# Non-Exclusive Competition under Adverse Selection*

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**Abstract**

Consider a seller of a divisible good, facing several identical buyers. The quality of the good may be low or high, and is the seller's private information. The seller has strictly convex preferences that satisfy a single-crossing property. Buyers compete by posting arbitrary menus of contracts. Competition is non-exclusive in that the seller can simultaneously and secretly trade with several buyers. We fully characterize conditions for the existence of an equilibrium. Equilibrium aggregate allocations are unique. Any traded contract must yield zero profit. If a quality is indeed traded, then it is traded efficiently. Depending on parameters, both qualities may be traded, or only one of them, or the market may break down completely to a no-trade equilibrium.

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# 1 Introduction

The recent financial crisis has spectacularly recalled that the liquidity of financial markets cannot be taken for granted, even for markets that attract many traders and on which exchanged volumes are usually very high. For instance, the issuance of asset-backed securities declined from over 300 billion dollars in 2007 to only a few billion in 2009.[1] Indeed, structured financial products such as mortgage-backed securities, collateralized debt obligations, and credit default swaps, often involve many different underlying assets, and their designers clearly have more information about their quality; this may create an adverse selection problem and reduce liquidity provision.[2] Similarly, the interbank market experienced a severe liquidity dry-up over the 2007–2009 period, with many banks choosing to keep their liquidity idle instead of lending it even at short maturities.[3] One interpretation of this behavior is that banks became increasingly uncertain about their counterparties' exposure to risky securities.[4] There is also evidence that lending standards and the intensity of screening have been progressively deteriorating with the expansion of the securitization industry in the pre–2007 years.[5] Overall, many attempts at interpreting the recent crisis put at the center stage the difficulties raised by a lack of information on the quality of securities, or on the net position of counterparties. Notice also that most of these securities were traded outside of organized exchanges on over-the-counter markets, with poor information on the trading volume or on the net position of traders. Hence agents were able to interact secretly with multiple partners, at the expense of information release.[6]

What economic theory tells us about the impact of adverse selection on competitive outcomes has mainly been developed in the context of two alternative paradigms. Akerlof (1970) studies an economy where privately informed sellers and uninformed buyers act as price-takers. All trades are assumed to take place at the same price. Competitive equilibria typically exist, and feature a form of market failure: because the market clearing price must be equal to the average quality of the goods that are offered by sellers, the highest qualities are generally not traded in equilibrium. It seems therefore natural to investigate whether such a drastic outcome can be avoided by allowing buyers to screen the different qualities of

[1]See Adrian and Shin (2010).

[2]See Gorton (2009) and Krishnamurthy (2009).

[3]Brunnermeier (2009) provides some evidence for the liquidity squeeze in the interbank market. In the case of the sterling money markets, Acharya and Merrouche (2009) document an almost permanent 30 percent upward shift in banks' liquidity buffers starting from August 2007.

[4]See Heider, Hoerova, and Holthausen (2009), Philippon and Skreta (2010), and Taylor and Williams (2009).

[5]See Demyanyk and van Hemert (2009), and Keys, Mukherjee, Seru, and Vig (2010).

[6]See Acharya and Bisin (2010).

the goods. In this spirit, Rothschild and Stiglitz (1976) consider a strategic model in which buyers offer to trade different quantities at different unit prices, thereby allowing sellers to credibly communicate their private information. They show that high quality sellers end up trading a suboptimal, but nonzero quantity, while low quality sellers trade efficiently: for instance, in the context of insurance markets, high-risk agents are fully insured, while low-risk agents only obtain partial coverage. An equilibrium does not exist, however, if the proportion of high quality sellers is too high.

The present paper revisits these classical approaches by relaxing the assumption of exclusive competition, which states that each seller is allowed to trade with at most one buyer. This assumption plays a central role in Rothschild and Stiglitz's (1976) model, and it is also satisfied in the simplest versions of Akerlof's (1970) model, since sellers can only trade zero or one unit of an indivisible good. However, situations where sellers can simultaneously and secretly trade with several buyers naturally arise on many markets—one may even say that non-exclusivity is the rule rather than the exception. In addition to the contexts we have already mentioned, standard examples include the European banking industry, the US credit card market, and the life insurance and annuity markets of several OECD countries.[7] The structure of annuity markets is of particular interest since some legislations explicitly rule out the possibility to design exclusive contracts: for instance, on September 1, 2002, the UK Financial Services Authority ruled in favor of the consumers' right to purchase annuities from suppliers other than their current pension provider (Open Market Option).

Our aim is to study the impact of adverse selection in markets with such non-exclusive trading relationships. To do so, we allow for non-exclusive trading in a generalized version of Rothschild and Stiglitz's (1976) model. This exercise is interesting per se: as we shall see, the reasonings that lead to the characterization of equilibria are quite different from those put forward by these authors. The results are also different: the equilibria we construct typically involve linear pricing, possibly with a bid-ask spread, and trading is efficient whenever it occurs. On the other hand, equilibria may fail to exist, as in Rothschild and Stiglitz (1976), and some types may be excluded from trade, as in Akerlof (1970). It might even be that the only equilibrium is a no-trade equilibrium. The variety of these outcomes may help to better understand how financial markets react to informational asymmetries.

[7]Ongena and Smith (2000) and Detragiache, Garella and Guiso (2000) document that multiple banking relationships have become very widespread in Europe. Rysman (2007) provides recent evidence of multi-homing in the US credit card industry, while Cawley and Philipson (1999) and Finkelstein and Poterba (2004) report similar findings for the US life insurance market and the UK annuity market, respectively.

Our analysis builds on the following simple model of trade. There is a finite number of buyers, who compete for a divisible good offered by a single seller. The seller is privately informed of the quality of the good, which can be either low or high. The seller's preferences are strictly convex, but otherwise arbitrary, provided they satisfy a single-crossing property. Buyers compete by simultaneously posting menus of contracts, where a contract specifies both a quantity and a transfer. After observing the menus offered, and taking into account her private information, or type, the seller chooses which contracts to trade. Our model encompasses pure trade and insurance environments as special cases.[8]

In this context, we provide a full characterization of the seller's aggregate trades in any pure strategy equilibrium. First, we provide a necessary and sufficient condition for such an equilibrium to exist. This condition can be stated as follows: let $v$ be the average quality of the good. Then, a pure strategy equilibrium exists if and only if, at the no-trade point, the low quality type would be willing to *sell* a small quantity of the good at price $v$, while the high quality type would be willing to *buy* a small quantity of the good at price $v$. Second, we show that the aggregate equilibrium allocations are unique. Any contract traded in equilibrium yields zero profit, so that there are no cross-subsidies across types. In addition, if the willingness to trade at the no trade-point varies enough across types, equilibria are first-best efficient: the low quality type *sells* the efficient quantity, while the high quality type *buys* the efficient quantity. By contrast, if the two type have similar willingness to trade at the no-trade point, any equilibrium involves no trade. Finally, in intermediate cases, one type of the seller trades efficiently, while the other type does not trade at all.

These results suggest that, under non-exclusivity, the seller may only signal her type through the sign of the quantity she proposes to trade with a buyer. This is however a very rough signalling device, and it is only effective when one type acts as a seller, while the other one acts as a buyer. In particular, there is no equilibrium in which both types of the seller trade non-trivial quantities on the same side of the market. Finally, equilibrium allocations can be supported by simple menu offers. For instance, if only the low quality seller is actively trading in equilibrium, the corresponding allocation can be supported by having all buyers offering to purchase any nonnegative quantity at a unit price equal to the low quality. Overall, these findings suggest that non-exclusive competition exacerbates the adverse selection problem: if the first best cannot be achieved, a nonzero level of trades for

[8] The labels *seller* and *buyers* are only used for expositional purposes. Since offered contracts may well involve negative quantities, both the buyers and the seller can end up trading on any side of the market. In financial markets, a buyer trades a negative quantity when he is selling assets short. Similarly, one can think of insurance companies as buying risk from risk-averse agents who sell their risk for insurance purposes.

one type of the seller can be sustained in equilibrium only if the other type of the seller is left out of the market. That is, the market breakdown originally conjectured by Akerlof (1970) also arises when buyers compete in arbitrary non-exclusive menu offers. In financial markets, the buyers' fear that a seller's willingness to trade essentially reflects her need of getting rid of low quality assets leads to a low provision of liquidity. In the annuity market, consumers with a higher life expectancy will typically not annuitize their retirement savings.[9]

From a methodological standpoint, the analysis of non-exclusive competition under adverse selection gives rise to interesting strategic insights. On the one hand, each buyer can build on his competitors' offers by proposing additional trades that are attractive to the seller. Thus new deviations become available to the buyers compared to the exclusive competition case. On the other hand, the fact that competition is non-exclusive also implies that each buyer gets access to a rich set of devices to block such deviations and discipline his competitors. In particular, he can issue latent contracts, that is, contracts that are not traded by the seller on the equilibrium path, but which she finds it profitable to trade in case a buyer deviates from equilibrium play, so as to punish this deviating buyer. Such latent contracts are in particular useful to deter cream-skimming deviations designed to attract one specific type of the seller.

Formally, the best response of any single buyer could in principle be determined by looking at a situation where he would act as a monopsonist, facing a seller whose preferences would be represented by an indirect utility function depending on the profile of menus offered by his competitors. However, because we impose very little structure on the menus that can be offered by the buyers, we cannot assume from the outset that this indirect utility function satisfies useful properties such as, for instance, a single-crossing condition. Moreover, we do not assume that, if the seller has multiple best responses in the continuation game, she necessarily chooses one that is best from the deviator's viewpoint. This rules out using standard mechanism design techniques to characterize each buyer's best response, as Biais, Martimort, and Rochet (2000) do.

To develop our characterization, we consider instead a series of deviations by a single buyer who designs his own menu offer in such a way that a specific type of the seller will select a particular contract from this menu, along with some other contracts offered by the other buyers. We refer to this technique as *pivoting*, as the deviating buyer makes strategic

[9]Several recent attempts have been made at solving the puzzle of why only a small fraction of individuals purchase life annuities, despite their welfare enhancing role underlined in much of the economic literature (see Brown (2007) for an extensive discussion). To the best of our knowledge, the non-exclusive feature of competition in the annuity market has never been emphasized as a potential source of its breakdown.

use of his competitor's offer to propose attractive trades to the seller. Consider, as an example, the equilibrium allocation characterized by Rothschild and Stiglitz (1976), where the low-risk agent purchases less than full coverage to signal her quality, while the high-risk agent obtains full coverage. Our analysis shows that this allocation cannot be supported in equilibrium when competition is non-exclusive. The intuition for this result can be provided in the context of a free-entry equilibrium. Indeed, an entrant can earn a positive profit by offering the high-risk agent to purchase an additional quantity of insurance on top of what the low-risk type is trading in equilibrium; the corresponding transfer can be chosen in such a way that the high-risk agent will accept the deviating contract. While this intuition has already been suggested by Jaynes (1978), our paper generalizes this pivoting technique to get a full characterization of the set of equilibrium aggregate trades.

**Related Literature** The implications of non-exclusive competition have been extensively studied in moral hazard contexts. Following the seminal contributions of Hellwig (1983) and Arnott and Stiglitz (1993), many recent works emphasize that, in financial markets where agents can take some non contractible effort, the impossibility of enforcing exclusive contracts can induce positive profits for financial intermediaries and a reduction in trades. Positive profits arise at equilibrium since none of the intermediaries can profitably deviate without inducing the agents to trade several contracts and select inefficient levels of effort.[10] The present paper rules out moral hazard effects and argues that non-exclusive competition under adverse selection drives intermediaries' profits to zero.

The analysis of adverse selection has been initiated by Pauly (1974), Jaynes (1978) and Hellwig (1988). Pauly (1974) suggests that Akerlof outcomes can be supported at equilibrium in a situation where buyers are restricted to offer linear price schedules. As recalled above, Jaynes (1978) points out that the separating equilibrium characterized by Rothschild and Stiglitz (1976) is vulnerable to entry by an insurance company proposing additional trades that could be concealed from the other companies. He further argues that the non-existence problem identified by Rothschild and Stiglitz (1976) can be overcome if insurance companies can share the information they have about the agents' trades. Hellwig (1988) discusses the relevant extensive form for the inter-firm communication game.

Biais et al. (2000) study a model of non-exclusive competition among uninformed market-makers who supply liquidity to an informed insider whose preferences are quasilinear, and quadratic in the quantities she trades. Although our model encompasses their specification

[10] See for instance Parlour and Rajan (2001), Bisin and Guaitoli (2004), and Attar and Chassagnon (2009) for applications to loan and insurance markets.

of preferences, we develop our analysis in the two-type case, while Biais et al. (2000) consider a continuum of types. Despite the similarities between the two setups, however, the results of Biais et al. (2000) stand in sharp contrast with ours. Indeed, restricting attention to equilibria where market-makers post convex price schedules, they argue that non-exclusivity may lead to a Cournot-like equilibrium outcome, in which each market-maker earns a positive profit. This is very different from our Bertrand-like equilibrium outcomes, in which each traded contract yields zero profit for each buyer.

Attar, Mariotti, and Salanié (2009) consider a situation where a seller is endowed with one unit of a good whose quality she privately knows. This good is divisible, so that the seller may trade any quantity of it with any of the buyers, as long as she does not trade more than her endowment in the aggregate. Both the buyers' and the seller's preferences are linear in quantities and transfers. In this setting, Attar et al. (2009) show that pure strategy equilibria always exist, and that the corresponding aggregate allocations are generically unique. Depending on whether quality is low or high, and on the probability with which quality is high, the seller may either trade her whole endowment, or abstain from trading altogether. Buyers earn zero profit in any equilibrium. These results therefore offer a fully strategic foundation for Akerlof's (1970) classic study of the market for lemons, based on non-exclusive competition. Besides equilibrium existence, a key difference with our setting is that equilibria in Attar et al. (2009) may exhibit pooling and hence cross-subsidies across types. This reflects that, unlike in the present paper, trades are subject to an aggregate capacity constraint.

Ales and Maziero (2009) study non-exclusive competition in an insurance context similar to the one studied by Rothschild and Stiglitz (1976). Relying on free-entry arguments, they show that only the high-risk agent can obtain a positive coverage in equilibrium. This result is in line with those derived in the present paper, where free entry is not assumed from the outset. Our model is also more general than theirs in that we do not rely on a particular parametric representation of the seller's preferences, which allows us to uncover the common logical structure of a large class of potential applications. Finally, a distinctive feature of our analysis is that we fully characterize the set of aggregate allocations that can be supported in a pure strategy equilibrium, and that we provide necessary and sufficient conditions for the existence of such an equilibrium.

The paper is organized as follows. Section 2 describes the model. Section 3 characterizes pure strategy equilibria. Section 4 derives necessary and sufficient conditions under which such equilibria exist. Section 5 concludes.

# 2 The Model

Our model features a seller, who can simultaneously trade with several identical buyers. We put restrictions neither on the sign of the quantities of the good traded by the seller, nor on the sign of the transfers she receives in return. The labels *seller* and *buyers*, while useful, are therefore conventional.

## 2.1 The Seller

The seller is privately informed of her preferences. She may be of two types, $L$ or $H$, with positive probabilities $m_L$ and $m_H$ such that $m_L + m_H = 1$. Subscripts $i$ and $j$ are used to index these types, with the convention that $i \neq j$. When type $i$ trades an aggregate quantity $Q$, for which she receives in exchange an aggregate transfer $T$, her utility is $u_i(Q, T)$, where the function $u_i$ is strictly increasing in its second argument. The following regularity and convexity assumption will be useful at some point of the analysis.

**Assumption C** *For each $i$, the function $u_i$ is continuously differentiable and strictly quasi-concave in $(Q, T)$.*

Under Assumption C, each type's indifference curves are strictly convex. Moreover, for each $i$, the marginal rate of substitution

$$\tau_i(Q, T) \equiv -\frac{\frac{\partial u_i}{\partial Q}}{\frac{\partial u_i}{\partial T}}(Q, T)$$

is well defined and strictly increasing along type $i$'s indifference curves. Note that $\tau_i(Q, T)$ can be interpreted as the seller's marginal cost of supplying a higher quantity, given that she already trades $(Q, T)$. The following assumption is key to our results.

**Assumption SC** *For each $(Q, T)$, $\tau_H(Q, T) > \tau_L(Q, T)$.*

Assumption SC expresses a standard single-crossing condition: type $H$ is less eager to sell a higher quantity than type $L$ is. As a result, in the $(Q, T)$ plane, a type $H$ indifference curve crosses a type $L$ indifference curve only once, from below.

## 2.2 The Buyers

There are $n \geq 2$ identical buyers. If a buyer receives from type $i$ a quantity $q$ and makes a transfer $t$ in return, he obtains a profit $v_i q - t$. The following assumption will be maintained throughout the analysis.

**Assumption CV** $v_H > v_L$.

We let $v = m_L v_L + m_H v_H$ be the average quality of the good, so that $v_H > v > v_L$. Assumption CV reflects common values: the seller's type has a direct impact on the buyers' profits. Together with Assumption SC, Assumption CV captures a fundamental tradeoff of our model: type $H$ provides a more valuable good to the buyers than type $L$, but at a higher marginal cost. These assumptions are natural if we interpret the seller's type as the quality of the good she offers. Together, they create a tension that will be exploited later on: Assumption SC leads type $H$ to offer less of the good, but Assumption CV would induce buyers to demand more of the good offered by type $H$, if only they could observe quality.

## 2.3 The Non-Exclusive Trading Game

As in Biais et al. (2000), and Attar et al. (2009), trading is non-exclusive in that no buyer can control, and a fortiori contract on, the trades that the seller makes with his competitors. Buyers compete in menus for the good offered by the seller.[11] The timing of our trading game is thus as follows:

1. Each buyer $k$ proposes a menu of contracts, that is, a set $C^k \subset \mathbb{R}^2$ of quantity-transfer pairs that contains at least the no-trade contract $(0, 0)$.[12]

2. After privately learning her type, the seller selects one contract from each of the menus $C^k$'s offered by the buyers.

A pure strategy for type $i$ is a function that maps each menu profile $(C^1, \ldots, C^n)$ into a vector of contracts $((q^1, t^1), \ldots, (q^n, t^n)) \in C^1 \times \ldots \times C^n$. To ensure that type $i$'s problem

$$\max \left\{ u_i \left( \sum_k q^k, \sum_k t^k \right) : (q^k, t^k) \in C^k \text{ for all } k \right\}$$

has a solution for any menu profile $(C^1, \ldots, C^n)$, we suppose hereafter that the buyers' menus are compact sets. This allows us to use perfect Bayesian equilibrium as our equilibrium concept. Throughout the paper, we focus on pure strategy equilibria.

## 2.4 Applications

The following examples illustrate the range of our model.

[11] As shown by Peters (2001), and Martimort and Stole (2002), there is no need to consider more general mechanisms in this multiple-principal single-agent setting.

[12] The assumption that each menu must contain the no-trade contract allows one to deal with participation in a simple way: the seller cannot be forced to trade with any particular buyer.

### 2.4.1 Pure Trade

In the pure trade model, the seller's utility is quasilinear:

$$u_i(Q, T) = T - c_i(Q).$$

Assumption C is satisfied if the cost $c_i(Q)$ of delivering quantity $Q$ is strictly convex in $Q$. Assumption SC requires that $c'_H(Q) > c'_L(Q)$ for all $Q$. For instance, Biais et al. (2000) consider a parametric version of the pure trade model in which the cost function $c_i$ is quadratic, $c_i(Q) = \theta_i Q + \frac{\gamma}{2} Q^2$, for some positive constant $\gamma$.[13] Assumption SC then reduces to $\theta_H > \theta_L$. Coupled with the assumption $v_H > v_L$, this implies that a good of higher quality is more valuable, but has a higher marginal cost. Biais et al. (2000) also assume that $v_H - \theta_H < v_L - \theta_L$, which implies that the first-best quantities are implementable, a situation sometimes called *responsiveness* in the literature.[14] Our analysis does not rely on such an assumption. Finally, it should be noted that Attar et al. (2009) study a version of the pure trade model in which the seller's utility is linear in transfers and quantities, and a capacity constraint is imposed, in the form of an upper bound on aggregate quantities traded. As we shall see, the existence of this capacity constraint is the key difference between their model and the present one.

### 2.4.2 Insurance

In the insurance model, an agent can sell a risk to several insurance companies. As in Rothschild and Stiglitz (1976), the agent faces a binomial risk on her wealth, that can take two values $(W_G, W_B)$, with probabilities $(\pi_i, 1 - \pi_i)$ that define her type. Here $W_G - W_B$ is the positive monetary loss that the agent incurs in the bad state. A contract specifies a reimbursement $r$ to be paid in the bad state, and an insurance premium $p$. Let $R$ be the sum of the reimbursements, and let $P$ be the sum of the insurance premia. We assume that the agent's preferences have an expected utility representation

$$\pi_i u(W_G - P) + (1 - \pi_i) u(W_B - P + R),$$

where $u$ is a strictly concave von Neumann and Morgenstern utility function. The profit of an insurance company from selling the contract $(r, p)$ to type $i$ is $p - (1 - \pi_i) r$, which can be written as $v_i q - t$ if we set

$$v_i \equiv -(1 - \pi_i), \qquad q \equiv r, \qquad t \equiv -p,$$

[13] In Biais et al. (2000), the informed party is a buyer, but this difference with our model is just a matter of convention.

[14] See, in a different context, Caillaud, Guesnerie, Rey, and Tirole (1988).

so that $Q = R$ and $T = -P$. Hence the agent purchases for a transfer $-T$ a reimbursement $Q$ in the bad state, and her expected utility now writes as

$$u_i(Q, T) = \pi_i u(W_G + T) + (1 - \pi_i)u(W_B + Q + T).$$

Assumption C holds when the function $u$ is strictly concave and differentiable. In that case

$$\tau_i(Q, T) = -\frac{1}{1 + \frac{\pi_i}{1-\pi_i}\frac{u'(W_G+T)}{u'(W_B+T+Q)}},$$

so that Assumption SC requires that type $H$ has a lower probability of incurring a loss, $\pi_H > \pi_L$. Finally, we indeed have $v_H > v_L$, so that Assumption CV holds. Therefore our model encompasses the non-exclusive version of the Rothschild and Stiglitz's (1976) model considered by Ales and Maziero (2009); note that we could also allow for non-expected utility in the modeling of the agent's preferences.

# 3 Equilibrium Characterization

## 3.1 Preliminaries and Notation

An equilibrium specifies aggregate trades $(Q_i, T_i) = \left(\sum_k q_i^k, \sum_k t_i^k\right)$ for each type of the seller. It follows from Assumption SC that $Q_H \leq Q_L$ and $T_H \leq T_L$. We denote type by type individual and aggregate buyers' profits by

$$b_i^k = v_i q_i^k - t_i^k, \qquad B_i = \sum_k b_i^k,$$

respectively, and type averaged individual and aggregate buyers' profits by

$$b^k = m_L b_L^k + m_H b_H^k, \qquad B = \sum_k b^k,$$

respectively. Observe that we can also write

$$b^k = (vq_j^k - t_j^k) + m_i[v_i(q_i^k - q_j^k) - (t_i^k - t_j^k)].$$

The first term on the right-hand side of this expression is the profit from trading $(q_j^k, t_j^k)$ with both types, while the second term is the profit from further trading $(q_i^k - q_j^k, t_i^k - t_j^k)$ with type $i$ only, or, equivalently, the loss in buyer $k$'s profit from trading $(q_j^k, t_j^k)$ instead of $(q_i^k, t_i^k)$ with type $i$. For subsequent use, let us denote this quantity by

$$s_i^k = v_i(q_i^k - q_j^k) - (t_i^k - t_j^k), \qquad S_i \equiv \sum_k s_i^k,$$

so that

$$b^k = vq_j^k - t_j^k + m_i s_i^k, \qquad B = vQ_j - T_j + m_i S_i. \tag{1}$$

Therefore one can compute aggregate profits as if both types were trading $(Q_j, T_j)$, yielding aggregate profit $vQ_j - T_j$, while type $i$ were trading on top of this $(Q_i - Q_j, T_i - T_j)$, yielding with probability $m_i$ additional aggregate profit $S_i$. Finally, define the indirect utility functions

$$z_i^{-k}(q,t) = \max \left\{ u_i\left( q + \sum_{l \neq k} q^l, t + \sum_{l \neq k} t^l \right) : (q^l, t^l) \in C^l \text{ for all } l \neq k \right\},$$

so that, in equilibrium, one has, for each $i$ and $k$,

$$U_i \equiv u_i(Q_i, T_i) = z_i^{-k}(q_i^k, t_i^k).$$

Observe that the functions $z_i^{-k}$ are continuous by Berge's maximum theorem.[15]

## 3.2 Pivoting

In the remainder of this section, we assume that an equilibrium exists, and we characterize it. In line with Rothschild and Stiglitz (1976), we examine well-chosen deviations by a buyer, and we use the fact that in equilibrium deviations cannot be profitable. A key difference, however, is that in Rothschild and Stiglitz (1976) competition is exclusive, while in our setting competition is non-exclusive.

Under exclusive competition, what matters from the viewpoint of any given buyer $k$ is simply the maximum utility levels $U_L^{-k}$ and $U_H^{-k}$ that each type of the seller can get by trading with some other buyer. A deviation targeted at type $i$ by buyer $k$ is then a contract $(q_i^k, t_i^k)$ that gives type $i$ a strictly higher utility, $u_i(q_i^k, t_i^k) > U_i^{-k}$. Type $j$ may be attracted or not by this contract; in any case, one can compute the deviating buyer's profit.

By contrast, under non-exclusive competition, all the contracts offered by the other buyers matter from the viewpoint of buyer $k$. Suppose indeed that the seller can trade some pair $(Q^{-k}, T^{-k})$ with the buyers other than $k$. Then buyer $k$ can use this as an opportunity to build more attractive deviations. For instance, to attract type $i$, buyer $k$ can propose the contract $(Q_i - Q^{-k}, T_i - T^{-k} + \varepsilon)$, for some positive number $\varepsilon$: combined with $(Q^{-k}, T^{-k})$, this contract gives type $i$ a strictly higher utility than her equilibrium aggregate trade $(Q_i, T_i)$. In that case, we say that buyer $k$ *pivots on* $(Q^{-k}, T^{-k})$ *to attract type* $i$. Type $j$ may be

[15] This distinguishes our model from Attar et al. (2009), where the presence of a capacity constraint may induce discontinuities in the seller's indirect utility functions.

attracted or not by this contract; in any case, one can provide a condition on profits that ensures that the deviation is not profitable.

Formally, the key difference between exclusive and non-exclusive competition is thus that, in the latter case, each buyer $k$ faces at the deviation stage a single seller whose type is unknown, but whose preferences are defined by the indirect utility function $z_i^{-k}$, rather than by the primitive utility function $u_i$ as in the exclusive case. The difficulty stems from the fact that the functions $z_i^{-k}$ are endogenous, since they depend on the menus offered by the buyers other than $k$, on which we impose no restrictions besides compactness. As a result, there is no a priori guarantee that the functions $z_i^{-k}$ are well behaved: for instance, they could fail to satisfy a single-crossing condition, unlike the seller's utility function over aggregate trades. This prevents us from using standard mechanism techniques to characterize each buyer's best response.[16] Instead, we rely only on pivoting arguments to fully characterize candidate aggregate equilibrium allocations, as in Attar et al. (2009).

The following lemma encapsulates our pivoting technique.

**Lemma 1** *Choose $k$, $i$, $q$, and $t$ such that the quantity $Q_i - q$ can be traded with the buyers other than $k$, in exchange for a transfer $T_i - t$. Then*

$$v_i q - t > b_i^k \quad \textit{only if} \quad vq - t \leq b^k.$$

The intuition for this result is as follows. If the pair $(Q_i - q, T_i - t)$ can be traded with the buyers other than $k$, then buyer $k$ can pivot on it to attract type $i$, while still offering the contract $(q_j^k, t_j^k)$. If the contract $(q, t)$ allows buyer $k$ to increase the profits he makes with type $i$, it must be that type $j$ also selects it instead of $(q_j^k, t_j^k)$ following buyer $k$'s deviation; moreover, this contract cannot increase buyer $k$'s average profit if traded by both types $i$ and $j$, for otherwise we would have constructed a profitable deviation.

Now recall from (1) that one can compute aggregate profits as if both types were trading $(Q_j, T_j)$ in the aggregate, with type $i$ trading in addition $(Q_i - Q_j, T_i - T_j)$. A key implication of Lemma 1 is that, in the aggregate, buyers cannot earn positive profits from making this additional trade with type $i$. Let us first give an intuition for this result in the free-entry case. Notice that, under free entry, the seller can trade $(Q_j, T_j)$ with the existing buyers, so that an entrant can pivot on $(Q_j, T_j)$ to attract type $i$. That is, an entrant could simply propose to buy a quantity $Q_i - Q_j$ in exchange for a transfer slightly above $T_i - T_j$. This

[16] Unless one moreover assumes that the menus offered by the buyers are convex sets, as Biais et al. (2000) do. See Martimort and Stole (2009) for a recent exposition of the standard methodology for the analysis of common agency games with incomplete information.

contract would certainly attract type $i$; besides, if it also attracted type $j$, this would also be good news for the entrant, since $v_j(Q_i - Q_j) \geq v_i(Q_i - Q_j)$ as $v_H > v_L$ and $Q_L \geq Q_H$. In a free-entry equilibrium, it must therefore be that $v_i(Q_i - Q_j) \leq T_i - T_j$. The same result holds when the number of buyers is fixed, although the argument is a bit more involved.

**Proposition 1** *In any equilibrium,* $v_i(Q_i - Q_j) \leq T_i - T_j$*, that is,* $S_i \leq 0$*.*

As simple as it is, this result is powerful enough to rule out standard equilibrium outcomes that have been emphasized in the literature. Consider for instance the separating equilibrium of Rothschild and Stiglitz's (1976) exclusive competition model of insurance provision under adverse selection. In this equilibrium, insurance companies earn zero profit, and no cross-subsidization takes place. Using the parametrization of Section 2.4.2, this means that the equilibrium contract $(Q_i, T_i)$ of each type $i$ lies on the line with negative slope $v_i = -(1-\pi_i)$ going through the origin. Moreover, the high-risk agent, that is, in our parametrization, type $L$, is indifferent between the contracts $(Q_L, T_L)$ and $(Q_H, T_H)$. Since $Q_L > Q_H > 0$, it follows that the line connecting these two contracts has a negative slope strictly lower than $v_L$, that is, $T_L - T_H < v_L(Q_L - Q_H)$, in contradiction with the result in Proposition 1. Thus the Rothschild and Stiglitz's (1976) equilibrium is not robust to non-exclusive competition.

## 3.3 The Zero-Profit Result

In any Bertrand-like setting, the standard argument consists in making buyers compete for any profits that may result from serving the whole demand. This logic also applies to our setting. Indeed, suppose for instance that the aggregate profit from trading with type $j$ is positive, $B_j > 0$. Suppose also for simplicity that there is free entry. Then an entrant could propose to buy $Q_j$ in exchange for a transfer slightly above $T_j$. This contract would certainly attract type $j$, which benefits the entrant; in equilibrium, it must therefore be that this trade also attracts type $i$, and that $vQ_j - T_j \leq 0$.[17] Now recall that aggregate profits may be written as

$$B = vQ_j - T_j + m_i S_i.$$

Our first result in Proposition 1 was that $S_i \leq 0$, and we just have shown that $vQ_j - T_j \leq 0$ when $B_j > 0$. Hence aggregate profits must be zero. This result can be extended to the case where the number of buyers is fixed.

**Proposition 2** *In any equilibrium,* $b^k = 0$ *for all* $k$*.*

[17] This reasoning is once more an application of our pivoting technique. Here the entrant pivots on the no-trade contract $(0, 0)$ to attract type $j$.

**Remark** An inspection of the proofs reveal that Propositions 1 and 2 only require weak assumptions on feasible trades, namely that if the quantities $q$ and $q'$ are tradable, then so are the quantities $q + q'$ and $q - q'$. Hence, we allow for negative and positive trades, but we may for instance have integer constraints on quantities. Finally, we did use in Lemma 1 the fact that the functions $u_i$, and thus the functions $z_i^k$, are continuous with respect to transfers, but, for instance, we did not use the fact that the seller's preferences are convex.

### 3.4 Pooling versus Separating Equilibria

We say that an equilibrium is *pooling* if both types of the seller make the same aggregate trade, that is, $Q_L = Q_H$, and that it is *separating* if they make different aggregate trades, that is, $Q_L > Q_H$. We now investigate the basic price structure of these two kinds of candidate equilibria.

**Proposition 3** *The following holds*:

- *In any pooling equilibrium*, $T_L = vQ_L = T_H = vQ_H$.
- *In any separating equilibrium*,

  *(i) If* $Q_L > 0 > Q_H$, *then* $T_L = v_L Q_L$ *and* $T_H = v_H Q_H$.

  *(ii) If* $Q_L > Q_H \geq 0$, *then* $T_H = vQ_H$ *and* $T_L - T_H = v_L(Q_L - Q_H)$.

  *(iii) If* $0 \geq Q_L > Q_H$, *then* $T_L = vQ_L$ *and* $T_H - T_L = v_H(Q_H - Q_L)$.

The first statement of Proposition 3 is a direct consequence of the zero-profit result. Otherwise, the equilibrium is separating, and three possible cases may arise. In case (i), type $L$ *sells* a positive quantity $Q_L$, while type $H$ *buys* a positive quantity $|Q_H|$. No cross-subsidization takes place in equilibrium, so that $B_L = B_H = 0$. In case (ii), everything happens as if both types were selling an aggregate quantity $Q_H$ at unit price $v$, with type $L$ selling an additional quantity $Q_L - Q_H$ at unit price $v_L$. Thus, if $Q_H > 0$, cross-subsidization takes place in equilibrium, with $B_L < 0 < B_H$. Case (iii) is the mirror image of case (ii), with both types buying $|Q_L|$ at unit price $v$, and type $H$ buying an additional quantity $|Q_H - Q_L|$ at unit price $v_H$. If $Q_L < 0$, the cross-subsidization pattern is reversed, with $B_L > 0 > B_H$. Notice that, when both types trade nonzero quantities in the aggregate, the equilibrium price structure in cases (ii)–(iii) is similar to that described by Jaynes (1978) and Hellwig (1988) in a version of Rothschild and Stiglitz's (1976) model with non-exclusive competition where insurance companies can share information about their clients. By contrast, when only one type trades a nonzero quantity in the aggregate, the equilibrium price structure is similar to that which prevails in Akerlof (1970), or, in a model of non-exclusive competition, in Attar et al. (2009).

### 3.5 The No Cross-Subsidization Result

In this section, we prove that our non-exclusive competition game has no equilibria with cross-subsidies, that is, $B_L = B_H$ in any equilibrium. This drastically reduces the set of candidate equilibria. Indeed, by Proposition 3, this cross-subsidization result rules out pooling equilibria where $Q_L = Q_H \neq 0$, and separating equilibria where either $Q_L > Q_H > 0$ or $0 > Q_L > Q_H$.

The first step of the analysis consists in showing that, if buyers make positive aggregate profits when trading with type $j$, then type $j$ trades inefficiently in equilibrium. Specifically, her marginal rate of substitution at her equilibrium aggregate trade is not equal to the quality of the good she sells, but rather to the *average* quality of the good.

**Lemma 2** *If $B_j > 0$ for some $j$, then $\tau_j(Q_j, T_j) = v$.*

The intuition for Lemma 2 is as follows. If $\tau_j(Q_j, T_j)$ were different from $v$, then any buyer could attempt to reap the aggregate profit on type $j$, while making limited additional losses on his trades with type $i$. For this deviation not to be profitable, it must therefore be that, in equilibrium, the profit that each buyer $k$ makes with type $j$ is no less than the aggregate profit $B_j$ on type $j$. This, however, is impossible if the latter is positive, as assumed in Lemma 2.

The second step of the analysis consists in showing that, if buyers make positive aggregate profits when trading with type $j$, then the aggregate trade made by type $j$ in equilibrium must remain available if any buyer withdraws his menu offer. This would clearly be true under free entry. In our oligopsony model, this rules out Cournot-like outcomes in which the buyers share the market in such a way that each of them is needed to provide type $j$ with her equilibrium aggregate trade, as is the case in the equilibrium described in Biais et al. (2000). This makes our setting closer to Bertrand competition, and cross-subsidies are harder to sustain.

**Lemma 3** *If $B_j > 0$ for some $j$, then, for each $k$, the quantity $Q_j$ can be traded with the buyers other than $k$, in exchange for a transfer $T_j$.*

The proof of Lemma 3 proceeds as follows. First, we show that if $B_j > 0$, then the equilibrium utility of type $j$ must remain available following any buyer's deviation; the reason for this is that, otherwise, a buyer could deviate and reap the aggregate profits on type $j$. As a result, for any buyer $k$, there exists an aggregate trade $(Q^{-k}, T^{-k})$ with the buyers other than $k$ that allows buyer $j$ to achieve the same level of utility as in equilibrium, $u_j(Q^{-k}, T^{-k}) = U_j$. From Assumption C and Lemma 2, we get that if $Q^{-k} \neq Q_j$, then $T^{-k} > vQ^{-k}$. We finally show that this would allow buyer $k$ to profitably deviate by pivoting around $(Q^{-k}, T^{-k})$.

We are now ready to state and prove the main result of this section.

**Proposition 4** *In any equilibrium,* $B_L = B_H = 0$.

As mentioned above, the impossibility of cross-subsidization rules out many equilibrium candidates. To illustrate the main steps of the proof, consider for instance a candidate separating equilibrium with positive quantities $Q_L > Q_H > 0$, as illustrated on Figure 1.

—Insert Figure 1 Here—

According to Proposition 3(ii), we have $T_H = vQ_H < v_H Q_H$, so that there exists a buyer $k$ who earns a positive profit when he trades with type $H$. Because of the zero-profit result, this buyer must make a loss when he trades with type $L$. The key for this buyer is first to secure his profit on type $H$, which can be done by offering the contract $c_H^k = (q_H^k, t_H^k + \varepsilon_H)$, for $\varepsilon_H$ positive and small enough. Simultaneously, buyer $k$ would like to attract type $L$ on another contract that would make a negligible loss. Consider the contract $c_L^k = (Q_L - Q_H, T_L - T_H + \varepsilon_L)$. From Lemma 3, we know that type $L$ can trade $(Q_H, T_H)$ with the buyers other than $k$. By also trading $c_L^k$ with buyer $k$, type $L$ would increase her utility, as long as $\varepsilon_L$ is positive. If moreover $\varepsilon_L$ is high enough compared to $\varepsilon_H$, then type $L$ is indeed attracted by $c_L^k$. Finally, provided $\varepsilon_L$ is small enough, the loss for buyer $k$ from trading $c_L^k$ with type $L$ is small, since Proposition 3(ii) indicates that the slope of the segment between $(Q_H, T_H)$ and $(Q_L, T_L)$ is exactly $v_L$, as shown on Figure 2. Thus buyer $k$ can deviate by offering the two contracts $c_L^k$ and $c_H^k$. Now, we know that type $L$ is attracted by $c_L^k$. If type $H$ trades $c_H^k$, then the deviation is profitable because $c_H^k$ yields a positive profit when traded by type $H$, while the loss on type $L$ is reduced to a negligible amount. If type $H$ decides instead to trade $c_L^k$, then the deviation is profitable because $c_L^k$ yields a positive profit when sold to both types, since its unit price is close to $v_L$. This shows that there exists no separating equilibrium with positive quantities. The reasoning with a pooling equilibrium is slightly more involved, but reaches the same conclusion. Intuitively, equilibrium cross-subsidies are not sustainable because it is possible to neutralize the bad type, on which a buyer makes losses, by proposing her to mimic the behavior of the good type when facing the other buyers.

In the absence of cross-subsidies, Proposition 3 leads to the conclusion that one must have $Q_H \leq 0 \leq Q_L$ in any equilibrium. Thus two types of equilibrium outcomes that have been emphasized in the literature cannot occur in our model: first, pooling outcomes such as the one described in Attar et al. (2009), in which both types would trade the same nonzero quantity at a price equal to the average quality of the good; second, separating outcomes such as the one described by Jaynes (1978) and Hellwig (1988), and illustrated on Figure 1. If one leaves aside the case in which both types trade nonzero quantities on opposite sides of the market, the remaining possibilities for equilibrium outcomes have a structure reminiscent of Akerlof (1970): either there is no trade in the aggregate, or only one type actively trades in the aggregate, at a unit price equal to the quality of the good she offers.

### 3.6 Equilibrium Aggregate Trades

In this section, we fully characterize the candidate equilibrium aggregate trades, and we provide necessary conditions for the existence of an equilibrium. Given the price structure of equilibria delineated in Section 3.4, all that remains to be done is to give restrictions on each type's equilibrium marginal rate of substitution. Two cases need to be distinguished, according to whether a type's aggregate trade is zero or not in equilibrium.

Our first result is that, if type $j$ does not trade in the aggregate, then her equilibrium marginal rate of substitution must lie between $v$ and $v_j$.

**Lemma 4** *If $Q_j = 0$, then $v_j - \tau_j(0,0)$ and $\tau_j(0,0) - v$ have the same sign.*

The intuition for Lemma 4 is as follows. Suppose that $j = H$. If $v_H > \tau_H(0,0)$, then any buyer could attract type $H$ by proposing a contract offering to buy a small positive quantity at a unit price lower than $v_H$. For this deviation not to be profitable, type $L$ must also trade this contract, and one should have $\tau_H(0,0) \geq v$, so that the deviator makes losses when both types trade this contract. The same reasoning applies when $v_H < \tau_H(0,0)$, if one considers a contract offering to sell a small positive quantity at a unit price higher than $v_H$. The case $j = L$ can be handled in a symmetric way.

Our second result is that, if type $i$ trades a nonzero quantity in the aggregate, then she must trade efficiently in equilibrium.

**Lemma 5** *If $Q_i \neq 0$, then $\tau_i(Q_i, T_i) = v_i$.*

The intuition for Lemma 5 is as follows. Suppose that $i = L$. Since cross-subsidization cannot occur in equilibrium, $T_L = v_L Q_L > 0$ if $Q_L \neq 0$. If type $L$ were trading inefficiently in equilibrium, that is, if $\tau_L(Q_L, T_L) \neq v_L$, then there would exist a contract offering to buy a positive quantity at a unit price lower than $v_L$, and that would give type $L$ a strictly higher utility than $(Q_L, T_L)$. Any of the buyers could profitably attract type $L$ by proposing this contract, which would be even more profitable for the deviating buyer if traded by type $H$. Hence type $L$ must trade efficiently in equilibrium. The case $i = H$ can be handled in a symmetric way.[18]

To state our characterization result, it is necessary to define first-best quantities. The following assumption ensures that these quantities are well defined.

**Assumption FB** *For each $i$, there exists $Q_i^*$ such that $\tau_i(Q_i^*, v_i Q_i^*) = v_i$.*

Assumption FB states that $Q_i^*$ is the efficient quantity for type $i$ to trade at a unit price $v_i$ that gives an aggregate zero profit for the buyers. In the pure trade model, $Q_i^*$ is defined by $c_i'(Q_i^*) = v_i$. In the insurance model, because or the seller's risk aversion, efficiency requires full insurance for all types, so that $Q_i^* = W_G - W_B$.[19] An important consequence of Assumption C is that $Q_i^* \geq 0$ if and only if $\tau_i(0,0) \leq v_i$, and that $Q_i^* = 0$ if and only if $\tau_i(0,0) = v_i$. We can now state our main characterization result.

**Theorem 1** *If an equilibrium exists, then $\tau_L(0,0) \leq v \leq \tau_H(0,0)$. Moreover,*

- *If $v_L \leq \tau_L(0,0) \leq v \leq \tau_H(0,0) \leq v_H$, all equilibria are pooling, with $Q_L = Q_H = 0$.*
- *Otherwise, all equilibria are separating, and*
  - *(i) If $\tau_L(0,0) < v_L < v < v_H < \tau_H(0,0)$, then $Q_L = Q_L^* > 0$ and $Q_H = Q_H^* < 0$.*
  - *(ii) If $\tau_L(0,0) < v_L < v \leq \tau_H(0,0) \leq v_H$, then $Q_L = Q_L^* > 0$ and $Q_H = 0$.*
  - *(iii) If $v_L \leq \tau_L(0,0) \leq v < v_H < \tau_H(0,0)$, then $Q_L = 0$ and $Q_H = Q_H^* < 0$.*

The first message of Theorem 1 is a negative one: the non-exclusive competition game need not have an equilibrium. In the pure trade model, no equilibrium exists if the cost function of type $L$ is such that $c_L'(0) > v$, or if the cost function of type $H$ is such that $c_H'(0) < v$; for instance, this is the case in the Biais et al. (2000) setting if $\theta_L > v$, or if

[18] It should be noted that the proofs of Lemmas 4 and 5 involve no pivoting arguments—or, what amounts to the same thing, pivoting on the no-trade contract—and would therefore also go through in an exclusive competition context.

[19] A special feature of these two examples is that efficient quantities depend on the type of the seller, but not on the buyers' aggregate profit.

$\theta_H < v$, that is, if the low-cost type $L$ is not eager enough to sell, or if the high-cost type $H$ is too eager to sell. In the insurance model, no equilibrium exists if $\frac{\pi_H}{1-\pi_H}\frac{u'(W_G)}{u'(W_B)} < \frac{\pi}{1-\pi}$, where $\pi = m_L\pi_L + m_H\pi_H$, that is, if the low-risk type $H$ is too eager to buy insurance.[20] Overall, Theorem 1 reinforces the insight of the no cross-subsidization result: an equilibrium exists only if the adverse selection problem is severe enough, so that both types' incentives to trade are not too closely aligned. On a more positive note, as we will later show in Theorem 2, the necessary condition $\tau_L(0,0) \leq v \leq \tau_H(0,0)$ for the existence of an equilibrium also turns out to be sufficient. Thus Theorem 1 gives a complete description of the structure of aggregate equilibrium outcomes, which is summarized on Figure 2.

—Insert Figure 2 Here—

Second, Theorem 1 shows that pooling requires $v_L \leq \tau_L(0,0)$ and $v_H \geq \tau_H(0,0)$; by the no cross-subsidization result, we already know that a pooling equilibrium involves zero aggregate trade for both types. The conditions $v_L \leq \tau_L(0,0)$ and $v_H \geq \tau_H(0,0)$ together imply that $Q^*_L \leq 0 \leq Q^*_H$; when one of these inequalities is strict, the first-best quantities are not implementable. Thus pooling requires a strong form of nonresponsiveness: in the first-best scenario, type $L$ would like to buy, and type $H$ to sell. This cannot arise in the insurance model, for in this case $Q^*_L = Q^*_H = W_G - W_B$. Thus the insurance model admits no pooling equilibrium. In the pure trade model, a pooling equilibrium exists only if $c'_L(0) \geq v_L$ and $c'_H(0) \leq v_H$; for instance, this is the case in the Biais et al. (2000) setting if $\theta_L \geq v_L$ and $\theta_H \leq v_H$.[21]

Third, Theorem 1 states that in a separating equilibrium, at least one of the types trades efficiently. In case (i), types $L$ and $H$'s preferences are sufficiently far apart from each other, in the sense that $Q^*_L > 0 > Q^*_H$: in the first-best scenario, type $L$ would like to sell, and type $H$ to buy. In that case, both types end up trading their first-best quantities in equilibrium. Observe that the insurance model admits no equilibrium of this kind. In the pure trade model, a first-best equilibrium may exist if $c'_L(0) < v_L$ and $c'_H(0) > v_H$; for instance, this is the case in the Biais et al. (2000) setting if $\theta_L < v_L$ and $\theta_H > v_H$. In case (ii), both $Q^*_L$ and $Q^*_H$ are nonnegative: in the first-best scenario, both types would like to sell. The unique candidate equilibrium outcome is then similar to the one which prevails in Akerlof (1970): type $L$ trades efficiently, while type $H$ does not trade at all. This is the situation that prevails

[20]This was noted by Ales and Maziero (2009), assuming free entry. The condition $\tau_L(0,0) \leq v$, or, equivalently, $\frac{\pi_L}{1-\pi_L}\frac{u'(W_G)}{u'(W_B)} \leq \frac{\pi}{1-\pi}$, is automatically satisfied since $\pi > \pi_L$ and $u'(W_B) > u'(W_G)$.

[21]Notice, however, that, in their paper, Biais et al. (2000) explicitly rule out this parameter configuration for technical reasons.

in the insurance model, when an equilibrium exists at all, that is, if $\frac{\pi_H}{1-\pi_H}\frac{u'(W_G)}{u'(W_B)} \geq \frac{\pi}{1-\pi}$: in that case, the high-risk type $L$ obtains full insurance at an actuarially fair price, while the low-risk type $H$ purchases no insurance. In the pure trade model, this type of equilibrium may exist if $c'_L(0) < v_L$ and $c'_H(0) \leq v_H$; for instance, this is the case in the Biais et al. (2000) setting if $\theta_L < v_L$ and $\theta_H \leq v_H$. Finally, case (iii) is symmetric to case (ii), exchanging the roles of type $L$ and $H$. Observe that in any separating equilibrium, each type strictly prefers her equilibrium aggregate trade to that of the other type. This contrasts with the predictions of models of exclusive competition under adverse selection, such as Rothschild and Stiglitz's (1976), in which type $L$ is indifferent between her equilibrium contract and that of type $H$.

**Remark** It is interesting to compare the conclusions of Theorem 1 with those reached by Attar et al. (2009). As explained in Section 2.4.1, the two distinctive features of their model is that the seller has linear preferences, $u_i(Q,T) = T - \theta_i Q$, and makes choices under an aggregate capacity constraint, $Q \leq 1$. Observe that, in this context, type $i$'s marginal rate of substitution is constant and equal to $\theta_i$. In a two-type version of their model in which there are potential gains from trade for each type, that is, $v_L > \theta_L$ and $v_H > \theta_H$, Attar et al. (2009) show that the non-exclusive competition game always admits an equilibrium, that the buyers receive zero profits, and that the aggregate equilibrium allocation is generically unique. If $\theta_H > v$, the equilibrium is similar to the separating equilibrium found in case (ii) of Theorem 1: type $L$ trades efficiently, that is, $Q_L = 1$ and $T_L = v_L$, while type $H$ does not trade at all, that is, $Q_H = T_H = 0$.[22] By contrast, if $\theta_H < v$, the situation is markedly different from that described in Theorem 1. First, an equilibrium exists, while, in the analogous situation where $\tau_H(0,0) < v$, no equilibrium exists in our model. Second, any equilibrium is pooling and efficient, that is, $Q_L = Q_H = 1$ and $T_L = T_H = v$, while cross-subsidies, and therefore non-trivial pooling equilibria, are ruled out in our model. The key difference between the two setups that explains these discrepancies is that, unlike Attar et al. (2009), we do not require the seller's choices to satisfy an aggregate capacity constraint. This implies that some deviations that are crucial for our characterization result are not available in Attar et al. (2009). A case in point is the no cross-subsidization result: key to the proof of Proposition 4 is the possibility, for a deviator that makes profit when trading with type $j$, to pivot on $(Q_j, T_j)$ to attract type $i$, while preserving the profit he makes by trading with type $j$. However, for the argument to go through, there must be no restrictions on the signs of the quantities traded in such deviations; in particular, it is crucial that the deviator be

[22]In the non-generic case where $\theta_H = v$, there also exist separating equilibria in which $0 < Q_H \leq 1$.

able to induce type $i$ to consume more than $Q_i$ in the aggregate.[23] This, however, is precisely what is impossible to do in the presence of a capacity constraint, when both types trade up to capacity in the candidate equilibrium, as in the pooling equilibrium described in Attar et al. (2009). Thus it is the capacity constraint, and not the linearity of the preferences per se, that constitutes the key difference between their model and the one studied in this paper.

### 3.7 Equilibrium Individual Trades

So far, we have focused on the aggregate equilibrium implications of our model. In this section, we briefly sketch a few implications for individual equilibrium trades. First, we show that our no cross-subsidization result also holds at the level of individual buyers.

**Proposition 5** *In any equilibrium, $b_j^k = 0$ for all $j$ and $k$.*

Our second result states that aggregate and individual equilibrium trades have the same sign. This reinforces the basic insight of our model that, in equilibrium, the seller can signal her type only through the sign of the quantities she trades.

**Proposition 6** *In any equilibrium, $q_L^k \geq 0 \geq q_H^k$ for all $k$.*

It follows from Proposition 6 that if a type does not trade in the aggregate, then she does not trade at all, so that the pooling equilibrium, when it exists, is actually a no-trade equilibrium. Observe also that, when a type trades a nonzero quantity in the aggregate, there need not be more than one active buyer, as will be clear from considering the equilibria we now construct.

## 4 Equilibrium Existence

To establish the existence of an equilibrium, we impose the following technical assumption on preferences.

**Assumption T** *There exist $\overline{Q}_L$ and $\underline{Q}_H$ such that*

$$\tau_L(Q,T) > v_L \quad \textit{if} \quad Q > \overline{Q}_L, \quad \textit{and} \quad \tau_H(Q,T) < v_H \quad \textit{if} \quad Q < \underline{Q}_H,$$

*uniformly in $T$.*

[23] Formally, it follows from the proof of Proposition 4 that, if $B_H > 0$ in a pooling equilibrium where each type trades a positive aggregate quantity $Q$, then, for any small enough additional trade $(\delta_L, \varepsilon_L)$ such that $\tau_L(Q,T)\delta_L < \varepsilon_L$, and that would thus attract type $L$, one must have $v\delta_L \leq \varepsilon_L$. If there are no restrictions on the sign of $\delta_L$, this implies that $\tau_L(Q,T) = v$, from which a contradiction can be derived using Lemma 2. But if, for some reason, only nonpositive $\delta_L$'s are admissible, say, because the seller cannot trade more than $Q$ in the aggregate, then one can only conclude that $\tau_L(Q,T) \leq v$, from which no contradiction follows.

Assumption T ensures that equilibrium menus can be constructed as compact sets of contracts. It should be emphasized that the restrictions it imposes on preferences are rather mild. In the pure trade model, because of the quasilinearity of preferences, Assumption T follows from Assumption FB, and one can take $\overline{Q}_L = Q_L^*$ and $\underline{Q}_H = Q_H^*$. In the insurance model, Assumption T follows from the seller's risk aversion, and one can take $\overline{Q}_L = \underline{Q}_H = W_G - W_B = Q_L^* = Q_H^*$.

**Theorem 2** *An equilibrium exists if and only if* $\tau_L(0,0) \leq v \leq \tau_H(0,0)$.

Theorem 2 shows that the necessary conditions for the existence of an equilibrium given in Theorem 1 are also sufficient: indeed, in any of the scenarios identified in Theorem 1, one can construct menus of contracts for the buyers that support the candidate equilibrium allocation. While we make no general attempt at minimizing the size of equilibrium menus, the proof of Theorem 2 shows that different types of menus can be used depending on the scenario considered.

Whenever the equilibrium is separating, two situations can arise. If both types trade efficiently in equilibrium, as in case (i) of Theorem 1, the equilibrium can be supported by simple menu offers in which at least two buyers offer the aggregate equilibrium trades $(Q_L^*, v_L Q_L^*)$ and $(Q_H^*, v_H Q_H^*)$. This reflects that the standard Bertrand logic applies, because, in this case, the two types' preferences are sufficiently far apart from each other. If, by contrast, only one of the types, say type $i$, trades efficiently in equilibrium, as in cases (ii) and (iii) of Theorem 1, then the equilibrium can be supported by linear menus offers in which at least two buyers offer to trade any positive (in case (ii)) or negative (in case (iii)) quantities at a unit price $v_i$, up to some limit. These menus are similar to those derived by Attar et al. (2009) in a non-exclusive version of Akerlof's (1970) model. In particular, unlike in the first-best case (i), they contain *latent contracts*, that is, contracts that are not traded on the equilibrium path, but which the seller finds it profitable to trade at the deviation stage. As in Attar et al. (2009), the role of such contracts is to deter cream-skimming deviations. Consider for instance case (ii), and suppose that a buyer attempts to deviate and purchase from type $H$ only. To be successful, this cream-skimming deviation must involve trading a relatively small quantity at a relatively high price. However, this contract becomes also attractive to type $L$ if, along with it, she can make enough further trades at the equilibrium price $v_L$, so as to obtain a higher utility than in equilibrium. This implies that the deviating buyer can obtain at most the profit from a pooling deviation, which is easily shown to be nonpositive.

Whenever the equilibrium is pooling, two situations can arise. If the bounds $\overline{Q}_L$ and $\underline{Q}_H$ in Assumption T can be chosen in such a way that $\overline{Q}_L \leq 0 \leq \underline{Q}_H$, it is straightforward to show that even a monopsonist would be unable to improve over the no-trade outcome, and extract rents from the seller.[24] The equilibrium menus can then be reduced to the no-trade contract. Things are more complex when $\overline{Q}_L$ and $\underline{Q}_H$ cannot be chosen in such a way that $\overline{Q}_L \leq 0 \leq \underline{Q}_H$, for, in this case, there are situations where a monopsonist could make profits by offering each type to trade a specific contract, distinct from the trivial one. To block the corresponding deviations, latent contracts must be available in equilibrium. We construct the equilibrium menus in such a way that buyers offer to trade any positive quantity at a unit price $v_L$, and any negative quantity at a unit price $v_H$, up to some limits. Since $v_H > v_L$, this can intuitively be interpreted as a bid-ask spread.

A noticeable feature of our construction is that, in any scenario, no contract issued in equilibrium could potentially make losses. This reflects an extreme fear of adverse selection, and should be contrasted with the equilibrium of an exclusive competition game such as Rothschild and Stiglitz's (1976), in which the contract designed for the low-risk agent would make losses if traded by the high-risk agent.

# 5 Conclusion

In this paper, we analyzed the impact of adverse selection on markets where competition is non-exclusive. We fully characterized aggregate equilibrium allocations, which are uniquely determined, and we gave a necessary and sufficient condition for the existence of a pure strategy equilibrium. Our results show that, under non-exclusivity, market breakdown may arise in a competitive environment where buyers can compete through arbitrary menu offers: specifically, whenever first-best allocations cannot be achieved, equilibria when they exist involve no trade for at least one type of the seller.

These predictions contrast with those of standard competitive screening models, which typically focus on exclusive competition. In those settings, one type of the seller signals the

[24]This situation arises in the pure trade model, because in that case one can take $\overline{Q}_L = Q_L^*$ and $\overline{Q}_H = Q_H^*$, and $Q_L^* \leq 0 \leq Q_H^*$ by nonresponsiveness. The idea of the proof is as follows. By standard arguments, one can show that at least one type's participation constraint must be binding at the optimum. Suppose it is type $H$'s. Then, if $Q_H > 0$, type $L$'s incentive compatibility constraint must also be binding at the optimum. Since $\tau_H(0,0) \geq v$ and $Q_H > 0 \geq \overline{Q}_L$, it follows that $vQ_H - T_H < 0$ and $T_L - T_H > v_L(Q_L - Q_H)$. Thus, the monopsonist's profit, which can be rewritten as $vQ_H - T_H - [T_L - T_H - v_L(Q_L - Q_H)]$, is negative if $Q_H > 0$. If $Q_H < 0$, then type $L$'s participation contraint and type $H$'s incentive compatibility constraint together imply that $Q_L \geq 0$. Since $\tau_L(0,0) \geq v_L$ and $\tau_H(0,0) \leq v_H$, one obtains that $v_H Q_H - T_H < 0$ and $v_L Q_L - T_L \leq 0$, so that the the monopsonist's profit is negative if $Q_H < 0$. The argument when type $L$'s participation constraint is binding is symmetrical.

quality of the good she offers by trading an inefficient, but nonzero quantity of this good. When competition is non-exclusive, each buyer's inability to control the seller's trades with his opponents creates additional deviation opportunities. This makes screening more costly, and implies that the seller either trades efficiently, or does not trade at all.

There has been so far little investigation of the welfare implications of adverse selection in markets where competition is non-exclusive. A natural development of our analysis would be to study the decision problem faced by a planner who wants to implement an efficient allocation, subject to informational constraints, but also to the constraint that exclusivity be non-enforceable. It is unclear that such a planner can improve on the market allocations characterized in this paper. If he could, this would provide new theoretical insights in favor of welfare-based regulatory interventions, in particular in the context of financial markets.

# Appendix

**Proof of Lemma 1.** Let $k$, $i$, $q$, and $t$ satisfy the assumption of the lemma, and suppose that $v_i q - t > b_i^k$. Buyer $k$ can deviate by proposing a menu consisting of the no-trade contract and of the contracts $c_i^k = (q, t + \varepsilon_i)$ and $c_j^k = (q_j^k, t_j^k + \varepsilon_j)$, for $\varepsilon_i$ and $\varepsilon_j$ positive. Given the assumption in the lemma, by trading $c_i^k$ with buyer $k$ and $(Q_i - q, T_i - t)$ with the buyers other than $k$, type $i$ gets a utility $u_i(Q_i, T_i + \varepsilon_i) > U_i$. In equilibrium one has $U_i \geq z_i^{-k}(q_j^k, t_j^k)$, and the function $z_i^{-k}$ is continuous. Therefore $u_i(Q_i, T_i + \varepsilon_i) > z_i^{-k}(q_j^k, t_j^k + \varepsilon_j)$ for all small enough $\varepsilon_j$. Hence, for any such $\varepsilon_j$, type $i$ must select $c_i^k$ following buyer $k$'s deviation. By accepting $c_j^k$, type $j$ can get a utility $u_j(Q_j, T_j + \varepsilon_j) > U_j$. Hence type $j$ selects either $c_i^k$ or $c_j^k$ following buyer $k$'s deviation. If type $j$ selects $c_j^k$, then by deviating buyer $k$ obtains a profit

$$m_i(v_i q - t - \varepsilon_i) + m_j(v_j q_j^k - t_j^k - \varepsilon_j) = m_i(v_i q - t) + m_j b_j^k - (m_i \varepsilon_i + m_j \varepsilon_j).$$

However, from the assumption $v_i q - t > b_i^k$, this is strictly higher than $b^k$ when $\varepsilon_i$ and $\varepsilon_j$ are small enough, a contradiction. Therefore it must be that type $j$ selects $c_i^k$ following buyer $k$'s deviation. In equilibrium the deviation cannot be profitable, so that $vq - t - \varepsilon_i \leq b^k$. Letting $\varepsilon_i$ go to 0, the result follows. ■

**Proof of Proposition 1.** Choose $i$ and $k$ and set $q = q_j^k + Q_i - Q_j$ and $t = t_j^k + T_i - T_j$. Then the quantity $Q_i - q = \sum_{l \neq k} q_j^l$ can be traded with the buyers other than $k$, in exchange for a transfer $T_i - t = \sum_{l \neq k} t_j^l$. We can thus apply Lemma 1. One has

$$\begin{aligned}
v_i q - t - b_i^k &= v_i(q_j^k + Q_i - Q_j) - (t_j^k + T_i - T_j) - b_i^k \\
&= v_i(Q_i - Q_j) - (T_i - T_j) - [v_i(q_i^k - q_j^k) - (t_i^k - t_j^k)] \\
&= S_i - s_i^k
\end{aligned}$$

and

$$\begin{aligned}
v_j q - t - b_j^k &= v_j(q_j^k + Q_i - Q_j) - (t_j^k + T_i - T_j) - b_j^k \\
&= -[v_j(Q_j - Q_i) - (T_j - T_i)] \\
&= -S_j,
\end{aligned}$$

so that we get

$$S_i > s_i^k \quad \text{only if} \quad m_i(S_i - s_i^k) \leq m_j S_j. \tag{2}$$

by Lemma 1. We now show that $S_i \leq s_i^k$ for all $i$ and $k$, which implies the result by summing over $k$. Suppose by way of contradiction that there exists $i$ and $k$ such that $S_i > s_i^k$. Then, from (2), $S_j > 0$. Therefore there exists $l$ such that $S_j > s_j^l$. Now, (2) remains valid if one exchanges $i$ and $j$, so that $S_i > 0$. Since $S_i + S_j = (v_i - v_j)(Q_i - Q_j)$, one finally gets $Q_L < Q_H$, in contradiction with Assumption SC. Hence the result. ■

**Proof of Proposition 2.** We first prove that for each $j$ and $k$, one has

$$B_j > b_j^k \quad \text{only if} \quad B - b^k \leq m_i S_i. \tag{3}$$

Indeed, if $B_j > b_j^k$, buyer $k$ can deviate by proposing a menu consisting of the no-trade contract and of the contracts $c_i^k = (q_i^k, t_i^k + \varepsilon_i)$ and $c_j^k = (Q_j, T_j + \varepsilon_j)$, for $\varepsilon_i$ and $\varepsilon_j$ positive. Because $U_j \geq z_j^k(q_i^k, t_i^k)$ and the function $z_j^k$ is continuous, it is possible, given the value of $\varepsilon_j$, to choose $\varepsilon_i$ small enough, so that type $j$ trades $c_j^k$ following buyer $k$'s deviation. Turning now to type $i$, observe that she must trade either $c_i^k$ or $c_j^k$ following buyer $k$'s deviation: indeed, because $\varepsilon_i > 0$, type $i$ strictly prefers $c_i^k$ to any contract she could have traded with buyer $k$ before the deviation. If type $i$ selects $c_i^k$, then buyer $k$'s profit from this deviation is $m_i(b_i^k - \varepsilon_i) + m_j(B_j - \varepsilon_j)$, which, since $B_j > b_j^k$ by assumption, is strictly greater than $b^k$ for $\varepsilon_i$ and $\varepsilon_j$ close enough to zero, a contradiction. Therefore type $i$ must select $c_j^k$ following buyer $k$'s deviation, and for such a deviation not to be profitable one must have $vQ_j - T_j - \varepsilon_j \leq b^k$. From (1), this may be rewritten as $B - m_i S_i - \varepsilon_j \leq b^k$, from which (3) follows by letting $\varepsilon_j$ go to zero.

Now, it may be that for each $j$ and $k$, $B_j \leq b_j^k$. Summing over $k$ then yields $B_j \leq 0$ for all $j$, so that aggregate and individual profits must be equal to zero. Suppose alternatively that $B_j > b_j^k$ for some $j$ and $k$. Then, by (3) along with the fact that $S_i \leq 0$, $\sum_{l \neq k} b^l \leq 0$, and hence $b^l = 0$ for all $l \neq k$. There only remains to show that $b^k = 0$. If $B_i > b_i^l$ or $B_j > b_j^l$ for some $l \neq k$, then, by the same reasoning, $b^k = 0$. Otherwise, $B_i \leq b_i^l$ and $B_j \leq b_j^l$ for all $l \neq k$. By averaging over types, this yields $B \leq b^l$, and we know that $b^l = 0$ for all $l \neq k$. Therefore $B = 0$ and thus $b^k = 0$, from which the result follows. ■

**Proof of Proposition 3.** In the case of a pooling equilibrium, the conclusion follows immediately from the zero-profit result. Consider next a separating equilibrium, and let us start with case (ii): $Q_L > Q_H \geq 0$. We know from Lemma 1 that $S_L \leq 0$. Suppose $S_L < 0$. From (3) and the zero-profit result, we get that $B_H \leq b_H^k$ for all $k$, which implies that $B_H \leq 0$. Now notice from (1) that

$$B = vQ_H - T_H + m_L S_L = B_H + m_L[S_L - (v_H - v_L)Q_H].$$

Because $B_H \leq 0$, $S_L < 0$ and $Q_H \geq 0$, we get that $B < 0$, a contradiction. Therefore it must be that $S_L = 0$. It follows that $B = vQ_H - T_H$, so that $T_H = vQ_H$ since $B = 0$. Hence the result. Case (iii) follows in a similar manner, exchanging the roles of $L$ and $H$. Consider finally case (i): $Q_L > 0 > Q_H$. As above, $B = B_H + m_L[S_L - (v_H - v_L)Q_H] = 0$. Suppose that $B_H > 0$ and thus $B_H > b_H^k$ for some $k$. Again, from (3), this implies that $S_L = 0$ and thus that $B = B_H - m_L(v_H - v_L)Q_H$. Since $B_H > 0$, one must have $Q_H > 0$, a contradiction. Hence $B_H = 0$, and therefore $B_L = 0$ since $B = 0$. It follows that $T_L = vQ_L$ and $T_H = v_H Q_H$. Hence the result. ■

**Proof of Lemma 2.** If $B_j > 0$, then one must have $T_j = vQ_j$ by Proposition 3. Any buyer $k$ can deviate by proposing a menu consisting of the no-trade contract and of the contracts $c_i^k = (q_i^k, t_i^k + \varepsilon_i)$ and $c_j^k = (Q_j + \delta_j, T_j + \varepsilon_j)$, for some numbers $\varepsilon_i$, $\delta_j$, and $\varepsilon_j$. Suppose by way of contradiction that $\tau_j(Q_j, T_j) \neq v$. Then one can choose $\delta_j$ and $\varepsilon_j$ such that $\tau_j(Q_j, T_j)\delta_j < \varepsilon_j < v\delta_j$. For $\delta_j$ and $\varepsilon_j$ small enough, the first inequality guarantees that type $j$ can strictly increase her utility by trading $c_j^k$ with buyer $k$. It is then possible to choose $\varepsilon_i$ positive and small enough, so that, following buyer $k$'s deviation, type $j$ prefers trading $c_j^k$ to trading $c_i^k$. Turning now to type $i$, observe that she must trade either $c_i^k$ or $c_j^k$ following buyer $k$'s deviation: indeed, because $\varepsilon_i > 0$, type $i$ strictly prefers $c_i^k$ to any contract she could have traded with buyer $k$ before the deviation. If type $i$ selects $c_j^k$, then buyer $k$'s profit from this deviation is $v(Q_j + \delta_j) - (T_j + \varepsilon_j) = v\delta_j - \varepsilon_j > 0$, in contradiction with the zero-profit result. Therefore type $i$ must select $c_i^k$ following buyer $k$'s deviation, and for this deviation not to be profitable one must have

$$m_i(b_i^k - \varepsilon_i) + m_j[v_j(Q_j + \delta_j) - (T_j + \varepsilon_j)] \leq m_i b_i^k + m_j b_j^k.$$

Letting $\varepsilon_i$, $\varepsilon_j$, and $\delta_j$ go to zero yields that $B_j \leq b_j^k$. Since this holds for any buyer $k$, we can sum over $k$ to get $B_j \leq 0$, a contradiction. The result follows. ■

**Proof of Lemma 3.** Suppose first that $U_j > z_j^{-k}(0,0)$ for some $k$. Then buyer $k$ can deviate by proposing a menu consisting of the no-trade contract and of the contract $(Q_j, T_j - \varepsilon)$, with $\varepsilon$ positive. For $\varepsilon$ small enough, one has $u_j(Q_j, T_j - \varepsilon) > z_j^{-k}(0,0)$, so type $j$ trades the contract $(Q_j, T_j - \varepsilon)$ following buyer $k$'s deviation. If type $i$ does not trade the contract $(Q_j, T_j - \varepsilon)$, buyer $k$'s profit from this deviation is $m_j(v_j Q_j - T_j + \varepsilon) = m_j(B_j + \varepsilon) > 0$, in contradiction with the zero-profit result. If type $i$ trades the contract $(Q_j, T_j - \varepsilon)$, then, because $T_j = vQ_j$ by Proposition 3, buyer $k$'s profit from this deviation is $vQ_j - T_j + \varepsilon = \varepsilon > 0$, again in contradiction with the zero-profit result. Since in any case $U_j \geq z_j^{-k}(0,0)$, it must be that

$U_j = z_j^{-k}(0,0)$ for all $k$. It follows that, for any buyer $k$, there exists a trade $(Q^{-k}, T^{-k})$ with the buyers other than $k$ such that $u_j(Q^{-k}, T^{-k}) = U_j$.

Suppose now that $Q^{-k} \neq Q_j$. Then, from Assumption C and Lemma 2, one must have $T^{-k} > vQ^{-k}$. We now examine two deviations for buyer $k$ that both pivot around $(Q^{-k}, T^{-k})$. First, define $(q_1, t_1)$ such that $(q_1, t_1) + (Q^{-k}, T^{-k}) = (Q_j, T_j)$. Then the quantity $Q_j - q_1$ can be traded with the buyers other than $k$, in exchange for a transfer $T_j - t_1$. Moreover, using the fact that $T_j = vQ_j$ by Proposition 3, and that $T^{-k} > vQ^{-k}$, one gets

$$\begin{aligned} vq_1 - t_1 &= v(Q_j - Q^{-k}) - (T_j - T^{-k}) \\ &= T^{-k} - vQ^{-k} \\ &> 0. \end{aligned}$$

Therefore, by Lemma 1, one must have $v_j q_1 - t_1 \leq b_j^k$, that is, again using $T_j = vQ_j$, $T^{-k} - v_j Q^{-k} + (v_j - v)Q_j \leq b_j^k$. Because $T^{-k} > vQ^{-k}$, this implies that

$$(v_j - v)(Q_j - Q^{-k}) < b_j^k. \tag{4}$$

Second, define $(q_2, t_2)$ such that $(q_2, t_2) + (Q^{-k}, T^{-k}) = (Q_i, T_i)$. Then the quantity $Q_i - q_1$ can be traded with the buyers other than $k$, in exchange for a transfer $T_i - t_1$. Moreover, using the fact that $S_i = 0$ and $T_j = vQ_j$ by Proposition 3, and that $T^{-k} > vQ^{-k}$ and $(v - v_i)(Q_i - Q_j) \geq 0$, one gets

$$\begin{aligned} vq_2 - t_2 &= v(Q_i - Q^{-k}) - (T_i - T^{-k}) \\ &= T^{-k} - vQ^{-k} + vQ_i - [T_j + v_i(Q_i - Q_j) - S_i] \\ &= T^{-k} - vQ^{-k} + (v - v_i)(Q_i - Q_j) \\ &> 0. \end{aligned}$$

Therefore, by Lemma 1, one must have $v_i q_2 - t_2 \leq b_i^k$, that is, again using $S_i = 0$ and $T_j = vQ_j$, $T^{-k} - v_i Q^{-k} + (v_i - v)Q_j \leq b_i^k$. Because $T^{-k} > vQ^{-k}$, this implies that

$$(v_i - v)(Q_j - Q^{-k}) < b_i^k. \tag{5}$$

Since $v = m_i v_i + m_j v_j$, and $m_i b_i^k + m_j b_j^k = 0$ by the zero-profit result, averaging (4) and (5) yields $0 < 0$, a contradiction. Therefore it must be that $Q^{-k} = Q_j$. Since $u_j(Q^{-k}, T^{-k}) = U_j = u_j(Q_j, T_j)$, it follows that $T^{-k} = T_j$, which implies the result. ■

**Proof of Proposition 4.** Suppose by way of contradiction that $B_j > 0$ for some $j$. Then any buyer $k$ such that $b_j^k > 0$ can deviate by proposing a menu consisting of the no-trade contract and of the contracts $c_i^k = (Q_i - Q_j + \delta_i, v_i(Q_i - Q_j) + \varepsilon_i)$ and $c_j^k = (q_j^k, t_j^k + \varepsilon_j)$, for some numbers $\delta_i$, $\varepsilon_i$, and $\varepsilon_j$. Choose $\delta_i$ and $\varepsilon_i$ such that $\tau_i(Q_i, T_i)\delta_i < \varepsilon_i$. This ensures that, for $\delta_i$ and $\varepsilon_i$ small enough, type $i$ can strictly increase her utility by trading $c_i^k$ with buyer $k$, and $(Q_j, T_j)$ with the buyers other than $k$; according to Lemma 3, this is feasible, since $B_j > 0$. Because $U_i \geq z_i^k(q_j^k, t_j^k)$ and the function $z_i^k$ is continuous, it is possible, given the values of $\delta_i$ and $\varepsilon_i$, to choose $\varepsilon_j$ positive and small enough, so that type $i$ trades $c_i^k$ following buyer $k$'s deviation. Turning now to type $j$, observe that she must trade either $c_i^k$ or $c_j^k$ following buyer $k$'s deviation: indeed, because $\varepsilon_j > 0$, type $j$ strictly prefers $c_j^k$ to any contract she could have traded with buyer $k$ before the deviation. If type $j$ selects $c_j^k$, then buyer $k$'s profit from this deviation is $m_i(v_i\delta_i - \varepsilon_i) + m_j(v_j q_j^k - t_j^k - \varepsilon_j)$, which, since $v_j q_j^k - t_j^k = b_j^k > 0$ by assumption, is positive when $\delta_i$, $\varepsilon_i$, and $\varepsilon_j$ are small enough, in contradiction with the zero-profit result. Therefore type $j$ must select $c_i^k$ following buyer $k$'s deviation, and for this deviation not to be profitable one must have

$$v(Q_i - Q_j + \delta_i) - v_i(Q_i - Q_j) - \varepsilon_i \leq 0. \tag{6}$$

Now, recall that, as a consequence of Assumption SC, $(Q_i - Q_j)(v - v_i) \geq 0$. Therefore, letting $\delta_i$ and $\varepsilon_i$ go to zero in (6), we get $Q_i = Q_j$, so that the equilibrium must be pooling. Replacing in (6), what we have shown is that for any small enough $\delta_i$ and $\varepsilon_i$ such that $\tau_i(Q_i, T_i)\delta_i < \varepsilon_i$, one has $v\delta_i \leq \varepsilon_i$. Since $\delta_i$ can be positive or negative, it follows that $\tau_i(Q_i, T_i) = v$. However, according to Lemma 2, one also has $\tau_j(Q_j, T_j) = v$ since $B_j > 0$. Because $(Q_i, T_i) = (Q_j, T_j)$ as the equilibrium is pooling, this contradicts Assumption SC. The result follows. ■

**Proof of Lemma 4.** Suppose that $Q_j = 0$. If $\tau_j(0, 0) = v_j$, the result is immediate. Suppose then that $\tau_j(0, 0) \neq v_j$. Any buyer $k$ can deviate by proposing a menu consisting of the no-trade contract and of the contract $c_j^k = (\delta_j, \varepsilon_j)$ for some numbers $\delta_j$ and $\varepsilon_j$. Choose $\delta_j$ and $\varepsilon_j$ such that $\tau_j(0, 0)\delta_j < \varepsilon_j$. This ensures that, for $\delta_j$ and $\varepsilon_j$ small enough, type $j$ can strictly increase her utility by trading $c_j^k$ with buyer $k$. If moreover $v_j\delta_j > \varepsilon_j$, then type $i$ must also trade $c_j^k$ following buyer $k$'s deviation, and one must have $\varepsilon_j \geq v\delta_j$, for, otherwise, this deviation would be profitable. Thus we have shown that for any small enough $\delta_j$ and $\varepsilon_j$, $\tau_j(0, 0)\delta_j < \varepsilon_j < v_j\delta_j$ implies that $\varepsilon_j \geq v\delta_j$, which is equivalent to the statement of the lemma. Hence the result. ■

**Proof of Lemma 5.** By the no cross-subsidization result, if $Q_i \neq 0$, the equilibrium must be separating. Moreover, from Proposition 3, one must have $T_i = v_i Q_i$. Suppose by way of contradiction that $\tau_i(Q_i, T_i) \neq v_i$. Then any buyer $k$ can deviate by proposing a menu consisting of the no-trade contract and of the contract $c_i^k = (q_i, t_i)$, for some numbers $q_i$ and $t_i$. Since $\tau_i(Q_i, T_i) \neq v_i$, it follows from Assumption C that one can choose $(q_i, t_i)$ close to $(Q_i, T_i)$ such that $U_i < u_i(q_i, t_i)$ and $t_i < v_i q_i$, where $q_i$ is positive if $i = L$, and negative if $i = H$. The first inequality guarantees that type $i$ trades $c_i^k$ following buyer $k$'s deviation. Since $v_i q_i > t_i$, type $j$ must also trade $c_i^k$ following buyer $k$'s deviation, and one must have $t_i \geq v q_i$, for, otherwise, this deviation would be profitable. Overall, we have shown that $v_i q_i > v q_i$. Since $q_i$ is positive if $i = L$, and negative if $i = H$, and since $v_H > v > v_L$, we obtain a contradiction in both cases. The result follows. ■

**Proof of Theorem 1.** Suppose first that a pooling equilibrium exists. Then, according to the no cross-subsidization result, $Q_L = Q_H = 0$. Lemma 4 then implies that

$$v_L \leq \tau_L(0,0) \leq v \leq \tau_H(0,0) \leq v_H. \tag{7}$$

Suppose next that a separating equilibrium exists. Then, according again to the no cross-subsidization result, only three scenarios are possible.

(i) In the first case, $Q_H < 0 < Q_L$. Then, by Proposition 3, $T_L = v_L Q_L$ and $T_H = v_H Q_H$. Moreover, by Lemma 5, $\tau_L(Q_L, T_L) = v_L$ and $\tau_L(Q_H, T_H) = v_H$. As a result, $Q_L = Q_L^*$ and $Q_H = Q_H^*$, so that $Q_H^* < 0 < Q_L^*$. Assumption C then implies that

$$\tau_L(0,0) < v_L \quad \text{and} \quad \tau_H(0,0) > v_H. \tag{8}$$

(ii) In the second case, $Q_H = 0 < Q_L$. Then, by Lemma 4, $v \leq \tau_H(0,0) \leq v_H$. Moreover, by Proposition 3, $T_L = v_L Q_L$. Finally, by Lemma 5, $\tau_L(Q_L, T_L) = v_L$. As a result $Q_L = Q_L^*$, so that $Q_L^* > 0$. Assumption C then implies that

$$\tau_L(0,0) < v_L \quad \text{and} \quad v \leq \tau_H(0,0) \leq v_H. \tag{9}$$

(iii) In the third case, $Q_H < 0 = Q_L$. Then, by Lemma 4, $v_L \leq \tau_L(0,0) \leq v$. Moreover, by Proposition 3, $T_H = v_H Q_H$. Finally, by Lemma 5, $\tau_H(Q_H, T_H) = v_H$. As a result $Q_H = Q_H^*$, so that $Q_H^* < 0$. Assumption C then implies that

$$v_L \leq \tau_L(0,0) \leq v \quad \text{and} \quad \tau_H(0,0) > v_H. \tag{10}$$

To conclude the proof, observe that, from (7) to (10), an equilibrium exists only if $\tau_L(0,0) \leq v \leq \tau_H(0,0)$. Since conditions (7) to (10) are mutually exclusive, the characterization of the candidate equilibrium aggregate trades is complete. Hence the result. ■

**Proof of Proposition 5.** Suppose by way of contradiction that $b_j^k > 0$ for some $j$ and $k$. We first show that $S_i = S_j = 0$. To prove that $S_i = 0$, observe that, by the no cross-subsidization result, one has $b_j^l < 0 = B_j$ for some $l \neq k$. From (3), this implies that $m_i S_i \geq B - b^l$. Since $B - b^l = 0$ by the zero-profit result, and since $S_i \leq 0$ by Proposition 1, it follows that $S_i = 0$. To prove that $S_j = 0$, observe that if $b_j^k > 0$, then $b_i^k < 0 = B_i$ by the zero-profit result and the no cross-subsidization result. Arguing as for $S_i$, we obtain that $S_j = 0$. Hence $S_i = S_j = 0$, as claimed. Because $S_i + S_j = (v_i - v_j)(Q_i - Q_j)$, one must have $Q_i = Q_j$, and the equilibrium is pooling: $(Q_i, T_i) = (Q_j, T_j) = (0, 0)$. Now, since $b_j^k > 0$, and since $(Q_j, T_j) = (0, 0)$ can obviously be traded with the buyers other than $k$, one can show as in the proof of Proposition 4 that $\tau_i(0, 0) = v$. Finally, consider buyer $l$ as above. Since $b_j^l < 0$, one has $b_i^l > 0$ by the zero-profit result. Since $(Q_i, T_i) = (0, 0)$ can obviously be traded with the buyers other than $l$, it follows along the same lines that $\tau_j(0, 0) = v$ as well, which contradicts Assumption SC. The result follows. ■

**Proof of Proposition 6.** Consider first the case $Q_i = 0$. Then $T_i = 0$ and $T_j = v_j Q_j$ by the characterization of aggregate equilibrium trades; notice that one may have $Q_j = 0$ as well. It follows that $S_j = v_j(Q_j - Q_i) - (T_j - T_i) = 0$. Now, from the proof of Proposition 1, one has $S_j \leq s_j^k$ for all $k$, so that actually $s_j^k = 0$ for all $k$. Since

$$s_j^k = v_j(q_j^k - q_i^k) - (t_j^k - t_i^k) = b_j^k - b_i^k - (v_j - v_i)q_i^k = (v_i - v_j)q_i^k \tag{11}$$

as $b_i^k = b_j^k = 0$ by Proposition 5, it follows that $q_i^k = 0$ for all $k$. Hence the result.

Consider next the case $Q_L > 0$, the argument for $Q_H < 0$ being symmetrical. Suppose by way of contradiction that $q_L^k < 0$ for some $k$. Then, letting $i = L$ and $j = H$ in (11), it follows that $s_H^k > 0$. We now show that $s_i^k \leq 0$ for all $i$ and $k$, which concludes the proof by contradiction. We first prove in analogy with (2) that for each $i$, $k$, and $l \neq k$, one has

$$s_i^k > 0 \quad \text{only if} \quad m_i s_i^k \leq m_j(s_j^k + s_j^l). \tag{12}$$

Set $q = q_i^k + q_i^l - q_j^k$ and $t = t_i^k + t_i^l - t_j^k$. Then the quantity $Q_i - q = q_j^k + \sum_{m \neq k,l} q_i^m$ can be traded with the buyers other than $k$, in exchange for a transfer $T_i - t = t_j^k + \sum_{m \neq k,l} t_i^m$. We can thus apply Lemma 1. One has

$$\begin{aligned} v_i q - t - b_i^l &= v_i(q_i^k + q_i^l - q_j^k) - (t_i^k + t_i^l - t_j^k) - b_i^l \\ &= b_i^k - b_j^k - (v_i - v_j)q_j^k \\ &= s_i^k \end{aligned}$$

and

$$\begin{aligned} v_j q - t - b_j^l &= v_j(q_i^k + q_i^l - q_j^l) - (t_i^k + t_i^l - t_j^l) - b_j^k \\ &= -[b_j^k - b_i^k - (v_j - v_i)q_i^k + b_j^l - b_i^l - (v_j - v_i)q_i^l] \\ &= -(s_j^k + s_j^l), \end{aligned}$$

so that we get (12) by Lemma 1. Now, suppose by way of contradiction that $s_i^k > 0$ for some $i$ and $k$. Then, by (12),

$$m_i s_i^k \leq m_j(s_j^k + s_j^l) \tag{13}$$

for all $l \neq k$. Summing on $l \neq k$ yields

$$(n-1)m_i s_i^k \leq m_j[S_j + (n-2)s_j^k].$$

From Proposition 1, we know that $S_j \leq 0$. Hence, if $s_i^k > 0$, one must also have $s_j^k > 0$. Applying (12) once more yields

$$m_j s_j^k \leq m_i(s_i^k + s_i^l) \tag{14}$$

for all $l \neq k$. Combining (13) and (14) leads to $m_i s_i^k \leq m_j s_j^l + m_i(s_i^k + s_i^l)$, or, equivalently, $m_i s_i^l + m_j s_j^l \geq 0$ for all $l \neq k$. Note that we also have $m_i s_i^k + m_j s_j^k > 0$ as both $s_i^k$ and $s_j^k$ are positive. Summing all these inequalities yields $m_i S_i + m_j S_j > 0$, in contradiction with Proposition 1. Hence the result. ■

**Proof of Theorem 2.** Consider some buyer $k$ that attempts to deviate from a candidate equilibrium. When constructing his deviation, he can restrict himself to menus that contain, on top of the no-trade contract, at most two other contracts $(\tilde{q}_L^k, \tilde{t}_L^k)$ and $(\tilde{q}_H^k, \tilde{t}_H^k)$. For $(\tilde{q}_L^k, \tilde{t}_L^k)$ to be selected by type $L$, and $(\tilde{q}_H^k, \tilde{t}_H^k)$ to be selected by type $H$, the following incentive and participation constraints must hold:

$$z_L^{-k}(\tilde{q}_L^k, \tilde{t}_L^k) \geq z_L^{-k}(\tilde{q}_H^k, \tilde{t}_H^k), \tag{15}$$

$$z_H^{-k}(\tilde{q}_H^k, \tilde{t}_H^k) \geq z_H^{-k}(\tilde{q}_L^k, \tilde{t}_L^k), \tag{16}$$

$$z_L^{-k}(\tilde{q}_L^k, \tilde{t}_L^k) \geq U_L, \tag{17}$$

$$z_H^{-k}(\tilde{q}_H^k, \tilde{t}_H^k) \geq U_H. \tag{18}$$

Observe that, in formulating the participation constraints, we implicitly supposed that the equilibrium aggregate trade of each type remains available following buyer $k$'s deviation.

**Separating Equilibria** There are three subcases to examine.

(i) Suppose first that $\tau_L(0,0) < v_L$ and $\tau_H(0,0) > v_H$. Then $Q_L^* > 0 > Q_H^*$, and the candidate equilibrium aggregate trades are characterized by $Q_L = Q_L^*$, $T_L = v_L Q_L$, $Q_H = Q_H^*$, and $T_H = v_H Q_H$, with $\tau_L(Q_L, T_L) = v_L$ and $\tau_H(Q_H, T_H) = v_H$. We show that there exists an equilibrium in which each buyer offers the menu

$$C_{FB} = \{(0,0), (Q_L^*, v_L Q_L^*), (Q_H^*, v_H Q_H^*)\}.$$

By construction, the equilibrium aggregate trade of type $L$ remains available following buyer $k$'s deviation. An upper bound to buyer $k$'s profit from deviating is given by

$$\max \{m_L(v_L \tilde{q}_L^k - \tilde{t}_L^k) + m_H(v_H \tilde{q}_H^k - \tilde{t}_H^k)\}$$

subject to the participation constraints (17) and (18). For each $i$, one must thus solve

$$\max \{v_i \tilde{q}_i^k - \tilde{t}_i^k\}$$

subject to $z_i^{-k}(\tilde{q}_i^k, \tilde{t}_i^k) \geq U_i$. We claim that the value of this problem is zero. Indeed, let $(\tilde{Q}_i, \tilde{T}_i)$ be a final aggregate trade of type $i$ when trading $(\tilde{q}_i^k, \tilde{t}_i^k)$ with buyer $k$ and optimally choosing from the menus $C_{FB}$ offered by the buyers other than $k$. Then

$$\tilde{Q}_i = n_i Q_i^* + n_j Q_j^* + \tilde{q}_i^k,$$
$$\tilde{T}_i = n_i v_i Q_i^* + n_j v_j Q_j^* + \tilde{t}_i^k,$$

where $n_i$ and $n_j$ are the numbers of times type $i$ optimally trades the contracts $(Q_i^*, v_i Q_i^*)$ and $(Q_j^*, v_j Q_j^*)$, respectively, with the buyers other than $k$. Thus

$$v_i \tilde{q}_i^k - \tilde{t}_i^k = n_j(v_j - v_i)Q_j^* + v_i \tilde{Q}_i - \tilde{T}_i \leq v_i \tilde{Q}_i - \tilde{T}_i \leq 0,$$

where the first inequality reflects that $v_H > v_L$ and $Q_L^* > 0 > Q_H^*$, and the second that $u_i(\tilde{Q}_i, \tilde{T}_i) \geq u_i(Q_i^*, v_i Q_i^*) = \max_Q u_i(Q, v_i Q)$. Hence, given the menus $C_{FB}$ offered by the buyers other than $k$, a contract $(\tilde{q}_i^k, \tilde{t}_i^k)$ may attract type $i$ only if $\tilde{t}_i^k \geq v_i \tilde{q}_i^k$. As a result, there is no profitable deviation for buyer $k$. The result follows.

(ii) Suppose next that $\tau_L(0,0) < v_L$ and $v \leq \tau_H(0,0) \leq v_H$. Then $Q_L^* > 0$, and the candidate equilibrium aggregate trades are characterized by $Q_L = Q_L^*$, $T_L = v_L Q_L$, and $Q_H = T_H = 0$, with $\tau_L(Q_L, T_L) = v_L$. Observe in particular that $\overline{Q}_L \geq Q_L^* > 0$ for each $\overline{Q}_L$ that satisfies Assumption T. Fix one such $\overline{Q}_L$. We show that there exists an equilibrium in which each buyer offers the menu

$$C_L = \left\{(q,t) : 0 \leq q \leq \frac{\overline{Q}_L}{n-1} \text{ and } t = v_L q\right\}.$$

Since $\tau(Q_L, T_L) = v_L$, one has $Q_L < \overline{Q}_L$ by definition of $\overline{Q}_L$, so that the equilibrium aggregate trade of type $L$ remains available following buyer $k$'s deviation. An upper bound to buyer $k$'s profit from deviating is given by

$$\max\{m_L(v_L\tilde{q}_L^k - \tilde{t}_L^k) + m_H(v_H\tilde{q}_H^k - \tilde{t}_H^k)\}$$

subject to the incentive constraint (15), and the participation constraints (17) and (18). Since $z_L^{-k}$ and $z_H^{-k}$ are strictly decreasing with respect to transfers, (18) must be binding. That is, letting $(\tilde{Q}_H, \tilde{T}_H)$ be a final aggregate trade of type $H$ when trading $(\tilde{q}_H^k, \tilde{t}_H^k)$ with buyer $k$ and optimally choosing from the menus $C_L$ offered by the buyers other than $k$, one must have $u_H(\tilde{Q}_H, \tilde{T}_H) = U_H$. Two cases must be distinguished.

If $\tilde{Q}_H \leq 0$, then, since $\tilde{q}_H^k \leq \tilde{Q}_H$ and $\tau_H(0,0) \leq v_H$, $v_H\tilde{q}_H^k - \tilde{t}_H^k \leq 0$. Moreover, $z_L^{-k}(\tilde{q}_H^k, \tilde{t}_H^k) < U_L$, so that, by (17), (15) must be slack. Thus $(\tilde{q}_L^k, \tilde{t}_L^k)$ solves

$$\max\{v_L\tilde{q}_L^k - \tilde{t}_L^k\}$$

subject to the participation constraint (17). We claim that the value of this problem is zero. Indeed, let $(\tilde{Q}_L, \tilde{T}_L)$ be a final aggregate trade of type $L$ when trading $(\tilde{q}_L^k, \tilde{t}_L^k)$ with buyer $k$ and optimally choosing from the menus $C_L$ offered by the buyers other than $k$. Then

$$\begin{aligned}\tilde{Q}_L &= Q_L^{-k} + \tilde{q}_L^k,\\ \tilde{T}_L &= v_L Q_L^{-k} + \tilde{t}_L^k,\end{aligned}$$

where $(Q_L^{-k}, v_L Q_L^{-k})$, with $Q_L^{-k} \in [0, \overline{Q}_L]$, is the aggregate trade type $L$ makes with the buyers other than $k$. Thus

$$v_L\tilde{q}_L^k - \tilde{t}_L^k = v_L\tilde{Q}_L - \tilde{T}_L \leq 0,$$

where the inequality reflects that $u_L(\tilde{Q}_L, \tilde{T}_L) \geq u_L(Q_L^*, v_L Q_L^*) = \max_Q u_L(Q, v_L Q)$. Hence, given the menus $C_L$ offered by the buyers other than $k$, a contract $(\tilde{q}_L^k, \tilde{t}_L^k)$ may attract type $L$ only if $\tilde{t}_L^k \geq v_L\tilde{q}_L^k$. As a result, there is no profitable deviation for buyer $k$ such that $\tilde{Q}_H \leq 0$.

If $\tilde{Q}_H > 0$, then, since $\tau_H(\tilde{Q}_H, \tilde{T}_H) > \tau_H(0,0) \geq v > v_L$ as $u_H(\tilde{Q}_H, \tilde{T}_H) = u_H(0,0)$, one must have $\tilde{q}_H^k = \tilde{Q}_H$; for, if $\tilde{q}_H^k < \tilde{Q}_H$, then type $H$ could strictly increase her utility by trading $(\tilde{q}_H^k, \tilde{t}_H^k)$ with buyer $k$ and $(\tilde{Q}_H - \tilde{q}_H^k - \varepsilon, v_L(\tilde{Q}_H - \tilde{q}_H^k - \varepsilon))$ with the buyers other than $k$, for $\varepsilon$ positive and small enough. Define $(\tilde{Q}_L, \tilde{T}_L)$ as above. By Assumption SC, $\tilde{Q}_L \geq \tilde{Q}_H = \tilde{q}_H^k$. Now, suppose that $\tilde{T}_L - \tilde{t}_H^k \geq v_L(\tilde{Q}_L - \tilde{q}_H^k)$. Then, since $v_L\tilde{q}_L^k - \tilde{t}_L^k = v_L\tilde{Q}_L - \tilde{T}_L$, it follows by averaging that buyer $k$'s profit from deviating is at most $v\tilde{q}_H^k - \tilde{t}_H^k$, which is negative, since $u_H(\tilde{q}_H^k, \tilde{t}_H^k) = u_H(0,0)$, $\tilde{q}_H^k > 0$, and $\tau_H(0,0) \geq v$. Thus buyer $k$ can earn a positive profit only if $\tilde{T}_L - \tilde{t}_H^k < v_L(\tilde{Q}_L - \tilde{q}_H^k)$. If $\tilde{Q}_L \leq \tilde{q}_H^k + \overline{Q}_L$, this is impossible if type $L$ is optimizing when trading $(\tilde{Q}_L, \tilde{T}_L)$ in the aggregate, since she always has the option to trade $(\tilde{q}_H^k, \tilde{t}_H^k)$ with buyer $k$, and then to sell any positive quantity up to $\overline{Q}_L$ at a unit price $v_L$ to the buyers other than $k$. If $\tilde{Q}_L > \tilde{q}_H^k + \overline{Q}_L$, then, because $\tau_L(\tilde{q}_H^k + \overline{Q}_L, \tilde{t}_H^k + v_L\overline{Q}_L) > v_L$ by Assumption T, one has

$$\begin{aligned} u_L(\tilde{q}_H^k + \overline{Q}_L, \tilde{t}_H^k + v_L\overline{Q}_L) &> u_L(\tilde{q}_H^k + \tilde{Q}_L - \tilde{q}_H^k, \tilde{t}_H^k + v_L(\tilde{Q}_L - \tilde{q}_H^k)) \\ &> u_L(\tilde{Q}_L, \tilde{T}_L). \end{aligned}$$

Thus type $L$ would be strictly better off trading $(\tilde{q}_H^k, \tilde{t}_H^k)$ with buyer $k$, and then selling $\overline{Q}_L$ at a unit price $v_L$ to the buyers other than $k$, a contradiction. Hence there is no profitable deviation for buyer $k$ such that $\tilde{Q}_H > 0$. The result follows.

(iii) Suppose finally that $v_L \leq \tau_L(0,0) \leq v$ and $\tau_H(0,0) > v_H$. Then $Q_H^* < 0$, and the candidate equilibrium aggregate trades are characterized by $Q_L = T_L = 0$, $Q_H = Q_H^*$, and $T_H = v_H Q_H$, with $\tau_H(Q_H, T_H) = v_H$. Observe in particular that $\underline{Q}_H \leq Q_H^* < 0$ for each $\underline{Q}_H$ that satisfies Assumption T. Fix one such $\underline{Q}_H$. One can show as in (ii) above that there exists an equilibrium in which each buyer offers the menu

$$C_H = \left\{(q,t) : \frac{\underline{Q}_H}{n-1} \leq q \leq 0 \text{ and } t = v_H q\right\}.$$

The result follows.

**Pooling Equilibria** If $v_L \leq \tau_L(0,0) \leq v \leq \tau_H(0,0) \leq v_H$, then $Q_L^* \leq 0 \leq Q_H^*$, and the candidate equilibrium aggregate trades are characterized by $Q_L = T_L = Q_H = T_H = 0$. Suppose without loss of generality that $\overline{Q}_L > 0 > \underline{Q}_H$, where $\overline{Q}_L$ and $\underline{Q}_H$ satisfy Assumption T. Fix two such $\overline{Q}_L$ and $\underline{Q}_H$. We show that there exists an equilibrium in which each buyer offers the menu

$$C_{LH} = C_L \cup C_H.$$

The following result reflects how the structure of offers in the menus $C_{LH}$ affects the seller's behavior following a deviation by buyer $k$.

**Fact 1** *Let $(\tilde{Q}_i, \tilde{T}_i)$ be a final aggregate trade of type $i$ when trading $(\tilde{q}_i^k, \tilde{t}_i^k)$ with buyer $k$ and optimally choosing from the menus $C_{LH}$ offered by the buyers other than $k$. Then,*

*(i) If $\tau_i(\tilde{Q}_i, \tilde{T}_i) > v_L$, then $\tilde{q}_i^k \geq \tilde{Q}_i$ and $\tilde{T}_i - \tilde{t}_i^k = v_H(\tilde{Q}_i - \tilde{q}_i^k)$,*

*(ii) If $\tau_i(\tilde{Q}_i, \tilde{T}_i) < v_H$, then $\tilde{q}_i^k \leq \tilde{Q}_i$ and $\tilde{T}_i - \tilde{t}_i^k = v_L(\tilde{Q}_i - \tilde{q}_i^k)$.*

**Proof.** Consider first case (i). If $(\tilde{Q}_i, \tilde{T}_i)$ is a final aggregate trade of type $i$ when trading $(\tilde{q}_i^k, \tilde{t}_i^k)$ with buyer $k$, then there exist trades $(\tilde{q}_i^l, \tilde{t}_i^l)$ with the buyers other than $k$, such that $\tilde{Q}_i - \tilde{q}_i^k = \sum_{l \neq k} \tilde{q}_i^l$, $\tilde{T}_i - \tilde{t}_i^k = \sum_{l \neq k} \tilde{t}_i^l$, and $(\tilde{q}_i^l, \tilde{t}_i^l) \in C_{LH}$ for all $l \neq k$. Now, suppose by way of contradiction that $\tilde{q}_i^l > 0$ for some $l \neq k$. Then $(\tilde{q}_i^l, \tilde{t}_i^l) \in C_L$, and $(\tilde{q}_i^l - \varepsilon, \tilde{t}_i^l - v_L\varepsilon) \in C_L$ as long as $0 < \varepsilon < \tilde{q}_i^l$. By trading $(\tilde{q}_i^l - \varepsilon, \tilde{t}_i^l - v_L\varepsilon)$ with buyer $l$, instead of $(\tilde{q}_i^l, \tilde{t}_i^l)$, and by keeping all her other trades unchanged, type $i$ can trade $(\tilde{Q}_i - \varepsilon, \tilde{T}_i - v_L\varepsilon)$ in the aggregate. However, if $\tau_i(\tilde{Q}_i, \tilde{T}_i) > v_L$, one has $u_i(\tilde{Q}_i - \varepsilon, \tilde{T}_i - v_L\varepsilon) > u_i(\tilde{Q}_i, \tilde{T}_i)$ for $\varepsilon$ positive and small enough, contradicting the assumption that type $i$ is optimizing when trading $(\tilde{Q}_i, \tilde{T}_i)$ in the aggregate. Thus we have proved that $\tilde{q}_i^l \leq 0$ for all $l \neq k$. Then $(\tilde{q}_i^l, \tilde{t}_i^l) \in C_H$ for all $l \neq k$, so that $\tilde{q}_i^k \geq \tilde{Q}_i$ and $\tilde{T}_i - \tilde{t}_i^k = v_H(\tilde{Q}_i - \tilde{q}_i^k)$, as claimed. Case (ii) follows in a similar manner. Hence the result. ■

We can now go on with the proof. We first show that there exists no profitable pooling deviation for buyer $k$. Indeed, suppose that the contract $(\tilde{q}^k, \tilde{t}^k)$ is offered by buyer $k$. Then, if $\tilde{q}^k \geq 0$ and $v\tilde{q}^k - \tilde{t}^k > 0$, type $H$ does not want to trade $(\tilde{q}^k, \tilde{t}^k)$ given the menus $C_{LH}$ offered by the buyers other than $k$, because $v \leq \tau_H(0, 0) \leq v_H$. Similarly, if $\tilde{q}^k \leq 0$ and $v\tilde{q}^k - \tilde{t}^k > 0$, type $L$ does not want to trade $(\tilde{q}^k, \tilde{t}^k)$ given the menus $C_{LH}$ offered by the buyers other than $k$, because $v_L \leq \tau_L(0, 0) \leq v$. Hence, if both types trade the same contract $(\tilde{q}^k, \tilde{t}^k)$ with buyer $k$, the resulting profit for buyer $k$ is at most zero. This implies that, if buyer $k$ attempts to deviate by offering, on top of the no-trade contract, two contracts $(\tilde{q}_L^k, \tilde{t}_L^k)$ and $(\tilde{q}_H^k, \tilde{t}_H^k)$ such that both incentive constraints (15) and (16) of types $L$ and $H$ are binding, one can always construct the continuation equilibrium in such a way that both types select the same contract, resulting in at most a zero profit for buyer $k$. One can thus focus without loss of generality on deviations by buyer $k$ such that at least one incentive constraint (15) or (16) is slack. Since $z_L^{-k}$ and $z_H^{-k}$ are strictly decreasing with respect to transfers, at least one of the participation constraints (17) or (18) must then be binding. In what follows, we suppose that (18) is binding, that is, in the notation of Fact 1, $u_H(\tilde{Q}_H, \tilde{T}_H) = U_H$; the argument when (17) is binding is symmetrical. Two cases must be distinguished.

If $\tilde{Q}_H < 0$, then, since $u_H(\tilde{Q}_H, \tilde{T}_H) = u_H(0, 0)$, $\tau_H(\tilde{Q}_H, \tilde{T}_H) < \tau_H(0, 0) \leq v_H$. Thus, by Fact 1(ii), $\tilde{q}_H^k \leq \tilde{Q}_H$ and $v_L\tilde{q}_H^k - \tilde{t}_H^k = v_L\tilde{Q}_H - \tilde{T}_H$. Since $v_H\tilde{Q}_H - \tilde{T}_H < 0$, one thus has

$$\begin{aligned} v_H\tilde{q}_H^k - \tilde{t}_H^k &= (v_H - v_L)\tilde{q}_H^k + v_L\tilde{q}_H^k - \tilde{t}_H^k \\ &= (v_H - v_L)\tilde{q}_H^k + v_L\tilde{Q}_H - \tilde{T}_H \end{aligned}$$

$$= (v_H - v_L)(\tilde{q}_H^k - \tilde{Q}_H) + v_H\tilde{Q}_H - \tilde{T}_H$$
$$< 0.$$

Consider now $(\tilde{Q}_L, \tilde{T}_L)$, as defined in Fact 1. One must have $\tilde{Q}_L \geq 0$, for, otherwise, (17) along with the fact that (18) is binding would imply that (16) is violated. It is easy to deduce from this that (17) is binding. Indeed, since $z_L^{-k}$ and $z_H^{-k}$ are strictly decreasing with respect to transfers, (15) would otherwise be binding, which is impossible since $U_L > z_L^{-k}(\tilde{q}_H^k, \tilde{t}_H^k)$. To summarize, $\tilde{Q}_L \geq 0$ and (17) is binding if $\tilde{Q}_H < 0$. In particular, if $\tilde{Q}_L = 0$, then $\tilde{T}_L = 0$, so that $\tau_L(\tilde{Q}_L, \tilde{T}_L) \leq v < v_H$ and $v_L\tilde{q}_L^k - \tilde{t}_L^k = v_L\tilde{Q}_L - \tilde{T}_L = 0$ by Fact 1(ii). Finally, if $\tilde{Q}_L > 0$, then, since $u_L(\tilde{Q}_L, \tilde{T}_L) = u_L(0,0)$, $\tau_L(\tilde{Q}_L, \tilde{T}_L) > \tau_H(0,0) \geq v_L$. By Fact 1(i), $\tilde{q}_L^k \geq \tilde{Q}_L$ and $v_H\tilde{q}_L^k - \tilde{t}_L^k = v_H\tilde{Q}_L - \tilde{T}_L$. Since $v_L\tilde{Q}_L - \tilde{T}_L < 0$, one thus has

$$\begin{aligned}
v_L\tilde{q}_L^k - \tilde{t}_L^k &= (v_L - v_H)\tilde{q}_L^k + v_H\tilde{q}_L^k - \tilde{t}_L^k \\
&= (v_L - v_H)\tilde{q}_L^k + v_H\tilde{Q}_L - \tilde{T}_L \\
&= (v_L - v_H)(\tilde{q}_L^k - \tilde{Q}_L) + v_L\tilde{Q}_L - \tilde{T}_L \\
&< 0.
\end{aligned}$$

Overall, we have shown that, if $\tilde{Q}_H < 0$, then $v_H\tilde{q}_H^k - \tilde{t}_H^k < 0$ and $v_L\tilde{q}_L^k - \tilde{t}_L^k \leq 0$. Hence there is no profitable deviation for buyer $k$ such that $\tilde{Q}_H < 0$.

If $\tilde{Q}_H \geq 0$, then, since $u_H(\tilde{Q}_H, \tilde{T}_H) = u_H(0,0)$, $\tau_H(\tilde{Q}_H, \tilde{T}_H) \geq \tau_H(0,0) \geq v > v_L$. Thus, by Fact 1(i), $\tilde{q}_H^k \geq \tilde{Q}_H$ and $v_H\tilde{q}_H^k - \tilde{t}_H^k = v_H\tilde{Q}_H - \tilde{T}_H \leq 0$, so that it remains only to show that $v_L\tilde{q}_L^k - \tilde{t}_L^k \leq 0$. Note also that $\tilde{Q}_L \geq \tilde{Q}_H$ by Assumption SC. Two subcases must be distinguished. If $\tilde{q}_H^k > \tilde{Q}_H$, then (17) is slack, so that, since $z_L^{-k}$ and $z_H^{-k}$ are strictly decreasing with respect to transfers, (15) must be binding. Proceeding as in the end of case (ii) of the proof for separating equilibria, one can show that buyer $k$ can earn no profit from deviating if $\tilde{Q}_L \geq \tilde{q}_H^k$. Thus, if $\tilde{q}_H^k > \tilde{Q}_H$, buyer $k$ may earn a positive profit only if $\tilde{Q}_H \leq \tilde{Q}_L < \tilde{q}_H^k$. Observe that $\tilde{T}_L - \tilde{T}_H \geq v_H(\tilde{Q}_L - \tilde{Q}_H)$ since type $L$ always has the option to trade $(\tilde{q}_H^k, \tilde{t}_H^k)$ with buyer $k$, and then to sell any negative quantity up to $\tilde{Q}_H - \tilde{q}_H^k$ at a unit price $v_H$ to the buyers other than $k$. Since $\tilde{Q}_L \geq \tilde{Q}_H \geq 0$, it follows that $v_L\tilde{Q}_L - \tilde{T}_L \leq v_H\tilde{Q}_H - \tilde{T}_H \leq 0$. Hence, to complete the argument in the case where $\tilde{q}_H^k > \tilde{Q}_H \geq 0$, we only need to check that $v_L\tilde{q}_L^k - \tilde{t}_L^k \leq v_L\tilde{Q}_L - \tilde{T}_L$. Let $(\tilde{q}_L^l, \tilde{t}_L^l)$ be the trades by type $L$ with buyers $l \neq k$, while trading $(\tilde{q}_L^k, \tilde{t}_L^k)$ with buyer $k$. Then, proceeding as in the end of case (i) of the proof for separating equilibria, one can show that

$$v_L\tilde{q}_L^k - \tilde{t}_L^k \leq (v_H - v_L) \sum_{\{l \neq k \,:\, \tilde{q}_L^l \leq 0\}} \tilde{q}_L^l + v_L\tilde{Q}_L - \tilde{T}_L \leq v_L\tilde{Q}_L - \tilde{T}_L,$$

as claimed. Hence there is no profitable deviation for buyer $k$ such that $\tilde{q}_H^k > \tilde{Q}_H \geq 0$. To conclude the proof, consider the case where $\tilde{q}_H^k = \tilde{Q}_H \geq 0$. If (17) is slack, the same reasoning as above implies that there are no profitable deviation for buyer $k$. Thus such a deviation may exist only if (17) is binding. One must then have $\tilde{Q}_H = 0$, for, otherwise, (15) would be violated. If $\tau_L(\tilde{Q}_L, \tilde{T}_L) < v_H$, then, by Fact 1(ii), $v_L \tilde{q}_L^k - \tilde{t}_L^k = v_L \tilde{Q}_L - \tilde{T}_L$, which is at most zero since $u_L(\tilde{Q}_L, \tilde{T}_L) = u_L(0,0)$, $\tilde{Q}_L \geq 0$, and $\tau_L(0,0) \geq v_L$. If $\tau_L(\tilde{Q}_L, \tilde{T}_L) \geq v_H > v_L$, then, by Fact 1(i), $q_L^k \geq \tilde{Q}_L$ and $v_H \tilde{q}_L^k - \tilde{t}_L^k = v_H \tilde{Q}_L - \tilde{T}_L$. By now standard computations, one has

$$v_L \tilde{q}_L^k - \tilde{t}_L^k = (v_L - v_H)(\tilde{q}_L^k - \tilde{Q}_L) + v_L \tilde{Q}_L - \tilde{T}_L,$$

which again is at most zero. Hence there is no profitable deviation for buyer $k$ such that $\tilde{q}_H^k = \tilde{Q}_H \geq 0$. The result follows. ■

# References


[1] Acharya, V.V., and O. Merrouche (2009): "Precautionary Hoarding of Liquidity and Inter-Bank Markets: Evidence from the Sub-prime Crisis", Unpublished Manuscript, Stern School of Business, New York University.

[2] Acharya, V.V, and A. Bisin (2010): "Centralized versus Over The Counter Markets," Unpublished Manuscript, Stern School of Business, New York University.

[3] Adrian, T., and H.S. Shin (2010): "The Changing Nature of Financial Intermediation and the Financial Crisis of 2007–09", *Annual Review*: *Economics* (forthcoming).

[4] Akerlof, G.A. (1970): "The Market for "Lemons": Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 84(3), 488–500.

[5] Ales, L., and P. Maziero (2009): "Adverse Selection and Non-Exclusive Contracts," Unpublished Manuscript, Carnegie Mellon University.

[6] Arnott, R.J., and Stiglitz, J.E. (1993): "Equilibrium in Competitive Insurance Markets with Moral Hazard," Mimeo, Boston College.

[7] Attar, A., and A. Chassagnon (2009): "On Moral Hazard and Nonexclusive Contracts," *Journal of Mathematical Economics*, 45(9–10), 511–525.

[8] Attar, A., T. Mariotti, and F. Salanié (2009): "Non-Exclusive Competition in the Market for Lemons," Unpublished Manuscript, Toulouse School of Economics.

[9] Biais, B., D. Martimort, and J.-C. Rochet (2000): "Competing Mechanisms in a Common Value Environment," *Econometrica*, 68(4), 799–837.

[10] Bisin, A., and D. Guaitoli (2004): "Moral Hazard and Nonexclusive Contracts," *RAND Journal of Economics*, 35(2), 306–328.

[11] Brown, J. (2007): "Rational and Behavioral Perspectives on the Role of Annuities in Retirement Planning," Unpublished Manuscript, University of Illinois at Urbana-Champaign.

[12] Brunnermeier, M.K. (2009): "Deciphering the Liquidity and Credit Crunch 2007–2008," *Journal of Economic Perspectives*, 23(1), 77–100.

[13] Caillaud, B., R. Guesnerie, P. Rey, and J. Tirole (1988): "Government Intervention in Production and Incentives Theory: A Review of Recent Contributions," *RAND Journal of Economics*, 19(1), 1–26.

[14] Cawley, J., and T.J. Philipson (1999): "An Empirical Examination of Information Barriers to Trade in Insurance," *American Economic Review*, 89(4), 827–846.

[15] Demyanyk, Y., and O. Van Hemert (2009): "Understanding the Subprime Mortgage Crisis," *Review of Financial Studies* (forthcoming).

[16] Detragiache, E., P. Garella, and L. Guiso (2000): "Multiple versus Single Banking Relationships: Theory and Evidence," *Journal of Finance*, 55(3), 1133–1161.

[17] Finkelstein, A., and J.M. Poterba (2004): "Adverse Selection in Insurance Markets: Policyholder Evidence from the U.K. Annuity Market," *Journal of Political Economy*, 112(1), 183–208.

[18] Gorton, G. (2009): "Information, Liquidity, and the (Ongoing) Panic of 2007," *American Economic Review Papers and Proceedings*, 99(2), 567–572.

[19] Heider, F., M. Hoerova, and C. Holthausen (2009): "Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk," Working Paper No. 126, European Central Bank.

[20] Hellwig, M.F. (1983): "On Moral Hazard and Non-Price Equilibria in Competitive Insurance Markets," Discussion Paper No. 109, Sonderforschungsbereich 21, University of Bonn.

[21] Hellwig, M.F. (1988): "A Note on the Specification of Interfirm Communication in Insurance Markets with Adverse Selection," *Journal of Economic Theory*, 46(1), 154–163.

[22] Jaynes, G.D. (1978): "Equilibria in Monopolistically Competitive Insurance Markets," *Journal of Economic Theory*, 19(2), 394–422.

[23] Keys, B.J., T. Mukherjee, A. Seru, and V. Vig (2010): "Did Securitization Lead to Lax Screening? Evidence from Subprime Loans," *Quarterly Journal of Economics*, 125(1), 307-362.

[24] Krishnamurthy, A. (2009): "The Financial Meltdown: Data and Diagnoses," Unpublished Manuscript, Kellog School of Management, Northwestern University.

[25] Martimort, D., and L. Stole (2002): "The Revelation and Delegation Principles in Common Agency Games," *Econometrica*, 70(4), 1659–1673.

[26] Martimort, D., and L. Stole (2009): "Market Participation under Delegated and Intrinsic Common-Agency Games," *RAND Journal of Economics*, 40(1), 78–102.

[27] Ongena S., and D.C. Smith (2000): "What Determines the Number of Bank Relationships? Cross-Country Evidence," *Journal of Financial Intermediation*, 9(1), 26–56.

[28] Parlour, C.A., and U. Rajan (2001): "Competition in Loan Contracts," *American Economic Review*, 91(5), 1311–1328.

[29] Pauly, M.V. (1974): "Overinsurance and Public Provision of Insurance: The Roles of Moral Hazard and Adverse Selection," *Quarterly Journal of Economics*, 88(1), 44–62.

[30] Peters, M. (2001): "Common Agency and the Revelation Principle," *Econometrica*, 69(5), 1349–1372.

[31] Philippon, T., and V. Skreta (2010): "Optimal Interventions in Markets with Adverse Selection," Unpublished Manuscript, Stern School of Business, New York University.

[32] Rothschild, M., and J.E. Stiglitz (1976): "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics*, 90(4), 629–649.

[33] Rysman, M. (2007): "An Empirical Analysis of Payment Card Usage," *Journal of Industrial Economics*, 60(1), 1–36.

[34] Taylor, J.B., and J.C. Williams (2009): "A Black Swan in the Money Market," *American Economic Journal: Macroeconomics*, 1(1), 58–83.

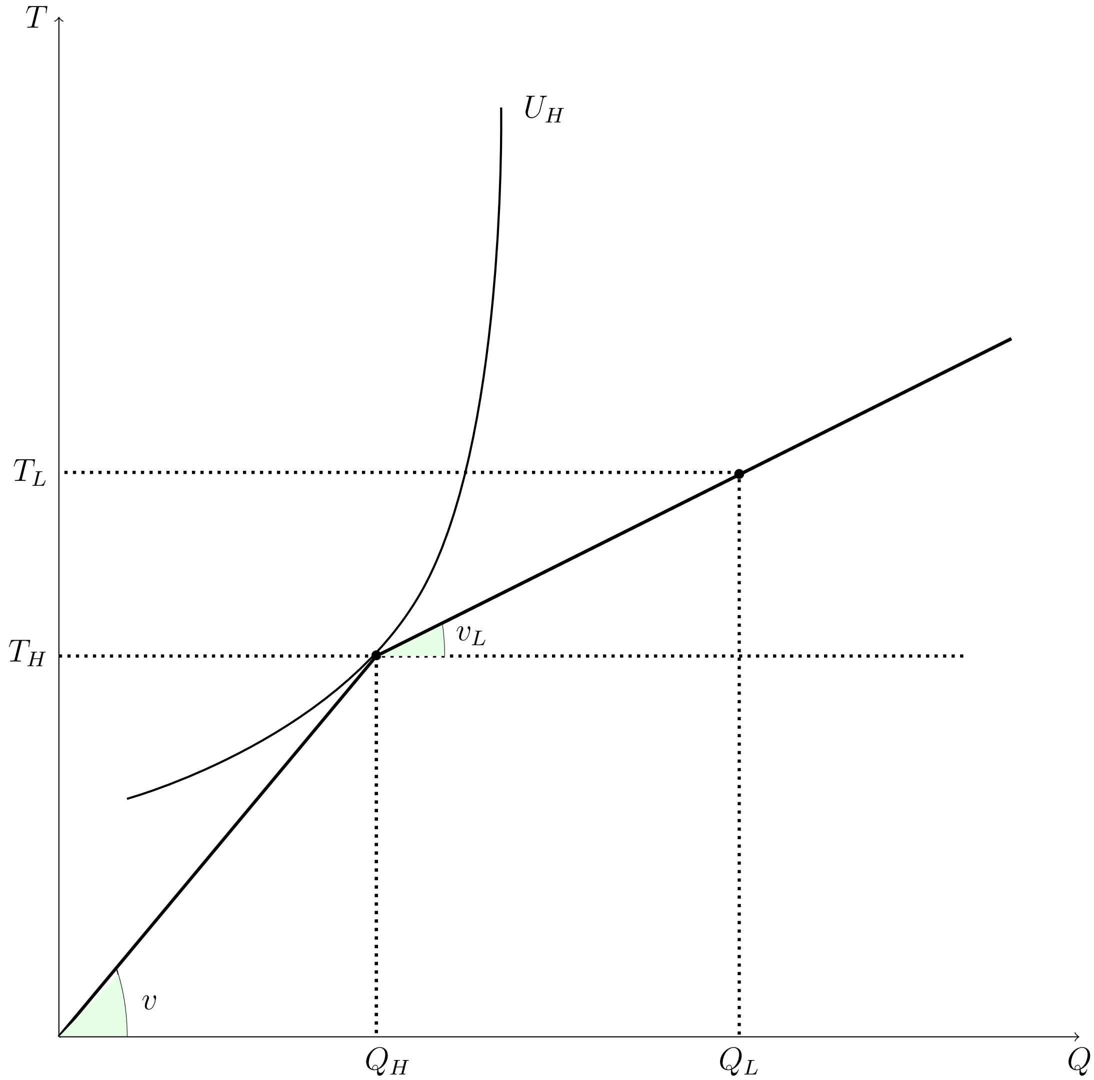


**Figure 1** This figure depicts a candidate separating equilibrium with $Q_L > Q_H > 0$.

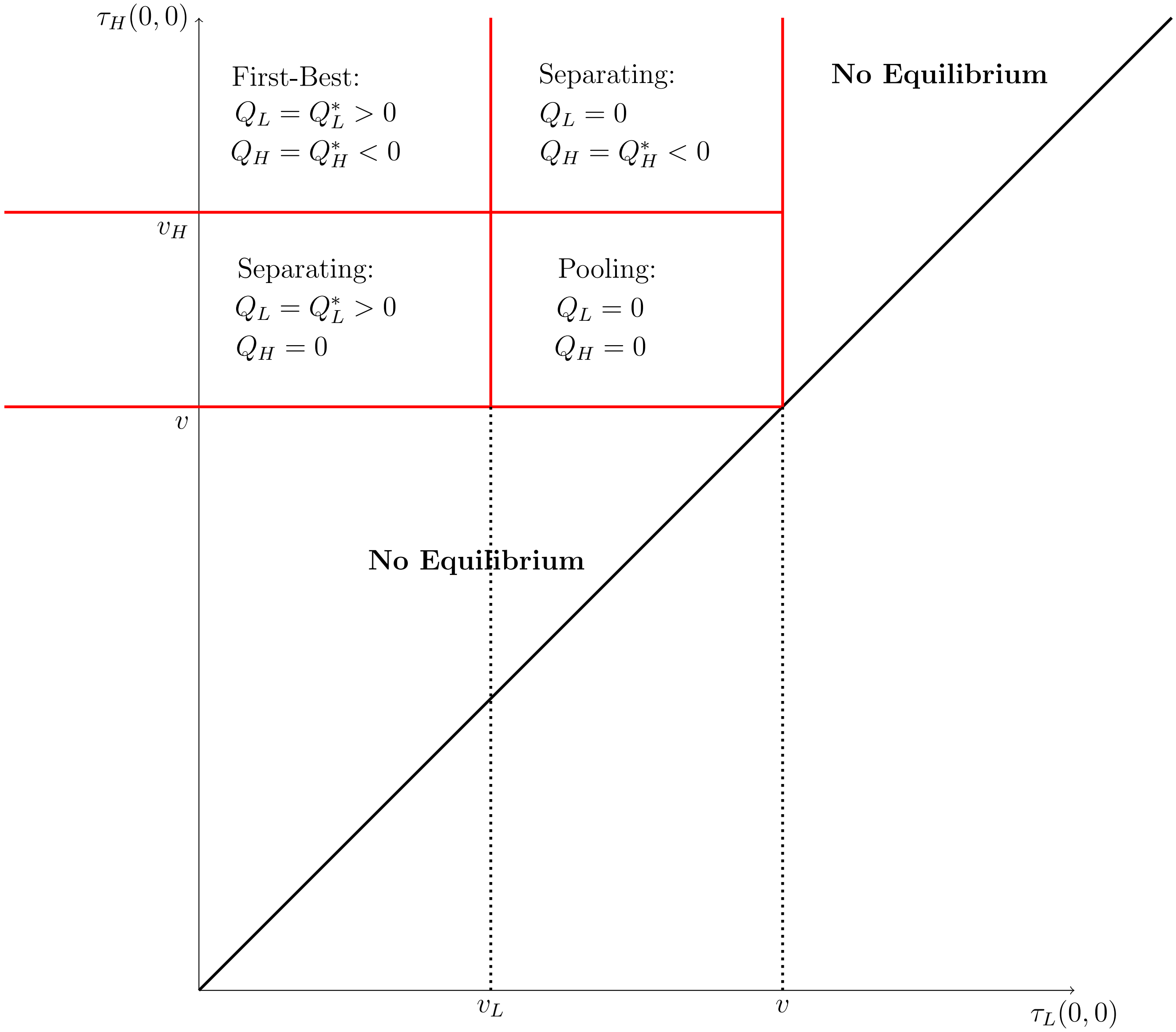


**Figure 2** This figure depicts the structure of equilibrium aggregate trades as a function of $\tau_L(0,0)$ and $\tau_H(0,0) > \tau_L(0,0)$, for fixed parameters $v_L$, $v_H$, and $v$.