Reserve Prices in Repeated Multi-unit Auctions: Theory and Estimation

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February 2006

Abstract

The major challenge posed by structural estimation in multi–unit auctions is the need to develop predictive models of bidding. The goal of this paper is to estimate the distribution of bidders' valuations for a high-tech product and the seller's optimal reserve price using data collected from repeated online auctions. We construct a model of search with discounting to estimate the distribution of the bidders' marginal valuations in repeated multi-unit auctions with multi-unit demands. The model of search is used to predict the effect on bidding of imposing a stationary reserve price. The estimated gain in revenue from imposing the optimal reserve price is about 25% for a subsample of the auctions in our data.

*This is a revision of the second chapter of my dissertation "Need for Speed: Demand Estimation Using Auction Data." I thank Colin Campbell, Sandra Campo, Ronald Harstad, Roger Klein, Stanley Liebowitz, Martin K. Perry, Michael Rothkopf, Alejandro Zentner and two anonymous referees for their comments and suggestions.

1 Introduction

Multi-unit Internet auctions have gained increased popularity over the last decade. Internet auction sites like ebay.com, ubid.com and overstock.com auction off daily volumes of goods in the tens of millions of dollars. The number of listings on Internet auction sites appears to be increasing exponentially over time. In the second quarter of 2005, the number of listings on e-bay alone exceeded 440 million, a 32% year-to-year increase, while at the same time the quarterly volume of sales increased by 36% to $10.9 billion (see [6]). Auction formats seem to differ somewhat by auction site. However, many auctions share a common feature: they are repeated. A bidder who did not win in an auction of a particular object knows that he can bid in subsequent auctions. Thus, the trade-off between a sure surplus in today's auction and a possibly larger, but uncertain surplus in subsequent auctions will determine the bidders' behavior. This type of behavior is no different from the behavior of consumers who search for the best price in stores that post different–indeed, random from the buyer's perspective–prices. Optimal search behavior is characterized by a reservation price property: buy only if the observed price is less than an endogenously determined reservation price. We view the optimal bidder behavior in repeated auctions as characterized by a reservation bid property. Therefore, a simple decision rule for a bidder in a sequence of repeated auctions is to establish a reservation bid and to advance his bid in each auction up to his reservation bid.

The main goal of the present paper is to estimate the distribution of bidders' valuations for a high-tech product and the seller's optimal reserve price using data collected from repeated ascending online auctions with multi-unit bidder demands. While our model and estimation procedure concern a particular auction format (the so-called Yankee auction, an ascending auction form in which bids consist of a single price-quantity pair), the model and the estimation procedure can be adapted to other multi-unit auction formats. If, for instance, entire bidding schedules are observed in the data (i.e., if bidding is not "lumpy" as in the Yankee auction), a bidder's reservation bid schedule can be obtained from his marginal valuation schedule using models and estimation procedures similar to ours.

Much of the body of empirical literature that has focused recently on estimating product demand seeks to provide a measure of the consumers' value (e.g., demand elasticities) for a particular product. Our methodology of computing the distribution of marginal valuations allows estimation of average (per-bidder) demands and price elasticities for products with different combinations of characteristics.

The approach of this paper relies on the observation that the auctions that constitute our sample are repeated. Unlike most of the previous empirical studies of auctions, which view auctions as one-shot interactions between bidders, we model the decision problem of impatient bidders who participate in repeated auctions. We show how the stop-out price (roughly speaking, the minimum price that guarantees a win in an auction) plays an important role in the determination of the bids. We use the terms "stop-out price" and "market-clearing price" interchangeably; a more precise definition of the stop-out price is in the beginning of section 4. We endow our bidders with rational expectations about the distribution of stop-out prices and view them as choosing their bids to maximize expected surplus from bidding in a sequence of ascending auctions.

The auctioneer can affect–to some degree–the distribution of stop-out prices. Specifically, by imposing a reserve price the auctioneer is able to change both the shape and the support of the distribution of stop-out prices. In this paper we provide a measure of the extent to which control over the distribution of transaction prices results in a higher expected revenue for the seller. Similar to the case of one-shot single-unit auctions, imposing a reserve price in our setting involves a trade-off between increased revenue in the event that the objects are sold and a lower likelihood of selling the objects. The goal of most empirical studies of one-shot, single-unit auctions is to use the bid function predicted by theory to obtain the distribution of the bidders' valuations, which in turn can be used to compute the optimal reserve price. Establishing such a theoretical relationship in multi-unit settings is problematic. Indeed, our task of recovering valuations from the observed bids is greatly hampered by the relative dearth in the literature of models that deliver predictive relationships between bids and valuations in multi-object auctions with multi-unit demands. The major challenge that structural estimation poses to the researcher in multi-unit settings is the need to develop a theoretical model that delivers a relationship between valuations and bids that can be taken to the data.

The theoretical model developed in this paper establishes a simple functional relationship that can be taken to the data between the bidders' valuation and their reservation bids. The ascending nature of the auctions in our sample makes unobservable the actual reservation bids of the bidders. However, reservation bids can be bracketed using the observed data. If the reservation bids were observed (e.g., when the auction form is sealed-bid), our model can be taken directly to the data to estimate the distribution of the bidders valuations and the seller's optimal reserve price.

We view both the number of bidders and the number of objects in each auction as random and exogenous. The distributions of both variables can be estimated using our data. However, we take a more direct approach and use the distribution of the stop-out price as the primitive for the theoretical model and for the empirical specification.

We collected data from repeated auctions of personal computer processors, or central processing units – CPUs. While it is likely that only few readers will have a particular interest in estimating demand for CPUs, our analysis of repeated auctions adds more general value with the construction of a simple model of search that may be applicable in a variety of settings involving repeated auctions. Perhaps more importantly, the model developed in this paper can be used to assess the effect on bidder behavior of imposing a reserve price.

Finding the relationship between reservation bids and the distribution of the stop-out price in the presence of a reserve price is somewhat problematic.[1] With a reserve price both the reservation bids and the distribution of the stop-out price change relative to the no-reserve case. The key observation that we make is that the imposition of a reserve price does not affect the ordering of reservation bids and values. Accordingly, the minimum price that guarantees a win in an auction is equal to the reserve if the stop-out bidder has a lower valuation, and equal to the bid of the stop-out bidder if his valuation is higher than the reserve. We use the predictions of the search model to compute the distribution of the stop-out valuations, and use that distribution to compute the reservation bid function in a reserve price environment.

We collected data from auctions of CPUs because they were easily accessible online and to minimize the effect of object heterogeneity on our estimates. We use CPU characteristics, all of which are assumed to be observed, to infer the bidders' valuations. Our approach can be viewed as hedonic, although we do not assume a particular functional form for the bidders' demand functions, nor do we need to consider the bidders' choice problem. The distribution of the bidders' valuations is found by inverting the monotonic reservation bid functions implied by our model.

The estimation procedure developed in the present paper makes use of the predicted one-to-one relationship between bids and valuations to recover the distribution of bidders' valuations and to compute the optimal reserve price.

We turn now to provide a brief survey of the related literature and some background on theory

[1] We have some, well, reservations about terminology. However, to be consistent with the terms used in search and auction theory and with the hope that this is not confusing to the reader, we use "reservation bid" to mean the highest bid a bidder is willing to submit and "reserve price" to mean the lowest acceptable bid imposed by the actioneer.

and on estimation in multi-unit auctions.

2 Background and related literature

In general, bidders in an auction of $K \geq 1$ identical and indivisible objects are characterized by a K-tuple of marginal valuations of the form $V = (V^1, ..., V^K)$. Any equilibrium model of bidding would specify, for each bidder, a bidding function that maps their vector of valuations into bids $(b^1, b^2, ..., b^K)$. When auction rules require bidders to submit 'lumpy' bids[2], as in our CPU auctions, both the quantity demanded and the per-unit price are specified in equilibrium as functions of the vector of marginal valuations V. Equilibrium bidding models provide results mainly for the case $K = 1$. Obtaining tractable results in the general case when $K > 1$ appears to be a formidably difficult analytical task. One can, in fact, find in the literature very few theoretical results concerning multi-object auctions with multi-unit demands. We turn next to briefly survey some of the available theoretical results.

Harris and Raviv [11] investigate optimal multi-object allocation mechanisms with two bidders and a discrete, equidistant support of the bidders' valuations and prove that, in the limit as the distance between the points in the support of the distribution of valuations approaches zero, the expected revenue from the uniform and discriminatory auction is the same. With more than two bidders, a uniform distribution of valuations and unit demands, they prove expected revenue equivalence in the risk-neutral case and establish the expected revenue ranking of the two auction formats under risk aversion and risk neutrality. Maskin and Riley [17] show that revenue equivalence with unit demands continues to hold for more general distributions of the bidders' valuations when bidders are risk neutral. In addition, they show that with downward sloping demands, the optimal mechanism is the nonlinear pricing mechanism, modified to account for a fixed supply. Wilson's [26] seminal paper concerning auctions of shares gave rise to a new strain of literature that relies on the good auctioned being perfectly divisible. In our case, however, the goods offered for sale are not divisible and bidders may demand more than one object. A model that obtains bid functions for both discriminatory and uniform auctions using the assumption that bidders are price-taking is by Nautz [20]. Tenorio [25] investigates the revenue ranking of discriminatory and uniform auctions using a simple two-bidder example. Engelbrecht-Wiggans and Khan [8] give a characterization of the equilibria of an uniform auction in which bidders make two bids. In a more recent paper,

[2] That is, instead of submitting a quantity-price schedule, bidders are required to submit a price-quantity pair.

Lebrun and Tremblay [16] provide closed-form solutions for the multi-object discriminatory auction in the case of two bidders whose valuations are piecewise-constant.

The empirical body of literature that discusses private-values multi-unit auctions focuses in general on the effect of changing a particular auction format (i.e., discriminatory or uniform) on revenue. Tenorio [24], for instance, investigates revenue equivalence in the context of multi-unit auctions using Zambian foreign exchange auction data. In an empirical analysis of bids to supply electricity, Wolfram [27] investigates the bidders' incentive to increase their bids at high quantities in an uniform auction. Using a multi-unit, private-values auction model in which bidders (electricity generators) have perfect information about costs, the author's empirical results confirm the theoretical prediction that bidders bid larger markups for units that are being produced at higher marginal cost. Cantillon and Pesendorfer [3] analyze the issue of identification and estimation in multi-unit auctions with synergies, with an application to auctions of bus routes. Their paper focuses on estimating the welfare effects of package bidding and on evaluating the bidders' cost synergies.

The two papers that are closest to our approach are by Hortaçsu [12] and Heller and Lengwiler [13] and concern Treasury auctions.

In an empirical analysis of the Turkish treasury market, Hortaçsu estimates the bidders' marginal valuations and shows that the revenue difference between the actual discriminatory auction employed by the Turkish Treasury and a counterfactual uniform auction is not statistically significant. Unlike our ascending auctions with lumpy bids, Turkish treasury auctions are discriminatory and bidders are required to submit an entire bidding schedule. In his paper, Hortaçsu uses resampling techniques to estimate the distribution of the market clearing price; in contrast, our approach uses semiparametric methods. The estimation method of Hortaçsu allows counterfactual comparisons between various auction formats. Since bidding rules in the Turkish treasury auctions allow bidders to place bidding schedules consisting of different combinations of quantities and prices, the bidders' valuations are nonparametric identified. In our case, since we only observe a unique combination of quantity and per-unit price for each bidder, the bidders' marginal valuation schedules are non-parametrically unidentified. To achieve identification we have to impose parametric restrictions on the marginal valuations of bidders.

Heller and Lengwiler [13] look for the auction format that maximizes the revenue from the Swiss treasury auctions. Their method uses models of bidding that are based on the assumption that bidders are price-taking and allows evaluation of the revenue performance of discriminatory and uniform auctions.

As in this paper, the close relationship between repeated auctions and search is the underpinning of Genesove's [9] analysis of search at wholesale auto auctions. Instead of considering the search behavior of sellers across auctions, we consider here the search behavior of buyers. Search theoretic models have been used extensively in the context of labor markets (see e.g., McCall [18] and Mortensen [19]; for a recent survey see Rogerson and Wright [21]).

The empirical part of this paper belongs to a large and growing body of literature that investigates Internet auctions. For a survey of recent studies of Internet auctions see Bajari and Hortaçsu [2].

Estimating the effect of bidder- and auction-specific variables on valuations is a difficult task because available to the researcher are only the bids, and not the actual valuations of the bidders. For isolated single-unit auctions, the (Bayesian-Nash) equilibrium relationship between bids and valuations can be, in principle, easily derived. Establishing a similar theoretical relationship between valuations and bids in multi-object auctions with multi-unit demands, however, appears to have been, to this point, beyond our abilities. We render unnecessary such a task by making use of a search theoretic model.

Before presenting our theoretical model we give a short description of the auction and of the data.

3 Description of the auction

We collected our data from auctions that took place in January and February 2001, during a time interval of roughly 30 days. Data were collected from the U-bid.com site, an Internet business-to-person auction house. U-bid employs a discriminatory auction in which bidders are required to submit "lumpy" bids consisting of a per-unit price and a quantity demanded. The auction is ascending because the price component of a bid can be increased in integer multiples of the minimum bid increment. The quantity component of a bid, however, is set at the beginning. Bidders cannot place two bids in the same auction (i.e., one cannot bid for more than one quantity). Bidding in all auctions in our sample was open for 24 hours, but if there was any bidding activity in a 10 minute time interval after the 24-hour period, the auction entered a "going-going-gone" stage that ended as soon as no bid had been received for 10 minutes. Bidding starts at a low per-unit price ($9) and the auction is ascending. Recall that the stop-out (or market clearing) price is the lowest per-unit price that guarantees a win. The smallest entry (or re-entry) bid is a minimum bid increment over the standing stop-out price. Bids can be updated only in multiples of a given minimum increment (usually $10). The order of precedence is price over quantity over time. That is, if the per-unit bids are the same the bidder demanding the highest quantity has precedence, and if both the price and quantity submitted are equal, the bidder whose first bid was placed earlier wins. The auction is discriminatory, i.e., each winning bidder gets the number of objects demanded[3] and pays her last per-unit price multiplied by the number of objects that she wins. Presented below is a stylized example of the typical auction, in which five bidders compete for three identical objects. The minimum bid increment is $10 and the starting bid is $9.

Table I: Stylized example of an auction

Bidder	Bid [$/unit]	Quantity demanded	Bidder winning (No. objects)
1	9	3	1(3)
2	19	2	2(2), 1(1)
3	299	1	3(1), 2(2)
4	359	1	4(1), 3(1), 2(1)
5	469	2	5(2), 4(1)
3	649	1	3(1), 5(2)

In our example, bidder 3 wins one object and pays $649 and bidder 5 wins two objects and pays $469. About one third of the bids in our data are jump bids,[4] a finding similar to that of Easley and Tenorio [7]. In the example, all bids except bidder 2's bid are jump bids.

Our data consist of the bidding histories for 89 auctions. By accessing on average once every minute the information posted on the merchant's website we collected data about the per-unit price, the quantity demanded and the time at which bids were placed. In addition, we recorded the number of objects for sale, as well as the characteristics of the CPUs (clock speed, cache size, data bus speed and manufacturer). The only bidder-specific information available consisted of the bidders' initials, their city and state. The starting bid in all auctions is $9. We found some missing bids in 9 out of the 89 auctions in the initial data set. Accordingly, we use data from 89 auctions to estimate the distribution of the stop-out price and data from 80 auctions (with a total of 2302

[3] Since bids may be submitted for more than one unit, there may be quantity rationing at the last winning bid.

[4] A *ratchet* or pedestrian bidding strategy requires bidders to submit the minimum accepted bid whenever their previous per-unit price is smaller than the bids placed by the current winners. Conversely, a *jump* bidding strategy involves bidding multiples of the minimum bid increment above the lowest winning bid.

bidders) to estimate the valuations.

The variables, their description and summary statistics are presented in Table II. The top part presents processor characteristics, in the middle are bidder-related variables and the bottom part of the table concerns auction-specific variables. All processors are new and are covered by a three-year warranty.

Table II: Description of variables and summary statistics

Variable	**Description**	**Mean**	**St. dev**	Min	Max
SPEED	Processor clock frequency (MHz)	964.92	174.16	733	1400
SPD2	$\text{SPEED}^2/1000$				
CACHE	Cache size (Kbytes)	296.73	141.01	128	512
CAC2	$\text{CACHE}^2/1000$				
INTEL	Manufacturer dummy (Intel=1)	0.35			
MEM	Bundled memory dummy	0.06			
BUSSPD	Frequency of front size bus (MHz)	200.68	47.56	66	400
SERVER	Processor for servers dummy	.045			
BUTLER	Proxy bidding dummy	0.32			
q	Quantity demanded	2.39	5.27	1	57
WIN	Winning dummy	0.27			
BID	Last recorded bid	143.71	85.69	9	788
NOBJ	Number of objects in an auction	9.90	13.10	1	72
INCR	Minimum bid increment	11.02	2.17	10	20
NBDRS	Number of bidders in an auction	28.24	32.21	3	177

A brief discussion about the CPU characteristics is useful at this point. The speed of a CPU (or the frequency of its internal clock) is a key determinant of the number of elementary arithmetic or logic operations that can be performed by the processor in a given period of time. While CPU speed is perceived by most users as the most important characteristic of a processor, the overall performance of a system may be significantly affected by the speed of the data bus that connects the central processing unit with the system memory, hard drive and peripherals. In general, bus speeds are of an order of magnitude less than CPU speeds, so that for memory intensive processes (large scale simulations, or even the use of Internet browsers and word processors) CPU speed is less relevant than bus speed. To help deal with this issue, manufacturers place a relatively small amount of high-speed "cache" memory on-the-chip to improve the overall performance of a system. In principle, a computer's perceived speed of operation is increasing in both processor speed and cache size, but the amount of system memory and the architecture of the system are also very important in determining a system's overall performance.

We present next our model of the economic environment and its predictions.

4 The model

We consider an infinite sequence of ascending multi-unit auctions indexed by $i \geq 1$, with $k_i \in \mathcal{K} = \{1, ..., \bar{k}\}$ identical objects sold in each auction and with $n_i \in N = \{2, ..., \bar{n}\}$ risk-neutral bidders in each auction. We take both the number of bidders and the number of objects to be exogenous and stochastic. Bidders draw the parameters of their marginal valuation schedules before they participate in the sequence of auctions. The winning bidders in an auction either draw new valuations in the next period, or they never return to participate in an auction. The following assumptions summarize our view of the economic environment.

Assumption (CMV) *For all $i \geq 1$, bidder $j's$ ($j \in \{1, ..., n_i\}$) marginal valuations are equal to v_j (i.e., constant) for a number $q_j \geq 1$ of objects, and zero otherwise. Thus, a bidder's marginal valuation schedule is characterized by the pair of numbers (v_j, q_j).*

Since our data consist of "lumpy" bids (i.e., pairs consisting of a per-unit price and a quantity demanded), individual bidder marginal valuation schedules are non-parametrically unidentifiable. A large fraction of bidders in our sample of auctions (about 70%) demand only one object, so for these bidders the constant marginal valuation assumption will cause no harm. We believe that most of the bidders who demand more than one unit buy CPUs to build computers with the intent of sale. If this is the case, the marginal valuations of a multi-unit bidder for a fixed number of objects interpreted as "capacity" are not likely to differ much.

Assumption (I) *The pairs (v_j, q_j) are independent draws from a time-invariant distribution $F_{v,q}$: $[\underline{v}, \bar{v}] \times \mathcal{K} \rightarrow [0, 1]$, with $0 \leq \underline{v} < \bar{v} < \infty$. A bid is a pair (b, q^*) consisting of a per-unit price and a quantity demanded.*

Since we assume that bidders have constant marginal valuations, a bidder will submit a bid whose quantity component q^* is the minimum between q, his quantity demanded, and k_i, the supply of objects in auction i. To simplify the discussion, we assign bidder indices in decreasing order of their valuations: $v_{ji} \geq v_{j+1,i}$ for all $j \in \{1, ..., n_i - 1\}$.

Definition 1 *The index ι_i of the stop-out bidder in auction i satisfies*

$$\iota_i = 1 + \min \left[t \in \{1, ..., n_i\} | \sum_{j=1}^{t} q_{ji} \geq k_i \right]. \tag{1}$$

We will assume, as it is the case in our data, that ι_i is well-defined for all i. We define the stop-out price as the highest non-winning bid in an auction. Since the supply in each auction is fixed, the last winner's quantity may be rationed; our definition of the stop-out price accounts for this fact.

Assumption (RP) *The behavior of bidders is governed by reservation price search, i.e., a bidder with value v will advance her bid up to a value $b(v)$ which we term that bidder's* reservation bid. *The bidders' common inter-period discount factor is $\delta \in (0, 1)$. Bidding in each auction is costless.*

Assumption (RP) implies that the last observed bid of the highest non-winning bidder (bidder ι_i) will be equal to his reservation bid. We therefore define the stop-out price s_i in auction i as the reservation bid of the highest non-winning bidder: $s_i = b(v_{\iota_i})$.

Assumption (SO) *The c.d.f. of stop-out prices F_s is continuous and strictly increasing on its support $[\underline{v}, \bar{s}]$. Stop-out prices in each auction are independent draws from F_s. The bidders in our sequence of auctions have rational expectations about the distribution of the stop-out price.*

Assumption (SO) is made to simplify our theoretical model and to allow the design of a feasible estimation strategy.[5] Since bidders can only advance their bids in integer multiples of the minimum bid increment, stop-out prices in our auctions have discrete support. However, since the minimum bid increment is small relative to the sale price of a CPU, the assumption is fairly mild. We assume that each period the new bidders draw their valuation parameters independently from the same distribution $F_{v,q}$. In each particular auction the bidders' valuations are i.i.d., so observing that the stop-out value is an order statistic (of random rank) of the i.i.d. sample of valuations and that the stop-out price is the reservation bid function evaluated at the stop-out value, stop-out prices over auctions are independent draws.

[5] It is possible to construct a model of search with stop-out prices that have discrete support. Moreover, estimation of the dependence between the stop-out price and auction characteristics is also possible. However, since the number of realizations of any particular stop-out price is relatively small in our data, the estimation of such a model of search is problematic.

Since the results of previously closed auctions were available online at the time our data were collected, it is reasonable to assume that bidders know the distribution of the stop-out price. The assumption that bidding is costless is justified by the observation that proxy (automatic) bidding was available as an option to all bidders. We observe that a bidder will win if her reservation bid is greater than the stop-out price in any particular auction.

The following proposition characterizes the relationship between a bidder's reservation bid and her value.

Proposition 1 *The reservation bid function $b(v)$ satisfies*

$$v = b(v) + \Delta \int_{\underline{v}}^{b(v)} F_s(p)\, dp, \tag{2}$$

where $\Delta = \frac{\delta}{1-\delta} > 0$.

Proof. A bidder chooses her reservation bid b to satisfy

$$v - b = \delta \int_{\underline{v}}^{b} (v - s)\, dF_s(s) + \delta (v - b)(1 - F_s(b)); \tag{3}$$

note that the value of search (surplus per object) is equal to $v - b$. If the next period stop-out price s is less than b, then the bidder will win in the auction and his per-unit expected payoff is $E_s[v - s|s \leq b]$. If the next-period stop-out price is greater than b, the bidder will continue to search. The expected surplus from continuing to search next period is equal to the right hand side of (3). Integrating by parts and collecting terms yields (2). ■

Proposition 2 *The function $b(\cdot)$ is strictly increasing, satisfies $b(v) \leq v$ (with strict inequality if $v > \underline{v}$), and it is strictly concave.*

Proof. To show that $b(\cdot)$ is strictly increasing choose $v_1 > v_2 \geq \underline{v}$ and suppose that $b(v_1) \leq b(v_2)$. Since for $b(v_1) \leq b(v_2)$ $\Delta \int_{\underline{v}}^{b(v_1)} F_s(p)\, dp \leq \Delta \int_{\underline{v}}^{b(v_2)} F_s(p)\, dp$, adding up the two inequalities yields $v_2 \geq v_1$, a contradiction, so $b(\cdot)$ is strictly increasing. Observe that $b(\underline{v}) = \underline{v}$ (if not, the equality in (2) is not satisfied). We wish to show next that $b(v) < v$ whenever $v > \underline{v}$. Choose $v > \underline{v}$ and suppose $b(v_1) \geq v_1$. It follows that $\int_{\underline{v}}^{b(v_1)} F_s(p)\, dp \leq 0$. Since $F_s(\cdot)$ satisfies $F_s(p) = (>)0$ when $p = (>)\underline{v}$ and since $b(v_1) > \underline{v}$, $\int_{\underline{v}}^{b(v_1)} F_s(p)\, dp > 0$, a contradiction, so $b(v) < v$ whenever $v > \underline{v}$. Note that $b(\cdot)$ is bounded and strictly increasing, so it is differentiable almost everywhere.

Observing that

$$b'(v) = 1/(1 + \Delta F_s(b(v))), \tag{4}$$

since both $b(\cdot)$ and $F_s(\cdot)$ are strictly increasing, $b'(v_2) > b'(v_1)$ whenever $v_1 > v_2$, so $b(\cdot)$ is strictly concave. ■

Since $b(\cdot)$ is strictly increasing, it is invertible and thus for x in the range of $b(\cdot)$, (2) implies that

$$b^{-1}(x) = x + \Delta \int_{\underline{v}}^{x} F_s(p)\, dp. \tag{5}$$

Since the last observed bid of the highest non-winning bidder is equal to his reservation bid, it follows that $F_s(x) = \Pr[b(v_{\iota_i}) \leq x] = F_{v_\iota}\left(b^{-1}(x)\right)$, where F_{v_ι} is the distribution of the stop-out value that corresponds to the stop-out price. Since stop-out prices are observed and $b(v)$ can be computed using (2), F_{v_ι} can be estimated using our data. We turn next to investigate the effect on bidding of imposing a reserve price.

4.1 Reserve prices

The auctions in our sample do not have a reserve price (minimum acceptable bid). While it is true that the starting bid is \$9 in all auctions, it is hard to believe that the starting bid could be construed as a binding reserve price. The lowest sale price for any of the objects sold in our sample is about seven times larger than the starting bid.

Nevertheless, the seller may have different reserve price policies. We focus here on the implication on bidding of imposing a stationary reserve price (i.e., a reserve price that does not depend on the number of objects for sale or the number of bidders). We note that, in principle, the reserve price could depend on the number of objects in an auction. The seller could also condition the reserve price on the number of bidders, if it is known (e.g., in sealed-bid auctions with pre-qualified bidders). In our auctions the seller cannot condition the reserve price on the number of bidders, for the number of bidders is revealed with certainty only at the end of the auction. We maintain the assumption that the number of objects for sale in each auction is exogenous and random. Hence, it remains a possibility that the seller condition the reserve price on the realized number of objects for sale in an auction. If so, one would need to impose (most likely not innocuous) assumptions about the effect of imposing quantity-dependent reserve prices on the number of bidders in an auction. Rather than following this route, we analyze the effect of imposing a stationary reserve price, without making any claim about the optimality of imposing quantity-invariant reserve prices. We leave for future research the answer to the question of whether or not such a reserve price policy is optimal.

To evaluate the effect of imposing a reserve price r on the bidders' search strategy, note first that both the distribution of the stop-out price and the function that maps valuations into reservation bids become functions of r. In what follows, we denote by $F_s(\cdot, r)$ the distribution of the stop-out price, by $b_r(v) = b(v, r)$ the reservation bid function. We start by assuming that $b_r(r) = r$ and that $b_r(\cdot)$ is strictly increasing, so that $b_r^{-1}(\cdot)$, its inverse with respect to v, is well defined. Imposing a reserve price adds a mass point (at r) to the density of the stop-out prices. To see why, note that if v_ι is greater than r, the stop-out price is equal to $b_r(v_\iota)$. If v_ι is lower than r, the stop out price is equal to r. Thus, the density of stop-out prices will have a mass $F_{v_\iota}(r)$ at r, and the probability that the stop-out price is less than some value $p \geq r$, $F_s(p, r)$, is equal to $F_{v_\iota}\left(b_r^{-1}(p)\right)$.

According to the definition in the last section, the stop-out price is truncated, not censored. However, the relevant question for a bidder is what is the minimum price that guarantees winning in an auction. When $v_\iota < r$, regardless of whether or not a trade occurs, that minimum price is equal to r. Without a reserve price, that minimum price is equal to the stop-out price. With a reserve price policy in effect, we use the term stop-out price to mean the lowest price that guarantees winning in an auction. The following proposition characterizes the reservation bid function in a reserve price environment.

Proposition 3 *Let $r \in (\underline{v}, \bar{v})$. The reservation bid function $b_r(v)$ satisfies*

$$b_r(v) = \begin{cases} r + \int_r^v \frac{dx}{1+\Delta F_{v_\iota}(x)} \text{ if } v \geq r \\ < r \text{ otherwise.} \end{cases} \tag{6}$$

Proof. A bidder with value $v \geq r$ chooses her reservation bid $b_r(v)$ so that

$$\begin{aligned} v - b_r(v) &= \delta \left(\begin{array}{c} E\left[(v-s)\,|s=r\right]\Pr\left[s=r\right] + E\left[(v-s)\,|s \in (r, b_r(v)]\right]\Pr\left[s \in (r, b_r(v)\right] \\ +\left(v - b_r(v)\right)\Pr\left[s > b_r(v)\right] \end{array} \right) \\ &= \delta \left((v-r) F_{v_\iota}(r) + \int_r^{b_r(v)} (v-s)\, dF_{v_\iota}\left(b_r^{-1}(s)\right) + \left(v - b_r(v)\right)\left(1 - F_{v_\iota}(v)\right) \right) \\ &= \delta \left(\int_r^{b_r(v)} F_{v_\iota}\left(b_r^{-1}(s)\right) ds + \left(v - b_r(v)\right) \right) \end{aligned}$$

it follows that $b_r(v)$ satisfies

$$v = b_r(v) + \Delta \int_r^{b_r(v)} F_{v_\iota}\left(b_r^{-1}(s)\right) ds. \tag{7}$$

To find the reservation bid function, differentiate (7) with respect to v to yield

$$1 = b_r'(v)\left(1 + \Delta F_{v_\iota}(v)\right); \tag{8}$$

integrating with initial condition $b_r(r) = r$ yields (6). Note that $b_r(\cdot)$ is strictly increasing, as assumed. ■

The next proposition characterizes some properties of the reservation bid function $b_r(\cdot)$.

Proposition 4 *For $r \in (\underline{v}, \bar{v})$ and $v \geq r$, the following hold:*

(i) $b_r(v) > b(v)$,

(ii) $\frac{\partial b_r(v)}{\partial r} > 0$ and

(iii) $b_r'(v) = b'(v)$.

Proof. Fix $r \in (\underline{v}, \bar{v})$ and choose $v \geq r$. Observe that $b_r(v) - b(v) = r + \int_r^v \frac{dx}{1+\Delta F_{v_\iota}(x)} - \int_{\underline{v}}^v \frac{dx}{1+\Delta F_{v_\iota}(x)} = r - b(r)$, which is greater than zero by proposition 2. Thus, with a reserve price, the reservation bid function is a vertical shift of the reservation bid function without a reserve price, with the difference between the two functions independent of v. Since $b(\cdot)$ is concave and strictly increasing, and since $b(\underline{v}) = \underline{v}$ and $b(v) < v$ for $v > \underline{v}$ (by proposition 2), $r - b(r)$ is strictly increasing in r, so (ii) follows. Since $b_r(v) - b(v)$ is not a function of v, $b_r'(v) = b'(v)$. ■

Note that the difference between the functions $b_r(v)$ and $b(v)$ is equal to the per-unit surplus of a winner with value r in the no-reserve auction. Having computed the bid function with a reserve price, we compute next the expected revenue in the auction with a reserve price r.

5 Estimation and results

Our estimation procedure consists of two steps. First, we estimate, using bidder- and auction-specific covariates, the bidders' reservation bids and quantities demanded. We do this by observing that the non-winners' reservation bids are less than the minimum bid increment over the stop-out price, while the winners' reservation bids are greater than their last observed bids. Thus, for each observation, we bracket the reservation bids using the observed stop-out price and the last observed bids. A bidder's quantity demanded may be censored, since the supply of objects in an auction is fixed and that bidder's "capacity" q_j may be less than the number of objects supplied.

In the second step we use a distribution-free procedure to estimate the distribution of the stop-out price. Since the discount factor is not known, we design a procedure to estimate the discount factor.

We conclude this section by providing the estimation results.

5.1 Estimation of the reservation bids and quantities

The theoretical relationship between reservation bids and valuations established above is based on the assumption that bidders are not constrained in their choice of per-unit prices and quantities. The first step in estimating the valuations is to estimate the reservation bids and quantities demanded. Because of the ascending nature of the auction, winning bidders have reservation bids that are greater than or equal to their last observed bid, whereas the reservation bids of non-winning bidders can be up to at least one bid increment greater than their last recorded bid and less than the sum between the stop-out price and the minimum bid increment.

We take the parametric relationships between the reservation bids b^* and the covariates Z_b (containing bidder-, auction- and object-specific variables) to be of the form $b^* = \epsilon_b g(Z_b)$. Since bids must be positive, a natural specification is to let $g(Z_b) = \exp(Z_b\beta_b)$ and to assume that the multiplicative error term ϵ_b is i.i.d. log-normal. Taking logs, the relationship between the reservation bids and the vector of covariates Z_b becomes $\ln b^* = Z_b\beta_b + \varepsilon_b$,where ε_b is i.i.d. normal and independent of the regressors.

Some bidders may demand more than the number of objects offered for sale in any particular auction. Therefore, bidders who have quantities demanded that are equal to the number of objects in a given auction may wish to purchase more objects than what we observe in the data. We assume that quantities demanded are governed by the latent variable $\hat{q} = Z_q\beta_q + \varepsilon_q$, where Z_q is a matrix of covariates and ε_q is i.i.d. normal and independent of Z_q. In addition, quantities demanded are $q^* = q$ if $\hat{q} \in (q - 0.5, q + 0.5]$, for $q > 1$, and $q^* = 1$ if $\hat{q} \leq \hat{q}_c = 1.5$. By employing this specification we are able to accommodate the large proportion – about 70% – of observations with unit demands. Note that this specification sets the cutoff for unitary demands $\hat{q}_c$

at 1.5 without loss of generality, and that observations from single-unit auctions (less than 100 data points) do not provide any information about the "true" (or uncensored) quantities demanded. The estimation method outlined above cannot jointly identify $\hat{q}_c$ and of the constant in $Z_q\beta_q$; however, our procedure requires that only their sum be identified.

It has been argued (see for instance Rothkopf and Harstad [22], Avery [1] and Daniel and Hirshleifer [5]), that in ascending auctions bidders have an incentive to jump the bid to signal that they are willing to pay a high price. Rothkopf and Harstad [22] show that pedestrian bidding (i.e., the strategy of advancing a competitor's bid by the minimum bid increment) is an equilibrium with two bidders. With affiliated values, Avery [1] shows that jump bids provide a possibility of an ex-ante Pareto improvement for bidders over the outcome of the ascending auction with no jump bidding. In the data collected from various Internet auctions by Easley and Tenorio [7], more than one third of the bids are jump bids. We take into account jump bidding in our specification by bracketing the reservation bid of a non-winning bidder between his last observed bid and the sum between the stop-out price and the minimum bid increment. In the case of an observed winning bid b, the corresponding reservation bid b^* can be placed with certainty in the interval $[b, \infty)$. Our estimation procedure takes into account the fact that values of the bid less than the starting bid ($9) are not possible.

Assuming that the error terms in the reduced-form bid and quantity equations are jointly normally distributed with covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_b^2 & \rho\sigma_b\sigma_q \\ \rho\sigma_b\sigma_q & \sigma_q^2 \end{pmatrix}$$

the likelihood contribution of observation i, depending on whether the observed bid is winning and whether the quantity observed is equal to the supply of objects (censored) can be summarized as follows.

- Non-winning bid ($w_i = 0$), non-censored quantity ($c_i = 0$). Denoting by $\phi^*(x, y; \Sigma)$ the density of a bivariate normal distribution centered in the origin, truncated to the left of $y = \ln(9) - Z_b\beta_b$ and with correlation matrix Σ and by p_i the stop-out price in auction i, the contribution to the likelihood of observation i is

$$L^i_{0,0} = \int_{\ln p_i - \beta_b' Z_{bi}}^{\ln(p_i+\iota_i)-\beta_b' Z_{bi}} \int_{\underline{q}_i - \beta_q' Z_{qi}}^{\bar{q}_i - \beta_q' Z_{qi}} \phi^*(x, y; \Sigma) dx\ dy.$$

- Non-winning bid ($w_i = 0$), censored quantity ($c_i = 1$)

$$L^i_{0,1} = \int_{\ln p_i - \beta_b' Z_{bi}}^{\ln(p_i+\iota_i)-\beta_b' Z_{bi}} \int_{\underline{q}_i - \beta_q' Z_{qi}}^{\infty} \phi^*(x, y; \Sigma) dx\ dy.$$

- Winning bid ($w_i = 1$), censored quantity ($c_i = 1$)

$$L^i_{0,1} = \int_{\ln p_i - \beta_b' Z_{bi}}^{\infty} \int_{\underline{q}_i - \beta_q' Z_{qi}}^{\infty} \phi^*(x, y; \Sigma) dx\ dy.$$

- Winning bid ($w_i = 1$), non-censored quantity ($c_i = 0$)

$$L^i_{0,1} = \int_{\ln p_i - \beta_b' Z_{bi}}^{\infty} \int_{\underline{q}_i - \beta_q' Z_{qi}}^{\bar{q}_i - \beta_q' Z_{qi}} \phi^*(x, y; \Sigma) dx\ dy,$$

where $(\underline{q}_i, \bar{q}_i) = (q_i - 0.5, q_i + .5)$ if $q_i > 1$ and $(\underline{q}_i, \bar{q}_i) = (-\infty, 1.5)$ if $q_i = 1$.

The log-likelihood contribution of observation i is therefore:

$$\begin{aligned} \ln L^i &= (1 - w_i)(1 - c_i) \ln L^i_{0,0} + (1 - w_i) c_i \ln L^i_{0,1} \\ &\quad + w_i c_i \ln L^i_{1,1} + w_i (1 - c_i) \ln L^i_{1,0}. \end{aligned} \tag{9}$$

The maximum likelihood estimates are below.[6]

[6] An alternative specification in which we placed the maximum bid of a non-winning bidder between his last observed bid and the next bid on the grid has a considerably reduced goodness of fit. If quantities and bids are estimated using separate equations involving a negative binomial specification for quantities (modified to account for a large proportion of observations with unit demands) and a log-normal specification for bids, the results are qualitatively similar, but the goodness of fit is diminished. The estimates in Table III are scaled by the standard deviation of the error terms.

Table III: Maximum likelihood estimates

	Price		Quantity	
Parameter	**Estimate**	**Std. dev.**	**Estimate**	**Std. dev.**
Constant	-78.4168	3.0202	25.7679	5.3566
NAUCT	-0.1216	0.0119	0.0890	0.0319
NAUCT2	0.0023	0.0003	-0.0091	0.0014
log(SPEED)	39.9220	1.2085	-6.9527	2.0402
ln(CACHE)	-0.9847	0.1483	-0.9814	0.1807
NOBJ	0.0037	0.0025		
INTEL	3.5361	0.1561	1.0701	0.6038
BUTLER	-0.5578	0.0624	-0.2472	0.1406
BUSSPD	-0.0055	0.0008	0.0142	0.0054
INCR	-0.0350	0.0237	0.0281	0.0787
σ	0.1520	0.0036	1.0290	0.0603
$N = 2302$	$\rho = -0.3869$ (0.0778)		mean $\ln L : -1.9076$	

As expected, higher processor speeds generate–ceteris paribus–higher bids. Somewhat surprisingly, the opposite is true about the size of a processor's cache memory and about bus speed. Our conjecture is that most bidders in our auctions purchase processors to build computers with the intention of selling them. While both bus speed and cache size are important determinants of the perceived speed of a computer, most computer buyers care only about processor speed. Moreover, higher bus speeds call for higher speed memory that is considerably more expensive. The negative sign on bus speed and cache size is an indication of a lower demand of bidders (and buyers of their computers) for processors with higher bus speeds or larger cache sizes. Our results indicate that bidders are willing to pay considerably more, all other things equal, for CPUs produced by one of the manufacturers. The number of objects for sale does not appear to significantly affect bids – a finding consistent with search behavior. The use of proxy bidding is associated on average with lower bids and lower quantities demanded. The size of the bid increment does not appear to significantly affect the bids. Quantities demanded appear to be increasing, albeit at a decreasing rate, in the number of auctions in our sample in which a bidder placed bids.[7] Higher speeds correspond on average to higher quantities demanded, and the opposite holds for the size of cache memory. Finally, we note that the estimated correlation between the error terms in the price and quantity

[7] The available data concerning the bidders' identities is their initials, city and state. We match – possibly with some noise – the bidders across auctions using the information available and use the resulting number as an independent variable in regression. The mean of variable NAUCT is 4.65, its standard deviation is 8.27, the minimum is 1 and maximum is 43.

equations is negative and significant.

We present next the estimation procedure and results for the distribution of the stop-out price.

5.2 The distribution of the stop-out price

The theoretical model developed in the previous section looks at the decision problem of a bidder who participates in a sequence of auctions of identical objects. In principle, it is possible to estimate, either semi- or non-parametrically, the discrete distribution of the stop-out price for any type of CPU, but the number of observations for any particular type of CPU in our data is too small to achieve any meaningful results. By pooling observations across different types of objects and by assuming that the distribution of the stop-out price has continuous support we render feasible the estimation of the reserve price. The relatively small size (compared to the stop-out price) of the minimum bid increment suggests that the gains from using a continuous support – materialized in a much cleaner formulation of the theoretical model and empirical strategy relative to the discrete case – greatly outweigh any potential loss. The assumption is relatively innocuous: a minimum bid increment of $10 (which is the increment in most of the auctions that we consider) represents about 1.5% of the price of a high speed processor. We use in estimation the following index assumption that reduces substantially the number of data points necessary to reliably estimate the distribution of the stop-out price.

Assumption (SIM) *The vector of characteristics Z_s affects the stop-out price through a linear combination of the form $Z_s\beta_s$, where β_s is a vector of parameters that need to be estimated.*

The following discussion draws on Ichimura [14]. Suppose that the stop-out price s in auction l is given by $s_l = \varphi(Z_{sl}) + \varepsilon_{sl}$, for all $l = \overline{1, L}$,where $\varphi(\cdot)$ is an unknown link function and ε_l are zero mean i.i.d. disturbances distributed according to a distribution function $F_\varepsilon(\cdot)$. Assumption (SIM) implies that $\varphi(Z_{sl}) = \varphi(Z_{sl}\beta_s)$ and in what follows we use the two members of the equality interchangeably. We also assume that the error term and the characteristics Z_s are orthogonal and that the error term is homoscedastic.

Fixing β_s, the expectation of the stop-out price conditional on the index $Z_s\beta_s$ can be computed using semi- or non-parametric methods (see for instance Silverman [23]). We denote by $\hat{E}\left[s_l|Z_s\beta_s\right]$ the semiparametric estimate of the expectation of the stop-out price conditional on the index $Z_s\beta_s$.

With the conditional expectation of the stop-out price in hand, β_s can be estimated as the solution to the following "non-linear least squares" problem:

$$\hat{\beta}_s \in \arg\min_{\beta_s} \sum_{l=1}^{L} \left(s_l - \hat{E}\left[s_l | Z_s \beta_s\right] \right)^2 . \tag{10}$$

The solution (unique under some mild conditions imposed on the conditional expectation) enables the computation of an estimate of the mean shift $\varphi(\cdot)$. To see how, observe that

$$\hat{\varphi}(Z_s \hat{\beta}_s) = \hat{E}[s | Z_s \hat{\beta}_s]. \tag{11}$$

The semiparametric least squares problem amounts to choosing the value of coefficients β_s so that a distance between the actual realizations of the stop-out price and their expectations is minimized. The problem is solved sequentially: for given values of β_s the conditional expectation of the stop-out price can be computed using kernel methods, and the result is fed into a non-linear least squares estimation procedure.

For an n-dimensional sample of i.i.d. random draws $\{X_i\}_{i=\overline{1,n}}$, the kernel estimator of the density f is defined by:

$$\hat{f}(x) = \frac{1}{nh} \sum_{l=1}^{n} K\left(\frac{x - X_l}{h} \right), \tag{12}$$

where $K(\cdot)$ is a kernel function. We use a normal kernel to compute the conditional expectations with the "rule-of-thumb" bandwidth suggested by Silverman for an underlying normal distribution (see [23], p. 45). We present the SLS estimates, up to an unknown scale (in our case, the coefficient associated with the size of the CPU cache memory), in Table IV.

Table IV - SLS estimates

Parameter	SLS estimate	Std. dev.
SPEED	2.9508	0.8915
BUSSPD	-1.7027	1.0214
INTEL	645.6596	185.1149
MEM	-350.4327	232.7220
SERVER	394.2365	211.1889

Since the sign of the coefficient associated with the amount of cache memory should clearly be positive, the true signs of the above coefficients are unchanged (note that the scale–and hence the sign–of the coefficients is not identified in the SLS procedure). Notable is that the stop-out price of an Intel processor is higher on average than the stop-out price of an AMD processor with the same characteristics. As expected, the stop-out price increases, on average, with the speed of the processor. Surprisingly, higher bus speeds, which are expected to improve considerably the performance of a PC, give rise on average to lower stop-out prices. Since higher bus speeds require more expensive high speed memory, the result may be indicate that consumers are mostly concerned with the speed of the processor; we note, however, that the coefficient associated with bus speed is only borderline significant at the 10% confidence level.

Figure 1 presents graphically the estimation results. We were concerned about the accuracy of results obtained with this type of procedure when the sample size is relatively small, like in our case. Results of Monte Carlo simulations (not reported here) suggests that the SLS method provides good estimates of the mean-shifting function and of the distribution of the error term for sample sizes comparable to ours (89 observations).

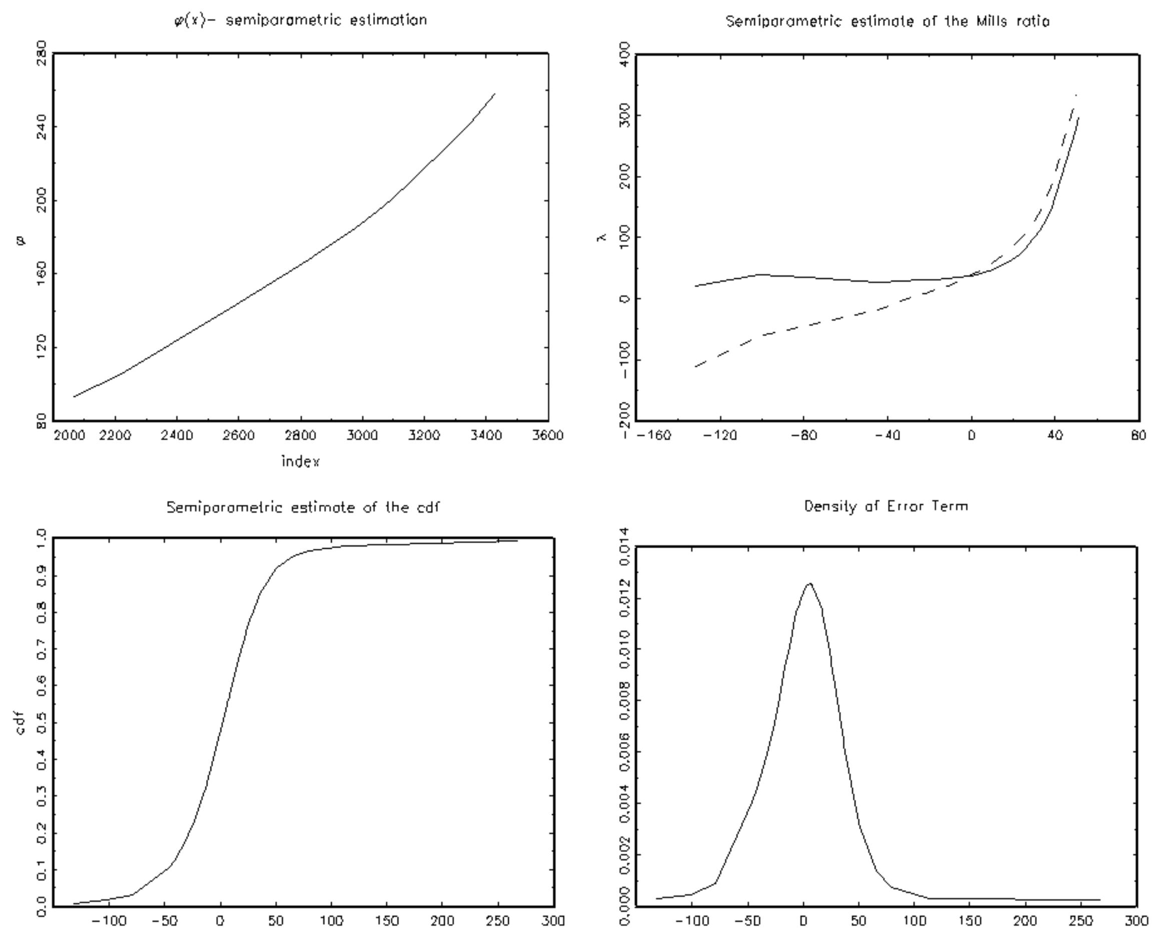

Figure 1: SLS estimation results.

The stop-out price appears to be determined in two different regimes. For low values of the index $Z_s\beta_s$, the mean stop-out price appears to increase linearly with a slope that is lower than the slope at which it increases at higher values of the index. This finding suggests that mean stop-out prices are proportionately higher for newer types of processors (i.e., processors with higher clock frequencies or cache sizes) than for relatively older types.

With estimates of the distribution of ε and of the mean-shift function $\varphi(\cdot)$ available, the conditional c.d.f. of stop-out prices can be computed using:

$$\hat{F}_s(b|Z_s) = \hat{F}_\varepsilon\left(b - \hat{\varphi}(Z_s\hat{\beta}_s)\right). \tag{13}$$

Note that, while the SLS procedure yields estimates of the parameters only up to an unknown location and scale, the index $Z_s\beta_s$ is identified.

5.3 Estimation of values and of the optimal reserve price

We are now ready to assemble the components needed to estimate the distribution of bidders' values and the optimal reserve price. We begin by making an observation about identification, then use the results of the theoretical model to construct our estimates.

5.3.1 Identification

If the discount factor δ were known, equation (5) would provide a straightforward way to express a bidder's constant marginal valuation as a function of his bid. It is useful to note that an estimation procedure that enables joint estimation of valuations and of the discount factor is not feasible. To see why, observe that equation (2) implies that a bid b can be rationalized by any pairs $(v_i, \Delta_i)_{i=\overline{1,2}}$ that satisfy $(v_1 - b)/(v_2 - b) = \Delta_1/\Delta_2)$.

Unlike the models of Guerre, Perrigne and Vuong [10] in the case of single-unit auctions, or Hortaçsu's [12] for multi-unit, discriminatory auctions, which are non-parametrically identified, our model is not. The fact that we only observe lumpy bids implies that, even though valuation are assumed to be private and independent, identification cannot be achieved without making additional assumptions about the functional form of the bidders' marginal valuations schedules.

Our identification problem resembles the problem faced by Campo, Guerre, Perrigne and Vuong [4] in jointly identifying the bidders' von Neumann-Morgenstern utility functions and the distribution of their valuations in first-price auctions.

We suggest next a way to get around the identification problem that concerns the discount factor, then use the estimated discount factor to compute the bidders' demand functions.

5.3.2 The discount factor

Since valuations and the discount factor are not identified, we produce an estimate of the discount factor at the expense of assuming that the log-marginal valuations for a particular type of processor are independently and identically normally distributed, with unknown mean μ and variance σ^2. The following is inspired by an observation of Klein and Sherman [15]. Since reservation bids are increasing in valuation (see proposition 2), the ordering of the bidders' marginal valuations and reservation bids is preserved. It turns out that, by exploiting the assumed distribution of valuations, the function that links reservation bids and values can be identified up to location and scale. The model developed in the previous section implies that, for a fixed discount factor $\delta_0 \in [0,1)$, letting $\Delta_0 = \frac{\delta_0}{1-\delta_0}$, the log of the inverse reservation bid function $M(b;\Delta_0)$ is given by

$$\ln M(b;\Delta_0) = \ln\left(b + \Delta_0 \int_{\underline{s}}^{b} F_s(p)dp\right), \tag{14}$$

where $\underline{s}$ is the lower boundary of the support of stop-out prices. Denoting by $\mathbb{P}(b)$ the empirical distribution of the reservation bids, the log of the inverse reservation bid function $b^{-1}(v)$ can be estimated (up to location and scale, i.e., up to μ and σ) by using:

$$\ln b^{-1}(b) = \mu + \sigma\Phi^{-1}(\mathbb{P}(b)), \tag{15}$$

where $\Phi(\cdot)$ is the standard normal c.d.f. It follows that, up to location and scale, the log of the inverse reservation bid function can be estimated by

$$\ln \hat{M}\,(b;\Delta_0) = \mu + \sigma\Phi^{-1}(\mathbb{P}(b)). \tag{16}$$

Note that (14) implies that $M_1(\underline{s};\Delta_0) = 1$ and that $M(\underline{s};\Delta_0) = \underline{s}$. We use these two observations to fix the location and scale of (16). To minimize the noise that may result from the estimation of the distribution of the stop-out price we use observations that correspond to bids in auctions of 950MHz AMD processors in which the number of auctioned objects is equal to 24 – a total of 87 observations. For each observation, we simulate the bids and the integral of the cumulative of the stop-out price using the parameter estimates computed above. We repeat the simulation 1,000 times and smooth out (16) at $\underline{s}$ to fix the scale and location parameters. An estimate of Δ_0 is then computed by minimizing the distance between the left- and right-hand sides of (16). The least squares point estimate is $\hat{\Delta} = 25.1101$ and the corresponding discount factor is $\hat{\delta} = 0.9617$. The estimate does not change significantly when we use different types of processors or different numbers of objects. We note that the estimated discount factor is significantly smaller than expected. We can speculate that its value is affected by the bidders being uncertain about when and whether an auction will be offered in the future, as well as by the bidders' high rates of time preference. The smooth of the bid function implied by the estimated discount factor is in Figure 2 (values are on the horizontal axis and bids on the vertical axis).

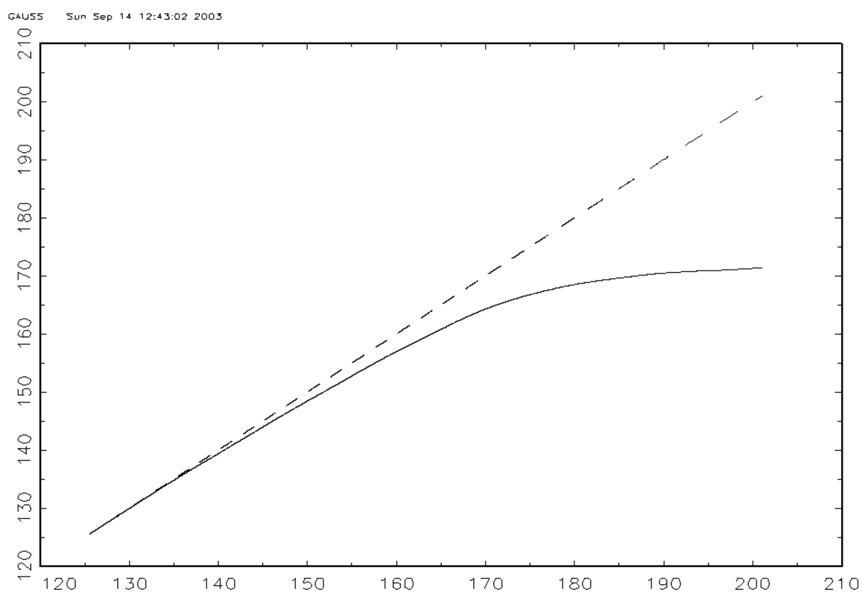

Figure 2: Smooth of bid function – search model

We compute next average demands using a simulation exercise. Specifically, we simulate the bids and the integral of the cumulative of the stop-out price using parameter estimates computed in the previous sections. A number of 5,000 replications is used to compute average demands. In Figure 3 we plot the average demand for an Athlon 950 MHz processor obtained using the predictions of the search model. Note that demand elasticities of the actual demand and average demand curves are the same. While we do not believe that auction markets of the kind studied here are representative for the entire PC processor market, auctions of other types of goods (timber, for instance) allocate a large fraction of supply; in that case, demands computed in a similar way as here could provide fairly accurate estimates of the price elasticity in the population.

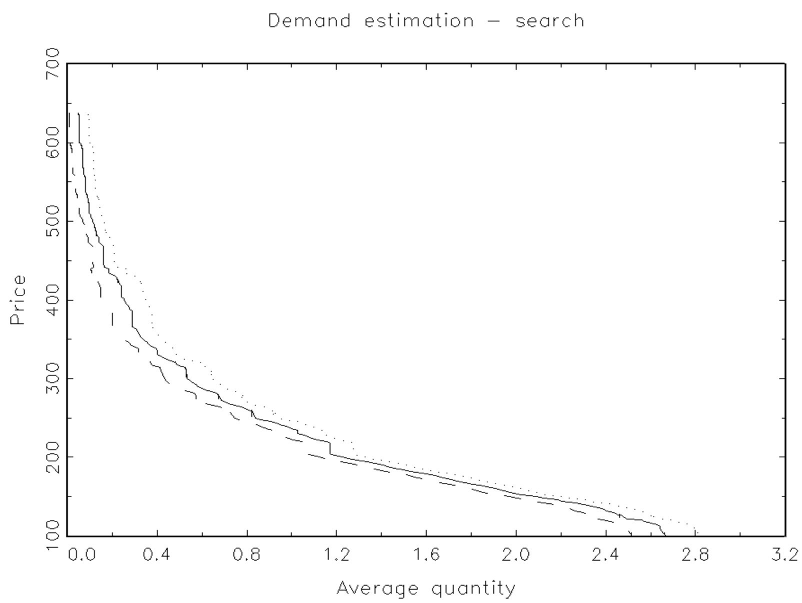

Figure 3: Average demand

5.4 The optimal stationary reserve price

A reserve price will increase the reservation bids of all bidders whose values are above the reserve price, but imposing a reserve price will decrease the likelihood that the objects are sold. To evaluate the trade-off between higher revenue in the event in which the objects are sold and a lower likelihood of selling the objects we use simulations. Specifically, we first use the data from our auctions and the coefficients estimated in section 5.1 to simulate the bidders' reservation bids. We use the data concerning auctions of the AMD950 CPUs (a total of 1011 final observed bids from 10 auctions) and replicate the process 1,000 times.

We then vary the reserve price r from \$100 to \$700 in increments of \$1 and compute the reservation bids in the auction with a reserve price r according to

$$\begin{aligned} \hat{b}_r(v) &= \hat{b}\,(v) + r - \hat{b}(r) \\ &= \hat{b}\,(v) + \hat{\Delta} \int_{\underline{s}}^{\hat{b}(r)} \hat{F}_s(p)dp, \end{aligned}$$

so that

$$\hat{b}_r(v) = \hat{b}\,(v) + \hat{\Delta} \int_{\underline{s}}^{\hat{b}(r)} \hat{F}_\varepsilon \left(p - \hat{\varphi}(Z_s \hat{\beta}_s)\right) dp. \tag{17}$$

Computing the new reservation bids is extremely fast, since the only computation needed is evaluating the integral in (17) once for any value of the reserve price.

Using the new reservation bids, we find the winners in each auction and the revenue that corresponds to a particular reserve price. We ignore jump bidding by assuming that all winning bidders pay the stop-out price; since jump bidding actually increases revenue, our simulations yield a lower bound for expected revenue.

Figure 4 depicts the relationship between per-unit revenue and the reserve price. Expected revenue is maximized by setting the reserve price approximately equal to \$306. The dashed horizontal line represents the per-unit revenue in the actual data (with no reserve price), equal to \$146.99. The per-unit revenue with reserve price dips below the horizontal line that represents revenue per unit with no reserve because our simulations do not take into account the possibility of jump bidding.

According to our simulations the expected gain per-unit of supply from using the optimal stationary reserve price is \$37.78. Therefore, using the optimal stationary reserve price results in an increase in per-unit revenue of about 25.7%. The increase in per-unit revenue due to the reserve price is quite substantial; in fact, as a result of jump bidding, the actual increase in per-unit revenue could be larger than 25%.

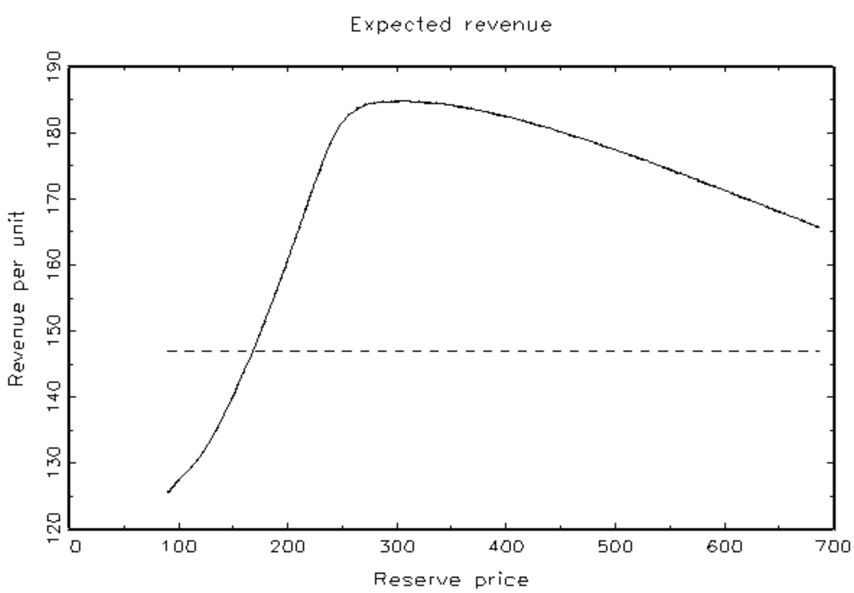

Figure 4: Reserve prices and per-unit revenue

While the number of bids in the data that underlie our simulations is relatively large, the observed number of auctions (10) concerning this particular type of processor (AMD Athlon 950

MHz) is relatively small. The small number of auctions in this sample captures the variation in the number of objects for sale or in the number of bidders with some noise. We take the results of the simulations as being indicative of significant revenue gains that could be achieved as a result of imposing a reserve price. However, the simulated revenue gain of 25% seems somewhat high and may, in part, be due to our data being a "lucky" draw from the sequence of number of bidders and number of objects for sale.

6 Conclusions

Equilibrium models of bidding in multi-unit auctions with multi-unit demands are notoriously difficult to obtain. In fact, equilibrium bid functions in multi-unit settings are only available for highly specialized auction formats, e.g., when bidders demand only one unit. In this paper we constructed a model of search that applies to bidding in ascending multi-unit auctions with multi-unit demands. Together with a suitable estimation procedure, our model enables estimation of the distribution of the bidders' valuations for a high-tech product. More importantly, the model can be used to evaluate the effect on revenue of imposing a stationary reserve price.

Our model relies on the assumption that bidders know the distribution of the stop-out price. Given the institutional details of the auctions that we investigate, this is a fairly realistic assumption. Since estimation results can be significantly affected by assuming a parametric distribution of the stop-out price, we estimate the distribution of the stop-out price using a semi-parametric method.

The auctions that we investigate are ascending; therefore, in general, the bid data that are available to the observer do not reveal the bidders' reservation bids precisely. However, by setting appropriate bounds that are based on the bidders' last recorded bids, we estimate the statistical relationship between bidder- and auction-specific characteristics and the bidders' reservation bids. In turn, this statistical relationship is used to evaluate the effect of imposing a reserve price on expected revenue.

Our main goals are the recovery of the demand schedules implied by the bidders' marginal valuations, as well as the computation of the optimal stationary reservation price. The methodology developed in the present paper may be applied to a variety of settings, e.g., repeated sealed-bid auctions. With some changes, the theoretical model developed in this paper applies to auctions in which bidders are allowed to place bids for more than one quantity.

The simulations in this paper show that by imposing a reserve price the auctioneer can increase the revenue per unit by 25%–a surprisingly large increase. Some part of this increase may be due to our data not capturing enough variation in the number of objects for sale and in the number of bidders.

Auction participants may constitute, in general, a non-randomly selected subset of CPU buyers. In environments in which the assumption is justified that auction participants are representative for the entire population of consumers, drawing from the estimated distribution of valuations could enable the researcher to infer the market demand for an object with a particular set of characteristics. Thus, questions related to the welfare effect of the introduction of new products or to the positioning of new products in the space of characteristics could be answered. Even when the sample of online auction bidders is not representative for the population of consumers, our methodology delivers empirical bid functions that can be useful for the design of multi-unit auctions.

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