Hiring in Labor Markets with Information Revelation

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Abstract

We investigate a common-value multi-stage labor market in which firms interview workers prior to hiring. When firms have private information about workers' quality and interview decisions are kept private, many firms may enter the market, interview, and hire. When firms' interview decisions are revealed, severe adverse selection arises. As a result, all firms except for the highest-ranked firm are excluded from the market.

1 Introduction

The hiring process in many firms includes several stages, at the end of which employment offers are made. This process often begins by conducting an initial evaluation of a potential worker's resumé and other credentials. If the evaluation proves favorable, the worker proceeds to the next stage, which may consist of an interview or fly-out, or a step of an administrative nature such as a "short list." At the end of the process, the firm may offer the worker a job. Many professionals, including academic economists, newly minted MBAs, law school graduates, and to some extent medical residents, are hired in this way.

We investigate a common-value setting in which several privately-informed firms may be interested in hiring a worker, and ask how making firms' intermediate decisions known to other firms affects the hiring process and the resulting allocation. To be more concrete, consider the academic job market for economists. In recent years, an online resource called Econjobmarket[1] started listing universities' interview and fly-out decisions, nearly in real time. A natural question is how this information revelation affects which interviews, fly-outs, and job offers a candidate gets. Because universities' intermediate decisions (interviews and fly-outs) contain some information about the candidate, making them known has the potential to increase the amount of information available to universities in the hiring process. This, in turn, could lead to a better hiring outcome. A closer inspection, however, shows that this intuition is incomplete. Although each university would benefit from knowing other universities' intermediate decisions, a university may or may not benefit from having its intermediate decisions revealed to other universities. Which universities benefit? Which candidates benefit? Should such information revelation be facilitated or prohibited?

Understanding the overall effect of revealing universities' intermediate decisions is complicated, because the amount of information revealed by these decisions is determined endogenously. If universities anticipate that their intermediate decisions and those of other universities will be revealed, they may adjust their intermediate decisions for two reasons. First, a university may advance a marginal candidate in the hiring process in the hopes of learning more from other universities' intermediate decisions. Second, a university may "give up" on a seemingly good candidate because it expects that a positive intermediate decision will result in an offer from a more attractive university.[2]

To analyze these issues, we investigate a simple three-stage model in which firms may be interested in hiring a worker whose value is common to all firms. In the first stage, each firm decides whether to pay a small cost to "enter" and participate in the hiring process.

[1] http://bluwiki.com/go/Econjobmarket.

[2] Thus, it is not clear whether seeing a low-ranked university interviewing a candidate constitutes "good news" or "bad news" about the candidate from a high-ranked university's point of view.

Each entering firm obtains some private information about the value of the worker. Entry decisions are made simultaneously, and a firm that does not enter cannot later interview or hire the worker. In the second stage, all entering firms simultaneously choose whether to pay a small cost to "interview" the worker. This decision is based on each firm's private information. An interview may be interpreted as a show of interest in the worker, as placing the worker on a "short list," as a purely administrative step in the hiring process, as a fly-out, or as an actual interview. Interviews are costly, reveal no additional information about the worker, and are a necessary step in the hiring process.[3] A firm that does not interview the worker cannot later hire the worker. In the third stage, after all interviews have taken place, all firms simultaneously decide whether to make employment offers to the worker. The worker has a common strict ranking over firms, and accepts the highest-ranked offer among those he receives. The worker's value is a function of all firms' private information. Therefore, each firm can make better hiring decisions if it has access to even coarse measures of other firms' private information. This, in turn, is determined by whether firms' interview decisions are revealed before employment offers are made.

When interview decisions are not revealed (no revelation), no learning takes place between the interviewing and the hiring stages. This means that lower-ranked firms can enter and make use of their private information. With two firms, for example, the strong firm will interview and hire the worker if its signal is sufficiently high, so the weak firm can interview and profitably hire the worker when the strong firm does not interview and the weak firm's signal is high enough to offset the "bad news" that the strong firm did not interview. Example 4.1 describes a setting in which for any n all n firms enter and with positive probability interview and profitably hire the worker. When interview decisions are revealed (revelation), a firm that interviews the worker can condition its hiring decision on the interview decisions of the other firms. Because this additional information improves the hiring decisions of all interviewing firms, it may seem that revelation is good for the firms. Proposition 2 shows that compared to no revelation, all firms are indeed weakly better off (and the worker is weakly worse off) with revelation when firms make their interview decisions anticipating no revelation.

In contrast, the main result of the paper shows that revelation is quite detrimental to firms (and the worker) when firms make their interview decisions anticipating that these decisions and those of the other firms will be revealed before the hiring stage. Two countervailing forces are at play. On the one hand, each firm but the top firm faces more severe adverse selection from higher-ranking firms as a result of revelation. On the other hand, each firm benefits from being able to condition its hiring decisions on the interview

[3]Section 7 discusses an extension in which interviews are informative. This extension does not change any of the results.

decisions of the other firms. Theorem 1 shows that the adverse selection that all firms except for the highest-ranked firm face as a result of revelation swamps the benefit of observing all firms' interview decisions. As a result, only the highest-ranked firm enters the market. All other firms do not interview and do not hire any workers. Compared to no revelation, all firms and the worker are weakly worse off. Any firm $2, \ldots, n$ that enters with no revelation is strictly worse off, and firm 1 is equally well off. If the worker is hired by a firm $2, \ldots, n$ with no revelation, he is strictly worse off with revelation.

At first blush, this result may seem obvious: if a low-ranked firm incurs a cost to interview the worker, then the firm must be interested in hiring the worker, so the common-value assumption implies that higher-ranked firms should hire the worker. This would certainly be the case if all firms' private information were revealed. In fact, the logic underlying the result is more intricate, because seeing a firm interview provides only a coarse measure of the firm's private information. Suppose, for example, that there are two firms and the value of the worker is the sum of the two firms' private signals. It may be that the low-ranked firm sees a very high signal and interviews. If the high-ranked firm sees a low signal, it may reason as follows: "I know that my signal is very low, and if I see the other firm interviewing I will only be able to deduce that its signal is in some range, whose expected value is not enough to offset my signal. Therefore, I will not interview." Similarly, the low-ranked firm only sees that the high-ranked firm does not interview, and concludes that in expectation the high-ranked firm saw a higher signal than the high-ranked firm actually did. As a result, the low-ranked firm is willing to make the worker an offer while the high-ranked firm is not. Our result shows that this reasoning is inconsistent with equilibrium.[4] Despite the fact that interviews only provide a coarse measure of a firm's private information, in equilibrium no firm can make use of its private information (except for the highest-ranked firm). Firms' entry choices are made as if they expect all their private information to be revealed. This is true even if a low-ranked firm has much better (or worse) private information about the worker's value than any other firm.

When the common value assumption is relaxed, exclusion of weaker firms will generally not occur. Indeed, suppose that for certain values of the worker a weak firm is interested in hiring the worker but stronger firms are not. Then, even if the value of the worker is known, the weak firm is not excluded. If, however, the values for which higher-ranked firms are interested in hiring the worker include those for which lower-ranked firms are interested in hiring the worker, then our exclusion result obtains. The result is also robust to the

[4] In addition to considering the worker's expected value from the perspective of different firms, the proof also takes into account that firms may use mixed strategies and that different firms may attribute different probabilities to the same event, because they have different information. The proof applies to affiliated signals and more than two firms.

hiring of multiple workers, provided there is sufficient separability across workers, and to informative interviews, provided that all interviewing firms gain the same information from interviewing.

There is an extensive literature on two-sided matching, beginning with the seminal work of Gale and Shapley (1962). The novelty of our paper is that firms have incomplete information about the value of the workers and this value is common to all firms. Also, we focus on a specific hiring process, which leads to new strategic considerations that influence firms' behavior. Such considerations do not arise in existing models of two-sided matching, both those that postulate complete information and those that postulate incomplete information of agents' preferences (see, for example, Roth and Sotomayor (1990), Sönmez (1999)). More recently, Masters (2009) studied hiring with interviews but did not consider revelation and the resulting interaction among firms. Coles and Niederle (2007) study a model in which students can use costly signals to indicate their interest in some universities. None of these models study the strategic interaction explored here.

The rest of the paper is organized as follows. Section 2 introduces the model and related notation. Section 3 conducts a preliminary analysis. Section 4 explores the setting with no revelation. Section 6 explores the setting with revelation, and states and proves the main result. Section 5 explores the setting in which firms anticipate no revelation but the interview decisions are nevertheless revealed. Section 7 discusses some extensions. Section 8 concludes. The Appendix contains statements and proofs of technical lemmas.

2 The Model and Notation

There are n risk-neutral firms and one worker. The set of firms $\{1, \ldots, n\}$ is denoted by N. The worker is characterized by a vector of (weakly) affiliated signals, one for each firm. The set of possible signal realizations for firm i, denoted S_i, is finite and linearly ordered, with generic element s_i'. The vector of firms' signals is drawn from a distribution F on $S = \times_i S_i$ with full support. We denote by s_i the random variable whose realization is an element in S_i, so $s_1, \ldots, s_n$ are affiliated.

The worker can work for only one firm, and has a commonly-known strict ranking over firms. Firm 1 is the workers' highest-ranked firm, firm 2 is the workers' second highest-ranked firm, etc. Once employed, the worker's net value, in monetary units, is common to all firms. This value is a function v of all firms' signals, and is strictly increasing in each firm's signal. The function v is normalized so that firms' outside option of not hiring the worker is 0.

The timing of the market is as follows. First, before observing the signals, all firms simultaneously choose whether to enter the market. Entry costs firm i $e_i > 0$, and provides the firm with its private signal of the workers' value. A firm that does not enter the market cannot participate in subsequent stages of the market. Entry decisions are commonly known. After the entry stage, all entering firms simultaneously decide whether to interview the worker. The cost of an interview to firm i is $c_i > 0$. An interview reveals no new information about the worker to the interviewing firm, but is a necessary step in the hiring process.[5] The worker cannot be hired by a firm that did not interview him. After all interviews take place, there are two possibilities. Either interview decisions are kept private, or they are revealed. We analyze these scenarios separately, and also consider a scenario in which firms expect interview decisions to be kept private, but they are nevertheless revealed. At the next stage, each firm decides whether to make an employment offer to the worker. The offers are made simultaneously.[6] The worker accepts the offer made by the firm he prefers most among those that made him an offer.[7] An entering firm i's payoff from hiring a worker with signals $s'_1, \ldots, s'_n$ is $v(s'_1, \ldots, s'_n) - c_i - e_i$. If the firm interviews but does not hire a worker, either because the firm does not make him an offer or because the worker does not accept the firm's offer, then the firm's payoff is $-c_i - e_i$. Firms' entry costs, and interview costs are commonly known.

Positive entry costs and interview costs guarantee that entry and interview decisions are not "cheap talk." Because we are interested in the informational effects of signals and interviews, we will typically consider small entry and interview costs. Small entry and interview costs lead to significantly different predictions than do costs of 0.

We analyze the game using the solution concept of sequential equilibrium (henceforth: equilibrium). Because the game is finite, a sequential equilibrium exists.

3 Preliminary Analysis

As a preliminary exercise, suppose that firms $2, \ldots, n$ do not enter. Then, conditional on entering, firm 1 interviews the worker (and later makes an offer that will be accepted) if

[5] Our analysis focuses on the effects of making interview decisions, which are based on firms' private information, known prior to the employment offer stage. This is why we chose to model interviews as non-informative. An extension to informative interviews is discussed in the Conclusion.

[6] If firms can make post-interview offers at any time up to a common time T, it is weakly dominant for all of them to make offers at time T.

[7] The assumption that the worker always accept the best employment offer he receives is without loss of generality. This is because if the workers prefers unemployment to working at a certain firm, then that firm will never enter the market, and can be ignored. Therefore, we assume that the set N of firms does not contain such firms.

and only if its signal s_1' satisfies

$$E\left[v|s_1 = s_1'\right] \geq c_1, \tag{1}$$

with possible mixing if the inequality is an equality. Because the worker's value increases in every firm's signal and firms' signals are affiliated, a higher signal makes firm 1 more optimistic about the worker's expected value.[8] This fact and the fact that the worker always accepts the firm's offer if it is made imply that the firm employs a threshold interviewing (and hiring) strategy. As the cost of interviewing c_1 decreases, the threshold decreases and the firm's expected profit increases. For the remainder of the paper, we make the following assumption.

A1 *Equation (1) holds with a strict inequality for at least one signal when* $c_1 = 0$*.*

Assumption A1 guarantees that for sufficiently small (but positive) interview costs, firm 1's post-entry expected profit is positive. Therefore, when the entry and interviewing costs are sufficiently small (but positive), the firm will enter and make positive profits even if it the only entering firm.

4 No Revelation

With no revelation, a firm makes an offer whenever it interviews. This is because interviewing is costly and no firm learns anything about the other firms' signals or interviewing decisions before it decides whether to make an offer. A firm's offer is accepted if and only if the firm is the highest-ranking firm that made an offer. As a result, every equilibrium can be solved for by proceeding from firm 1 to firm n and identifying each firm's interviewing strategy given those of all higher-ranked firms. Firm 1 behaves as described in Section 3. Conditional on entering, it employs a threshold interviewing strategy, interviewing and hiring for every signal above the lowest signal s_1' that satisfies Equation (1) (if such signals exist), with possible mixing at the lowest signal if the inequality is an equality.

Given firm 1's interviewing strategy, its entry decision depends on whether its expected profits conditional on entering offset the entry costs.[9] When the expected profits conditional on entering equal the entry cost, the firm may mix between entering and not entering. For low entry and interviewing costs e_1 and c_1, however, firm 1 has a unique optimal strategy. To see this, denote by T_1 the lowest signal for which Equation (1) holds with a strict inequality when $c_1 = 0$ (such a signal exists by Assumption A1). Then, for

[8] To see this, apply Lemma 7 in the Appendix with $Z_{-i} = S_{-i} \times \Omega_{-i}$.

[9] If firm 1 mixes at the lowest signal for which it interviews with positive probability, then it makes 0 profits there, so its behavior there does not affect the profitability of entry.

low e_1 and c_1, firm 1's unique optimal strategy is to enter with probability 1 and interview and hire with probability 1 at all signals greater or equal to T_1. The following result shows that for low entry and interviewing costs, there is in fact a unique equilibrium, in which every every entering firm employs a threshold interviewing strategy.

Proposition 1 *For low* $\max_{i\in N} e_i$ *and* $\max_{i\in N} c_i$ *there is a unique equilibrium, which is in pure strategies. In this equilibrium, every entering firm i interviews for all signals greater or equal to some signal T_i. The equilibrium can be found by iterated elimination of strictly dominated strategies.*

Proof. We prove the following claim by induction: for any $i \in N$, for low $\max_{j\leq i} e_j$ and $\max_{j\leq i} c_j$, every firm $j \leq i$ has a strictly dominant (pure) strategy once the strictly dominated strategies of higher-ranking firms have been iteratively eliminated. As we have seen, the claim is true for $i = 1$, because for low e_1 and c_1 firm 1 has a strictly dominant threshold interviewing strategy with threshold T_1. Now suppose that the claim is true for $i - 1 \geq 1$. Then, for low $\max_{j\leq i-1} e_j$ and $\max_{j\leq i-1} c_j$, in any equilibrium firms $1, \ldots, i-1$ play the strategies identified by the induction hypothesis. Given the strategies of firms $1, \ldots, i-1$, what should firm i do? Conditional on entering, firm i will succeed in hiring the worker when it makes an offer if and only if firms $1, \ldots, i-1$ do not make make an offer or, equivalently, do not interview. Because firms $1, \ldots, i-1$ play pure strategies, the event that none of these firms interview the worker is the set $B = \times_{j\in N} B_j$ such that

$$B_j = \begin{cases} \text{signals in } S_j \text{ for which firm } j \text{ does not interview} & j \leq i-1 \\ S_j & j > i-1 \end{cases}. \tag{2}$$

Therefore, conditional on entering, firm i's net profit if it interviews and makes an offer to a worker at signal s_i' is

$$\Pr\left(B|s_i = s_i'\right) E\left[v|B, s_i = s_i'\right] - c_i. \tag{3}$$

Conditional on entering, for low c_i it is uniquely optimal for firm i to interview with probability 1 at precisely all signals s_i' for which the expression in Equation (3) is strictly positive when c_i is replaced with 0. If there is at least one such signal, then for low e_i it is strictly optimal for firm i to enter. If there are no such signals, then it is strictly optimal for firm i not to enter. This shows that the induction hypothesis holds for i. Moreover, if the the expression in Equation (3) is strictly positive for some signal s_i' when c_i is replaced with 0, then the expression is also strictly positive for all signals $s_i'' > s_i'$. This is because (i) by affiliation and because v is strictly increasing $E\left[v|B, s_i = s_i'\right]$ increases with s_i' (Lemma 7 in the Appendix) and (ii) $\Pr\left(B|s_i = s_i'\right)$ is strictly positive for all signals s_i' if it is strictly positive for one signal s_i' (F has full support). Therefore, if firm i enters for low e_i, then for low c_i it interviews for all signals greater or equal to some signal T_i. ■

Costs and strategies of firms ranked lower than i do not appear in Equations (2) and (3). Therefore, the unique equilibrium when entry and interview costs are sufficiently low can be solved for by iteratively applying the process described in the proof of Proposition 1, proceeding from firm 1 to firm N.

The requirement that interview costs be sufficiently small is necessary to conclude that firms employ threshold interviewing strategies. To see this, suppose that e_1 and c_1 are low enough for firm 1 to use a threshold interviewing strategy with threshold T_1. Consider firm 2 and Equation (3). As s_2' increases, $\Pr(s_1 < T_1 | s_2 = s_2')$ decreases (affiliation) and $E[v | s_1 < T_1 | s_2 = s_2']$ strictly increases (v is strictly increasing and affiliation - Lemma 7). Therefore, the expression in Equation (3) may be a non-monotonic function of s_2'. This means that the set of signals for which firm 2 interviews need not correspond to a threshold interviewing strategy when c_2 is not sufficiently small.

Nevertheless, when firms' signals are independent, each firm employs a threshold interviewing strategy regardless of interviewing costs. This is because when firms' signals are independent, $\Pr(B | s_i = s_i')$ is independent of s_i', so the expression in Equation (3) strictly increases in s_i'.[10]

4.1 Example 1 - No Revelation

Suppose each firm's signal is drawn uniformly and independently from the set $\left\{-\frac{1}{2} + \varepsilon, \frac{1}{2} - \varepsilon\right\} \cup \left\{-\frac{1}{2} + \frac{i}{2^k} - \varepsilon, -\frac{1}{2} + \frac{i}{2^k} + \varepsilon : i = 1, \ldots, 2^k - 1\right\}$ for some $k \geq n$ and positive $\varepsilon < \frac{1}{2^{k+2}}$ (this approximates the uniform distribution on $\left[-\frac{1}{2}, \frac{1}{2}\right]$). Suppose that $v = \sum_{i=1}^{n} s_i$. Then, if entry and interview costs are sufficiently small, for a firm operating alone in the market it is uniquely optimal to interview and hire at any signal greater than or equal to ε, because the expected value of other firms' signals is 0.

With no revelation and sufficiently small entry and interview costs firm 1 enters and interviews and hires at any signal greater or equal to ε. Therefore, the expected value of firm 1's signal conditional on not interviewing is $-\frac{1}{4}$. As a result, for sufficiently small entry and interview costs, firm 2 enters and interviews and hires at any signal greater or equal to $\frac{1}{4} + \varepsilon$. Proceeding in this way, we see that with no revelation and sufficiently small entry and interview costs there is a unique equilibrium. In this equilibrium, all firms enter and every firm i interviews with probability 1 at all signals greater or equal to $T_i = \frac{1}{2} - \frac{1}{2^i} + \varepsilon$.

[10] If this expression equals 0 for some signal s_i', then the firm may mix between interviewing and not interviewing at s_i'.

5 Unexpected Revelation

The intuition that revealing firms' interviewing decisions should improve things for at least one side of the market could perhaps be traced to the following result.

Proposition 2 *Suppose firms' interview decisions are made assuming no revelation. Then, revealing firms' interview decisions (a) weakly decreases the set of signals for which the worker is hired, (b) may shift the worker to lower-ranked firms but does not shift the worker to higher-ranked firms, and (c) weakly increases the utility of all firms.*

Proof. By assumption, revealing firms' interview decisions does not affect the set of workers each firm interviews. With no revelation, every worker who is interviewed is hired by the highest-ranking firm that interviews him. Therefore, no new workers are hired as a result of revelation.

Now consider the movement of workers between firms. With no revelation, a firm hires a worker it interviews if and only if no higher-ranked firm interviews the worker, regardless of what lower-ranked firms do. This means that no worker can move up to a better firm because of revelation: if a firm hires a worker with revelation who is not hired by the firm with no revelation, the firm must have interviewed with no revelation (since the firm interviews the same set of workers with revelation), and the reason the worker is not hired by the firm with no revelation is that the worker is interviewed and hired by a higher-ranked firm, so the worker shifts down to be hired by the firm with revelation.

To see that all firms are weakly better off compared to no revelation, note that since there is no movement of workers to higher-ranked firms, with revelation each firm can hire the same set of workers it hires with no revelation by hiring if and only if all higher-ranking firms do not interview. By taking other firms' interviewing decisions into account, a firm can choose not to hire some of the workers it hires with no revelation, and can also choose to hire workers that higher-ranked firms interview but choose not to hire based on other firms' interview decisions. ∎

Proposition 2 suggests that the setting with no revelation is unstable. Indeed, suppose that firms make their interview decisions expecting no revelation. After the interview stage and before the hiring stage, the firms can be made collectively better off by revealing their interview decisions. No firm would be worse off and some firms could be strictly better off compared to no revelation. Small transfers between firms could make all firms strictly better off.

5.1 Example 2 - Unexpected Revelation

Consider the setting of Example 4.1 with two firms, $k \geq 3$, and low entry and interview costs. Firm 1's interview threshold is ε, and firm 2's interview threshold is $\frac{1}{4} + \varepsilon$. If firms' interviewing decisions are revealed unexpectedly, then when firm 1 interviews and firm 2 does not, firm 1 makes offers and hires for all signals greater or equal to $\frac{1}{8} + \varepsilon$. This contrasts with the no-revelation setting, in which firm 1 makes an offer whenever it interviews. When both firms interview, firm 1 hires, and when only firm 2 interviews, firm 2 hires, just as with no revelation. A worker with $s_1 = \varepsilon$ and $s_2 < \frac{1}{4} + \varepsilon$ is hired by firm 1 with no revelation, but is not hired by any firm with unexpected revelation.

6 Revelation

With revelation, each firm can condition its hiring decision on all other firms' interviewing decisions. As a result, a firm's interviewing strategies may depend on all other firms' interviewing strategies, and not only on those of higher-ranked firms. A firm may choose to interview at a signal because it expects to learn something about the worker's value from the other firms' interviewing decisions. But the firm also knows that the other firms will learn something from its interviewing decision, which may affect its probability of hiring the worker and his value conditional on being hired.

More concretely, suppose that m firms enter in an equilibrium with revelation. Assume for simplicity that firms use pure strategies, so that each firm has an "interview set" of signals for which it interviews. Consider the behavior of an entering firm. Conditional on interviewing, the firm can condition its hiring decision on each of the 2^{m-1} possible combinations of the other entering firms' interviewing decisions. For each such combination, the firm determines a "hiring set" of signals for which it makes an offer conditional on interviewing. These hiring sets, which depend on the other firms' interviewing sets, determine the firm's interviewing set. Because of this interdependence, all firms' hiring sets for each combination of the other firms' interviewing decisions and all interview sets are determined jointly. A sequential procedure like the the one described in Proposition 1 can therefore not be used to solve for an equilibrium.

The analysis is further complicated because mixed strategies (which may depend on entry and interview costs) and non-threshold interview and hiring sets cannot be easily ruled out. An argument like the one used in Proposition 1 to show that firms use pure strategies and that entering firms use threshold interviewing strategies, all of which are independent of costs when costs are sufficiently small, does not work with revelation. And if firms use non-threshold interviewing strategies, then seeing another firm interview is not necessarily "good news" about the worker's value. Despite these difficulties, the following result fully characterizes equilibrium behavior with revelation.

Theorem 1 *Choose $M > 1$. If entry and interview costs are low and the ratio between any two firms' interview costs is at most M, then in any equilibrium with revelation the* *only firm that enters is firm* 1.[11]

At the core of Theorem 1 lies a relatively simple argument. The essence of this argument can be explained by considering one worker and two firms that employ pure strategies and observe independent signals (which implies that they use threshold interviewing strategies).[12] To illustrate the argument, suppose that interview costs are 0, and that a firm interviews only if there is a positive probability that it can hire the worker and that conditional on hiring the worker the firm makes positive profits. By Condition A1, firm 1 enters and interviews for some signals, because it can always ignore firm 2. Suppose firm 2 enters. Since entry is costly, there are signals for which firm 2 interviews and hires. When both firms interview, firm 1 hires. This is because when firm 1 interviews it intends to hire for at least some interview decision of firm 2, and threshold interviewing strategies imply that when firm 2 interviews, the worker's value is higher than when firm 2 does not interview. Therefore, for firm 2 to be able to hire there must be signals for which firm 1 does not interview. Suppose that firm 2 observes the lowest signal s_2' for which it interviews, and firm 1 observes the highest signal s_1' for which it does not interview. Then firm 1's expectation of the worker's value is higher than that of firm 2: firm 1 observes s_1' and knows only that firm 2's signal is greater or equal to s_2', whereas firm 2 observes s_2' and knows only that firm 1's signal is at most s_1'. Because firm 2 is willing to hire the worker when it observes s_2' and firm 1 does not interview, firm 1 would be willing to hire the worker when it observes s_1' and firm 2 interviews. But then firm 1 would deviate and interview at s_1'.

The proof of Theorem 1 formalizes this argument and extends it to multiple firms, affiliated signals, positive interview costs (which imply that probabilities of certain events must be taken into account, and not only the worker's expected value), and mixed and non-threshold strategies (which imply that interviewing is not necessarily good news about the worker's value).

6.1 Proof of Theorem 1

Choose $M > 1$, and consider a sequence of strictly positive interviewing costs $c^k = \left(c_1^k, \ldots, c_n^k\right)$ whose maximal element approaches 0 and which satisfy $\max_{i,j \in N} \frac{c_i^k}{c_j^k} < M$. Choose a sequence of strictly positive entry fees $e^k = \left(e_1^k, \ldots, e_n^k\right)$ (that need not approach

[11] When there are only two firms ($n = 2$), the restriction on the ratio between firms' interview costs is not necessary for the result.

[12] When signals are independent and a firm interviews for some signal, which implies that its expected payoff conditional on interviewing at the signal is non-negative, then pursuing the same behavior at any higher signal delivers a strictly higher payoff.

0). Choose the entry fees and interviewing costs low enough so that firm 1 enters in any equilibrium with revelation.

Lemma 1 *For low entry and interviewing costs e_1 and c_1, with revelation it is strictly optimal for firm 1 to enter with probability 1, regardless of other firms' strategies.*

Proof. With no revelation, Assumption A1 guarantees that for low interviewing costs firm 1 enters with probability 1. With revelation, firm 1 is weakly better off conditional on entering than with no revelation, regardless of other firms strategies (because its offer is always accepted, it can mimic its no-revelation outcome by ignoring other firms' interviewing decisions). Therefore, for low interviewing costs firm 1 enters with probability 1 with revelation. ■

To model mixed strategies, we assume that each firm i observes the outcome of a uniform lottery over $\Omega_i = [0, 1]$, and denote by ω_i the realization of this lottery. The lotteries of different firms are statistically independent, and are also independent of all firms' signals.

We use the following notation for post-entry interviewing and hiring strategies parameterized by k, that is, strategies that take the set of entering firms as given. Firm i chooses a measurable set $\tilde{I}_i^k \subset S_i \times \Omega_i$ following whose elements it interviews the worker. We define $\sigma_i^k(s_i) = \text{Prob}(\{\omega_i : (s_i, \omega_i) \in \tilde{I}_i^k\})$ as the probability that firm i interviews after observing the signal s_i. For each subset $\mathcal{I} \subset \{1, \ldots, N\}$ such that $i \in \mathcal{I}$, firm i chooses a measurable set $\tilde{O}_{i,\mathcal{I}}^k \subset S_i \times \Omega_i$ following whose elements it makes an offer if it interviewed and observed interview schedule $\mathcal{I}$ (that is, if it observed precisely the firms in $\mathcal{I}$ interviewing). For every interview schedule $\mathcal{I}$ such that $i \in \mathcal{I}$, we define $\tau_i^k(s_i; \mathcal{I}) = \text{Prob}\{\omega_i : (s_i, \omega_i) \in \tilde{O}_{i,\mathcal{I}}^k\}$ as the probability that firm i makes an offer if it (i) interviewed after observing signal s_i and (ii) observed interview schedule $\mathcal{I}$. We denote by $\underline{s}_i^k = \min\{s_i : \sigma_i^k(s_i) > 0\}$ the lowest signal for which firm i interviews with positive probability, by $\bar{s}_i^k = \max\{s_i : \sigma_i^k(s_i) < 1\}$ the highest signal for which firm i interviews with probability less than one, and by $\bar{s}_{i,\mathcal{I}}^k = \max\{s_i : \tau_i^k(s_i; \mathcal{I}) < 1, \sigma_i^k(s_i) > 0\}$ the highest signal for which firm i interviews with positive probability and makes an offer with probability less than one after interviewing and observing interview schedule $\mathcal{I}$.

Let $I_i^k = \tilde{I}_i^k \times \prod_{j \neq i}(S_j \times \Omega_j)$ and $O_{i,\mathcal{I}}^k = \tilde{O}_{i,\mathcal{I}}^k \times \prod_{j \neq i}(S_j \times \Omega_j)$. For a set of firms $\mathcal{I}$, we denote by $\hat{\mathcal{I}} = \cap_{j \in \mathcal{I}} I_i^k \cap_{j \notin \mathcal{I}} \neg I_j^k$ the event that exactly this set of firms interviews. The set $\Phi_{i,\mathcal{I}}^k = \cap_{j \in \mathcal{I}, j < i} \neg O_{j,\mathcal{I}}^k$ is the event at which firm i could possibly have its offer accepted if precisely the firms in $\mathcal{I}$ interview (because all stronger interviewing firms do not make offers).

Because signals are affiliated and v is increasing, a higher signal is good news about a worker's value for any interview schedule of the other firms. This implies the following result.

Lemma 2 *For any $s_i'' > s_i'$ such that $\sigma_i^k(s_i') > 0$ and $\sigma_i^k(s_i'') > 0$, If $\tau_i^k(s_i';\mathcal{I}) > 0$, then $\tau_i^k(s_i'';\mathcal{I}) = 1$.*

Proof. Because $\sigma_i^k(s_i') > 0$ and $\tau_i^k(s_i';\mathcal{I}) > 0$, conditional on observing s_i', interviewing, and observing interview schedule $\mathcal{I}$, firm i weakly prefers making an offer and succeeding in hiring to not making an offer. Therefore, $\mathbf{E}(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k, s_i = s_i') \geq 0$. By Lemma 7 in the Appendix, $s_i'' > s_i'$ implies that $\mathbf{E}(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k, s_i = s_i'') > 0$, so conditional on observing s_i'', interviewing, and observing interview schedule $\mathcal{I}$, firm i is strictly better off making an offer than not making an offer. ■

We will show by reverse induction on $i \in \{2, \ldots, n\}$ that for low maximal interviewing costs (large enough k) firm i enters with probability 0 in any equilibrium with revelation, entry costs e^k, and interviewing costs c^k. This will prove Theorem 1. Choose $i \in \{2, \ldots n\}$, and suppose that for large enough k all firms $j > i$ enter with probability 0 in any equilibrium with revelation, entry costs e^k, and interviewing costs c^k. It suffices to show that for large enough k firm i enters with probability 0. Suppose in contradiction that there exists a subsequence of interviewing costs, without loss of generality the sequence itself, such that for any e^k and c^k in the sequence there exists a corresponding equilibrium q^k with revelation in which firm i enters with positive probability. Because entry is costly, for every k and equilibrium q^k in the sequence, firm i must make strictly positive expected profits conditional on entering.

Consider the following preliminary observation: If, given a set of entering firms, a firm interviews with sufficiently small probability, which depends only on the distribution F of the signals, then the firm does not interview with probability 1 conditional on any signal, so conditional on interviewing, the firm expects a profit of 0. This observation is true because F has full support. Because firm i makes strictly positive expected profits conditional on entering in q^k, the preliminary observation means that for every k there is some set J^k of firms that enter in q^k with positive probability, with $i \in J^k$, such that when the set of firms that enter is precisely J^k, firm i interviews with a probability that is uniformly bounded away from 0 for all k.

Consider firm i's strategy in the equilibrium q^k when the set of entering firms is the set J^k specified above. By Lemma 1, $1 \in J^k$. Because firm i interviews with positive probability at signal $\underline{s}_i^k$, there is an interview schedule $\mathcal{I}$ such that $\Pr\left(\hat{\mathcal{I}} \cap \Phi_{i,\mathcal{I}}^k | s_i = \underline{s}_i^k\right) > 0$ and

$$\mathbf{E}(v|\hat{\mathcal{I}}, \Phi_{i,\mathcal{I}}^k, s_i = \underline{s}_i^k) > 0. \tag{4}$$

If not, then conditional on interviewing with $\underline{s}_i^k$, firm i could not cover its interviewing costs. We now show that this $\mathcal{I}$ can only be the singleton $\{i\}$. Let $j = \min \mathcal{I}$ be the highest-ranked firm that interviews in $\mathcal{I}$, and suppose $j \neq i$. Because $\Pr\left(\hat{\mathcal{I}} \cap \Phi_{i,\mathcal{I}}^k\right) > 0$, the signal $\bar{s}_{j,\mathcal{I}}^k$ is well defined (there is at least one signal for which firm j interviews with positive probability and hires with a probability less than 1). Because firms' signals are affiliate and v is increasing,

$$
\begin{aligned}
0 &< \mathbf{E}(v|\hat{\mathcal{I}}, \Phi^k_{i,\mathcal{I}}, s_i = \underline{s}^k_i) \\
&\leq \mathbf{E}(v|\hat{\mathcal{I}}, \Phi^k_{i,\mathcal{I}}) && (5)\\
&\leq \mathbf{E}(v|\hat{\mathcal{I}}, \Phi^k_{i,\mathcal{I}}, s_j = \bar{s}^k_{j,\mathcal{I}}) && (6)\\
&\leq \mathbf{E}(v|\hat{\mathcal{I}}, s_j = \bar{s}^k_{j,\mathcal{I}}). && (7)
\end{aligned}
$$

The first inequality between conditional expectations follows from the definition of $\underline{s}^k_i$ as i's lowest signal consistent with $\hat{\mathcal{I}}$,[13] the second from the definition of $\bar{s}^k_{j,\mathcal{I}}$ as the highest signal of j consistent with $\Phi^k_{i,\mathcal{I}}$,[14] and the third from the fact that $\Phi^k_{i,\mathcal{I}}$ is bad news about the worker's value (Lemma 2).[15]

The inequality $0 < \mathbf{E}(v|\hat{\mathcal{I}}, s_j = \bar{s}^k_{j,\mathcal{I}})$ implies that in the positive-probability event in which firm j sees signal $\bar{s}^k_{j,\mathcal{I}}$ and interview schedule $\mathcal{I}$ (at which firm j interviews, because $j \in \mathcal{I}$), firm j would profit from hiring the worker. Because j is the strongest firm in $\mathcal{I}$, it would hire the worker if it made him an offer. Thus, j strictly prefers to make an offer at $\bar{s}^k_{j,\mathcal{I}}$, whereas by definition it makes an offer at $\bar{s}^k_{j,\mathcal{I}}$ with a probability less than 1, a contradiction. This shows that $j = i$, so $\mathcal{I} = \{i\}$ and $\Pr\left(\Phi^k_{i,\mathcal{I}}\right) = 1$.

Because $\mathcal{I} = \{i\}$ is the only schedule satisfying Equation (4), this schedule arises with positive probability conditional on firm i seeing the signal $\underline{s}^k_i$, as discussed above. This means that every entering firm $j \in J^k \backslash \{i\}$ interviews with probability less than 1. Recall that $\bar{s}^k_j$ is the highest signal for which firm j interviews with probability less than 1. From Equation (4) and because $\Pr\left(\Phi^k_{i,\mathcal{I}}\right) = 1$, for any $j \in J^k \backslash \{i\}$ we have

$$
\begin{aligned}
0 &< \mathbf{E}(v|\hat{\mathcal{I}}, s_i = \underline{s}^k_i) && (8)\\
&\leq \mathbf{E}(v|\hat{\mathcal{I}}) \\
&\leq \mathbf{E}(v|\hat{\mathcal{I}}, s_j = \bar{s}^k_j).
\end{aligned}
$$

These inequalities follow, as above, from the assumption that firms' signals are affiliated and v is increasing.

Lemma 3 *There exists some $\delta > 0$ and a subsequence, without loss of generality the*

[13] For every s^k_i with $\sigma^k_i\left(s^k_i\right) > 0$, apply Lemma 7 with $Z_{-i} = \hat{\mathcal{I}}_{-i} \cap \Phi^k_{i,\mathcal{I}}$, $s'_i = \underline{s}^k_i$, $A = \sigma^k_i\left(\underline{s}^k_i\right)$, $s''_i = s^k_i$, and $B = \sigma^k_i\left(s^k_i\right)$.

[14] For every s^k_j with $\sigma^k_j\left(s^k_j\right) > 0$ and $\tau^k_j(s^k_j; \mathcal{I}) < 1$, apply Lemma 7 with $Z_{-j} = \hat{\mathcal{I}}_{-j} \cap \Phi^k_{i,\mathcal{I},-j}$, $s'_i = s^k_j$, $A = \{\omega_j : (s^k_j, \omega_j) \notin \tilde{O}^k_{j,\mathcal{I}}\}$, $s''_i = \bar{s}^k_{j,\mathcal{I}}$, and $B = \{\omega_j : (\bar{s}^k_{j,\mathcal{I}}, \omega_j) \notin \tilde{O}^k_{j,\mathcal{I}}\}$.

[15] Apply Lemma 7 iteratively, for every $l \in \mathcal{I} \backslash \{i\}$, as in the previous footnote.

sequence itself, such that for all large enough k,

$$\mathbf{E}(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) > \delta \tag{9}$$

for some $j \in J^k \backslash \{i\}$.

Proof. By Equation (8), the claim is clearly true If there exists some $\delta > 0$ and a subsequence such that for all large enough k either $\mathbf{E}(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) - \mathbf{E}(v|\hat{\mathcal{I}}) \geq \delta$ for some $j \in J^k$, or $\mathbf{E}(v|\hat{\mathcal{I}}) - \mathbf{E}(v|\hat{\mathcal{I}}, s_i = \underline{s}_i^k) \geq \delta$. Suppose to the contrary that for every $\delta > 0$ there exists some $K(\delta)$ such that for all $k > K(\delta)$ and every firm $j \in J^k \backslash \{i\}$ we have (i) $\mathbf{E}(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) - \mathbf{E}(v|\hat{\mathcal{I}}) < \delta$ and (ii) $\mathbf{E}(v|\hat{\mathcal{I}}) - \mathbf{E}(v|\hat{\mathcal{I}}, s_i = \underline{s}_i^k) < \delta$. The inequality (i) implies that for every firm $j \in J^k$

$$\frac{\Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right)}{\Pr\left(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k\right)} \underset{k\to\infty}{\to} 0,$$

because

$$\mathbf{E}(v|\hat{\mathcal{I}}) = \frac{\Pr\left(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k\right)}{\Pr\left(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k\right) + \Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right)} \mathbf{E}(v|\hat{\mathcal{I}}, s_j = \bar{s}_j^k) \tag{10}$$

$$+ \frac{\Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right)}{\Pr\left(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k\right) + \Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right)} \mathbf{E}(v|\hat{\mathcal{I}}, s_j \neq \bar{s}_j^k) \tag{11}$$

whenever $\Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right) \neq 0$, and v is strictly increasing. By Corollary 3 in the Appendix,

$$\frac{\Pr\left(\hat{\mathcal{I}} \cap s_j \neq \bar{s}_j^k\right)}{\Pr\left(\hat{\mathcal{I}} \cap s_j = \bar{s}_j^k\right)} = K \frac{\Pr\left(\neg \tilde{I}_j^k \backslash \left(\bar{s}_j^k \times \Omega_j\right)\right)}{\Pr\left(\neg \tilde{I}_j^k \cap \left(\bar{s}_j^k \times \Omega_j\right)\right)}$$

for some constant $K > 0$ that depends only on the distribution F. Therefore,

$$\frac{\Pr\left(\neg \tilde{I}_j^k \backslash \left(\bar{s}_j^k \times \Omega_j\right)\right)}{\Pr\left(\neg \tilde{I}_j^k \cap \left(\bar{s}_j^k \times \Omega_j\right)\right)} \underset{k\to\infty}{\to} 0. \tag{12}$$

Similarly, (ii) implies that

$$\frac{\Pr\left(\tilde{I}_i^k \backslash \left(\underline{s}_i^k \times \Omega_i\right)\right)}{\Pr\left(\tilde{I}_i^k \cap \left(\underline{s}_i^k \times \Omega_i\right)\right)} \underset{k\to\infty}{\to} 0. \tag{13}$$

For every $l \in J^k \backslash \{i\}$, by (repeatedly) decomposing $\mathbf{E}(v|\hat{\mathcal{I}}, s_l = \bar{s}_l^k)$ as we did $\mathbf{E}(v|\hat{\mathcal{I}})$ in Equation (10) and applying Corollary 3 using Equation (12) for all $j \in J^k \backslash \{i, l\}$ and Equation (13), we obtain

$$\mathbf{E}(v|\hat{\mathcal{I}}, s_l = \bar{s}_l^k) \underset{k\to\infty}{\to} \mathbf{E}(v|s_{-i} = \bar{s}_{-i}^k, s_i = \underline{s}_i^k), \tag{14}$$

where $-i$ is the set of indices $J^k\setminus\{i\}$. Now consider two possibilities. The first is that for some subsequence, without loss of generality the sequence itself,

$$\mathbf{E}(v|s_{-i}=\bar{s}_{-i}^k, s_i=\underline{s}_i^k) \underset{k\to\infty}{\to} x, x\leq 0.$$

Because the number of signals is finite, the inequality holds for all large enough k. But then for large enough k we have

$$\mathbf{E}(v|\hat{\mathcal{I}}, s_i=\underline{s}_i^k)\leq \mathbf{E}(v|s_{-i}=\bar{s}_{-i}^k, s_i=\underline{s}_i^k)\leq 0,$$

a contradiction to Equation (8). The second possibility is that for some subsequence, without loss of generality the sequence itself,

$$\mathbf{E}(v|s_{-i}=\bar{s}_{-i}^k, s_i=\underline{s}_i^k)\to x, x>0.$$

Because the number of signals is finite, the inequality holds when 0 is replaced by some fixed $2\delta>0$ for all large enough k. This, together with Equation (14), implies that Equation (9) holds for large enough k and any $j\in J^k\setminus\{i\}$. ■

Now suppose that for the $j\in J^k$ specified in Lemma 3, $\Pr\left(I_i^k\cap_{m\in J^k\setminus\{i,j\}}\neg I_m^k|s_j=\bar{s}_j^k\right)$ is bounded away from 0 along some subsequence, without loss of generality the sequence itself. Lemma 3 shows that for some $\alpha>0$ and all large enough k we would have

$$\Pr\left(I_i^k\cap_{m\in J^k\setminus\{i,j\}}\neg I_m^k|s_j=\bar{s}_j^k\right)\mathbf{E}(v|\hat{\mathcal{I}}, s_j=\bar{s}_j^k)\geq\alpha.$$

But for large enough k, $c_j^k<\alpha$, so it is strictly optimal for firm j to interview with probability 1 at $\bar{s}_j^k$ (and make an offer, which will be accepted, when firm i interviews and all other firms do not interview). This contradicts the definition of $\bar{s}_j^k$ as the highest signal for which firm j interviews with probability less than 1.

Therefore,

$$\Pr\left(I_i^k\cap_{m\in J^k\setminus\{i,j\}}\neg I_m^k|s_j=\bar{s}_j^k\right)\underset{k\to\infty}{\to}0 \tag{15}$$

for some $j\in J^k$. Equation (15) and the fact that $\Pr\left(I_i^k\right)$ is bounded away from 0 imply that $\Pr\left(\neg I_l^k\right)\to 0$ for some firm $l\in J^k\setminus\{j,i\}$.[16] (if $J^k=\{i,j\}$, which happens for example if $n=2$, we have a contradiction and we are done.) For this firm $l\in J^k\setminus\{j,i\}$, therefore, $\frac{\Pr\left(I_i^k\right)}{\Pr\left(\neg I_l^k\right)}\underset{k\to\infty}{\to}\infty$. By definition of conditional expectation and Corollary 3,

$$\frac{\Pr\left(I_i^k\cap_{m\in J^k\setminus\{i,l\}}\neg I_m^k|s_l=\bar{s}_l^k\right)}{\Pr\left(_{m\in J^k\setminus\{i\}}\neg I_m^k|s_i=\underline{s}_i^k\right)}=\frac{\Pr\left(s_i=\underline{s}_i^k\right)}{\Pr\left(s_l=\bar{s}_l^k\right)}\frac{\Pr\left(I_i^k\cap_{m\in J^k\setminus\{i,l\}}\neg I_m^k\cap s_l=\bar{s}_l^k\right)}{\Pr\left(_{m\in J^k\setminus\{i\}}\neg I_m^k\cap s_i=\underline{s}_i^k\right)}$$

$$\geq\frac{\Pr\left(s_i=\underline{s}_i^k\right)}{\Pr\left(s_l=\bar{s}_l^k\right)}\frac{\Pr\left(I_i^k\right)\Pi_{m\in J^k\setminus\{i,l\}}\Pr\left(\neg I_m^k\right)\Pr\left(s_l=\bar{s}_l^k\right)}{\Pr\left(\neg I_l^k\right)\Pi_{m\in J^k\setminus\{i,l\}}\Pr\left(\neg I_m^k\right)\Pr\left(s_i=\underline{s}_i^k\right)}K=\frac{\Pr\left(I_i^k\right)}{\Pr\left(\neg I_l^k\right)}K$$

[16] To see why, apply Corollary 4 in the Appendix to Equation (15), and then use Corollary 2 in the Appendix.

for some constant $K > 0$ that depends only on the distribution F. We conclude that

$$\frac{\Pr\left(I_i^k \cap_{m\in J^k\setminus\{i,l\}} \neg I_m^k | s_l = \bar{s}_l^k\right)}{\Pr\left(_{m\in J^k\setminus\{i\}} \neg I_m^k | s_i = \underline{s}_i^k\right)} \underset{k\to\infty}{\to} \infty\,. \tag{16}$$

Because firm i interviews at $\underline{s}_i^k$ with positive probability,

$$\Pr\left(_{m\in J^k\setminus\{i\}} \neg I_m^k | s_i = \underline{s}_i^k\right) \mathbf{E}(v|\hat{\mathcal{I}}, s_i = \underline{s}_i^k) \geq c_i^k.$$

Together with Equation (8) for $j = l$ and the fact that $\frac{c_l^k}{c_i^k} < M$, this implies that

$$\Pr\left(_{m\in J^k\setminus\{i\}} \neg I_m^k | s_i = \underline{s}_i^k\right) \mathbf{E}(v|\hat{\mathcal{I}}, s_l = \bar{s}_l^k) > \frac{c_l^k}{M}.$$

For large enough k, Equation (16) gives us

$$\Pr\left(I_i^k \cap_{m\in J^k\setminus\{i,l\}} \neg I_m^k | s_l = \bar{s}_l^k\right) \mathbf{E}(v|\hat{\mathcal{I}}, s_l = \bar{s}_l^k) > c_l^k.$$

But then, for large enough k, it is strictly optimal for firm l to interview with probability 1 at $\bar{s}_l^k$. This contradicts the definition of $\bar{s}_l^k$, and therefore shows that for large enough k there is no equilibrium with revelation and costs c^k in which firm i enters with positive probability.

6.2 Discussion

Theorem 1 implies that when entry and interview costs are low, revelation leads to unraveling that excludes all but the top-ranking firm from interviewing and hiring. The outcome is as if the set of firms included only firm 1. This is the outcome that would arise if every firm's private information was revealed following an an interview. The result shows that merely revealing whether an interview took place, which is an endogenous binary statistic of a firm's private information, leads to the same outcome.

Compared to the setting with no revelation, no firm is better off with revelation, and any firm $2, \ldots, n$ that enters with no revelation is strictly worse off with revelation. In the setting of Example 4.1 above, in which all firms enter with no revelation, revelation makes firms $2, \ldots, n$ strictly worse off because they are excluded. Theorem 1 also implies that no worker is better off with revelation. Firm 1 hires the same set of workers with revelation as it does with no revelation, and any worker who is hired by some firm $2, \ldots, n$ with no revelation is unemployed with revelation. Thus, revelation lowers virtually any measure of welfare and efficiency.

We find this result surprising for several reasons. First, a firm's signal provides it with private information, and only a coarse measure of this information is made public when interviewing decisions are revealed. Second, with revelation, although each entering firm faces adverse selection from higher-ranked firms, it gains valuable information from the interview decisions of all other entering firms. Third, each firm may use non-interval interviewing strategies and employ mixed strategies. Fourth, because the function v is not assumed symmetric, the impact of one firm's signal on the worker's value may be high, while that of another firm is low. Thus, how informative a firm's signal is may vary across firms. In particular, when the number of firms is large, it may seem that at least some firms' interview decisions are not so informative. What Theorem 1 shows is that the adverse selection is so strong that firms $2, \ldots, n$ cannot make any use of their private information with revelation, regardless of n.

Because the game is finite, a sequential equilibrium is guaranteed to exist. How is on-path equilibrium behavior supported? First suppose that firm 1 does not enter, and choose some set of entering firms. This is a proper subgame, and so has a sequential equilibrium. Any sequential equilibrium will do, because by Lemma 1, firm 1 will never find a deviation to not entering attractive, regardless of what other firms do. Now suppose that at least 3 firms enter, including firm 1. This is a proper subgame in which any sequential equilibrium will do, because no firm can reach this subgame by deviating unilaterally. Finally, suppose that two firms enter, firm 1 and firm $j \neq 1$, and consider a sequential equilibrium of this proper subgame. The proof of Theorem 1 applied to $n = 2$ shows that firm j cannot make strictly positive profits net of entry costs in the subgame. Therefore, firm j makes non-positive profits net of entry costs in this subgame, and will not deviate to entering (because entry is costly).[17]

7 Extensions

7.1 Informative Interviews

Suppose that an interview conveys additional information about the worker to the interviewing firm. As long as all interviewing firms obtain the same information from the interview, the results of the paper apply without change. Formally, this information is an additional signal s_0 that is affiliated with the other signals and has full finite support, such that v is increasing in s_0. The realization of s_0 is revealed during the interview.

[17] In the subgame, both firms will interview with a positive probability that is strictly less than 1. Firm j will interview with low probability, and will interview with probability strictly less than 1 at any signal. It will therefore have expected profits of 0 net of entry costs.

7.2 Multiple workers

The analysis applies to multi-worker markets in which all interviews are conducted simultaneously before all offers are made simultaneously as long as there is enough separability across workers. For this we require that the vectors of signals for each worker be independent across workers, that each firm can hire any number of workers, and that the value of the workers hired by a firm be additively separable across workers. These assumptions imply that with no revelation we can analyze firms' interviewing (and hiring) decisions for each worker separately, so Proposition 1 holds.[18]

With revelation things are more delicate, because a firm's decision whether to interview a worker could depend on the signals it observes for other workers. To see why, suppose that the other firms believe that firm i decides whether to interview worker k based on the signals firm i observes for other workers. Then, the other firms will infer something about the value of workers other than k from firm i's interview decision regarding worker k. If firm i were to then decide whether to interview worker k only based on the signal it observes for worker k, then the other firms' hiring decisions regarding the other workers may be affected to the detriment of firm i. Thus, firm i may optimally condition its interview decision regarding worker k on the signals it observes for other workers.[19]

We would like to rule out such behavior, because the signal a firm observes for one worker contains no information about the value of other workers. We therefore consider separable equilibria, which are defined as follows.

Definition 1 *A* separable equilibrium *is a sequential equilibrium in which each firm's decision whether to interview a worker does not depend the firm's signals for other workers.*

In a separable equilibrium, a firm's interviewing decision regarding worker k does not contain any information about the value of other workers. Therefore, when analyzing separable equilibria we need only consider strategies in which a firm's hiring decision regarding worker k does not depend on the other firms' interviewing decisions regarding the other workers.[20] Because firms' signals are independent across workers and the value of the

[18] In principle, a firm could choose whether to interview a worker based on the signals it observes for other workers. These signals, however, do not tell the firm anything about the worker's value, so the firm could optimally use them only as a randomization device. For low enough entry and interview costs, Proposition 1 rules out such behavior because it shows that firms use pure strategies.

[19] With no revelation, such behavior does not arise, because no firm observes the other firms' interview decisions when it makes its hiring decisions.

[20] If a firm is indifferent between hiring and not hiring a worker, it may use the other firms' interviewing decisions regarding the other workers as a randomization mechanism. This does not change the statement or proof of Theorem 1.

workers to a firm is additively separable across workers, the continuation of any separable equilibrium conditional on entry is the conjunction of sequential equilibria conditional on entry in markets with one worker, one for each of the workers in the original market. In particular, for low entry costs a firm finds it optimal to enter in a sequential equilibrium of the original market if and only if it finds it profitable to enter in the sequential equilibrium of at least one of the markets corresponding to a single worker. Therefore, Theorem 1 characterizes all separable equilibria when entry and interview costs are sufficiently low.

7.3 Heterogeneity of Worker Value across Firms

The assumption that the worker's value is common to all firms is key to the analysis. If lower-ranked firms have a positive probability of hiring the worker when all firms' signals are made public, we would not expect revelation to exclude these firms (see Example 3 below). With this in mind, the common value assumption can be relaxed in the following way without changing the analysis or the results. Let every firm i's valuation for the worker be $g_i(v(s_1, \ldots, s_n))$, such that $g_i : \mathbb{R}^+ \to \mathbb{R}^+$ is strictly increasing and for any $i < j$ and x if $g_j(x) > 0$ then $g_i(x) > 0$. That is, if the net value of the worker to a lower-ranked firm is positive, then the same is true for any higher-ranked firm.[21] This maintains the property that if all firms' signals are public, a higher-ranked firm wants to hire the worker if a lower-ranked firm does.

7.3.1 Two Firms and Independent Signals

With only two firms and independent signals, we can compare the setting with revelation to that with no revelation even if the net value of the worker to firm 2 is higher than his net value to firm 1. Specifically, let the net value of the worker be $v(s_1, s_2)$ to firm 1 and $v(s_1, s_2) + w_2$ to firm 2, $w_2 > 0$. If v can take positive values lower than w_1, then it may be that firm 2 wants to hire the worker and firm 1 does not. With no revelation, Proposition 1 and its proof hold without change.

Now consider the setting with revelation, and suppose that both firms enter. For expositional simplicity, suppose that each firm interviews whenever it is indifferent between interviewing and not interviewing, and makes an offer whenever it is indifferent between making an offer and not making an offer. Because signals are independent, each firm employs a threshold interviewing strategy. Therefore, seeing the worker interviewed by firm 2 is "good news" and seeing the worker not interviewed by firm 2 is "bad news" for firm 1 about the worker's value, regardless of firm 2's interviewing threshold. This means

[21] One example is adding a firm-specific constant to v with higher-ranked firms having a higher constant. Another example is any firm-specific strictly increasing transformation for which 0 is a fixed point.

that for any signal s'_1, firm 1's estimation of the worker's value is higher when it sees firm 2 interviewing than what it would be with no revelation, which in turn is higher than its estimation when it sees firm 2 not interviewing. That is,

$$E\left[v|s_1 = s'_1, s_2 \geq T_2^0\right] \geq E\left[v|s_1 = s'_1\right] \geq E\left[v|s_1 = s'_1, s_2 < T_2^0\right],$$

where T_2^0 is firm 2's interview threshold (the lowest signal for which it interviews) with revelation. In particular,

$$E\left[v|s_1 = T_1, s_2 \geq T_2^0\right] \geq E\left[v|s_1 = T_1\right] \geq E\left[v|s_1 = T_1, s_2 < T_2^0\right],$$

where T_1 is firm 1's interview threshold with no revelation. So the lowest signal at which firm 1 is willing to interview and make the worker an offer if firm 2 interviews is weakly lower than T_1. Denote this signal by T_1^1. The lowest signal at which firm 1 is willing to interview and make the worker an offer if firm 2 does not interview is weakly higher than T_1. Denote this signal by T_1^0. Because $T_1^0 \geq T_1^1$, firm 1's interview threshold with revelation is T_1^1. If $s_1 \in [T_1^1, T_1^0)$, then firm 1 makes the worker an offer only if firm 2 interviews. In this interval of signals, firm 1 interviews the worker for the "option value" of hiring him.

How does T_2^0, firm 2's interview threshold with revelation, compare to T_2, firm 2's interview with no revelation? Suppose firm 2 interviews the worker. Because $T_1^1 \leq T_1^0$, firm 2 can only hire the worker when firm 1 doesn't interview. And because $T_1^1 \leq T_1$, firm 2 faces greater adverse selection with revelation than with no revelation. That is, for any signal s'_2, firm 2's expected value of the worker conditional on being able to hire him is lower with revelation than with no revelation, so

$$E\left[v|s_1 \leq T_1^1, s_2 = s'_2\right] \leq E\left[v|s_1 \leq T_1, s_2 = s'_2\right].$$

In particular,

$$E\left[v|s_1 \leq T_1^1, s_2 = T_2\right] \leq E\left[v|s_1 \leq T_1, s_2 = T_2\right].$$

Therefore, with revelation firm 2's interviewing (and hiring) threshold T_2^0 is weakly higher than T_2. The following figure summarizes these results.

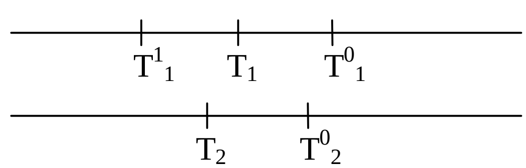

Figure 1: Interview and hiring thresholds with and without revelation for two firms with independent signals

The unambiguous location of the interviewing and hiring thresholds with revelation relative to those with no revelation leads to the following observations. First, if the worker is hired with revelation, then he is also hired with no revelation. This is because with revelation the worker is hired by firm 1 if $s_1 \geq T_1^1$ and $s_2 \geq T_2^0 \geq T_2$ (in which case with no revelation the worker is hired by firm 2 if not by firm 1) or if $s_1 \geq T_1^0 \geq T_1$ (in which case with no revelation the worker is hired by firm 1), and by firm 2 if $s_1 \leq T_1^1 \leq T_1$ and $s_2 \geq T_2^0 \geq T_1$ (in which case with no revelation the worker is hired by firm 2). This also shows that if the worker is hired by firm 2 with revelation, then he is also hired by firm 2 with no revelation, so the only possible "movement" of the worker as a result of revelation is from firm 2 to firm 1 (when $s_1 \in [T_1^1, T_1)$ and $s_2 \geq T_2^0$), and from both firms to unemployment (when $s_1 \in [T_1, T_1^0)$ and $s_2 \leq T_2^0$, or $s_1 \leq T_1^0$ and $s_2 \in [T_2, T_2^0)$). As a result, firm 1 is weakly better off and firm 2 is weakly worse off with revelation. The worker may be better off or worse off. The following proposition summarizes these results.[22]

Proposition 3 *Suppose that $n = 2$ and the two firms have independent signals. When entry and interview costs are low so that Proposition 1 holds, the following statements are true for any equilibrium with revelation.*

1. *Revelation makes firm 1 weakly better off. Firm 1's interview threshold is weakly lower with revelation than with no revelation.*
2. *Revelation makes firm 2 weakly worse off. Firm 2's interview threshold is weakly higher with revelation than with no revelation.*
3. *If the worker is hired by firm 2 with no revelation, he may be hired by firm 1 with revelation.*
4. *If the worker is hired by firm 2 with revelation, then he is hired by firm 2 with no revelation.*
5. *If the worker is hired with no revelation, he may be unemployed with revelation.*
6. *If the worker is hired with revelation, he is hired with no revelation.*

The proposition holds even without assuming that both firms enter. Indeed, if firm 1 enters, it is weakly better off if firm 2 enters, so by Assumption A1 firm 1 always enters and is therefore at least as well off with revelation as it is with no revelation. If firm 2

[22] Although we explicitly considered equilibria in which each firm interviews whenever it is indifferent between interviewing and not interviewing, and makes an offer whenever it is indifferent between making an offer and not making an offer, the same analysis holds for all equilibria, as does Proposition 3.

does not enter, then it is weakly worse off than with no revelation. The following example shows that both firms may enter with revelation.

7.3.2 Example 3 - The Net Value of the Worker to Firm 2 is Higher than to Firm 1

Suppose that there are two firms, and each firm's signal is drawn uniformly and independently from the set $\left\{-\frac{1}{2}\_\varepsilon, \frac{1}{2}-\varepsilon\right\} \cup \left\{-\frac{1}{2}+\frac{i}{12k}-\varepsilon, -\frac{1}{2}+\frac{i}{12k}+\varepsilon : i=1,\ldots,12k-1\right\}$ for some $k \geq 1$ and positive $\varepsilon < \frac{1}{48k}$ (this approximates the uniform distribution on $[0,1]$). Suppose that the net value of the worker is s_1+s_2 to firm 1 and $s_1+s_2+\frac{1}{2}$ to firm 2. Without revelation, if entry and interview costs are sufficiently small, there is a unique equilibrium. In this equilibrium, both firms enter and interview with probability 1 at all signals greater or equal to their interview thresholds, which are $T_1=\varepsilon$ and $T_2=-\frac{1}{4}+\varepsilon$. With revelation, if entry and interview costs are sufficiently small, there is a unique equilibrium in which both firms enter. This equilibrium has thresholds $T_1^1=T_2^0=-\frac{1}{6}+\varepsilon$ and $T_1^0=\frac{1}{3}+\varepsilon$. In particular, $T_1^1<T_1<T_1^0$ and $T_2<T_2^0$, as in Figure 1. A worker with s_i in $\left[-\frac{1}{6}+\varepsilon,\varepsilon\right)$ and $s_2 \geq -\frac{1}{6}+\varepsilon$ is hired by firm 2 with no revelation and by firm 1 with revelation. A worker with s_1 in $\left[\varepsilon,\frac{1}{3}+\varepsilon\right)$ and $s_2 \leq -\frac{1}{6}+\varepsilon$, or with $s_1 \leq \frac{1}{3}+\varepsilon$ and s_2 in $\left[-\frac{1}{4}+\varepsilon,-\frac{1}{6}+\varepsilon\right)$, is employed with no revelation but is unemployed with revelation.

7.3.3 More than Two Firms

Even when there are more than two firms, firm 1 is always made weakly better off by revelation compared to no revelation. This is because firm 1 benefits from more information and faces no adverse selection. In contrast to the two-firms case, however, the effect of revelation on firms other than firm 1 when a lower-ranked firm may have a higher value for the worker is no longer unambiguous. In particular, revelation may make a low-ranked firm better off. To see this, consider three firms, with firm 3's signal so uninformative that firms 1 and 2 ignore firm 3's interviewing decision when they make their hiring decisions. Our results from the two-firm setting suggest that sometimes the worker hired by firms 1 and 2 with no revelation may not be hired with revelation. When firm 3's wage is low enough, it would like to hire the worker when it is not hired by firms 1 and 2, so revelation may make it better off. This is what happens in the following example.

7.3.4 Example 4 - A Low-Ranked Firm is Better Off with Revelation

Modify Example 7.3.2 above by adding firm 3 with the same signal structure as firms 1 and 2, and set $v=s_1+s_2+\delta s_3+w_3$ for $\delta<\frac{\varepsilon}{2}$ and $w_3>0$ to be specified below. For sufficiently small entry and interview costs, we still have that with no revelation $T_1=\varepsilon$ and $T_2=-\frac{1}{4}+\varepsilon$.

Because firm 3 can hire when the other firms do not interview, the expected net value of a worker that firm 3 can hire with no revelation at signal s_3 is $-\frac{1}{4}-\frac{3}{8}+\delta s_3+w_3 = -\frac{5}{8}+\delta s_3+w_3$. With revelation, for sufficiently small entry and interview costs, firms 1 and 2 ignore firm 3's interviewing decision, so $T_1^1 = T_2^0 = -\frac{1}{6}+\varepsilon$ and $T_1^0 = \frac{1}{3}+\varepsilon$. In this case, firm 3 can hire when firm 1's signal is below T_1^0 and firm 2 doesn't interview. The expected value of the worker is then $-\frac{1}{12}-\frac{1}{3}+\delta s_3+w_3 = -\frac{5}{12}+\delta s_3+w_3 > -\frac{5}{8}+\delta s_3+w_3$. Moreover, the probability that firm 3 is able to hire with revelation is higher than with no revelation. If $w_3 \geq \frac{5}{12}+\varepsilon$, then for sufficiently small entry and interviewing costs, with revelation firm always 3 interviews the worker and is strictly better off than with no revelation: it succeeds in hiring with higher probability and obtains a worker with higher expected value whenever it hires. Moreover, if w_3 is in $\left[\frac{5}{12}+\varepsilon, \frac{5}{8}-\varepsilon\right)$, then firm 3 does not enter with no revelation, whereas with revelation, for sufficiently small entry and interviewing costs, it enters and makes strictly positive profits.

8 Conclusion

This paper has investigated a model in which privately-informed firms interview a worker whose value is common to all firms before making their hiring decisions. When firms' interview decisions are kept private, each firm can make use of its private information, even though all but the highest-ranked firm face adverse selection akin to a "winner's curse." When firms' interview decisions are revealed, the adverse selection becomes so strong that only the top firm can make use of its private information - all other firms stay out of the market. Revelation of firms' interview decisions, which has the potential to improve market outcomes through the sharing of private information, leads to complete unraveling and less usage of information than with no revelation. The outcome with revelation is worse than with no revelation according to virtually any efficiency or social welfare criterion. This effect may be less pronounced when complementarity/substitution among workers, private value components, and other real-world features are introduced.[23] We view our result as indicative of the potential for adverse selection in markets with intermediate, coarse information disclosure, even when there are many firms and the information structure is fairly general.

Our analysis assumed that whether firms' interview decisions are revealed is determined exogenously. An interesting question is under what circumstances we would expect revelation to arise endogenously. Suppose that the only way an interview is revealed is if the worker or firm that participated in the interview reveal the fact of the interview (no lying is possible). Then it is an equilibrium for all workers and firms to reveal all interviews,

[23] Of course, such features make the model much less tractable.

because a unilateral deviation by a worker or a firm does not affect revelation. A full characterization of all equilibria when revelation is endogenized is beyond the scope of this paper. We intend to explore this issue in future work.

A Appendix

Denote by G_i the uniform CDF on $\Omega_i = [0,1]$. Endow $\Omega = \times_{i\in N}\Omega_i$ with the product CDF $G = \times G_i$. Denote by μ^G the probability measure on Ω induced by G, by μ_i^G the probability measure on Ω_i induced by G_i,and by μ_{-i}^G the probability measure on Ω_{-i} induced by G_{-i}, where $-i$ is the set of indices other than i. Consider the probability space defined by $S\times\Omega$ and the probability measure $\mu^{F\times G}$ induced by $F\times G$. Denote by $\mu_i^{F\times G}$ and $\mu_{-i}^{F\times G}$ the induced probability measures on the measurable spaces $S_i\times\Omega_i$ and $S_{-i}\times\Omega_{-i}$.

Lemma 4 *Every measurable set $Z_i \subseteq S_i\times\Omega_i$ can be represented uniquely as $\cup_{s_i'\in S_i}\{s_i'\}\times A(s_i')$, where $A(s_i')$ are measurable subsets of Ω_i.*

Proof. The set $\Delta_i = \left\{\cup_{s_i'\in S_i}\{s_i'\}\times A(s_i') : A(s_i') \text{ are measurable subsets of } \Omega_i\right\}$ is a σ-algebra: $S_i\times\Omega_i$ is an element of Δ_i, the complement of an element of Δ_i is in Δ_i, and a countable union of elements in Δ_i are in Δ_i. Moreover, Δ_i is the smallest σ-algebra of $S_i\times\Omega_i$ with respect to which the projection mappings $\pi_1 : S_i\times\Omega_i \to S_i$ and $\pi_2 : S_i\times\Omega_i\to\Omega_i$ are continuous. To see that the projection mappings are continuous, note that for any $B\subseteq S_i$,

$$\pi_1^{-1}(B) = \cup_{s_i'\in B}\{s_i'\}\times\Omega_i \cup_{s_i'\notin B}\{s_i'\}\times\phi,$$

and for any $C\subseteq\Omega_i$,

$$\pi_2^{-1}(C) = \cup_{s_i'\in S_i}\{s_i'\}\times C.$$

Now consider some σ-algebra $\tilde{\Delta}_i$ of $S_i\times\Omega_i$ with respect to which the projection mapping are continuous. By continuity, for any $s_i'\in S_i$ and measurable $B\subseteq\Omega_i$, the sets $\pi_1^{-1}(\{s_i'\}) = \{s_i'\}\times\Omega_i$ and $\pi_2^{-1}(B) = \cup_{s_i'\in S_i}\{s_i'\}\times B$ are elements of $\tilde{\Delta}_i$. Because $\tilde{\Delta}_i$ is closed under finite intersections, $\{s_i'\}\times\Omega_i\cap\cup_{s_i'\in S_i}\{s_i'\}\times B = \{s_i'\}\times B$ is an element of $\tilde{\Delta}_i$. Because $\tilde{\Delta}_i$ is closed under countable unions, $\Delta_i\subseteq\tilde{\Delta}_i$. By definition as the smallest σ-algebra with respect to which the projection mappings are continuous, the product σ-algebra on $S_i\times\Omega_i$ is therefore Δ_i, so every measurable subset of $S_i\times\Omega_i$ is an element of Δ_i. Uniqueness of the representation follows from the fact that every $s_i'\in S_i$ appears only once in the representation. ■

Consider sets $Z_1,\ldots,Z_n$ such that for every $i\in N$, Z_i is a positive-measure subset of $S_i\times\Omega_i$. Let

$$\tilde{S}_i = \left\{s_i'\in S_i : \mu_i^G(A(s_i')) > 0\right\},$$

where $A(s_i')$ is such that $\{s_i'\}\times A(s_i')$ appears in the unique representation of Z_i from Lemma 4. The set $\tilde{S}_i$ is comprised of the signals in S_i that appear in Z_i with positive probability. Let $\tilde{S} = \times_{i\in N}\tilde{S}_i$, and for every $s' = (s_1',\ldots,s_n')\in\tilde{S}$, let

$$\delta(s') = f(s')\,\Pi_{i\in N}\mu_i^G(A(s_i')) > 0.$$

For every $s'\in\tilde{S}$, let $h(s') = \frac{\delta(s')}{\sum_{s''\in\tilde{S}}\delta(s'')}$. Then h induces a probability distribution on $\tilde{S}$. Denote the CDF of this probability distribution by H. For every $i\in N$, let $\tilde{s}_i$ be the random variable induced by H on $\tilde{S}_i$.

Lemma 5 *If the random variables $s_1, \ldots, s_n$ are affiliated (under F), then so are $\tilde{s}_1, \ldots, \tilde{s}_n$ (under H).*

Proof. Choose s', s'' in $\tilde{S} \subseteq S$. Because $\tilde{S} = \times_{i \in N} \tilde{S}_i$, $(s' \vee s'') \in \tilde{S}$ and $(s' \wedge s'') \in \tilde{S}$, where $\vee$ is the component-wise maximum and $\wedge$ is the component-wise minimum. It remains to show that $h(s' \vee s'') h(s' \wedge s'') \geq h(s') h(s'')$. We have

$$h(s' \vee s'') h(s' \wedge s'') = \frac{\delta(s' \vee s'') \delta(s' \wedge s'')}{\left(\sum_{\bar{s} \in \tilde{S}} \delta(\bar{s})\right)^2}$$

$$= \frac{1}{\left(\sum_{\bar{s} \in \tilde{S}} \delta(\bar{s})\right)^2} f(s' \vee s'') \Pi_{i \in N} \mu_i^G (A(\max(s_i', s_i''))) f(s' \wedge s'') \Pi_{i \in N} \mu_i^G (A(\min(s_i', s_i'')))$$

$$= \frac{f(s' \vee s'') f(s' \wedge s'')}{\left(\sum_{\bar{s} \in \tilde{S}} \delta(\bar{s})\right)^2} \Pi_{i \in N} \mu_i^G (A(s_i')) \mu_i^G (A(s_i''))$$

$$\geq \frac{f(s') f(s'')}{\left(\sum_{\bar{s} \in \tilde{S}} \delta(\bar{s})\right)^2} \Pi_{i \in N} \mu_i^G (A(s_i')) \mu_i^G (A(s_i'')) = h(s') h(s''),$$

where the inequality follows from affiliation under F. ■

In what follows, we will use the following well-known property of affiliation.

Lemma 6 *If $s_1, \ldots, s_n$ are affiliated and $v(s_1, \ldots, s_n)$ is non-decreasing in each of its arguments, then $E(v(s_1, \ldots, s_n) | s_1 = s_1')$ is non-decreasing in s_1'.*

Proof. Milgrom & Weber (1982), Theorem 5 (page 1100). ■

Corollary 1 *If $s_1, \ldots, s_n$ are affiliated and $v(s_1, \ldots, s_n)$ is strictly increasing in each of its arguments, then $E(v(s_1, \ldots, s_n) | s_1 = s_1')$ is strictly increasing in s_1'. If s_1' strictly increases, then so does the expectation.*

Proof. Let $s_1'' \geq s_1'$. We have

$$E(v(s_1 = s_1', \ldots, s_n) | s_1 = s_1') \leq E(v(s_1 = s_1', \ldots, s_n) | s_1 = s_1'')$$

$$< E(v(s_1 = s_1'', \ldots, s_n) | s_1 = s_1''),$$

where the first inequality is an application of the lemma, and the second inequality follows because it holds for every realization of $s_2, \ldots, s_n$. ■

Suppose that a firm has some conjecture about the realization of other firms' signals. The next lemma shows that regardless of this conjecture, seeing a higher signal makes the firm more optimistic about the value of the worker.

Lemma 7 *Suppose that $s_1, \ldots, s_n$ are affiliated, and $v(s_1, \ldots, s_n)$ is strictly increasing in each of its arguments. For every $j \neq i$, let Z_j be a positive-measure subset of $S_j \times \Omega_j$, and let $Z_{-i} = \times_{j \neq i} Z_j$. If s_i' and s_i'' are elements of S_i such that $s_i'' \geq s_i'$ and A and B are positive-measure subsets of Ω_i, then $E(v| \{s_i''\} \times A, Z_{-i}) \geq E(v| \{s_i'\} \times B, Z_{-i})$. If the first inequality is strict, then so is the second.*

Proof. Because G_i is statistically independent of F and G_{-i}, and v is not a function of Ω_i, we have $E(v|\{s_i'\} \times A, Z_{-i}) = E(v|s_i', Z_{-i})$. Let $Z_i = \{s_i', s_i''\} \times \Omega_i$, and define $\tilde{S}$ from (Z_i, Z_{-i}) as described above. By Lemma 5 and Corollary 1, $E_H(v|\tilde{s}_i = s_i'', \tilde{s}_{-i}) \geq E_H(v|\tilde{s}_i = s_i', \tilde{s}_{-i})$, with a strict inequality if $s_1'' > s_1'$. Therefore, it suffices to show that $E(v|s_i', Z_{-i}) = E_H(v|\tilde{s}_i = s_i', \tilde{s}_{-i})$ and $E(v|s_i'', Z_{-i}) = E_H(v|\tilde{s}_i = s_i'', \tilde{s}_{-i})$. We will show the first equality; the second follows by replacing s_i' with s_i''. Using the notation introduced above, we have

$$E_H(v|\tilde{s}_i = s_i', \tilde{s}_{-i}) = \frac{1}{\sum_{s'_{-i} \in \tilde{S}_{-i}} h(s_i', s'_{-i})} \sum_{s'_{-i} \in \tilde{S}_{-i}} h(s_i', s'_{-i}) v(s_i', s'_{-i})$$

$$= \frac{\sum_{s' \in \tilde{S}} \delta(s')}{\sum_{s'_{-i} \in \tilde{S}_{-1}} \delta(s_i', s'_{-i})} \sum_{s'_{-i} \in \tilde{S}_{-i}} \frac{\delta(s_i', s'_{-i})}{\sum_{s' \in \tilde{S}} \delta(s')} v(s_i', s'_{-i})$$

$$= \frac{1}{\sum_{s'_{-i} \in \tilde{S}_{-1}} \delta(s_i', s'_{-i})} \sum_{s'_{-i} \in \tilde{S}_{-i}} \delta(s_i', s'_{-i}) v(s_i', s'_{-i})$$

$$= \frac{1}{\sum_{s'_{-i} \in \tilde{S}_{-1}} f(s_i', s'_{-i}) \mu_i^G \underbrace{(A(s_i'))}_{\Omega_i} \Pi_{j \neq i} \mu_j^G (A(s_j'))}.$$

$$\sum_{s'_{-i} \in \tilde{S}_{-i}} f(s_i', s'_{-i}) \mu_i^G \underbrace{(A(s_i'))}_{\Omega_i} \Pi_{j \neq i} \mu_j^G (A(s_j')) v(s_i', s'_{-i})$$

$$= \frac{1}{\mu^{F \times G}(\{s'_{-i}\} \times \Omega_i \times Z_{-i})} \sum_{s'_{-i} \in \tilde{S}_{-i}} \mu^{F \times G}(\{s_i', s'_{-i}\} \times \Omega_i \times_{j \neq i} A(s_j')) v(s_i', s'_{-i})$$

$$= E(v|s_i', Z_{-i}).$$

■

For any $i \in N$ and $s_i' \in S_i$, denote by $f_i(s_i') = \sum_{s'_{-i} \in S_{-i}} f(s_i', s'_{-i})$ the marginal probability of s_i'. For any $s' \in S$, let $\widetilde{f}(s') = \prod_{i \in N} f_i(s_i') > 0$, and denote by $\widetilde{F}$ the CDF on S corresponding to $\widetilde{f}$. Denote by $\mu^{\widetilde{F} \times G}$ the measure on $S \times \Omega$ induced by $\widetilde{F} \times G$. By definition, under $\mu^{\widetilde{F} \times G}$ the measurable events in $S_i \times \Omega_i$ are statistically independent of those in $S_j \times \Omega_j$, for any $i \neq j$. Clearly, a set $X \subseteq S \times \Omega$ is $\mu^{F \times G}$-measurable if and only if it is $\mu^{\widetilde{F} \times G}$-measurable. By definition, for any measurable subset $Z_i \subseteq S_i \times \Omega_i$ we have $\mu^{F \times G}(Z_i \times S_{-i} \times \Omega_{-i}) = \mu^{\widetilde{F} \times G}(Z_i \times S_{-i} \times \Omega_{-i})$. For any $s' \in S$, let $\phi(s) = \frac{f(s')}{\widetilde{f}(s')} > 0$. Let $\phi_{\min} = \min_{s' \in S} \phi(s')$ and $\phi_{\max} = \max_{s' \in S} \phi(s')$.

Lemma 8 *If X is a measurable subset of $S \times \Omega$, then*

$$\phi_{\min} \mu^{\widetilde{F} \times G}(X) \leq \mu^{F \times G}(X) \leq \phi_{\max} \mu^{\widetilde{F} \times G}(X).$$

Proof. For any $s' \in S$ and every measurable set $A \subseteq \Omega$, we have

$$\mu^{F\times G}(\{s'\}\times A) = f(s')\,\mu^{G}(A) = \phi(s')\,\widetilde{f}(s')\,\mu^{G}(A) = \phi(s')\,\mu^{\widetilde{F}\times G}(\{s'\}\times A). \qquad (17)$$

A proof similar to that of Lemma 4 shows that every measurable subset of $S\times\Omega$ can be represented uniquely as $\cup_{s'\in S}\{s'\}\times A(s')$, where $A(s')$ are measurable subsets of Ω. This observation, together with equation (17) implies the result. ■

Corollary 2 *Suppose $X_1, X_2, ...$ is sequence of measurable subsets of $S\times\Omega$. Then $\mu^{F\times G}(X_k) \underset{k\to\infty}{\to} 0$ if and only if $\mu^{\widetilde{F}\times G}(X_k) \underset{k\to\infty}{\to} 0$.*

Proof. Immediate from Lemma 8. ■

Corollary 3 *A measurable subset X of $S\times\Omega$ has positive measure under $\mu^{F\times G}$ if and only if it has positive measure under $\mu^{\widetilde{F}\times G}$. For such a positive-measure set,*

$$\phi_{\min} \leq \frac{\mu^{F\times G}(X)}{\mu^{\widetilde{F}\times G}(X)} \leq \phi_{\max}.$$

In particular, if $X = \times_{i\in N} Z_i$ for positive-measure sets $Z_i \subseteq S_i \times \Omega_i$, then

$$\phi_{\min} \leq \frac{\mu^{F\times G}(X)}{\prod_{i\in N}\mu^{F\times G}(Z_i\times S_{-i}\times\Omega_{-i})} \leq \phi_{\max}.$$

Proof. The first two claims are Immediate from Lemma 8 and Corollary 2. The last claim follows from the definition of $\mu^{\widetilde{F}\times G}$. ■

Corollary 4 *Suppose $X_1, X_2, ...$ and $Y_1, Y_2, \ldots$ are sequences of measurable subsets of $S\times\Omega$, and $\mu^{F\times G}(Y_k)$ is bounded away from 0 for all k. Then (i) $\mu^{\widetilde{F}\times G}(Y_k)$ is bounded away from 0 for all k, and (ii) $\mu^{F\times G}(X_k|Y_k) \underset{k\to\infty}{\to} 0$ if and only if $\mu^{\widetilde{F}\times G}(X_k|Y_k) \underset{k\to\infty}{\to} 0$.*

Proof. Part (i) is an immediate application of the lemma. For part (ii), let $C_k = X_k \cap Y_k$. Recall that $\mu^{F\times G}(X_k|Y_k) = \frac{\mu^{F\times G}(C_k)}{\mu^{F\times G}(Y_k)}$ and $\mu^{\widetilde{F}\times G}(X_k|Y_k) = \frac{\mu^{\widetilde{F}\times G}(C_k)}{\mu^{\widetilde{F}\times G}(Y_k)}$. Because both $\mu^{F\times G}(Y_k)$ and $\mu^{\widetilde{F}\times G}(Y_k)$ are at most 1 and are bounded away from 0, $\mu^{F\times G}(X_k|Y_k) \underset{k\to\infty}{\to} 0$ if and only if $\mu^{F\times G}(C_k) \underset{k\to\infty}{\to} 0$, and $\mu^{\widetilde{F}\times G}(X_k|Y_k) \underset{k\to\infty}{\to} 0$ if and only if $\mu^{\widetilde{F}\times G}(C_k) \underset{k\to\infty}{\to} 0$. Now apply the Corollary 2 to the sequence $C_1, C_2, \ldots$. ■

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