ON THE RELATIONSHIP BETWEEN HISTORICAL COST, FORWARD-LOOKING COST AND MARGINAL COST WHEN PRODUCTION REQUIRES SUNK INVESTMENTS IN LONG-LIVED ASSETS

by
William P. Rogerson*
Northwestern University
September 13, 2005

* I would like to thank Debra Aron, Kathleen Hagerty, Jack Hughes, Volcker Nocke, and Stefan Reichelstein and for helpful discussions and comments.

INTRODUCTION

Under traditional rate of return regulation, the per period cost of using long-lived assets to produce goods and services is calculated by allocating the original purchase cost of each asset across all of the periods that the asset will be used. Since the cost of using long-lived assets in any given period will therefore depend on the original purchase costs of all of the assets being used in that period, such costs are often referred to as "historically-based costs" or simply "historical costs." In the telecommunications industry, the replacement cost of many of the long-lived assets that firms use to produce telecommunications services has been dropping dramatically over time due to technological progress. Motivated by the intuition that historical costing methods may overstate the current long run marginal cost of production in this case, because *current* long run marginal cost should depend on the *current* cost of replacing assets and not on the *historical* cost of purchasing assets, regulators in many developed countries, including the United States and most countries in Western Europe, have recently begun to base prices on costs calculated using the current replacement cost of assets instead of their original purchase cost. Costs calculated under such a methodology are often referred to as "forward looking costs."[1] Under a forward looking methodology, the regulator estimates the hypothetical cost of replacing the existing assets of the firm with functionally equivalent new assets and then allocates a share of the hypothetical total replacement cost to the current period.

---

[1]In the United States this cost concept is often referred to as Total Element Long Run Incremental Cost (TELRIC) and in Western Europe, New Zealand, and Australia it is often referred to as Total Service Long Run Incremental Cost (TSLIRIC). See Salinger (1998), Hausman(2000), Falch(2002), Rosston and Noll(2003), and Federal Communications Commision(2003) as well as the references cited below for further discussion and institutional background.

There has been considerable controversy and confusion over the related issues of whether or not it is desirable to base prices on forward looking costs instead of historical costs, and, if so, what share of the hypothetical replacement cost ought to be allocated to the current period. Economic theory has not provided any definitive answers to these questions. The purpose of this paper is to fill this void by constructing a theory that sorts out the relationship between various accounting measures of historical and forward looking cost and the true long run marginal cost of production. The theory will also have applications to the study of how unregulated for-profit firms use accounting data to organize and guide their decision-making.

The immediate practical question that regulators face when they decide to base prices on forward looking cost is deciding what share of the hypothetical total replacement cost to allocate to the current period for purposes of calculating forward looking cost. This will be called the current period allocation share. Most regulators, including the FCC, have adopted what will be called an annuity approach to calculating this value. Namely, they allocate a share of cost to the current period such that the firm would be fully reimbursed for the hypothetical replacement cost if it received a payment equal to this amount for every period of the asset's life. A number of papers[2] have investigated this pricing rule in simple models where it is assumed that the replacement cost of assets is falling over time at some constant percentage rate. These papers have pointed out that the rule used by the FCC under-compensates the firm for its investment costs, essentially because the calculation does not correctly anticipate that future output prices will continue to decline when this rule is used every period. They show how to calculate a

---

[2]See Hausman (1997), Laffont and Tirole (2003), Mandy (2002), and Mandy and Sharkey (2003).

current period allocation share that will result in the firm earning zero economic profit. Because this calculation correctly anticipates the effect of the rule on future prices, this corrected value of the current period allocation share will be called the "time consistent current period allocation share" and forward looking cost calculated using the time consistent current period allocation share will be called "time consistent forward looking cost."

While these papers show how to construct a forward looking pricing rule that results in the firm earning zero economic profit, they do not attempt to directly investigate whether there is any first or second best sense in which this rule is efficient. In particular, although setting prices equal to time-consistent forward looking cost results in the firm being fully compensated for its investments, this is also true by construction for any historical pricing rule using any allocation rule. Therefore this result, alone, does not explain why setting prices equal to time-consistent forward looking cost is necessarily superior to setting prices equal to historic cost calculated using any allocation rule, even if one imposes the requirement that the firm must earn zero economic profit. Furthermore, these papers certainly do not shed any light on the issue of whether prices that induce even more efficient outcomes could be set if the zero-profit requirement was not imposed. Laffont and Tirole (2003) also point out that the vector of time consistent forward looking costs of using assets can be interpreted as the rental prices that would be charged for using assets, if assets could be costlessly switched between uses and a perfectly competitive rental market for assets existed. However, it is not clear why this result is necessarily relevant to determining what the efficient prices should be in the real situation of interest where these assumptions are clearly not satisfied.

This paper considers a simple model of the sort used by the existing literature on forward looking pricing where a firm must invest in finite-but-long-lived assets in order to produce output, the purchase price of new assets is falling over time at some known constant percentage rate, and there are constant returns to scale in capacity each period in the sense that one unit of the asset allows the firm to produce one unit of output in every period of the asset's life. The main result is to show that, so long as the demand curve is weakly shifting out over time, setting price each period equal to time consistent forward looking cost is first-best in the sense that these prices simultaneously maximize total surplus ignoring the zero profit constraint and result in the firm earning zero profit.

The basic explanation for the result is as follows. Define the "true cost function" to be the function that gives the present discounted value of producing any given vector of outputs given that the firm must purchase assets to produce output. Define the "accounting cost function" to be the function giving the present discounted cost under the hypothetical assumption that the firm has a constant marginal cost of production each period equal to the time consistent forward looking cost for that period. The main result is to show that the true cost of production is exactly equal to the accounting cost of production so long as output is weakly increasing over time and that the true cost of production is always greater than or equal to the accounting cost of production. It follows directly from this result that the vector of outputs that maximizes total surplus when costs are calculated using the accounting cost function will also maximize total surplus when costs are calculated using the true cost function, so long as output is weakly increasing over time. This latter condition is implied by the assumption that the demand curves shift weakly out over time.

The result that the true cost function is linear and separable over a broad range of outputs and that first best prices exist is perhaps somewhat surprising given that each asset is a joint cost of production across multiple periods. A general principle from both the economics and managerial accounting literatures that study cost allocation[3] is that there is generally no economically meaningful way to assign a joint cost of producing multiple products to individual products. Thus we might expect that the cost function for producing a vector of joint products would inherently not be additively separable in each product. Yet this is precisely what happens in the model of this paper. The resolution to this apparent conflict lies in the fact that there are "multiple overlapping" joint costs in the model of this paper, instead of a single joint cost. When there is a single joint cost for all products, the only way to increase the output of a single product is to increase investment in the joint cost and this results in increased output of all products. Thus increasing the production of one good necessarily results in increases in the production of all goods. However, in the model of this paper, where there are multiple overlapping joint costs, this is not necessarily true.

For example, suppose that the firm has made investment plans to produce a particular vector of outputs over time and that the firm plans to purchase at least one unit of the asset every period. Suppose that each asset has a life of ten years and let period 0 be the current period. Then the firm can increase output in the current period by one unit while holding output in all other periods constant by implementing the following series of adjustments to it investment plans. The firm must purchase an additional unit of the asset in period 0 to increase production

[3]See, for example, Biddle and Steinberg (1985), Demski(1981), Thomas(1978), and Young(1985).

by one unit in this period. However, it will now be able to reduce its asset purchases by one unit in period 1. Now when period 10 arrives, the extra asset that the firm purchased in period 0 will no longer be available, so the firm will have to purchase an extra unit of the asset in that period to maintain its level of production at the previously planned level. However, as before, it will now be able to reduce its asset purchases by one unit in period 11. This process continues indefinitely. That is, the firm can produce exactly one more unit of output in the current period and hold output in all other periods fixed by shifting the purchase of one unit of the asset forward in time from period 1 to 0, 11 to 10, 21 to 20, etc. It is straightforward to calculate the present discounted value of the cost of these adjustments and this is the marginal cost of increasing output by one unit in the current period. Thus, even though each asset can be viewed as a joint cost of production over multiple periods, it is still possible to increase production in one period while holding output in all other periods constant by adjusting the entire vector of overlapping joint costs. This paper shows that in the model of this paper, the result of this is that the cost function defined over outputs is actually linear and separable so long as output is weakly increasing over time.

This explanation of how the marginal cost of increasing output in a single period is determined also provides an explanation for one of the potentially puzzling properties of time consistent forward looking cost. This is that an increase in the future rate of technological progress holding the current cost of purchasing assets constant increases time consistent forward looking cost in the current period. That is, while many regulators had the intuition that higher rates of future technological progress ought to allow them to reduce current prices, the correct rule will actually increase prices. The economic explanation for this is simply that the way that the firm increases production in the current period while holding output in all other periods constant is to shift asset purchases forward in time. Higher rates of technological progress increase the cost of shifting asset purchases forward in time because more assets must be purchased in earlier periods when they are more expensive. Thus higher future rates of technical progress increase the current cost of producing output.

The paper also goes on the investigate whether historical pricing rules can be used to calculate marginal cost and induce efficient consumption decisions. It is shown that historic cost calculated using an allocation rule which will be called the relative replacement cost (RRC) allocation rule is exactly equal to time consistent forward looking cost. Therefore, setting price equal to historic cost calculated using the RRC allocation rule will also produce the efficient outcome so long as demand weakly shifts out over time. In particular, then, regulators' intuition that a forward looking pricing rule must be used when asset prices are changing over time is not correct. There is actually a historical rule which produces the same result as the correct forward looking rule. It will be argued in the body of this paper that this is an important point, because an advantage that historically based costing rules have over forward looking rules is that they are likely to be based on more objective data.

Finally, in the model of this paper where one unit of the asset is required to produce one unit of output in a given period, the cost of producing one unit of output can equally well be interpreted as the cost of using one unit of capital for a given period. Therefore, even in more complex models where there are additional inputs and where output is not necessarily linear in capacity, the results of this paper can be interpreted as explaining how relatively simple accounting rules can be used to calculate the per period cost of using long-lived assets. Thus, accounting rules that allocate investment costs across periods can quite generally be used to simplify complex multi-period optimization problems by reducing them to a series of unrelated single-period problems.

Although the primary focus of this paper will be on the problem of choosing efficient prices for a regulated firm, the result of this paper that relatively simple accounting rules can be used to allocate investment costs across periods in order to calculate marginal cost is also very relevant to the study of how unregulated firms use accounting information to organize and guide their decision-making.[4] For example, the model as described above can be reinterpreted as a model of transfer pricing where an upstream division must make sunk investments in long-lived assets to produce a good used an input by a downstream division. Under this interpretation, the paper's result is that cost-based transfer prices can be calculated that simultaneously allow the upstream firm to break even and induce the downstream division to make efficient use of the input.

The papers in the economics literature most closely related to this paper are the previously mentioned papers that show that prices set equal to time consistent forward looking cost fully compensate the firm for its investments.[5] This paper provides a stronger justification for using this rule by formally showing that this rule induces efficient consumption decisions. Furthermore, it shows that the same result can be achieved by setting prices equal to historic cost

---

[4]See Balakrishnan and Sivaramakrishnan (2002) for an overview of the literature specifically focused on the issue of long run marginal cost and Demski and Kreps (1982) for a more general overview. To the best of my knowledge the result of this paper is new to this literature.

[5]See Hausman (1997), Laffont and Tirole (2003), Mandy (2002), and Mandy and Sharkey (2003).

calculated using the RRC allocation rule. The main formal model in the economics literature that analyzes the welfare effects of inter-temporal cost allocation rules in the context of cost-based regulation is due to Baumol(1971).[6] Baumol suppresses the issue of long run efficiency by simply assuming that the firm has already made a single fixed exogenous expenditure on long-lived assets and no further investment of any sort is possible. It is assumed that the firm can vary its output from period to period only by varying the amount of non-capital inputs it uses each period. In this analysis, it would be efficient to set price each period equal to short run marginal cost and, in general, setting prices at this level would not allow the firm to recover its investment costs. Baumol solves for a second best price path that maximizes total surplus subject to the constraint that the firm must be allowed to recover its investment cost. Therefore in Baumal's model, allocating the cost of long-lived investments across time is a sort of "necessary evil" that has to be endured in order that the firm be reimbursed for its investment expenses. This paper shows that a dramatically different sort of result can occur in a model where it is assumed that investment occurs every period and investment is modeled as being endogenously determined by the level of demand. Namely, allocating the cost of long-lived investments across time can help play a role in making consumption decisions more efficient by ensuring that prices reflect long run marginal cost and, at least in some cases, a first best solution is possible.

The paper is organized as follows. Section I presents the basic model. Section II presents the result of the existing literature that prices set equal to forward looking cost allow the firm to break even if and only if forward looking cost is calculated using a particular current

---

[6]Also see Crew and Kleindorfer (1992) who consider the issue that it may be necessary to front-load the reimbursement of a firm's investment if future entry of competitors is expected.

period allocation share which will be called the time consistent current period allocation share. Section III presents the main result of this paper which is that prices set equal to time consistent forward looking cost are first best so long as demand weakly shifs out over time. Section IV considers pricing rules based on historical costs shows that the same result can be achieved by setting prices equal to historical cost calculated using the RRC allocation rule. Section V briefly discusses how the model of the paper can be interpreted to apply to the study of how managers of unregulated firms use accounting information to organize and guide their activities. Section VI briefly explains how the results of the paper can be interpreted to apply to more complex cases where there are other inputs in addition to the long-lived assets and output each period in not necessarily linear in each period's capacity. Section VII draws a brief conclusion.

I. THE MODEL

A. Costs and Production

Suppose that there are an infinite number of periods indexed by $t \in \{0, 1, 2, \ldots\}$ where period 0 is the current period. Let q_t denote the number of units of output that the firm produces in period t and let $q = (q_0, q_1, \ldots)$ denote an entire vector of output levels.

Assume that the assets used to produce output have a life of L periods (where L is a positive integer) and one unit of the asset purchased in period t allows production of one unit of output in every period of the asset's life. Specifically, assume that purchase of one unit of an asset in period t allows production of one unit of output in periods $\{t, t+1, \ldots, t+L-1\}$. To reduce notational complexity, it will be assumed that the firm begins period 0 with no existing assets.[7] Let x_t denote the number of units of the asset the firm chooses to purchase in period t for $t \in \{0, 1, 2, . . . \}$ and let $x = (x_0, x_1, x_2, ., . .)$ denote the entire vector of the firm's asset purchases.

Let a_t denote the price of a unit of the asset in period t. Assume that the asset price in period 0, a_0, is a given positive number and that asset prices decline over time at a constant percentage rate. Formally, assume that

$$(1) \qquad a_t = \gamma^t a_0$$

where $\gamma \in [0, 1]$. The parameter γ determines the rate at which technological progress reduces expected asset prices over time. The case of $\gamma = 1$ corresponds to the situation where there is no technical progress and asset prices stay constant over time. Lower values of γ correspond to situations where there is more technical progress and asset prices drop over time.

Suppose that the firm discounts future cash flows according to a discount rate $\delta \in (0, 1)$. Since the firm discounts future cash flows and since the price of assets falls over time, the cost minimizing method of producing any given vector of outputs is straightforward to calculate. Namely, beginning with period 0, the firm considers each period sequentially and plans to purchase the minimum number of assets necessary to produce the required output in that period. (If the firm can produce a level of output greater than or equal to the required level of output without purchasing any assets, then it purchases no assets in that period.) Let $\phi_t(q)$ denote the

[7]It is straightforward to derive natural generalizations of the results of this paper when it is assumed that the firm begins period 0 with a vector of assets of varying vintages.

function giving the number of units of assets the firm purchases in period t to produce the output vector q and let $\phi(q) = (\phi_0(q), \phi_1(q), \ldots)$ denote the entire vector of investments. Then the discounted cost of producing q is given by

$$(2) \qquad C(q) = \sum_{t=0}^{\infty} \gamma^t \delta^t \phi_t(q) a_0.$$

For any vector of investments x, the maximum amount of output that could be produced in period t if the firm purchased assets according to x is given by

$$(3) \qquad \sum_{i=0}^{L-1} x_{t-i}$$

where x_j is interpreted to be zero for $j \leq -1$. It will be said that a vector of outputs, q, satisfies the fully utilized investment (FUI) property if the firm operates with no excess capacity every period when it produces q in a cost-minimizing manner. Formally, q satisfies FUI if and only if

$$(4) \qquad \sum_{i=0}^{L-1} \phi_{t-i}(q) = q_t \quad \text{for every } t \in \{0, 1, 2, \ldots\}$$

where $\phi_j(q)$ is interpreted to be zero for $j \leq -1$. Since the firm never purchases more capacity than it needs in any period, the only way for a firm to wind up with excess capacity in a given period is if it inherited more capacity from the previous period than it actually needs in the given period. In particular, then, a sufficient condition for a vector of outputs to satisfy FUI is simply that output be weakly increasing over time. For future reference this fact will be recorded as a lemma.

Lemma 1:

Suppose that $q_{t-1} \geq q_t$ for every $t \in \{0, 1, 2, \ldots\}$ Then q satisfies FUI.

proof:

As above. QED

B. Demand

Let p_t denote the price of output in period t and let $p = (p_0, p_1, p_2, \ldots)$ denote a vector of prices for all periods. It will be assumed that demand for the product in any period depends only on that period's price. Let $D_t(p_t)$ denote the demand function in period t. Let D(p) denote the entire vector of quantities demanded over time given the vector of prices p, i.e., $D(p) = (D_0(p_0), D_1(p_1), \ldots)$. It will be assumed that the demand functions are "well-behaved" in the sense that they are strictly decreasing and differentiable. It will also be assumed that they map the interval $(0, \infty)$ onto the interval $(0, \infty)$ to avoid the extra notation required to deal with corner solutions. Let $P_t(q_t)$ denote the inverse demand curve in period t.

Finally, the following condition on demand will not always be assumed to hold true, but will often be referred to in the analysis. It will be said that the demand curves weakly shift out over time if they satisfy

(5) $D_{t+1}(p) \geq D_t(p)$ for every $t \in \{0, 1, \ldots\}$ and for every $p \in (0, \infty)$.

C. Profit

Let $\pi(p)$ denote the present discounted value of the firm's profit if it produces the levels of output demanded by consumers when prices are p. Formally, $\pi(p)$ is defined by

$$(6) \qquad \pi(p) = \sum_{t=0}^{\infty} [p_t D_t(p_t) - \phi_t(D(p))a_t]\,\delta^t$$

When D(p) satisfies FUI, it is possible to rewrite equation (6) in a more useful form. Let $I_t(p)$ denote the present discounted value to the firm calculated in period t if it purchases one unit of the asset in period t, uses it to produce one unit of output in each period of the asset's life, and sells the output at prices determined by p. This will be referred to as profit of a fully utilized unit of investment in period t and is formally defined by

$$(7) \qquad I_t(p) = \sum_{i=0}^{L-1} p_{t+i}\,\delta^i - a_t.$$

When the firm is fully utilizing all of its investment, the profit the firm's earns on each unit of investment it undertakes is obviously equal to the profit of a fully utilized unit of investment. Therefore, the firm's total discounted profit can be rewritten as the discounted sum of the number of units of investment each period times the profit of a fully utilized unit of investment in each period. This result will also be recorded as a lemma for future reference.

Lemma 2:

Suppose that D(p) satisfies FUI. Then the firm's profit given p can be written as

$$(8) \qquad \pi(p) \quad = \quad \sum_{t=0}^{\infty} I_t(p)\, \phi_t(D(p))\delta^t$$

proof:

As above. QED

II. RESULTS OF THE EXISTING LITERATURE ON FORWARD LOOKING COST

This section will present the results of the existing literature[8] that show that there is a unique method of calculating forward looking cost that results in the regulated firm earning zero profit. Let f_t denote the forward looking cost of producing a unit of output in period t and let f = (f_0, f_1, . . .) denote an entire vector of forward looking costs for every period. In the simple model of this paper, a rule for calculating the forward looking cost of producing a unit of output in period t is simply a rule of the form

$$(9) \qquad f_t = k\, a_t.$$

where k is a non-negative constant and a_t is the cost of purchasing a unit of capacity in period t. The variable k denotes the share of the total replacement cost that is allocated to the current period for purposes of calculating forward looking cost and will be called the current period

[8]See Hausman (1997), Laffont and Tirole (2003), Mandy (2002), and Mandy and Sharkey (2003).

allocation share. Therefore if a regulator sets prices equal to forward looking cost, prices will be determined by the rule

$$(10) \qquad p_t = k\, a_t$$

where k denotes the current period allocation share.

Proposition 1 presents the observation of the existing literature that, under the assumption that the demand curves weakly shift out over time, prices set equal to forward looking cost will cause the firm to earn positive (zero; negative) profit if and only if k is greater than (equal to; less than) the value k* defined by equation (12), below.

Proposition 1:

Suppose that prices are set according to equation (10) and that the demand curves weakly shift out over time. Then

$$(11) \qquad \pi(p) \begin{matrix} > \\ = \\ < \end{matrix} 0 \qquad \text{iff} \qquad k \begin{matrix} > \\ = \\ < \end{matrix} k^*$$

where k* is defined by

$$(12) \qquad k^* = [1 - \delta\gamma] / [1 - (\delta\gamma)^L]$$

Proof:

Straightforward algebra shows that

$$(13) \qquad I_t(p) \begin{matrix} > \\ = \\ < \end{matrix} 0 \text{ for every } t \qquad \text{iff} \qquad k \begin{matrix} > \\ = \\ < \end{matrix} k^*$$

Since asset prices are weakly decreasing, this implies that forward looking prices as determined by equation (10) are weakly decreasing. This, together with the assumption that the demand curves weakly shift out over time therefore implies that output is weakly increasing over time which in turn means that the vector of outputs demanded by consumers satisfies FUI by Lemma 1. Therefore, by Lemma 2, profit can be calculated according to (8). Proposition 1 then follows immediately from (13). QED

Most regulators that have decided to base regulated prices on forward looking cost, including the FCC, have used a value of k which will be called the "annuity current period allocation share" and be denoted by k^A. It is defined as the allocation share that would fully reimburse the firm for its investment if the firm received this share for every period of the asset's life. Simple algebra shows that it is determined by the formula

$$k^A = (1-\delta)/(1-\delta^L). \tag{14}$$

If γ is strictly less than 1, then k^A is obviously less than k^*. Therefore, for the case where technological progress is causing the replacement cost of assets to fall over time, if prices are set equal to forward looking cost calculated using k^A, the result will be that the firm is under-reimbursed for its new investments. One possible explanation of regulators' motivation for choosing k^A is that they wanted to choose a current period allocation share that would result in the firm being fully reimbursed but "forgot" to take account of the fact that the price the firm receives for output will continue to fall over time if this rule is used every period. Consistent with this interpretation, k* will be called the time consistent current period allocation share since it corrects this calculation by correctly anticipating the path of future prices. Similarly forward looking cost calculated using k* will be called time consistent forward looking cost.

Since time consistent forward looking cost will play a special role in the rest of the analysis, notation will be introduced to denote these values of cost. Let f_t^* denote period t forward looking cost calculated using k*,

(15) $$f_t^* = k^* a_t$$

and let $f^* = (f_0^*, f_1^*, \ldots)$ denote the entire vector of such costs.

While the existing literature shows that the rule of setting prices equal to time consistent forward looking costs will result in the firm earning zero economic profit, it does not attempt to directly investigate whether there is any first or second best sense in which this rule is efficient. In particular, although setting prices equal to time-consistent forward looking cost results in the firm being fully compensated for its investments, this is also true by construction for any historical pricing rule using any allocation rule. Therefore this result, alone, does not explain why setting prices equal to time-consistent forward looking cost is necessarily superior to setting prices equal to historic cost calculated using any allocation rule, even if one imposes the requirement that the firm must earn zero economic profit. Furthermore, the existing literature certainly does not shed any light on the issue of whether prices that induce even more efficient outcomes could be set if the zero-profit requirement was not imposed. Laffont and Tirole (2003) also point out that the vector of time consistent forward looking costs of using assets can be interpreted as the rental prices that would be charged for using assets, if assets could be costlessly switched between uses and a perfectly competitive rental market for assets existed. However, it is not clear why this result is necessarily relevant to determining what the efficient prices should be in the real situation of interest where these assumptions are clearly not satisfied.

The next section of this paper will directly investigate the issue of efficiency.

III. USING FORWARD LOOKING COST TO CALCULATE EFFICIENT PRICES

This section will show that setting price each period equal to time consistent forward looking cost will induce consumers to choose the efficient vector of outputs so long as the demand curve is weakly shifting out over time. The basic structure of the proof is as follows. Define the "true cost function" to be the function that gives the present discounted cost of producing any given vector of outputs given that the firm must purchase assets to produce output. Define the "accounting cost function" to be the function giving the present discounted cost under the hypothetical assumption that the firm has a constant marginal cost of production each period equal to the time consistent forward looking cost for that period. The main result is to show that the true cost of production is exactly equal to the accounting cost of production so long as the vector of outputs satisfies FUI and that the true cost of production is always greater than or equal to the accounting cost of production. It follows directly from this result that the vector of outputs that maximizes total surplus when costs are calculated using the accounting cost function will also maximize total surplus when costs are calculated using the true cost function, so long as the solution to the former problem satisfies FUI. This condition is implied by the assumption that the demand curves shift weakly out over time.

Subsection A will formally define the accounting cost function and explain its relationship to the true cost function. Subsection B will provide some intuition for the result that the true cost function is linear and separable over a broad range of outputs by directly calculating the marginal cost of increasing output in a single period. Subsection C will define a class of optimization problems general enough to include the welfare maximization problem as a special case and show that the result of subsection A implies that a vector of outputs that solves the optimization problem when costs are calculated using the accounting cost function also solves the true optimization problem so long as the solution to the former problem satisfies FUI. Subsection D considers the special case of main interest to this paper where the benefit of output is defined to be discounted consumer surplus so the optimization problem is the welfare maximization problem and shows that a sufficient condition for the solution to satisfy FUI is that demand shift weakly out over time. Finally, Subsection E will discuss the result.

A. Comparing the Accounting Cost Function to the True Cost Function

Recall that the function C(q) defined by equation (2) provides the present discounted value of producing a vector of outputs given that the firm must purchase assets in order to produce the vector of outputs. This will often be referred to as the "true cost function" to distinguish it from another cost function which will now be defined. Define the "accounting cost function" to be the function giving the total discounted cost of producing any vector of outputs under the hypothetical assumption that the firm's cost of production is linear and separable and the unit cost of production in period t is equal to the time consistent cost in period t.[9] Let A(q)

---

[9]One way to think about this is that this would be the cost if the firm could rent assets on a period-by-period basis and the rental cost in period t was equal to the time consistent forward looking cost.

denote this accounting cost function. Formally, it is defined by

(17) $$A(q) = \sum_{t=0}^{\infty} f_t{}^* q_t \, \delta^t.$$

Proposition 2 shows that the true cost function is equal to the accounting cost function over the set of outputs satisfying FUI and that the true cost is strictly greater than the accounting cost in all other cases.

Proposition 2:

(18) $C(q) = A(q)$ if and only if q satisfies FUI

(19) $C(q) > A(q)$ for every q which does not satisfy FUI

Proof:

See Appendix. QED

The intuition for Proposition 2 is as follows. The cost function A(q) can be interpreted as giving the "hypothetical" cost of producing a vector of outputs under the assumption that the firm can rent assets on a period by period basis and the rental price in period t is given by $f_t{}^*$. The cost function C(q) gives the "true" cost of producing the vector of outputs given that the firm must purchase assets. The rental prices are constructed to have the property that the cost of purchasing an asset is exactly equal to the cost of renting the same asset if the asset is rented for *every* period of the asset's life. By definition, when q satisfies FUI, the rental services the firm would choose in the hypothetical case where it can rent assets could be produced by purchasing assets and fully utilizing them. Therefore the cost should be the same. When q does not satisfy FUI, the firm would take advantage of a rental market to use fewer assets in some periods than would be possible in the case where it had to purchase its own assets. This means that the hypothetical cost is then lower than the true cost.

Intuitively, the fact that firm must purchase its own assets and cannot resell them places limits on the extent to which it can reduce its cost by reducing asset usage from period to period. The hypothetical cost function is calculated under the assumption that this constraint does not exist. When the constraint does not bind, whether or not the constraint is relaxed does not mater, and the hypothetical cost is therefore equal to the true cost. However, when the constraint binds, the firm can reduce its costs when this constraint is relaxed, and the hypothetical cost is less than the true cost.

B. Directly Calculating Marginal Cost

The result of Proposition 2 that the true cost function is linear and separable over a broad range of vectors of outputs might seem somewhat surprising in light of the fact that each asset is represents a joint cost of production across multiple periods. A widely accepted general principle in both the economics and managerial accounting literatures that study cost allocation is that, there is generally no economically meaningful way assign a joint cost to individual products.[10] Thus we might expect that the cost of producing a vector of joint products would inherently not be additively separable in each product. Yet this is precisely what happens in the

[10]See, for example, Biddle and Steinberg (1985), Demski(1981), Thomas(1978), and Young(1985).

model of this paper. The resolution to this apparent conflict lies in the fact that there are "multiple overlapping" joint costs in the model of this paper instead of a single joint cost. When there is a single joint cost for all products, the only way to increase the output of a single product is to increase investment in the joint cost and this results in increased output of all products. Thus increasing the production of one good necessarily results in increases in the production of all goods. However, in the model of this paper, where there are multiple overlapping joint costs, this is not necessarily true.

An illuminating way to see this point is to directly calculate marginal cost by directly determining the adjustments in asset purchases that are necessary to produce a small increment of output in the current period while holding output in all other periods fixed. The present discounted value of these adjustments is, of course, by definition the marginal cost of production. For example, suppose that the firm has made investment plans to produce a particular vector of outputs over time and that the firm plans to purchase at least one unit of the asset every period.[11] Then the firm can increase output in period 0 by one unit while holding output in all other periods constant by implementing the following series of adjustments to it investment plans.[12] The firm must purchase an additional unit of the asset in period 0 to increase production by one unit in this period. However, it will now be able to reduce its asset

---

[11]The assumption that the firm is purchasing at least one unit of the asset every period will be necessary to calculate the cost of increasing output by one unit in the current period. Correspondingly smaller increases in output require correspondingly smaller levels of investment every period. Since investment is positive every period, the vector of outputs satisfies FUI. Thus the assumption that there exists some $\varepsilon > 0$ such that investment every period is greater than or equal to ε is slightly stronger than FUI.

[12]Obviously the same construction could be applied to calculate the cost of increasing output in any given period.

purchases by one unit in period 1. Now when period L arrives, the extra asset that the firm purchased in period T will no longer be available, so the firm will have to purchase an extra unit of the asset in that period to maintain its level of production at the previously planned level. However, as before, it will now be able to reduce its asset purchases by one unit in period L+1. This process continues indefinitely. That is, the firm can produce exactly one more unit of output in the current period and hold output in all other periods fixed by shifting the purchase of one unit of the asset forward in time from period 1 to 0, L+1 to L , 2L+1 to 2L, etc. The present discounted value of the cost of these adjustments is, by definition, the marginal cost of increasing output by one unit in the current period. It is straightforward to directly calculate this value and show that it is equal to k^*a_0.

C. Substituting the Accounting Cost Function for The True Cost Function

Consider an abstract optimization problem of the following form. Suppose that the present discounted value of the benefit from a vector of outputs is given by some specified function B(q). Define the "true optimization problem" to be the problem of maximizing the present discounted value of benefits minus the *true* cost of production, C(q). Similarly define the "accounting optimization problem" to be the problem of maximizing the present discounted value of benefits minus the *accounting* cost of production, A(q).

The special case of the true optimization problem that is of direct interest to this paper is course the case where B(q) is defined to be the discounted value of consumer surplus so that the objective function for the optimization problem is total surplus. This subsection will present a basic result that applies to the general optimization problem for any definition of B(q). Then the results that follow specifically for the case when B(q) is defined to be discounted consumer surplus will be described in the next subsection. There are two reasons for describing how the result applies for any definition of B(q). First, it makes the nature of the result clearer. Second, although the main interest of this paper is for the case when B(q) is defined to be consumer surplus, another potential application of interest is to the case where B(q) is defined to be total discounted revenue so the optimization problem being studied is the profit maximization problem of an unregulated firm. This subject will be returned to in Section V.

It follows immediately from Proposition 2 that a sufficient condition for a solution to the accounting optimization problem to also be a solution to the true optimization problem is that the solution to the accounting problem satisfy FUI. Furthermore, if there is a unique solution to the accounting optimization problem and it satisfies FUI then this is also the unique solution to the true problem. These results are stated as Proposition 3.

Proposition 3:

(i) Suppose that q* is a solution to the hypothetical optimization problem. Then if q* satisfies FUI, it is also a solution to the true optimization problem.

(ii) Furthermore, if q* is the unique solution to the accounting optimization problem and it satisfies FUI then it is also the unique solution to the true optimization problem.

proof:

See Appendix. QED

Of course Proposition 3 will only be useful if it can be shown that there are natural plausible conditions under which the solution to the accounting optimization problem satisfies FUI. The next subsection will show this is true for the special case where B(q) is defined to be the discounted total surplus and also describe the nature of the solution in this case.

D. Application of the Result to the Welfare Maximization Problem

In the welfare maximization problem, the benefit function B(q) is defined to be the discounted sum of consumer surplus. Formally, let $u_t(q_t)$ denote the consumer surplus that consumers in period t receive from consuming q_t units of output. It is defined by

$$(20) \qquad u_t(q_t) = \int_{y=0}^{q_t} P_t(y)dy.$$

Then B(q) is defined by

$$(21) \qquad B(q) = \sum_{t=0}^{\infty} u_t(q_t)\, \delta^t .$$

Note that the firm's discount rate is used in this definition of benefit. With a risk neutral firm, it is of course necessary to assume that discount rate for consumer surplus is the same as the firm's discount rate in order for there to be a well-defined solution to the optimization problem. Otherwise, one could create arbitrarily large values of total surplus by shifting income between periods.

Related to this, note also that, although consumer surplus is discounted using the firm's discount rate to create the social benefit function, this definition of efficiency does not necessarily require the assumption that consumers themselves discount consumer surplus at this rate or even necessarily that individual consumers live for more than a single period. Rather, this definition of efficiency can be interpreted as simply requiring that the vector of outputs satisfies a Pareto-efficiency requirement where consumers each period are viewed as separate actors for purposes of the Pareto calculation, compensating transfers from period t consumers to the firm are possible, and period t consumers pay their transfer in period t.[13] In this framework, a vector of outputs can be defined to be Pareto- efficient if it is not possible to choose some other vector of outputs and a vector of compensating transfers such every actor is weakly better off and at least one actor is strictly better off. It is straightforward to show that q satisfies this definition of Pareto-efficiency if and only if it maximizes B(q) - C(q) where B(q) is defined by (21).

The accounting optimization problem is to maximize B(q) - A(q) where B(q) is defined by (21) and A(q) is the accounting cost defined by (17). Since the marginal cost of production is constant each period and equal to time consistent forward looking cost and since it has been assumed that demand is "well-behaved"[14] it is immediate that there is a unique solution to the accounting problem given by the vector of outputs demanded by consumers when price each period is set equal to time consistent forward looking cost. This observation will stated as a lemma for future reference. To do so, it will be useful to introduce some additional notation. Formally, define the price vector $p^F = (p^F_0, p^F_1, \dots)$ to be the vector of prices set equal to time consistent forward looking cost.

(22) $$p^F_t = f^*_t = k^* a_t$$

---

[13]Even if consumers live for many periods one can still view their multiple "selves" in different periods as separate actors for purposes of a Pareto calculation.

[14]See section I.B.

and define $q^F = (q^F_0, q^F_1, \ldots)$ to be the vector of outputs that consumers demand at these prices.

(23) $$q^F_t = D_t(p^F_t).$$

The result can now be stated.

Lemma 3:

The unique solution to the accounting optimization problem when benefit is defined by (21) is q^F.

proof:

Straightforward. QED

Since it is straightforward to show that q^F satisfies FUI when the demand curve weakly shifts out over time, it follows immediately from Proposition 3 and Lemma 3 that q^F is also the unique solution to the true optimization problem. This result is stated as Proposition 4.

Proposition 4:

Suppose that demand weakly shifts out over time. Then q^F is the unique vector of outputs that maximizes total surplus.

Proof:

As described above it is sufficient to show that q^F satisfies FUI. The assumption that asset prices are weakly decreasing implies that p^F_t is weakly decreasing in t. Since demand weakly shifts out over time this implies that q^F_t is weakly increasing in t. Then Lemma 1 implies that q^F satisfies FUI. QED

E. Discussion

This subsection will make three comments on this result. First, the result that setting prices equal to time consistent forward looking cost induces the efficient level of outputs is likely to hold true in a wide variety of circumstances. The sufficient condition stated in proposition 4 that demand weakly shift out over time essentially requires that the market be weakly growing over time and not be subject to large periodic or random fluctuations. From an intuitive perspective, we would expect the problem of determining the per period cost of using assets to be much more complex when either or both of these conditions was not met. The firm would face the dilemma that, at least in some periods, it would ideally like to purchase and use assets today that it will have no use for tomorrow and one would expect that the per period cost of using assets would have to be modified to reflect this fact. The result of this paper is that, so long as this complication does not arise, that a relatively simple formula exists for calculating the per period cost of using long-lived assets. Note also that to the extent that the replacement cost of assets is falling over time, the result will continue to hold true so long as demand does not drop "too rapidly."

Second, one way to interpret the result of this paper is that it describes a set of conditions under which a problem that appears to be a relatively complex multi-period optimization problem with very demanding informational requirements actually collapses into a series of fairly simple single period problems with more modest informational requirements. In general, the problem of choosing a level of investment every period in order to maximize total discounted surplus is a complicated multi-period optimization problem and solving for the optimal level of investment in any given period generally requires solving for the optimal level of investment over the entire time horizon since costs between periods are inter-related. In addition to being complicated, this means that, in general, a decision-maker must know the demand for all future periods in order to solve for the optimal level of investment in any given period. The result of this paper shows that so long as the decision-maker knows (or is at least willing to assume) that demand will shift weakly out over time, the decision-maker does NOT need to have any other information about the nature of future demand to solve for the optimal level of investment in the current period. Rather, the decision-maker can simply decide how much to produce (and therefore how much to invest) by choosing the level of output that would be desirable if the decision-maker assumed that he was solving a single period problem and that the cost of using assets was equal to the time consistent forward looking cost. The idea that in some cases accounting allocations of cost can dramatically simplify complex multi-period problems by reducing them to a series of separable single-period problems will be returned to in section V which discusses applications of the results of this paper to managerial decision-making.

Third, the result that time consistent forward looking cost is literally the present discounted value of the series of adjustments to future asset purchases that would result in one more unit of output being produced in the current period holding output in all other periods constant, provides an explanation for one of the puzzling features of time consistent forward looking cost. As mentioned in the introduction, one of the motivations of regulators for shifting to forward looking cost was their intuition that current prices should be lower if the rate of technical progress is higher. However, examination of the formula for k^* in equation (12) shows that k* grows larger when the rate of technical progress is faster. That is, holding all other factors equal, a faster rate of technical progress should result in higher prices. The explanation, of course, is that the firm produces more output in the current period by essentially shifting its planned asset purchases forward in time. When asset prices are dropping more rapidly over time, the cost of shifting asset purchases forward in time is greater because the drop in price from waiting is higher. Thus holding all other factors (including the current price of assets) constant, an increase in the future rate of technical progress will increase the current incremental cost of producing output and current price should be raised to reflect this fact.

Thus, somewhat paradoxically, an exogenous increase in the future rate of technical progress can actually make current period consumers worse off if prices each period are set equal to marginal cost. Mathematically, the price of output each period is set equal to a constant, k*, times the price of purchasing assets that period. When the future rate of technical progress changes there are two effects. First, the constant, k*, is higher. Second, after the initial period, all future prices of purchasing assets are lower. The first effect increases the price of output and the second effect decreases the price of output. For the initial period the second effect is zero so the net effect must be positive. As time goes on, the second effect will eventually dominate the first effect. However, the effect may be negative for a number of early periods and is definitely negative in the current period.

IV. USING HISTORICAL COST TO CALCULATE EFFICIENT PRICES

This section will investigate the extent to which historical costing methods can be used to calculate efficient prices. Sub-section A will introduce notation to describe historical costing rules. Sub-section B will show that historical cost calculated using an allocation rule which will be called the relative replacement cost (RRC) allocation rule results in exactly the same result as time consistent forward looking cost. Section C will discuss policy implications of this result.

A. Notation and Definitions to Describe Historical Cost

This section will begin by introducing notation to describe allocation and depreciation rules. Formally define a depreciation rule to be a vector $d = (d_0, d_1, \ldots, d_{L-1})$ that satisfies $d_t \geq 0$ for every t and

$$(24) \qquad \sum_{t=0}^{L-1} d_t = 1.$$

where d_t is interpreted as the share of depreciation allocated to the t^{th} period of the asset's life. Define an allocation rule to be a vector $s = (s_0, s_1, \ldots, s_{L-1})$ that satisfies $s_t \geq 0$ for every t and

$$(25) \qquad \sum_{t=0}^{L-1} s_t \delta^t = 1$$

where s_t is interpreted as the share of the asset's cost allocated to the t^{th} period of the asset's life.

Regulators and the firms they regulate generally think of themselves as directly choosing a depreciation rule for assets. The cost allocated to each period is then calculated as the sum of the depreciation allocated to that period plus a rate of return on the remaining (non-depreciated) book value of the asset. Formally, for any depreciation rule, d, the corresponding allocation rule is given by

$$(26) \qquad s_t = d_t + \{(1-\delta)/\delta\}\sum_{j=t+1}^{L-1} d_j$$

It is straightforward to show that equation (26) can be inverted to yield

$$(27) \qquad d_t = \sum_{j=t}^{L-1} \delta^{j-t} s_j - \sum_{j=t+1}^{L-1} \delta^{j-t-1} s_j$$

Therefore, for any fixed discount rate, δ, there is a one-to-one correspondence between depreciation rules and allocation rules and one can equivalently think of the regulator or firm as choosing either. For the purposes of this paper, it will be more convenient to view the regulator and firm as directly choosing an allocation rule.[15]

The total cost of using assets in any period can be calculated by determining the cost of using each individual asset and summing across all assets. The per unit cost of production is then obtained by dividing the total cost by the number of units produced. Let $h_t(q, s)$ denote the historic cost of a unit of output in period t when the vector of outputs q is produced and the allocation rule s is used. Formally it is defined by

$$(28) \qquad h_t(q, s) = \{ \sum_{i=0}^{L-1} x_{t-i} a_{t-i} s_i \} / q_t$$

where

$$(29) \qquad x_t = \phi_t(q) \quad \text{for } t \geq 0$$

---

[15]See Rogerson (1992) for a fuller discussion of the relationship between depreciation and allocation rules and their properties.

(30) $x_t = 0$ for $t \leq -1$.

The numerator of (28) in brackets gives the total cost of using all assets in period t. The denominator give the total output in that period. The ratio of these is the historic cost per unit of output.

A regulatory equilibrium given an allocation rule will now be defined.

Definition:

An ordered pair of prices and outputs (p, q) will be said to be a regulatory equilibrium given the allocation rule s if consumers demand q_t in each period given p_t, i.e., if

(31) $$q_t = D_t(p_t)$$

each if price each period is equal to the historic cost calculated using the allocation rule s, i.e., if

(32) $$p_t = h_t(q, s) \qquad \text{for every } t \in \{0, 1, 2, \ldots\}$$

Note that the definition of a regulatory equilibrium for a historic pricing rule is somewhat more complicated than the analogous notion for a forward pricing rule, because calculation of prices under a forward pricing rule does not depend on the vector of outputs demanded. Thus there is no "equilibrium feedback" to take account of. That is, a forward pricing rule determines prices which in turn determine demand and that is the end of the story. For a historic pricing rule, the prices determined by a given allocation rule are dependent on the vector of outputs produced. Therefore this feedback must be taken into account as part of the definition.

B. Regulatory Equilibrium Under the RRC Allocation Rule

One particular allocation rule, which will be called the Relative Replacement Cost (RRC) rule, will now be described.. This is the unique allocation rule that satisfies the requirement that the allocations across periods decrease at the same rate that replacement costs are expected to decrease at, given by

$$(33) \qquad s_t = \gamma\ s_{t-1} \text{ for } t \in \{1, 2, \ldots, L-1\}$$

and the requirement (that all allocation rules must satisfy) that the discounted sum of the allocations be equal to one, given by (25). Let $s^{RRC} = (s^{RRC}_0, s^{RRC}_1, \ldots, s^{RRC}_{L-1})$ denote the unique allocation rule that satisfies (33) and (25). It is straightforward to show that it is defined by

$$(34) \qquad s^{RRC}_t = \gamma^t\ k^*$$

where k* is the time consistent current period allocation share defined by equation (12).

Examination of the formula for calculating the allocation shares for the RRC rule reveals that they have a very interesting property. Namely, in any given period, if we were to calculate the cost of using any particular vintage of asset, we would find that the cost is independent of the vintage of the asset and is equal to the time consistent forward looking cost. This result will be stated as Lemma 4.

Lemma 4:

Consider any period t and an asset which was purchased in period t-i for some i ∈ {0, 1, 2,. .. L-1}. Then the cost of using this asset in period t calculated using the RRC allocation rule does not depend on i and is equal to the time consistent forward looking cost in period t given by k^*a_t.

Proof:

Consider an asset purchased in period t-i for some i ∈ {0, 1, 2,. .. L-1}. The purchase price of the asset is a_{t-i} which, by equation (1) can be written as

(35) $$a_t/ \gamma^i.$$

The share of cost allocated to period t is s_i, which, by equation (34) can be written as

(36) $$\gamma^i k^*.$$

The cost of using the asset is period t is the product of (35) and (36) which is equal to $a_t k^*$. QED

Lemma 4 immediately implies that the historic cost of using a unit of the asset in period t calculated using the RRC allocation rule will always be equal to the time consistent forward looking cost. Therefore so long as the firm is operating at full capacity, the historic cost of producing a unit of output will also be equal to the time consistent forward looking cost. It follows immediately from this that (p^F, q^F) is a regulatory equilibrium given the RRC allocation rule so long as the demand curves weakly shift out over time. That is, the sufficient condition to guarantee that the prices p^F induces the efficient vector of outputs is also sufficient to guarantee that the result is a regulatory equilibrium using the RRC allocation rule.

This result is stated as Proposition 5.

Proposition 5:

Suppose that demand weakly shifts out over time. Then (p^F, q^F) is a regulatory equilibrium given the RRC allocation rule.

Proof:

See Appendix. QED

C. Discussion

Proposition 5 shows that to the extent that regulators thought that the rapid pace of technological progress in the telecommunications industry required them to switch from basing prices on historical costs to basing prices on forward looking costs, they were incorrect. Either method can be used to calculate efficient prices when technological progress is causing the replacement cost of assets to fall over time.

Furthermore, recent history suggests that basing prices on forward looking cost is likely to create a whole host of extra problems that do not arise when prices are based on historical cost because of a factor not captured in the formal model. Namely, in reality, calculations of historical cost are likely to be based on much more objective data that are less subject to manipulation than are calculations of forward looking cost. Historical cost is based on the amount of money that was actually spent to purchase an asset. However, forward looking cost is based on the amount of money that the regulator *estimates* that it would cost to purchase *functionally equivalent* assets. In the formal model of this paper these problems are glossed over because it is assumed that the asset is a simple homogenous commodity that does not change over time that is sold at some easily measured market price. The reality of the situation of course likely to be quite different. This creates two related problems. First, at a minimum, it is very expensive to conduct the sort of investigations required to determine what the current replacement cost of assets is. It is widely recognized that the regulatory proceedings in the United States used to determine forward looking cost have become highly adversarial and very expensive. Second, to the extent that forward looking cost is manipulable, this allows regulators the opportunity to essentially attempt to reneg *ex post* on their commitment to reimburse the firm for its investments in sunk assets. To the extent that some sort of *ex ante* commitment is necessary and desirable in order to alleviate the hold-up problem, the fact that a forward pricing rule weakens this commitment ability may be undesirable.

V. APPLICATIONS TO THE THEORY OF THE FIRM AND MANAGERIAL DECISION-MAKING

This section will briefly discuss how the results of this paper can be interpreted to apply to the study of how private unregulated firms use accounting data to organize and guide their decision-making activities. In particular, when the benefit function of output is defined to be total discounted revenue instead of discounted surplus, the optimization problem described in

Section III of this paper can be interpreted as the profit maximization problem of an unregulated firm. Therefore the following result follows from Proposition 3. Suppose that the firm calculates the profit maximizing solution under the assumption that the marginal cost of production is constant each period and equal to time consistent forward looking cost. Then, so long as the solution to this problem exhibits weakly increasing output over time, the solution is also a solution to the true profit maximization problem.

The only slight complication in applying this result is that a slightly stronger assumption is required to guarantee that the solution to the accounting optimization problem exhibits weakly increasing output. Namely, it must now be assumed that marginal revenue is weakly increasing over time instead of merely that demand is weakly increasing over time. However, this is an assumption that we would expect to see met in most growing markets. That is, we would expect that a firm is a growing market with constant marginal costs would weakly increase output over time.

There are two qualitatively different ways that one could imagine accounting information being "useful" to a firm. First, if one viewed the firm as consisting of a number of separate actors each with potentially with their own private information and incentives, one could investigate how accounting information could help the firm decentralize and coordinate the actions of its various members. Second, if one viewed the firm as a single decision-maker with limited analytic or computational ability, one could investigate whether accounting information might help simplify problems so that they were easier to solve. Most modern research on the theory of the firm and how firms use accounting information takes the first point of view and I firmly agree that this is a valuable and important perspective. However, I believe that in some cases, it may also be the case that accounting information can be understood as playing the second role. I will briefly describe two examples of how accounting information might be "useful" to the firm, based on the results of this paper, one of each type.

An example of the first type is transfer pricing. Suppose that an upstream division produces a good using long-lived assets that is then used by a downstream division in some profit-making activity. This paper shows that transfer prices can be chosen to simultaneously allow the upstream firm to break even and give the downstream firm incentives to use the input efficiently.

As has already been discussed in Section III, it also clear that one could interpret the result of this paper as showing that, in some cases, the solution to what appears to be a very complex multi-period profit maximization problem can be calculated by using a simple rule to allocate costs across periods to create a series of simple one period profit maximization problems and solving these.

VI. MORE COMPLEX PRODUCTION FUNCTIONS

The result of this paper that the firm breaks even when price each period is set equal to marginal cost clearly depends on the assumption that there are constant returns to scale in capacity each period. However, the result that simple accounting formulas can be used to allocate the cost of investment across periods so that a complex multi-period optimization problem collapses into a series of unrelated single period problems is more general. In the formal model of this paper where one unit of capacity was needed to produce one unit of output, we could equivalently view the forward looking cost of producing a unit of output as being the forward looking cost of using a unit of capacity. One could continue to use the formula to calculate the time consistent forwarding looking cost of a unit of capacity in a more general model where there were other inputs and output was not necessarily linear in capacity. The more general result in such a model is as follows. Suppose that one calculates the solution to an optimization problem (which could be either a welfare maximization or a profit maximization problem) under the assumption that units of capacity in any period can be used at a constant marginal cost equal to the time consistent forward looking cost. Then, so long as the solution to this "hypothetical" problem exhibits weakly increasing capacity usage over time, it is also a solution to the "true" problem. Note that costs in the hypothetical problem are additively separable over time. Therefore the result shows that accounting rules that allocate investment costs across periods can quite generally be used to simplify complex multi-period optimization problems by reducing them to a series of unrelated single-period problems.

VII. CONCLUSION

This paper has shown that when production requires sunk investments in long-lived assets, that fairly simple rules can quite generally be used to calculate the per period cost of using these assets for purposes of solving either welfare maximization or profit maximization problems. The basic conditions that must be met are that the replacement cost of assets is weakly falling over time and that the market as a whole is weakly growing over time. Somewhat surprisingly, even though each asset is a joint cost of production across multiple periods, the fact that there are "multiple overlapping" joint costs means that the cost of using capacity is linear and separable in each period's capacity over a broad range of capacities. The rule for calculating the per period cost of using assets can be interpreted either as a rule for calculating time consistent forward looking cost or as a rule for calculating historical cost using the RRC allocation rule.

When applied to the particular policy issue of setting regulated prices in telecommunications markets where technological progress is causing the replacement cost of assets to fall over time, the results of this paper show the following. The rule for calculating time consistent forward looking cost identified by previous papers as allowing the firm to break even also induces consumers to demand efficient outputs. Therefore the rule most regulators currently use to calculate forward looking cost (the annuity rule) is flawed, not only because it does not reimburse the firm for its investments, but also because it induces inefficiently high levels of consumption by pricing telecommunications services below their incremental cost. In addition, there was no need to switch from a historic methodology to a forward looking methodology in order to properly capture the effect of technological progress on the cost of production. In theory, this could be accomplished equally well by using a historic methodology with an appropriately chosen allocation rule. Furthermore, the switch to a forward looking methodology may have actually created some additional problems, since, in practice, calculation of forward looking cost is based on less objective data than is historical cost.

APPENDIX
PROOFS OF PROPOSITIONS

Proof of Proposition 2:

Consider any vector of outputs q and suppose that x is the vector of investments that produces q, i.e., $x = \phi(q)$. Let $q^{max} = (q_0^{max}, q_1^{max}, \ldots)$ denote the vector of outputs that would be produced is x was fully utilized. That is,

$$(A.1) \qquad q_t^{max} = \sum_{i=0}^{L-1} x_{t-i}$$

where x_t is interpreted to be zero for $t \leq -1$. By construction and by the definition of FUI, q and q^{max} satisfy the following two conditions.

$$(A.2) \qquad q_t \leq q_t^{max} \quad \text{for every } t$$

$$(A.3) \qquad q_t = q_t^{max} \text{ for every } t \text{ if and only if } q \text{ satisfies FUI.}$$

By definition, C(q) is given by

$$(A.4) \qquad C(q) \quad = \quad \sum_{t=0}^{\infty} a_t x_t \delta^t.$$

The middle row (corresponding to the equal sign) of equation (16) can be written as

$$(A.5) \qquad a_t = \quad \sum_{i=0}^{L-1} f^*_{t-i} \delta^i$$

Substitution of (A.4) into (A.5) yields

$$\text{(A.6)} \qquad C(q) = \sum_{t=0}^{\infty} \left[\sum_{i=0}^{L-1} f^*_{t-i}\, \delta^i \right] x_t \delta^t.$$

Regrouping the summations and substitution of the definition of q_t^{max} given by equation (A.1) into the result yields

$$\text{(A.7)} \qquad C(q) = \sum_{t=0}^{\infty} f^*_t\, q_t^{max}\, \delta^t.$$

Equation (17) provides the formula for A(q). The result then follows directly from equations (A.2)-(A.3). QED

Proof of Proposition 3:

Part (i):

Suppose that q* is a solution to the accounting optimization problem and satisfies FUI. Now suppose for contradiction that q* is not a solution to the true optimization problem. Then there exists a vector of quantities q** such that

$$\text{(A.8)} \qquad B(q^{**}) - C(q^{**}) > B(q^*) - C(q^*).$$

Since q* satisfies FUI,

$$\text{(A.9)} \qquad C(q^*) = A(q^*).$$

All output vectors, including q** must satisfy

$$\text{(A.10)} \qquad C(q^{**}) \geq A(q^{**})$$

Substitution of (A.9) and (A.10) into (A.8) yields

$$\text{(A.11)} \qquad B(q^{**}) - A(q^{**}) > B(q^{*}) - A(q^{*})$$

which contradicts the assumption that q* is a solution to the hypothetical problem.

Part (ii):

Suppose that q* is the unique solution to the accounting optimization problem and satisfies FUI. By the result of part 1, q* is also a solution to true optimization problem. Now suppose for contradiction that q* is not the unique solution to the true optimization problem. Then there exists a vector of quantities q** such that

$$\text{(A.12)} \qquad B(q^{**}) - C(q^{**}) = B(q^{*}) - C(q^{*}).$$

Equations (A.9) and (A.10) are still true for the same reasons. Substitution of (A.9) and (A.10) into (A.12) yields

$$\text{(A.13)} \qquad B(q^{**}) - A(q^{**}) = B(q^{*}) - A(q^{*})$$

which contradicts the assumption that q* is the unique solution to the accounting problem. QED

Proof of Propostion 5:

Substitution of $q = q^F$ and $s = s^{RRC}$ into equations (28)-(30) yields

(A.14) $$h_t(q^F, s^{RRC}) = \{\sum_{i=0}^{L-1} x_{t-i} a_{t-i} s^{RRC}{}_i\} / q^F{}_t$$

where

(A.15) $$x_t = \phi_t(q^F) \quad \text{for } t \geq 0$$

(A.16) $$x_t = 0 \quad \text{for } t \leq -1.$$

By Lemma 4

(A.17) $$a_{t-i} s^{RRC} = k^* a_t$$

for every $t \in \{0, 1, \ldots\}$ and every $i \in \{0, 1, \ldots, L-1\}$.

The assumption that the demand curves weakly shift out over time implies that $q^F{}_t$ is weakly increasing in t which in turn implies that q^F satisfies FUI. Therefore,

(A.18) $$q^F{}_t = \sum_{i=0}^{L-1} x_{t-i}$$

for every $t \in \{0, 1, 2, \}$ where x_t is interpreted to be zero for $t \leq -1$. Substitution of (A.17) and (A.18) into (A.14) yields

(A.19) $$h_t(q^F, s^{RRC}) = k^* a_t$$

which is the desired result. QED

REFERENCES

Balakrishnan, Ramji and K. Sivaramakrishnan. "A Critical Overview of the Use of Full-Cost Data for Planning and Pricing." *Journal of Management Accounting Research*, 14, 2002, pp. 3-32.

Baumol, William J. "Optimal Depreciation Policy: Pricing the Products of Durable Assets." *Bell Journal of Economics,* 1971, 2, pp638-656.

Biddle, Gary C. and Richard Steinberg, "Common Cost Allocation in the Firm." In H. Peyton Young. ed., *Cost Allocation: Methods Principles, Applications*. Amsterdam: North Holland, 1985. pp. 31-54.

Crew, Micahael A. and Paul Kleindorfer, "Economic Depreciation and the Regulated Firm Under

Competition and Technical Change." *Journal of Regulatory Economics*, 1992, 4(1), 51-62.

Demski, J.S.. "Cost Allocation Games." In S. Moriarity, ed., *Joint Cost Allocations*. Norman, OK: University of Oklahoma Press, 1981, pp. 142-73.

Demski, J.S. and D.M. Kreps. "Models in Managerial Accounting." *Journal of Accounting Research*: Supplement, 1982, 20, 117-148.

Federal Communications Commission. *Notice of Proposed Rulemaking In the Matter of Review of the Commission's Rules Regarding the Pricing of Unbundled Network Elements and the Resale of Service by Incumbent Local Exchange Carriers,* WC Docket No. 03-173, September 15, 2003.

Hausman, Jerry A. "Regulated Costs and Prices In Telecommunications." in Gary Madden and

Scott J. Savage, eds., *The International Handbook of Telecommunications Economics, Volume II*. Edward Elgar Publishers, 2000.

Laffont, Jean-Jacques, and Jean Tirole. *Competition in Telecommunications*. Cambridge: MIT Press, 2003.

Falch, Morten. "TELRIC - The Way Towards Competition? The European Point of View." *Review of Network Economics*, September 2002, 1(2), pp. 147-154.

Mandy, David M. "TELRIC Pricing With Vintage Captial." *Journal of Regulatory Economics*, 2002, 22(3), pp. 215-249.

Mandy, David M. and Sharkey, William W. Federal Communications Commission Office of Strategic Planning and Policy Analysis Working Paper 40, September 2003.

Rosston, Gergory L. and Roger G. Noll. "The Economics of the Supreme Court's Decision on Forward-Looking Costs." *Review of Network Economics*, September 2002, 1(2), pp. 81-89.

Rogerson, William P. "Optimal Depreciation Schedules for Regulated Utilities." *Journal of Regulatory Economics*, 1992, 4(1), pp. 5-34.

Salinger, Michael A. "Regulating Prices to Equal Forward-Looking Costs: Cost-Based Prices or Price-Based Costs?" *Journal of Regulatory Economics*, 1998, 14, pp. 149-163.

Thomas, A.L. "Aribtrary and Incorrigible Allocations: A Comment." *Acccounting Review,* 1978, 52, pp. 263-9.

Young, H. Peyton. "Methods and Principles of Cost Allocation." In H. Peyton Young. ed., *Cost Allocation: Methods Principles, Applications*. Amsterdam: North Holland, 1985. pp. 3-29.