Vertical Integration and Risk Management in Competitive Markets of Non-Storable Goods

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Outline

Motivation

- Study vertical integration from the perspective of risk management
- Compare to forward hedging
- Understand the relationship between retail, forward and spot prices

Vertical Integration in the litterature

→ Vertical integration often studied in the context of market/contract imperfections

Inspired by some papers on forward equilibrium:

- Allaz: Oligopoly , Uncertainty and Strategic Forward Transactions (1992)
- Bessembinder and Lemmon: Equilibrium Pricing and Optimal Hedging in Electricity Forward Markets (2002)

But few references including a retail market.

Actors

We consider a set $\mathcal{K}$ of actors:

- Subset $\mathcal{P}$ of producers: cost c_k, generation level S_k
- Subset $\mathcal{R}$ of retailers: market share α_k (in % of total demand)
- All actors are traders: buy f_k forward, G_k spot
- Mean-variance utility: $\mathrm{MV}_{\lambda_k}[\Pi_k] = \mathbb{E}[\Pi_k] - \lambda_k \mathrm{Var}[\Pi_k]$

A 2-step model (equiv to 3-step):

- Retail and forward decisions at $t = 0$, spot decisions at $t = 1$
- Inelastic and random demand D at $t = 1$
- Competitive equilibrium

Market Equilibrium

- Retail: retail price p and market shares α_k s.t.

$$1 = \sum_{k \in \mathcal{R}} \alpha_k.$$

- Forward: forward price q and forward positions f_k s.t.

$$0 = \sum_{k \in \mathcal{K}} f_k.$$

- Spot: spot price P and spot positions G_k s.t.

$$0 = \sum_{k \in \mathcal{K}} G_k.$$

- Generation: generation levels S_k s.t.

$$D = \sum_{k \in \mathcal{P}} S_k$$

Profit

- Actor k's profit

$$p\alpha_k D\mathbf{1}_{\{k\in\mathcal{R}\}} - qf_k - PG_k - c_k\left(S_k\right)\mathbf{1}_{\{k\in\mathcal{P}\}}$$

- Non-storability condition at $t = 1$

$$\alpha_k D\mathbf{1}_{\{k\in\mathcal{R}\}} = f_k + G_k + S_k\mathbf{1}_{\{k\in\mathcal{P}\}}$$

- Profit thus reads

$$(p-P)\alpha_k D\mathbf{1}_{\{k\in\mathcal{R}\}} + (P-q)f_k + \left(PS_k - c_k\left(S_k\right)\right)\mathbf{1}_{\{k\in\mathcal{P}\}}$$

- Sum of retail, forward and spot profits

Spot Market Equilibrium and Profit Function

- Spot market equilibrium

$$P^* = C'(D)\ ,\ S_k^* = (c_k')^{-1}(P^*)$$

where C is the aggregated cost function

- Actor k's generation profit

$$\Pi_k^g \quad := \quad \left(P^* S_k^* - c_k\left(S_k^*\right)\right)\mathbf{1}_{\{k\in\mathcal{P}\}}$$

- Actor k's profit function

$$\Pi_k(p, q, \alpha_k, f_k) \quad = \quad \Pi_k^r(p, \alpha_k) + \Pi_k^t(q, f_k) + \Pi_k^g$$

with $\Pi_k^s := (p - P^*)\alpha_k D\mathbf{1}_{\{k\in\mathcal{R}\}}$ and $\Pi_k^t := (P^* - q)f_k$

Finding the Equilibrium

- 2 cases: without and with a forward market
- Quadratic utility + Linear constraints $\Rightarrow$ Explicit solution

$\longrightarrow$ Interpretation of equations and fast computation

Equilibrium

Proposition

The equilibrium is given by:

$$\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\Lambda_{\mathcal{R}}}{\lambda_k} \frac{\mathrm{Cov}[\Pi^r, \Pi_l^g]}{\mathrm{Var}[\Pi^r]} - \frac{\mathrm{Cov}[\Pi^r, \Pi_k^g]}{\mathrm{Var}[\Pi^r]}$$

and p^* is the smallest root of:

$$0 = \mathbb{E}[(p^* - P^*)D] - 2\Lambda_{\mathcal{R}} \mathrm{Cov}[(p^* - P^*)D, (p^* - P^*)D + \Pi_l^g]$$

$$\Lambda_{\mathcal{R}}^{-1} := \sum_{k \in \mathcal{R}} \lambda_k^{-1},\ \Pi_l^g := \sum_{k \in \mathcal{P} \cap \mathcal{R}} \Pi_k^g,\ \Pi^r := \sum_{k \in \mathcal{R}} \Pi_k^r$$

1^{st} Comments

- Risk neutral retail price: $p^0 = \frac{\mathbb{E}[P^* D]}{\mathbb{E}[D]}$
- No integration $\Rightarrow \alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k}$
- Presence of integrated producers $\Rightarrow p^*$ decreases
- Integrated actors have higher market shares

$\longrightarrow$ No intuition on the utility of the actors

Equilibrium on the forward market

Proposition

The equilibrium on the forward market is given by:

$$f_k^* = \frac{\Lambda}{\lambda_k}\frac{\mathrm{Cov}\left[P^*, \Pi^e\right]}{\mathrm{Var}\left[P^*\right]} - \frac{\mathrm{Cov}\left[P^*, \Pi_k^g\right]}{\mathrm{Var}\left[P^*\right]} - \alpha_k^* \frac{\mathrm{Cov}\left[P^*, \Pi^r\right]}{\mathrm{Var}\left[P^*\right]}$$

$$q^* = \mathbb{E}[P^*] - 2\Lambda \mathrm{Cov}[P^*, p^* D - C(D)]$$

$$\Pi^e := \sum_{k \in \mathcal{K}} \Pi_k = p^* D - C(D), \ \Lambda^{-1} := \sum_{k \in \mathcal{K}} \lambda_k^{-1}$$

- Classical formula for q^* (as in Allaz or B.&L.), independent of market shares
- Forward positions split in trading, generation and retail components

Equilibrium on the retail market

Proposition (end)

The equilibrium on the retail market is given by:

$$\alpha_k^* = \frac{\Lambda_{\mathcal{R}}}{\lambda_k} + \frac{\mathrm{Cov}[P^*, \Pi^r]}{\Delta}\mathrm{Cov}\left[P^*, \Pi_k^g - \frac{\Lambda_{\mathcal{R}}}{\lambda_k}\Pi_l^g\right] - \frac{\mathrm{Var}[P^*]}{\Delta}\mathrm{Cov}\left[\Pi^r, \Pi_k^g - \frac{\Lambda_{\mathcal{R}}}{\lambda_k}\Pi_l^g\right]$$

$$0 = \mathbb{E}[(p^* - P^*)D] - 2\Lambda_{\mathcal{R}}\mathrm{Cov}[(p^* - P^*)D, (p^* - P^*)D + \Pi_l^g] + 2\Lambda_{\mathcal{R}}\frac{\mathrm{Cov}[P^*, (p^* - P^*)D]}{\mathrm{Var}[P^*]}\mathrm{Cov}\left[P^*, (p^* - P^*)D + \Pi_l^g\right] - 2\Lambda_{\mathcal{R}}\frac{\mathrm{Cov}[P^*, (p^* - P^*)D]}{\mathrm{Var}[P^*]}\mathrm{Cov}\left[P^*, \frac{\Lambda}{\Lambda_{\mathcal{R}}}(p^* D - C(D))\right]$$

$$\Delta := \mathrm{Var}[P^*]\mathrm{Var}[\Pi^r] - \mathrm{Cov}^2[P^*, \Pi^r]$$

Comments

$\longrightarrow$ Retail equilibrium difficult to analyze

Nonetheless, we can show that:

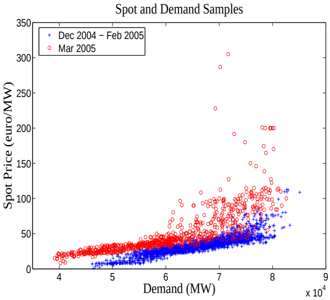
- $\mathcal{P} \subset \mathcal{R} = \mathcal{K} \Rightarrow$ No impact of forward market on retail price
- No integration $\Rightarrow$ Forward market decreases retail price
- If quadratic cost functions, integration decreases retail price

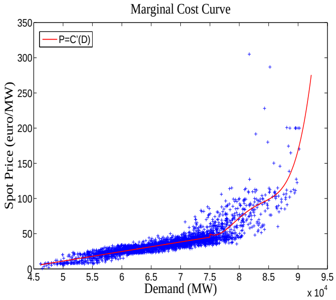
Moreover the model shows a strong asymmetry between retailers and suppliers:

- Forward hedging always profitable for producers, not the case for retailers! Downward impact on retail price.
- Π_k^g independent of p, Π_k^r dependent of P

Data set

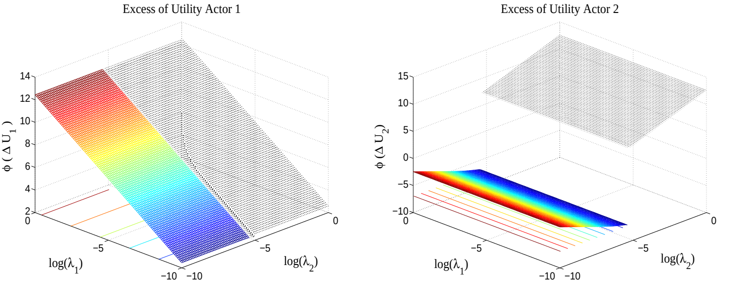
- Spot and Demand hourly data from Dec 2004 to Mar 2005
- Regress cost curve C
- Test different configurations





Asymmetry Retailer-Producer

- Pure retailer vs Pure producer
- Excess of utility due to forward trading



Similarities between VI & FW

- Downward impact on retail price
- Upward impact on market shares
- Tend to decrease retailers' utility (trade-off Gain-Risk)
- The presence of 1 lever drastically reduces the impact of the 2^{nd}

$\longrightarrow$ Little impact of VI on price and utility in the presence of a forward market, only on market shares

Discrepancies

- VI restores symmetry between actors (equilibrium always exists)
- Retail contract = non-linear contract in D. Better to hedge a non-linear profit, but less flexible.
- Under high risk aversion, VI is more robust (existence of equilibrium) and can increase retailers' utility $\Rightarrow$ incentive to integrate
- No trading is never an equilibrium, whereas no integration can be

Conclusion

Under perfect competition:

- FW and VI have similar impact on equilibrium
- Downward impact on retail price that can offset risk reduction
- No clear advantage of VI in the presence of a forward market
- Study shows strong asymmetry between retailers and producers $\longrightarrow$ decreased by VI

Extensions:

- Market power
- True utility function