Modeling and Computing Two-settlement Oligopolistic Equilibrium in a Congested Electricity Network

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Objectives and Scope

- Generalize the Allaz Vila model to a realistic constrained network setting
  - Explore formulation issues.
  - Does the basic intuition still hold?
  - Explore computational feasibility.
  - Develop special purpose computational tools.
  - Test cases.
Electricity Market Model

Generators
(Cournot Players)
Load nodes
(Demand functions)
Transmission network
(Lossless DC approximation of Kirchhoff laws)
System Operator
(Set import and export quantities and nodal prices so as to meet transmission constraints)
Probabilistic Contingencies
(Load uncertainty, Gen outages, Line outages)
Two Settlement Market Equilibrium

- Generators enter into forward contracts to supply specified quantities at agreed upon prices (forward markets) and decide in real time (spot market) how much to produce.

- Forward and spot markets may have different granularly of settlement points.
  - Nodal spot market
  - Forward contracts are settled at clusters of nodes (Hubs) based on a weighted average of the nodal spot prices.
Two-settlement Model Structure

Forward market (upper level)

Firms sign forward contracts (Financial CFDs)

Spot market (Lower level)

Nature picks a state \( c \).

Firms produce & ISO re-dispatches

Nodal prices & Congestion rents

Forward prices

Expected zonal settlement prices

Zonal settlement prices

Solution Concept: Subgame Perfect Nash equilibrium

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The ISO Problem

- Conducts the energy redispach
- Sets locational prices and transmission charges
- Maximizes social welfare

\[
\begin{align*}
\max_{\{r^c_i, i \in N\}} & \quad \sum_{i \in N} \left( \int_0^{r^c_i} \left[ P_i(\tau_i) d\tau_i - C_i(q^c_i) \right] \right) \\
\text{subject to:} & \\
\sum_{i \in N} r^c_i &= 0 & \text{Balancing constraint} \\
-K^c_l & \leq \sum_{i \in N} D^c_{l,i} r^c_i & \leq K^c_l, \quad l \in L & \text{Flows constraints}
\end{align*}
\]
KKT conditions for the ISO problem

\[ P_i(q_i + r_i) - p - \varphi_i = 0 \quad i \in N \]
\[ \varphi_i = \sum_{m \in L} (\lambda^+_m - \lambda^-_m)D_{m,i} \]
\[ \sum_{j \in N} r_j = 0 \]
\[ 0 \leq \lambda^-_l \perp \sum_{j \in N} D_{l,j}r_j + K_l \geq 0 \quad l \in L \]
\[ 0 \leq \lambda^+_l \perp K_l - \sum_{j \in N} D_{l,j}r_j \geq 0 \quad l \in L \]
Modeling Choices

- Cournot generation firms act as multi-Stackelberg leaders anticipating the outcome of the ISO redispach.
- Cournot generation firms and ISO move simultaneously as Nash players taking each other’s strategic variables as parameters in their optimization problem.
  - The ISO’s strategic variables are import/export quantities for each node (Cournot-Cournot).
  - The ISO’s strategic variables are the nodal injection charges = nodal premium relative to the slack bus price (Cournot-Bertrand).
Generation Firms Anticipate ISO Redispachtch

$$\max_{q_i; i \in N_g} \sum_{i \in N_g} \left( P_i(q_i + r_i)q_i - C_i(q_i) \right)$$

subject to:

$$q_i \leq q_i \leq \bar{q}_i \quad i \in N_g$$

$$P_i(q_i + r_i) - p - \varphi_i = 0 \quad i \in N$$

$$\varphi_i = \sum_{m \in L} (\lambda^+_m - \lambda^-_m)D_{m,i}$$

$$\sum_{j \in N} r_j = 0$$

$$0 \leq \lambda^-_i \perp \sum_{j \in N} D_{l,j}r_j + K_l \geq 0 \quad l \in L$$

$$0 \leq \lambda^+_i \perp K_l - \sum_{j \in N} D_{l,j}r_j \geq 0 \quad l \in L$$
Problem with Sequential Move Formulation

- A two settlement model will result in a non-tractable three level optimization problem.
- Generation firms in spot market have incentive to induce degeneracy in ISO problem (Induce flow just below constraints to avoid congestion rents):
  - Non-uniqueness
  - Discontinuity in reaction functions
  - Possible non-existence of pure strategy equilibrium
Simultaneous Move Model (generation firms do not account for impact on congestion)

$$\max_{q_i:i \in N_g} \sum_{i \in N_g} \left( P_i(q_i + r_i)q_i - C_i(q_i) \right)$$

subject to:

$$q_i \leq q_i \leq \overline{q}_i \quad i \in N_g$$

$$P_i(q_i + r_i) - p - \varphi_i = 0 \quad i \in N$$

$$\varphi_i = \sum_{m \in L} (\lambda^+_m - \lambda^-_m)D_{m,i}$$

$$\sum_{j \in N} r_j = 0$$

$$0 \leq \lambda^-_l \perp \sum_{j \in N} D_{l,j}r_j + K_l \geq 0 \quad l \in L$$

$$0 \leq \lambda^+_l \perp K_l - \sum_{j \in N} D_{l,j}r_j \geq 0 \quad l \in L$$
Market Equilibrium when ISO’s Strategic Variables are Import/Export Quantities \( \{r_i\} \)

- **Mixed Nonlinear Complementarity Problem (NCP)**

\[
P_i(q_i + r_i) + q_i \frac{\partial P_i(q_i + r_i)}{\partial q_i} - \frac{dC_i(q_i)}{dq_i} + \rho_i^- - \rho_i^+ = 0 \quad i \in N
\]

\[
0 \leq \rho_i^- \perp q_i - q_i^i \geq 0 \quad i \in N
\]

\[
0 \leq \rho_i^+ \perp \overline{q_i} - q_i \geq 0 \quad i \in N
\]

\[
P_i(q_i + r_i) - p - \varphi_i = 0 \quad i \in N
\]

\[
\varphi_i = \sum_{m \in L} (\lambda_m^+ - \lambda_m^-)D_{m,i}
\]

\[
\sum_{j \in N} r_j = 0
\]

\[
0 \leq \lambda_l^- \perp \sum_{j \in N} D_{l,j} r_j + K_l \geq 0 \quad l \in L
\]

\[
0 \leq \lambda_l^+ \perp K_l - \sum_{j \in N} D_{l,j} r_j \geq 0 \quad l \in L
\]

- When demand function are linear and supply functions quadratic it can be reduced to an LCP

\[
w = t + My, 0 \leq w \perp y \geq 0
\]

M is symmetric or bisymmetric PSD
Implications

- No effect of multiple ownership (problem is separable so that each unit is priced independently)
- When there is no congestion import/exports variables are selected so as to equalize nodal prices across nodes.
  - The resulting market equilibrium is different than the Cournot equilibrium when nodal demand is aggregated (residual demands at each node retain the slope of the local demand function)
Firms’ Optimization when ISO Strategic Variables are the Nodal Price Premiums

\[
\begin{align*}
\max_{q_i: i \in N_g, p} & \quad \sum_{i \in N_g} (p + \varphi_i) q_i - \sum_{i \in N_g} C_i(q_i) \\
\text{subject to:} & \\
q_i & \geq q_{\underline{i}} \quad i \in N_g \\
q_i & \leq q_{\bar{i}} \quad i \in N_g \\
\sum_{j \in N} q_j & = \sum_{j \in N} P_j^{-1}(p + \varphi_i) \quad \text{Implicit residual demand function}
\end{align*}
\]

Firms do not see transmission constraints only nodal price premiums
Market equilibrium for simultaneous move model where ISO strategic variables are the locational premiums (Cournot-Bertrand)

\[ p + \varphi_i - \beta - \frac{dC_i(q_i)}{dq_i} + \rho_i^- - \rho_i^+ = 0 \quad i \in N_g \]

\[ \beta \sum_{j \in N} dP_j^{-1}(p + \varphi_j) + \sum_{j \in N_g} q_j = 0 \]

\[ \sum_{j \in N} q_j = \sum_{j \in N} P_j^{-1}(p + \varphi_j) \]

\[ 0 \leq \rho_i^- \perp q_i - q_i \geq 0 \quad i \in N_g \]

\[ 0 \leq \rho_i^+ \perp q_i - q_i \geq 0 \quad i \in N_g \]

\[ P_i(q_i + r_i) - p + \sum_{m \in L} (\lambda_m^- - \lambda_m^+)D_{m,i} = 0 \quad i \in N \]

\[ \sum_{j \in N} r_j = 0 \]

\[ 0 \leq \lambda_i^- \perp \sum_{j \in N} D_{l,j}r_j + K_l \geq 0 \quad l \in L \]

\[ 0 \leq \lambda_i^+ \perp K_l - \sum_{j \in N} D_{l,j}r_j \geq 0 \quad l \in L \]

- Can be reduced to an LCP when demand functions are linear and cost functions are quadratic

\[ w = t + My, \quad 0 \leq w \perp y \geq 0 \]

\[ M \text{ is a bisymmetric PSD matrix} \]

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Implications

- Ownership structure affects results
- When there is no congestion nodal price premiums go to zero and market equilibrium is identical to the single node oligopoly solution.
- The market equilibrium will reflect an oligopoly solution even when the market is separated (strategically decoupled) due to a thin line (zero capacity) or permanently congested line (fixed imports/exports)
Firms’ programs with forward contracts for the Cournot-Bertrand Case

\[
\max_{q^c: i \in N_g, p^c} \sum_{i \in N_g} [(p^c + \varphi_i^c) \cdot q_i^c - C_i(q_i^c)] + \sum_{z \in Z} (F_z - u_z^c) \cdot x_{g,z}
\]

subject to:

\[
q_i^c \geq \underline{q}_i^c \quad i \in N_g
\]

\[
q_i^c \leq \overline{q}_i^c \quad i \in N_g
\]

\[
\sum_{j \in N} q_j^c = \sum_{j \in N} (P_j^c)^{-1} \left( p^c + \varphi_j^c \right)
\]

\[
u_z^c = \sum_{j: z(j) = z} \left( p^c + \varphi_j^c \right) \delta_j \quad z \in Z
\]
Forward Market Decisions

EPEC Formulation – Each firm solves an MPEC

\[
\max_{x_{g,z} \in Z} \sum_{z \in Z} F_z x_{g,z} + \sum_{c \in C} \Pr(c) \left( \sum_{i \in N_g} \left( \sum_{m \in L} (\lambda^c_{m+} - \lambda^c_{m-}) D^c_{m,i} \right) q^c_i - C_i(q^c_i) \right) - \sum_{z \in Z} u^c_z x_{g,z} \]

subject to:

\[
F_z = \sum_{c \in C} \Pr(c) u^c_z \quad z \in Z
\]

\[
u^c_z = \sum_{j: z(j) = z} (p + \varphi_i) \delta_i \quad z \in Z
\]

and

Complementarity conditions characterizing spot market
The EPEC problem structure

- The MPECs for each firm

\[
\begin{align*}
\text{min} & \quad f_g(x_g, y, w, x_{-g}) \\
\text{s.t.} & \quad x_g \in X_g \\
& \quad w = t + A^g x_g + A^{-g} x_{-g} + M y, \quad 0 \leq w \perp y \geq 0
\end{align*}
\]

\(x_g\): decision variable, \(y, w\): state variables, \(x_{-g}\): parameters

- The EPEC

\[
\begin{align*}
\text{min} & \quad f_1(x_1, y, w, x_{-1}) & \text{min} & \quad f_2(x_2, y, w, x_{-2}) & \text{min} & \quad f_3(x_3, y, w, x_{-3}) & \cdots \\
\text{s.t.} & \quad x_1 \in X_1 & \text{s.t.} & \quad x_2 \in X_2 & \text{s.t.} & \quad x_3 \in X_3 \\
& \quad w = t + \sum_g A^g x_g + M y, \quad 0 \leq w \perp y \geq 0
\end{align*}
\]
The EPEC Algorithm

Round 0

Round 1

Round 2

\[ \min_{x_g} f_g(x_g, y, w, x_{-g}) \]

s.t

\[ x_g \in X_g \]

\[ y_\alpha = M^{-1}_g a_\alpha + A^{-g}_g x_g \]

\[ w_\pi = -M^{-1}_g a_\pi (a_\pi + A^-g x_{-g} + A^{g}_g x_g) + (a_\pi + A^{-g}_g x_{-g} + A^{g}_g x_g) \]

\[ w_\alpha = 0 \]

\[ y_\pi = 0 \]
Stylized Belgian System

53 nodes,
71 lines,
19 generators,
6 contingency states
# Contingency states

<table>
<thead>
<tr>
<th>State</th>
<th>Prob</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.2</td>
<td>On-peak state: All demands are on the peak.</td>
</tr>
<tr>
<td>2</td>
<td>.5</td>
<td>Normal state: Demands are at shoulder.</td>
</tr>
<tr>
<td>3</td>
<td>.03</td>
<td>Shoulder demands with line breakdown: Line [31,52] goes down.</td>
</tr>
<tr>
<td>4</td>
<td>.03</td>
<td>Shoulder demands with generation outage: Plant at node 10 goes down.</td>
</tr>
<tr>
<td>5</td>
<td>.04</td>
<td>Shoulder demands with generation outage: Plant at node 41 goes down.</td>
</tr>
<tr>
<td>6</td>
<td>.2</td>
<td>Off-peak state: All demands are off-peak.</td>
</tr>
</tbody>
</table>
Impact of Forward Contracting on Spot Prices (in normal state)

Single Settlement

Two Settlement
Firm’s Forward Commitments

### 2 Firms

<table>
<thead>
<tr>
<th>Outer iteration</th>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>1</td>
<td>-513.063752</td>
<td>575.219726</td>
</tr>
<tr>
<td>2</td>
<td>-331.223467</td>
<td>1546.721883</td>
</tr>
<tr>
<td>3</td>
<td>-545.254227</td>
<td>1747.692181</td>
</tr>
<tr>
<td>4</td>
<td>-552.287608</td>
<td>1747.692181</td>
</tr>
<tr>
<td>5</td>
<td>-552.287608</td>
<td>1747.692181</td>
</tr>
</tbody>
</table>

### 3 Firms

<table>
<thead>
<tr>
<th>Outer iteration</th>
<th>Firm 1</th>
<th>Firm 2</th>
<th>Firm 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>1</td>
<td>6739.889190</td>
<td>-16.249658</td>
<td>-288.471837</td>
</tr>
<tr>
<td>2</td>
<td>6739.889190</td>
<td>246.601419</td>
<td>-103.536223</td>
</tr>
<tr>
<td>3</td>
<td>6851.687937</td>
<td>556.357457</td>
<td>71.319790</td>
</tr>
<tr>
<td>4</td>
<td>7001.487699</td>
<td>849.405693</td>
<td>154.719273</td>
</tr>
<tr>
<td>5</td>
<td>7154.268773</td>
<td>1001.093059</td>
<td>149.846951</td>
</tr>
<tr>
<td>6</td>
<td>7237.416442</td>
<td>1006.167745</td>
<td>149.619740</td>
</tr>
<tr>
<td>7</td>
<td>7239.775870</td>
<td>1006.342137</td>
<td>149.611431</td>
</tr>
<tr>
<td>8</td>
<td>7239.859233</td>
<td>1006.348165</td>
<td>149.611440</td>
</tr>
<tr>
<td>9</td>
<td>7239.862110</td>
<td>1006.348374</td>
<td>149.611129</td>
</tr>
<tr>
<td>10</td>
<td>7239.862110</td>
<td>1006.348382</td>
<td>149.611130</td>
</tr>
<tr>
<td>11</td>
<td>7239.862110</td>
<td>1006.348382</td>
<td>149.611130</td>
</tr>
</tbody>
</table>
The California Network
## Test Case – WECC Light Summer 2005

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of buses</td>
<td>2161</td>
</tr>
<tr>
<td>Number of generation buses</td>
<td>401</td>
</tr>
<tr>
<td>Number of consumption buses</td>
<td>1205</td>
</tr>
<tr>
<td>Number branches/transformers</td>
<td>3398</td>
</tr>
<tr>
<td>Number of firms</td>
<td>16 (9 strategic players)</td>
</tr>
<tr>
<td>Total demand</td>
<td>22700 MW</td>
</tr>
</tbody>
</table>
# Spot and forward trading

<table>
<thead>
<tr>
<th></th>
<th>Total installed capacity (MW)*</th>
<th>Spot output under two settlements (MW)</th>
<th>Forward contracts (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Southern California Edison</td>
<td>22407</td>
<td>4887.1</td>
<td>444.2</td>
</tr>
<tr>
<td>SDG&amp;E</td>
<td>3205</td>
<td>2132.2</td>
<td>0</td>
</tr>
<tr>
<td>WAPA – SNR</td>
<td>825.8</td>
<td>825.8</td>
<td>825.8</td>
</tr>
<tr>
<td>Bureau of Reclamation (PG&amp;E)</td>
<td>1439</td>
<td>1208.6</td>
<td>0</td>
</tr>
<tr>
<td>PG&amp;E customer owned facilities</td>
<td>16720</td>
<td>4770.6</td>
<td>508</td>
</tr>
<tr>
<td>Department of Water Resources</td>
<td>914.3</td>
<td>914.3</td>
<td>914.3</td>
</tr>
<tr>
<td>Sacramento Utility District</td>
<td>2119.5</td>
<td>1893.4</td>
<td>98.8</td>
</tr>
<tr>
<td>PG&amp;E</td>
<td>7921</td>
<td>3320.2</td>
<td>76.1</td>
</tr>
<tr>
<td>Northern California Power Agency</td>
<td>633.4</td>
<td>633.4</td>
<td>633.4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>56185</strong></td>
<td><strong>20847.3</strong></td>
<td><strong>3500.5</strong></td>
</tr>
</tbody>
</table>

- Small units (with capacities less than 10MW) are ignored.
- The total number of iterations is 79 (stopped with a relative error of 1e-5).
Nodal Prices

<table>
<thead>
<tr>
<th></th>
<th>Single settlement</th>
<th>Two settlement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range of nodal prices($/MWh)</td>
<td>27.9 ~ 179.6</td>
<td>28.0 ~ 117.4</td>
</tr>
<tr>
<td>% Change in nodal prices</td>
<td></td>
<td>-56.5 ~ 29.9</td>
</tr>
<tr>
<td>Average nodal prices</td>
<td>87.9</td>
<td>65.6</td>
</tr>
</tbody>
</table>

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Questions?