What is a QALY Worth?

Admissible Utility Functions for Health, Longevity, and Wealth

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Abstract

Health risks are commonly quantified using either of two alternative measures, willingness to pay (WTP) and quality-adjusted life years (QALYs). The relationship between these measures is of interest for comparing benefit-cost analysis (which uses WTP) with cost-effectiveness analysis (which uses QALYs), identifying a threshold for determining which health interventions are cost-effective, and estimating WTP for health improvement from expected QALY gains. WTP and QALYs rest on different theoretical foundations. Utility functions that are consistent with both concepts are derived. Some commonly used utility functions (e.g., expected present value of period utility) are inconsistent with these assumptions except in special cases. The admissible utility functions imply that an individual's marginal WTP per QALY is not constant but depends on baseline and incremental QALYs, wealth, and aversion to longevity risk, which implies that cost-effectiveness analysis using a fixed threshold value per QALY is inconsistent with individuals' preferences. Moreover, the admissible utility functions imply that value per statistical life is weakly increasing in future health and life expectancy, in conflict with empirical evidence. These results suggest that QALYs are not a valid measure of individuals' health and longevity preferences.

Keywords: quality adjusted life year, willingness to pay, value per statistical life, risk aversion
JEL classification: D61, D81, I10

1. Introduction

An individual's utility depends on health, longevity, and wealth that can be used for consumption or as a bequest. The effects of these factors are of interest both for modeling individual behavior and for evaluating policies that affect risks to health and longevity. Parameters of particular interest include individuals' rates of substitution between health, longevity, and wealth.

In cost-effectiveness analysis (CEA), it is conventionally assumed that preferences for health and longevity can be represented by quality-adjusted life years (QALYs) and policies are evaluated in terms of the cost incurred per expected QALY gained. Determining whether the cost per QALY is acceptable or excessive requires comparison with some measure of willingness to pay (WTP) per QALY. The 'threshold' value that distinguishes interventions that are cost-effective for a society from those that are not is generally assumed to be constant across health interventions (Gold et al., 1996).[1] Whether CEA is consistent with economic welfare theory and with benefit-cost analysis depends in part on whether individual WTP per QALY is constant across individuals and for different changes in QALYs (Johannesson, 1995; Garber and Phelps, 1997).[2]

Estimates of WTP per QALY are also useful for combining results from the QALY and WTP literatures to value changes in health. For example, French and Mauskopf (1992), Tolley et al. (1994), and Cutler and Richardson (1997) calculate the monetary value of health by multiplying changes in QALYs by a constant WTP per QALY. Johnson et al. (1997) and Van Houtven et al. (2006) predict WTP to avoid morbidity using less restrictive nonlinear functions of a QALY-based measure of health quality and illness duration. The U.S. Food and Drug Administration and Department of Transportation estimate the benefits of averted morbidity by multiplying the expected QALY gain by a constant monetary value and adding expected medical costs (Adler, 2006; Robinson, 2007). Hence, for interpreting cost-

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[1] For example, the U.K. National Institute for Clinical Excellence states 'Generally, however, if a treatment costs more than £20,000-30,000 per QALY, then it would not be considered cost effective.' (www.nice.org.uk/newsroom/features/measuringeffectivenessandcosteffectivenesstheqaly.jsp, accessed 12 May 2012).

[2] Johannesson (1995: 485) writes "the difference between cost-benefit analysis and cost-effectiveness analysis is that in cost-effectiveness analysis the willingness to pay per QALY gained is assumed to be the same for all individuals under all circumstances and for all sizes of the change in QALYs." Dolan and Edlin (2002) show that an individual's WTP per QALY cannot be constant if illness hinders the ability to enjoy consumption.

effectiveness analysis and for evaluating current approaches to valuing morbidity that rely on QALYs, it is important to examine the theoretical relationship between WTP and QALY measures of health.

In previous work, the effects of health, longevity, and consumption on individual utility have been represented by a variety of alternative utility functions (Rey and Rochet, 2004). Grossman (1972) assumed a general function,

$$U = u(h_1, h_2, ..., h_T, c_1, c_2, ..., c_T), \qquad (1.1)$$

where h_t is the flow of health services and c_t is other consumption in period t. Subsequent authors have imposed more structure, often assuming that lifetime utility is the expected discounted sum of period utilities,

$$U = \sum_{t=0}^{T} \delta^t s_t u(h_t, c_t), \qquad (1.2)$$

where δ is a constant discount factor,[3] s_t is the probability of surviving to age t, and T is a maximum possible age (e.g., Cutler and Richardson, 1997; Meltzer, 1997). The period-utility function in equation (1.2) is often assumed to be multiplicative in health and consumption,

$$u(h_t, c_t) = q(h_t)\, v(c_t), \qquad (1.3)$$

where q(·) and v(·) are monotonically increasing (e.g., Garber and Phelps, 1997; Murphy and Topel, 2006[4]). Bleichrodt and Quiggin (1999) provide an axiomatic basis for the representation in equations (1.2–1.3) under both expected-utility and rank-dependent expected-utility theories. Alternative representations include the monetary-loss-equivalent model, in which the effect of impaired health is equivalent to the effect of reduced consumption,

$$u(h, c) = u[h^*, c - m(h^* - h)], \qquad (1.4)$$

where h* represents full health and m(h* – h) is a monetary value (Evans and Viscusi, 1990) and an additively separable model (Eeckhoudt et al., 1998),

$$u(h, c) = q(h) + v(c), \qquad (1.5)$$

in which q(·) and v(·) are monotone increasing in their arguments. The literature on valuing mortality risk typically uses a health-state-dependent specification,

$$u(h, c) = u_h(c), \qquad (1.6)$$

[3] While constant exponential discounting using discount factors δ^t is most common, other discounting functions can be represented by replacing δ^t by δ_t (Harvey, 1994).
[4] Murphy and Topel (2006) define v(·) as a function of both consumption and leisure.

where the health state is restricted to two values, alive and dead (e.g., Drèze, 1962; Jones-Lee, 1974; Weinstein et al., 1980; Pratt and Zeckhauser, 1996). Equation (1.6) is also used to model preferences for health while living (Evans and Viscusi, 1990; Sloan et al., 1998).

In this paper, I characterize the specifications of the lifetime utility function for health, longevity, and wealth that are consistent with the assumption that preferences over health and longevity can be represented by QALYs (including generalizations of the standard QALY). I examine the properties of these admissible specifications, including their implications for willingness to pay to increase health, longevity, and probability of surviving the current period (the 'value per statistical life' or VSL). I find that the notion that WTP per QALY is constant for an individual is inconsistent with the admissible utility functions. This result implies that the conventional application of cost-effectiveness analysis using a constant threshold value for cost-effectiveness is inconsistent with individual preferences. In addition, the admissible utility functions have implications for VSL that are inconsistent with many empirical estimates (e.g., that VSL is weakly increasing in future health and life expectancy). This suggests that QALYs are not a valid measure of individuals' preferences for health and longevity, and that conventional cost-effectiveness analysis is inconsistent with economic welfare theory.

The paper is organized as follows. Section 2 describes the assumptions under which (generalized) QALYs describe preferences for health and longevity and derives the utility functions for health, longevity, and wealth that are consistent with these assumptions. Section 3 characterizes the implications of the admissible utility functions for the marginal utility of wealth and for risk postures with respect to longevity and wealth. Marginal WTP per QALY and its dependence on health, longevity, and wealth are examined in Section 4 and marginal WTP to decrease mortality risk (VSL) is examined in Section 5. Section 6 concludes.

2. Utility for Health, Longevity, and Wealth

Let lifetime utility $u(h, t, w)$ depend on health h, longevity t, and wealth w (including labor and other income). Utility may be defined over an entire lifetime or the part remaining at the individual's current age. The value of h may be constant or represent an average or representative lifetime value. This approach contrasts with previous work that assumes lifetime utility can be represented as a discounted sum of period utilities (equation (1.2)), which requires assumptions about intertemporal separability, additivity, and the form of the discounting function (e.g., Shepard and Zeckhauser, 1984; Rosen, 1988; Ng, 1992; Harvey, 1994; Bleichrodt and Quiggin, 1999). Moreover, it obviates the need to consider the extent to which individuals can allocate consumption and health spending over the lifecycle. For example, Bleichrodt's and Quiggin's (1999) estimates of WTP per QALY assume that individuals allocate medical and other spending to obtain constant health and consumption over their lifetimes.[5]

Although the standard QALY ubiquitous in the literature assumes risk neutrality over longevity, generalizations exist that allow for other risk postures with respect to longevity. Miyamoto et al. (1998) characterize generalized QALYs by

$$u(h,t) = q_s(h)\, v(t) \tag{2.1}$$

where $q_s(h)$ is the health-related quality of life (HRQL) associated with health h and v(t) is a function of longevity with v(0) = 0. HRQL is normalized to $q_s(h) = 0$ for health states indifferent to death and $q_s(h) = 1$ for 'full' or 'perfect' health. Under expected utility, equation (2.1) is a valid utility function for health and longevity if and only if preferences satisfy a 'zero condition' (all values of h are equally preferred when t = 0) and 'standard-gamble invariance' (preferences between (h, t) and a lottery offering (h, t') with probability p and (h, t'') with probability 1 – p are independent of h, for all health states preferred to death). Risk posture with respect to longevity is unrestricted; v(t) may exhibit risk aversion, risk neutrality, and risk proneness for different values of t. Indeed, v(t) need not even be monotone; e.g., it may decrease for t greater than some value.

Bleichrodt et al. (1997) proposed a special case of equation (2.1) assuming risk neutrality with regard to longevity, which implies v(t) = t. Pliskin et al. (1980) identified a set of assumptions (mutual utility independence of health and longevity, constant proportional tradeoff of longevity for health) which imply constant relative risk aversion (or risk proneness) for longevity, i.e.,

$$v(t) = \frac{t^{1-r}}{1-r}, \tag{2.2}$$

which is risk neutral for r = 0, risk seeking for r < 0, and risk averse for r > 0 (with v(t) = ln(t) for r = 1). In practice, QALYs are usually discounted at some constant rate ρ, which implies constant absolute risk aversion with respect to longevity and that

$$v(t) = \int_0^t e^{-\rho s} ds = \frac{1}{\rho}\left(1 - e^{-\rho t}\right), \tag{2.3}$$

[5] The assumption that individuals can allocate medical spending to achieve constant health over the lifecyle appears very strong, given that health tends to deteriorate with age and that not all impairments can be eliminated by treatment.

which is risk neutral for $\rho = 0$ (for which $v(t) = t$), risk seeking for $\rho < 0$, and risk averse for the conventional case with $\rho > 0$. Discounted QALYs can also be interpreted as risk neutral over the present value of longevity (Johannesson et al., 1994).[6]

In contrast to wealth, for which risk-seeking preferences are considered unusual if not implausible, risk-seeking preferences with respect to longevity appear to be normatively acceptable. Moreover, there is descriptive evidence that all three types of risk preference exist. Pliskin et al. (1980) surveyed ten health-utility experts and report that four expressed risk-seeking preferences, four expressed risk-neutral preferences, and two expressed risk-averse preferences with respect to longevity. Miyamoto and Eraker (1985) surveyed 46 patients with symptomatic coronary artery disease and estimated values of $1 - r$ (equation (2.2)) from more than 10 to less than 0.3. In a general-population survey, Corso and Hammitt (2001) asked respondents to choose their preferred lottery from each of five pairs of lotteries on longevity. Only 15 percent of respondents made all five choices consistent with any global risk posture, but 43 percent had at least four responses that were consistent with a global risk posture. Of the full sample, 16 percent gave at least four responses consistent with risk proneness, 0.5 percent gave responses consistent with risk neutrality, and 27 percent gave responses consistent with risk aversion.

An alternative formulation to equation (2.1) is (Pliskin et al., 1980)

$$u(h,t) = v[q_t(h) \cdot t]. \qquad (2.4)$$

The measure of HRQL in equation (2.4), q_t, differs from the measure in equation (2.1), q_s, though each can be obtained from the other if the function $v(\cdot)$ is known.[7] The measure incorporated in equation (2.1), $q_s(h)$, is the answer to a standard-gamble question in which the subject reports that he is indifferent between living t years with health h and a lottery offering probability $q_s(h)$ of surviving t years in full health and complementary probability $1 - q_s(h)$ of immediate death. The measure incorporated in equation (2.4), $q_t(h)$, is the answer to a time-

[6] Note that if t represents future longevity from the individual's current age, constant proportional risk aversion and risk proneness (equation (2.2)) are dynamically inconsistent. Consider a choice between living 20 years longer and a lottery offering equal chances of living 16 or 25 years longer (where health is held constant). An individual with $v(t) = t^{1/2}$ would prefer the lottery. But 16 years later he would reverse his preference, since he would prefer living 4 years longer to the lottery offering equal chances of living 0 and 9 more years. In contrast, constant absolute risk aversion and risk proneness (equation (2.3)) are dynamically consistent.

[7] Note that $q_s(h) = v[q_t(h) \cdot t] / v(t)$. If v is geometric then $q_s(h) = v[q_t(h)]$ and is independent of t.

tradeoff question in which a subject reports that he is indifferent between living t years with health h and $q_t(h) \cdot t$ years in full health.

The formulations in equations (2.1) and (2.4) are equivalent. In the remainder of the paper I adopt the formulation (2.4) and omit the subscript t on q(h), yielding the following definition.

Definition 1: Generalized QALYs are defined as

$$Q = v[q(h) \cdot t] \qquad (2.5)$$

where q(h) is the HRQL associated with health h, t is longevity, and v(0) = 0.

This choice is consistent with the conventional practice when QALYs are discounted and with the formulation derived by Pliskin et al. (1980: equations (4a) and (4b)). It has the advantage of treating HRQL and longevity symmetrically, which simplifies notation in the following sections. Common specifications for the function $v(\cdot)$ include constant absolute risk aversion (which is equivalent to discounted QALYs),

$$v(t) = sgn(r)[1 - e^{-rt}], \qquad r \neq 0$$

$$v(t) = t, \qquad r = 0 \qquad (2.6a)$$

and constant relative risk aversion,

$$v(t) = sgn(1 - r)t^{1-r}, \qquad r \neq 1$$

$$v(t) = ln(t), \qquad r = 1. \qquad (2.6b)$$

For both specifications (2.6a) and (2.6b), v(t) is risk averse for $r > 0$, risk neutral for $r = 0$, and risk prone for $r < 0$. An increase in r increases risk aversion.

The literature on QALYs is virtually silent on the extent to which HRQL depends on wealth, income, or consumption (Hammitt, 2002). In practice, HRQL is elicited with no attention to income, wealth, or consumption and is assumed (at least implicitly) to be independent of these factors (Dolan, 2000; Schulpher and O'Brien, 2000; Lawrence et al., 2006). Indeed, the notion that the benefits of health interventions can be evaluated without considering income or consumption of the affected individuals is likely to be one reason that QALYs have become so widely used in evaluation of public-health and medical interventions.

This practice motivates the following definition.

Definition 2: The HRQL for health h, q(h), satisfies HRQL invariance if its value is independent of w.

Assume that preferences over health and longevity are consistent with generalized QALYs (equation (2.5), that q(h) satisfies HRQL invariance, and that, holding wealth constant at any value w', the individual prefers more QALYs to fewer. Then the conditional utility functions u(h, t | w) are related as positive affine transformations (Keeney and Raiffa, 1976). Hence the utility function for health, longevity, and wealth can be written as

$$u(h, t, w) = u(h, t, w_0) \cdot a(w) + b(w) \qquad (2.7)$$

where a(w) > 0 (so more QALYs are preferred to fewer).[8] If there is a subsistence wealth level w_s below which the individual does not prefer survival to death, a(w) may be non-positive for $w \leq w_s$. Substituting equation (2.5) into equation (2.7) yields the primary result of this paper:

Proposition 1. If preferences for health and longevity conditional on wealth are consistent with generalized QALYs Q, health-related quality of life q(h) satisfies HRQL invariance, and, for $w > w_s$, more QALYs are preferred to fewer, then

$$u(h, t, w) = Q \cdot a(w) + b(w) \qquad (2.8)$$

where $a(w) > 0$ for $w > w_s$.

Equation (2.8) describe the utility functions for health, longevity, and wealth that are admissible under the assumptions that preferences for health and longevity can be represented using generalized QALYs and that q(h) satisfies HRQL invariance. The following sections investigate the implications of these utility functions for: (a) the relationship between the risk postures for longevity and wealth, (b) willingness to pay per QALY, and (c) willingness to pay to reduce current mortality risk.

[8] Bleichrodt and Quiggin (1999) derive a similar result for period-utility $u(h_t, c_t)$ then argue that the utility of a bequest is additively separable from utility of consumption, which implies $b(c_t) = 0$ for the period-utility function (1.3). They do not consider the effects of adding a term representing the utility of a bequest to the lifetime expected utility function (1.2).

3. Properties of the Admissible Utility Functions

In this section, properties of the admissible utility functions are considered, including the marginal utility of wealth, risk postures with respect to longevity and wealth, and their dependence on health and longevity.

3.1. Marginal Utility of Wealth

To explore the implications of the admissible utility functions (equation (2.8)) for the marginal utility of wealth, first consider the case of death, for which t = 0. Then u(h, 0, w) = b(w), so b(w) is the utility for wealth conditional on death (i.e., the utility of a bequest). For the admissible utility functions, the utility of a bequest must be equal to the utility of wealth for all health states indifferent to dead (for which q(h) = 0). In the literature on valuing mortality risk, it is conventionally assumed that b'(w) ≥ 0, i.e., the marginal utility of a bequest is non-negative (e.g., Jones-Lee, 1974; Weinstein et al., 1980), and I adopt that assumption here.

The marginal utility of wealth is given by

$$\frac{\partial}{\partial w}u(h,t,w)=Qa'(w)+b'(w)\,, \qquad (3.1)$$

where prime indicates first derivative. I assume that the marginal utility of wealth is strictly positive for health states preferred to dead, which implies

$$a'(w) > -b'(w)/Q. \qquad (3.2)$$

If b'(w) = 0, then a'(w) > 0 and the marginal utility of wealth increases with QALYs. This implies that the marginal utility of wealth increases with both health and longevity. If b'(w) > 0, then a'(w) may be less than or equal to zero so long as its absolute value is not too large. If a' < 0, the marginal utility of wealth decreases with health and longevity. The literature on valuing mortality risk assumes that the marginal utility of wealth is greater in the event of survival than in the event of death, which implies a'(w) > 0 and hence that the marginal utility of wealth increases with health and longevity. Limited empirical evidence (Viscusi and Evans, 1990; Sloan et al., 1998; Domeij and Johannesson, 2006; Finkelstein et al., 2008) and the frequent adoption of the multiplicative utility function (1.3) also support the notion that the marginal utility of wealth increases with health and longevity, and I adopt that assumption here.

3.2. Risk Postures for Longevity and Wealth

There is no necessary relationship between the risk postures for longevity and for wealth. The longevity risk posture is determined by v(t) and is independent of wealth. This follows immediately from the assumption that preferences for health and longevity are independent of wealth.

In contrast, the risk posture with respect to wealth may depend on health and longevity. The Arrow-Pratt measure of local absolute risk aversion $\pi(w)$ (Pratt, 1964) is given by

$$\pi(w) = -\frac{\frac{\partial^2}{\partial w^2} u(h,t,w)}{\frac{\partial}{\partial w} u(h,t,w)} = -\frac{Qa''(w) + b''(w)}{Qa'(w) + b'(w)} \qquad (3.3)$$

where single and double primes denote first and second derivatives, respectively. If the individual is indifferent to the level of his bequest (b' = 0), then the measure of local risk aversion is independent of QALYs and equal to the Arrow-Pratt measure for the function a(w), i.e., $\pi_a(w) = -a''(w)/a'(w)$.[9] Alternatively, b' > 0 and differentiating equation (3.3) with respect to Q yields

$$\frac{\partial \pi}{\partial Q} = [\pi_a - \pi_b] \frac{a'b'}{[Qa' + b']^2} \qquad (3.4)$$

where $\pi_b(w)$ is the Arrow-Pratt measure for the function b(w). Hence risk aversion with respect to wealth increases, is constant, or decreases with QALYs as the Arrow-Pratt measure for a(w) is respectively greater than, equal to, and less than the Arrow-Pratt measure for b(w). If a'' and b'' are of opposite sign, wealth risk posture can change sign as QALYs increase. Hammitt et al. (2009) report survey evidence showing that wealth risk aversion decreases with health and life expectancy (and increases with age, as also found by Barsky et al., 1997). These results suggests that individuals are more risk averse with respect to their bequests than their wealth given survival and that $\pi_a < \pi_b$.

The effect of aversion to longevity risk on aversion to financial risk is indeterminate and depends on what values are held constant as longevity risk aversion increases. Assuming one of the specific functional forms for v(·) (i.e., equations (2.6a) or (2.6b)) and differentiating equation (3.3) and with respect to the measure of risk aversion r yields

$$\frac{\partial \pi}{\partial r} = [\pi_a - \pi_b] \frac{a'b'}{[v(Q)a' + b']^2} \frac{\partial Q}{\partial r}. \qquad (3.5)$$

The sign of the last term, $\partial Q/\partial r$, depends on the value of Q and on what is held constant as risk aversion is increased. For the function v(·) that captures risk posture in equation (2.5), we require that v(0) = 0 but there is no obvious normalization at any other value of Q. If we

[9] Except at Q = 0 where $\pi(w) = 0$.

require v(1) = 1 (so that 1 QALY has the same value regardless of the degree of risk aversion), then $\partial Q/r > 0$ for $0 < Q < 1$ and $\partial Q/r < 0$ for $Q > 1$.[10]

It is widely assumed that individuals are risk averse with respect to wealth (conditional on survival) and there is substantial empirical support for this assumption (e.g., the existence of insurance and risk-dependence of returns on financial instruments). Demand for life insurance suggests risk aversion with respect to bequests. In the remainder of the paper, I assume

$$a(w) > 0, \tag{3.6a}$$

$$a'(w) > 0, \tag{3.6b}$$

$$b'(w) \geq 0, \tag{3.6c}$$

$$a''(w) \leq 0, \text{ and} \tag{3.6d}$$

$$b''(w) \leq 0, \tag{3.6e}$$

consistent with conventional assumptions about utility increasing with QALYs, the marginal utilities of wealth and of bequests, and weak financial risk aversion.

4. Willingness to Pay per QALY

This section examines the implications of the admissible utility functions for WTP per QALY and its dependence on health, longevity, and wealth. Let V denote the individual's marginal WTP per QALY. V is obtained by totally differentiating equation (2.8) holding utility constant to obtain

$$V = -\frac{dw}{dQ} = \frac{a(w)}{Qa'(w)+b'(w)} + \frac{\partial w}{\partial Q}. \tag{4.1}$$

In general, marginal WTP per QALY depends on wealth and QALYs. The first term in equation (4.1) represents pure WTP for improvements in health and longevity and is positive under assumptions (3.6). The second term represents the feedback effect of changes in health and longevity on lifetime wealth. The sign and magnitude of $\frac{\partial w}{\partial Q}$ may depend on whether QALYs are gained by improved health or increased longevity (e.g., fewer sick days while working or a longer retirement). For example, Meltzer (1997) and Bleichrodt and Quiggin

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[10] For illustration, substitute the specification (2.6b) into the formula for generalized QALYs (equation (2.5)) to obtain $Q = \text{sgn}(1 - r)\,[q(h)\,t]^{(1-r)}$. Then $\partial Q/r > 0$ for $0 < Q < 1$ and $\partial Q/r < 0$ for $Q > 1$.
Using the specification (2.6a), $\partial Q/r > 0$ for all Q, but this is due to the normalization (which sets the limit of u(Q) to 1 as $Q \rightarrow \infty$. This specification can be renormalized to $v(Q) = \text{sgn}(r)\,[1 - e^{-rQ}] / [1 - e^{-r}]$ for which v(1) = 1 and the sign of $\partial Q/r > 0$ for $0 < Q < 1$ and $\partial Q/r < 0$ for $Q > 1$.

(1999) assume lifetime wealth depends on longevity but not on health. In the remainder of this section, I focus on pure WTP for health and longevity, neglecting any feedback effect. Setting $\frac{\partial w}{\partial Q} = 0$, equation (4.1) simplifies to

$$V = -\frac{dw}{dQ} = \frac{a(w)}{Qa'(w) + b'(w)}. \qquad (4.2)$$

Hence WTP per QALY decreases with QALYs. If the individual is indifferent to his bequest, b' = 0 and V is inversely proportional to total QALYs. Otherwise, WTP decreases with QALYs but less than proportionally. To obtain that WTP per QALY is independent of QALYs requires a'(w) = 0, but as discussed above this is implausible because it implies the marginal utility of wealth is independent of both health and survival.[11]

If b' = 0, V is proportional to a(w)/a'(w), which is the 'fear of ruin' (the reciprocal of 'boldness'), where ruin is defined as the level of w at which a(w) = 0 and hence the individual is indifferent between life and death. Fear of ruin measures the individual's willingness to risk financial ruin in exchange for a marginal increase in wealth (Aumann and Kurz, 1977; Foncel and Treich, 2005). WTP per QALY increases with fear of ruin: when the marginal utility of wealth is small, the individual is unwilling to accept a small risk of ruin to increase wealth and is also willing to spend more for health and longevity.

Note that pure WTP per QALY is the same whether QALYs are gained through increased longevity or improved health. (Except in the case of risk neutrality with respect to longevity, this result depends on the choice of specification (2.4) over (2.1), i.e., on using time-tradeoff rather than standard-gamble measures of HRQL.) This result contrasts with the result of Bleichrodt and Quiggin (1999), who found that marginal WTP per QALY differs for health and longevity gains, even in the case of risk neutrality.[12]

Some empirical studies support the result that marginal WTP per QALY decreases with QALYs. With respect to incremental QALYs, stated-preference studies (e.g, Tolley et al., 1994; Pinto-Prades et al., 2009; Haninger and Hammitt, 2011) and the Johnson et al.

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[11] If actuarially fair insurance were available, however, optimal insurance would lead to equal marginal utility of income in all states.

[12] As an example, for certain longevity t, annual consumption c, and elasticity of utility with respect to consumption ε, their equations (12) and (13) imply marginal pure WTP per QALY equals ct/ε for an incremental gain in health and (1 – ε) ct/ε for an incremental gain in longevity.

(1997) meta-analyses find that WTP is an increasing, concave function of QALYs gained.[13] In addition, Krupnick et al. (2002) and Smith et al. (2004) report evidence that people suffering health impairments that reduce future health and longevity (e.g., cancer, angina) have larger VSL than those without these impairments. Since people with these impairments presumably have both fewer baseline QALYs and smaller QALY gains from reducing current mortality risk, their average WTP per QALY is larger but the effects of baseline and incremental QALYs cannot be distinguished.

Marginal WTP per QALY increases with wealth. Under assumptions (3.6), an increase in wealth increases the numerator and decreases the denominator of equation (4.2). This finding is unsurprising and has been anticipated (e.g., Gold et al., 1996; Garber and Phelps, 1997).

The effect of longevity risk aversion on WTP per QALY is ambiguous. Substituting a specific functional form for Q (e.g., equation (2.6a) or (2.6b) into equation (4.2) and differentiating with respect to r yields

$$\frac{\partial V}{\partial r} = -\frac{aa'}{\left[Qa' + b'\right]^2}\frac{\partial Q}{\partial r}. \qquad (4.3)$$

As described in Section 3, the sign of the last term depends on the value of Q and on what is held constant as risk aversion increases.

5. Willingness to Pay to Reduce Mortality Risk

WTP to reduce mortality risk, often described as the ‘value per statistical life’ (VSL), is the marginal rate of substitution between wealth and the probability of surviving the current time period. The individual’s expected utility is given by

$$EU = (1 - p)\, u_a(w) + p\, u_d(w) \qquad (5.1)$$

where p is the probability of dying in the current period and $u_a(w)$ and $u_d(w)$ represent utility of wealth conditional on surviving and not surviving the period, respectively (e.g., Drèze, 1962; Jones-Lee, 1974; Weinstein et al., 1980). Total differentiation of equation (5.1) yields

$$VSL = \frac{dw}{dp} = \frac{u_a(w) - u_d(w)}{(1-p)u_a'(w) + pu_d'(w)}. \qquad (5.2)$$

Intuition suggests VSL should increase with future health and longevity; estimates of WTP per QALY or value per statistical life year (VSLY) are often based on dividing VSL by future life years or QALYs (e.g., Hirth et al., 2000). Yet in the standard model (equation

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[13] In contrast, the Van Houtven et al. (2006) meta-analysis finds that average WTP per QALY falls with duration but increases with health-quality gain.

(5.2)) the effects of health and longevity on VSL are ambiguous. Better health and greater life expectancy conditional on surviving the current period increase the utility of survival $u_a(w)$ and may increase the marginal utility of wealth conditional on survival $u_a'(w)$. Reductions in life expectancy and health clearly limit the opportunities for gaining utility from wealth (Dolan and Edlin, 2002) and there is some empirical evidence that impaired health reduces the marginal utility of wealth (Viscusi and Evans, 1990; Sloan et al., 1998; Domeij and Johannesson, 2006; Finkelstein et al., 2008). Depending on the magnitudes of the effects on the total and the marginal utilities of wealth given survival, better health and increased longevity may increase, decrease, or not affect VSL.

For the admissible utility function (2.8), $u_a(w) = Q \cdot a(w) + b(w)$ and $u_d(w) = b(w)$. Substituting these expressions into equation (5.2) yields

$$VSL = \frac{dw}{dp} = \frac{Qa(w)}{(1-p)Qa'(w)+b'(w)}. \qquad (5.3)$$

The effects of health and longevity on VSL can be seen by inspection of equation (5.3). If the individual is indifferent to the level of his bequest (b'(w) = 0), then VSL = $(1\text{-}p)^{-1}$ a(w)/a'(w) (i.e., fear of ruin divided by the survival probability) and is independent of health and longevity. If the marginal utility of the bequest is positive (b'(w) > 0), then the proportionate effect of an increase in Q is larger in the numerator than in the denominator of equation (5.3) and so VSL increases with health and longevity. These results can be verified by differentiating equation (5.3) to obtain

$$\frac{\partial}{\partial Q}VSL = \frac{ab'}{[(1-p)Qa'+b']^2}. \qquad (5.4)$$

The value of equation (5.4) is zero when b'(w) = 0 and positive when b'(w) > 0. The admissible utility functions constrain the relationship between the effects of health and longevity on the utility and the marginal utility of wealth in such a way as to remove the ambiguity about the effects of increased longevity and health on VSL in the standard model (equation (5.2)).

The effect of longevity risk aversion on VSL is ambiguous. Substituting a specific functional form for Q (e.g., equation (2.6a) or (2.6b)) into equation (5.3) and differentiating with respect to r yields

$$\frac{\partial}{\partial r}VSL = \frac{ab'}{[(1-p)Qa'+b']^2}\frac{\partial Q}{\partial r}. \qquad (5.5)$$

As discussed in Section 3, the sign of the last term is ambiguous.[14]

Krupnick et al. (2002) and Smith et al. (2004) report empirical evidence suggesting that reduced health increases VSL, which is inconsistent with the admissible utility functions. In addition, empirical estimates suggest that VSL varies little with age or initially rises then falls with age (Aldy and Viscusi, 2007, 2008; Krupnick, 2007). Increasing VSL with age is inconsistent with the admissible utility functions, and a constant VSL is consistent only if the individual is indifferent to his bequest.

6. Conclusion

If an individual's preferences for health and longevity can be represented using QALYs, including generalized QALYs Q that are discounted for time or that reflect risk aversion or risk proneness with regard to longevity, then his utility function for health, longevity, and wealth is tightly constrained: it must be a positive affine transformation of QALYs in which the slope and intercept may depend on wealth, i.e., $u(h, t, w) = Q \cdot a(w) + b(w)$, where $a(w) > 0$ (so utility is increasing in QALYs). Many of the utility functions used in the literature are inconsistent with these assumptions. Significantly, commonly used utility functions that are additive over time (equation (1.2)) are inconsistent with these assumptions except in the special case where the individual is risk-neutral with respect to longevity and the discount rate is zero (i.e., the discount factor $\delta = 1$).

Standard assumptions about the marginal utility of wealth and of bequest imply $a'(w) > 0$ and $b'(w) \geq 0$. Standard assumptions about financial risk aversion imply $a''(w) \leq 0$ and $b''(w) \leq 0$. Under these assumptions, I obtain the following results:

1. Risk posture with respect to longevity and to wealth are independent. Risk posture with respect to longevity is independent of wealth. Risk aversion with regard to wealth can depend on health and longevity, consistent with survey evidence (Barsky et al., 1997; Hammitt et al., 2009)

2. Marginal willingness to pay (WTP) per QALY decreases with QALYs and increases with wealth. (Any effect of health or longevity on wealth supplements this effect.) These results accord with most empirical results.

3. The value per statistical life (VSL) or marginal WTP to reduce current mortality risk increases with QALYs (consistent with common intuition but contrary to much empirical

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[14] Eeckhoudt and Hammitt (2004) show that the effect on VSL of risk aversion with respect to wealth is also ambiguous, again because it depends on what is held constant when risk aversion changes.

evidence), except that VSL is independent of longevity and health when the individual is indifferent to the level of bequest.

4. The effects of risk posture with respect to longevity on WTP per QALY and on VSL are ambiguous. They depend on what is held constant when longevity risk aversion changes and on total QALYs.

The result that individual WTP per QALY is not constant implies that cost-effectiveness analysis using QALYs and a fixed threshold value per QALY is inconsistent with standard welfare economics and benefit-cost analysis (Dolan and Edlin, 2001). Instead, the threshold for an intervention should decrease with both baseline QALYs and the expected increase. Moreover, WTP to reduce mortality risk is not proportional to life expectancy, and so an individual's value per statistical life year (VSLY) is not constant but increases with wealth and decreases with health and life expectancy. Estimates of WTP to avoid mortality or morbidity calculated using a constant WTP per life year or per QALY are consequently inconsistent with the economic theory underlying WTP and QALYs. These results imply that QALYs are not a valid measure of individual preferences for health and longevity, and that cost-effectiveness analysis is more appropriately viewed as an alternative (Williams, 1993) rather than an implication of benefit-cost analysis.

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