# The Logistic Function Approach to Discriminatory and Uniform Price Treasury Auctions

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**Abstract**

There has been considerable debate in the literature concerning whether uniform or discriminatory pricing raise more revenue in Treasury Bill auctions. A standard approach has been to examine empirically how revenue changes given a switch from one type to the other. The weakness of this approach is that such a revenue change may be due to changes in economic conditions. This paper is the first to examine the two methods while taking into account changes in economic conditions. To do this, it adopts a three-stage procedure. First, it fits a logistic function to the cumulative bid distribution for each auction. Second, it estimates two sets of equations relating the logistic function parameters to economic conditions, one for uniform and one for discriminatory pricing, on the assumption that T-bill offerings by the government are exogenous. And third, using the estimated equations, it predicts how much revenue would have been raised from discriminatory price auctions if instead uniform pricing had been used, holding economic condition constant, and vice versa. The data employed are for Turkish Treasury auctions from 2000-2002, in the middle of which the Turkish Treasury switched auction types. The results point to the superiority of the discriminatory price auction, but also cast doubt on the assumption that government T-bill offerings are exogenous.

**Keywords**: Treasury auctions, multi-unit auctions, discriminatory and uniform-price auctions, market bid, logistic function
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# 1 Introduction

In a multiunit auction problem, multiple units of identical objects are sold through auctions. A typical example is that of Treasury bills, which are sold by this method in many countries. There are two most commonly used auction methods;[1] discriminatory and uniform price auctions.[2] In the former, a bidder pay *her* bid amount if the bid is accepted. In the latter, however, *the minimum accepted* bid, rather than the bid itself, is paid by the bidder. Although both methods have been used for many years, there is neither theoretical nor empirical consensus on which auction method raises higher revenue.

A standard empirical approach in comparing the revenue raised from the two auction types has been to compare how revenue changes given a switch from one auction type to the other. The weakness of this approach is that the change in revenue may be due to changes in economic conditions rather than the change in auction type. This paper is the first to examine such revenue changes while taking into account changes in economic conditions. To do this, it adopts a three-stage procedure. First, it fits a logistic function to the cumulative bid distribution for each auction. Second, it estimates two sets of equations relating the logistic function parameters to economic conditions, one for uniform and one for discriminatory pricing. And third, using the estimated equations, it predicts how much revenue would have been raised from each auction during the period when discriminatory pricing was employed if instead uniform pricing had been employed, holding economic conditions constant, and vice versa. To implement this procedure, it employs

[1] A hybrid of these two methods used in Spain is beyond the scope of this paper.

[2] In both auction methods, bidders submit multiple price-quantity pairs, and then the seller allocates the supply to those bid pairs whose price bid is higher than the minimum accepted price bid.

Turkish treasury auction data for the period 2000-2002, in the middle of which the Turkish Treasury switched auction types.

In the first part of the paper, by estimating the parameters of the logistic growth curve for the Turkish Treasury auction data, I show that logistic growth curves represent the market bid functions fairly well, a result confirmed in the literature for some other countries.[3] In the second part of the paper, I estimate the fluctuations in the parameters of the logistic function from one auction to another by using economic variables like Preget and Waelbroeck (PW) (2003) do for *only* discriminatory auctions. In addition to finding the economic determinants of the estimated logistic function parameters under discriminatory auctions as do PW (2003), I show that this can be also done for uniform auctions. I also find the estimation equations for the parameters under uniform auctions. The estimation equations for the parameters are different for the two auction methods. This suggests that under different auction types bidders pay attention to different market variables. Thus the approach used in this paper differs that of Boukai and Landsberger (BL) (1999) and Berg, Boukai, and Landsberger (BBL) (1999), who treat fluctuations in the parameters of the logistic function from one auction to another as random shocks instead of estimating these fluctuations from economic variables.

Estimating the logistic curves allows us to determine the results of both auction formats under the same economic environment. The estimation results are somewhat contradictory. According to the estimation results with the discriminatory auctions data, discriminatory auctions raise much

[3] Boukai and Landsberger (1999) show it for Israeli auctions data, Preget and Waelbroeck (2003) for French data, and Berg, Boukai, and Landsberger (1999) for Norwegian and Swiss data. The first two studies show that the aggregate demand functions for discriminatory auctions can be well approximated by the logistic growth function. The last one finds that the market demand function of Norwegian bonds – long-term bills – issued with uniform price auctions is also in the form of a logistic curve.

more revenue than uniform auctions. However, on the uniform auctions data, the estimated revenue of discriminatory auctions is a little lower than that of uniform auctions. This result might be due to two reasons. First, the Treasury decides the optimal supply in an auction given the auction method, so supply may vary in the two types. Second, the frequency of auctions is also determined given the auction method. Both of these issues are left for future study. In light of these results, I conclude that the discriminatory format raises more revenue for the Treasury.

The data set employed in this paper is prepared in Ozcan (2004). It contains bidder-level data for both discriminatory and uniform auctions undertaken by the Turkish Treasury from January 2000 to February 2002. The Turkish Treasury switched from discriminatory to uniform auctions in February 2001. The data set also contains all the secondary market transactions either done in the Istanbul Stock Exchange (ISE) Bonds and Bills Market, or conducted through bilateral trade, which has to be registered to the ISE Settlement and Custody Bank.

The findings of this paper appears contradictory to those of BBL (1999), who argue in favor of uniform auctions. The approach taken here is superior to theirs, since they don't account economic conditions. Moreover, using *both* auction methods during one time period, which is the case in BBL's (1999) data,[4] is significantly different from using only one auction method for one time period and the other method for another time period, which is the case in this paper's data. The strategic decisions of bidders might very well be different under these two different auction environments. One thus may naturally expect different comparative revenues for each auction method

[4] In their data for Norwagian Treasury auctions, bills are the Treasury's short-term securities whereas bonds are the long-term securities. For a given time period, both short- and long-term securities co-exist in their data. Unlike the approach taken in this paper, they compare the Treasury's revenue for short-term and long-term securities issued with discriminatory and uniform price auctions, respectively.

under these two *distinct* types of conditions. Therefore, the results of BBL (1999) may not be directly comparable with the results of this paper.[5]

Among others, Wilson (1979), Nautz (1995), and Heller and Lengwiller (1998) present theoretical studies in favor of uniform auctions. Back and Zender (1993), Binmore and Swierzbinski (2000), and Wang and Zender (2002) provide some of the theoretical studies in favor of discriminatory auctions. Empirical studies also produce mixed results. For example, using U.S. data Simon (1994a) finds that borrowing costs are lower in discriminatory auctions. Adopting a structural approach to estimate nonparametrically a symmetric private-values model for Turkish Treasury auctions data, Hortacsu (2003) finds that discriminatory auctions yield higher revenues than uniform auctions. In contrast, Umlauf (1993) finds using Mexican data that uniform auctions seem to raise more revenue than discriminatory auctions. Such mixed theoretical and empirical results concerning the choice of auction method in selling Treasury bills motivate further investigation of the question of discriminatory versus uniform price auctions.

The paper is organized as follows: Section 2 describes the methodology. Section 3 gives a brief description of the auction data as well as other economic data. Section 4 presents estimation results of the logistic function parameters for each auction. Section 5 estimates the economic determinants of the logistic function parameters. Section 6 produces results for both types of auction under the same economic conditions. It also compares the Treasury's revenue for both auction methods under the same economic conditions. Section 7 concludes the paper.

[5] In most countries, Treasury departments use one auction method in issuing the short- or long-term bills for a period of time, and then might switch to the other method. Thus, in addition to the approach taken here, this paper's data set and hence its comparison produce better insights regarding the choice of auction method.

# 2 Methodology

## 2.1 Fitting the market demand with the logistic function

Appendix A presents few examples of aggregate quantity demand curves plotted against the ratio of the price bids to the first day of the secondary market price of the bill. Note that all the graphs are sigmoidal in shape. A sigmoidal curve can be approximated by the following three-parameter logistic function

$$y = \frac{\alpha}{1 + \exp\left(-\lambda(p - \tau)\right)}, \tag{1}$$

where $y$ is the aggregate quantity demand as a function of price bid $p$. $\alpha > 0, \tau > 0$, and $\lambda > 0$ are the parameters of the function. However, instead of using price bids, equation (1) can be written in terms of the normalized price bids $t = p/p^s$, where $p^s$ is the first-day secondary market price of the bill. Hence, I use the following form of the logistic function:

$$y = \frac{\alpha}{1 + \exp\left(-\lambda(t - \tau)\right)}. \tag{2}$$

I give the observed cumulative aggregate demand curve and the curve estimated by equation (2) in Appendix A. I provide three randomly chosen graphs for discriminatory auctions (DAs) and uniform auctions (UAs) as a sample. The number of bids greater than the first-day secondary market price is higher in UAs. The reason is that bidders pay the minimum accepted bid instead of their individual bids, and hence the marginal cost of submitting a high bid is lower in UAs than in DAs. This behavior shows up as in the graphs on UAs a shift in the curve to the right.

The interpretation of the logistic growth function of the Treasury auctions setup is the following:

— At level $\alpha$, the market demand is satisfied to the fullest: there is no market demand beyond $\alpha$. On the graph, it is the height of the S-shape curve. Mathematically speaking, $y \to \alpha$ as $t \to \infty$. Higher demand in the auction means a higher $\alpha$.

— $\tau$ doesn't change the shape of the curve but determines the inflection point and the position of the curve. It shows how much price bids deviate from the first-day secondary market price of the bill. When $t = \tau$, $y = \alpha/2$, i.e. the inflection point of the curve is at the halfway point of the market satisfaction level.

— $1/\lambda$ is the measure of the dispersion of the normalized price bids, hence of the price bids themselves. Higher uncertainty leads to more dispersion, meaning a lower $\lambda$.

The parameter vector of the auction $i$ is denoted as $\Theta_i = (\alpha_i, \tau_i, \lambda_i)'$. Let $t_{ij}$ be the $j^{th}$ observation of the normalized price bid in auction $i$. I then estimate the parameters of the logistic function through the following specification:

$$y_{ij} = f(t_{ij}, \Theta_i) + \varepsilon_{ij}, \tag{3}$$

where $f(t_{ij}, \Theta_i)$ is defined by equation (2), and $\varepsilon_{ij}$ is a random error.

Equation (3) is estimated by using nonlinear least squares. In order to show that the logistic function is a good representation not only visually but also statistically, I regenerate the stop-out and the quantity-weighted average prices for comparison with the observed values. The stop-out rate, $t_{si}$, is the price at which $f(t_{ij}, \Theta_i) = y_i$, where $y_i$ represents the supply in the auction $i$, i.e.

$$t_{si} = \tau_i - \frac{1}{\lambda_i} \log\left(\frac{\alpha_i - y_i}{y_i}\right). \tag{4}$$

The quantity-weighted average rate $t_{wi}$ is calculated by

$$t_{wi} = \frac{1}{y_i} \int_{t_{si}}^{t_{mi}} t_i \frac{\partial y_i}{\partial t_i}(t_i) dt_i, \tag{5}$$

where $t_{mi}$ is the maximum $t_{ij}$.

I compare the estimated stop-out and quantity-weighted price bids with the observed values to show the goodness-of-fit. The next section describes the estimation procedure of the parameters from the economic variables.

## 2.2 Estimating the economic determinants of the estimated logistic function parameters

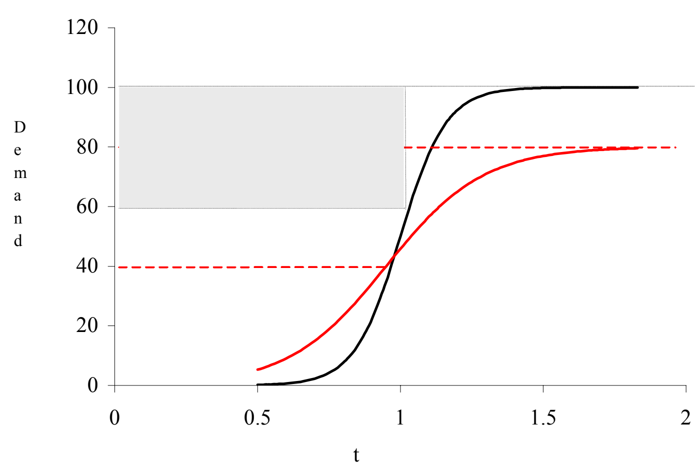
In this section, I describe the methodology of estimating the parameter vector, $\Theta_i = (\alpha_i, \tau_i, \lambda_i)'$, using various economic variables. BL (1999) and BBL (1999) treat the fluctuations in the market demand function from one auction to another as random events. However, PW (2004) argue that those fluctuations, rather than being random events, depend on the economic situation at the time of the auction. In this paper, fluctuations are also interpreted as functions of economic variables. In other words, the fluctuations are determined by the economic conditions. Using variables representative of the economic environment at the time of the auction, I estimate the parameter vector for the logistic function.

In this regard, I estimate parameters using Zellner's Seemingly Unrelated Regression (SUR) model, which controls for the economic variables. Causality relationships between variables justify the use of SUR. Significance levels are computed using the bootstrap method, which also controls for the small sample bias. This analysis is done separately for DAs and UAs. Finally, I derive an estimation equation for each parameter of the logistic function under DAs as well as under UAs. The next section depicts how the simulations are done to calculate the revenues under both auction methods by using the parameter estimation equations given in this section.

## 2.3 Simulation method

Using parameter estimation equations for both auction types, this section describes the method used to compare the two types in terms of their revenue extraction ability. I consider the equations for each parameter for one type of auction and use the data for the other type of auction to simulate the stop-out price. In other words, I take the equations for the parameters that are estimated for the UAs and calculate the stop-out price using the DAs' economic conditions. This simulates the UAs under the same economic situation that existed for the DAs. I also take the equations for DAs and simulate the stop-out price using the UAs' data. Appendix B gives an outline of the estimation procedure step-by-step.

**Graph 2.** This figure shows the graphical representation of revenue calculation under discriminatory and uniform auctions using the market demand curves.




Graph 2 shows how the revenues are calculated under DAs and UAs. There are two hypothetical market demand curves, the higher is for uniform and the lower is for discriminatory auctions. In this example, supply is assumed to be the same for both auction types, to be consistent with the approach employed throughout this paper. The revenue for the uniform auction is the rectangular gray area, which is the supply amount – 40 in this example – times the minimum accepted price bid. The revenue for the discriminatory auction is the area between the y-axis and the market demand curve. As seen from the graph, one cannot trivially do the comparison of the revenues, since the mentioned areas depend on the curvature and positions of the market demand curves.

As mentioned, the assumption here is that the supply for two auctions is the same, which is 40 in this graphical example. One may argue that supply might be different under different auction rules. However, the supply amount is announced prior to the auction and the Turkish Treasury has very little room to change it since the country is suffering from high debt. Hence, in the case of the Turkish Treasury, this assumption might be a good generalization.

# 3 Data

## 3.1 Description of the auction data

I use the data set of Ozcan (2004) to calculate various variables that are crucial to this study. The data set of Ozcan (2004) consists of auction codes, the date of the auction, the issue, the maturity date of the T-bills, and price-quantity bid pairs[6] as well as the accepted bid pairs. The data set also contains the secondary market price of the T-bills, and Turkish Lira (TL) and USD exchange rates. The stock market index is obtained from the Istanbul Stock Exchange (ISE).

The data set spans the period of January 2000 to February 2002. There are 82 auctions during this period. Of these 82 auctions, half are DAs, the

[6] There is also a code for each bidder which is carried out through the auctions. However, we don't need this information to carry out the approach in this paper.

other half are UAs. Bidders submit price-quantity pairs for pure discount Treasury bills with a face value of TL100,000. Bills do not bear any coupons. As stated earlier, the Treasury changed the auction format in February 2001 from discriminatory to uniform pricing. The number of price bids for each auction ranges from 42 to 473 for the DAs, and 92 to 477 for the UAs.

Table 1 shows the maturity structure of the auctions dividing the sample into discriminatory and uniform auctions. As seen from the table, bills don't have a regular maturity structure. Unlike some other countries, in Turkey there is no regular pattern to issuing the bills. Since Turkey had been going through a series of economic and financial crises and suffering high domestic debt with short maturity, it seems that the Treasury conducts auctions just before high debt payback. Table 1 also shows the total demand and supply and the averages for each of the maturities. The cover ratio, the satisfied portion of the demand, is given in percentages. Both demand and supply are lower for the uniform auction period. The cover ratio is higher in the uniform auction period. There are also more bills with short maturities in the uniform auctions.

I calculate the cumulative aggregate quantity bids from the individual price-quantity bids for each price bid. Unlike PW (2004) who use interest rate bids, I use price bids in the analysis. As do BL (1999), I normalize the price bids by the first-day secondary market price of that bill. I calculate auction-specific variables. The variables include the maturity of the bill auctioned, total supply, the first-day secondary market price of the auctioned bill, whether the auction is undertaken at the end of the month, whether there is another auction at the same day, and whether the bill is auctioned for the first time. In addition, the trading volume of the stock market and the variance of the overnight interest rates prior to the auction are included in the variables in order to fully simulate the economic conditions at the

**Table 1**. Summary statistics of the data. The top panel reports results for discriminatory auctions. The bottom panel reports results for uniform auctions. Maturity is in months, and number of auctions shows the corresponding number of auctions. Aggregate demand and aggregate supply are in columns three and five respectively. The averages are given in columns four and six. All the monetary values are given in million $US by using the exchange rate at the time of the auction. The cover ratio is calculated as the percentage of the satisfied demand in the last column.

| **Discriminatory Auctions** | | | | | | |
|---|---|---|---|---|---|---|
| Maturity | Number of Auctions | Aggregate Demand | Average Demand | Aggregate Supply | Average Supply | Cover Ratio (%) |
| 1 | 1 | 257.68 | 257.68 | 232.73 | 232.73 | 90.32 |
| 3 | 10 | 1691.88 | 169.19 | 944.52 | 94.45 | 55.83 |
| 6 | 1 | 352.46 | 352.46 | 222.25 | 222.25 | 63.06 |
| 12 | 3 | 793.49 | 264.50 | 339.03 | 113.01 | 42.73 |
| 13 | 3 | 1214.38 | 404.79 | 559.33 | 186.44 | 46.06 |
| 14 | 6 | 1480.70 | 246.78 | 909.99 | 151.66 | 61.46 |
| 16 | 5 | 2270.71 | 454.14 | 997.05 | 199.41 | 43.91 |
| 18 | 3 | 665.45 | 221.82 | 270.59 | 90.20 | 40.66 |
| 24 | 9 | 1604.18 | 178.24 | 583.14 | 64.79 | 36.35 |
| TOTAL | 41 | 10330.92 | 251.97 | 5058.63 | 123.38 | 48.97 |
| **Uniform Auctions** | | | | | | |
| Maturity | Number of Auctions | Aggregate Demand | Average Demand | Aggregate Supply | Average Supply | Cover Ratio (%) |
| 3 | 11 | 1717.99 | 156.18 | 892.62 | 81.15 | 51.96 |
| 4 | 4 | 527.30 | 131.82 | 424.20 | 106.05 | 80.45 |
| 5 | 5 | 571.71 | 114.34 | 374.66 | 74.93 | 65.53 |
| 6 | 6 | 726.10 | 121.02 | 505.78 | 84.30 | 69.66 |
| 7 | 3 | 331.32 | 110.44 | 283.94 | 94.65 | 85.70 |
| 8 | 4 | 465.79 | 116.45 | 319.09 | 79.77 | 68.51 |
| 10 | 4 | 385.83 | 96.46 | 260.90 | 65.22 | 67.62 |
| 12 | 1 | 190.02 | 190.02 | 91.49 | 91.49 | 48.15 |
| 13 | 3 | 213.75 | 71.25 | 168.71 | 56.24 | 78.93 |
| 24 | 1 | 179.76 | 179.76 | 89.41 | 89.41 | 49.74 |
| TOTAL | 42 | 5309.56 | 126.42 | 3410.81 | 81.21 | 64.24 |

time of the auction. I calculate the minimum accepted price bid (the stop-out price) and the quantity-weighted average of the accepted price bids in order to compare these with the estimation results.

There are two arguments by PW (2004) against using the individual bid functions. First, there are few observations with which to estimate individual bid functions. Second, the Treasury uses the market bid function, not the individual bid functions. Hortacsu and Ozcan (2004) also use the data set prepared by Ozcan (2004) to show that individual bid functions can be approximated by linear functions. Hence the first argument is not valid for this paper's data set. However, given BL's (1999) argument – central banks in almost all countries record the data by aggregating rather than keeping the individual bids – and the fact that the given Treasury uses the market bid function, we may have sufficient motivation to study the aggregate bid functions.

There are on average 192 price bids in the DA part of the data, and 247 in the UA part of the data. The average number of winning price bids is 88 in DAs and 136 in UAs. The average number of bidders in DAs is 56, whereas in UAs it is 80. This finding shows that the Treasury achieved an increase in participation by switching to uniform auctions. This is consistent with the argument that UAs increase participation since in DAs it is costlier to bid, both because bid decision requires more specialization and because the winners' curse is high (if one wins, one pays more than the stop-out price). Indeed, the same observation is also valid for the average number of the winners: 37 in the DAs and 67 in the UAs.

### 3.2 Other economic data

Before I calculate the aggregate quantity demand and plot it against the ratio of the price bids to the first-day price of the bill on the secondary market, I drop upper and lower 0.5% of the observations in each auction to eliminate the outliers such as bidding mistakes and unrealistic bid pairs. This eliminates 100 observations out of 18,330.

The definition of variables is given in Table 2. Some of the variables are defined as by PW (2004) and BL (1999). *Supply* is announced before the auctions, and hence taken to be exogenous. For a larger *supply* announcement, we would expect a larger demand, $\alpha$, as the Treasury can anticipate the market demand and decide the supply accordingly. Since $\tau$ is the ratio of price bid to the first-day price of the bill on the secondary market, *supply* should not have any impact on it. However, we may expect higher dispersion for a higher *supply*. $Firstdayprice$ is published by the ISE Bonds and Bills market at the end of each trading day. This variable is received from the ISE.

**Table 2**. Definition of the variables used in the estimations.

| |
|---|
| ***Supply*** is the total amount of the issue in a particular auction in trillion Turkish Lira. |
| ***Firstdayprice*** is the published first-day price of the auctioned bill on the first trading day on the secondary market. |
| ***Inoneday*** takes the value *1* if there is another auction on the same day, and *0* otherwise. |
| ***Notreissue*** is set to *1* if the bill has NOT been issued before, and *0* if it is a reopening. |
| ***Endofmonth*** is *1* if the maturity date of the bill is in the last 6 days of a month, *0* otherwise. |
| ***Maturity*** shows the maturity of bills in weeks. |
| ***Volume*** is the trading volume of the ISE one day ahead of an auction. |
| ***Close*** is the ISE index one day ahead of an auction. |
| ***Repovariance*** is the variance of the overnight interest rates over a five-day window prior to an auction. |

When more than one auction is conducted on the same day, $\alpha$ decreases. This is because bidders have to allocate their budget between the auctions, which therefore decreases the amount they bid in each auction. Hence we would expect a negative relationship between $\alpha$ and *inoneday*.

Since the first-day secondary market price is more uncertain for a bill issued for the first time than for a bill issued before, *notreissue* has a negative impact on $\tau$. We may also argue that as the reissued bill is readily available on the secondary market, bidders may choose to buy it on the secondary market instead of participating in the auction. Hence $\alpha$ goes down for the reissues. Park and Reinganum (1986) show that bills with a maturity date in the last week of a month have lower yields. In addition, Simon (1994b) finds that bills maturing at month-ends tend to have lower yields than those maturing earlier in the month.

A negative relation between *maturity* and the parameter $\tau$ is expected. This is because the variance of the bidders' belief of the price of a security with longer maturity is higher than it is for one with shorter maturity. Hence, the difference between the first-day price of the security and the price bids at an auction increases, and the price bids decrease as well, leading to a negative relation.

Stocks are considered as an alternative investment venue to bills. If the *volume* of the stock market is high, we would expect less demand for the bills. As the ISE index one day ahead of an auction, *close*, goes up, we may expect less demand in the auction. Bidders may choose to invest instead in the stock market, which will lower $\alpha$. The *repovariance* variable is used as an indicator of the financial and economic situation at the time of the auction.

# 4 Estimating the Logistic Function Parameters for Each Auction

I first estimate the parameters of equation (2) following the procedure described in section 2.1. The results indicate that the logistic function approach describes the aggregate market demand very well.

The graphs of aggregate market demand versus the price ratio and the estimated logistic function suggest that the logistic functional approach provide a good description of the data. However, we need to show that the correlation between the estimated curve and the data points is high in order to validate the visual judgment. To this end, I calculate the stop-out and quantity-weighted winning price bids from both the data and the estimated curve just as described in equations (4) and (5). The results are given in Table 3.

**Table 3**. Stop-out price is calculated from the estimated curve as the minimum accted price bid. Min accepted price is the observed minimum accepted price bid in the data. Fourth and fifth columns show the quantity-weighted prices calculated from the estimated curve and the data, respectively.

| | ***Stop-out Price*** | ***Min Accepted Price*** | ***Observed Q-Weighted Price*** | ***Estimated Q-Weighted Price*** |
|---|---|---|---|---|
| **Discriminatory Auctions** | | | | |
| mean | 82,256 | 82,322 | 82,662 | 82,718 |
| Stdev | 13,983 | 13,755 | 13,720 | 13,789 |
| **Uniform Auctions** | | | | |
| mean | 76,546 | 76,252 | 76,818 | 76,846 |
| Stdev | 9,842 | 9,892 | 9,697 | 9,718 |

As seen from Table 3, the estimated and observed minimum and quantity-weighted prices are close to each other. In fact, the correlation coefficients between the estimated and observed prices are 0.997 and 0.999 for the minimum and weighted prices under the UA rule, respectively. If we perform hypothesis testing for the difference of the means of minimum and weighted prices, we find that the hypothesis that the means are equal cannot be rejected. Therefore, I conclude that the logistic function approach describes the aggregate market demand very well.

The next section estimates the parameters of the logistic function from economic variables using Zellner's SUR regressions.

## 5 Estimating the Economic Determinants of the Logistic Function Parameters

I estimate the parameters of equation (3) from a set of economic variables in order to explain the parameters' fluctuations from one auction to another. The economic variables used in the Zellner's SUR model are the maturity in weeks, total supply in the auction, first-day price of the auctioned bill on the secondary market, the stock market index and volume, the five-day variance of the overnight interest rates prior to the auction date, and two dummy variables. One dummy variable takes the value of 1 if there is another auction on the same day, and 0 otherwise. The other takes the value of 1 if the bill is a reissue, and 0 otherwise. I also include a dummy variable showing that the maturity date of the security is in the last 6 days of the month.

I use bootstrap regressions to correct for the small sample bias. This is done for DAs and UAs separately. For the DAs, I use the same regression equation for all three parameters. However, we get a better fit by changing the regression equations for UAs. I add the five-day overnight interest rate variance prior to an auction, and replace the stock market volume with the stock market index in the regression equations of $\alpha$ and $\lambda$. Zellner's SUR regressions reveal that the parameters of the logistic function can be estimated using the economic variables, and that the economic variables can explain the fluctuations in the parameters from one auction to another. The results are shown in Table 4.

**Table 4.** The results of Zellner's SUR model for estimating the parameters of the logistic function from economic variables. The left-hand side gives the results for the discriminatory auctions; the right-hand side is for the uniform auctions. *, **, and *** show significance at the 10%, 5%, and 1% level, respectively.

| **Discriminatory Auctions** | | | **Uniform Auctions** | | |
|---|---|---|---|---|---|
| | Coefficient | Standard Error | Coefficient | Standard Error | |
| **Alpha** | | | | | **Alpha** |
| maturity (in weeks) | 0.6596 ** | 0.4018 | 0.7378 * | 0.6023 | maturity (in weeks) |
| inoneday | -30.8344 | 29.2073 | 53.5474 * | 26.9011 | inoneday |
| supply | 1.4356 *** | 0.2201 | 1.1460 *** | 0.1435 | supply |
| firstdayprice | -0.0012 | 0.0008 | 0.0013 * | 0.0010 | firstdayprice |
| notreissue | 26.4436 | 25.2112 | 23.5837 | 20.3843 | notreissue |
| voltrillion | 0.0245 | 0.0561 | -0.0152 *** | 0.0057 | close |
| endofmonth | -19.8911 | 54.2471 | 13.4314 | 22.6497 | endofmonth |
| constant | 128.2182 | 75.1781 | 79974.8300 | 23863.79 | repovariance |
| | | | 33.0355 | 90.5279 | constant |
| **Lambda** | | | | | **Lambda** |
| maturity(in weeks) | -3.3148 ** | 1.2984 | -6.8593 *** | 2.2961 | maturity(in weeks) |
| inoneday | 5.8864 | 94.3900 | -174.1590 * | 102.0565 | inoneday |
| supply | -0.6479 | 0.7115 | -0.6050 | 0.5445 | supply |
| firstdayprice | 0.0076 *** | 0.0027 | 0.0102 | 0.0038 | firstdayprice |
| notreissue | 75.5355 | 81.4759 | -200.4339 ** | 77.3294 | notreissue |
| voltrillion | -0.0556 | 0.1812 | 0.0566 ** | 0.0225 | close |
| endofmonth | -55.5670 | 175.3118 | -84.9092 | 85.9285 | endofmonth |
| constant | 19.2797 | 242.9552 | 151426.6000 | 91019.70 | repovariance |
| | | | -615.2171 | 346.8189 | constant |
| **Tau** | | | | | **Tau** |
| maturity(in weeks) | -0.0000099 | 0.0001154 | -0.0001063 | 0.0000966 | maturity(in weeks) |
| inoneday | -0.0056025 | 0.0083919 | -0.0009264 | 0.0046349 | inoneday |
| supply | 0.0000418 | 0.0000633 | -0.0000011 | 0.0000242 | supply |
| firstdayprice | 0.0000005 ** | 0.0000002 | 0.0000001 | 0.0000002 | firstdayprice |
| notreissue | -0.0047261 | 0.0072437 | -0.0031210 | 0.0034705 | notreissue |
| voltrillion | -0.0000127 | 0.0000161 | 0.0000154 ** | 0.0000078 | voltrillion |
| endofmonth | -0.0044249 | 0.0155864 | -0.0066317 | 0.0037770 | endofmonth |
| constant | 0.9648085 **** | 0.0216003 | -1.6064300 | 3.8033740 | repovariance |
| | | | 0.9943357 **** | 0.0141440 | constant |

As expected, the supply amount is the most important variable affecting market saturation level, $\alpha$, in both DAs and UAs. Similar to the results of PW (2004), BL (1999) and BBL (1999), it has a positive coefficient, i.e. as supply increases the market saturation level increases. Maturity also plays a role in determining the value of $\alpha$. It has a positive coefficient. In UAs, the stock market index affects $\alpha$ inversely. This can be explained as follows: as the ISE index goes up, investors lean more toward buying stocks than participating in auctions. This variable is significant only in the case of UAs, supporting the argument that UAs attract more investors and that generally these investors have smaller portfolios. As some investors move toward buying stocks, their budget is depleted at a low level of $\alpha$.

Under DAs, maturity affects the price bid dispersion negatively. This negative relation indicates that as maturity increases, the price bids become more dispersed. Either every bidder increases the difference between her maximum and minimum price bids, or, even if they don't change the difference between their maximum and minimum price bids, the variance across bidders increases, and hence the overall dispersion on the aggregate market demand curve increases. The first-day price of a bill on the secondary market, however, has a positive effect on the price bid dispersion (and a negative effect on $\lambda$ as dispersion is $1/\lambda$), i.e. as the first-day secondary market price of a bill increases, price dispersion in the auction decreases. One explanation of this might be that because of the bad performance of the Turkish economy during the data period, a higher first-day price on the secondary market might signal a better economic outlook. Hence, bidders can make better forecasts, and this leads to lower levels of price bid dispersion in the auction.

For UAs, maturity has the same effect as it does for DAs. There is a negative relation between $\lambda$ and the auctions done in the same day (*inoneday*) and the reissued auctions. It seems that having more than one auction on the same day increases the price bid spread. If a bill is issued for the first time ($notreissue = 1$), the uncertainty about its price is high compared to the price of a bill that has been issued before and traded in the secondary market. Hence the sign of the coefficient follows.

Under DAs, people care about the first-day price of the security, i.e it has a positive significant coefficient. As the first-day price rises, $\tau$ rises, and hence the logistic curve shifts to the right. One interpretation of this might be that in DAs bidders have to pay their bids, and hence they watch the secondary market price more closely in order to avoid the winner's curse. However, in UAs, instead of the first-day price, the volume of the stock market has a positive and significant coefficient.

Using estimation equations, I calculate the parameters. Then equation (2) gives the aggregate market demand function. I calculate the stop-out and weighted average prices using these equations. The correlation coefficient of the in-sample forecasts of the stop-out price and the observed minimum price is 0.997; the estimated weighted price and the observed weighted price is 0.999 for the DAs. The corresponding values for the UAs are 0.996 and 0.999, respectively.

### 5.1 Out-of-sample forecast

I forecast the stop-out and weighted prices of a few auctions using a part of the data set. In order to make the out-of-sample forecast, I first exclude 5 auctions from the DAs' data. Following the above lines,[7] I estimate the parameters. Then, using these parameter values, I calculate the stop-out and weighted prices. The corresponding correlation coefficients are 0.997

[7] I do all the bootstrap and Zellner's SUR regressions with a smaller number of auctions. I then estimate the parameters, $\alpha, \lambda$, and $\tau$.

for the observed minimum and the estimated stop-out price; and 0.999 for the observed and estimated weighted prices for the DAs.

The same procedure is also undertaken for the UAs. The correlation coefficients are 0.757 and 0.983 for the estimated and observed minimum prices and the estimated and observed weighted prices.

# 6 Simulations

In this section, I implement the results from Section 5. The first part of the analysis is as follows: using the parameter estimates and using the DAs' part of the data, I calculate the stop-out and the weighted average prices.

**Table 5**. In Panel A, there are minimum and q-weighted prices both observed in the DA and estimated for UA. In Panel B, we have the observed variables for the UA prices, and estimated DA prices.

| Panel A Discriminatory Part | | | Panel B Uniform Part | | |
|---|---|---|---|---|---|
| **Variable** | **Mean** | **Std. Dev.** | **Mean** | **Std. Dev.** | **Variable** |
| **Minimum DA price** | 80,846 | 15,195 | 75,835 | 10,226 | **Stop-out DA price** |
| **Weighted observed DA price** | 81,234 | 15,125 | 76,629 | 9,763 | **Weighted estimated DA price** |
| **Stop-out UA price** | 80,656 | 15,215 | 76,232 | 9,769 | **Minimum UA price** |
| **Weighted estimated UA price** | 81,175 | 15,092 | 76,784 | 9,578 | **Weighted observed UA price** |

Panel A of Table 5 summarizes the estimated and observed prices for the DAs' part of the data. In Panel A, the mean of the minimum and weighted average prices observed in the data for DAs are 80,846 and 81,234, which are shown in the first and second rows, respectively. The mean of the calculated minimum price and the weighted average prices, estimated using the uniform pricing rule, are 80,656 and 81,175, respectively. The details can be seen in the third and fourth rows of Panel A. The mean of the observed minimum prices for DAs, 80,846, is greater than the mean of the estimated stop-out rate, 80,656, if we had used UAs. The same relation is also true for the mean of the weighted average prices. This result contradicts the claim that in DAs bidders shade their bids more than they do in UAs. This empirical finding is important because, to the best of our knowledge, this is the first comparison of the two auction types under the same economic conditions. The only assumption is that the supply for both types of auctions is the observed amount, as mentioned in section 2.3.

The second part of the analysis involves reverse calculations. Reverse calculations may give us more insight into the comparison of both auction types. I consider the equations, which are used to acquire parameter estimates for DAs, and use them to estimate parameters with the economic variables during the UA period. I calculate the estimated stop-out and the weighted average prices under DAs instead of UAs. The results are in Table 5, Panel B. The mean of the estimated weighted average prices under DAs, which is 76,629, is less than the mean of the observed weighted average prices under UAs, 76,784. This result contradicts the first finding. The mean of the estimated stop-out prices under DAs is 75,835, which is less than the mean of the observed UAs' minimum prices, 76,232. Hence, this finding supports the claim that the minimum price is less under DAs than it is under UAs.

I also calculate the revenue of the Treasury under the two auction types by using the calculated minimum prices and the aggregate market demand curves. Table 6 shows the simulated revenues under different auction types.

**Table 6.** The estimated revenues of the different auction types under the same economic conditions.

| | Revenue under Disc. Auction | Revenue under Uniform Auction |
|---|---|---|
| Disc. Auc. Data | 93.680 | 82.580 |
| Unif. Auc. Data | 58.039 | 61.271 |

During the actual discriminatory auctions period, the revenue estimations reveal that DAs generate more revenue than UAs. The revenue under discriminatory auctions is 93 million \$US on the average, whereas it would have been 82 million \$US if the uniform auction type were employed during that period. However, during the actual uniform auctions period, the average revenue is 61 million \$US, which is a little greater than the average revenue, 58 million \$US, if discriminatory auctions had been employed during that period. This result may be due to the fact that in the estimations the supply is not changed to reflect a change in auction type. The Treasury might choose to change the supply, or the frequency of auctions depending on the type of the auction employed; these questions are left for future research. Although the discriminatory versus uniform auction puzzle remains, it can be concluded that discriminatory auctions raise more revenue for the Treasury than uniform auctions. This is because the discriminatory auctions data so strongly favor discriminatory auctions, whereas the uniform auctions data favor uniform auctions by a smaller margin.[8].

[8] One may argue that revenue is not the only concern. There may be other concerns such as maintaining the liquidity of the secondary market, or protecting small investors, etc. However, in Turkish auctions, we may assume the Threasury cares more about revenue, as the country suffers high domestic and foreign debt.

## 7 Conclusion

This paper employs an empirical method proposed by BL (1999) to analyze Treasury auctions. Instead of considering individual bid functions and bidders, I consider the aggregate market bid functions. Like BL (1999), BBL (1999), and PW (2004), I show that the logistic growth curve represents the market bid function very well. Unlike BL (1999) and BBL (1999), which treat the fluctuations from one auction to another as random, I consider, like PW (2004), the fluctuations to be a function of economic variables. Hence, I estimate the parameters the logistic function by using the economic and auction-specific variables.

First, I estimate the parameters of the logistic function. I then show that in-sample and out-of-sample forecasts fit the data very well. Therefore, I conclude the aggregate market bid function can be approximated fairly well using the logistic function.

The comparison between discriminatory and uniform auctions is an ongoing debate that requires much research. However, this paper diverges from the literature in that it compares the auction types while *economic conditions constant*. This paper is the first study in the literature that empirically compares the amount of revenue extracted for the Treasury by simulating the same economic environment under both auction types. Having both types of auction in the data set enables us to perform such a comparison. Elimination of any change in the economic conditions yields a much better comparison. The revenue comparison results are contradictory; however, as the revenues of both types are very close during the uniform auctions period, whereas the discriminatory auctions produce much higher revenue than would the uniform auctions during the discriminatory auctions period, I conclude that discriminatory pricing is better for the Treasury. However, this conclusion should be supported in future studies by taking into account the trade-off between frequency and volume of auctions, and the maturity of the security, and the possibility of changing supply in different auction types.

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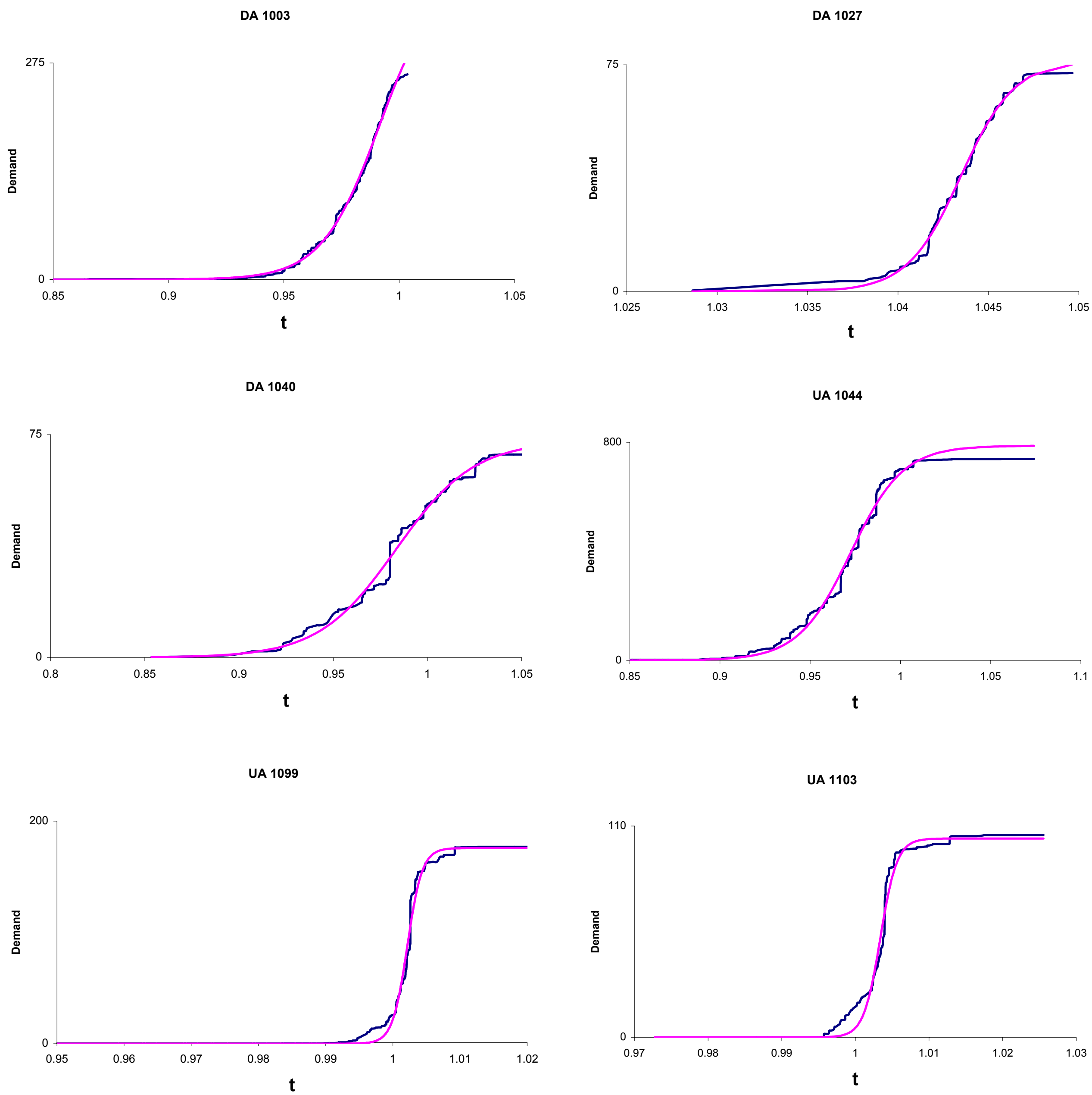
# Appendix A

**Graph 1**. I present six graphs; three for DAs and three for UAs. The title in each graph shows the type of the auction and the auction number in the data. Price bid to the first-day secondary market price ratio is on the x-axis, and the cumulative demand is on the y-axis. The smooth curve is the estimated curve; the other is the observed one in the data.



## Appendix B

### Outline of the Empirical Strategy

This appendix presents the outline of the estimation procedure step-by-step. Since this paper's data set has both discriminatory and uniform auctions, I divide the data into two subsets, and do the estimations of the logistic curves as well as the estimations of the parameters of the logistic curves and the revenues for each subset.

| ***Discriminatory Auctions Data Period*** | ***Uniform Auctions Data Period*** |
|---|---|
| **D1)** Plot market demand curves. | **U1)** Plot market demand curves. |
| **D2)** Estimate the logistic curve, calculate minimum accepted price and quantity-weighted average price bids. | **U2)** Estimate the logistic curve, calculate minimum accepted price and quantity-weighted average price bids. |
| **D3)** Estimate the parameters of the logistic curve from economic variables. | **U3)** Estimate the parameters of the logistic curve from economic variables. |
| **D4)** Simulate the market demand curves by the estimated parameters. | **U4)** Simulate the market demand curves by the estimated parameters. |
| **D5)** Simulate the revenue under discriminatory auctions. | **U5)** Simulate the revenue under uniform auctions. |
| **D6)** Take parameter estimates from (U4) and use the economic environment of discriminatory auctions data period to simulate the uniform auction's revenue. | **U6)** Take parameter estimates from (D4) and use the economic environment of uniform auctions data period simulate the discriminatory auction's revenue. |
| **D7)** Compare the discriminatory auctions revenue from (D5) with the uniform auction's revenue from (D6). | **U7)** Compare the uniform auctions revenue (U5) with the discriminatory auction's revenue from (U6). |