}{\partial n}\left[\frac{\left(1+\rho\left(n-1\right)\right)\sigma_\theta^2}{n}+\frac{\sigma_\varepsilon^2}{n}\right]+a_n^2\left[-\frac{\left(1-\rho\right)\sigma_\theta^2}{n^2}-\frac{\sigma_\varepsilon^2}{n^2}\right]-2\frac{\partial a_n}{\partial n}a\rho\sigma_\theta^2\right\}}{-\frac{1}{n^2}}=$$

$$=\beta^2 a^2\left[\left(1-\rho\right)\sigma_\theta^2+\sigma_\varepsilon^2\right].♦$$

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