Nonparametric Identification and Estimation of a common value auction model

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Abstract

Structural econometric studies on auctions have mainly focused on the independent private value paradigm. In this paper, we are interested in the "opposite" case : the common value model. More precisely, we restrict our attention to a common value model defined by two functions : the density of the true value of the auctioned good and a unique function that appears in the definition of the conditional densities of the signals. We establish that this common value model is nonparametrically identified without any further restrictions. We then propose a one-step nonparametric estimation method and prove the consistency of our estimators. We apply our method on simulated data and show that the technique we propose is adequate to recover the distribution functions of interest.

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1 Introduction

Structural econometric approaches have been successfully applied during the last decade to study auction data. The aim of such analyzes is to recover the structural parameters of a theoretical model from the data using econometric methods. In the case of auctions, the econometrician is interested in estimating the distribution of the value of the good for each participant from the observed bids. It relies on the equilibrium that defines how bids depend on this distribution.

Previous studies mostly focused on the independent private value paradigm (IPV) (Laffont et al. (1995), Donald and Paarsch (1996), Elyakime et al. (1994, 1997), Guerre et al. (2000)). In these models, each bidder knows his own private value for the auctioned good but ignores others' valuations. Furthermore, the valuations are independent from each other. Some recent papers extend these analyzes for affiliated private values (Li et al. (2002)) and for asymmetric or risk-averse bidders ((Campo et al. (2003), Campo et al. (2002)).

The "opposite" case is known as the common value paradigm (CV). In this model, the value of the auctioned good is unknown but the same for each bidder. The participants receive a signal correlated with this value. It turns out that identification and estimation for CV models are more complicated than for IPV models. The main reason behind these difficulties comes from the nonparametric identification of the CV model from observed bids (Laffont and Vuong (1996)). As a consequence, one has to impose some further restrictions to obtain identification results. Paarsch (1992) proposes a parametric approach, whereas Li et al. (2000) develop a nonparametric one to analyze the CV model. In their paper, the authors assume a multiplicative decomposition of the signals into a common component (the value of the good) and an indiosyncratic one (a specific signal) for each bidder. Adding some further restrictions, Li et al. show that the CV model is identifiable and propose a two-step nonparametric procedure to estimate the densities of both components.

In this paper, we analyze a CV model in which the knowledge of all densities of the signals conditionally on the value of the good when this value varies reduces to the knowledge of a unique function. In this context, the CV model is defined by two functions only : the density of the true value and the unique function that enters in the conditional density of the signals. This model is a particular case of the CV models studied by Fevrier (2006) in which he proves that these models are nonparametrically identified without any further restrictions. Unfortunately, his proof does not allow us to derive an estimation method. Hence, we propose another way to prove identification on which our nonparametric estimation method will be based. Contrary to most of the studies, we show that it is possible to use directly the observed bids instead of using a two-step method as in Guerre et al. (2000). We prove the consistency of our one-step nonparametric estimator and apply our method on simulated data. We show that our method is feasible and recover correctly the distribution functions of interest.

The paper is organized as follows. Section 2 presents the CV model and our nonparametric identification results. Section 3 describes the estimation method we propose. Section 4 applies this method to simulated data. Section 5 concludes.

2 The CV model and the Structural Approach

2.1 The CV model

In the Common Value model (Rothkopf (1969), Wilson (1977)), a single and indivisible good is auctioned to n bidders. The value V of the good, unknown to the bidders, is distributed following a distribution function $F_V(.)$ and a density function $f_V(.)$ on the support $[\underline{V}, \overline{V}]$ with $(\underline{V}, \overline{V}) \in \mathbb{R}^{+^2}$. Each bidder i receives a private signal S_i. The signals are conditionally independent given the common value V. We note $F_{S|V}(.|.)$ the distribution function of the signals given V and $f_{S|V}(.|.)$ the associated density function. Its support is $[\underline{S}_V, \overline{S}_V]$ with $(\underline{S}_V, \overline{S}_V) \in \mathbb{R}^{+^2}$. We suppose that $f_{S|V}(.|.)$ satisfies the monotone likelihood ratio property.[1] Each player knows his private signal as well as the distribution functions. He does not know however the private signals of the other bidders.

We study first auction auctions in which each bidder submits a bid. The winner is the one who submits the highest bid. He obtains the object and pay his bid.

A strategy for a player i is a function $b_i(.)$ that associates to each signal S_i the amount $b_i(S_i)$ that player i wants to bid. As shown by Milgrom and Weber (1982), a symmetric equilibrium exists in first price common value auctions. To describe this equilibrium, it is useful to introduce the following functions. We note $Y_i = \max_{j \neq i} S_j$ and $F_{Y_i|S_i}(.|.)$ (resp. $f_{Y_i|S_i}(.|.)$) the associated distribution function (resp. density function) conditionally on

[1] The density $f_{S|V}$ has the monotone likelihood ratio property if for all $s' > s$ and $v' > v$, $f_{S|V}(s|v)/f_{S|V}(s|v') \geq f_{S|V}(s'|v)/f_{S|V}(s'|v')$.

the signal S_i of player i. We also introduce the function $V(s,y) = E[V|S_i = s, Y_i = y]$ that is the expected value of the good conditionally of the signal S_i of player i and the highest signal Y_i of the other players.

Proposition 1. *(Milgrom and Weber, 1982) In a common value first price auction, a symmetric equilibrium strategy is given by:*

$$b(s) = V(s,s) - \int_{\underline{S}}^{s} L(\alpha|s)dV(\alpha,\alpha)$$

where $L(\alpha|s) = exp[-\int_{\alpha}^{s} f_{Y_i|S_i}(u|u)/F_{Y_i|S_i}(u|u)du]$ *and* $\underline{S}$ *is the minimum value that a signal can take.*

2.2 Nonparametric Identification

2.2.1 Our model

We consider a CV model with $n \geq 3$ bidders where the density $f_V(.)$ of the value V has a support $[\underline{V}, \overline{V}]$ and where the density of the signals conditionally on the value $V = v$ is distributed over $[\overline{S}(\underline{V}), \overline{S}(v)]$ and takes the form:

$$f_{S|V}(.|v) = \frac{h(.)}{H(\overline{S}(v))}$$

where $h(.)$ is the derivative of $H(.)$ and $H(\overline{S}(\underline{V})) = 0$.

Because $f_{S|V}(.|.)$ satisfies the monotone likelihood property, $\overline{S}(.)$ has to be increasing. We will suppose that this function is even strictly increasing.

In this model, no restriction is imposed on the value V whereas the distributions of the signals conditionally on the value V are supposed to be representable by a unique function $h(.)$. A natural example is a model in which the value of the good is distributed uniformly on $[\underline{V}, \overline{V}]$ and the signals are distributed uniformly on $[\underline{V}, 2v - \underline{V}]$ conditionally on $V = v$. This is the case when $h = 1$ and $\overline{S}(v) = 2v - \underline{V}$. More generally, the function $h(.)$ and the interval $[\overline{S}(\underline{V}), \overline{S}(v)]$ define the amount of information that the signals carry over the value and play therefore a key role in the analysis.

It is important to study if this model is identified nonparametrically or not i.e. to analyze if the observation of the bids determines uniquely the functions $F_V(.)$, $h(.)$ and $\overline{S}(.)$. Of course, what is observed is important and we will suppose that, in every auction, all bids are available.

In the general case, Laffont and Vuong (1996) (see also Athey and Haile (2002)) have shown that the CV model is not identifiable. Fevrier (2006) proved however that the mineral rights model is identified if there are some variations in the bounds of the conditional distribution functions of the signals. In our case, proposition 2 of Fevrier (2006) applies and we can conclude that our model is nonparametrically identified.

Proposition 2. *Fevrier (2006). The model is nonparametrically identified.*

Unfortunately, Fevrier's identification result does not help us to find a tractable way to estimate the distributions functions. For this reason, we propose another way to prove identification upon which our estimation method will be based. We proceed in three steps.

- We first prove that for each $s' \in [\overline{S}(\underline{V}), \overline{S}(\overline{V})]$ and each $s \in [\overline{S}(\underline{V}), s']$, $f_{S|V}(s|\overline{S}^{-1}(s'))$ is identified.
- We then show that for each $s \in [\overline{S}(\underline{V}), \overline{S}(\overline{V})]$, $(\overline{S}^{-1})'(s) f_V(\overline{S}^{-1}(s))$ is identified
- Finally, using the first order condition, we prove that $\overline{S}^{-1}(.)$ is identified over its support $[\overline{S}(\underline{V}), \overline{S}(\overline{V})]$.

The identification of $f_V(.)$ and $f_{S|V}(.|.)$ is proved by combining these results.

Identification of $f_{S|V}(.|\overline{S}^{-1}(.))$

First, one has to remark that the model is defined up to a transformation of the signals. Indeed, observing $k(s)$ instead of s is equivalent to replace $H(.)$ by $H \circ k^{-1}(.)$ that is defined over the segment $[k \circ \overline{S}(\underline{V}), k \circ \overline{S}(\overline{V})]$. A natural normalization is $b(s) = s$ which means that what we observe in the data are the signals.

The density of $s_1 \leq s_2 \leq s_3$ is thus identified and is given by

$$f_S(s_1, s_2, s_3) = h(s_1)h(s_2)h(s_3) \int_{\overline{S}^{-1}(s_3)}^{\overline{V}} \frac{f_V(v)}{H^3(\overline{S}(v))} dv$$

Hence, for all $(s, s') \in [\overline{S}(\underline{V}), \overline{S}(\overline{V})]^2$, we identify

$$\frac{h(s)}{h(s')} = \frac{f_S(s', s, s)}{f_S(s', s', s)} \tag{1}$$

as well as the bounds $\overline{S}(\underline{V})$ and $\overline{S}(\overline{V})$.

The function $h(.)$ can be recovered using equation (1) up to a constant by fixing s'. Hence, for each $s \in [\overline{S}(\underline{V}), \overline{S}(\overline{V})]$, $f_{S|V}(.|\overline{S}^{-1}(s)) = \frac{h(.)}{H(s)}$ is identified over its support $[\overline{S}(\underline{V}), s]$.

Identification of $(\overline{S}^{-1})'(.)f_V(\overline{S}^{-1}(.))$

Similarly, the density of a signal s is given by

$$f_S(s) = h(s)\int_{\overline{S}^{-1}(s)}^{\overline{V}} \frac{f_V(v)}{H(\overline{S}(v))}dv$$

Deriving this equation for $s \in [\overline{S}(\underline{V}), \overline{S}(\overline{V})]$, one obtains

$$(\overline{S}^{-1})'(s)f_V(\overline{S}^{-1}(s)) = \frac{\frac{h'(s)}{h(s)}f_S(s) - f'_S(s)}{\frac{h(s)}{H(s)}} \tag{2}$$

The right hand side of equation (2) is identified. Hence $(\overline{S}^{-1})'(.)f_V(\overline{S}^{-1}(.))$ also is.

Identification of $\overline{S}^{-1}(.)$

We prove in appendix A that the first order condition can be rewritten for all $s \in [\overline{S}(\underline{V}), \overline{S}(\overline{V})]$ as

$$\begin{aligned}\overline{S}^{-1}(s) =& s + \frac{1}{(n-1)f_{S|V}(s|\overline{S}^{-1}(s))} \\ & - \left[\frac{n}{n-1} - \frac{h'(s)/h(s)}{(n-1)f_{S|V}(s|\overline{S}^{-1}(s))}\right] \frac{F_{Y,S}(s,s)}{(\overline{S}^{-1})'(s)f_V(\overline{S}^{-1}(s))f_{S|V}(s|\overline{S}^{-1}(s))}\end{aligned} \tag{3}$$

where $F_{Y,S}(.,.)$ is the joint density of the signal S of a player and the highest signal Y of his opponents.

The functions that appear in the right hand side of equation (4) are all identified either directly from the data or from the previous results. Hence, $\overline{S}(.)$ also is.

Combining the three previous result, we can conclude that the distribution function of the values and the conditional distributions of the signals are identified. This identification result is important and gives with Li et al. (2000) another nonparametric identification result for CV auctions upon which an estimation method can be based. Our result has also some nice properties.

- First, our model is a simple and intuitive common value model that is easy to interpret. The idea is to reduce the infinite number of density functions of the signals to a single function $h(.)$. No other restriction is needed and full nonparametric identification is achieved.

- Second, the way we prove the identification result allows us to derive in the next subsection some new results about the link between the pure Common Value model and the Conditionally Independent Private Value model.
- Finally, the model we consider is overidentifies. It implies several restrictions on the distribution of the bids that are easy to determine and test.

2.2.2 The conditionally independent private information model

Common Value auctions are part of a larger class of auctions (conditionally independent private information (CIPI) model) where the value of the good for player i is a function $U(.,.)$ of the value V and the signal S_i received by the player. The common value model corresponds to $U(v, s_i) = v$. Another special case is the conditionally independent private value (CIPV) model where $U(v, s_i) = s_i$. Li et al. study the identification of this class of model. They prove (see their proposition 1) that any CIPI model, in particular the CV model, is observationally equivalent to a CIPV model. They mention however that nothing is known about the converse.

The following proposition gives a condition under which the converse is true for our CIPI model.

Proposition 3. *Given our assumptions, any CIPI model (in particular any CV model) is observationally equivalent to a CIPV model.*

Given our assumptions, any CIPI model (in particular any CIPV model) is observationally equivalent to a CV model if and only if the function $\overline{S}(.)$ *defined by (4) is increasing.*

Proof See Appendix A.

This result shows that despite the similar probabilistic structure of CIPV and CV models, they are not necessarily equivalent. To find a CIPV model that generates data that are not "compatible" with a CV model, it is sufficient to find two functions $f_V(.)$ and $h(.)$ such that $\overline{S}(.)$ is not increasing.

As in Li et al. (2000), one can also be interested by the properties of the CIPV model. It is well known that the joint distribution of the signals is, in the CIPV model, nonparametrically identified (Li et al. (1999)). This distribution corresponds to the distribution of $(\xi(b_1, G_b), ..., \xi(b_n, G_b))$ where $G_b(.)$ is the distribution of the bids and $\xi(b, G_b) = b + \frac{G_{B|b}(b|b)}{g_{B|b}(b|b)}$. $G_{B|b}(.|.)$ (resp. $g_{B|b}(.|.)$) is the distribution function (resp. density) of the highest bid of the opponents of a player conditionally on the fact that this player bids b. Applying the previous reasoning, we derive the properties of the CIPV model.

Proposition 4. *Given our assumptions, the CIPV model is nonparametrically identified.*

Proof See appendix A.

This result proves that full nonparametric identification is also achieved for the CIPV model. The nonparametric estimation of this model follows the same logic as the one we propose for the common value model in the next section. It is however based on the transformation $\xi(b, G_b)$ of the bids.

3 Estimation

The estimation method we propose is based on our identification result and will follow the same logic. It consists in estimating some distribution functions of the bids and to use them to construct estimates for $f_{S|V}(.|\overline{S}^{-1}(.))$, $(\overline{S}^{-1})'(.)f_V(\overline{S}^{-1}(.))$ and $\overline{S}^{-1}(.)$.

Let n be a given number of bidders. Let L be the number of auctions indexed by $l = 1, .., L$. We note $\{s_{il};\ i = 1, ..., n;\ l = 1, ..., l\}$ the observed signals.

We note, for each k, $f_S(s_1, ..., s_k)$ the joint density of $(s_1, ..., s_k)$, $F_S(s_1, ..., s_k)$ the joint distribution function.

3.1 Estimation of $f_{S|V}(.|\overline{S}^{-1}(.))$

We first estimate nonparametrically $f_S(., ., .)$ by

$$\widehat{f}_S(s_1, s_2, s_3) = \frac{1}{Lh_3^3} \sum_{l=1}^{L} \frac{1}{n(n-1)(n-2)} \sum_{1 \leq i \neq j \neq k \leq n} K\left(\frac{s_1 - s_{il}}{h_3}\right) K\left(\frac{s_2 - s_{jl}}{h_3}\right) K\left(\frac{s_3 - s_{kl}}{h_3}\right)$$

where h_3 is some bandwidth and $K(.)$ a kernel.

Using this estimation and equation (1), we can estimate $h(.)$ up to a constant for different values of s'. We fix T values $(s'_1, ..., s'_T)$ for s' and combine these estimations to obtain:

$$\widehat{h}(s) = \frac{1}{T}\sum_{i=1}^{T} c_i \frac{\widehat{f}_S(s'_i, s, s)}{\widehat{f}_S(s'_i, s'_i, s)}$$

with $c_i = \frac{\widehat{f}_S(s'_i, s'_i, s'_1)}{\widehat{f}_S(s'_i, s'_1, s'_i)}$. The constants c_i appear to normalize $\widehat{h}(s'_1)$ to be equal to 1.[2] Using T values for s'_i improves the estimation of $h(.)$. Their choice depends on the data and is discussed in the next section.

To estimate $f_{S|V}(.|\overline{S}^{-1}(.))$, one also needs to estimate $H(.)$. A natural solution is to integrate $\widehat{h}$. However, it is better to estimate directly this function using $H(s) = h(s)\frac{\frac{\partial F_S}{\partial s_2}(s,s)}{f_S(s,s)}$. Hence, the estimator $\widehat{f^{\overline{S}}}_{S|V}(.|.)$ of $f_{S|V}(.|\overline{S}-1)(.)$ that we propose is

$$\widehat{f^{\overline{S}}}_{S|V}(s|s') = \frac{\widehat{h}(s)\widehat{f}_S(s', s')}{\widehat{h}(s')\widehat{\frac{\partial F_S}{\partial s_2}}(s', s')}$$

where

$$\widehat{f}_S(s_1, s_2) = \frac{1}{Lh_2^2}\sum_{l=1}^{L}\frac{1}{n(n-1)}\sum_{1\leq i\neq j\leq n} K\left(\frac{s_1 - s_{il}}{h_2}\right) K\left(\frac{s_2 - s_{jl}}{h_2}\right)$$

and

$$\widehat{\frac{\partial F_S}{\partial s_2}}(s_1, s_2) = \frac{1}{Lh_1}\sum_{l=1}^{L}\frac{1}{n(n-1)}\sum_{1\leq i\neq j\leq n} 1\left(s_{il} \leq s_1\right) K\left(\frac{s_2 - s_{il}}{h_1}\right)$$

It is possible to use different kernels for each estimation. However, to simplify, we will always use the same kernel $K(.)$. h_1 and h_2 are some bandwidths for univariate and bivariate densities.

3.2 Estimation of $(\overline{S}^{-1})'(.)f_V(\overline{S}^{-1}(.))$

Equation (2) can be rewritten for $s' > s$ as

$$(\overline{S}^{-1})'(s)f_V(\overline{S}^{-1}(s)) = \frac{\frac{\frac{\partial f_S}{\partial s_2}(s,s')}{f_S(s,s')} f_S(s) - f'_S(s)}{f_{S|V}(s|\overline{S}^{-1}(s))}$$

[2] If $n = 2$, the identification is obtained by a similar reasoning on $f_S(s, s')$. However, one needs to distinguish the two cases $s > s'$ and $s < s'$.

Using

$$\widehat{f}_S(s) = \frac{1}{Lh_1}\sum_{l=1}^{L}\frac{1}{n}\sum_{1\le i\le n} K\left(\frac{s_1 - s_{il}}{h_1}\right)$$

$$\widehat{f'}_S(s) = \frac{1}{Lh_1^2}\sum_{l=1}^{L}\frac{1}{n}\sum_{1\le i\le n} k\left(\frac{s_1 - s_{il}}{h_1}\right)$$

where $k(.)$ is the derivative of $K(.)$, and

$$\widehat{\frac{\partial f_S}{\partial s_2}}(s_1, s_2) = \frac{1}{Lh_2^3}\sum_{l=1}^{L}\frac{1}{n(n-1)}\sum_{1\le i\neq j\le n} K\left(\frac{s_1 - s_{il}}{h_2}\right) k\left(\frac{s_2 - s_{jl}}{h_2}\right)$$

we construct the following estimator for $(\overline{S}^{-1})'(.) f_V(\overline{S}^{-1}(.))$:

$$\widehat{f^{\overline{S}}}_V(s) = \frac{\frac{1}{M}\sum_{i=1}^{M}\frac{\widehat{\frac{\partial f_S}{\partial s}}(s,s'_i)}{\widehat{f}_S(s,s'_i)}\widehat{f}_S(s) - \widehat{f}'_S(s)}{\widehat{f^{\overline{S}}}_{S|V}(s|s)}$$

In this estimation, we estimate $\frac{\frac{\partial f_S}{\partial s_2}(s,s')}{f_S(s,s')}$ for M different values $(s'_1(s), ..., s'_M(s))$ of s' greater than s. Using M signals improves the estimation of the density $f_V(.)$. Their choice is discussed in the next part.

3.3 Estimation of $\overline{S}^{-1}(.)$

Introducing

$$\widehat{F}_{Y,S}(s_1, s_2) = \frac{1}{Lh_1}\sum_{l=1}^{L}\frac{1}{n}\sum_{i=1}^{n} 1\left(y_{il} \le s_1\right) K\left(\frac{s_2 - s_{il}}{h_1}\right)$$

we can estimate $\overline{S}^{-1}(.)$ using equation (4):

$$\begin{aligned}\widehat{\overline{S}^{-1}}(s) =& s + \frac{1}{(n-1)\widehat{f^{\overline{S}}}_{S|V}(s|s)} \\ & - \left[\frac{n}{n-1} - \frac{\frac{1}{M}\sum_{i=1}^{M}\widehat{\frac{\partial f_S}{\partial s_2}}(s, s'_i)}{(n-1)\widehat{f^{\overline{S}}}_{S|V}(s|s)}\right]\frac{\widehat{F}_{Y,S}(s,s)}{\widehat{f^{\overline{S}}}_V(s)\widehat{f^{\overline{S}}}_{S|V}(s|s)}\end{aligned}$$

Finally, we estimate the distribution function of the values by

$$\widehat{f}_V(v) = \widehat{f^S}_V\left(\left(\widehat{\overline{S}^{-1}}\right)^{-1}(v)\right)$$

and the conditional distribution function of the signals by

$$\widehat{f}_{S|V}(s|v) = \widehat{f^S}_{S|V}\left(s|\left(\widehat{\overline{S}^{-1}}\right)^{-1}(v)\right)$$

As in Guerre et al. (2000), our estimation method relies heavily on the distributions of the bids that are observed in the data. However, most of the structural papers that study auctions propose two-step approaches. This is not the case here. Our method is a one step nonparametric estimation method and is in that sense easier than the method proposed by Li et al. (2000).

Under our assumptions, we can prove the convergence of our estimators.

Proposition 5. *$\widehat{f}_V(.)$ and $\widehat{f}_{S|V}(.|.)$ are uniformly consistent estimators for $f_V(.)$ and $f_{S|V}(.|.)$*

Proof See appendix A.

4 Simulations

The estimation method we proposed in the previous section requires the estimation of several functions and may seem difficult to apply. In this section, we simulate data using a particular model and apply our estimation method to prove that this is indeed not the case.

More precisely, we suppose that the value of the good is uniformly distributed on $[0, 2]$ and that the signals are uniformly distributed on $[0, 2v]$ conditionally on v. We simulate $L = 1000$ auctions with $n = 3$ bidders.

The equilibrium strategy is given by proposition (1) and simplifies in this case to (see Appendix A):

$$b(s) = 4 - \frac{32}{s} + \frac{128}{s^2}\ln\left(1 + \frac{s}{4}\right)$$

We thus observe b_{il} for $l = 1, ..., 1000$ and $i = 1, ..., 3$.

When normalizing our model by $b(s) = s$, we transform this model into the following one: V is distributed uniformly on $[0, 2]$; the density of the signals conditionally on $V = v$ is $f_{S|V}(s|v) = \frac{1}{2v}(b^{-1})'(s)$ defined on the interval $[0, b(2v)]$.

First, we need to address the choice of the kernel functions and of the bandwidths. We choose a kernel that satisfies our hypotheses. This is the case of the triweight kernel defined as

$$K(u) = \frac{35}{32}(1-u^2)^3 1(|u| \le 1)$$

We use the so-called rule of thumb (Scott, 1992) to define our bandwidths. In our data, we find $h_1 = 2.978 \times 1.06\widehat{\sigma_S}(nL)^{-1/5} = 0.24$, $h_2 = 2.978 \times 1.06\widehat{\sigma_S}(nL)^{-1/6} = 0.31$ and $h_3 = 2.978 \times 1.06\widehat{\sigma_S}(nL)^{-1/7} = 0.38$. The factor 2.978 follows from the use of a triweight kernel instead of the gaussian kernel. $\widehat{\sigma_S}$ is the standard deviation of the signals.

Figure 1 represents the estimated density of the signals for the simulated data and the theoretical density. It appears that $f_S(.)$ is well estimated on the interval $[h_1, b(4) - h_1]$. This is due to the boundary effect in the kernel estimation. Hence, to understand how precise our estimation method is, we will restrict our analysis to this interval where the boundary effects disappear.

Estimation of $f_{S|V}(.|\overline{S}^{-1}(.))$

The estimation of $f_{S|V}(.|\overline{S}^{-1}(.))$ depends on the signals (s'_i). We take $K = 7$ and $(s'_1, ..., s'_7) = (0.8, 0.5, 0.6, 0.7, 0.9, 1, 1.1)$. This choice is somehow arbitrary. We use several signals to improve the stability of the estimation and use signals that appear frequently in the data in order to have good estimations. Figures 2 and 3 show the functions $f_{S|V}(.|\overline{S}^{-1}(s'))$ and $\widehat{f^{S}}_{S|V}(.|s')$ for $s' = 1.1$ on the segment $[h_3, 1.1]$ as well as the function $h(.)$ and its estimation on the interval $[h_3, S_{max} - h_3]$. It appears that our estimations are quite good .

Estimation of $(\overline{S}^{-1})'(.)f_V(\overline{S}^{-1}(.))$

We first have to define the signals $(s'_1(s), ..., s'_M(s))$. This choice is arbitrary and several hypotheses can be made. Taking $M = 4$ and $(s'_1, ..., s'_4) = (s + (S_{max} - h_3 + s), s + (S_{max} - h_3 + s)/2, s + (S_{max} - h_3 + s)/4, s + (S_{max} - h_3 + s)/8)$, figure 4 shows that the estimated function $\widehat{f^{\overline{S}}}_V(.)$ is close to the theoretical one on the interval $[h_3, S_{max} - h_3]$.

Estimation of $\overline{S}^{-1}(.)$

Finally, the estimation of $\overline{S}^{-1}(.)$ and the true function are represented in figure 5 whereas the estimations $\widehat{f}_V(.)$ and $\widehat{f}_{S|V}(.)$ are represented in figure 6.

It shows that our estimation method is easy to implement and that, in this example, it allows us to recover the true densities.

5 Conclusion

In this paper, we studied a common value model defined by two functions: the distribution function of the value of the good and a unique function that enters in the definition of the conditional densities of the signals. We proved that this model is nonparametrically identified without any further restriction. We proposed a one-step nonparametric estimation method and applied it to simulated data. We show that our method is easy to implement and that our estimators predict correctly the true densities.

This paper gives with Li et al. (2000) a second class of common value models that are identified and easy to estimate nonparametrically.

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Appendix A

Proof of proposition 5

To be completed

Derivation of equation (4)

Deriving the first order condition given in equation (1) and using $b(s) = s$, we have

$$V(s,s) = s + \frac{F_{Y|S}}{f_{Y|S}}$$

i.e.

$$\begin{aligned}(n-1)h^2(s)H^{n-2}(s)\int_{\overline{S}^{-1}(s)}^{\overline{V}} v\frac{f_V(v)}{H^n(\overline{S}(v))}dv =&(n-1)sh^2(s)H^{n-2}(s)\int_{\overline{S}^{-1}(s)}^{\overline{V}} \frac{f_V(v)}{H^n(\overline{S}(v))}dv \\ &+ h(s)H^{n-1}(s)\int_{\overline{S}^{-1}(s)}^{\overline{V}} \frac{f_V(v)}{H^n(\overline{S}(v))}dv\end{aligned}$$

or equivalently

$$\left[s + \frac{H(s)}{(n-1)h(s)}\right]\int_{\overline{S}^{-1}(s)}^{\overline{V}} \frac{f_V(v)}{H^n(\overline{S}(v))}dv = \int_{\overline{S}^{-1}(s)}^{\overline{V}} \frac{f_V(v)}{H^n(\overline{S}(v))}dv$$

Deriving this equation in s and rearranging the terms allow us to find $\overline{S}^{-1}$:

$$\begin{aligned}\overline{S}^{-1}(s) =&s + \frac{1}{(n-1)f_{S|V}(s|\overline{S}^{-1}(s))} \\ &- \left[\frac{n}{n-1} - \frac{h'(s)/h(s)}{(n-1)f_{S|V}(s|\overline{S}^{-1}(s))}\right]\frac{F_{Y,S}(s,s)}{(\overline{S}^{-1})'(s)f_V(\overline{S}^{-1}(s))f_{S|V}(s|\overline{S}^{-1}(s))}\end{aligned} \tag{4}$$

Proof of proposition 4

To be completed

Derivation of the equilibrium strategy for the simulations

By proposition (1),

$$b(s) = V(s,s) - \int_{\underline{S}}^{s} L(\alpha|s) dV(\alpha,\alpha)$$

When V is uniformly distributed on $[0,2]$ and when the signals conditionally on $V = v$ are uniformly distributed on $[0, 2v]$, straightforward calculus lead to

$$\begin{aligned} V(s,s) &= \frac{\int_{s/2}^{2} 2v(\frac{1}{2v})^2 \frac{s}{2v} \frac{1}{2} dv}{\int_{s/2}^{2} 2(\frac{1}{2v})^2 \frac{s}{2v} \frac{1}{2} dv} \\ &= \frac{4s}{4+s} \end{aligned}$$

Similarly,

$$\begin{aligned} L(\alpha,s) &= \exp\left[-\int_{\alpha}^{s} \frac{\int_{s/2}^{2} 2(\frac{1}{2v})^2 \frac{s}{2v} \frac{1}{2} dv}{\int_{s/2}^{2} 2\frac{1}{2v} \left(\frac{s}{2v}\right)^2 \frac{1}{2} dv} ds \right] \\ &= \exp\left[-\int_{\alpha}^{s} \frac{2}{s} ds \right] \\ &= \left(\frac{\alpha}{s}\right)^2 \end{aligned}$$

Combining both results, we have

$$\begin{aligned} b(s) &= \int_{0}^{s} \frac{4\alpha}{4+\alpha} \frac{2\alpha}{s^2} d\alpha \\ &= 4 - \frac{32}{s} + \frac{128}{s^2} \ln\left(1 + \frac{s}{4}\right) \end{aligned}$$

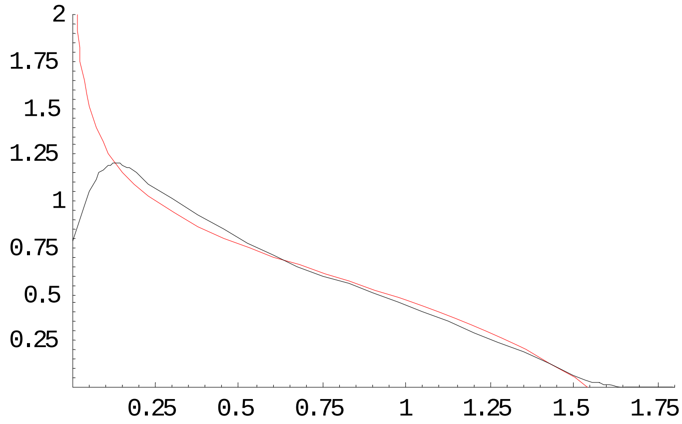

Figure 1. The theoretical and estimated densities of the signals

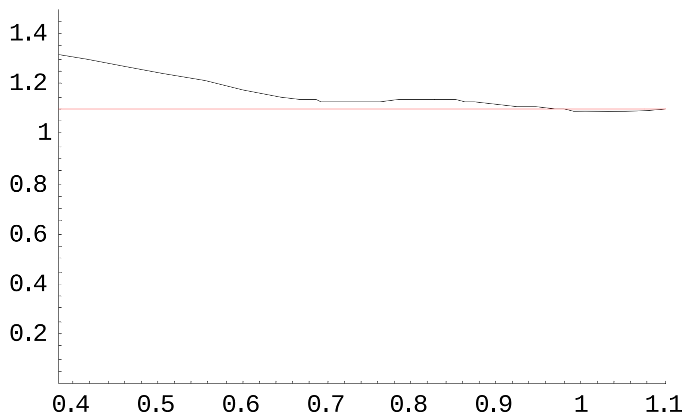

Figure 2. The theoretical and estimated conditional densities $f_{S|V}(.|S^{-1}(1.1))$

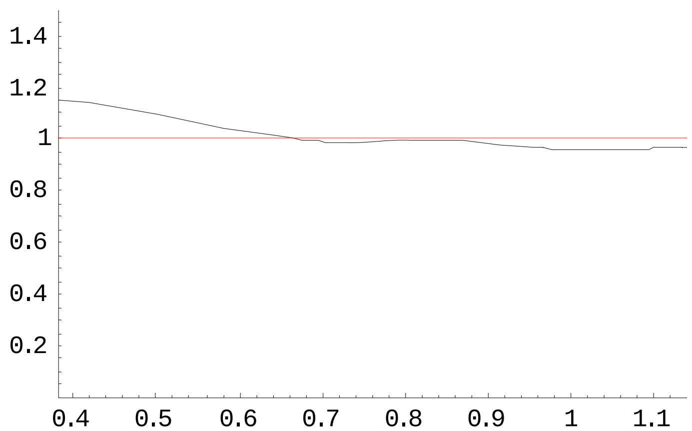

Figure 3. The theoretical and estimated function h(.)

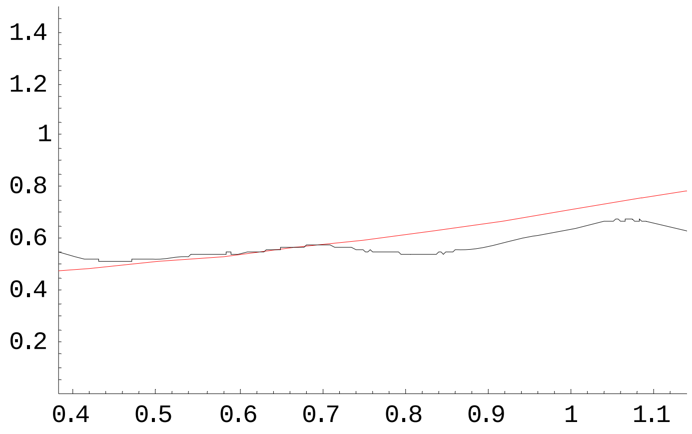

Figure 4. The theoretical and estimated function $f_V(S^{-1}(.))$

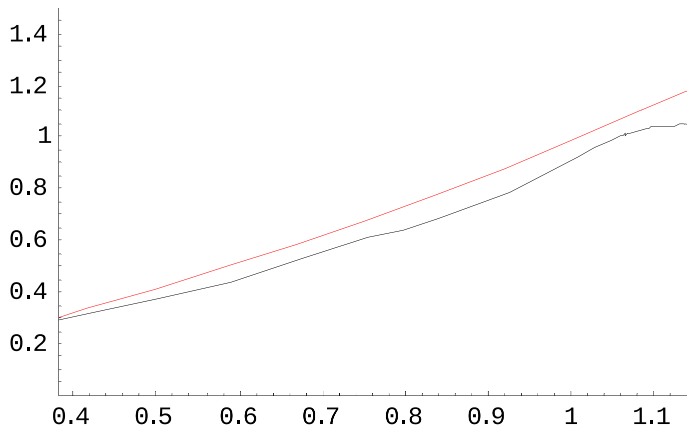

Figure 5. The theoretical and estimated function $S^{-1}(.)$