# Modeling and Computing Two-settlement Oligopolistic Equilibrium in a Congested Electricity Network

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**Abstract**

A model of two-settlement electricity markets is introduced, which accounts for flow congestion, demand uncertainty, system contingencies and market power. We formulate the subgame perfect Nash equilibrium for this model as an equilibrium problem with equilibrium constraints (EPEC), in which each firm solves a mathematical program with equilibrium constraints (MPEC). The model assumes linear demand functions, quadratic generation cost functions and a lossless DC network, resulting in equilibrium constraints as a parametric linear complementarity problem (LCP). We introduce an iterative procedure for solving this EPEC through repeated application of an MPEC algorithm. This MPEC algorithm is based on solving quadratic programming sub-problems and on parametric LCP pivoting. Numerical examples demonstrate the effectiveness of the MPEC and EPEC algorithms and the tractability of the model for realistic size power systems.

## 1 Introduction

Electricity restructuring aims at creating new competitive environments that provide long-term consumer benefits. A major obstacle to this goal is market power, both vertical and horizontal. Vertical market power in electricity markets has been substantially mitigated through the unbundling of the generation, transmission and distribution sectors, and through "open access" to transmission grids. However, horizontal and locational market power remains an important issue to policy makers due to the non-storability of electricity, the lack of demand elasticity, high market concentration and limited transmission capacities.

Among the many proposed and implemented economic means of mitigating horizontal market power is a two-settlement approach where forward contracts and real-time balancing transactions are settled at different prices. Both theoretical analysis and empirical evidences in [1, 2, 8, 14, 26, 30] have suggested that forward contracting decreases the incentives of sellers to manipulate spot market prices since, under two settlements, the volume of trading that can be affected by spot prices is reduced. Allaz [1] assumes a two-period market and demonstrates that, if all producers have access to a forward market, it leads to a prisoners' dilemma type of game among them. Allaz and Vila [2] show that, as the number of forward trading periods increases, producers lose their ability to raise energy prices above their marginal cost. Kamat and Oren [22] analyze two-settlement markets over two- and three-node networks, and extend the results in [1, 2] to a system with uncertain transmission capacities in the spot market.

Recent work in [37] and [38] further extends the above results to more realistic multi-node and multi-zone networks. Yao, Oren and Adler [37] consider flow constraints, system contingencies and demand uncertainties in the spot market. Their numerical tests show that, like in the simple cases, generation firms have incentives to engage in forward contracting, which increases social surplus and reduces spot prices. Yao et al. [38] consider two alternative mechanisms for capping prices. They observe that a forward cap, which may be induced by free entry of new generation capacity, increases firms' incentives for forward contracting, whereas a regulatory spot price cap reduces such incentives.

In this paper, we continue the study of two-settlement electricity systems. Our objective here is two-fold. First, we introduce a new model of Cournot equilibrium in two-settlement markets which overcomes some shortcomings of the formulations in [37, 38]. As before, the model is formulated as an equilibrium problem with equilibrium constraints (EPEC) where each generation firm solves a mathematical problem with equilibrium constraints (MPEC, [23]) parameterized by the other firms' forward commitments. The model assumes linear demand functions, quadratic generation cost functions and a lossless DC network, resulting in the preceding equilibrium constraints in the form of a parametric linear complementarity problem (LCP, [6]). When applied to realistic size systems, this EPEC model presents a computational challenge. Therefore, the second goal of this paper is to study the computational aspect of our EPEC model and, by exploiting the problem structure, present in detail the solution approach for the EPEC and MPECs arising in our formulation.

Solving an EPEC problem amounts to solving simultaneously a set of MPEC problems, each parameterized by the other MPECs' decision variables (see [29] for more discussions on related topics). One solution approach is to derive the optimality conditions for the regularization scheme of the MPECs (see [9, 32, 33]), then either solve the nonlinear complementarity conditions of the EPEC as a whole [19, 34] or iteratively solve the nonlinear complementarity conditions of individual MPECs [19, 34].

The second approach which we will follow in this research is to iteratively solve MPECs using MPEC-based algorithms. There has been a growing literature on MPEC algorithms. The monograph by Luo, Pang and Ralph [23] present a comprehensive study of MPEC problems and provides first- and second-order optimality conditions; it also describes some iterative algorithms, such as the penalty interior point algorithm (PIPA) and the piecewise sequential quadratic programming (PSQP, see also [21]) algorithm. More recent advances in MPEC algorithms can be found, for example, in [5, 7, 11, 12, 13, 20, 31]. Fukushima, Luo and Pang [11] present a sequential quadratic programming approach, through a reformulation of the complementarity condition as a system of semismooth equations by means of Fischer-Burmcister functionals. This algorithm shares several common features with the PIPA in terms of computational steps and convergence properties; however, it differs from the PIPA in the way of updating the penalty parameters and determining the step sizes. Chen and Fukushima [5] consider MPECs whose lower constraints are a parametric P-matrix LCP. They smooth out the complementarity constraints through the use of Fischer-Burmcister functionals, from which the state variables are viewed as implicit functions of the decision variables. The MPECs can thus be solved by a sequence of well-behaved, though non-convex, nonlinear programs. Fukushima and Tseng [12] propose an $\epsilon$-active set algorithm for solving MPECs with linear complementarity constraints and establish convergence to B-stationary points under the uniform linear independence constraint qualification on the feasible set. This algorithm generates a sequence of variable value sets such that the objective value is almost decreasing, while maintaining the $\epsilon$-feasibility of the complementarity constraints.

The remainder of this paper is organized as follows. The next section introduces the EPEC model of two-settlement markets. In Section 3, we give a compact representation of the computational problem underlying our model. Section 4 proposes the MPEC and EPEC algorithms. We perform a number of numerical tests in Section 5. Finally, we explore some economic implications of our test cases and draw conclusions.

## 2 The Model

We view two-settlement markets as a two-period Nash-Cournot game: the forward market (period zero) and the spot market (period one) and we characterize the equilibrium of this game as a sub-game perfect Nash equilibrium (SPNE, [10]).

In period zero, rational firms enter into forward contracts, forming rational expectations regarding the forward commitments of their rivals and the period-one equilibrium outcomes. Period-one is a subgame with two stages. In stage one, Nature picks a state defined by a realization of the uncertain demand and system contingencies. In stage two, the firms whose information sets include the state of nature and all forward commitments, compete in a Nash-Cournot manner while the Independent System Operator (ISO) transmits electricity and sets congestion prices so as to maximize social surplus across the entire system. The dynamics of this model is illustrated in figure 1, where the solid lines represent time progress and the dashed lines denote rational expectations.

From a mathematical perspective, the model is formulated as an EPEC, which comprises a set of MPECs that characterize the decisions of the individual firms. In each MPEC, the upper level is the firm's profit-maximization problem in the forward market, and the lower level, shared by all MPECs, consists of the period-one equilibrium conditions.

The following summarizes main features of our model that will be elaborated in the rest of this section.

- We consider a lossless DC approximation of an electricity network where flows on transmission lines are constrained by thermal capacities and random line outages.

- The demand side is price taking with elastic demand functions subject to random shocks (in the form of quantity shifts) at each node.

- The supply side consists of Cournot producers with multiple generators at various nodes that are subject to random outages, who sell energy to a Pool at uniform locational marginal prices (LMPs) set by the ISO.

- Generator outages, transmission line outages and demand shocks are represented in terms of system contingent states that have known probabilities in the forward market and are realized before the spot market commences.

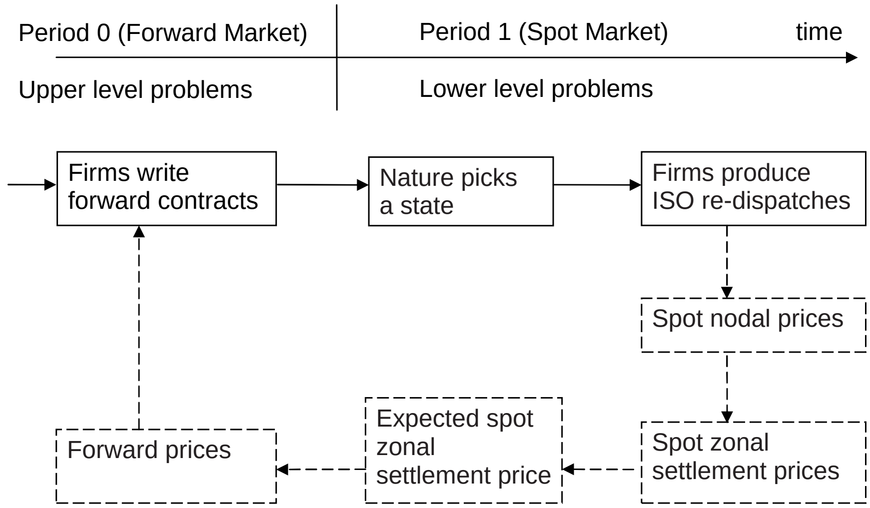



Figure 1: The model dynamics

- The forward market is organized at zonal hubs as financial contracts traded at uniform market clearing prices and settled at spot hub settlement prices based on the nodal LMPs.
- In the spot market, producers engage in a Nash-Cournot competition (i.e., setting quantities) while the ISO who maintains feasibility of the transmission constraints behaves a la Bertrand by setting nodal price premiums or equivalently congestion charges between nodes.
- The market is efficient, i.e., risk neutral speculators will arbitrage away any difference between the forward hub prices and expected spot hub settlement prices.

## 2.1 Period-one: the Spot Market

Electricity restructuring in different markets has been following different blueprints. In the US, one prevailing design is the so-called centrally-dispatched market. This type of market usually consists of a pool run by an ISO that serves as a broker, or auctioneer, for wholesale spot electricity market transactions. The ISO leases the transmission system from transmission owners and controls flows so as to maintain feasibility of the network by redispatching generators and simultaneously setting nodal price premiums and implied congestion charges for bilateral energy transactions.

In this paper, we consider a centrally-dispatched wholesale spot market with demand uncertainty, flow constraints and system contingencies. The network underlying the spot market consists of a set $N$ of nodes, and a set $L$ of transmission lines. There is a set $G$ of generation firms competing in the market, each operating the units at a subset of locations $N_g \subseteq N$. We assume that at most one generation firm operates at a node and, if necessary, we can introduce artificial nodes to meet this assumption. We also assume, for convenience, that there is elastic demand at each node so that pure generation nodes are represented by a demand function intersecting the quantity axis at a very small value.

### 2.1.1 The ISO's Decision Making.

In each state $c \in C$, the ISO controls the import/export $r_i^c$ at each node $i \in N$ (using the convention that positive quantities represent imports) and sets the corresponding locational marginal prices. These quantities must satisfy the network feasibility constraints, that is, the resulting power flows should not exceed the thermal limits $K_l^c$ of the transmission lines in both directions. The transmission network is modeled in terms of lossless DC (i.e. linear) approximation of Kirchhoff's laws (see [4]). Specifically, flows on lines can be calculated using power transfer distribution factor (PTDF) $D_{l,i}^c$ which specifies the proportion of flow on a line $l \in L$ resulting from an injection of one-unit electricity at a node $i \in N$ and a corresponding one-unit withdrawal at some fixed reference node (also known as the slack bus). Moreover, because electricity is not economically storable, the load and generation must be balanced at all times so the sum of all import and export quantities must add up to zero.

The ISO's objective is to maximize social welfare of the entire system. That is the aggregated area under the nodal inverse demand functions (IDFs) $P_i^c(\cdot)$, which represent the total consumer willingness-to-pay, less the sum of all generation costs $C_i(\cdot)$. Mathematically, the ISO solves the following problem parametric on the firms' production decisions ($\{q_i^c\}_{i\in N}$):

$$\max_{r_i^c : i \in N} \quad \sum_{i\in N} \left( \int_0^{r_i^c + q_i^c} P_i^c(\tau_i) d\tau_i - C_i(q_i^c) \right)$$

subject to:

$$\sum_{i\in N} r_i^c = 0 \tag{1}$$

$$\sum_{i\in N} D_{l,i}^c r_i^c \geq -K_l^c, \quad l \in L \tag{2}$$

$$\sum_{i\in N} D_{l,i}^c r_i^c \leq K_l^c, \quad l \in L \tag{3}$$

In the above formulation, we have excluded the non-negativity constraints $r_i^c + q_i^c \geq 0, \quad i \in N$, by implicitly assuming an interior solution with respect to these constraints. The numerical results in Section 5 validate this simplification but that may not be true in general. Let $p^c$, $\lambda_{l-}^c$ and $\lambda_{l+}^c$ be the Lagrange multipliers corresponding to (1)-(3), the first order necessary conditions (the Karush-Kuhn-Tucker, KKT conditions) for the ISO's problem are:

$$
\begin{aligned}
& P_i^c(q_i^c + r_i^c) - p^c - \varphi_i^c \quad i \in N \\
& \varphi_i^c = \sum_{l \in L} (\lambda_{l+}^c D_{l,i}^c - \lambda_{l-}^c D_{l,i}^c), \quad i \in N \\
& \sum_{i \in N} r_i^c = 0 \\
& 0 \leq \lambda_{l-}^c \quad \perp \quad \sum_{i \in N} D_{l,i}^c r_i^c + K_l^c \geq 0, \quad l \in L \\
& 0 \leq \lambda_{l+}^c \quad \perp \quad K_l^c - \sum_{i \in N} D_{l,i}^c r_i^c \geq 0, \quad l \in L
\end{aligned}
$$

The first KKT condition herein implies that

$$
q_i^c + r_i^c = (P_i^c)^{-1} (p^c + \varphi_i^c), \quad i \in N
$$

and consequently, due to (1),

$$
\sum_{i \in N} q_i^c = \sum_{i \in N} (P_i^c)^{-1} (p^c + \varphi_i^c) \tag{4}
$$

This equation represents the aggregate demand function in the network relating the total consumption quantity to the reference node price $p^c$ and the nodal price premiums $\{\varphi_i^c\}_{i \in N}$ which determine the relative nodal prices. The corresponding congestion charges for transmission from node $i \in N$ to node $j \in N$ that will prevent arbitrage between nodal energy transactions and bilateral transactions among nodes must be $\varphi_j^c - \varphi_i^c$.

#### 2.1.2 The Firms' Decision Making.

In the spot market, each firm $g \in G$ determines the outputs from its units at $N_g$. A variety of modeling approaches have been proposed to simulate generation firms' decision making (see, for example, [18, 27, 35, 36]). One modeling consideration regarding the suppliers' strategic behaviors in these models is whether or not they game the congestion prices set by the ISO. Following [18] and [27] we classify spot market models into two basic approaches.

The first approach assumes that generation firms anticipate the impact of their production on the congestion prices set by the ISO and take that effect into account in their production decisions. The resulting formulation of the spot market is a multi-leader one-follower Stackelberg game [27]. Each producer $g$ solves the following MPEC, in which the optimality conditions for the ISO's program are the constraints shared by all the firms:

$$\max_{q_i^c : i \in N_g} \quad \sum_{i \in N_g} P_i^c(r_i^c + q_i^c) q_i^c - \sum_{i \in N_g} C_i(q_i^c)$$

subject to:

$$0 \le q_i^c \le \overline{q}_i^c, \quad i \in N_g$$

$$P_i^c(q_i^c + r_i^c) - p^c + \sum_{l \in L} (\lambda_{l-}^c D_{l,i}^c - \lambda_{l+}^c D_{l,i}^c) = 0, \quad i \in N$$

$$\sum_{i \in N} r_i^c = 0$$

$$0 \le \lambda_{l-}^c \quad \perp \quad \sum_{i \in N} D_{l,i}^c r_i^c + K_l^c \ge 0, \quad l \in L$$

$$0 \le \lambda_{l+}^c \quad \perp \quad K_l^c - \sum_{i \in N} D_{l,i}^c r_i^c \ge 0, \quad l \in L$$

The equilibrium problem among the above MPECs represents a "generalized Nash game" (see [16]), and it may have zero or multiple equilibria (see [3]). On the other hand, even if some pure-strategy equilibrium is found, it can be degenerate, that is, firms will find it optimal to barely congest some transmission lines so as to avoid congestion charges (see [28]). Moreover, this formulation would lead to a two-settlement model with three levels of decision, which makes an equilibrium solution for the two-settlement market computationally intractable.

The second approach assumes that the firms do not fully anticipate the impact of their production decisions on congestion charges (see, for example, [25]) which can be interpreted as a "bounded rationality" assumption. In this approach, the ISO is a Nash player that moves simultaneously with the generation firms. The firms determine their supply quantities so as to maximize their profits but they act as price takers with respect to transmission cost. The market equilibrium is then determined by aggregating the optimality conditions for the firms' and the ISO's problems, which result in a mixed complementarity problem or a variational inequality problem.

There are still two modeling options within this simultaneous-move framework. The first option assumes that the ISO like the generation firms is a Cournot player whose strategic variables are the import/export quantities at each node (see [27, 37, 38]). Hence, each firm $g \in G$ solves the following profit-maximization problem:

$$\max_{q_i^c : i \in N_g} \quad \sum_{i \in N_g} P_i^c(r_i^c + q_i^c) q_i^c - \sum_{i \in N_g} C_i(q_i^c)$$

subject to:

$$0 \le q_i^c \le \overline{q}_i^c, \quad i \in N_g$$

Notice that, since this program is parameterized by $\{r_i^c\}_{i \in N}$, it can be decomposed into $|N_g|$ subproblems, each corresponding to the production decision at one node. Therefore, this model will yield a spot market equilibrium that is invariant to the generation resource ownership structure (i.e, it doesn't matter whether a firm owns one or multiple generators). Moreover, under this formulation, when the network constraints (2)-(3) are nonbinding, the equilibrium solution predicts uniform nodal prices that are systematically higher than the Cournot equilibrium price corresponding to a single market with the aggregated system demand function [27]. These aspects make the choice of import/export quantities as the ISO's strategic variables (which we have used in our previous work [37, 38]) unsatisfactory.

The second option which we employ in this paper is to use the locational price premiums as the ISO's strategic variables. This option can be viewed as a mixed Cournot-Bertrand model where the ISO behaves a la Bertrand while the generation firms are Cournot players with respect to each other (i.e., set quantities) but treat the ISO as a price setter. Thus, each firm chooses its production quantities so as to maximize profits with respect to the residual demand defined implicitly by (4). In this formulation the reference bus price $p^c$ is determined implicitly by the aggragate production decisions of all the generation firms just as in a regular Cournot game. However, these production decisions and the implied reference node price also depend on the nodal premiums $\{\varphi_i^c\}$ set by the

ISO. The resulting problem solved by each generation firm is

$$\max_{q_i^c : i \in N_g, p^c} \quad \sum_{i \in N_g} (p^c + \varphi_i^c)\, q_i^c - \sum_{i \in N_g} C_i(q_i^c)$$

subject to:

$$0 \le q_i^c \le \overline{q}_i^c, \quad i \in N_g$$

$$\sum_{i \in N} q_i^c = \sum_{i \in N} (P_i^c)^{-1} (p^c + \varphi_i^c)$$

This modeling option takes account of the resource ownership structure and, when the network constraints are relaxed, the locational price premiums go to zeros so that the model produces the same equilibrium solution as the Cournot equilibrium applied to the aggregate system demand. Unfortunately, this approach has another shortcoming which manifests itself if we reduce the capacity of a radial transmission line to zero or, more realistically, if it is common knowledge that a radial line is constantly congested. In such situations, subnetworks connected by saturated radial lines are effectively decoupled from a competitive interaction point of view. The demand functions on both sides of the saturated line will be shifted by the import/export quantities but their slope stays the same so generators will behave as local monopolists. For instance, in the case of a symmetric two-node one-line network, reducing the line capacity to zero creates two symmetric local monopolies. However, in this situation, our model will produce a symmetric duopoly equilibrium with prices that are systematically lower than the locational monopoly prices. Unfortunately, there is no satisfying solution to this problem since a Nash equilibrium in a congestion prone network depends on the conjectured common knowledge with regard to the extent of possible competition across transmission lines. Such conjectures affect the perceived elasticity of the residual demand by the competing firms and hence their strategic behavior. The discontinuities in reaction functions and resulting equilibrium prices when a single transmission line separating two competitors in a two node system switches from a congested to an uncongested regime, have been eloquently demonstrated in [3]. Such discontinuities become intractable in a meshed system with multiple nodes.

We partially address the above issue in a sequal paper [39] through a hybrid approach that requires apriority identification of "systematically congested" links (e.g. path 15 in California or the link between France and the UK across the English Channel) which effectively decouples the network into strategic subnetworks. In this paper, however, we will assume that the network is fully connected physically and strategically so that competing firms behave as if the demand at all nodes is contestable.

In our two-settlement model, we allow different granularity in the forward and spot markets. This is achieved by dividing the network into a set $Z$ of zones (or trading hubs), each consisting of a cluster of nodes. In particular, the spot market supply and demand at each node are settled at the nodal prices, whereas the forward contracts are traded at zonal hub forward prices and settled at corresponding spot hub settlement prices $\{u_z^c\}_{z\in Z}$, which are defined as weighted averages of the nodal prices in the respective zones. Thus, the nodal spot prices resulting from the strategic interaction in the spot market will affect the settlement of the forward contracts, which is debited from the firms' spot market profits, through the zonal hub prices. The nodal weights $\{\delta_i\}_{i\in N}$ are assumed to be exogenous parameters based on historical load shares at the nodes. This assumption is consistent with common practice, for instance, at the Pennsylvania-Jersey-Maryland (PJM) Western Hub. In mathematical terms, each firm $g \in G$ solves in the spot market the following profit-maximization problem parametric on the locational price premiums $\{\varphi_i^c\}_{i\in N}$ and on its own forward contract commitments $\{x_{gz}\}_{z\in Z}$:

$$\max_{q_i^c: i\in N_g, p^c} \quad \sum_{i\in N_g} (p^c + \varphi_i^c)\, q_i^c - \sum_{z\in Z} u_z^c x_{gz} - \sum_{i\in N_g} C_i(q_i^c)$$

subject to:

$$u_z^c = \sum_{i: z(i)=z} (p^c + \varphi_i^c)\, \delta_i$$

$$q_i^c \geq 0, \quad i \in N_g \tag{5}$$

$$q_i^c \leq \overline{q}_i^c, \quad i \in N_g \tag{6}$$

$$\sum_{i\in N} q_i^c = \sum_{i\in N} (P_i^c)^{-1} (p^c + \varphi_i^c) \tag{7}$$

Let $\rho_{i-}^c$, $\rho_{i+}^c$ and $\beta_g^c$ be the Lagrange multipliers corresponding to (5)-(7), then the KKT conditions for firm $g$'s program are (after substituting the first constraint into the objective function)

$$
\begin{aligned}
& p^c + \varphi_i^c - \beta_g^c - \frac{dC_i(q_i^c)}{dq_i^c} + \rho_{i-}^c - \rho_{i+}^c = 0 && i \in N_g \\
& \beta_g^c \sum_{i \in N} \frac{d\left(P_i^c\right)^{-1}(p^c + \varphi_i^c)}{dp^c} + \sum_{i \in N_g} q_i^c - \sum_{i \in N} \delta_i x_{gz(i)} = 0 && \\
& \sum_{i \in N} q_i^c = \sum_{i \in N} \left(P_i^c\right)^{-1}(p^c + \varphi_i^c) && \\
& 0 \le \rho_{i-}^c \quad \perp \quad q_i^c \ge 0 && i \in N_g \\
& 0 \le \rho_{i+}^c \quad \perp \quad \overline{q}_i^c - q_i^c \ge 0 && i \in N_g
\end{aligned}
$$

Here, the first two conditions are the derivatives of the Lagrangian function with respect to $q_i^c$ and $p^c$, respectively.

### 2.1.3 Period-one Equilibrium Conditions.

Aggregating the KKT conditions for the firms' and the ISO's programs yields the spot market equilibrium conditions, which, in general, form a mixed nonlinear complementarity problem. It becomes a mixed LCP when both the nodal demand functions and the marginal cost functions are linear as we assume in the remainder of this paper.

Let the inverse demand functions and the cost functions be, respectively,

$$
\begin{aligned}
P_i^c(q) &= a^c - b_i^c q, \quad i \in N, \\
C_i(q) &= d_i q + \frac{1}{2} s_i q^2, \quad i \in N,
\end{aligned}
$$

then the market equilibrium conditions become

$$
p^c + \varphi_i^c - \beta_g^c - d_i - s_i q_i^c + \rho_{i-}^c - \rho_{i+}^c = 0 \qquad i \in N_g, g \in G \tag{8}
$$

$$
p^c = a^c - \frac{\sum_{i \in N} \frac{\varphi_i^c}{b_i^c}}{\sum_{i \in N} \frac{1}{b_i^c}} - \frac{\sum_{i \in N} q_i^c}{\sum_{i \in N} \frac{1}{b_i^c}} \tag{9}
$$

$$
-\beta_g^c \sum_{i \in N} \frac{1}{b_i^c} + \sum_{i \in N_g} q_i^c - \sum_{i \in N} \delta_i x_{gz(i)} = 0 \qquad g \in G \tag{10}
$$

$$
0 \le \rho_{i-}^c \quad \perp \quad q_i^c \ge 0 \qquad i \in N \tag{11}
$$

$$0 \leq \rho^c_{i+} \quad \perp \quad \overline{q}^c_i - q^c_i \geq 0 \qquad i \in N \tag{12}$$

$$\sum_{i \in N} r^c_i = 0 \tag{13}$$

$$a^c - (q^c_i + r^c_i)b^c_i - p^c - \varphi^c_i = 0 \qquad i \in N \tag{14}$$

$$\varphi^c_i = \sum_{l \in L} (\lambda^c_{l+} D^c_{l,i} - \lambda^c_{l-} D^c_{l,i}) \qquad i \in N \tag{15}$$

$$0 \leq \lambda^c_{l-} \quad \perp \quad \sum_{i \in N} D^c_{l,i} r^c_i + K^c_l \geq 0 \qquad l \in L \tag{16}$$

$$0 \leq \lambda^c_{l+} \quad \perp \quad K^c_l - \sum_{i \in N} D^c_{l,i} r^c_i \geq 0 \qquad l \in L \tag{17}$$

Here, (8)-(12) are the aggregated KKT conditions for the firms' problems, while (13)-(17) are the KKT conditions for the ISO's problem. Under the assumption of linear demand functions and quadratic convex cost functions, the firms' and the ISO's programs are strictly-concave-maximization problems so (8)-(17) are also sufficient. Note that, since (9) is implied by (13) and (14), it can be excluded from the preceding market equilibrium conditions.

### 2.2 Period Zero: the Forward Market

In period zero, we assume that the forward market is a standardized liquid market such that all forward contracts in a zone are settled at equal prices. It is also assumed that there are enough risk-neutral arbitrageurs in the markets and they will eliminate any profitable arbitrage opportunity arising from the difference between the forward prices and the expected spot zonal settlement prices; this is referred to as a "no-arbitrage", or "perfect-arbitrage" condition. Consequently, the forward price ($h_z$) in each zone $z \in Z$ equal to the expected values of the corresponding spot hub prices over all contingent states ($c \in C$) with respective probabilities ($Pr(c)$) (we assume for simplicity that the state probabilities and the market risk neutral probabilities are identical).

The risk-neutral firms simultaneously determine their forward contract quantities $\{x_{gz}\}_{g \in G, z \in Z}$ so as to maximize the total profit from both the forward contracts and the spot productions, while anticipating the forward commitments of their rivals as well as the equilibrium outcome in period one. In mathematical terms, each firm $g \in G$ solves the following MPEC program, where

$\{(8)-(17)\}_{c\in C}$ form the inner problem:

$$
\begin{aligned}
&\max_{\Phi_g} \quad \sum_{z\in Z} h_z x_{g,z} + \sum_{c\in C} Pr(c)\pi_g^c \\
&\text{subject to:} \\
&\Phi_g = \{\{x_{g,z}\}_{z\in Z}, \{r_i^c, q_i^c, \rho_{i+}^c, \rho_{i-}^c\}_{i\in N, c\in C}, \{\lambda_{l-}^c, \lambda_{l+}^c\}_{l\in L, c\in C}\} \\
&\pi_g^c = \sum_{i\in N_g} (p^c + \varphi_i^c) q_i^c - \sum_{z\in Z} u_z^c x_{g,z} - \sum_{i\in N_g} C_i(q_i^c), \quad c\in C \\
&h_z = \sum_{c\in C} Pr(c) u_z^c, \quad z\in Z \\
&u_z^c = \sum_{i:z(i)=z} (p^c + \varphi_i^c)\delta_i, \quad z\in Z, c\in C \\
&\text{and } (8)-(17), \quad c\in C
\end{aligned}
$$

The equilibrium problem in period zero is an EPEC. A solution to this EPEC is a set of the variables, including the firms' forward and spot decisions, the ISO's redispatch decisions and the aforementioned Lagrange multiplies, at which all firms' MPEC problems are simultaneously solved, and no market participant is willing to unilaterally change its decisions in either market.

It is worth noting that, from a philosophical point of view, the above formulation may appear internally inconsistent since firms seem to base their decisions in the forward market on information which is not available to them in the spot market. To resolve this inconsistency we might assume that forward commitments are based on correct forecast of the expected spot market outcomes rather then on the detailed information that we use to replicate that forecast. Furthermore, it is also reasonable to assume that forward contracting decisions and spot market production decisions are made by functionally independent entities within a firm operating on different time horizons and employing different forecasting tools. So while the decisions made in the spot market are informed of the forward contracting positions of the firm they do not necessarily account for all the global information that led to these contracting decisions.

# 3 A Compact Representation of the Model

In this section, we compact the notation so as to streamline the subsequent algorithmic presentation by grouping and relabelling the variables, including the dual variables, as follows:

- $x_g$ ($\in R^{|Z|}$): The vector of the forward commitments by firm $g \in G$.
- $r^c$ ($\in R^{|N|}$): The vector of the ISO's import/export quantities in state $c \in C$.
- $q^c$ ($\in R^{|N|}$): The vector of the firms' generation quantities in state $c \in C$.
- $\rho_-^c, \rho_+^c$ ($\in R^{|N|}$): The vectors of the Lagrange multipliers associated with the generation capacity constraints in state $c \in C$.
- $\lambda_-^c, \lambda_+^c$ ($\in R^{|L|}$): The vectors of the Lagrange multipliers associated with the flow capacity constraints in state $c \in C$.

In addition, the parameters are relabelled as

- $\Delta$ ($\in R^{|N|\times|Z|}$): A matrix where the $(i, z)$-th element is $-1$ if $z(i) = z$, and 0 otherwise.
- $\overline{q}^c$ ($\in R^{|N|}$): The vector of the generator capacity bounds in state $c \in C$.
- $B_c$ ($\in R^{|N|\times|N|}$): A diagonal matrix for state $c \in C$ where the $(i, i)$-th element is $b_i^c$ .
- $d$ ($\in R^{|N|}$): The vector of the marginal generation costs.
- $D_c$ ($\in R^{|L|\times|N|}$): A PTDF matrix for state $c \in C$ where the $(l, i)$-th element is $D_{l,i}^c$.
- $k^c$ ($\in R^{|L|}$): The vector of the flow capacities of the transmission lines in state $c \in C$.
- $X_g$ ($\in R^{|Z|}$): The feasible region of $x_g$ for each firm $g \in G$.

## 3.1 Compact Representation of the Inner Problem $\{(8) - (17)\}_{c\in C}$

Let $e \in R^{|N|}$ be a vector with all 1's, then (13) and (14) become

$$\begin{bmatrix} a^c e \\ 0 \end{bmatrix} - \begin{bmatrix} B_c \\ 0 \end{bmatrix} q^c - \begin{bmatrix} B_c & e \\ e^T & 0 \end{bmatrix} \begin{bmatrix} r^c \\ p^c \end{bmatrix} + \begin{bmatrix} D_c^T \\ 0 \end{bmatrix} \lambda_- - \begin{bmatrix} D_c^T \\ 0 \end{bmatrix} \lambda_+ = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solving $r^c$ and $p^c$ yields

$$\begin{bmatrix} r^c \\ p^c \end{bmatrix} = \begin{bmatrix} Q_c & \frac{B_c^{-1}e}{e^T B_c^{-1} e} \\ \frac{e^T B_c^{-1}}{e^T B_c^{-1} e} & \frac{-1}{e^T B_c^{-1} e} \end{bmatrix} \left( \begin{bmatrix} a^c e \\ 0 \end{bmatrix} - \begin{bmatrix} B_c \\ 0 \end{bmatrix} q^c + \begin{bmatrix} D_c^T \\ 0 \end{bmatrix} \lambda_-^c - \begin{bmatrix} D_c^T \\ 0 \end{bmatrix} \lambda_+^c \right)$$

where

$$Q_c = B_c^{-1} - \frac{B_c^{-1} e e^T B_c^{-1}}{e^T B_c^{-1} e}$$

Hence,

$$\begin{aligned} r^c =& - Q_c B_c q^c + Q_c \left( D_c^T \lambda_-^c - D_c^T \lambda_+^c \right) \\ p^c =& a^c - \frac{e^T}{e^T B_c^{-1} e} q^c + \frac{e^T B_c^{-1}}{e^T B_c^{-1} e} \left( D_c^T \lambda_-^c - D_c^T \lambda_+^c \right) \end{aligned}$$

Now, consolidating conditions (8)-(10), we have

$$\rho_-^c = -a^c e + d + H_c q^c + B_c Q_c \left( D_c^T \lambda_-^c - D_c^T \lambda_+^c \right) + \rho_+^c + \frac{1}{e^T B_c^{-1} e} \Delta \sum_{g \in G} x_g$$

where $H_c$ is a matrix such that

$$(h_c)_{ij} = \begin{cases} \frac{2+s_i}{e^T B_c^{-1} e} & \text{if } i = j \\ \frac{2}{e^T B_c^{-1} e} & \text{if } i \neq j, \text{ and the units at nodes } i \text{ and } j \text{ belong to the same firm} \\ \frac{1}{e^T B_c^{-1} e} & \text{otherwise} \end{cases}$$

Next, let $w^c$ and $y^c$ be two variable vectors, and $t^c$, $A_c$ and $M_c$ be constants such that

$$w^c = \begin{bmatrix} \bar{q}^c - q^c \\ \rho_-^c \\ k^c + D_c r^c \\ k^c - D_c r^c \end{bmatrix}, \quad y^c = \begin{bmatrix} \rho_+^c \\ q^c \\ \lambda_-^c \\ \lambda_+^c \end{bmatrix}, \quad t^c = \begin{bmatrix} \bar{q}^c \\ -a^c e + d \\ k^c \\ k^c \end{bmatrix},$$

$$A_c = \begin{bmatrix} 0 \\ \frac{\Delta}{e^T B_c^{-1} e} \\ 0 \\ 0 \end{bmatrix}, \quad M_c = \begin{bmatrix} 0 & -I & 0 & 0 \\ I & H_c & B_c Q_c D_c^T & -B_c Q_c D_c^T \\ 0 & -D_c Q_c B_c & D_c Q_c D_c^T & -D_c Q_c D_c^T \\ 0 & D_c Q_c B_c & -D_c Q_c D_c^T & D_c Q_c D_c^T \end{bmatrix}$$

The preceding applied to (8)-(17) leads to

$$w^c = t^c + A_c \sum_{g \in G} x_g + M_c y^c, \quad w^c \geq 0, \quad y^c \geq 0, \quad (y^c)^T w^c = 0 \tag{18}$$

Finally, aggregating (18) for all states $c \in C$, we present the inner problem $\{(8) - (17)\}_{c \in C}$ as

$$w = t + A \sum_{g \in G} x_g + My, \quad w \geq 0, \quad y \geq 0, \quad y^T w = 0$$

where $y$ and $w$ are variables, and $t$, $A$ and $M$ are constants as follows:

$$y = \begin{bmatrix} y^c & c \in C \end{bmatrix}, \quad w = \begin{bmatrix} w^c & c \in C \end{bmatrix}, \quad t = \begin{bmatrix} t^c & c \in C \end{bmatrix},$$

$$A = \begin{bmatrix} A_c & c \in C \end{bmatrix}, \quad M = \begin{bmatrix} M_1 & & 0 \\ & M_2 & \\ & & \ldots \\ 0 & & M_{|C|} \end{bmatrix}.$$

## 3.2 Compact Representation of the MPEC Problems

In period zero, each firm $g \in G$ solves the following MPEC problem:

$$\begin{aligned} \mathcal{F}_g(\bar{x}_{-g}) : \quad & \min_{x_g, y, w} \quad f_g(x_g, y, w, \bar{x}_{-g}) \\ & \text{subject to :} \\ & x_g \in X_g \\ & w = t + A\bar{x}_{-g} + Ax_g + My, \quad w \geq 0, \quad y \geq 0, \quad y^T w = 0 \end{aligned} \tag{19}$$

In this program, $x_g$ is the decision variable, $(y, w)$ are the state variables, and $\bar{x}_{-g} = \sum_{k \in G \setminus \{g\}} \bar{x}_k$ is a parameter that is the sum of other firms' forward contract quantities .

We denote the EPEC problem in period zero as $\{\mathcal{F}_g(\cdot)\}_{g \in G}$. An equilibrium of this EPEC problem in period zero is a set $(\{\bar{x}_g\}_{g \in G}, y, w)$ that solves $\mathcal{F}_g(\bar{x}_{-g})$ for all $g \in G$, i.e., $(\bar{x}_g, y, w) \in \mathrm{SOL}\left(\mathcal{F}_g(\bar{x}_{-g})\right), \quad g \in G$, where $\mathrm{SOL}\left(\mathcal{F}_g(\bar{x}_{-g})\right)$ denotes the solution set of $\mathcal{F}_g(\bar{x}_{-g})$.

# 4 Solution Approach

To solve the EPEC as stated above, we propose an iterative scheme that solves in turn the MPEC problems $\mathcal{F}_g(\bar{x}_{-g})$ by holding fixed the decision variables of the other MPEC problems. To avoid ambiguity, we refer to the iterations for solving the MPECs as the *inner iterations*, while the iterations for solving the EPEC problem are referred to as *outer iterations*. Below, we first observe some properties of the MPEC problems; then exploiting these properties, we propose an MPEC algorithm; and finally develop the EPEC solution scheme.

## 4.1 Properties of the MPEC Problems

We observe the following properties of $\mathcal{F}_g(\bar{x}_{-g})$.

1. $f_g(x_g, y, w, \bar{x}_{-g})$ is quadratic with respect to $(x_g, y, w)$.

2. $M$ is positive semi-definite. To show this, we first notice that $H_c$ is symmetric positive-definite. Secondly,

$$\begin{aligned} v^T Q_c v &= v^T B_c^{-1} v - \frac{v^T B_c^{-1} e e^T B_c^{-1} v}{e^T B_c^{-1} e} \\ &= \frac{\|B_c^{-\frac{1}{2}} v\|^2 \|B_c^{-\frac{1}{2}} e\|^2 - \|v^T B_c^{-\frac{1}{2}} B_c^{-\frac{1}{2}} e\|^2}{\|B_c^{-\frac{1}{2}} e\|^2} \\ &\geq 0, \quad v \in R^{|N|}. \end{aligned}$$

Hence, $Q_c$ is symmetric positive semi-definite. Now, since

$$\frac{M_c + M_c^T}{2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & H_c & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ D_c \\ -D_c \end{bmatrix} Q_c \begin{bmatrix} 0 \\ 0 \\ D_c \\ -D_c \end{bmatrix}^T,$$

we conclude that $M_c$ is positive semi-definite.

3. Given $\bar{x}_{-g}$, the constraint set (19) is an LCP parameterized by $x_g$. Moreover, for any $x_g$,

$$w_c = \begin{bmatrix} \bar{q}^c \\ \left(-a^c e + d + \frac{\Delta x_{-g} + \Delta x_g}{e^T B_c^{-1} e}\right)^+ \\ k^c \\ k^c \end{bmatrix}, \quad y_c = \begin{bmatrix} \left(-a^c e + d + \frac{\Delta x_{-g} + \Delta x_g}{e^T B_c^{-1} e}\right)^- \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad c \in C$$

satisfy the linear constraints of this LCP. By Theorem 3.1.2 in [6], the LCP problem (19) is always solvable.

From an economic perspective, it reasonable to assume that, for each state in period one, there is a unique market equilibrium, that is, (19) has unique solution $(w, y)$ for all $x_g \in X_g$. By Theorem 3.1.7 in [6], this is equivalent to assuming that the active constraints at the optimal solutions to the period-one problems are linearly independent.

## 4.2 The MPEC Algorithm

The uniqueness of solution $(w, y)$ in (19) implies its solution $(y, w)$ is an implicit functions of $x_g$. Consequently, $\mathcal{F}_g(\bar{x}_{-g})$ can be reduced to an optimization problem with respect only to $x_g$. This feature enables us to develop an algorithm for solving $\mathcal{F}_g(\bar{x}_{-g})$ via a "divide-and-conquer" approach. The proposed MPEC algorithm is a variant of the PSQP algorithm in [21, 23], but it specializes the PSQP algorithm by taking advantage of the properties of $\mathcal{F}_g(\bar{x}_{-g})$ identified above. Specifically, we partition $X_g$ into a set of polyhedra according to feasible complementary bases of (19). In each polyhedron, we derive explicitly the affine functions for the state variables in terms of $x_g$, and solve a quadratic program involving only $x_g$. Through parametric LCP pivoting, the proposed MPEC algorithm searches in the space of feasible $x_g$ for a B-stationary point of $\mathcal{F}_g(\bar{x}_{-g})$ along adjacent polyhedra.

### 4.2.1 Partition of $X_g$

The partition of $X_g$ is determined by the feasible complementary bases (see Definition 1.3.2 in [6]) of the LCP problem (19). Let $n$ and $m$ be the dimensions of $x_g$ and $y$ (and $w$), respectively. Consider (19), given a partition $(\alpha, \bar{\alpha})$ of $\{1, 2, ..., m\}$, we define matrix $C_M(\alpha) \in R^{m\times m}$ as

$$C_M(\alpha)_{\bullet i} = \begin{cases} -M_{\bullet i} & \text{if } i \in \alpha \\ I_{\bullet i} & \text{if } i \in \bar{\alpha} \end{cases}.$$

$C_M(\alpha)$ is called a *complementary matrix* of $[-M, I]$ with respect to $\alpha$; it is a *complementary basis* if nonsingular; it is a *feasible complementary basis* with respect to $x_g$ if $C_M^{-1}(\alpha)(q + Ax_g) \geq 0$, where $q = t + A\bar{x}_{-g}$.

Now, given the partition $(\alpha, \bar{\alpha})$, let $w_\alpha = 0$ and $y_{\bar{\alpha}} = 0$, then (19) is reduced to

$$v^\alpha = C_M^{-1}(\alpha)(q + A^g x_g) \geq 0, \qquad \text{where } v_i^\alpha = \begin{cases} y_i & \text{if } i \in \alpha \\ w_i & \text{if } i \in \bar{\alpha} \end{cases}.$$

Note that the orthogonality of $w$ and $y$ is guaranteed for $w_\alpha = 0$ and $y_{\bar{\alpha}} = 0$. The preceding is equivalent to

$$\begin{aligned} y_\alpha &= M_{\alpha\alpha}^{-1}(q_\alpha + A_{\alpha\bullet} x_g) \geq 0 \\ w_{\bar{\alpha}} &= -M_{\bar{\alpha}\alpha} M_{\alpha\alpha}^{-1}(q_\alpha + A_{\alpha\bullet} x_g) + q_{\bar{\alpha}} + A_{\bar{\alpha}\bullet} x_g \geq 0. \end{aligned}$$

When $C_M(\alpha)$ is a feasible complementary basis with respect to $x_g$, these two nonnegative constraints are satisfied.

As a result, the polyhedron

$$\tilde{P}_g(\alpha) = \left\{ x_g \in R^n : C_M^{-1}(\alpha)(q + A^g x_g) \geq 0 \right\}$$

defines a set of $x_g$ with respect to which $C_M(\alpha)$ is feasible. Moreover, the state variables $y$ and $w$

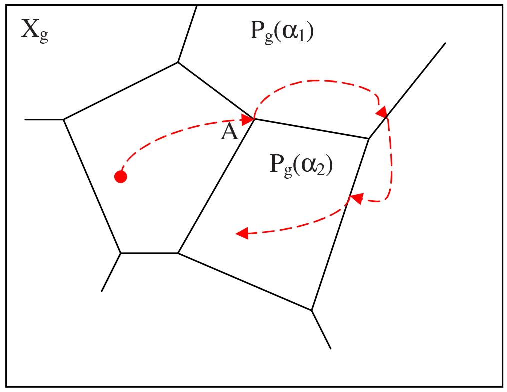



Figure 2: A typical partition of $X_g$

are affine functions of $x_g \in \tilde{P}_g(\alpha)$:

$$y_\alpha = M_{\alpha\alpha}^{-1}(q_\alpha + A_{\alpha\bullet} x_g) \tag{20}$$

$$y_{\bar{\alpha}} = 0 \tag{21}$$

$$w_\alpha = 0 \tag{22}$$

$$w_{\bar{\alpha}} = -M_{\bar{\alpha}\alpha} M_{\alpha\alpha}^{-1}(q_\alpha + A_{\alpha\bullet} x_g) + q_{\bar{\alpha}} + A_{\bar{\alpha}\bullet} x_g \tag{23}$$

Equations (20)-(23) imply that $x_g \in Int(\tilde{P}_g(\alpha))$ if and only if $(y_\alpha, w_{\bar{\alpha}}) \in R^m_{++}$ ($y$ and $w$ are non-degenerate), and that $x_g \in Bd(\tilde{P}_g(\alpha))$ if and only if there exists some $i \in \alpha$ such that $y_i = w_i = 0$ ($y$ and $w$ are degenerate).

Enumerating all feasible complementary bases $C_M(\alpha)$, one can partition $X_g$ into a set of polyhedra $P_g(\alpha) = X_g \bigcap \tilde{P}_g(\alpha)$. The uniqueness of the solution $(y, w)$ to (19) guarantees that such partition is also unique (but with respect to a fixed $\bar{x}_{-g}$). Figure 2 illustrates a sample partition for the case of $n = 2$.

#### 4.2.2 Stationary Point

Equations (20)-(23) imply that, whenever $C_M(\alpha)$ is a feasible complementary basis, $f_g(x_g, y, w, \bar{x}_{-g})$ is reduced to a quadratic function with respect to $x_g \in P_g(\alpha)$. We denote this function as $f_{g,\alpha}(x_g, \bar{x}_{-g})$. Now, limiting $\mathcal{F}_g(\bar{x}_{-g})$ to $x_g \in P_g(\alpha)$ leads to the following program parameterized by $\bar{x}_{-g}$:

$$
\begin{aligned}
QP_g(\alpha): \quad & \min_{x_g} \quad f_{g,\alpha}(x_g, \bar{x}_{-g}) \\
& \text{subject to:} \\
& x_g \in X_g \\
& M_{\alpha\alpha}^{-1}(q_\alpha + A_{\alpha\bullet} x_g) \geq 0 \\
& -M_{\bar{\alpha}\alpha} M_{\alpha\alpha}^{-1}(q_\alpha + A_{\alpha\bullet} x_g) + q_{\bar{\alpha}} + A_{\bar{\alpha}\bullet} x_g \geq 0
\end{aligned}
$$

Notice that this problem does not involve $y$ and $w$.

We call $\alpha$ the *associated (index) basis* of polyhedron $P_g(\alpha)$. Let $x_g \in X_g$ be given, and $(y, w)$ be the corresponding solution to (19), equations (20)-(23) hold for all associated bases in

$$
B_g(x_g, \bar{x}_{-g}) = \{\alpha \subseteq \{1, 2, ..., m\} : \{i : y_i > 0\} \subseteq \alpha \subseteq \{i : w_i = 0\}\}.
$$

We refer to this set as the *association (basis) set* at $x_g$. Clearly, $x_g \in P_g(\alpha)$ for all $\alpha \in B_g(x_g, \bar{x}_{-g})$.

We are now ready to characterize the B-stationary points of $\mathcal{F}_g(\bar{x}_{-g})$. Following [23], a vector $(\bar{x}_g, y, w)$ is called a B-stationary point of $\mathcal{F}_g(\bar{x}_{-g})$ if, for all feasible directions (with respect to (19)) $u \in R^{n+2m}$ at $(\bar{x}_g, y, w)$, the directional derivative $\nabla_u f_g(x_g, y, w, \bar{x}_{-g}) \geq 0$.

Thus, a point $\bar{x}_g \in X_g$ is a B-stationary point of $\mathcal{F}_g(\bar{x}_{-g})$ if and only if, for all $\alpha \in B_g(\bar{x}_g, \bar{x}_{-g})$, either of the following holds

1. $P_g(\alpha)$ is a singleton containing only $\bar{x}_g$, i.e., $P_g(\alpha) = \{\bar{x}_g\}$;

2. for any unit-vector direction $u \in R^n$ such that there exists a sufficiently small scaler $\epsilon > 0$ satisfying $\bar{x}_g + \epsilon u \in P_g(\alpha)$, the directional derivative of $f_g(x_g, y, w, \bar{x}_{-g})$ at $\bar{x}_g$ with respect to $u$ is non-negative, i.e.,

$$
\nabla_u f_g(\bar{x}_g, y, w, \bar{x}_{-g}) = \left.\frac{\partial f_g}{\partial x_g}\right|_{\bar{x}_g} u + \left.\frac{\partial f_g}{\partial y}\frac{dy}{dx_g}\right|_{\bar{x}_g} u + \left.\frac{\partial f_g}{\partial w}\frac{dw}{dx_g}\right|_{\bar{x}_g} u \geq 0,
$$

   where $y$ and $w$ are as in (20)-(23).

The above B-stationary conditions suggest that, if a local minimum or stationary point $\bar{x}_g$ of

$QP_g(\alpha)$ yields non-degenerate $(y, w)$ in (19), it is a B-stationary point of $\mathcal{F}_g(\bar{x}_{-g})$; otherwise, one should identify whether this point is a B-stationary point of $\mathcal{F}_g(\bar{x}_{-g})$ by checking whether it is a local minimum or stationary point with respect to all polyhedra associated with $B_g(\bar{x}_g, \bar{x}_{-g})$.

#### 4.2.3 The MPEC Algorithm

Let $\bar{x}_g \in X_g$ be a given starting point. If there exists an $\alpha \in B_g(\bar{x}_g, \bar{x}_{-g})$ such that $QP_g(\alpha)$ is unbounded, then $\mathcal{F}_g(\bar{x}_{-g})$ is also unbounded. If $\bar{x}_g$ is a local minimum or stationary point of problems $QP_g(\alpha)$ for all $\alpha \in B_g(\bar{x}_g, \bar{x}_{-g})$, it is a B-stationary point of $\mathcal{F}_g(\bar{x}_{-g})$. Otherwise, there exists an $\alpha^* \in B_g(\bar{x}_g, \bar{x}_{-g})$ for which $QP_g(\alpha^*)$ yields a solution different than $\bar{x}_g$. Let this point be $x_g^*$, then its corresponding state variables $y^*$ and $w^*$ are as in (20)-(23) with $\alpha$ replaced with $\alpha^*$. If $y^*$ and $w^*$ are non-degenerate, $x_g^* \in Int(\tilde{P}_g(\alpha^*))$ and hence a B-stationary point of $\mathcal{F}_g(\bar{x}_{-g})$; otherwise, it serves as the starting point for the next (inner) iteration. The dashed lines in figure 2 illustrate such a sample path.

*The MPEC algorithm*

Input: $\bar{x}_g, \bar{x}_{-g}$

0. (Initialization) Set $\alpha^* := \emptyset$.

1. (Subroutine call) Call the *search subroutine*.

2. (Termination check)

    If the subroutine reports unboundedness,

        report the problem $\mathcal{F}_g(\bar{x}_{-g})$ as unbounded, stop.

    else if the subroutine reports $\bar{x}_g$ as a B-stationary point,

        $\bar{x}_g$ is a B-stationary point of $\mathcal{F}_g(\bar{x}_{-g})$, stop.

    else

        let $x_g^*$ and $\alpha^*$ be the returned point and the associated basis, respectively.

        let $y^*$ and $w^*$ solve (19) with $x_g^*$.

        If $(y^*_{\alpha^*}, w^*_{\bar{\alpha}^*}) \in R^m_{++}$,

            $x_g^*$ is a B-stationary point of $\mathcal{F}_g(\bar{x}_{-g})$, stop.

        else

set $\bar{x}_g := x_g^*$, go to step 1.

The *search subroutine* (input: $\alpha^*, \bar{x}_g, \bar{x}_{-g}$)

0. (Initial pivoting) Pivot to an associated basis $\alpha \in B_g(\bar{x}_g, \bar{x}_{-g}) \backslash \{\alpha^*\}$ at $\bar{x}_g$.

1. (Search)

   Call a *quadratic programming subroutine* to solve $QP_g(\alpha)$.

   If $QP_g(\alpha)$ has an unbounded direction,

   report unboundedness.

   If $QP_g(\alpha)$ yields a point $x_g^* \neq \bar{x}_g$ with a decreased objective value,

   set $\alpha^* := \alpha$, return $x_g^*$ and $\alpha^*$.

2. (Termination Check)

   If all bases in $B_g(\bar{x}_g, \bar{x}_{-g}) \backslash \{\alpha^*\}$ have been visited,

   return $\bar{x}_g$ as a B-stationary point.

   else

   pivot (at $\bar{x}_g$) to the next $\alpha \in B_g(\bar{x}_g, \bar{x}_{-g}) \backslash \{\alpha^*\}$, go to step 1.

Here, one can use any available quadratic programming solver as the *quadratic programming subroutine*.

#### 4.2.4 Remarks

- Because the number of zones is typically much smaller than the number of nodes, the dimension of $QP_g(\alpha)$, $|x_g|$, is usually much smaller than that of $\mathcal{F}_g(\bar{x}_{-g})$. Therefore, the proposed MPEC algorithm improves the performance of the general PSQP method [23].

- The MPEC algorithm maintains feasibility with respect to all constraints (including the complementarity constraint) in (19).

- If $|B_g(\bar{x}_g, \bar{x}_{-g})| \leq 2$ throughout the course of the MPEC algorithm, then no basis will be repeated. This, combined with the fact that there exists a finite number of partition of $X_g$ (bounded by the number of feasible complementary bases in (19)), establishes the finite global convergence of the MPEC algorithm. If the preceding condition is violated (that is, if $|B_g(\bar{x}_g, \bar{x}_{-g})| > 2$ for some $\bar{x}_g$), one can use any of the standard lexicographic schemes (in the context of LCP pivoting; see, for example, [6]) to avoid cycling. It should be noted that different lexicographic schemes might lead to different search paths of the MPEC algorithm and thus possibly to different B-stationary points. For example, if the lexicographic scheme selects the basis $\alpha_2$, instead of $\alpha_1$, at point A in figure 2, the algorithm terminates immediately.

- Note that, to solve $QP_g(\alpha)$, we need to compute $M_{\alpha\alpha}^{-1}$ and possibly, depending on the quadratic programming subroutine in use, a starting point. $M_{\alpha\alpha}^{-1}$ can be computed efficiently from the corresponding matrix of the previously-visited basis, which differs from $\alpha$ by one index. The solution to the quadratic program with respect to the previous-visited basis can be used as the starting point.

## 4.3 The EPEC Scheme

### 4.3.1 B-stationary equilibrium

To define the B-stationary equilibrium for $\{\mathcal{F}_g(\cdot)\}_{g\in G}$, we extend the definition of association set for the MPECs to the EPEC. Because the association set depends on $(y, w)$, which is determined through (19) jointly by all MPECs' design variables, we state the following equivalence property.

Let $(y, w)$ solve (19) with given $\{\bar{x}_g \in X_g\}_{g\in G}$ and consider any two firms $g$ and $g'$, then the association set at $\bar{x}_g$ for $\mathcal{F}_g(\bar{x}_{-g})$ and the association set at $\bar{x}_{g'}$ for $\mathcal{F}_{g'}(\bar{x}_{-g'})$ are equivalent, i.e.,

$$B_g(\bar{x}_g, \bar{x}_{-g}) = B_{g'}(\bar{x}_{g'}, \bar{x}_{-g'}), \quad g, g' \in G.$$

The above equivalence of the association sets among all MPECs implies that

- if $\bar{x}_g \in Int(\tilde{P}_g(\alpha))$ for some $\alpha$, then $\bar{x}_{g'} \in Int(\tilde{P}_{g'}(\alpha))$;
- if $\bar{x}_g$ is in the boundaries of polyhedra $\tilde{P}_g(\alpha_1)$, $\tilde{P}_g(\alpha_2)$, ..., $\tilde{P}_g(\alpha_k)$, $\bar{x}_{g'}$ is also in the boundaries of polyhedra $\tilde{P}_{g'}(\alpha_1)$, $\tilde{P}_{g'}(\alpha_2)$, ..., $\tilde{P}_{g'}(\alpha_k)$.

We define the association set of $\{\mathcal{F}_g(\cdot)\}_{g\in G}$ as follows. Given $\{\bar{x}_g\}_{g\in G}$, let $(y, w)$ solve (19), then the association set for the EPEC $\{\mathcal{F}_g(\cdot)\}_{g\in G}$ is

$$B(\{\bar{x}_g\}_{g\in G}) = \{\alpha \subseteq \{1,2,...,m\} : \{i : y_i > 0\} \subseteq \alpha \subseteq \{i : w_i = 0\}\}.$$

The association set of $\{\mathcal{F}_g(\cdot)\}_{g\in G}$ allows us to characterize the B-stationary equilibria of $\{\mathcal{F}_g(\cdot)\}_{g\in G}$ as follows. A set $\{\bar{x}_g\}_{g\in G}$ is a *B-stationary equilibrium* of $\{\mathcal{F}_g(\cdot)\}_{g\in G}$ if $\bar{x}_g$ is a B-stationary point of $\mathcal{F}_g(\bar{x}_{-g})$ for all $g \in G$, i.e., $\bar{x}_g$ is a local minimum or stationary point of $QP_g(\alpha)$ for all $\alpha \in B(\{\bar{x}_g\}_{g\in G})$.

### 4.3.2 The Scheme

To solve $\{\mathcal{F}_g(\cdot)\}_{g\in G}$, we start with an arbitrary set $\{\bar{x}_g^0 \in X_g\}_{g\in G}$. At each outer iteration $k$, we compute $\bar{x}_g^k$ from $\mathcal{F}_g(\bar{x}_{-g}^k)$ for each $g \in G$ while taking $\bar{x}_{-g}^k$ as given. The algorithm terminates when the improvement of the design variables in two consecutive iterations is reduced to a predetermined limit, or when the number of iterations reaches a predetermined upper bound.

*The EPEC Scheme*

0. (Initialization) Select an arbitrary $\{\bar{x}_g^0 \in X_g\}_{g\in G}$. Let $k := 1$.

1. (Loop) Let $\{\bar{x}_g^k\}_{g\in G} := \{\bar{x}_g^{k-1}\}_{g\in G}$.

   For each $g \in G$,

   apply the MPEC algorithm to $\mathcal{F}_g(\bar{x}_{-g}^k)$.

   if $\mathcal{F}_g(\bar{x}_{-g}^k)$ is unbounded,

   report the failure of finding an equilibrium, stop.

   else

   let $\bar{x}_g^k$ and $(y, w)$ be the returned decision and state variables.

2. (Termination check)

   If $\|\{\bar{x}_g^k - \bar{x}_g^{k-1}\}_{g\in G}\|$ is within a given error bound,

   report $(\{\bar{x}_g^k\}_{g\in G}, y, w)$ as a B-stationary equilibrium, stop.

   else if the predetermined bound of the number of iterations is reached,

   stop.

else

go to step 1 with $k := k + 1$.

#### 4.3.3 Remarks

- The termination basis of an MPEC problem can be used as the starting basis for the next MPEC problem.
- The termination point for an MPEC problem can be used as the starting point for solving the next MPEC problem.

## 5 Computational Results

We implemented in MATLAB the MPEC and EPEC algorithms which utilize the optimization toolbox for solving quadratic programs. In the implementation, we treat any number below $10^{-16}$ as zero to account for roundoff errors. Tests of the algorithms are performed on both randomly generated problems and representative test cases specific to the context of electricity markets.

### 5.1 Tests of the MPEC Algorithm

The main computational effort involved in the EPEC scheme is to solve the MPECs. While Our MPEC algorithm is guaranteed to terminate in finite number of steps (see Section 4.2.4), it is not known whether it can be solved in polynomial time. In this section, we test the actual performance of the algorithm on a randomly generated set of generic MPEC problems with quadratic objective functions. Specifically, these MPEC problems are of the form:

$$\min_{x,y} \quad \frac{1}{2}\begin{bmatrix} x & y \end{bmatrix} P \begin{bmatrix} x \\ y \end{bmatrix} + c^T \begin{bmatrix} x \\ y \end{bmatrix}$$

subject to:

$$Ax + a \leq 0$$

$$w = Nx + My + q,$$

$$w \geq 0, \quad y \geq 0, \quad w^T y = 0$$

where $P$, $A$, $B$, $M$ (a positive-semidefinite matrix), $N$, $c$, $a$ and $q$ are constant matrices and vectors with suitable dimensions. We use the "QPECgen" package by Jiang and Ralph [21] to generate these MPEC programs.

In the tests, we launch the MPEC algorithm from random starting points. Table 1 summarizes the test results. The first three columns list the dimensions of the decision and state variables, and columns 5 to 7 report the minimum, maximum and average numbers of iterations, respectively. We observe that

- The average number of iterations increases moderately as the dimension of the MPEC problems grows (except for the case of $n = 150$ and $m = 100$); but, there does not exist such a trend for the minimum and maximum numbers of iterations.
- The algorithm is able to effectively solve MPEC problems with relatively large dimensions. Note that all instances in table 1 have greater dimensions than those reported in [21].

Table 1: Test results of the MPEC algorithm

| Dim($x$) | Dim($y$) | Dim($w$) | Total dimension | Iterations | | |
|---|---|---|---|---|---|---|
| | | | | min | max | average |
| 25 | 50 | 50 | 125 | 3 | 34 | 16 |
| 50 | 50 | 50 | 150 | 7 | 35 | 18 |
| 50 | 100 | 100 | 250 | 2 | 49 | 22 |
| 100 | 100 | 100 | 300 | 10 | 43 | 23 |
| 150 | 100 | 100 | 350 | 2 | 30 | 14 |
| 100 | 200 | 200 | 500 | 3 | 38 | 23 |
| 200 | 200 | 200 | 600 | 2 | 88 | 29 |
| 200 | 500 | 500 | 1200 | 2 | 76 | 43 |

## 5.2 Tests of the EPEC Scheme

We now test the MPEC/EPEC algorithms on an EPEC problem derived from the stylized Belgian electricity system which was also used in our previous work [38]. This system is originally composed of 92 380kv and 220kv transmission lines including some lines in neighboring countries for capturing the effect of loop flow. Parallel lines between the same pairs of nodes have been collapsed into single lines with equivalent electric characteristics. In total, the stylized network comprises 71 transmission lines and 53 nodes (see figure 3). Generation units in this system are located, respectively, at

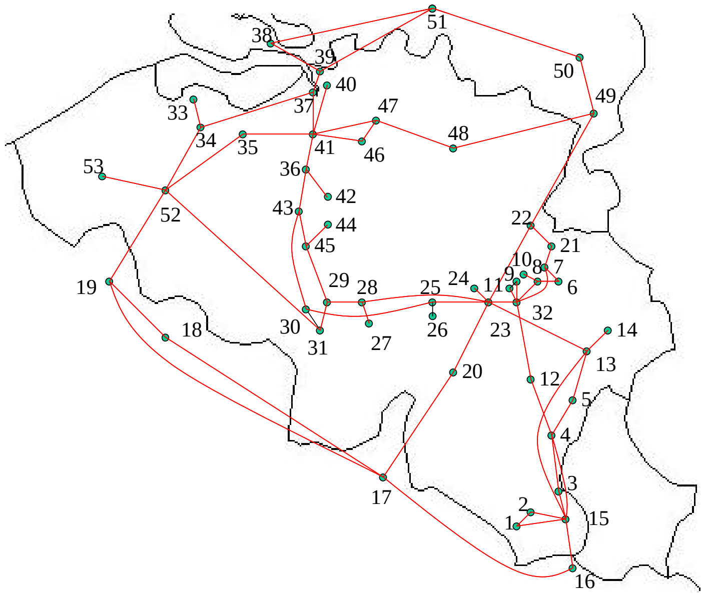


Figure 3: Belgian high voltage network

the nodes $\{7, 9, 10, 11, 14, 22, 24, 31, 33, 35, 37, 40, 41, 42, 44, 47, 48, 52, 53\}$. The ownership structure, zonal aggregation in the forward market and contingency states are fictitious and so are the nodal demand functions, although they are calibrated to actual demand information.

Table 2 lists the nodal information for this test problem, including the IDF slopes, the marginal generation costs (marginal costs are constant in this example), and the capacity bounds of the generation units. The network data is summarized in table 3 which lists the impedance of the transmission line and the corresponding thermal limits. Only the lines 22-49, 29-45, 30-43 and 31-52 are prone to congestion in this example. The method for calculating the state-dependent PTDF matrices from the network data can be found in standard electrical engineering textbooks (e.g. [15]) and will be omitted here due to space limitation.

We assume six independent contingency states in the spot market. The first three states correspond to demand uncertainty, while all generation units and all transmission lines are rated at their full capacities. State 4, 5 and 6 have the same demand levels as state 2, but they represent the system contingencies resulting from transmission or generator outages. State 4 denotes a transmission breakdown on line 31-52. State 5 and 6 capture the unavailability of two generation units at nodes 10 and 41, respectively. The price intercepts of the hypothetical IDFs and the probabilities of the six states are given in table 4.

Table 2: Nodal data

| Node | IDF slope $/MW$^2$ | Marg. cost $/MW | Capac. MW | Node | IDF slope $/MW$^2$ | Marg. cost $/MW | Capac. MW | Node | IDF slope $/MW$^2$ | Marg. cost $/MW | Capac. MW |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | – | 0 | 19 | 0.68 | – | 0 | 37 | 1 | 10 | 1399 |
| 2 | 0.82 | – | 0 | 20 | 1.05 | – | 0 | 38 | 0.85 | – | 0 |
| 3 | 1.13 | – | 0 | 21 | 1 | – | 0 | 39 | 1 | – | 0 |
| 4 | 1 | – | 0 | 22 | 1.1 | 19 | 602 | 40 | 1.15 | 10 | 1378* |
| 5 | 0.93 | – | 0 | 23 | 1 | – | 0 | 41 | 1 | 21 | 522 |
| 6 | 0.85 | – | 0 | 24 | 0.75 | 10 | 2985 | 42 | 0.79 | 18 | 385 |
| 7 | 1 | 45 | 70 | 25 | 1 | – | 0 | 43 | 0.68 | – | 0 |
| 8 | 1 | – | 0 | 26 | 0.8 | – | 0 | 44 | 1.03 | 20 | 538 |
| 9 | 0.88 | 18 | 460 | 27 | 1.13 | – | 0 | 45 | 1 | – | 0 |
| 10 | 0.9 | 18 | 121* | 28 | 1 | – | 0 | 46 | 1 | – | 0 |
| 11 | 1 | 20 | 124 | 29 | 0.93 | – | 0 | 47 | 1 | – | 0 |
| 12 | 0.73 | – | 0 | 30 | 0.85 | – | 0 | 48 | 0.73 | 22 | 258 |
| 13 | 1 | – | 0 | 31 | 1 | 18 | 712 | 49 | 1.2 | – | 0 |
| 14 | 0.85 | 13 | 1164 | 32 | 1 | – | 0 | 50 | 1.5 | – | 0 |
| 15 | 1 | – | 0 | 33 | 0.88 | 20 | 496 | 51 | 1 | – | 0 |
| 16 | 1.3 | – | 0 | 34 | 0.5 | – | 0 | 52 | 1 | 20 | 879 |
| 17 | 1 | – | 0 | 35 | 1 | 25 | 1053 | 53 | 0.7 | 58 | 95 |
| 18 | 0.79 | – | 0 | 36 | 0.73 | – | 0 | | | | |

* These numbers are zeros in states 5 and 6 respectively.

It is worth a mention that the stylized system has the dimension $2|C|\times(|N|+|L|) = 684$ of $y$ (and $w$), and the total number of possible partitions is $2^{684}$. In this implementation, we terminate the EPEC algorithm at an outer iteration $k$ if the relative improvement of the MPECs' decision variables (forward commitments) is no greater than $10^{-8}$, i.e., $\|\{\bar{x}_g^k - \bar{x}_g^{k-1}\}_{g\in G}\| \leq 10^{-8}\|\{\bar{x}_g^{k-1}\}_{g\in G}\|$.

We ran the tests with different numbers of zones and firms, and, for each test, we start with randomly generated decision variables of the MPECs. In the implementation, the MPEC algorithm was limited to execute a single inner iteration. (Note that, if the MPEC algorithm terminates before it reaches a B-stationary point, the LCP constraints (19) are still satisfied; this allows us to trade off the accuracy of the MPEC solution against speed of the overall EPEC scheme. We also tried some other rules for terminating the MPEC algorithm; however, they don't provide comparable results.) The test results are summarized in table 5. Columns 4 to 9 show the minimum, maximum and average numbers of outer iterations and quadratic programs, respectively. In addition, tables 6 and 7 report the outer iterations of the firms' total forward commitments for the cases of two zones and two or three firms. We find that

Table 3: Belgian transmission network data

| Line | Imped. (Ohm) | Capac. (MW) | Line | Imped. (Ohm) | Capac. (MW) | Line | Imped. (Ohm) | Capac. (MW) |
|---|---|---|---|---|---|---|---|---|
| 1-2 | 23716 | 345 | 16-17 | 2633 | 5154 | 34-37 | 7048 | 1350 |
| 1-15 | 6269 | 345 | 17-18 | 4236 | 1715 | 34-52 | 12234 | 1350 |
| 2-15 | 8534 | 345 | 17-19 | 1939 | 5140 | 35-41 | 14204 | 1350 |
| 3-4 | 5339 | 240 | 17-20 | 8071 | 1179 | 35-52 | 9026 | 1420 |
| 3-15 | 11686 | 240 | 18-19 | 1465 | 13710 | 36-41 | 15777 | 2770 |
| 4-5 | 6994 | 510 | 19-52 | 11321 | 1179 | 36-42 | 11186 | 2840 |
| 4-12 | 5887 | 405 | 20-23 | 13165 | 1316 | 36-43 | 15408 | 2770 |
| 4-15 | 3644 | 240 | 21-22 | 47621 | 1420 | 37-39 | 66471 | 1420 |
| 5-13 | 6462 | 510 | 22-23 | 11391 | 1350 | 37-41 | 21295 | 1350 |
| 6-7 | 23987 | 300 | 22-49 | 9138 | 1350 | 38-39 | 10931 | 1650 |
| 6-8 | 9138 | 400 | 23-24 | 41559 | 5540 | 38-51 | 17168 | 946 |
| 7-21 | 14885 | 541 | 23-25 | 16982 | 1420 | 39-51 | 8596 | 1650 |
| 7-32 | 5963 | 410 | 23-28 | 8610 | 1350 | 40-41 | 11113 | 2770 |
| 8-9 | 45360 | 400 | 23-32 | 33255 | 1350 | 41-46 | 11509 | 2840 |
| 8-10 | 26541 | 800 | 25-26 | 134987 | 1420 | 41-47 | 13797 | 1420 |
| 8-32 | 11467 | 400 | 25-30 | 11991 | 1420 | 43-45 | 34468 | 1350 |
| 9-11 | 20157 | 410 | 27-28 | 64753 | 1420 | 44-45 | 47128 | 1420 |
| 9-32 | 10012 | 375 | 28-29 | 38569 | 1350 | 46-47 | 34441 | 1420 |
| 11-32 | 18398 | 375 | 29-31 | 284443 | 1350 | 47-48 | 14942 | 1420 |
| 12-32 | 4567 | 405 | 29-45 | 14534 | 1350 | 48-49 | 6998 | 1420 |
| 13-14 | 121410 | 2700 | 30-31 | 269973 | 1420 | 49-50 | 5943 | 3784 |
| 13-15 | 5094 | 790 | 30-43 | 10268 | 1420 | 50-51 | 2746 | 5676 |
| 13-23 | 5481 | 2770 | 31-52* | 1453 | 400 | 52-53 | 1279 | 2840 |
| 15-16 | 8839 | 400 | 33-34 | 40429 | 1420 | | | |

* This line breaks down in state 4.

Table 4: States of the Belgian spot market

| State | IDF intercept ($/MW$^2$) | Probability | Type and description |
|---|---|---|---|
| 1 | 1000 | 0.20 | Demand uncertainty: Demands are on the peak. |
| 2 | 500 | 0.50 | Demand uncertainty: Demands are at shoulder. |
| 3 | 250 | 0.20 | Demand uncertainty: Demands are off-peak. |
| 4 | 500 | 0.03 | Contingency of line breakdown: Line 31-52 goes down. |
| 5 | 500 | 0.03 | Contingency of generation outage: Plant at node 10 goes down. |
| 6 | 500 | 0.04 | Contingency of generation outage: Plant at node 41 goes down. |

- For all test problems, the EPEC scheme converges rapidly.
- There exists no clear relationship between the problem dimensions and the number of iterations. However, the total number of quadratic programs grows as the number of firms increases.
- In the tests, the EPEC scheme quickly reaches the proximity of the B-stationary equilibrium, after which it only improves the significant decimal digits (see, for example, tables 6 and 7).

Table 5: Test results of the EPEC algorithm

| $\lvert Z\rvert$ | $\lvert G\rvert$ | Dim($y$) | Outer iterations | | | Quadratic programs | | |
|---|---|---|---|---|---|---|---|---|
| | | | min | max | average | min | max | average |
| 2 | 2 | 684 | 4 | 8 | 6 | 15 | 29 | 24 |
| 2 | 3 | 684 | 8 | 11 | 9 | 47 | 65 | 55 |
| 2 | 4 | 684 | 7 | 12 | 10 | 55 | 94 | 77 |
| 3 | 2 | 684 | 2 | 4 | 3 | 7 | 15 | 11 |
| 3 | 3 | 684 | 4 | 6 | 5 | 23 | 34 | 29 |
| 3 | 4 | 684 | 4 | 10 | 8 | 32 | 79 | 62 |
| 4 | 2 | 684 | 3 | 9 | 7 | 11 | 36 | 27 |
| 4 | 3 | 684 | 7 | 21 | 13 | 41 | 126 | 80 |
| 4 | 4 | 684 | 9 | 25 | 15 | 70 | 197 | 122 |

Table 6: Iterations of the firms' total forward commitments (2 firms)

| Outer iteration | Firm 1 | Firm 2 |
|---|---|---|
| 0 | 0.000000 | 0.000000 |
| 1 | -513.063752 | 575.219726 |
| 2 | -331.223467 | 1546.721883 |
| 3 | -545.254227 | 1747.692181 |
| 4 | -552.287608 | 1747.692181 |
| 5 | -552.287608 | 1747.692181 |

# 6 Economic Interpretation of the Results

The EPEC algorithm is not guaranteed to locate a (global) Nash equilibrium; however, as we will demonstrate in this section, it produced results that are consistent with economic intuition.

In particular, we considered two hypothetical generator ownership structures with two zones in the stylized Belgian network: node 1 through 32 belongs to zone #1, and the remaining nodes to

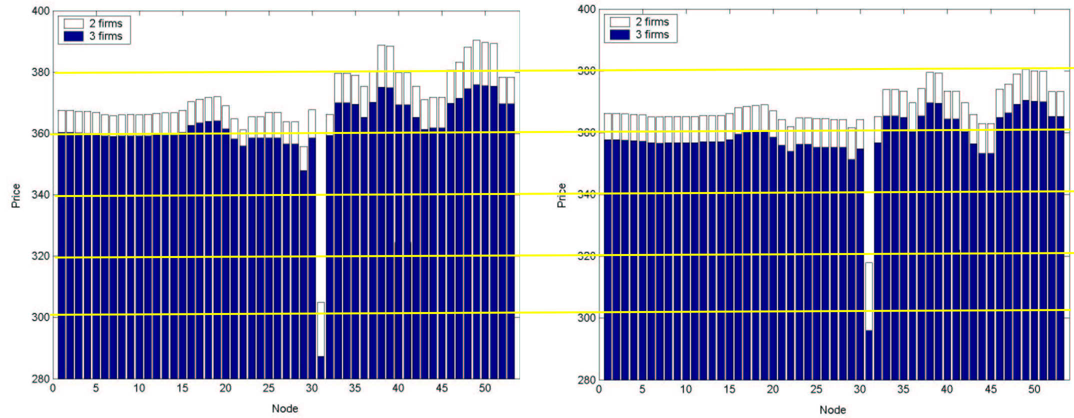
Table 7: Iterations of the firms' total forward commitments (3 firms)

| Outer iteration | Firm 1 | Firm 2 | Firm 3 |
|---|---|---|---|
| 0 | 0.000000 | 0.000000 | 0.000000 |
| 1 | 6739.889190 | -16.249658 | -288.471837 |
| 2 | 6739.889190 | 246.601419 | -103.536223 |
| 3 | 6851.687937 | 556.357457 | 71.319790 |
| 4 | 7001.487699 | 849.405693 | 154.719273 |
| 5 | 7154.268773 | 1001.093059 | 149.846951 |
| 6 | 7237.416442 | 1006.167745 | 149.619740 |
| 7 | 7239.775870 | 1006.342137 | 149.611431 |
| 8 | 7239.859233 | 1006.348165 | 149.611140 |
| 9 | 7239.862110 | 1006.348374 | 149.611129 |
| 10 | 7239.862110 | 1006.348382 | 149.611130 |
| 11 | 7239.862110 | 1006.348382 | 149.611130 |

zone #2. The first structure has two firms, where the units at the node set {9, 11, 22 31, 35, 37, 41, 47, 52, 53} belong to the first firm and the remaining units to the second firm. The second structure is composed of three firms, operating the units at {7, 11, 33, 37, 41, 52, 53}, {10, 14, 24, 40, 44, 48} and {9, 22, 31, 35, 42, 47}, respectively.

We observe that, under both resource structures, firms have strategic incentives for forward contracting as reported in tables 6 and 7. While some firms in our example have taken short forward positions, the total forward commitment in the entire market is positive.

We plot in figure 4 the expected spot nodal prices under two settlements and contrast them with the corresponding nodal prices in the equilibrium of a single-settlement market which is obtained by constraining all firms' forward positions to zero. We first notice that, whether or not there exists a forward market, the three-firm structure yields lower spot equilibrium prices than the Duopoly structure, as one would expect. Moreover, under both the two- and three-firm structures, a two-settlement equilibrium results in lower spot equilibrium prices at most nodes than a single settlement. However, nodes #29 and #31 do not follow this trend. Consequently, two settlements increase social welfare and consumer surplus. These results suggest that the welfare-enhancing effect described in [1] and [2] generalizes to the case with flow congestion, system contingency and demand uncertainty, although that effect is quantitatively different due to generator capacities and transmission limits.




Single Settlement

Two Settlements

Figure 4: Expected spot nodal prices

# 7 Concluding Remarks

We study the Nash-Cournot equilibrium in two-settlement electricity markets. We develop an EPEC model of this equilibrium, in which each firm solves an MPEC problem parameterized by the design variables of the other MPECs.

We propose an MPEC algorithm by taking advantage of the special properties of the problems at hand. This algorithm partitions the feasible region of the decision variables into a set of polyhedra, and projects the state variables into the space of the decision variables. The algorithm solves a quadratic program for a stationary point in each polyhedron, and pivots through adjacent polyhedra while maintaining feasibility of the linear complementarity constraints. We establish the finite global convergence of this MPEC algorithm. An EPEC scheme is constructed by deploying the MPEC algorithm iteratively. Numerical tests on randomly generated quadratic MPECs and on the EPEC derived from a stylized Belgian electricity network demonstrate the effectiveness of the algorithms.

One limitation of our model is the assumption of risk neutrality on the part of the generating firms. Unfortunately, introducing risk aversion will make the objective functions of the MPECs non-quadratic which significantly increases the computational complexity of the model.

On the other hand we like to point out that although the MPEC and EPEC algorithms are presented here in the context of two-settlement electricity markets, they can be applied to other quadratic EPEC problems provided that the linear complementarity constraints yield unique values of the state variables.

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