Arrow’s theorem of the deductible: moral hazard and stop-loss in health insurance

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Arrow’s theorem of the deductible

Theorem

“If an insurance company is willing to offer any insurance policy against loss desired by the buyer at a premium which depends only on the policy’s actuarial value, then the policy chosen by a risk-averting buyer will take the form of 100% coverage above a deductible minimum” (Arrow, 1963).
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Logic is obvious (and robust): since it is better for the consumer to insure expenditures when disposable income is low rather than high, insurance funds should be spent on the highest expenditures.
Moral hazard in health insurance

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- Focus on the balance between the welfare loss of moral hazard, calling for a larger out-of-pocket share for the insured, and the welfare gain of risk sharing, calling for a more generous reimbursement (Pauly, 1968; Zeckhauser, 1970).
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- Most popular model has a fixed coinsurance rate. Non-linear model (Blomqvist, 1997): “alas, a complicated problem, whose algebra is not particularly revealing” (Cutler and Zeckhauser, 2000).
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• Real world insurance policies often feature explicit deductibles (the Netherlands, Switzerland), or a stop-loss (Belgian maximum billing system). Partial first-dollar insurance and stop loss in RAND-experiment.
This paper

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4. Ex ante moral hazard.
FIRST BEST: structure of the model

- $S$ states of health $s = 1, \ldots, S$. 
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- Individuals have separable preferences over vectors $(M_s, C_s) \in \mathbb{R}_+^2$ of medical expenditures $M_s$ and consumption $C_s$:

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U_s(M_s, C_s) = f_s(M_s) + g(C_s)
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- Functions $f_s$ and (state-independent) $g$ are continuously differentiable and strictly concave.
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- Resources are state-independent: $W_s = W_t = W$ for all $s, t = 1, \ldots, S$.
- Individual may buy insurance at a premium

$$\pi = (1 + \lambda) \sum_s p_s \alpha_s M_s$$
Optimal policy

Optimal policy problem

$$\max_{\alpha_1, \ldots, \alpha_S, M_1, \ldots, M_S} V(M, C) = \sum_s p_s \left[ f_s(M_s) + g(W - \pi - (1 - \alpha_s)M_s) \right]$$

subject to $\pi = (1 + \lambda) \sum_s p_s \alpha_s M_s$. 
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\]

subject to \( \pi = (1 + \lambda) \sum_s p_s \alpha_s M_s \).

First-order conditions:

\[
\frac{dV}{dM_s} = p_s \left[ f'_s - (1 - \alpha_s) g'_s \right] - (1 + \lambda) p_s \alpha_s \sum_t p_t g'_t = 0,
\]

\[
\frac{dV}{d\alpha_s} = p_s M_s \left[ g'_s - (1 + \lambda) \sum_t p_t g'_t \right] \leq 0, \quad \alpha_s \frac{dV}{d\alpha_s} = 0.
\]
Arrow's result

(1) Level of medical expenditures is set optimally:

for all \( s = 1, \ldots, S, \ f'_s = g'_s \)
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\[ f'_s = g'_s \]

for all \( s = 1, \ldots, S \), \( f'_s = g'_s \)

(2) **Optimality of the deductible:**

either \( \alpha_s = 0 \) or \( g'_s = (1 + \lambda) \sum_t p_t g'_t := (1 + \lambda) \bar{g}' \).

or (with the deductible \( D := (1 - \alpha_s) M_s \) and \( g'_D \) for marginal utility of wealth at \( C = W - \pi - D \)),

\[ \alpha_s = \max(0, \frac{M_s - D}{M_s}), \quad g'_D = (1 + \lambda) \bar{g}' \].
SECOND BEST: ex post-moral hazard

Choice of treatment after observing the state (without regard for the impact of $M_s$ on premium $\pi$):

$$\max_{M_s} f_s(M_s) + g(W - \pi - (1 - \alpha_s)M_s)$$

leading to “overconsumption”,

$$f'_s = g'_s(1 - \alpha_s).$$
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leading to “overconsumption”,

$$f'_s = g'(1 - \alpha_s).$$

Define

$$\eta_s = \frac{\alpha_s}{M_s} \frac{dM_s}{d\alpha_s} > 0$$
Optimal policy

Optimal policy problem

\[
\max_{\alpha_1, \ldots, \alpha_S} \Lambda = \sum_s p_s \left[ f_s(M_s(\alpha_s)) + g(W - \pi - (1 - \alpha_s) M_s(\alpha_s)) \right]
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subject to \( \pi = (1 + \lambda) \sum_s p_s \alpha_s M_s(\alpha_s) \).
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subject to $\pi = (1 + \lambda) \sum_s p_s \alpha_s M_s(\alpha_s)$.

First-order conditions

$$\frac{\partial \Lambda}{\partial \alpha_s} = p_s M_s \left[ g'_s - \bar{g}' (1 + \lambda) (1 + \eta_s) \right]$$

$$\alpha_s \frac{\partial \Lambda}{\partial \alpha_s} = 0.$$
“Implicit deductible” property

Rewriting, we obtain

either \( \alpha_s = 0 \) or \( g_s' = (1 + \lambda)g'(1 + \eta_s) \)

**Proposition.** If resources are state-independent, preferences are separable with state-independent consumption preferences and the probabilities of the different states cannot be influenced by the consumer, the optimal insurance contract results in the same indemnities as a contract with 100% insurance above a variable deductible positively related to \( \eta_s \), the elasticity of medical expenditures with respect to the insurance rate \( \alpha_s \).
Interpretation

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- Policy implemented through variable insurance rates $\alpha_s$, NOT through the explicit announcement of a deductible $D$. Assumption of state-specific insurance rates is unrealistic.
- Qualitative finding 1: our results validate the practice of higher insurance rates (not only indemnities) for major medical expenses. (If $\eta_s = \eta_t$, then $(1 - \alpha_s)M_s = (1 - \alpha_t)M_t$).
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- Qualitative finding 1: our results validate the practice of higher insurance rates (not only indemnities) for major medical expenses. (If $\eta_s = \eta_t$, then $(1 - \alpha_s) M_s = (1 - \alpha_t) M_t$).
- Qualitative finding 2: optimal medical insurance scheme will in general be nonlinear. Our vector of insurance rates $(\alpha_1, ..., \alpha_S)$ can be seen as discrete approximation of non-linear model of Blomqvist (1997).
THIRD BEST: explicit stop-loss arrangement
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$$\max_{\alpha_s, D} \Lambda = \sum_{M_s < D} p_s \left[ f_s(M_s(\alpha_s)) + g(W - \pi - (1 - \alpha_s)M_s(\alpha_s)) \right]$$

$$+ \sum_{M_s \geq D} p_s \left[ f_s(M_s) + g(W - \pi - D) \right]$$

under the constraints

$$\pi = (1 + \lambda) \left[ \sum_{M_s < D} p_s \alpha_s M_s(\alpha_s) + \sum_{M_s \geq D} p_s (M_s - D) \right]$$

$$f_s' = (1 - \alpha_s)g_s' \text{ if } M_s < D, \quad f_s' = 0 \text{ if } M_s \geq D.$$
Solution

First-order conditions for $\alpha_s$ (states with $M_s < D$)

$$\text{either } \alpha_s = 0 \text{ or } g_s' = (1 + \lambda)g'(1 + \eta_s).$$
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either $\alpha_s = 0$ or $g'_s = (1 + \lambda)\bar{g}'(1 + \eta_s)$.

First-order condition for $D$

$$\frac{\partial \Lambda}{\partial D} = - \sum_{M_s \geq D} p_s \left[ g'_s - (1 + \lambda) \sum_t p_t g'_t \right] \leq 0, \quad D \frac{\partial \Lambda}{\partial D} = 0.$$  

Writing $g'_D$ for $g'(W - \pi - D)$, this gives

either $D = 0$ or $g'_D = \bar{g}'(1 + \lambda)$.  \hspace{1cm} (1)
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Combining

if $\alpha_s D > 0$, then $g'_s = g'_D(1 + \eta_s) > g'_D$.  

Result

Conclusion: if $D > 0$, then $\alpha_s = 0$.

Proposition If resources are state-independent, preferences are separable with state-independent consumption preferences and the probabilities of the different states cannot be influenced by the consumer, an optimal stop-loss insurance policy takes the form of a deductible, i.e. there is no reimbursement for expenses below the stop-loss amount and full reimbursement of the excess of expenses over the deductible.
EX ANTE MORAL HAZARD: treatment as prevention

General preventive behavior (lowering probability of expensive states) should be subsidized.

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- Model with explicit deductible $D$.
- Only two states of health: $s$ (standard) and $t$ (calling for expensive therapy).
- Consulting GP in state $s$ may lead to early detection of severe diseases and may help avoiding severe complications: $p_t = p_t(M_s)$ with $dp_t/dM_s < 0$. 
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- Model with explicit deductible $D$.
- Only two states of health: $s$ (standard) and $t$ (calling for expensive therapy).
- Consulting GP in state $s$ may lead to early detection of severe diseases and may help avoiding severe complications: $p_t = p_t(M_s)$ with $d p_t / d M_s < 0$.
- Preventive and curative aspects from regular doctor visits cannot be distinguished.
Optimal policy

Policy problem:

$$\max_{\alpha_s, D} \Lambda = (1 - p_t(M_s)) \left[ f_s(M_s(\alpha_s)) + g(W - \pi - (1 - \alpha_s)M_s(\alpha_s)) \right]$$

$$+ p_t(M_s) \left[ f_t(M_t) + g(W - \pi - D) \right]$$

subject to

$$\pi = (1 + \lambda) \left[ (1 - p_t(M_s))\alpha_s M_s(\alpha_s) + p_t(M_s)(M_t - D) \right] .$$
**Optimal policy**

**Policy problem:**

\[
\begin{align*}
\max_{\alpha_s, D} \Lambda &= (1 - p_t(M_s)) \left[ f_s(M_s(\alpha_s)) + g(W - \pi - (1 - \alpha_s)M_s(\alpha_s)) \right] \\
&\quad + p_t(M_s) \left[ f_t(M_t) + g(W - \pi - D) \right] \\
\text{subject to} & \\
\pi &= (1 + \lambda) \left[ (1 - p_t(M_s))\alpha_s M_s(\alpha_s) + p_t(M_s)(M_t - D) \right].
\end{align*}
\]

Define the elasticity of \( p_s \) with respect to \( M_s \):

\[
\eta_{p_s M_s} = \frac{M_s dp_s}{p_s dM_s} > 0
\]
Optimality conditions

Behavior insured patient, who disregards the impact of $M_t - D$ on the premium $\pi$:

$$\frac{\partial \Lambda}{\partial M_s} \bigg|_\pi = (1 - p_t) \left[ f_s' - g_s'(1 - \alpha_s) \right] + \frac{dp_t}{dM_s} \left[ f_t + g_t - (f_s + g_s) \right] = 0.$$
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Behavior insured patient, who disregards the impact of $M_t - D$ on the premium $\pi$:

$$\frac{\partial \Lambda}{\partial M_s} \bigg|_{\pi} = (1 - p_t) \left[ f'_s - g'_s (1 - \alpha_s) \right] + \frac{dp_t}{dM_s} \left[ f_t + g_t - (f_s + g_s) \right] = 0.$$  

Condition defining a socially efficient level of $M_s$:

$$\frac{\partial \Lambda}{\partial M_s} = \frac{\partial \Lambda}{\partial M_s} \bigg|_{\pi} - g' \left( 1 + \lambda \right) \left[ (1 - p_t) \alpha_s + \frac{dp_t}{dM_s} (M_t - D - \alpha_s M_s) \right] = 0.$$
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$$\left. \frac{\partial \Lambda}{\partial M_s} \right|_{\pi} = (1 - p_t) \left( f_s' - g_s'(1 - \alpha_s) \right) + \frac{dp_t}{dM_s} \left( f_t + g_t - (f_s + g_s) \right) = 0.$$  

Condition defining a socially efficient level of $M_s$:

$$\frac{\partial \Lambda}{\partial M_s} = \frac{\partial \Lambda}{\partial M_s} \left( \pi - \bar{g}'(1 + \lambda) \left[ (1 - p_t)\alpha_s + \frac{dp_t}{dM_s} (M_t - D - \alpha_s M_s) \right] \right) = 0.$$  

Optimal $\alpha_s$:

$$\alpha_s = \frac{\eta p_s M_s}{1 + \eta p_s M_s} \frac{(M_t - D)}{M_s}$$


Result

Proposition

If resources are state-independent and preferences are separable with state-independent consumption preferences, the desirability of preventive behaviour (lowering the probability of the expensive health states) justifies some insurance below the deductible (i.e. $\alpha_s > 0$) if health care expenditures in a state of standard health have a negative effect on the probability of getting into a state with large medical expenses, but the preventive component of these expenditures cannot be identified as such.
Result

**Proposition**  If resources are state-independent and preferences are separable with state-independent consumption preferences, the desirability of preventive behaviour (lowering the probability of the expensive health states) justifies some insurance below the deductible (i.e. $\alpha_s > 0$) if health care expenditures in a state of standard health have a negative effect on the probability of getting into a state with large medical expenses, but the preventive component of these expenditures cannot be identified as such.

Strong analogy with literature on complementarity/substitution relationships between different health care commodities (e.g. Goldman and Philipson, 2007): subsidizing medicines to lower hospital expenditures.
Conclusion

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Common practice of first-dollar insurance in a model with stop-loss is not optimal in standard model: a straight deductible is optimal.
Conclusion

- Logic of Arrow’s theorem of the deductible remains at work in a model with ex post moral hazard. Strong arguments in favour of stop-loss arrangement.
- Common practice of first-dollar insurance in a model with stop-loss is not optimal in standard model: a straight deductible is optimal.
- However, some insurance below deductible is optimal if health care expenditures in relatively healthy states have a negative effect on the probability of getting into a state with large medical expenses.
Important open issues

- Time-dimension: what about the chronically ill?
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- Redistributive considerations in public health insurance schemes. Relationship with other redistributive instruments (e.g. nonlinear income tax).