Multi-Round Procurement Auctions with Secret Reserve Prices: Theory and Evidence*

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Abstract

When a secret reserve price is used in an auction, the auctioneer cannot guarantee that the good can be sold out at the auction, and can re-auction the unsold objects in the next round. Motivated by this interesting feature observed in the procurement auctions organized by the Indiana Department of Transportation, we construct a bidding model in multi-round procurement auctions with secret reserve prices and evaluate how the release of the auctioneer's reserve price affects bidders' bidding behavior and the auctioneer's expected payment. Our theoretical model predicts that the equilibrium bids uniformly decline over stages, and the numerical analysis of our model indicates that hiding the reserve price may be better than announcing it for the auctioneer under some specifications of underlying distributions. We develop an empirical model to recover the unknown structutral parameters and to conduct counterfactual analyses.

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1 Introduction

This paper is motivated by an interesting feature we observed from the procurement auctions organized by the Indiana Department of Transportation (INDOT): many of these auctions are held with multiple rounds. This feature is attributed to the use of secret reserve prices in these auctions. Prior research has indicated that auctions with reserve prices sometimes lead to no transaction if no bidder can propose a price better than the reserve price.[1] However, there are still chances of trade if bidders' values for the unsold objects change. Thus the seller can continue auctioning the unsold objects from previous auctions. Previous research, however, has not paid much attention to this feature in auctions. In this paper, we propose a game-theoretic bidding model in multi-round procurement auctions with secret reserve prices and evaluate how the release of the auctioneer's reserve price affects bidders' bidding behavior and auctioneer's expected payment. Then we carry out a structural econometric analysis on the multi-round procurement auction data from the INDOT. Using the structural estimates, we evaluate how the release of the reserve price affects the government's expected payment through counterfactual analysis.

To model the multiple stages and secret reserve price, we focus our first-price sealed-bid auction model on a simple environment – the independent private value (IPV) paradigm. We also restrict our attention to bidders' strategic changes over stages, while assuming that the government's reserve price is exogenous and private over stages.[2] This assumption, although restrictive, is consistent with our data.[3]

While our model focuses on the procurement auctions that are low-bid auctions, as it is motivated by the procurement data from the INDOT, it can be readily extended to high-bid auctions. Our model yields some interesting predictions and implications. First, the bidding prices uniformly

[1] See, for example, Elyakime, Laffont, Loisel and Vuong (1997), Bajari and Hortasu (2003), and Li and Perrigne (2003).

[2] The theoretical study of the seller's optimal reserve price strategy is often in a relatively simple environment. See e.g., Riley and Samuelson (1981) and Laffont and Maskin (1980). The empirical study of auction data in complicated environment, on the other hand, often treats the seller's reserve exogenous and concentrates on analyzing bidding. See e.g., Bajari and Hortacsu (2003)'s study of eBay auctions with reserve prices and endogenous entry, and Jofre-Bonet and Pesendorfer (2003)'s study of repeated games of highway auctions.

[3] According to the officials at the INDOT, generally no change is made in the engineer estimate (as the reserve price) after a round of unsuccessful auction and in practice there were very few changes made. Hence the reserve price is the government's knowledge rather than its dynamic strategy.

decline over stages, because of the information about the secret reserve prices revealed in the previous stage. Second, under some conditions, hiding the secret reserve price is better for the government than announcing it. This result provides an explanation as to why secret reserve prices are commonly used in auctions from a new perspective.

Note that use of secret reserve prices in auctions has been studied in empirical work. See, e.g., Hendricks, Porter and Wilson (1994) for the Outer Continental Shelf auctions, Ashenfelter (1989) for wine and arts auctions, Elyakime, Laffont, Loisel and Vuong (1994, 1997) and Li and Perrigne (2003) for timber auctions, and Bajari and Hortacsu (2003) for eBay auctions. Theoretical work in studying secret reserve prices, however, has been limited, with exceptions such as Vincent (1995) using risk aversion to explain the use of secret reserve prices in a common value paradigm and Li and Tan (2000) in an independent private value paradigm. They show that in the presence of risk aversion, using a secret reserve price is better for the seller than using the optimal public reserve price for single-round auctions under some conditions. Alternative explanations have also been provided through the seller's objectives other than maximizing profits such as maximizing the expected sales as in Elyakime, Laffont, Loisel and Vuong (1994).[4]

On the other hand, studies of multi-round auctions have been quite limited. Elyakime, Laffont, Loisel and Vuong (1997) study a two-round auction game where the first round is conducted as a first-price sealed bid auction with a secret reserve price, and if the object is not sold, the second round is conducted through bargaining between the seller and the bidder with the highest bid from the first round. Horstmann and LaCasse (1997) propose a common value second-price bidding model in which the seller is assumed to know the true common value and has the option of holding the auction for a one-time resale. The seller announces a reserve price for screening inferior bids but does not guarantee a sale in the first round auction. Evidently, these two models do not fit with the multi-round procurement auctions organized by the INDOT as these auctions can be held for more than two rounds if the project is not sold in the previous round, and the government does not strategically choose to re-auction the project.

To analyze the procurement auction data, and in addition to provide an empirical framework

[4] Hiding reserve prices may also help the auctioneer deter collusion from bidders as explained in Ashenfelter (1989). In addition, as argued in Bajari and Hortacsu (2003), secret reserve prices may be used to encourage participation in auctions with entry.

within which the multi-round model with secret reserve prices can be analyzed, we develop a structural model from the theoretical model that we propose. Our structural approach takes into account of the unobserved auction heterogeneity, the existence of which in procurement auctions has been documented (e.g. Krasnokutskaya (2002) and Li and Zheng (2005)). Failing to control for the unobserved auction heterogeneity can cause severe bias in structural estimation, and hence result in misleading policy evaluations and recommendations.

We adopt the method of simulated moments (MSM) to estimate the underlying structural parameters. This approach provides a unified framework within which some interesting hypotheses can be tested, in addition to the computational advantage in obtaining consistent estimates. For example, we can test whether the private cost distribution varies across stages. We use our structural approach to analyze the INDOT data. Using the structural estimates, we carry out a counterfactual analysis by simulating the auctions with different government's reserve price release policies in the multi-round scenario. We find that the government could save more than \$13,000 (or about 2.5% of the project value) on average on a representative bridge work auction by hiding the engineer estimate rather than disclosing it. Hence the use of secret reserve price may be a good policy in practice in procurement auctions.

This paper is organized as follows. In Section 2, we present the data to motivate the model. In Section 3, we construct the model of multi-round procurement auctions with secret reserve prices, and solve the Bayesian Nash equilibrium. We also investigate the implications from our model. In Section 4, we compare the effects of different information release policies. In Section 5, we conduct a reduced-form econometric analysis of the data. In Section 6, we provide a structural econometric framework for analyzing multi-round auction data. In Section 7, we apply the structural approach to analyze the INDOT data. In Section 8, we use counterfactual analysis to evaluate the government's reserve price policy. Section 9 concludes. All technical proofs are included in the Appendix.

2 Data

This paper analyzes a data set of highway auctions held by the INDOT. The INDOT lets highway construction contracts through auctions. The auctions are held as first-price sealed-bid auctions where the INDOT reserve prices are unknown to bidders. Each contract specifies the construction work on highways within Indiana State undertaken by the winner of the auction. The winner of each auction performs projects described in the contract and is paid by the government. The prices bid by all participants are the amount that they ask for compensation.

An auction proceeds as follows. The INDOT posts the notice to contractors to invite bids five weeks prior to the bidding day. The notice includes simple information such as the type of projects in each contract, date of completion requested, and the length or area of the projects. Bid proposals and plans for the contracts that consist of more information on characteristics of the projects are also available upon request. Next with the advent of the bidding day, each bidder submits a sealed bid to an electronic bidding system under the government's secret reserve price. Finally on the bidding day, the received bids are unsealed and ranked by the government publicly. If the lowest bid in the auction is lower than the reserve price, the contract is then awarded to the bidder. Otherwise the contract will be readvertised and reauctioned in the following month. This feature makes the data unique.

The INDOT lets four types of construction work: road work, bridge work, traffic facilities and highway maintenance. We select one specific type of bridge work, which is called bridge rehabilitation, to analyze for two reasons. First, there exists large heterogeneity across different auctions. The characteristics of bridges are relatively more observable to econometricians among all work. Second, among all bridge work, bridge rehabilitation work reveals most characteristics to econometricians and occurs most frequently.

The sample analyzed in this paper is from INDOT monthly lettings from September 1996 to December 2004. For each auctioned contract, we have the following observations: the identity of each bidder, all bids, the reserve price, the number of bidders, the number of projects, the length of projects, the number of working days (or the completion date), the DBE goal and the structure of the bridge.[5] Before we exclude the lettings whose descriptive variables are missing, we have 37

[5] DBE (short for disadvantaged business enterprise program) is committed by the INDOT to implement to ensure nondiscrimination in the award and administration of USDOT-assisted contracts. DBE goal is expressed as a percentage. This percentage, when applied to the total federal highway construction funds received by the INDOT during the year, represents the amount of dollars that DBE firms working on INDOT contracts as prime contractors, subcontractors, or truckers should receive. Hence in a particular letting, the primary contractor if not a DBE firm, has the responsibility for contracting all ready, willing and able DBE firms who express a desire to work on any of the pay items of the contract; and must subcontract at least as much as the required percentage of the total value to

lettings that have two rounds of auctions. In a majority of 34 lettings, the contractors in the second round are a subset of the contractors in the first round. There are three lettings, however, all of which have one single bidder in the first round and one new single bidder in the second round. We exclude them from our sample. We also exclude from the sample the lettings whose descriptive variables are missing. As a result, our final sample consists of 273 lets and 1428 bids in total. Among the 273 lets, 243 were sold in the first round that involve 1261 bids. There are 30 lets unsold in the first round (near 12.5%) but sold in the second round with totally 167 bids in both rounds, and 102 bids in the first round, and 65 bids in the second round, respectively.

Table 1 and Table 2 give the description of the variables and the summary statistics of the data. On average, DBE percentage is 7.52 which means 7.52% of the total value of the contract is operated by DBE firms. DBE is regulated by the government hence it is not the choice of bidders. The average number of working days for completing the bridge work is around 138. The average length of the projects is 79.21 meters (about 260 feet). Intuitively, the longer a project takes and/or the longer the bridge is, the more work needs to be done and hence the higher cost it could result in. The average number of projects in each contract is 1.18, meaning that there can be multiple projects on vicinity sites. Multiple projects could potentially affect the capacity as well as the share ability of the facilities of the firms. 38% of the bridges have a steel structure, with the rest having structures of concrete, wood and others.

The summary statistics also reveal several important features of the data. On average, the number of bidders in the first round is 4.99 whereas the number of bidders in the second round is 2.23. Second, the average reserve price is \$855,615 and the average bid is \$839,506 for those with only one round. The former is greater than the latter meaning that the secret reserve price is effectively binding. On the other hand, they are very close. Third, if we concentrate on the auctions with two rounds, we find that on average, the bid is \$ 638,917 in the first round and \$588,992 in the second round, with a difference of \$49,925. This indicates that bids on average are lower in the second round than in the first round.

one or more DBE firms.

3 The Model for Multi-Round Auctions with Secret Reserve Prices

In this section, we propose a game-theoretic model for multi-round procurement auctions with secret reserve prices, and derive the corresponding Bayesian-Nash equilibrium.

3.1 Setup of the Game

The government lets a single and indivisible contract to firm contractors. There are N potential contractors who are interested in bidding for the contract. Each potential bidder is risk-neutral with a disutility equal to his private cost c. The government has an engineer estimate that is kept secret and serves as a reserve price in that the lowest bid has to be below it to become the winning bid. Because of the secret reserve price, it is possible for a project not to be awarded in an auction. If this is the case, the project will be re-auctioned later. Thus the game has multiple stages.

The government's secret reserve price r_0 is drawn from a distribution $G(\cdot)$ with support $[\underline{c}, \overline{c}]$ where $\underline{c} \geq 0$. $G(\cdot)$ is twice continuously differentiable and has a density $g(\cdot)$ that is strictly positive on the support. Potential bidders draw their private costs independently at stage j from a common distribution denoted $F_j(\cdot)$ with support $[\underline{c}, \overline{c}]$ and the corresponding density $f_j(\cdot)$ that is strictly positive on the support.[6] Thus we focus on the independent private value paradigm. When forming his bid, each bidder knows his private cost c, but does not know r_0 as well as others' private costs. On the other hand, each bidder knows that r_0 is drawn from $G(\cdot)$ and all private costs are independently drawn from $F_j(\cdot)$. $G(\cdot)$ and $F_j(\cdot)$ are common knowledge to all bidders. As a result, all bidders are identical *a priori* and the game is symmetric.

More specifically, the game can be characterized in the following order. In the first stage, the government has an engineer estimate that serves as the reserve price. The reserve price is kept fixed and secret until the contract is sold. It is exogenous in that it is not related to the government's optimal and strategic decision. As a result, we can focus on the strategic changes of the bidders' strategies across stages. Without knowing the reserve price, all N potential bidders participate in the game in the first stage and submit their bids. At the end of the first stage, all bids are opened, ranked and released. The reserve price, however, is not made public until after the contract is sold out. If the lowest bid, which requests the least compensation of cost from the government, is

[6] While we assume that $G(\cdot)$ and $F_j(\cdot)$ have a common support for simplicity, our approach can be readily generalized to the general case where $G(\cdot)$ and $F_j(\cdot)$ have different supports.

lower than the reserve price, the contract is awarded to the associated bidder and the game ends. Otherwise, the game continues to the next stage.

In the second stage, there are two main changes. First, each contractor re-draws his private cost from a common distribution $F_2(\cdot)$, which in general can be different from $F_1(\cdot)$; his re-drawn cost is independent of his cost from the preceding stage. This is a key assumption in our model, and will be labeled as the "random cost replacement" assumption hereafter. This assumption implies each bidder's cost in one auction round is independent of his in another. Being endowed with the lowest cost in the first round does not mean being endowed with the lowest in other rounds. This assumption can be used to justify our observation that in most of the auctions in our data, the actual bidders of the second round are a subset of the bidders in the first round. Moreover, the assumption that each bidder re-draws his private cost in a different round is reasonable. Each firm can participate in several different auctions in one month. They may lose in some auctions while winning in others. In a later round, the firm's private cost for the same project can change from the previous round because the firm may face different capacity constraints and may have different opportunity costs.[7]

Another important feature of our model is that there is a Bayesian updating on the reserve price from the bidders. Specifically, when an unsold project is re-auctioned, though the engineer's estimate is still kept secret, bidders have more information about this secret reserve price in this round than in the preceding one as they know the lowest bid from the preceding round. Therefore, they take this lowest bid into their strategy calculation as additional information as they know the secret reserve price has to be below this lowest bid. If a potential bidder's private cost he re-draws in this new round is above the lowest bid from the preceding round, he will not submit his bid. Thus this lowest bid plays a similar role to that of a public reserve price in screening bids. Thus, though we assume that there is no entry problem in the first round in that all potential bidders submit their bids, in the subsequent rounds, a potential bidder will not submit his bid if his private cost is higher than the lowest bid he observes from the preceding round. As a result, the actual bidders in the subsequent rounds must be a subset of the potential bidders of the first round.

[7] Statistical tests we perform in the data offer support for the random replacement assumption. The correlation of the ranks of the bidders in the first round and in the second round is about 0.30. Further test of the correlation of same bidders' bids across auction rounds shows that the correlation is 0.17.

3.2 The Bayesian-Nash Equilibrium Bidding Strategy

Denote a bidder's cost at the j-th stage c_j, and the associated bidding strategy b_j. We focus on the symmetric increasing Bayesian-Nash bidding equilibrium. Define the equilibrium bidding function as $b_j = \beta(c_j)$ such that $\beta'(\cdot) > 0$. We also use s_j^* to denote the lowest bid in the j-th stage.

In this paper, we assume that bidders solve their bidding strategies stage by stage without considering possible future rounds at the current round.[8] Under this assumption and the random cost replacement assumption, we can derive the bidder's Bayesian-Nash equilibrium across stages as follows.

Proposition 1 *The Bayesian-Nash equilibrium strategies are*

$$\beta_1(c) = c + \frac{\int_c^{\overline{c}}[1 - F_1(x)]^{N-1}[1 - G(\beta_1(x))]dx}{[1 - F_1(c)]^{N-1}[1 - G(\beta_1(c))]}, \tag{1}$$

for the first round and

$$\beta_j(c) = c + \frac{\int_c^{s_{j-1}^*}[1 - F_j(x)]^{N-1}[G(s_{j-1}^*) - G(\beta_j(x))]dx}{[1 - F_j(c)]^{N-1}[G(s_{j-1}^*) - G(\beta_j(c))]}. \tag{2}$$

for the j-th reauction round respectively.

3.3 Comparing Bidding Strategies across Stages

Expressions (1) and (2) indicate that the bidding strategies differ from stage to stage in our model, and the interval over which the bidding strategy is defined also changes over stages. For instance while the first round equilibrium is defined on $[\underline{c}, \overline{c}]$, the second round equilibrium is defined on the interval $[\underline{c}, s_1^*]$, which is truncated from above compared to the first round. While the secret reserve price is not revealed, the rejected lowest bid from the previous round gives bidders information that the secret reserve price is below this bid; bidders will not submit their bids above this lowest bid. Intuitively, this would make bidders bid more aggressively and reduce their bids over stages. This is indeed the case, as shown in the next proposition.

Proposition 2 *In the multi-round auction model, the equilibrium bid in stage j is less than or equal to the equilibrium bid in the previous stage everywhere on* $[\underline{c}, s_{j-1}^*]$*; i.e.,* $\beta_j(c) \leq \beta_{j-1}(c)$.

[8] This assumption rules out forward-looking bidders. On the other hand, it has a generality in that it allows for different private cost distributions across different rounds, while one has to assume the same private cost distribution across stages in a dynamic game with forward-looking bidders.

A few remarks follow. First, this proposition shows that the equilibrium bidding strategies are indeed decreasing from stage to stage if the contract is not sold out. Moreover, the reduction is universal on the whole common interval. Second, this result is established allowing for the private cost distributions to vary across stages. Thus it is a strong prediction from the model as it is robust to the change of the bidders' cost distribution over stages. Third, this result is empirically testable and can be used for testing rationality of bidders in real auctions.

3.4 Numerical Examples

To explore more properties of the bidding functions across stages, we give some numerical examples. We specify different distributions and vary the number of potential bidders. As the analytical solutions are in general not attainable, we numerically solve for equilibrium bids. Without loss of generality, we illustrate $\beta_1(c)$ and $\beta_2(c)$. The bidding functions under different specifications are depicted in figure set 1 (figure 1-1 – 1-3).

The depicted curves reinforce two main findings from the theoretical model. First, the bidding functions are strictly increasing. Second, the bids in the second round are everywhere below the bids in the first round on the common support. We conduct a large number of numerical specifications and these findings are generally consistent.

The graphs also reveal some other interesting patterns. First, the bids are negatively related to the number of potential bidders in every round. This is simply because of the *competition effect*. Second, the disparity between $\beta_1(c)$ and $\beta_2(c)$ are affected by two factors. On one hand, it is affected by the number of potential bidders. The difference between them shrinks as the number of potential bidders increases. This is reasonable because as the number of potential bidders increases, the *competition effect* becomes more intense, which makes a bidder's mark-up in every auction round small and converging. On the other hand, it is affected by the lowest bid in the first round of unsold auction. The smaller this lowest bid is, the larger the difference between $\beta_1(c)$ and $\beta_2(c)$. We label this effect as *boundary effect*. In the first round, the boundary condition is at the upper bound of the cost distribution, while in the second round the boundary condition is at the previous lowest bid (upper bound of a truncated distribution). The lower the truncated bound is, the lower the maximum possible bid in the second round is. Hence the *boundary effect* tends to enlarge the difference in bidding across stages.

4 Government's Information Revelation

In this section we compare welfare impacts of different reserve price release policies from the government. Motivated by the INDOT data feature, our model has focused on the use of the secret reserve price by the government. Alternatively, the government can make the engineer's estimate public and use it as a public reserve price. In this scenario, the government can find no bids submitted if all bidders' private costs are above the public reserve price. If we maintain the random cost replacement assumption, then the government can re-auction the project in the next round with the same public reserve price. As a result, under the random cost replacement assumption, the multi-round feature can be accommodated by both secret and public reserve prices. It would be interesting to compare the welfare implications of these two mechanisms and gain insights on why secret reserve prices are used in auctions.

4.1 Multi-Round Auctions with Public Reserve Prices

We maintain all the assumptions made in Section 2.1 except that now the reserve price r_0 is public. In the j-th round, the Bayesian-Nash equilibrium strategy, as shown in Riley and Samuelson (1981), is given by

$$\beta_{pj}(c) = c + \frac{\int_c^{r_0} [1 - F_j(x)]^{N-1} dx}{[1 - F_j(c)]^{N-1}} \tag{3}$$

for $c < r_0$; for a potential bidder whose private cost is above r_0, he will not bid.

If all potential bidders' private costs are above r_0, no bids are submitted at the current round, and the project can be re-auctioned in the next round. As in the secret reserve price case, we assume that the set of potential bidders remains the same across stages. At each round, however, a bidder's bidding strategy as defined in (3) may change because of the new private cost he re-draws from $F_j(\cdot)$.

4.2 The Comparison of Mechanisms

We compare the government's ex ante expected payments under the two reserve price policies, assuming that bidders re-draw their private costs at each round, and the government will re-auction the unsold contract in the next round until it is sold out. Since it is infeasible to make such a comparison generally as the ex ante expected payments in these two cases do not have closed form expressions in general, we conduct some simulation studies by assuming that the private cost distribution remains the same across stages, and by considering some commonly used functional forms for the private cost distribution and the reserve price distribution such as the uniform and exponential distributions.

We specify different distributions and vary the number of potential bidders to carry out a group of simulations. We then compare the government's expected expenditures under the two mechanisms.

We first plot the simulated expected expenditure as a function of the reserve price. As can be seen from the graphs, under some specifications, the secret reserve price policy dominates the public reserve price in that the (ex post) expected expenditure under the secret reserve price is below that under the public reserve price. Some other specifications, however, yield the opposite findings. We further compute the ex ante expected expenditures by integrating out the reserve prices and report them in Table 3, which reveals that the expected expenditure under secret reserve prices is sometimes lower and sometimes higher than under public reserve prices.

The graphs also reveal some other interesting patterns. First, the (ex post) expected expenditure as a function of either a secret or public reserve price is almost increasing with the reserve price[9]. This is reasonable because the higher is the reserve price, the less restrictive is the auction to the bidders. The acceptable bids are high when the reserve prices are high. Second, the effect of the number of bidders is complicated. On one hand, the larger the number of bidders, the closer the two curves are because of the competition that tends to offset the different effects of different reserve prices on bidding. This competition effect is in analogy to that in the earlier numerical results on equilibrium bids between stages. On the other hand, the minimum bid in the previous round is lowered by the intensity of competition in the case of a secret reserve price. This is a factor that drags down the bids in the auctions with secret reserve prices, which does not exist in the public reserve price mechanism. The boundary effect favors the secret reserve price. It affects the position of the intersection and the difference of the two curves. Consequently, the boundary effect tends to enlarge the favorable range for the secret reserve price and increase the distance between the two curves in its favorable range. The net effect is determined by the combination. It seems that the

[9] The sampling variation resulting from simulations causes the small fluctuations on the curve; otherwise the curve could be more monotone.

competition effect often dominates from the graphs.

5 Reduced-Form Empirical Analyses

In this section, we provide a preliminary analysis of our data, trying to relate our theoretical model to the data by justifying some assumptions, and to test some predictions from the model.[10]

5.1 Exogeneity of Number of Potential bidders in the First Round and the Reserve Price

In our model, we assume that there is no entry in the first round. In other words, the set of potential bidders is identical to that of actual bidders. To justify this assumption, we take a look at the number of bidders in the first round and test the exogeneity of this variable. To this end, we use both the Poisson model and the negative binomial model as the number of bidders is a count variable. Using all data in the first round including both sold lettings and unsold lettings, we estimate both models. Since in our data set, we do not have any auction that has no bidder participation, the number of bidders in our data is truncated from zero. Thus we use the truncated Poisson and negative binomial models.

The ML estimation results of both models are reported in Table 4. The results show that no covariates used in the regression are statistically significant in explaining the number of bidders. Thus, the number of actual bidders can be treated exogenous and considered the same as the number of potential bidders.

Another important assumption in the theoretical model is that the government's reserve price is exogenous in that it is not related to the number of bidders and does not change across different rounds. To test the exogeneity of the reserve price in our data, we run a regression of the logarithm of the reserve price on a set of covariates including the number of bidders. From the results reported in Table 5, we can see that interestingly, both the number of bidders and the round-two dummy are not significant in the regression.[11] That both the number of bidders and round-two dummy have

[10] While our model is general enough to allow for possibility of infinite rounds, we can only focus on analyzing the first two rounds because of our data limitation.

[11] "round-two" is a dummy variable equal to 0 when an auction is in the first round and 1 when an auction is in the second round.

no effect on the reserve price provides support for the exogeneity of the reserve price.

5.2 Regression Analysis of Bids

There are two empirically testable implications about the equilibrium bids from our theoretical model. First, it can be easily verified that the Bayesian-Nash equilibrium strategies given by (1) and (2) are monotone decreasing with the number of potential bidders. Intuitively, the larger the number of bidders, the more competitive the auction. The competition drives the bidders to bid more aggressively. Second, the theoretical model predicts that the equilibrium bids are lower in the second stage.

To test these two implications, we run a pooled regression of the logarithm of bids on a set of covariates. To allow for structural change in bid over the two auction rounds, which is indicated by the theoretical model, we include the round-two dummy variable and its interactions with other variables. We report the regression results in Table 6. It turns out that the number of bidders is strongly significant and negatively related to bids. The round-two dummy and some interactive terms are strongly significant, meaning that there exists structural change in bid across auction stages. Furthermore, we calculate the marginal effect of the round-two dummy on the bids. The marginal effect is -\$53004 and strongly significant, meaning that on average the bidders tend to lower their bids in the second round by \$53,004 which is about 8.3% of the project value. This result is quite close to the outcome in the summary statistics. These findings offer support to our theoretical model.

The R^2 of the pooled regression is 0.51, indicating that on the one hand the model fits moderately well, on the other hand we may ignore some unobserved auction heterogeneity. To further ascertain the existence of unobserved auction heterogeneity, we conduct a random-effect panel data analysis using only the first round auction data, as the auction data have a panel feature. We report the regression results in Table 7. The results strongly indicate that there exists unobserved auction heterogeneity as the error variance from the unobserved heterogeneity accounts for 95% of the total error variance. Hence it calls for controlling the unobserved heterogeneity in the structural inference.

6 Structural Inference of Muti-Round Auction Models

6.1 The Parameterization of the Structural Model

Based on the theoretical auction model, there are three primitives, namely, the government's reserve price distribution $G(\cdot)$ and the private cost distributions $F_j(\cdot)$, $j = 1, 2$. $F_1(\cdot)$ can be in general different from $F_2(\cdot)$. Nonparametrically, $G(\cdot)$ can be identified from the observed reserve prices as they are assumed to be random draws from $G(\cdot)$. Moreover, following Guerre, Perrigne and Vuong (2000) and Li and Perrigne (2003), it can be verified that $F_1(\cdot)$ is identified over its entire support $[\underline{c}, \overline{c}]$ by the observed bids in the first round, and $F_2(\cdot)$ is identified over $[\underline{c}, s_1^*]$ by the observed bids in the second round. In this paper, however, we adopt the parametric approach because we only observe 30 auctions in the second round, which makes nonparametric estimation problematic.

In an econometric framework, asymptotic statistical inference is based on a large number of auctions. Let L_1 be the number of auctions that transact in the first round, L_2 be the number of auctions that transact in the second round. For the ℓ-th auction at the j-th round, let $G_\ell(\cdot)$, $F_{j\ell}(\cdot)$ and $F_{j\ell}(\cdot)$ denote each primitive distribution respectively with corresponding densities $g_\ell(\cdot)$ and $f_{j\ell}(\cdot)$, $j = 1, 2$. Assume that $G_\ell = G(\cdot|x_\ell, u_\ell, \gamma)$ and $F_{j\ell} = F(\cdot|x_\ell, u_\ell, \theta_j)$, where x_ℓ is a vector of variables that we use to control for the observed auction heterogeneity, and u_ℓ is a scalar variable that represents the unobserved auction heterogeneity, both affecting the government's reserve price as well as the bidders' costs, γ is a vector of unknown parameters in $\Gamma \subset \mathbb{R}^K$, and θ is a vector of unknown parameters in $\Theta \subset \mathbb{R}^K$. We assume that u is independent of x, and has a distribution $W(\cdot|\sigma)$ with $w(\cdot|\sigma)$ being the density function, where σ is a vector of unknown parameter in $\Sigma \subset \mathbb{R}^m$.

Conditional on both observed and unobserved heterogeneity x and u, we specify the reserve price distribution and the cost distribution as exponential as follows

$$g_\ell(r|x_\ell, u_\ell, \gamma) = \frac{1}{\exp(x_\ell\gamma + u_\ell)} \exp\left(\frac{-r}{\exp(x_\ell\gamma + u_\ell)}\right) \tag{4}$$

$$f_{j\ell}(c|x_\ell, u_\ell, \theta_j) = \frac{1}{\exp(x_\ell\theta_j + u_\ell)} \exp\left(\frac{-c}{\exp(x_\ell\theta_j + u_\ell)}\right) \quad j = 1, 2 \tag{5}$$

where $c \in (0, \infty)$ and $r \in (0, \infty)$. By including the intercept in x, we normalize the unobserved heterogeneity term u such that $E[u] = 0$. We assume that $u \sim N(0, \sigma^2)$, where σ^2 is an unknown parameter.

6.2 Structural Equilibrium Solutions

Next we need to solve the theoretical auction model for equilibrium solutions with the above specified distributions. The Bayesian Nash equilibrium bidding strategy in the first round is given as follows, which is a closed-form solution

$$\beta_1(c) = c + \frac{1}{\dfrac{N-1}{\exp(x_\ell\theta_1 + u_\ell)} + \dfrac{1}{\exp(x_\ell\gamma + u_\ell)}}. \tag{6}$$

The Bayesian Nash equilibrium bidding strategy in the second round, which is a solution to the equation given below, does not have a closed form.

$$\beta_2(c) = c + \frac{\int_c^{s_1^*}[\exp\left(\frac{-z}{\exp(x_\ell\theta_2+u_\ell)}\right)]^{N-1}[\exp\left(\frac{-\beta_2(z)}{\exp(x_\ell\gamma+u_\ell)}\right)-\exp\left(\frac{-s_1^*}{\exp(x_\ell\gamma+u_\ell)}\right)]dz}{[\exp\left(\frac{-c}{\exp(x_\ell\theta_2+u_\ell)}\right)]^{N-1}[\exp\left(\frac{-\beta_2(c)}{\exp(x_\ell\gamma+u_\ell)}\right)-\exp\left(\frac{-s_1^*}{\exp(x_\ell\gamma+u_\ell)}\right)]} \tag{7}$$

6.3 Estimation and Testing for Changes of Private Cost Distributions

In our auction data, at round j, $j = 1, 2$, we observe reserve prices, number of potential bidders and a set of auction heterogeneities (r_ℓ, N_ℓ, x_ℓ). We also observe bids in round 1 and round 2, respectively. Our estimation of the structural parameters is based on the likelihood function of r given in (4) and the moment conditions of $b_{ji\ell}$ $(j = 1, 2)$, where $b_{ji\ell}$ denoted the i-th bid in the ℓ-th auction at the j-th round. Specifically, from (6) we obtain the moment condition

$$\begin{aligned} E[b_{1i\ell}|N_\ell, g_\ell(\cdot), x_\ell, u_\ell] \quad &\equiv \quad m_1(b_{1i\ell}, x_\ell, u_\ell, \gamma, \sigma; \theta_1) = E[c|N_\ell, g_\ell(\cdot), x_\ell, u_\ell] \\ &\quad + \frac{1}{\dfrac{N_\ell - 1}{\exp(x_\ell\theta_1 + u_\ell)} + \dfrac{1}{\exp(x_\ell\gamma + u_\ell)}} \\ &= \exp(x_\ell\theta_1 + u_\ell) + \frac{1}{\frac{N_\ell-1}{\exp(x_\ell\theta_1+u_\ell)} + \frac{1}{\exp(x_\ell\gamma+u_\ell)}} \end{aligned} \tag{8}$$

for the equilibrium bids $b_{1i\ell}$ in the first auction round. Similarly, from (7) we can obtain

$$E[b_{2i\ell}|c \leq s_1^*, N_\ell, g_\ell(\cdot), x_\ell, u_\ell] \equiv m_2(b_{2i\ell}, x_\ell, u_\ell, \gamma, \sigma; \theta_2) \tag{9}$$

for the equilibrium bids $b_{2i\ell}$ in the second round, where $m_2(b_{2i\ell}, x_\ell, u_\ell, \gamma, \sigma; \theta_2)$ does not have a closed form because the second round bidding function does not have a closed form.

Note (4), (8) and (9) are all conditional on u_ℓ which is not observed. For estimation, however, we need to obtain conditions that depend only on observables. In order to derive such conditions, we integrate u_ℓ out of (4), (8) and (9) and get the followings.

$$g_\ell(r|x_\ell,\gamma,\sigma) = \int_{-\infty}^{\infty} \frac{1}{\exp(x_\ell\gamma + u_\ell)} \exp\left(\frac{-r}{\exp(x_\ell\gamma + u_\ell)}\right) \cdot w(u_\ell|\sigma)du_\ell \tag{10}$$

$$E[b_{1i\ell}|N_\ell, g_\ell(\cdot), x_\ell] \equiv M_1(b_{1i\ell}, x_\ell, \gamma, \sigma; \theta_1) = \int_{-\infty}^{\infty} m_1(b_{1i\ell}, x_\ell, u_\ell, \gamma, \sigma; \theta_1) \cdot w(u_\ell|\sigma)du_\ell \tag{11}$$

$$E[b_{2i\ell}|c \le s_1^*, N_\ell, g_\ell(\cdot), x_\ell] \equiv M_2(b_{2i\ell}, x_\ell, \gamma, \sigma; \theta_2) = \int_{-\infty}^{\infty} m_2(b_{2i\ell}, x_\ell, u_\ell, \gamma, \sigma; \theta_2) \cdot w(u_\ell|\sigma)du_\ell \tag{12}$$

The parameters of primary interests are γ, θ_1, θ_2 and σ. We estimate them using (10), (11) and (12).

6.3.1 A Two-Step Estimation Approach

We adopt a two-step estimation strategy. In the first step, we recover γ and σ using likelihood function (10) to get $\widehat{\gamma}$ and $\widehat{\sigma}$. In the second step, we estimate θ_1 and θ_2 using moment conditions (11) and (12) as well as the estimates $\widehat{\gamma}$ and $\widehat{\sigma}$.

Since we fully specify the distribution of the reserve price and we observe reserve prices, in the first step, γ and σ can be efficiently estimated by maximum likelihood (ML) approach. A complication arises from the feature that there is no closed form likelihood function because of the integration with respect to u_ℓ. Thus, we use a simulated maximum likelihood (SML) estimation approach (Gourieroux and Monfort (1996)). Specifically, the SML estimator is defined by

$$(\widehat{\gamma}, \widehat{\sigma})_{SML} = \arg\max_{\gamma,\sigma} \sum_{\ell=1}^{L} \log\left[\frac{1}{S}\sum_{s=1}^{S} \frac{g_\ell(r_\ell|x_\ell, u_\ell^s, \gamma)w(u_\ell^s|\sigma)}{\phi(u_\ell^s)}\right] \tag{13}$$

where $L = L_1 + L_2$. As indicated in (13), we use the importance sampling technique in the numerical integration. The importance density function is the standard normal $\phi(\cdot)$. We draw S of u_ℓ^ss from $\phi(\cdot)$ in simulation, where S is sufficiently large compared to the sample size L. As $S, L \to \infty$ and $\sqrt{L}/S \to 0$, the SML estimator is asymptotically equivalent to the ML estimator (Gourieroux and Monfort (1996)).

In the second step, we separately estimate θ_1 and θ_2, using moment conditions (11) and (12), respectively, and the estimates $\widehat{\gamma}$ and $\widehat{\sigma}$ obtained in the first step. Again because of the presence of the unobserved heterogeneity, we propose a method of simulated moments estimator (MSM). Let

$Y_{ji\ell}(\theta_j) = b_{ji\ell} - M_j(b_{ji\ell}, x_\ell, \widehat{\gamma}, \widehat{\sigma}; \theta_j)$, j $(= 1, 2)$. We need to simulate $M_j(b_{ji\ell}, x_\ell, \widehat{\gamma}, \widehat{\sigma}; \theta_j)$, hence we draw u_ℓ^s from $w(\cdot|\widehat{\sigma})$ and define $y_{ji\ell}^s(\theta_j) = b_{ji\ell} - m_j(b_{ji\ell}, x_\ell, u_\ell^s, \widehat{\gamma}, \widehat{\sigma}; \theta_j)$. For each j $(= 1, 2)$, a MSM estimator can be defined by

$$\widehat{\theta}_{jMSM} = \arg\min_{\theta_j} \left(\sum_{\ell=1}^{L_j} \frac{1}{n_\ell} \sum_{i=1}^{n_\ell} \left(x_{j\ell} \frac{1}{S} \sum_{s=1}^{S} y_{ji\ell}^s(\theta_j) \right) \right)' \Omega \left(\sum_{\ell=1}^{L_j} \frac{1}{n_\ell} \sum_{i=1}^{n_\ell} \left(x_{j\ell} \frac{1}{S} \sum_{s=1}^{S} y_{ji\ell}^s(\theta_j) \right) \right) \quad (14)$$

where Ω is a $K \times K$ symmetric positive-definite weighting matrix.

Additional difficulty in computation arises from the fact that the simulated $M_j(b_{ji\ell}, x_\ell, \widehat{\gamma}, \widehat{\sigma}; \theta_j)$ involves the Bayesian-Nash equilibrium strategy, which is especially cumbersome for $j = 2$, because it does not have a closed form solution. We follow Elyakime, Laffont, Loisel and Vuong (1994) to numerically recover the bidding function by a recursive procedure. Starting from the boundary condition, the equilibrium bidding strategy can be numerically solved in a recursive manner. Note that the resulting MSM estimator is consistent given that the first-step estimators $\widehat{\gamma}$ and $\widehat{\sigma}$ are consistent.

Noting the complexity involved in our two-step estimation procedure, we use bootstrap to obtain variance-covariance matrices of the estimates. Because of the panel feature of the auction data, we adopt a block bootstrap (e.g. Andrews (2002)) to obtain the standard errors for our two-stage MSM estimator.

6.3.2 Testing for Cross-Stage Change of Private Cost Distributions

In the previous section, we develop a framework of estimating the structural model of multi-round auctions separately round by round, allowing for the underlying cost distributions to change across two rounds. It would be interesting to test whether the underlying cost distributions change or not across stages. If it turns out that the distributions do not change, it means that bidders re-draw their costs from the same distribution across stages. Moreover, in this case, we can more efficiently estimate the private cost distribution parameters by jointly estimating both auction rounds.

We propose a formal test following Andrews and Fair (1988), who extend the Chow test of structural changes in classical linear models (Chow (1960)) to test structural changes in nonlinear models. The null hypothesis here is $H_0 : \theta_1 = \theta_2$, which is the case of testing for pure structural change (see Andrews and Fair (1988)). The Wald test statistic is applicable to our MSM estimator, which can be implemented as follows. First, the MSM in (14) is implemented as discussed previously.

Let $\pi_1 = L_1/L$ and $\pi_2 = L_2/L$. Then, the Wald test statistic is given by

$$W = L(\widehat{\theta}_1 - \widehat{\theta}_2)'(\widehat{V}_1/\pi_1 + \widehat{V}_2/\pi_2 - 2\widehat{V}_{12}/\sqrt{\pi_1\pi_2})^{-}(\widehat{\theta}_1 - \widehat{\theta}_2)$$

where $\widehat{V}_1$ and $\widehat{V}_2$ are the estimated asymptotic variances matrices of $\widehat{\theta}_1$ and $\widehat{\theta}_2$ respectively, $\widehat{V}_{12}$ is the matrix of the estimated asymptotic covariances between $\widehat{\theta}_1$ and $\widehat{\theta}_2$. The general inverse $(\cdot)^{-}$ of covariance term in the middle equals the regular inverse $(\cdot)^{-1}$ with probability going to one as $L \longrightarrow \infty$. W follows a χ^2 distribution with the dimension of the structural parameter vector θ as its degrees of freedom.

7 Results

In this section, we apply our structural econometric approach to analyze the data from the INDOT, so as to uncover the underlying private cost and reserve price distributions. By concentrating on a specific type of bridge work, we choose a set of observed covariates $x = \{$dbe, time, np, steel, length, intercept$\}$. First we use the two-stage estimation to estimate the model under unobserved auction heterogeneity. Then we test the cross-stage change of private cost distributions. Lastly, we conduct a robustness check.

7.1 SML and MSM Estimates for the Structural Parameters and the Unobserved Heterogeneity

The parameters of the reserve price distribution γ and the parameter of the unobserved heterogeneity σ can be jointly estimated based on (13). We draw a large sample, namely $S = 1000$, of u_ℓ^ss from $N(0,1)$, *i.e.*, $\phi(u_\ell)$, and adopt importance sampling to implement the SML. Furthermore, we gain the standard errors through bootstrap. The results are reported in Table 8.

Next we estimate parameters in private cost distributions θ_j (for $j = 1, 2$) based on (14). To gain the simulated moments, we recursively solve for equilibrium bids and calculate the numerical integration. We simulate u_l from $N(0, \widehat{\sigma}^2)$. Here the number of u_ℓ^ss that we draw is $S = 100$, a number relatively smaller than the one we use in implementing SML, as an MSM estimator is consistent for any fixed number of simulations (Gourieroux and Monfort (1996)). Furthermore, we use the identity matrix as the weighting matrix. Using bootstrap, we obtain the standard errors of the estimates. Moreover, as we need to incorporate the variation from the estimation of $\widehat{\gamma}$ and

$\widehat{\sigma}$, we jointly resample the auction data including both reserve prices and bids and repeat the SML estimation and MSM estimation simultaneously. The results of the estimation are reported in the first four columns of Table 9.

The results indicate that all variables that we pick up have significant effects on private costs. Evaluated at the sample mean of the observed and unobserved auction characteristics, the mean private cost is about \$641,000. Increases in the length of the bridges and the time needed to accomplish the projects raise private costs, and in turn increase bids, as expected. Specifically, holding all the other factors constant, increasing the length of the project by one meter (or 3.28 feet) will increase the mean private cost by 0.21% or about \$1,350. One more working day needed for a project will increase the mean private cost approximately by 0.42% or \$2,700. Furthermore, rising in the DBE percentage results in higher private cost. This is reasonable because higher DBE percentage increases the primary contractor's transaction cost in a project by finding and subcontracting partial work to a DBE firm. More specifically, one unit increase in DBE will increase the mean private cost by about 4% or slightly more than \$25,700. An interesting pattern shows different effects of the number of projects (np) on the government's reserve price and the bidders' costs. Increasing the number of projects involved in one contract tends to raise the government's valuation of the work, but to lower the bidders' costs. This is because a bidder, while undertaking the projects, will consider the economic scale of taking multi-projects on multi-sites in neighborhood that reduces his cost. The government may not take the effect of economic scale into account since it does not assume the work anyway. This explains why we see a negative effect of np on the private costs, but a positive effect on the reserve price. Moreover, one unit increase in the number of projects can save the firm's private cost on average by about 3.9% which is slightly less than \$25,000. Bridges of a steel structure cause about \$180,000 more than bridges of other structures on the mean private cost. Furthermore, the estimate of the unobserved heterogeneity parameter is strongly significant, meaning that there exists unobserved auction heterogeneity in our data set.

7.2 Testing for Cross-Stage Change of Private Cost Distributions

To implement the test, we estimate the bidding equations separately round by round and obtain $\widehat{\theta}_1$ and $\widehat{\theta}_2$. We then compute the W statistic, which is 1.45. Thus the null hypothesis is not rejected at a 5% significance level. It implies that a bidder re-draws his private cost from the same cost distribution across different auction rounds.

In this case, we re-estimate θ in view of $\theta_1 = \theta_2 = \theta$ by utilizing this restriction in the MSM estimation to obtain a more efficient estimate. The results are reported in the last two columns of Table 9. In sections that follow we use these estimates for inference.

7.3 Robustness Check

We take the estimate of the parameter of the unobserved auction heterogeneity $\widehat{\sigma}$ from the first-stage estimation for granted in the second-stage, assuming that the unobserved auction heterogeneities are from the same distribution for both the auctioneer and the bidders. Although this is mainly for simplifying computation (particularly for those of the second round auctions), we can empirically check its validity. We estimate the parameter of the unobserved heterogeneity in (14) jointly with the parameters of the private cost distribution and obtain $\widetilde{\sigma}$. We then compare $\widehat{\sigma}$ and $\widetilde{\sigma}$. It turns out that $\widehat{\sigma}$ and $\widetilde{\sigma}$ are very close (0.054 and 0.055), which validates the assumption.

8 Counterfactual Analysis

In this section we investigate welfare impacts of different reserve price release policies on the government. Motivated by the INDOT data feature, our model has focused on the use of the secret reserve price by the government. Alternatively, the government can make the engineer's estimate public and use it as a public reserve price. In this scenario, the government can find no bids submitted if all bidders' private costs are above the public reserve price. If we maintain the random cost replacement assumption, then the government can re-auction the project in the next round with the same public reserve price. As a result, under the random cost replacement assumption, the multi-round feature can be accommodated by both secret and public reserve prices. It would be interesting to compare the welfare implications of these two mechanisms using a counterfactual analysis. Such a comparison allows us to evaluate the INDOT's auction mechanism and assess the efficiency of its current reserve price policy. Moreover, it offers insight on why secret reserve prices are used in auctions. Since we have uncovered the underlying structural elements, we can conduct simulations under the two different reserve price release policies and compare the government's payment under the two different scenarios.

We construct a representative auction by setting all observed characteristics at the sample means of the corresponding covariates. The simulation results are reported in Table 10. The expected procurement cost is \$537,689 under the public reserve price, \$524,048 under the secret reserve price. The difference is strongly significant. The INDOT on average can save \$13,641, or 2.5% of the project value, on a typical bridge work auction by adopting a secret reserve price, thereby saving millions of dollars of budgets on all highway projects yearly. On the other hand, the difference in the probability of no sale is 10%. The INDOT undergoes a no sale risk of 10% greater by adopting a secret reserve price. However, it comes with a large standard error and therefore statistically insignificant. Moreover, in practice the highway contracts are often sold out within two rounds, the cost saved by adopting the secret reserve price outweighs the no sale risk caused. Hence our findings indicate that the use of secret reserve price may be a good policy in practice in procurement auctions.[12]

9 Conclusion

In this paper, we study multi-round auctions with secret reserve prices. Our model yields some predictions that can be empirically tested, such as that the equilibrium bids decline uniformly over various stages. Also, our simulation study of the model demonstrates that depending on the specifications of the underlying distributions, the auctioneer may be better off by keeping the reserve price secret, which is the case in our data that motivates our study. Thus our model has the potential to be used to explain why, in some real world auctions, secret reserve prices are used.

We develop a structural approach to analyze the INDOT data. The structural approach recovered the distributions of the reserve prices and the private cost. The estimates for structural parameters allow us to conduct counterfactual analyses. We find that the INDOT could have significantly saved budgets by adopting a secret reserve price rather than using a public reserve price. Our paper offers insight on the use of secret reserve prices in multi-round auctions and the strategic changes in bidders' bidding strategies. It is worth noting, on the other hand, that our model is a static model with non-forward looking bidders. We make this assumption to simplify the analysis and to accommodate the flexibility of allowing for changes of bidders' private cost distributions

[12] McAfee and McMillan (1992) have argued that secret reserve prices can be used for preventing collusions.

across stages. Alternatively, one can introduce dynamic features into the model by assuming that bidders are forward looking and their private cost distributions do not change across stages. Ji and Li (2006) propose a dynamic model of multi-round auctions with secret reserve prices and develop a structural approach.

Appendix

1. Lemma 1 In the first stage, bidder $i's$ probability of winning is $[1 - F(c_{i1})]^{N-1}[1 - G(b_{i1})]$; while in the j-th stage, bidder $i's$ probability of winning is $\dfrac{[1 - F_j(c_{ij})]^{N-1}[G(s^*_{j-1}) - G(b_{i1})]}{G(s^*_{j-1})}$.

Proof bidder i wins the auction if his bid is less than the other $N - 1$ bids as well as the reserve price. The probability of winning can be described by occurrence of the following event, $\Pr(b_{i1} < \min_{k \neq i}(b_{k1})$ and $b_{i1} < r_0)$, which is a joint probability. Because the pair wise independence of agents, it is a product of $\Pr(b_{i1} < r_0)$ and $\Pr(b_{i1} < b_{k1}), \forall k \neq i$. At equilibrium if the bidders play the symmetric bidding strategy, then $\beta(c_{i1}) < \beta(c_{k1})$ implies $c_{i1} < c_{k1}$. Hence it follows that $\Pr(c_{i1} < c_{k1}) = 1 - \Pr(c_{k1} > c_{i1}) = 1 - F_1(c_{i1})$, $\Pr(b_{i1} < r_0) = 1 - \Pr(r_0 < b_{i1}) = 1 - G(b_{i1})$.

In the j-th stage, the information from previous auction rounds enables the bidders to form a Bayesian updated belief of r_0. Therefore the probability that b_{ij} is less than r_0 is contingent on the past information set Λ_{j-1}, i.e., $\Pr(b_{ij} < r_0|\Lambda_{j-1})$, where s^*_{j-1} is the lowest bid from the previous auction round. Henceforth $r_0 < s^*_{j-1}$, is the information set Λ_{j-1}. bidders bid as if they saw r_0 drawn from a truncated distribution $G(r|r < s^*_{j-1})$. It then leads to the following result. $\Pr(b_{ij} < r_0|\Lambda_{j-1}) = 1 - \Pr(r_0 < b_{ij}|r_0 < s^*_{j-1}) = 1 - \dfrac{G(b_{ij})}{G(s^*_{j-1})} = \dfrac{G(s^*_{j-1}) - G(b_{ij})}{G(s^*_{j-1})}$. Similarly, the probability of winning is based on $\Pr(b_{ij} < \min_{k \neq i}(b_{kj})$ and $b_{ij} < r_0)$, a joint probability. Thus the result immediately follows.

2. Proof of Proposition 1 Define $\beta(\cdot)$ as the symmetric increasing Bayesian-Nash equilibrium bidding strategy. Since it is the same function for each bidder, we could suppress the subscript i. We index the strategy in j-th stage by β_j. We solve the game stage by stage to obtain the separate equilibrium. In the first stage, the bidder chooses b_1 to maximize his expected payoff $\pi_1 = (b_1 - c_1)[1 - F_1(\beta_1^{-1}(b_1))]^{N-1}[1 - G(b_1)]$.

$$\underset{b_1}{Max}(b_1 - c_1)[1 - F_1(\beta_1^{-1}(b_1))]^{N-1}[1 - G(b_1)]$$

Note that the probability depends upon b_1 through both $F_1(\cdot)$ and $G(\cdot)$ because it is b_1 not c that determines the probability of winning the auction. So we should treat c in $F_1(\cdot)$ endogenously through the inverse function $c = \beta_1^{-1}(b_1)$. The first order condition is as follows.

$$[1 - F_1(\beta_1^{-1}(b_1))]^{N-1}[1 - G(b_1)] - (b_1 - c)[1 - F_1(\beta_1^{-1}(b_1))]^{N-1}g(b_1)$$

$$-(b_1-c)[1-G(b_1)](N-1)[1-F_1(\beta_1^{-1}(b_1))]^{N-2}f_1(\beta_1^{-1}(b_1))\frac{1}{\beta'(\beta_1^{-1}(b_1))}=0$$

Where $b_1 = \beta_1(c)$ and $c = \beta_1^{-1}(b_1)$. After we replace b_1 with the function of c, we get the differential equation for β_1.

$$[1-F_1(c)]^{N-1}[1-G(\beta_1)]\beta_1'(c)-(\beta_1-c)[1-F_1(c)]^{N-1}g(\beta_1)\beta_1'(c)$$

$$-(\beta_1(c)-c)[1-G(\beta_1(c))](N-1)[1-F_1(c)]^{N-2}f_1(c)=0$$

Further algebraic manipulation turns the differential equation into

$$\frac{d}{dc}\{\beta_1(c)[1-F_1(c)]^{N-1}[1-G(\beta_1(c))]\}=c\frac{d}{dc}\{[1-F_1(c)]^{N-1}[1-G(\beta_1(c))]\}$$

The boundary condition is $\beta_1(\overline{c}) = \overline{c}$. The probability of winning for the bidder with $\overline{c}$ is zero because obviously $\overline{c}$ is the highest possible cost a bidder may have. As $F_1(\cdot)$ is a well defined cumulative probability distribution function, $F_1(\overline{c}) = 1$. Bidding more than $\overline{c}$ definitely loses the auction , while bidding less than $\overline{c}$ can incur strictly negative payoff. Therefore $\beta_1(\overline{c}) = \overline{c}$ is weakly dominant strategy. Integrate it over $[c, \overline{c}]$, using the boundary condition, we get

$$\beta_1(c)=c+\frac{\int_c^{\overline{c}}(1-F_1(x))^{N-1}[1-G(\beta_1(x))]dx}{[1-F_1(c)]^{N-1}[1-G(\beta_1(c))]}$$

$\beta_1(c)$ is increasing, under regular conditions, second order condition satisfied, $\beta_1(c)$ is the optimal bidding strategy. This implies that given all bidders bid following this bidding function, no one can be better off by deviation. Therefore it is a Bayesian Nash equilibrium.

In any j-th reauction stage, the bidder maximize his expected payoff in the j-th round, as follows.

$$\underset{b_j}{Max}(b_j-c_j)\frac{[1-F_j(\beta_j^{-1}(b_j))]^{N-1}[G(s_{j-1}^*)-G(b_j)]}{G(s_{j-1}^*)}$$

the first order condition for maximizing π_j is

$$[1-F_j(\beta_j^{-1}(b_j))]^{N-1}\frac{[G(s_{j-1}^*)-G(b_j)]}{G(s_{j-1}^*)}-(b_j-c)[1-F_j(\beta_j^{-1}(b_j))]^{N-1}g(b_j)\frac{1}{G(s_{j-1}^*)}$$

$$-(b_j-c)\frac{[G(s_{j-1}^*)-G(b_j)]}{G(s_{j-1}^*)}(N-1)[1-F_j(\beta_j^{-1}(b_j))]^{N-2}f_j(\beta_j^{-1}(b_j))\frac{1}{s'(\beta_j^{-1}(b_j))}=0$$

In equilibrium, we obtain the differential equation for β_j

$$[1-F_j(c)]^{N-1}[G(s_{j-1}^*)-G(\beta_j)]\beta_j'(c)-(\beta_j-c)[1-F_j(c)]^{N-1}g(\beta_j)\beta_j'(c)$$

$$-(\beta_j(c)-c)[G(s^*_{j-1})-G(\beta_j(c))](N-1)[1-F_j(c)]^{N-2}f_j(c)=0$$

Then it can be written as

$$\frac{d}{dc}\{\beta_j(c)[1-F_j(c)]^{N-1}[G(s^*_{j-1})-G(\beta_j)]\}=c\frac{d}{dc}\{[1-F_j(c)]^{N-1}[G(s^*_{j-1})-G(\beta_j)]\}$$

The boundary condition is different here. It involves participation decision of the bidder. Entry to the j-th round auction occurs only if a bidder's private cost in j-th round is less than s^*_{j-1}. Hence the above strategy is conditional on that the private cost is less than s^*_{j-1}. With the same argument as in the first stage, we can establish the boundary condition $\beta_j(s^*_{j-1})=s^*_{j-1}$ for this stage. Integrate over $[c,s^*_{j-1}]$, using the boundary condition, to get the following.

$$\beta_j(c)=c+\frac{\int_c^{s^*_{j-1}}[1-F_j(x)]^{N-1}[G(s^*_{j-1})-G(\beta_j(x))]dx}{[1-F_j(c)]^{N-1}[G(s^*_{j-1})-G(\beta_j(c))]}$$

$\beta_j(c)$ is increasing, under regular conditions, second order condition satisfied, $\beta_j(c)$ is a Bayesian Nash equilibrium.

3. **Proof of Proposition 2** By the uniqueness of symmetric Bayesian Nash equilibrium solution of (1) and (2), we have

$$\frac{1}{G^*_{j-2}}(\beta_{j-1}-c)[1-F_{j-1}(c)]^{N-1}[G^*_{j-2}-G(\beta_{j-1})]$$
$$\geq\frac{1}{G^*_{j-2}}(\beta_j-c)[1-F_{j-1}(c)]^{N-1}[G^*_{j-2}-G(\beta_j)]$$

where G^*_{j-2} is short for $G(s^*_{j-2})$, and

$$\frac{1}{G^*_{j-1}}(\beta_j-c)[1-F_j(c)]^{N-1}[G^*_{j-1}-G(\beta_j)]$$
$$\geq\frac{1}{G^*_{j-1}}(\beta_{j-1}-c)[1-F_j(c)]^{N-1}[G^*_{j-1}-G(\beta_{j-1})]$$

Since $s^*_{j-1}<s^*_{j-2}<\bar{c}$, it immediately follows that

$$(\beta_{j-1}-c)[G^*_{j-2}-G(\beta_{j-1})]\geq(\beta_j-c)[G^*_{j-2}-G(\beta_j)]$$

$$(\beta_j-c)[G^*_{j-1}-G(\beta_j)]\geq(\beta_{j-1}-c)[G^*_{j-1}-G(\beta_{j-1})]$$

Furthermore, from the first inequality above we can get

$$(\beta_{j-1}-c)[G^*_{j-2}+G^*_{j-1}-G^*_{j-1}-G(\beta_{j-1})]\geq(\beta_j-c)[G^*_{j-2}+G^*_{j-1}-G^*_{j-1}-G(\beta_j)]$$

and that is

$$(\beta_{j-1} - c)[G^*_{j-1} - G(\beta_{j-1})] + (\beta_{j-1} - c)(G^*_{j-2} - G^*_{j-1})$$
$$\geq (\beta_j - c)[G^*_{j-1} - G(\beta_j)] + (\beta_j - c)(G^*_{j-2} - G^*_{j-1})$$

for which to hold, it must be that

$$(\beta_{j-1} - c)(G^*_{j-2} - G^*_{j-1}) \geq (\beta_j - c)(G^*_{j-2} - G^*_{j-1})$$

Therefore it must be $\beta_j(c) \leq \beta_{j-1}(c)$.

4. **Derivation of the Structural Equilibrium Bidding Function** For the exponential distribution, $F(c) = 1 - \exp(-c/\exp(x\theta + u))$, $G(r) = 1 - \exp(-r/\exp(x\gamma + u))$, $1 - F(c) = \exp(-c/\exp(x\theta + u))$, $1 - G(r) = \exp(-r/\exp(x\gamma + u))$, substitute them into equation (1).

$$\begin{aligned} \beta_1(c) &= c + \frac{\int_c^\infty [\exp(-x/\exp(x\theta + u))]^{N-1} \exp(-\beta_1(x)/\exp(x\gamma + u))dx}{\exp[-c/\exp(x\theta + u))]^{N-1} \exp(-\beta_1(c)/\exp(x\gamma + u))} \\ &= c + \frac{\int_c^\infty \exp(-[(N-1)x/\exp(x\theta + u) + \beta_1(x)/\exp(x\gamma + u)])dx}{\exp(-[(N-1)c/\exp(x\theta + u) + \beta_1(c)/\exp(x\gamma + u)])} \end{aligned}$$

We use contraction mapping to solve it. We start with a conjecture of $\beta_1(c)$, say $\beta_1^0(c)$, which is the left hand side function. Then we substitute it into the right hand side to compute. This yields the right hand side function $\beta_1^1(c)$. If our conjecture is right, then $\beta_1^0(c) = \beta_1^1(c)$. Otherwise replace our conjecture with $\beta_1^1(c)$ and start iteration until the left hand side function equals the right hand side function, say $\beta_1^i(c) = \beta_1^{i+1}(c)$. Start with $\beta_1^0(c) = c$, calculate right hand side function as

$$\begin{aligned} \beta_1^1(c) &= c + \frac{\int_c^\infty \exp(-[(N-1)x/\exp(x\theta + u) + x/\exp(x\gamma + u)])dx}{\exp(-[(N-1)c/\exp(x\theta + u) + c/\exp(x\gamma + u)])} \\ &= c - \frac{1}{(N-1)/\gamma + 1/\theta} \frac{\exp(-[(N-1)x/\exp(x\theta + u) + x/\exp(x\gamma + u)])}{\exp(-[(N-1)c/\exp(x\theta + u) + c/\exp(x\gamma + u)])} |_c^\infty \\ &= c + \frac{1}{(N-1)/\exp(x\theta + u) + 1/\exp(x\gamma + u)} \end{aligned}$$

The last equality obtained when we use the boundary condition of $\beta_1(\bar{c}) = \bar{c}$, particularly in limit in this case $\lim_{\bar{c}\to\infty}(\beta_1(\bar{c}) - \bar{c}) = 0$. This is a weakly dominant strategy for the bidder with upper bound cost. Hence it guarantees that the solution is unique. Then with $\beta_1^1(c)$, we compute $\beta_1^2(c)$. It follows that $\beta_1^2(c) = \beta_1^1(c)$.

$$\beta_1^2(c) = c + \frac{1}{(N-1)/\exp(x\theta + u) + 1/\exp(x\gamma + u)}$$

Therefore the solution is given by $\beta_1^2(c)$, *i.e.*

$$\beta_1(c) = c + \frac{1}{(N-1)/\exp(x\theta + u) + 1/\exp(x\gamma + u)}$$

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Table 1 Variables and Number of Observations

number of lets with one round	243
number of lets with two round	30

Variable	Description	NOBS
characteristics of lets		
dbe	DBE percentage goal	273
time	number of working days needed	273
np	number of projects	273
steel	whether the bridge is of steel structure	273
length	length of the bridge(s), meter	273
prices		
rp	government's engineer estimate	273
bid1	for lets with one round	1261
bid1&2	for lets with two round (both rounds)	167
bid1-2	bid in the first round auction that is unsold	102
bid2-2	bid in the second round	65
participation		
nb	number of potential bidders	273
nb2	number of bidders in 2nd round	30

Table 2 Summary Statistics

variable	mean	std.dev.	min	max
dbe	7.52	3.16	0	15
time	137.53	65.76	20	451
np	1.18	0.60	1	5
steel	0.38	0.49	0	1
length	79.21	82.27	3.22	607.31
rp	855614.8	895489.2	70671.35	6742284
bid1	839506	869855.5	65325.78	6684512
bid1&2	619485.5	398895.8	94853	2230051
bid1-2 (first round)	638917.3	427400.6	94853	2230051
bid2-2 (second round)	588992.4	350553.3	97637.2	1505183
nb	4.99	1.98	1	10
nb2	2.23	1.01	1	5

Table 3 Results of Simulation Study

$F(c)$	$G(r)$	N	public rp	secret rp
exp (1)	exp (1)	4	0.0388	0.0341
exp (1)	exp (1)	7	0.024	0.0228
exp (1)	exp (1)	10	0.0174	0.0172
exp(2)	exp (1)	10	0.0306	0.0281
exp(0.5)	exp (1)	10	0.0092	0.0096
exp(0.3)	exp (1)	10	0.0056	0.0062
exp(0.3)	exp(1)	7	0.0082	0.0086
unif [0,1]	unif [0,1]	4	0.2911	0.2662
unif [0,1]	unif [0,1]	7	0.2032	0.1958
unif [0,1]	unif [0,1]	10	0.1554	0.1543
unif [0,0.3]	unif [0,1]	10	0.0519	0.0586
unif [0,0.5]	unif [0,1]	10	0.0838	0.0886
unif [0,0.7]	unif [0,1]	10	0.1138	0.1174

exp: exponential distribution, mean in parentheses

unif: uniform distribution, bounds in brackets

Table 4-1 Poisson Regression Model of NB

nb (Dept. Var.)	Coef.	Std. Err.
dbe	0.0161	0.01
time	5.07e-04	4.31e-04
np	-0.0505	0.0513
steel	0.042	0.0608
length	3.59e-04	3.82e-04
_cons	1.42*	0.104

Observations: 273 Log likelihood: -567.24

restricted log likelihood -573.42

chi2(d.f.=5) = 12.37 p-value = 0.03

left truncated data, at nb=0

*: significant at 5%

Table 4-2 Negative Binomial Model of NB

nb (Dept. Var.)	Coef.	Std. Err.
dbe	0.016	0.01
time	5.08e-04	5.21e-04
np	-0.0504	0.0653
steel	0.042	0.073
length	3.59e-04	3.37e-04
_cons	1.42*	0.13

Observations: 273 Log likelihood: -567.24

left truncated data, at nb=0

*: significant at 5%

Table 5 OLS Estimates of Regression of Reserve Prices

log(rp) (Dept. Var.)	Coef.	Std. Err.
nb	-0.011	0.011
round-two	-0.12	0.12
dbe	0.045*	0.011
time	0.005*	0.0005
np	0.204*	0.060
steel	0.273*	0.073
length	0.0024*	0.0004
_cons	11.84*	0.149
Number of Observations: 273		$R^2 = .52$
*: significant at 5%		

Table 6 OLS Estimates of Regression of Bids

log(bid) (Dept. Var.)	Coef.	Std. Err.
round-two	-0.826*	0.4113
nb	-0.020*	0.0068
dbe	0.037*	0.0053
time	0.0048*	0.0002
np	0.191*	0.0387
steel	0.272*	0.0337
length	0.0024*	0.0002
nb*round-two	0.0504	0.0481
dbe*round-two	0.006	0.0197
time*round-two	-0.001	0.0016
np*round-two	0.681*	0.1560
steel*round-two	-0.340	0.2276
length*round-two	0.0005	0.0005
_cons	11.927*	0.0809
marginal effect $(b_2 - b_1)$	-5.3e+04*	3.1e+03
Number of Observations: 1428		$R^2 = 0.51$
*: significant at 5%		

Table 7 Random Effect Analysis of Bids

log(bid) (Dept. Var.)	Coef.	Std. Err.
nb	-0.0214	0.016
dbe	0.039*	0.011
time	0.0047*	0.0005
np	0.201*	0.060
steel	0.265*	0.073
length	0.0024*	0.0005
_cons	11.91*	0.14
$\sigma_u^2 = 0.5346$	$\sigma_\varepsilon^2 = 0.1179$	
$\rho = 0.9536$ (fraction of variance due to u_ℓ)		
Number of Groups: 273		
Model: $\log(bid_{i\ell}) = x_{i\ell}\beta + u_\ell + \varepsilon_{i\ell}$		
*: significant at 5%		

Table 8 SML Estimates of Reserve Prices Distribution and Unobserved Heterogeneity

Variable	Coef.	Std. Err.
dbe	0.0514*	0.0018
time	0.0052*	0.0002
np	0.299*	0.0082
steel	0.300*	0.0106
length	0.00212*	0.0001
_cons	11.469*	0.0923
σ^2	0.054*	0.0016

*: significant at 5%

Table 9 MSM Estimates of Private Distribution

Variable	Separate Estimates				More Efficient Estimates	
	θ_1	Std. Err	θ_2	Std. Err	θ	Std. Err
dbe*	0.0398	0.0010	0.0393	0.0037	0.0401	0.0009
time*	0.0044	0.0001	0.0041	0.0005	0.0042	0.0001
np*	-0.0343	0.0008	-0.0394	0.0077	-0.0388	0.0009
steel*	0.2947	0.0098	0.2440	0.0529	0.2809	0.0071
length*	0.0021	0.0001	0.0019	0.0002	0.0021	0.0001
_cons*	12.1394	0.0336	16.8904	2.2696	12.2670	0.0654

σ^2(robustness check): 0.0548

*: significant at 5%

Table 10 The Comparison of Policies by Simulations

difference in government's payment ($public - secret$)	Std. Err	difference in probability of no sale ($public - secret$)	Std. Err
13641*	5876	-0.1	0.06

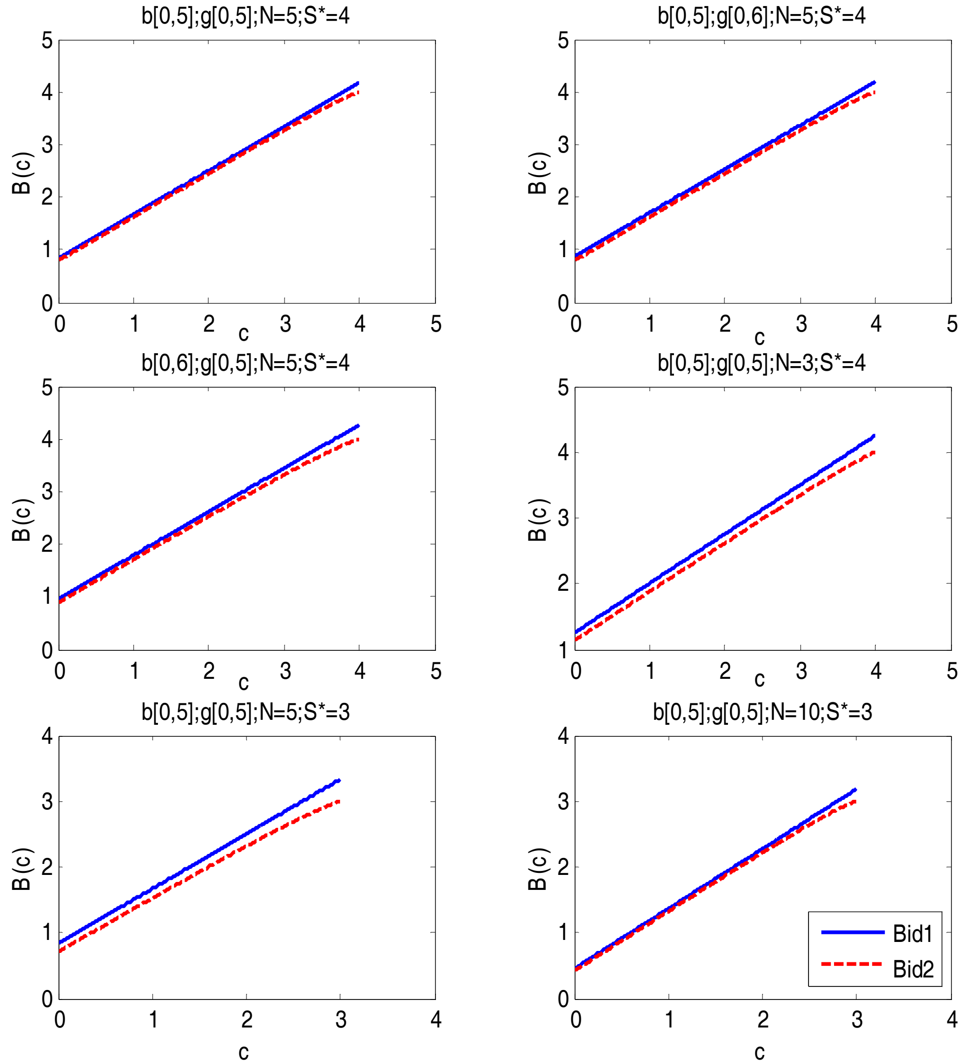

Figure 1-1 Bidding Functions of the First Two Stages in Multi-Round Auctions with Secret Reserve Prices: Uniform Distributions

Note: The contractors' private cost and the governmental reserve price are uniformly distributed. In each subplot, b refers to contractor, g refers to government, N refers to number of contractors and S* refers to the lowest bid in the first round of an unsold contract. We maintain the convention of notations throughout the figure set 1.

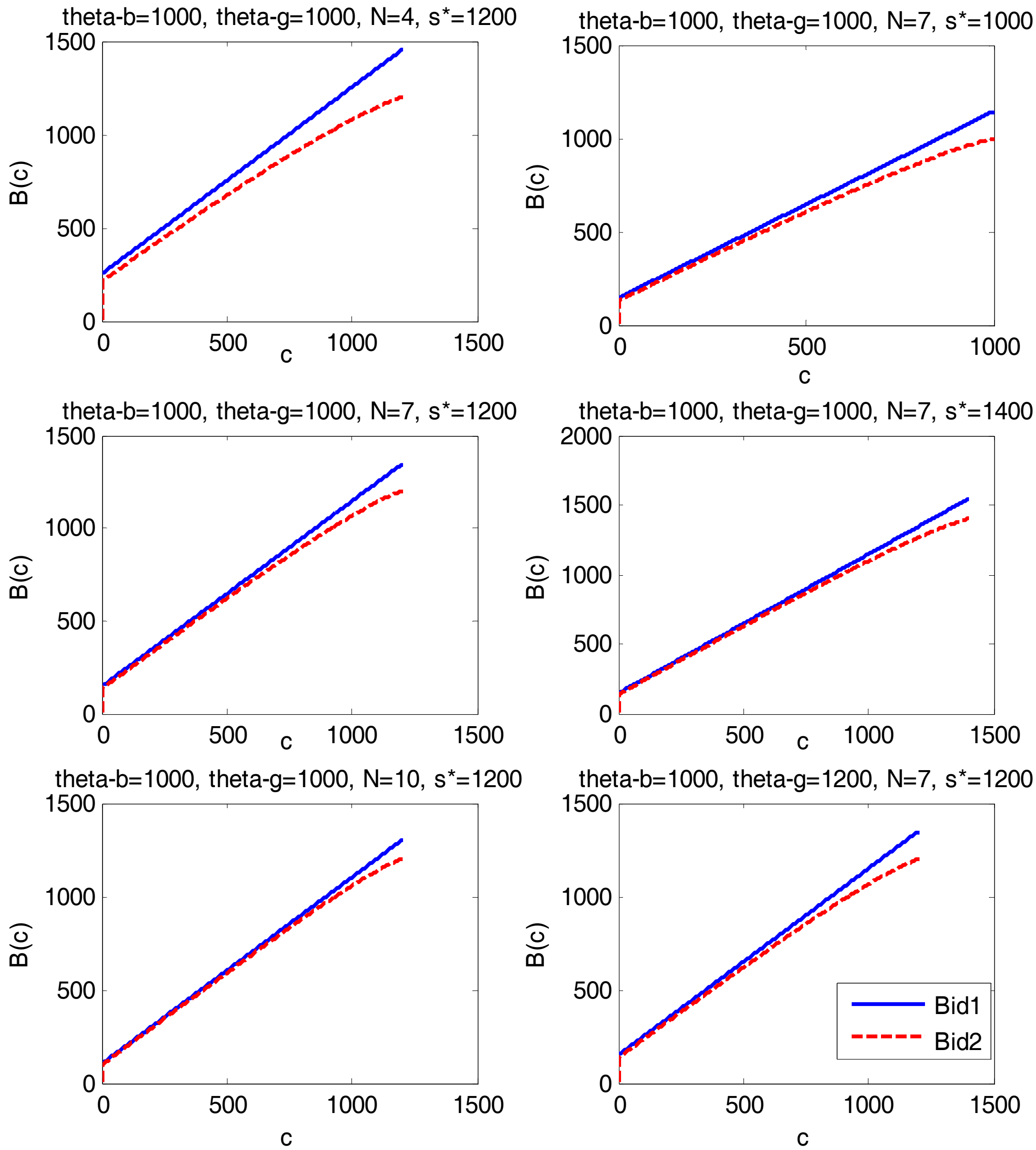

Figure 1-2 Bidding Functions of the First Two Stages in Multi-Round Auctions with Secret Reserve Prices: Exponential Distributions

Note: The contractors' private cost and the governmental reserve price are exponentially distributed.

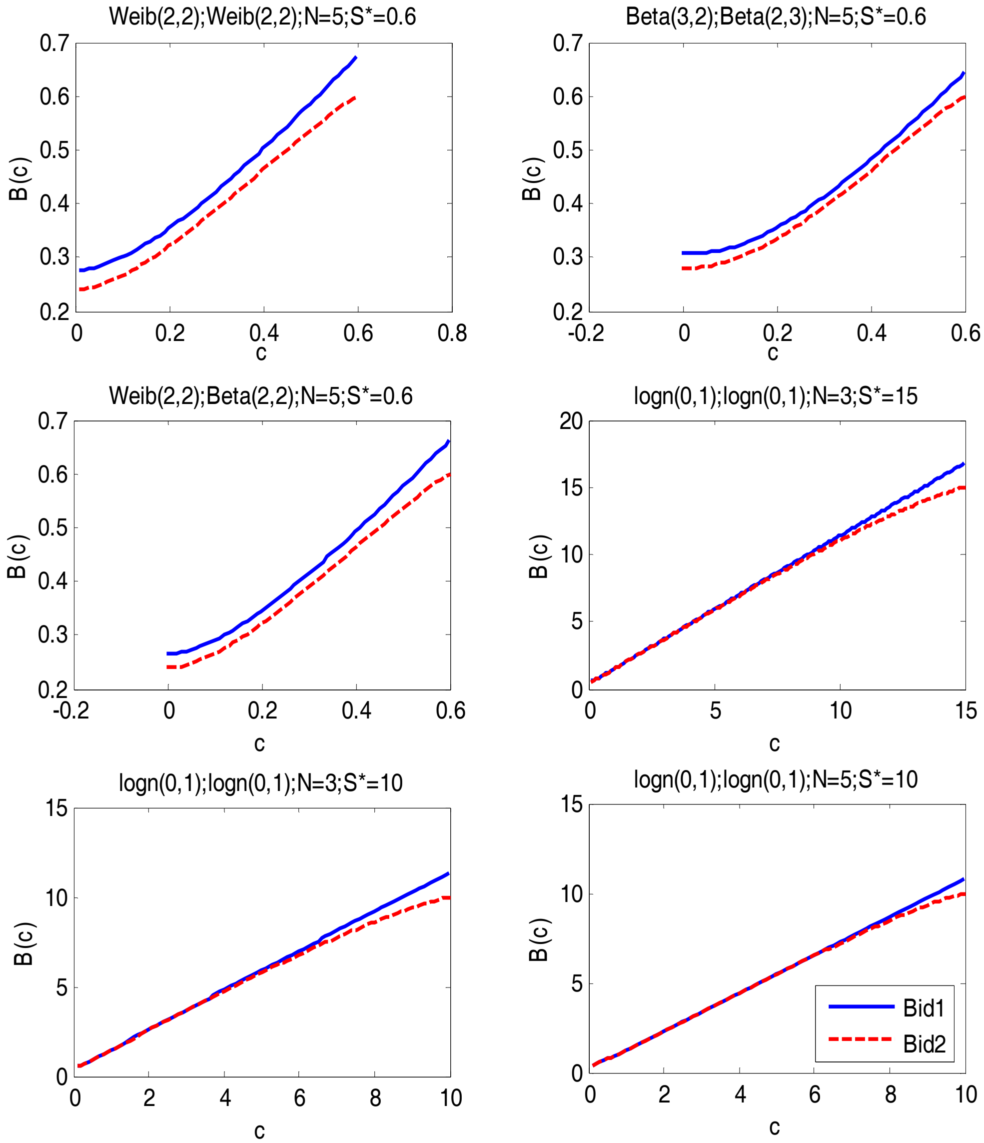

Figure 1-3 Bidding Functions of the First Two Stages in Multi-Round Auctions with Secret Reserve Prices: Mixed Distributions

Note: Weib refers to weibull distribution; Beta refers to Beta distribution; logn refers to log normal distribution. In each subplot, the contractor's distribution is put in the first place.

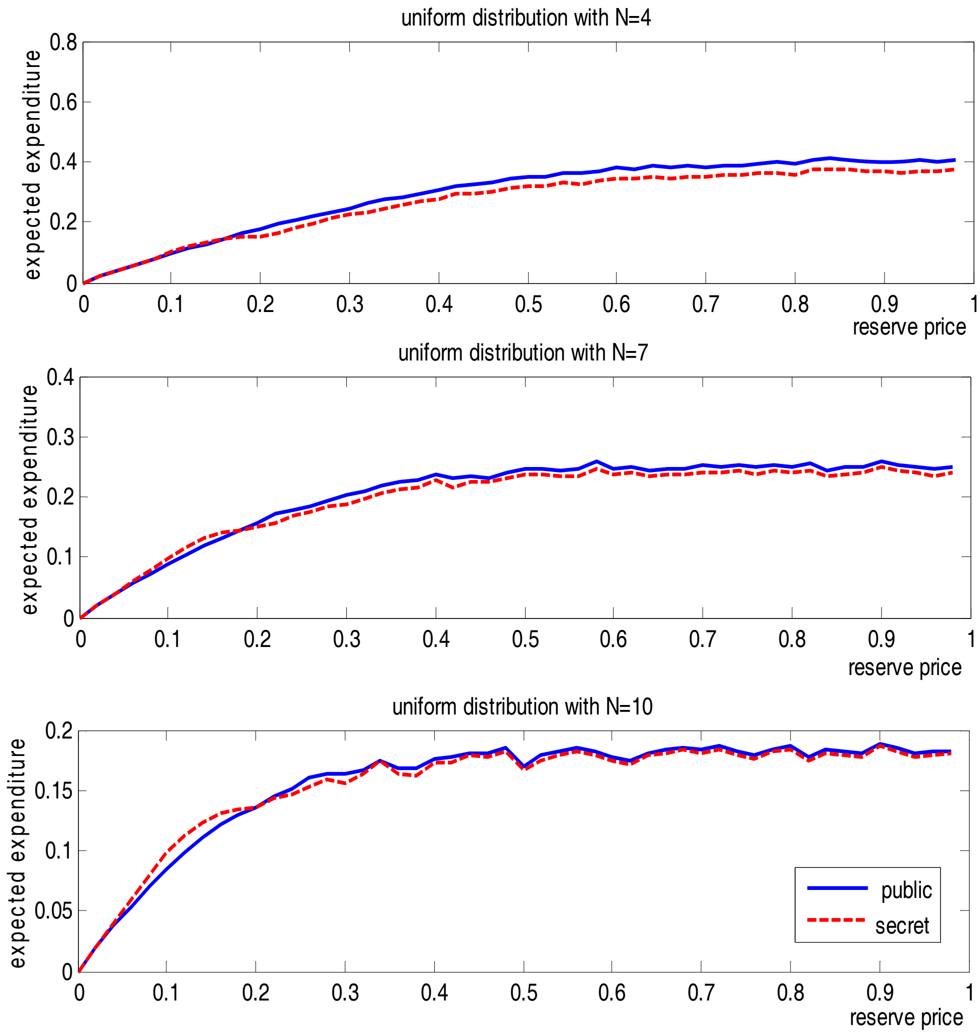

Figure 2-1 The Comparison of Governmental Expenditures under Uniform Cost Distributions

Note: solid lines represent (ex post) governmental expenditures under public reserve prices, dash lines are under secret reserve prices.

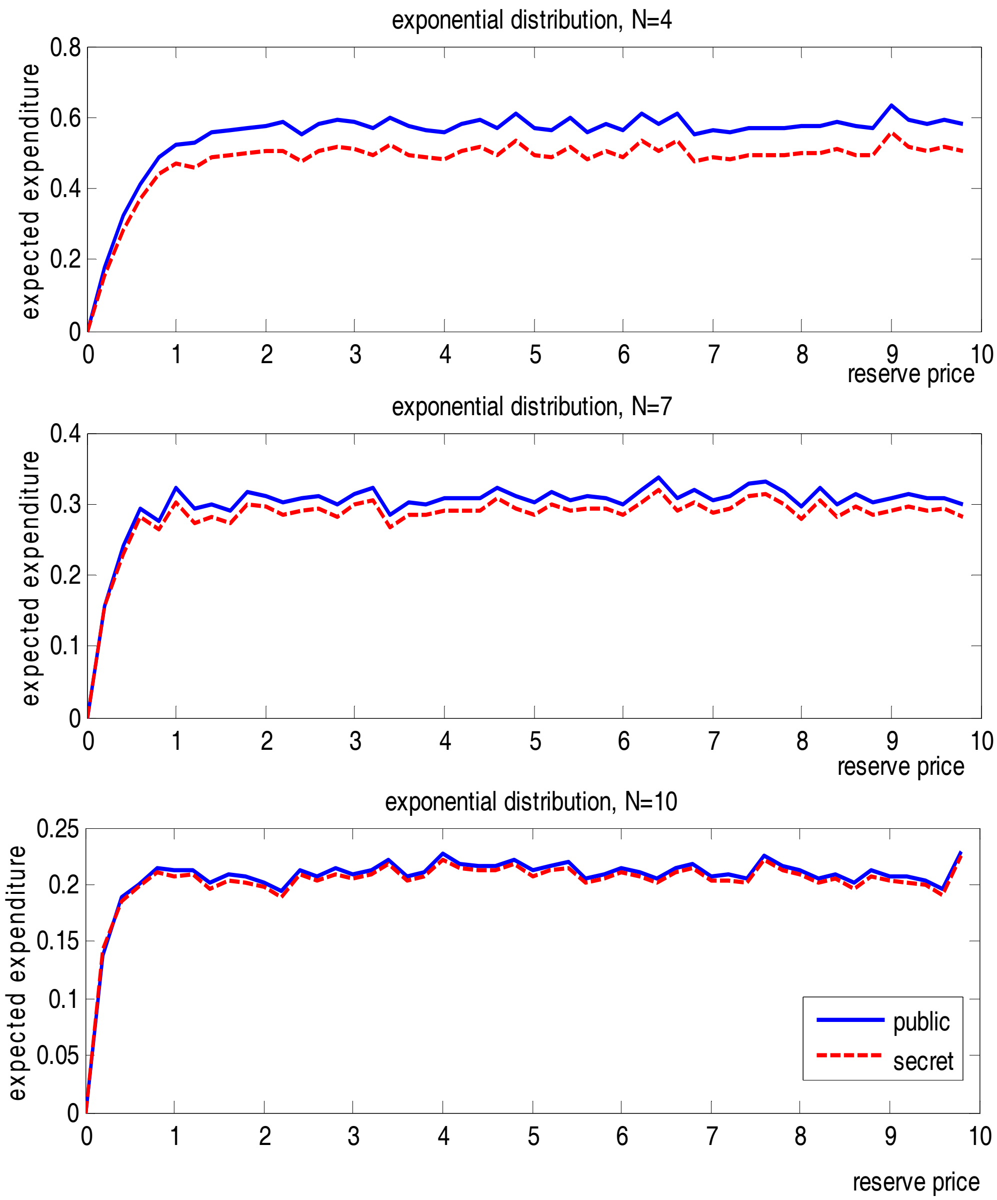

Figure 2-2 The Comparison of Governmental Expenditures under Exponential Cost Distributions