Risk Sharing with Formal and Informal Contracts: Theory, Semi-Parametric Identification and Estimation

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Abstract

We present a theoretical model which shows how households can insure through formal and informal contracts when some verifiable production takes place in an environment of incomplete markets. We construct a theoretical setting which nests the case of complete markets when all risks can be insured by formal contracts (because all states of nature would be verifiable) and the case where only informal agreements are available (agreements specifying informal transfers that needs to be self-enforceable). We derive two equations of interest, an Euler-type equation and an equation of determination of the formal contract. We then study the semi-parametric identification of the model and show how it can be estimated. We estimate both equations using data of village economies in Pakistan. Empirical results are consistent with the model[1].

JEL Classification: C14, D12, D13, D91, L14, O12, O17, Q12, Q15

Key Words: Risk, Insurance, Contracts, Incomplete Markets, Sharecropping, Informal Transfers, Rural Households, Semi-parametric Identification, Average Derivatives, GMM, Pakistan.

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1 Introduction

Following the seminal paper of Townsend (1994), the empirical testing of whether or not markets are complete in village economies have proved to be a fertile and valuable line of research. It led to a better understanding of market failures and a better targeting of households in the village who are most affected by these failures (Deaton, 1990, Morduch 1995, 1999, Fafchamps, 1997). These results were paralleled by tests of complete markets in developed economies at the aggregate level (see Attanasio and Ríos-Rull, 2001) and the micro level (Cochrane, 1991, and Mace, 1991). In both literatures, most papers report rejections of the null hypothesis of complete markets and much effort is now put on looking at alternative credible models of partially insured agents. This is where the two literatures, the one on village economies and the otheron developed economies, depart (Mace, 1991, Attanasio and Ríos-Rull, 2001, Dubois, 2000). Because village economies seem a priori to be less prone to imperfect information problems, the models that were developed in that literature, put emphasis on contract enforcement. Village economies lack institutions that are able to enforce the whole set of contracts that would permit complete risk sharing. Villagers are bound to enter agreements that are informal. These informal contracts can be Pareto-improving because they permit risk sharing but they also need to be self-enforced (Thomas and Worrall, 1988, Coate and Ravallion, 1993, Fafchamps, 1992, Kimball, 1988). The latter requirement restricts the set of informal agreements which may not be rich enough to lead to complete risk sharing in the village. A few recent papers show the empirical credibility of such alternatives in static or dynamic cases (Ligon, Thomas and Worrall, 2000, 2002).

Although these self-enforcing contracts play their part in sharing risk within extended families or within networks of households formed by kinship, ethnicity and so on, (Grimard, 1999, Fafchamps and Lund, 2000), some contracts may be much easier to enforce. In particular, sharecropping and fixed rent formal contracts are commonly observed in villages of LDCs and their role in allocating risk have been repetitively emphasized. It is why in this paper, we consider the case where both types of contracts coexist. Risks that households face are of many kinds. Because formal enforceable contracts on production are observed, we take the assumption that formal contracts on production are possible that is we assume that some subset of the set of states of nature are observable and verifiable. Other states of nature however may not be contractible like those related to health problems or to returns to individual activities. Hence, formal contracts are allowed to be contingent on agricultural risk while other risks can only be shared through the use of implicit informal agreements that need to be self-enforced. Of course, informal transfers and contracts can also be contingent on verifiable risks. Informal agreements are decided *given* the formal agreements and would explain why formal contracts are accompanied by informal transfers that can attenuate their effects in bad states of nature (Udry, 1994).

Generally speaking, modeling formal and informal transfers amounts to take seriously the problem of the ex ante diversification of risks that households routinely perform and that, as we will show, makes income endogenous. Random shocks affecting preferences that are observed by the household but unobserved by the econometrician can also determine income. The interpretation of the test of complete markets may therefore be different. It is quite similar to the common case of non separability between leisure and consumption, leisure being determined by random shocks and determining income. It goes however through a different route closer to an insurance mechanism.

We construct a theoretical setting which nests the case of complete markets when all risks can be insured by formal contracts (because all states of nature would be verifiable), the case where only one non contingent transfer is allowed ($\{s\} = \Omega$) as in Gauthier, Poitevin and Gonzalez (1997) and the case where only informal agreements are available. We derive two equations of interest, an Euler-type equation of consumption dynamics and the equation of determination of the formal contract. This theoretical model proves to be quite general and makes a new step in the modelling of incomplete risk sharing with formal (enforceable) and informal (that needs to be self-enforceable) contracts.

Then, we study the semi-parametric identification of the model and show how to implement an empirical estimation. We estimate both equations using data of village economies in Pakistan. This setting yields a richer test of complete markets since we are able to cope with the problem of endogeneity of income using the structural model.

The paper is organized as follows. Section 2 presents the theoretical model and the main propositions obtained. Section 3 studies the theoretical identification and econometric estimation of the model. Section 4 presents the data used and the empirical estimation results. Section 5 concludes and appendices are at the end.

2 Theoretical Model of Risk Sharing with Formal and Informal Contracts

Consider an economy with two agents and states of nature indexed by σ_t for date $t = 1, ..., \infty$. At every date the state of nature σ_t belongs to some finite set Ω, and the distribution of σ_t is i.i.d. We denote by σ a generic element of Ω and by π_σ the probability of state σ. Assume that the income process of agent i is z_σ^i in state σ, the total resources being $z_\sigma = z_\sigma^1 + z_\sigma^2$. Agent 2 has a fixed Von-Neuman Morgenstern utility $u_2(.)$. To account for random preferences in the empirical analysis, we assume that the utility function follows some stochastic process. The utility of agent 1 at date t is equal to $u_{1t}(.) = \eta_t u_1(.)$, where $\eta_1 = 1$, $\eta_t = \widetilde{\varepsilon}_t \eta_{t-1}$. $\widetilde{\varepsilon}_t$ stands for random preference shocks and is independently and identically distributed (i.i.d) with mean 1 and positive variance and whose support is an interval of $\mathbb{R}^+$. We assume that η_t is observed by the two agents at the very beginning of date t before endowment shocks σ_t that are observed only at the end of period t. The ex-ante utility of agent 1 is then $E\left[\sum_{t=1}^{\infty} \beta^{t-1} \eta_t u_1(c_t^1)\right]$, while it is $E\left[\sum_{t=1}^{\infty} \beta^{t-1} u_2(c_t^2)\right]$ for agent 2. As there are only two agents, we assume that the second agent have non-stochastic preferences. As we show below, what matters are ratios of marginal utilities and this assumption is therefore a simple normalization.[2]

[2] The analysis can be extended to accommodate additional shocks on preferences and resources (see the end of the theoretical section).

The ex-ante utility of agent i under autarchy v_a^i is (using $E(\eta_t) = 1$ and $c_t^i = z_{\sigma_t}^i$):

$$\forall\ i = 1, 2 : v_a^i = \frac{1}{1-\beta} \sum_{\sigma} \pi_\sigma u_i(z_\sigma^i)$$

Consider the benchmark case of complete contracts. In this case optimal insurance is achieved. The consumption in date t depends only on the realization of total resources and according to Borch rules, for all states the ratio of marginal utilities is the same. Thus, under a full contracting setting, the stochastic dynamics of consumption is given by

$$\frac{\eta_{t+1}}{\eta_t} \frac{u_1'(c_{t+1}^1)}{u_2'(c_{t+1}^2)} = \frac{u_1'(c_t^1)}{u_2'(c_t^2)}.$$

Now, we introduce limitations on the possibility for agents to sign formal contracts. Incompleteness is modeled here by two restrictions.

1. Contracts are short term and they are signed at the beginning of the period for the on-going period. Thus, prior to the realization of the period shocks but after the realization of preference shocks, individuals can sign a contract on how resources will be shared. At this stage they are not allowed to contract on the sharing of income for the subsequent periods.[3]

2. Second, contracts cannot be contingent to all components of the states of nature σ_t but only to some sets of states of nature. There is a set of events $s \in S$, where S is a partition of Ω and s is interpreted as random shocks affecting the realization of some (say "agricultural") production that is verifiable. We denote π_s the probability of event s: $\pi_s = \sum_{\sigma \in s} \pi_\sigma$. The formal contracts specify a reallocation of resources between agent 2 and agent 1 which can be contingent only on s in the current period. A contract is thus represented by a vector $T = (t_1, ..., t_S)$ of transfers. We assume that T belongs to some set $\mathcal{T} = \times_s [\underline{t}_s, \bar{t}_s]$, where $\underline{t}_s < 0 < \bar{t}_s$.[4]

[3] This first restriction is important only in the case where there is aggregate risk in the economy otherwise spot contracts are sufficient to provide full insurance.

[4] For what follows what really matters is that $\mathcal{T}$ has a lattice structure.

The model extends Gauthier, Poitevin and Gonzalez (1997) by allowing to account for random preferences and formal contracting on verifiable production. In Gauthier, Poitevin and Gonzalez (1997), only one ex-ante transfer is allowed such that it corresponds to the case where a non contingent transfer only is allowed ($\{s\} = \Omega$ i.e. $card(S) = 1$) and one agent is risk neutral. Because of convexity issues we will allow for some randomization beyond the fundamentals[5]. We assume that at every period, there is a public random variable, ε_t, the intrinsic shock, that is uniformly distributed and whose realization occurs at the beginning of date t. The precise timing of realization of the various events within period t, is the following.

- At t : the random preference, η_t, and intrinsic shock, ε_t, are realized and observed by both agents. The contract $T_t \in \mathcal{T}$ is signed, valid for period t.

- At $t + 1/2$: the income shock, state σ_t, is realized and observed. The contract T_t is enforced. However, the parties are free to complement it by voluntary transfers. Then consumptions take place.

To fix idea, one can assume that at date t, agent 1 makes a take-it or leave-it offer T_t to agent 2. If the agent 2 rejects the offer then no contract is signed for the ongoing period.[6]

With such a timing, current preferences are known when the contract is signed. The contract T_t can thus be contingent on η_t. Moreover the random component ε_t allows a stochastic link between the taste parameter and the contract.

With such a formulation we obtain the standard model of informal risk sharing when no contract is feasible (as a limit) like in Thomas and Worrall (1988), and the complete markets hypothesis when S is the set of individual events $\{\sigma\}$.

[5] The reason is that the value function may not be concave. Gauthier, Poitevin and Gonzalez (1997) assume that the value function is concave because they cannot prove it. However, in our case, we show that randomization over utilities is enough to obtain a convex program even with a non-concave value function.

[6] For the analysis, the precise bargaining game is not important, apart from the fact that it may affect the minimal equilibrium payoff of the agents for low values of the discount factor. An alternative would be to assume that there is a planer who propose the contract.

Let $H_t = (\sigma_1, .., \sigma_{t-1}, \eta_1, .., \eta_t, \varepsilon_1, .., \varepsilon_t)$ be the history of the states of nature up to t, and $h_t = (\sigma_1, .., \sigma_t, \eta_1, .., \eta_t, \varepsilon_1, .., \varepsilon_t) = (H_t, \sigma_t)$ the history up to $t + 1/2$. An allocation is a random consumption profile c_t^i and contract profile T_t that is measurable with respect to history: $c_t^i = c^i(h_t)$ and $T_t = T(H_t)$. The allocation is feasible if in all states, $c_t^1 + c_t^2 = z_t$ and $T_t \in \mathcal{T}$.

The expected utility of the agents are then

$$\begin{aligned} v^1 &= E\left[\sum_{t=1}^{\infty} \beta^{t-1} \eta_t u_1\left(c_t^1\right)\right] \\ v^2 &= E\left[\sum_{t=1}^{\infty} \beta^{t-1} u_2\left(c_t^2\right)\right] \end{aligned}$$

Because of the presence of the random taste parameter, our model is not truly a repeated game. However, it is stationary in the sense that the sub-game starting at date t is identical to the game starting at date 1 up to a re-normalization of utilities. To see that define the expected utility at the beginning of date t normalized by η_t as v_t^i:

$$\begin{aligned} v_t^1 &= E\left[\sum_{r=1}^{\infty} \beta^{r-1} \frac{\eta_{t+r-1}}{\eta_t} u_1(c_{t+r-1}^1) \mid H_t\right] \\ v_t^2 &= E\left[\sum_{r=1}^{\infty} \beta^{r-1} u_2(c_{t+r-1}^2) \mid H_t\right]. \end{aligned}$$

Notice that

$$\begin{aligned} v_t^1 &= E\left[u_1\left(c_t^1\right) + \beta \frac{\eta_{t+1}}{\eta_t} v_{t+1}^1 \mid H_t\right] \\ v_t^2 &= E\left[u_2\left(c_t^2\right) + \beta v_{t+1}^2 \mid H_t\right] \end{aligned}$$

Now consider the subgame starting at date t with η_t known and expected utilities v_t^i. Denote $\hat{\eta}_r^t = \frac{\eta_{t+r}}{\eta_t}$. Given that $\frac{\hat{\eta}_{r+1}^t}{\hat{\eta}_r^t} = \widetilde{\varepsilon}_{t+r+1}$ is i.i.d., the distribution of $\left\{\hat{\eta}_r^t\right\}_{r\geq 1}$ is the same as the distribution of $\{\eta_r\}_{r\geq 1}$. Thus the subgame starting at date t is identical to the initial game. This means that the sets of equilibria of the two games coincide. In other words, using normalized utilities v_t^i we can solve the game using the same tools as for a repeated game.

In particular there are minimal and maximal expected utility levels of agent i, denoted $\underline{v}^i$ and $\bar{v}^i$, that can be supported in equilibrium (up to the normalization). In the case of bilateral limited commitment, the allocations has to be self sustainable. In order to prevent a party reneging on the agreement, it is optimal to coordinate in such a way that if agent i deviates, the equilibrium that follows is the worst equilibrium for agent i. In other words one should apply an optimal penal code as defined by Abreu (1988) and Abreu, Pearce and Stacchetti (1986, 1990). When no contract is feasible, this means that the agent will receive its autarchic consumption. With short-term contracts however, autarchy may not be an equilibrium outcome for low discount factors.[7] Fortunately to solve and estimate the model, we don't need to derive the maximal punishment. For what follows, all that we need to know is that the minimal utility that a deviant agent can obtain from date t on is $\underline{v}_t^1 = \underline{v}^1$ for agent 1 and $\underline{v}_t^2 = \underline{v}^2$ for agent 2.

It is shown in appendix that with the take-it or leave-it bargaining game, then $\underline{v}^2$ is indeed the autarchy level v_a^2, while $\underline{v}^1$ coincides with v_a^1 when $\beta \geq \frac{1}{2}$ **(to be included)**.

Since the game is one with symmetric information, an allocation can be supported in equilibrium provided that at any point in time both agents prefer to abide to the informal agreement rather than to renege and be punished by receiving his minimal equilibrium utility. Thus at date t, it must be the case that the agent is willing to sign the contract[8]

$$v_t^i \geq \underline{v}^i, \tag{1}$$

and at date $t + 1/2$, the agent must prefer to make the informal transfer rather than to enforce the formal contract:

$$\begin{aligned} u_1(c_t^1) + \beta E\left[\frac{\eta_{t+1}}{\eta_t} v_{t+1}^1 \mid h_t\right] &\geq u_1(z_t^1 + t_t) + \beta \underline{v}^1, &(2)\\ u_2(c_t^2) + \beta E\left[v_{t+1}^2 \mid h_t\right] &\geq u_2(z_t^2 - t_t) + \beta \underline{v}^2. &(3)\end{aligned}$$

Following the standard approach to the problem we derive the set of Pareto optimal equilibria. For a given expected utility v of agent 2 at date 1, let $P(v)$ denote the maximal expected

[7] This is because with non-degenerate partition S, there will be some short-run gains in sharing the risk with a formal contract.

[8] To support the equilibrium at date t with the take-it or leave-it bargaining game, we can assume the following. If agent 2 rejects the offer $T(H_t)$, there is no informal transfer and the next period equilibrium is the worst for agent 2 so that he receives $E\{u^2(z_\sigma^2)\} + \beta\underline{v}^2 = \underline{v}^2$. If agent 1 doesn't offer $T(H_t)$, equilibrium strategies are those that support the worst equilibrium for him so that he cannot obtain more than $\underline{v}^1$.

utility that the agent 1 can obtain in equilibrium. Then $P(v)$ solves

$$P(v) = \max_{c_t^1, c_t^2, T_t} E\left[\sum_{t=1}^{\infty} \beta^{t-1} \eta_t u_1\left(c_t^1\right)\right] \tag{4}$$

s.t.

$$E\left[\sum_{t=1}^{\infty} \beta^t u_2\left(c_t^2\right)\right] \geq v,\ c_t^1 + c_t^2 = z_t, (1)\,,\ (2)\,,\ (3)\,. \tag{5}$$

A standard argument (see Thomas and Worrall, 1988) shows that the function $P(v)$ is decreasing and continuous. Clearly the optimal contract is such that conditional on H_t, the agent 1 should receive the maximal expected utility given that agent 2 receives at least v_t^2. Notice that under our assumptions on the stochastic processes of σ_t and η_t, the problem of maximizing v_t^1 conditional on H_t and v_t^2 is the same as the problem of maximizing the ex-ante utility of agent 1 subject to giving an ex-ante utility of at least v_t^2 to agent 2. Thus we must have $v_t^1 = P(v_t^2)$.

Then, the standard arguments apply and the allocation of consumption is the solution to the program

$$P(v) = \max_{\left(c_\sigma^1, c_\sigma^2, t_s, v_{\sigma\eta\varepsilon}\right)} E\left[u_1\left(c_\sigma^1\right) + \beta\eta P\left(v_{\sigma\eta\varepsilon}\right)\right]$$

s.t.

$$\begin{aligned}
u_1(c_\sigma^1) + E\left[\beta\eta P(v_{\sigma\eta\varepsilon}) \mid \sigma\right] &\geq u_1(z_\sigma^1 + t_s) + \beta\underline{v}^1\ \ \forall\sigma \\
u_2(c_\sigma^2) + E\left[\beta v_{\sigma\eta\varepsilon} \mid \sigma\right] &\geq u_2(z_\sigma^2 - t_s) + \beta\underline{v}^2\ \ \forall\sigma \\
E\left[u_2\left(c_\sigma^2\right) + \beta v_{\sigma\eta\varepsilon}\right] &\geq v \\
c_\sigma^1 + c_\sigma^2 &= z_\sigma\ \ \forall\sigma \\
v_{\sigma\eta\varepsilon} &\in \left[\underline{v}^2, \bar{v}^2\right]\ \ \forall\sigma, \eta, \varepsilon
\end{aligned}$$

In the program, $v_{\sigma\eta\varepsilon}$ is the agent 2 promised utility in date 2, conditional on a realization σ in date $1 + 1/2$, taste parameter η in date 2 and random shock ε in date 2. The expectation operator refers to the joint probability distribution of σ, η and ε. The optimal allocation can thus be described by consumption levels c_σ^i for each agent at date $1 + 1/2$, contract T and continuation expected utility $v_{\sigma n\varepsilon}^i$ at date 2 contingent on the realization of the shocks.

When there is no contract T, it is known (Thomas and Worrall) that the function P is decreasing, concave and differentiable. However, when contracts T are allowed, $P(.)$ need not be concave. Still we show that the problem is convex. Notice that if $P(.)$ is not concave, it is optimal for the agents to randomize between several date 2 utilities. Let us denote $v_{\sigma\eta} = E\left[v_{\sigma\eta\varepsilon} \mid \sigma, \eta\right]$. Given $v_{\sigma\eta}$, choosing $v_{\sigma\eta\varepsilon}$ is equivalent to choosing a distribution of utility on $[\underline{v}^2, \bar{v}^2]$. Denote Δ the set of probability distributions on $[\underline{v}^2,\, \bar{v}^2]$ and F a generic element of this set. Then an optimal allocation is such that conditional on σ and η, the distribution of the future utility solves the program

$$\hat{P}(v_{\sigma\eta}) = \max_{F \in \Delta} E\left[\int P(v) dF(v)\right] \; s.t. \; \int v dF(v) = v_{\sigma\eta}$$

$\hat{P}(.)$ is a concave decreasing function since the program is linear. $P(.)$ and $\hat{P}(.)$ coincide whenever $P(.)$ is concave.

Consider now the choice of $v_{\sigma\eta}$. Here again, given an expected utility $v_\sigma = E\left[v_{\sigma\eta} \mid \sigma\right]$, it is optimal to choose $v_{\sigma\eta}$ so as to maximize agent 1 utility. Define then

$$Q(v_\sigma) = \max_{v_{\sigma\eta} \in [\underline{v}_2, \bar{v}^2]} E\left[\eta \hat{P}(v_{\sigma\eta}) \mid \sigma\right] \; s.t. \; E\left[v_{\sigma\eta}\right] \geq v_\sigma$$

The function $Q(.)$ is decreasing and concave since $\hat{P}(.)$ is decreasing and concave.

The value function $P(.)$ can then be written as the solution of:

$$P(v) = \max_{(c_\sigma^1, c_\sigma^2, t_s, v_\sigma)} E\left[u_1\left(c_\sigma^1\right) + \beta Q\left(v_\sigma\right)\right] \tag{6}$$

s.t.

$$\begin{aligned}
E\left[u_2\left(c_\sigma^2\right) + \beta v_\sigma\right] &\geq v && (7)\\
u_1\left(c_\sigma^1\right) + \beta Q\left(v_\sigma\right) &\geq u_1\left(z_\sigma^1 + t_s\right) + \beta \underline{v}^1 \qquad \forall s, \sigma \in s && (8)\\
u_2\left(c_\sigma^2\right) + \beta v_\sigma &\geq u_2\left(z_\sigma^2 - t_s\right) + \beta \underline{v}^2 \qquad \forall s, \sigma \in s && (9)\\
c_\sigma^1 + c_\sigma^2 &= z_\sigma \qquad \forall \sigma && (10)\\
\underline{v}_2 &\leq v_\sigma \leq \bar{v}^2 \qquad \forall \sigma && (11)
\end{aligned}$$

This shows that, for a fixed contract $T = \{t_1, ..., t_S\}$, the program is convex although $P(.)$ may not be concave.

The argument developed in Poitevin et Al. (1997) for the case where the contract is a fixed transfer can be used similarly in our case to show that $P(.)$ is continuously differentiable because we proved that $Q(.)$ is concave. This in turn implies that $Q(.)$ and $\hat{P}(.)$ are continuously differentiable. For what follows we need to assume in addition that they are not linear:

Assumption: $\hat{P}'(\underline{v}^2) > \hat{P}'(\bar{v}^2)$.

This ensures that at the solution of $Q(v_\sigma)$, we have $\underline{v}^2 < v_{\sigma\eta} < \bar{v}^2$ with positive probability. The assumption is thus a non-triviality assumption. It rules out a situation where $P(.)$ is convex everywhere, in which case optimality would require to alternate between corner solutions. This is clearly a degenerate case which is not interesting for estimation purpose nor from a theoretical perspective. We however failed so far to prove that it cannot occur. This is thus a very weak assumption that ensures a rich equilibrium dynamics.

Now, to describe the dynamics of the system, we don't need to describe the whole frontier P but only those points that can occur in equilibrium, in other words the supports of the distribution F that solves $\hat{P}(.)$. It is immediate that (we skip the proof as this is standard):

If a point $v^2 \in [\underline{v}^2, \bar{v}^2]$ occurs with positive probability, then $P(v^2) = \hat{P}(v^2)$ and $v^2 = \arg\max_x \left\{ P(x) - \hat{P}'(v^2)x \right\}$.

Let $\mathcal{W}$ be the set of solutions $\Phi(.)$ of the program

$$\Phi(\mu) = \max_{v \in [\underline{v}^2, \bar{v}^2]} P(v) + \mu v$$

when the weight μ varies continuously between $-\hat{P}'(\underline{v}^2)$ and $-\hat{P}'(\bar{v}^2)$. Then the set of utility $v_{\sigma\eta\varepsilon}$ that can obtain with positive probability in equilibrium is included in $\mathcal{W}$. We shall solve this program $\Phi(\mu)$ to derive the equilibrium. This amounts to maximize $E\left[u_1\left(c_\sigma^1\right) + \beta Q\left(v_\sigma\right)\right] + \mu E\left[u_2\left(c_\sigma^2\right) + \beta v_\sigma\right]$ subject to constraints (8) to (11). To solve this program, notice that it is separable between events s. In other words

$$\Phi(\mu) = \max_{T=\{t_1,..,t_S\}} \sum_s \pi_s \Phi_s(\mu, t_s)$$

where $\Phi_s(\mu, t_s)$ is the solution for a fixed value of t_s of the maximization of

$$E\left[u_1\left(c_\sigma^1\right) + \beta Q(v_\sigma) \mid s\right] + \mu E\left[u_2\left(c_\sigma^2\right) + \beta v_\sigma \mid s\right]$$

subject to the constraints (8) to (11) in event s. $\Phi_s(\mu, t_s)$ is a concave program and we show that due to the preference shock η_t, it is a strictly concave problem with a unique solution. Working with this program we obtain the main result that will be used for estimation:

Proposition 1 *Let's note $r_\sigma = z_\sigma^1 + t_s$ the agent 1 income in state σ. There exists functions $\underline{\mu}(z_\sigma, r_\sigma) \leq \bar{\mu}(z_\sigma, r_\sigma)$ with values in $[-\hat{P}'(\underline{v}^2), -\hat{P}'(\bar{v}^2)]$, decreasing in r_σ whenever interior such that:*

When $\bar{\mu}(z_\sigma, r_\sigma) > -\hat{P}'(\underline{v}^2)$ and $\underline{\mu}(z_\sigma, r_\sigma) < -\hat{P}'(\bar{v}^2)$, then $\bar{\mu}(z_\sigma, r_\sigma) > \underline{\mu}(z_\sigma, r_\sigma)$ and:

$$\frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} = -Q'(v_\sigma) = \bar{\mu}(z_\sigma, r_\sigma) \text{ if } \mu \geq \bar{\mu}(z_\sigma, r_\sigma) \text{ ((37) binds)} \tag{12}$$

$$\frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} = -Q'(v_\sigma) = \underline{\mu}(z_\sigma, r_\sigma) \text{ if } \mu \leq \underline{\mu}(z_\sigma, r_\sigma) \text{ ((38) binds)} \tag{13}$$

$$\frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} = -Q'(v_\sigma) = \mu \text{ if } \underline{\mu}(z_\sigma, r_\sigma) \leq \mu \leq \bar{\mu}(z_\sigma, r_\sigma) \tag{14}$$

In addition

$\frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} \leq -Q'\left(\underline{v}^2\right)$ *and* $v_\sigma = \underline{v}^2$ *if* $\bar{\mu}(z_\sigma, r_\sigma) = -\hat{P}'(\underline{v}^2)$

$\frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} \geq -Q'\left(\bar{v}^2\right)$ *and* $v_\sigma = \bar{v}^2$ *if* $\underline{\mu}(z_\sigma, r_\sigma) = -\hat{P}'(\bar{v}^2)$.

Proof. See Appendix A. ■

For a given formal contract T, this result is a generalization of Thomas and Worrall (1988) where one agent is risk neutral to the case of bilateral limited commitment, risk aversion of both players and with formal contracts allowed. Also, it extends the results of Gauthier, Poitevin and Gonzalez (1997) where one agent has a constant endowment and T is a unidimensional unconditional transfer. The proposition thus defines the current and future ratio of marginal utilities as a function of the multiplier μ and ex-post resources (which depend on the contract t_s). Using this we can fully characterize the solution as a function of the contract.

Then, the main contribution is that this theoretical model allows us to derive a property of the optimal endogenous contract T shown in a second step. Actually, the second step is to show that the optimal contract T is monotone in μ. The problem is not concave in T so that there may be multiple solutions for T. Multiple solutions arise when the frontier $P(v)$ is not concave or when no incentive constraint is binding in some event s. However intuition suggests that when μ increases, $T(\mu)$ should decrease as v moves along the Pareto frontier toward higher utility for agent 2 (since μ is the slope of the frontier).

Proposition 2 *The mapping $\bar{T}(\mu)$: $\mu \rightarrow \arg\max_T E\{\Phi_s(\mu, t_s)\}$ is a monotone decreasing correspondence in μ (according to the strong order set).*

Proof. See Appendix B. ■

To summarize, as we move along the frontier $\hat{P}(v)$ toward higher absolute slopes (and thus higher v), the contract becomes uniformly more favorable to the agent 2. Notice that the same holds true for the allocation of consumptions and future utilities (c_σ^2, v_σ).[9]

Let us now turn to the implications of the results for the dynamics of consumption and contracts. For the estimation we assume that corner solutions never arise:

Assumption: $prob\{\underline{v}^2 < v_t^2 < \bar{v}^2\} = 1$.

The dynamics can be described by mean of the evolution of the weight $\mu_t = -P'(v_t^2)$ associated with the point in $\mathcal{W}$ chosen after history H_t.

At these stage, agents sign a contract $T_t \in \bar{T}(\mu_t)$. At date $t + 1/2$ consumption is given as a function of μ_t, the contract T_t and σ_t by Proposition 1. This also defines the slope $Q'(v^2(h_t))$ at this interim stage. Then at date $t + 1$, H_{t+1} is realized and thus v_{t+1}^2. This

[9] This follows from the fact that at the solution of $\Phi_s(\mu, t_s)$, both c_σ^2 and v_σ are non-decreasing with μ and non-increasing with t_s.

gives the new value of the weight μ_{t+1}. The intertemporal link is provided by the relation $\frac{\eta_{t+1}}{\eta_t}P'(v^2_{t+1}) = \frac{\eta_{t+1}}{\eta_t}\hat{P}'(v^2_{t+1}) = Q'(v^2(h_t))$. The dynamics thus verifies

$$
\begin{aligned}
T_t &= \{t_t(s)\}_s \in \bar{T}(\mu_t) && (15)\\
r_t &= z^1_t + t_t(s_t) && (16)\\
\frac{u'_1(c^1_t)}{u'_2(c^2_t)} &= \bar{\mu}\left(z_t, r_t\right)\ if\ \mu_t \geq \bar{\mu}\left(z_t, r_t\right) && (17)\\
\frac{u'_1(c^1_t)}{u'_2(c^2_t)} &= \mu_t,\ if\ \underline{\mu}\left(z_t, r_t\right) \leq \mu_t \leq \bar{\mu}\left(z_t, r_t\right) && (18)\\
\frac{u'_1(c^1_t)}{u'_2(c^2_t)} &= \underline{\mu}\left(z_t, r_t\right)\ if\ \mu_t \leq \underline{\mu}\left(z_t, r_t\right) && (19)\\
\mu_{t+1} &= \frac{\eta_t}{\eta_{t+1}}\frac{u'_1(c^1_t)}{u'_2(c^2_t)} && (20)
\end{aligned}
$$

Whenever the Pareto frontier is concave this defines exactly the whole dynamics as $\bar{T}(\mu)$ is single valued. If $P(.)$ is not concave $\bar{T}(\mu)$ can be multi-valued. Notice that it is single valued for all values μ_t where $\Phi(\mu_t)$ has a unique solution. This corresponds to values where $-\hat{P}'(v_t) = \mu_t$ has a unique solution.[10] But we have shown in the proof of Proposition 1 (in lemma 3) that this occurs with probability 1 due to the effect of the preference shock η_t. Thus in equilibrium $\bar{T}(\mu_t)$ is single valued with probability 1. We can thus ignore the issue of equilibrium randomization over utilities and contracts for estimation purpose.

Additional sources of observed heterogeneity

In what preceded we assume fixed utilities and a stationary resource process. Suppose that the utility is $u_i(c^i; x^i_t)$ where x^i_t follows a Markov process. Suppose that the resources depend on σ_t and q_t, where q_t follows a Markov process. Suppose also that q_t and x^i_t are learned at the beginning of period t. Let $y_t = (x^1_t, x^2_t, q_t)$ the information at the beginning of period t. Then the value function at date t is a function $P(v, y_t)$. The interim value function is $Q(v; y_t) = \max E\left\{\frac{\eta_{t+1}}{\eta_t}\hat{P}(v(y_{t+1}, \frac{\eta_{t+1}}{\eta_t}); y_{t+1}) \mid y_t\right\}$ subject to $E\left\{v(y_{t+1}, \frac{\eta_{t+1}}{\eta_t}) \mid y_t\right\} \geq v$. The program $P(v; y_t)$ then is the solution of $\max E\left[u_1\left(c^1(y_t, \sigma_t); x_t\right) + \beta Q\left(v(y_t, \sigma_t); y_t\right) \mid y_t\right]$ subject to incentive and participation constraints. In this set-up all the proofs generalize. The functions

[10] $t(s)$ may still be undetermined if the probability that an incentive constraint binds in event s is zero. We rule out such possibility.

$\bar{\mu}$ and $\underline{\mu}$ depend only on z_t, r_t and y_t (but not on η_t): $\bar{\mu}(z_t, r_t; y_t)$ and $\underline{\mu}(z_t, r_t; y_t)$. The ratio $\frac{u_1'(c_t^1)}{u_2'(c_t^2)}$ has to be conditioned on x_t^i only: $\frac{u_1'(c_t^1;x_t^1)}{u_2'(c_t^2;x_t^2)}$. The contract depends on μ_t and y_t: $\bar{T}(\mu_t, y_t)$. But the dynamics of the multiplier μ_t is unchanged since $-\frac{\eta_{t+1}}{\eta_t}\hat{P}'(v(y_{t+1}, \frac{\eta_{t+1}}{\eta_t}), y_{t+1}) = -Q'(v(y_t, \sigma_t); y_t) = \frac{u_1'(c_t^1;x_t^1)}{u_2'(c_t^2;x_t^2)}$ with probability 1 and $\mu_{t+1} = \hat{P}'(v(y_{t+1}, \frac{\eta_{t+1}}{\eta_t}), y_{t+1})$.

3 Econometric Specification, Identification and Estimation

We first state the structural form of the econometric model by specifying the two equations of interest: consumption dynamics and the income process. We assume that all functions of interest are linear or log-linear and we investigate identification of the model in the leading case developed in the theoretical model, where random shocks are independent of explanatory variables. These restrictions are strong enough to get identification of the main parameters of interest. We estimate the model by GMM using even stronger identifying assumptions as they are testable.

3.1 Consumption Dynamics

We start from equations (17, 18, 19, 20) describing the dynamics of the ratios of the marginal utilities of consumption for a pair of households. As we do not observe pairs of households engaged into formal contracts but only individual households, we assume (as is common in this literature, see Ligon, Thomas and Worrall, 2002), that the ratio of marginal utilities between household i and its partner can be written as:

$$\eta_t \tau(c_{it}, x_{it}) \exp(\delta_{vt}) \tag{21}$$

where c_{it} is household i 's consumption and x_{it} are household demographic variables that affect preferences and where the "partner" household is assumed to be the whole village (or district). Its marginal utility is summarized by a village-and-period effect δ_{vt}. We also assume that households have constant relative risk aversion, θ:

$$\tau(c_{it}, x_{it}) = \exp(x_{it}\theta\beta).c_{it}^{-\theta}$$

where demographics are permitted to affect the slope of marginal utilities only[11]. The logarithm of marginal utility is therefore assumed to be log-linear. The consumption dynamics depends on the multiplier μ_t which is given by:

$$\ln \mu_t = \ln \frac{\eta_{it-1}}{\eta_{it}} + \ln \tau(c_{it-1}, x_{it-1}) + \delta_{vt-1}.$$

Taking logarithms in equation (21) we get consumption dynamics in the three regimes, the regimes being defined by whether or not incentive constraints are binding:

$$\ln \tau(c_{it}, x_{it}) + \delta_{vt} = \ln \frac{\eta_{it-1}}{\eta_{it}} + \ln \tau(c_{it-1}, x_{it-1}) + \delta_{vt-1}$$
$$\text{if } \ln \frac{\eta_{it-1}}{\eta_{it}} + \ln \tau(c_{it-1}, x_{it-1}) + \delta_{vt-1} \in [\ln \underline{\mu}(r_{it}, y_{it}), \ln \overline{\mu}(r_{it}, y_{it})] \quad \text{(R1)}$$

$$\ln \tau(c_{it}, x_{it}) + \delta_{vt} = \ln \underline{\mu}(r_{it}, y_{it})$$
$$\text{if } \ln \frac{\eta_{it-1}}{\eta_{it}} + \ln \tau(c_{it-1}, x_{it-1}) + \delta_{vt-1} < \ln \underline{\mu}(r_{it}, y_{it}) \quad \text{(R2)}$$

$$\ln \tau(c_{it}, x_{it}) + \delta_{vt} = \ln \overline{\mu}(r_{it}, y_{it})$$
$$\text{if } \ln \frac{\eta_{it-1}}{\eta_{it}} + \ln \tau(c_{it-1}, x_{it-1}) + \delta_{vt-1} > \ln \overline{\mu}(r_{it}, y_{it}) \quad \text{(R3)}$$

where r_{it} is agricultural and non agricultural profit net of input costs including labor, and where y_{it} is the vector of variables in the information set at the end of period t. This vector comprises any variable that affect current preferences or help to predict future preferences and income processes. In particular, y_{it} includes current taste shifters, x_{it}, asset variables such as owned land, and other variables that affect agricultural production and that are known when the contract is signed. All the latter variables are denoted q_{it} so that $y_{it} = (x_{it}, q_{it})$.

Whether incentive constraints are binding or not, are not observable events, and the three regimes giving consumption dynamics, are therefore not observable. As a consequence, the system of equations (R1 to R3) is equivalent to a single equation, describing the dynamics of marginal utilities as:

$$\Delta \ln \tau(c_{it}, x_{it}) + \Delta \delta_{vt} + \ln \tilde{\varepsilon}_{it} = \underline{\phi}_{it} . \mathbf{1}_{\{\underline{\phi}_{it} > 0\}} + \overline{\phi}_{it} \mathbf{1}_{\{\overline{\phi}_{it} < 0\}} \tag{22}$$

[11] The relative risk aversion parameter could also be made a function of observed characteristics as in Dubois (2000). See the empirical section.

where $\tilde{\varepsilon}_{it} = \frac{\eta_{it}}{\eta_{it-1}}$ are the random preference shocks and where:

$$\Delta \ln \tau(c_{it}, x_{it}) = \ln \tau(c_{it}, x_{it}) - \ln \tau(c_{it-1}, x_{it-1})$$

$$\underline{\phi}_{it} = \ln \underline{\mu}(r_{it}, y_{it}) + \ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}$$

$$\overline{\phi}_{it} = \ln \overline{\mu}(r_{it}, y_{it}) + \ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}$$

This is the first equation of the structural model and this equation describes consumption dynamics. The pair of endogenous variables are consumption and non-labor income (c_{it}, r_{it}) and the explanatory variables are (x_{it-1}, y_{it}) consisting of preference shifters at time $t-1$ and t and of other information variables. Non-labor income is endogenous in this model because it depends on formal contracts that are endogenous. There are two sources of stochastic shocks in the model. The random shocks that are unobserved by the econometrician are preference shocks, $\ln \tilde{\varepsilon}_{it}$, revealed at the beginning of the period and income shocks on r_{it}, revealed at the mid-period. The specification of the income variable r_{it} as a function of formal contracts, is the object of the next subsection.

We have to specify the upper and lower bound functions $\underline{\mu}$ and $\overline{\mu}$ related to incentive constraints to finish to write the consumption equation. To conform with our idea of exploring identification under linearity assumptions, we assume that the upper and lower constraints in (22) are semi-log-linear:

$$\begin{cases} \ln \underline{\mu}(r_{it}, y_{it}) = \underline{\mu}_0 r_{it} + \underline{\mu}_y y_{it} + \underline{\mu}_{vt} \\ \ln \overline{\mu}(r_{it}, y_{it}) = \overline{\mu}_0 r_{it} + \overline{\mu}_y y_{it} + \overline{\mu}_{vt} \end{cases} \tag{23}$$

where $\underline{\mu}_{vt}$ and $\overline{\mu}_{vt}$ are village effects summarizing the effects on incentives, of global resources available at the village level as in the theoretical setting. Parameters $\underline{\mu}_0$ and $\overline{\mu}_0$ are **negative** as shown in the structural model (consequence of Proposition 2).

Some comments are in order. First, this specification could implicitly take randomness into account if y_{it} was allowed to contain such unobserved heterogeneity components. As noted above, the structure of stochastic shocks is already sufficiently rich to permit these bounds to be random because of the income variable r_{it}. As we assume some measurement errors in income, the fact that we won't allow for unobserved heterogeneity in y_{it} does not seem to be too tight an assumption for this model. A more difficult issue that we do not treat here, is the presence of individual effects in y_{it} but individual effects are notoriously difficult to handle in non-linear dynamic settings (for an analysis of identification, see Magnac and Thesmar, 2002). Second, we do not impose for the moment that for any r_{it}, y_{it}, any village and any period, the constraint $\underline{\mu}(r_{it}, y_{it}) \leq \overline{\mu}(r_{it}, y_{it})$ is verified. We shall return to this point in the section related to identification.

An interesting particular case of this model is the case of complete markets. It amounts to assume that the incentive constraints never bind in this model, that is $\underline{\phi}_{it} < 0$, $\overline{\phi}_{it} > 0$ so that:

$$\Delta \ln \tau(c_{it}, x_{it}) + \Delta\delta_{vt} + \ln \tilde{\varepsilon}_{it} = 0$$

Two remarks are in order. First, the event $\underline{\phi}_{it} < 0$, $\overline{\phi}_{it} > 0$ can have probability 1 only if all variables (including those that are unobserved) appearing in the expressions of $\underline{\phi}_{it}$ and $\overline{\phi}_{it}$ are bounded or if some parameters take infinite values. It does not favor the use of a full parametric test of the hypothesis of complete markets. This consequence agrees well however with the general prediction of a model with self-enforcing constraints. In this model, the dynamics is at times consistent with the hypothesis of complete markets and at times not consistent. Secondly, the right hand side of (22) is a function of r_{it} while the left hand side is not. If r_{it} were exogenous, the standard test of the hypothesis of complete markets broadly would consist in looking at the significance of the correlation between the residuals under the null hypothesis and the income process r_{it}. This test is correct provided that income be excluded from preferences or, more precisely, from the marginal utility of consumption (or related variables such as hours of work). In the present model, income r_{it} is endogenous because it depends on formal contracts that depend themselves on random preference shocks. This is why we now specify the other equation determining the income process r_{it}. It is clear enough that a test of complete markets can be constructed if there are exogenous variables that affect income and are independent of random preference shocks and therefore of formal contracts[2].

3.2 Formal Contracts and the Income Process

A formal contract is described by Proposition 2 or equation (15). The vector of formal transfers (*i.e.* for any state s) is a function of the following form:

$$T = T(\ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}, y_{it})$$

which makes clear that formal contracts are dependent of preference shocks and where y_{it} includes L_{it} the quantity of owned land, for instance. These formal transfers T are supposed to be supported by land-leasing contracts: a sharecropping contract involves M_{it} units of land with an output share α_{it}; a fixed rent-contract concern F_{it} units of land at a fixed price, set at the village level. We freely consider that M_{it} and F_{it} can be positive or negative depending on whether land-leasing is in or out. Moreover, agricultural profits are necessarily a function of these land inputs:

$$\pi_{it} = \pi_{it}(M_{it}, F_{it}, \alpha_{it}, y_{it}, z_{it}, \xi_{it})$$

where z_{it}, ξ_{it} are variables or shocks revealed after the signature of the contracts (see below). Depending on available data, we could presumably estimate a production function and input demands including labor in order to derive this profit function. Given the complicated endogenous structure of land exploitation, results will not be robust to specification errors on the production side. This is why we model directly the dependence of profits on the marginal utility of consumption and the information variables, skipping the relationships between the quantities of land under sharecropping and fixed-rent, and the marginal utility[3]. We then write agricultural profits as a linear function:

$$\pi_{it} = \pi_{vt} + \pi_0(\ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}) + y_{it}\pi_y + z_{it}\pi_z + \xi_{it}$$

[2] In some papers, the issue of income endogeneity is treated in a reduced-form setting (Jacoby and Skoufias, 1998, Jalan and Ravallion, 1999, Kochar, 1999).

[3] We shall however test in the empirical section that these quantities are related to marginal utilities.

where parameter π_0 is **positive** by Proposition 2. Risks, summarized by state s in the theoretical model, are assumed to be translated by the village intercept π_{vt} and the household random shock ξ_{it}. Other risks, summarized by state σ in the theoretical model, are described by the same random shock ξ_{it} and are also described and determined by variables z_{it}, such as days of sickness and so on. We shall assume that random shocks ξ_{it} are independent across households and therefore, that the village effects perfectly take into account any dependence across households.

To close the gap with the income variable that appear in the equation of consumption dynamics, household net income r_{it} is written as the sum of agricultural profits, π_{it}, and non-agricultural profits or other exogenous income, π_{it}^e:

$$r_{it} = \pi_{it} + \pi_{it}^e$$

Exogenous income could be other non-labor income or exogenous transfers such as exogenous remittances from abroad if they are independent of random preference shocks, $\ln \tilde{\varepsilon}_{it}$, and income shocks, ξ_{it}. They exclude informal transfers from the extended families studied for instance by Foster and Rosenzweig (2001) because these transfers are linked to the endogenous informal contracts modeled here and obviously dependent on both types of random shocks. This income equation defines the income variable, r_{it}, appearing in (22). With no loss of generality, we include π_{it}^e among the z_{it} variables already defined above and the profit equation above determining π_{it} is also the income equation giving r_{it}. We finish by adding some measurement errors, ς_{it}, to profits (or income) to obtain measured profits:

$$\tilde{\pi}_{it} = \pi_{vt} + \pi_0(\ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}) + y_{it}\pi_y + z_{it}\pi_z + \xi_{it} + \varsigma_{it} \tag{24}$$

The structural form of the model therefore consists of equations (22) and (24). We now write the reduced form.

3.3 Identifying Restrictions and the Reduced Forms

We state the identifying restrictions and write the reduced form. Recall that all variables x entering preferences are also included in the information set $y = (x, q)$ where q are all other variables known before the signature of the contract (owned land for instance). Recall also that z_{it} denote the variables known after the signature of the contract[4]. We shall therefore accordingly denote that coefficients, say, $\pi_y = (\pi_x, \pi_q)$ whenever necessary. The two endogenous variables at time t are consumption growth, $\Delta \ln c_{it}$, and agricultural income, $\tilde{\pi}_{it}$. We also have the following list of (weakly) exogenous variables, $\ln c_{it-1}$, x_{it-1}, x_{it}, q_{it} and z_{it} that appear in both equations.

The identifying restrictions are given by the following assumptions.

Assumption: While structural random shocks are $(\ln \tilde{\varepsilon}_{it}, \xi_{it})$ and measurement errors are described by ς_{it}, we assume that:

i/ the vector $(\ln \tilde{\varepsilon}_{it}, \xi_{it})$ is independent of $(\ln c_{it-1}, x_{it-1}, x_{it}, q_{it}, z_{it})$ and of ς_{it},and is identically distributed and independent across households and periods.

ii/ measurement error ς_{it} is mean-independent of $(\ln c_{it-1}, x_{it-1}, x_{it}, q_{it}, z_{it})$ and is independent across households and periods, with bounded variance.

iii/ the support of the distribution function of conditioning variables $(\ln c_{it-1}, x_{it-1}, x_{it}, q_{it}, z_{it})$ has a non-empty interior.

Assumption **i/** is slightly stronger than the ones generally used in linear dynamic models. It is a very usual assumption in non-linear dynamic models. Non-linearities, due here to the presence of bounds, require more than mean-independence assumptions. We could relax them somehow to get identification of some subsets of parameters but we do not investigated thoroughly this point. Assumption **ii/** is weaker as it takes advantage of linearity. Assumption **iii/** has two consequences. It first implies that the distribution function of conditioning variables $(\ln c_{it-1}, x_{it-1}, x_{it}, q_{it}, z_{it})$ is not degenerate. It is not innocuous because the "other" variables, q_{it}, in the information set could include x_{it-1} only (or c_{it-1}) which might lead to a violation of this assumption. Technically, assumption **iii/** opens the door to the analysis of derivatives of estimable equations with respect to the covariates.

[4] At the identification stage, we can assume that z_{it} and y_{it} are independent. Actually, if they are not, we could replace z_{it} by the innovation in z_{it} independent of y_{it} since we are only interested by the identification of π_0 and β. See Kochar (1999).

After using and reshuffling terms in the different equations, the income equation can be written as:

$$\tilde{\pi}_{it} = \tilde{\pi}_{vt} + \pi_0\theta(\ln c_{it-1} - x_{it-1}\beta) + x_{it}\pi_x + q_{it}\pi_q + z_{it}\pi_z + \pi_0 \ln \tilde{\varepsilon}_{it} + \xi_{it} + \varsigma_{it}$$

where the village effect, $\tilde{\pi}_{vt}$, includes preference, δ_{vt-1}, income, and village effects π_{vt}, and where we used that:

$$\ln \tau(c_{it-1}, x_{it-1}) = -\theta(\ln c_{it-1} - x_{it-1}\beta)$$

The reduced form of the income process is therefore:

$$\begin{aligned}\tilde{\pi}_{it} &= \tilde{\pi}_{vt} + \pi_0\theta \ln c_{it-1} + x_{it-1}(\pi_x - \pi_0\theta\beta) + \Delta x_{it}\pi_x \\ &\quad + q_{it}\pi_q + z_{it}\pi_z + \pi_0 \ln \tilde{\varepsilon}_{it} + \xi_{it} + \varsigma_{it}\end{aligned} \tag{25}$$

where we distinguished x_{it-1} and Δx_{it} instead of (x_{it-1}, x_{it}) because of the consumption equation. Namely, the reduced form of the consumption equation (22) can be written as the sum of a linear function of Δx_{it} and of other terms related to incentive constraints:

$$-\theta(\Delta \ln c_{it} - \Delta x_{it}\beta) + \Delta\delta_{vt} + \ln \tilde{\varepsilon}_{it} = \underline{\phi}_{it}\mathbf{1}_{\{\underline{\phi}_{it}>0\}} + \overline{\phi}_{it}\mathbf{1}_{\{\overline{\phi}_{it}<0\}} \tag{26}$$

The upper and lower bounds are given by (23) where we replaced income:

$$\begin{aligned}\phi_{it} &= \mu_0\left(\pi_{vt} + \pi_0(\ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}) + y_{it}\pi_y + z_{it}\pi_z + \xi_{it}\right) \\ &\quad + y_{it}\mu_y + \mu_{vt} + \ln \tilde{\varepsilon}_{it} - \ln \tau(c_{it-1}, x_{it-1}) - \delta_{vt-1}\end{aligned}$$

where we implicitly denoted, $\phi_{it} \in \{\underline{\phi}_{it}, \overline{\phi}_{it}\}$ and where μ_0, μ_y and μ_{vt} are defined accordingly (*i.e.* $\mu_i \in \{\underline{\mu}_i, \overline{\mu}_i\}$ for the index i taking "values" 0, y, vt, see equation (23)). Then, reshuffling:

$$\begin{aligned}\phi_{it} &= \phi_{vt} - (\mu_0\pi_0 + 1)\ln \tau(c_{it-1}, x_{it-1}) + y_{it}\left(\mu_0\pi_y + \mu_y\right) + z_{it}\mu_0\pi_z \\ &\quad + (\mu_0\pi_0 + 1)\ln \tilde{\varepsilon}_{it} + \mu_0\xi_{it}\end{aligned}$$

where ϕ_{vt} is the composition of different village effects:

$$\phi_{vt} = \mu_0(\pi_{vt} - \pi_0\delta_{vt-1}) - \delta_{vt-1} + \mu_{vt}.$$

Replacing the marginal utility function and using the different exogenous variables, we get:

$$\begin{aligned}\phi_{it} &= \phi_{vt} + (\mu_0\pi_0 + 1)\theta(\ln c_{it-1} - x_{it-1}\beta) + \\ &\quad + z_{it}\mu_0\pi_z + x_{it}(\mu_0\pi_x + \mu_x) + q_{it}(\mu_0\pi_q + \mu_q) + u_{it}\end{aligned}$$

where we denote $u_{it} \in \{\underline{u}_{it}, \overline{u}_{it}\}$ the random terms in ϕ_{it} defined by:

$$u_{it} = (\mu_0\pi_0 + 1)\ln\tilde{\varepsilon}_{it} + \mu_0\xi_{it}$$

Linearity implies that unobserved heterogeneity only enters the intercept in indices, ϕ_{it}, which is the central piece of identifying restrictions. A slight generalization of this setting could permit parameters μ_0 and π_0 or other parameters to depend on exogenous variables. Generalizing to functions which slopes depend on unobserved heterogeneity is a much more difficult task.

If $\underline{\mu}_0 \neq \overline{\mu}_0$, there is a one-to-one mapping between $(\ln\tilde{\varepsilon}_{it},\ \xi_{it})$ and $(\underline{u}_{it},\ \overline{u}_{it})$. It is equivalent to assume that the pair $(\ln\tilde{\varepsilon}_{it},\ \xi_{it})$ is identically and independently distributed and independent of the exogenous variables, or that the pair $(\underline{u}_{it},\ \overline{u}_{it})$ is identically and independently distributed independent of the exogenous variables. It is therefore identical at this stage to fix one or the other of these distribution functions.

Indices ϕ_{it} can be written as:

$$\begin{aligned}\phi_{it} &= \phi_{vt} + (\mu_0\pi_0 + 1)\theta\ln c_{it-1} + x_{it-1}(-(\mu_0\pi_0 + 1)\theta\beta) \\ &\quad + x_{it}(\mu_0\pi_x + \mu_x) + q_{it}(\mu_0\pi_q + \mu_q) + z_{it}\mu_0\pi_z + u_{it} \\ &= \phi^*_{it} + u_{it}\end{aligned} \tag{27}$$

Replacing indices ϕ_{it}, the consumption equation (26) is now given by:

$$-\theta(\Delta\ln c_{it} - \Delta x_{it}\beta) + \Delta\delta_{vt} + \ln\tilde{\varepsilon}_{it} = (\underline{\phi}^*_{it} + \underline{u}_{it})\mathbf{1}_{\{\underline{u}_{it} > -\underline{\phi}^*_{it}\}} + (\overline{\phi}^*_{it} + \overline{u}_{it})\mathbf{1}_{\{\overline{u}_{it} < -\overline{\phi}^*_{it}\}} \tag{28}$$

where the linearity of the intensities of the incentive constraints as a function of heterogeneity has now been made explicit.

The system of equations (25) and (28) defines the endogenous variables as functions of the (weakly) exogenous variables:

$$w_{it} = (\ln c_{it-1}, x_{it-1}, \Delta x_{it}, q_{it}, z_{it})$$

3.4 Semi-parametric Identification

We are interested by the identification of the following parameters: β,θ, π_0, π_x, π_q, π_z, μ_0, μ_x, μ_q (the last three with upper and lower bars). The identification analysis proceeds as follows. First, the parameters of the reduced form of the income equation (25) are trivially identified since $Ew'_{it}w_{it}$ has full rank, that is to say parameters $\tilde{\pi}_{vt}$, $\pi_0\theta$, $\pi_x - \beta\pi_0\theta$,π_x, π_z, and π_q are identified. Thus, β is also identified. Because measurement errors are only mean-independent of covariates, this is the only piece of identifying power that one can get from the income equation.

Turning to the consumption equation, it is easy to show that θ is not identified. The transformation from the vector of parameters (θ, π_0, μ_0, μ_x, μ_q, $\Delta\delta_{vt}$, ϕ_{vt}) into (1, $\pi_0\theta$, $\frac{\mu_0}{\theta}$, $\frac{\mu_x}{\theta}$, $\frac{\mu_q}{\theta}$, $\frac{\Delta\delta_{vt}}{\theta}$, $\frac{\phi_{vt}}{\theta}$) (leaving unchanged the other parameters) and the vector of random shocks ($\ln\tilde{\varepsilon}_{it}$, ξ_{it}, ς_{it}) into ($\frac{\ln\tilde{\varepsilon}_{it}}{\theta}$, ξ_{it}, ς_{it}) is invariant for the two equations of interest. We shall therefore normalize $\theta = 1$ without loss of generality and change the interpretation of other parameters accordingly. It is not a surprise since, in a usual Euler framework, the relative risk aversion or intertemporal substitution parameter is not identified if there is no information on the true interest rate.

The consumption equation becomes:

$$\begin{aligned} \Delta \ln c_{it} &= \Delta x_{it}\beta + \Delta\delta_{vt} + \ln\tilde{\varepsilon}_{it} \\ &\quad -(\underline{\phi}^*_{it} + \underline{u}_{it})\mathbf{1}_{\{\underline{u}_{it} > -\underline{\phi}^*_{it}\}} - (\overline{\phi}^*_{it} + \overline{u}_{it})\mathbf{1}_{\{\overline{u}_{it} < -\overline{\phi}^*_{it}\}} \end{aligned} \tag{29}$$

We can finally remark that if $\{\underline{\phi}_{it}, \overline{\phi}_{it}\}$ are identified from the consumption equation (as we will show below), then $\underline{\phi}_{vt}$, $\overline{\phi}_{vt}$ are identified. Moreover, as π_{vt} is identified from the income equation and $\Delta\delta_{vt}$ from the consumption equation, there cannot be cross constraints between village effects appearing in the different terms because the structure is sufficiently flexible. The presence of village effects does not yield identification power, quite the contrary.

We now study the identification of the remaining parameters $\underline{\mu}_0$, $\underline{\mu}_x$, $\underline{\mu}_q$ and $\overline{\mu}_0$, $\overline{\mu}_x$, $\overline{\mu}_q$.

3.4.1 The Derivatives of the Consumption Equation

Using the conditional expectation of the consumption growth equation (29) only is the way we proceed because this equation could include additional random shocks apart from the preference shock, $\ln \tilde{\varepsilon}_{it}$, as briefly sketched at the end of the theory section. We can also show that a different specification for the upper and lower constraints (equation 23) lead to the same type of equation (see Appendix C).

Write the conditional expectation of equation (29) conditional on w_{it} and on (v, t) which is left implicit:

$$\begin{aligned} E(\Delta \ln c_{it} \quad &| \quad w_{it}) = \Delta x_{it}\beta + \Delta\delta_{vt} \qquad (30) \\ -E((\underline{\phi}^*_{it} + \underline{u}_{it})\mathbf{1}_{\{\underline{u}_{it} > -\underline{\phi}^*_{it}\}} \quad &| \quad w_{it}) - E((\overline{\phi}^*_{it} + \overline{u}_{it})\mathbf{1}_{\{\overline{u}_{it} < -\overline{\phi}^*_{it}\}} \mid w_{it}) \end{aligned}$$

The last terms are written as, for instance:

$$E((\underline{\phi}^*_{it} + \underline{u}_{it})\mathbf{1}_{\{\underline{u}_{it} > -\underline{\phi}^*_{it}\}} \mid w_{it}) = \int_{\underline{u}_{it} > -\underline{\phi}^*_{it}(w_{it})} (\underline{\phi}^*_{it}(w_{it}) + \underline{u}_{it}) d\underline{F}(\underline{u}_{it})$$

where $\underline{F}(u_{it})$ is independent of w_{it} and (v, t) using the identifying restrictions. Function $\underline{\phi}^*_{it}(w_{it})$ is the only function that depends on w_{it} in this expression. Therefore, for any continuous variable in w_{it}, the derivatives of these terms are:

$$\begin{aligned} \frac{\partial}{\partial w_{it}} E((\underline{\phi}^*_{it} + \underline{u}_{it})\mathbf{1}_{\{\underline{u}_{it} > -\underline{\phi}^*_{it}\}} \quad | \quad w_{it}) &= \int_{\underline{u}_{it} > -\underline{\phi}^*_{it}(w_{it})} \frac{\partial}{\partial w_{it}} \underline{\phi}^*_{it}(w_{it}) d\underline{F}(\underline{u}_{it}) \\ &= \frac{\partial}{\partial w_{it}} \underline{\phi}^*_{it}(w_{it}). \int_{\underline{u}_{it} > -\underline{\phi}^*_{it}(w_{it})} d\underline{F}(\underline{u}_{it}) = \frac{\partial}{\partial w_{it}} \underline{\phi}^*_{it}(w_{it}).\underline{\Psi}(w_{it}) \end{aligned}$$

and similarly

$$\frac{\partial}{\partial w_{it}} E((\overline{\phi}^*_{it} + \overline{u}_{it})\mathbf{1}_{\{\overline{u}_{it} < -\overline{\phi}^*_{it}\}} \mid w_{it}) = \frac{\partial}{\partial w_{it}} \overline{\phi}^*_{it}(w_{it}).\overline{\Psi}(w_{it})$$

using the smoothness of distribution functions of random terms and where

$$\underline{\Psi}(w_{it}) = \int_{\underline{u}_{it} > -\underline{\phi}^*_{it}(w_{it})} d\underline{F}(\underline{u}_{it}) \tag{31}$$

$$\overline{\Psi}(w_{it}) = \int_{\overline{u}_{it} < -\overline{\phi}^*_{it}(w_{it})} d\overline{F}(\overline{u}_{it}) \tag{32}$$

The derivative of the equation of interest (30) becomes:

$$\frac{\partial}{\partial w_{it}} E(\Delta \ln c_{it} \mid w_{it}) = \frac{\partial \Delta x_{it}\beta}{\partial w_{it}} - \frac{\partial \underline{\phi}^*_{it}}{\partial w_{it}} \underline{\Psi}(w_{it}) - \frac{\partial \overline{\phi}^*_{it}}{\partial w_{it}} \overline{\Psi}(w_{it}) \tag{33}$$

which yields the moment conditions that form the basis of the analysis of identification. First, these derivatives are identified (Pagan and Ullah, 1999). Second, $\underline{\Psi}$ and $\overline{\Psi}$ are functions of linear indices $\phi^*_{it}(w_{it})$ that are left implicit. Note also the implicit dependence on village and period specific indicators.

To proceed, the different relevant derivatives are given in the following table:

w_{it}	$\frac{\partial \Delta x_{it}\beta}{\partial w_{it}}$	$\frac{\partial \phi^*_{it}}{\partial w_{it}}$
$\ln c_{it-1}$	0	$\mu_0\pi_0 + 1$
x_{it-1}	0	$-\beta(\mu_0\pi_0 + 1) + \mu_0\pi_x + \mu_x$
Δx_{it}	β	$\mu_0\pi_x + \mu_x$
q_{it}	0	$\mu_0\pi_q + \mu_q$
z_{it}	0	$\mu_0\pi_z$

Under the linearity assumptions, these derivatives are independent of exogenous variables. We can therefore use weighted average derivatives as well to recover the parameters (Stoker, 1990). Denote $\rho(w_{it})$ any function depending on w_{it} (and possibly on (v,t)) such that:

$$\left| E(\rho(w_{it}) \frac{\partial}{\partial w_{it}} E(\Delta \ln c_{it} \mid w_{it})) \right| < \infty$$

the estimable equation (33) can be written as:

$$D^w_\rho \equiv E(\rho(w_{it}) \frac{\partial}{\partial w_{it}} E(\Delta \ln c_{it} \mid w_{it})) = \frac{\partial \Delta x_{it}\beta}{\partial w_{it}} - \frac{\partial \underline{\phi}^*_{it}}{\partial w_{it}} \underline{\Psi}_\rho - \frac{\partial \overline{\phi}^*_{it}}{\partial w_{it}} \overline{\Psi}_\rho \tag{34}$$

where there is a slight abuse of notations, $\Psi_\rho = E(\rho(w_{it})\Psi(w_{it}))$, $\Psi(.) \in \{\underline{\Psi}(.), \overline{\Psi}(.)\}$. We use this notation in order to emphasize that D^w_ρ could stand for "straight" average derivatives (Stoker, 1990) or point-wise derivatives as well if $\rho(w_{it})$ is the Dirac measure at w_{it}.

This equation can be used to prove identification in various ways. We first study the case where we choose a single weight function $\rho(.)$ then turn to the general case.

3.4.2 Identification from Single-Weight Average Derivatives

As already said π_0, π_z, π_x, π_q and β are identified from the income equation. If we use a single weight function $\rho(.)$, for instance, $\rho = 1$,we have therefore five (formal) relationships on average derivatives in order to identify $\underline{\Psi}_\rho$, $\overline{\Psi}_\rho$, and the six (formal) parameters of interest, $\overline{\mu}_0$, $\underline{\mu}_0$, $\overline{\mu}_x$, $\underline{\mu}_x$, $\overline{\mu}_q$, $\underline{\mu}_q$. The degree of underidentification is at least equal to 3.

Notice that

$$\beta = \frac{D_\rho^{\Delta x} - D_\rho^x}{1 + D_\rho^c}.$$

which is a structural restriction on average derivatives that does not depend on still unknown parameters.[5] In particular note that in a complete markets structure $\beta = D_\rho^{\Delta x}$.

The other first-order average derivatives are written as follows:

$$\begin{aligned}
-D_\rho^c = &\ \pi_0(\underline{\mu}_0\underline{\Psi}_\rho + \overline{\mu}_0\overline{\Psi}_\rho) + \underline{\Psi}_\rho + \overline{\Psi}_\rho \\
-D_\rho^z = &\ \pi_z(\underline{\mu}_0\underline{\Psi}_\rho + \overline{\mu}_0\overline{\Psi}_\rho) \\
-D_\rho^{\Delta x} + \beta = &\ \pi_x(\underline{\mu}_0\underline{\Psi}_\rho + \overline{\mu}_0\overline{\Psi}_\rho) + \underline{\mu}_x\underline{\Psi}_\rho + \bar{\mu}_x\overline{\Psi}_\rho \\
-D_\rho^q = &\ \pi_q(\underline{\mu}_0\underline{\Psi}_\rho + \overline{\mu}_0\overline{\Psi}_\rho) + \underline{\mu}_q\underline{\Psi}_\rho + \bar{\mu}_q\overline{\Psi}_\rho
\end{aligned} \tag{35}$$

The system of average derivatives allows to identify the parameters $\underline{\Psi}_\rho + \overline{\Psi}_\rho$, $\underline{\mu}_0\underline{\Psi}_\rho + \overline{\mu}_0\overline{\Psi}_\rho$, $\underline{\mu}_x\underline{\Psi}_\rho + \bar{\mu}_x\overline{\Psi}_\rho$, $\underline{\mu}_q\underline{\Psi}_\rho + \bar{\mu}_q\overline{\Psi}_\rho$.

Testable restrictions on the identified parameters are:

$$\begin{aligned}
\pi_0 &\geq 0 \text{ (monotonicity of the contract from proposition 2)} \\
\underline{\Psi}_\rho + \overline{\Psi}_\rho &\geq 0 \text{ (probability that an incentive constraint binds)} \\
\underline{\mu}_0\underline{\Psi}_\rho + \overline{\mu}_0\overline{\Psi}_\rho &\leq 0 \text{ (monotonicity from proposition 1)}
\end{aligned}$$

3.4.3 Full identification

The deeper parameters are still not identified and other weight functions have to be used. From the previous system of equations (35) and using that $\underline{\mu}_0 \neq \overline{\mu}_0$, we can eliminate $\underline{\Psi}_\rho$ and $\overline{\Psi}_\rho$ to get an estimable equation that is true for any weight function:

$$\forall \rho; H(D_\rho^w, \underline{M}, \overline{M}) = 0$$

[5] The restriction may not be robust to a mistake in the choices of the variables put in q. Suppose for instance that either x_{it-1} (or c_{it-1}) adds some information, then we should have coefficient π or μ for these variables as well. Then β would not be identified in the profit equation nor by the above relation.

where $M = (\mu_0, \mu_x, \mu_q)$. The necessary and sufficient condition for identification of $\underline{M}$ and $\overline{M}$ is therefore that the set:

$$\cap_\rho \{\underline{M}, \overline{M}; H(D_\rho^w, \underline{M}, \overline{M}) = 0\}$$

is reduced to one point that is the true value of the parameters. Some overidentifying restrictions are also derived from this condition.

3.5 Estimation using a Generalized Method of Moments

As the previous objects are identified in quite general conditions, we shall further strengthen the identifying assumptions to:

$$\underline{u}_{it} \text{ and } \overline{u}_{it} \text{ are normally distributed conditional to } w_{it}$$

and $(\underline{\sigma}^2, \overline{\sigma}^2)$ are their respective variances. This assumption is testable as shown above. When using the normality assumption for the non-linear terms of the consumption growth equation (30), we get:

$$\begin{aligned} E((\overline{\phi}_{it}^* + \overline{u}_{it})\mathbf{1}_{\{\overline{u}_{it} < -\overline{\phi}_{it}^*\}} \quad | \quad w_{it}) &= \overline{\sigma}[\frac{\overline{\phi}_{it}^*}{\overline{\sigma}}\Phi(\frac{-\overline{\phi}_{it}^*}{\overline{\sigma}}) - \varphi(\frac{\overline{\phi}_{it}^*}{\overline{\sigma}})] = -\overline{\sigma}h(-\frac{\overline{\phi}_{it}^*}{\overline{\sigma}}) \\ E((\underline{\phi}_{it}^* + \underline{u}_{it})\mathbf{1}_{\{\underline{u}_{it} > -\underline{\phi}_{it}^*\}} \quad | \quad w_i) &= \underline{\phi}_{it}^*.\Phi(\frac{\underline{\phi}_{it}^*}{\underline{\sigma}}) + \underline{\sigma}\varphi(\frac{\underline{\phi}_{it}^*}{\underline{\sigma}}) = \underline{\sigma}h(\frac{\underline{\phi}_{it}^*}{\underline{\sigma}}) \end{aligned}$$

where Φ and φ are the cumulative and density functions of a standard normal random variable. Note that $h(x) = x.\Phi(x) + \varphi(x)$ is a positive, increasing and convex function.

Moment conditions are derived from equation (30):

$$E(m_{it}' \ln \tilde{\varepsilon}_{it}) = 0$$

where m_{it} are the variables w_{it} and village-and-period indicators. Thus:

$$E(m_{it}^{'}(\Delta \ln c_{it} - \Delta x_{it}\beta - \Delta\delta_{vt} + \underline{\sigma}h(\frac{\underline{\phi}_{it}^*}{\underline{\sigma}}) - \overline{\sigma}h(-\frac{\overline{\phi}_{it}^*}{\overline{\sigma}}))) = 0 \tag{36}$$

The other moment conditions related to the profit linear equation is the second estimating equation. The parameters imposing the structural restrictions are estimated in an iterative procedure in two steps. In the first step, we use the weighting matrix corresponding to linear 2SLS. In the second step, we compute an estimator of the weighting matrix using the standard arguments.

In the case of normality and using definitions (31) and (32) we have $\underline{\Psi}(w_{it}) = 1-\Phi(-\frac{\underline{\phi}_{it}^{*}}{\underline{\sigma}}(w_{it}))$ and $\overline{\Psi}(w_{it}) = \Phi(\frac{-\overline{\phi}_{it}^{*}}{\overline{\sigma}}(w_{it}))$. Then, we can test

$$\underline{\mu}_0 \leq 0 \text{ and } \overline{\mu}_0 \leq 0 \text{ (monotonicity from proposition 1)}$$

4 Empirical Estimation

4.1 Data

The data come from a survey conducted by IFPRI (International Food Policy Research Institute) in Pakistan between 1986 and 1989 (see Alderman and Garcia, 1993). The survey consists of a stratified random sample interviewed 12 times of around 900 households from four districts of three regions (Attock and Faisalabad in Punjab, Badin in the Sind, and Dir in the North West Frontier Province, NWFP). For each of the four districts, the villages were chosen randomly from an exhaustive list of villages classified in three sets according to their distances to two markets (*mandis*). In each village, households were randomly drawn from an exhaustive list of village households. The attrition observed in the data (927 households at the beginning and only 887 at the end) seems to come from administrative and political problems rather than from self-selection of households (Alderman and Garcia, 1993). We consider that attrition is exogenous. These rich data contain information on household demographics, income from various sources, individual labor supply, endowments and owned assets, agrarian structure, crops and productions, and finally land contracts such as sharecropping and fixed rent. Sources of income are wages, agricultural profits, rents from property rights, pensions, informal transfers (from relatives or others).

We had to construct some of the variables of interest from the different data files that were available. Household demographics are directly available from the individual data. Household food consumption is initially reported by food item, in quantity and value, or quantity and price. It comprises meals at home including home-produced goods, and meals taken outside for all household members but the meals that were the result of invitation or rewards in kind, because the information was not available. Household agricultural income consists of cash income from staples, milk products, animal poultry and livestock production, net of total input expenditures including wage costs, feeding costs of productive animals, fertilizers and pesticides (net of household handicraft income). Household wage income consists of wages received in agricultural and non-agricultural off-farm activities. Asset income come from property rents, fixed pensions regularly received from the government and rentals of different productive assets. Transfers correspond to transfers received from relatives, friends and from solidarity funds of local mosques (*zakat*). Descriptive statistics are presented in Table 1.

Table 1: Descriptive statistics

Descriptive statistics on the full sample (all periods)			
Variable	Average	Std Err.	Obs.
Food consumption	197.9	151.4	9990
Other non durable expenditures (heating, ..)	47.3	196.1	9991
Total owned land area (acres)	9.42	21.81	10083
Irrigated land (acres)	4.19	11.25	10083
Non irrigated land (acres)	5.24	17.09	10083
Household size	8.64	4.23	9987
Number of children (<=15years)	4.08	2.91	9987
Pensions	70.5	450.5	9906
Agricultural profits	109.26	1095.6	9906
Transfers	106	974	9906
Total income (without transfers)	321.7	1291.1	9906
Sharecropping dummy variable (renting in)	0.35	0.47	10083
Fixed rent dummy variable (renting in)	0.08	0.26	10083

Looking at the variability of the log of household food consumption $\ln c_{it}$, one finds that the within household variability explains 39.4% of the overall variability, while considering changes of the log of household food consumption $\Delta \ln c_{it}$, the within household variability explains only 7.7% of the overall variability.

4.2 Empirical Results[6]

We estimate the parameters of the structural model using the consumption growth equation:

$$E(\Delta \ln c_{it} \mid w_{it}) = \Delta x_{it}\beta + \Delta\delta_{vt} - \underline{\sigma} h(\frac{\underline{\phi}^*_{it}}{\underline{\sigma}}) + \overline{\sigma} h(-\frac{\overline{\phi}^*_{it}}{\overline{\sigma}})$$

where from (27)

$$\begin{aligned} \underline{\phi}^*_{it} &= \phi_{vt} + (\mu_0\pi_0 + 1)\ln c_{it-1} + x_{it-1}(-(\mu_0\pi_0+1)\beta) \\ &\quad + x_{it}(\mu_0\pi_x + \mu_x) + q_{it}(\mu_0\pi_q + \mu_q) + z_{it}\mu_0\pi_z \end{aligned}$$

and the income equation (25):

$$\tilde{\pi}_{it} = \tilde{\pi}_{vt} + \pi_0 \ln c_{it-1} + x_{it}\pi_x - x_{it-1}\pi_0\beta + q_{it}\pi_q + z_{it}\pi_z + \pi_0 \ln \tilde{\varepsilon}_{it} + \xi_{it} + \varsigma_{it}$$

We chose to begin by estimating a model with a limited number of variables of type x, q and z. First, other investigations using the same data (Dubois, 2000) showed that household size is among the main preference shifters. These are the variables that we consider among variables x_{it}. We could also consider age of the household head which is a continuous variable and very much related to the household demographics dynamics. Second, various empirical analyses (see Jalan and Ravallion, 1999) give evidence that the main cause for rejection of the hypothesis of complete markets come from contrasting rich and poor households. Whereas income is endogenous in our model, the quantity of owned land seems to be a good indicator of household wealth in productive assets and therefore a good predictor of income. The quantity of owned irrigated land that is available in the survey (or the complement, rain-fed land) should give additional information about the quality and price of productive land. These two variables are the ones that we include in the so-called information variables, denoted q_{it}, in the theoretical section. Finally, it seems to be interesting to explore the influence of exogenous income sources or of variables related to illnesses of household members, as they affect agricultural production and as they are likely to be non-contractible. It means that

[6] **The empirical results presented here are preliminary and incomplete. Do not quote.**

(at least the non-predictable part of it) they do not affect the formal contracts but do affect the informal arrangements between households.

Table 2 provides the estimates of the structural parameters of the profit equation.

Table 2: Income Profit Equation[7]

Parameter	Coeff.	(t-stat)	Coeff.	(t-stat)	Coeff.	(t-stat)	Coeff.	(t-sta
Dependent variable: $\tilde{\pi}_{it}$	(1)		(2)		(3)		(4)	
π_0	0.050	(2.21)	0.045	(2.05)	0.081	(3.18)	0.079	(3.07
π_x								
(log) household size	0.309	(0.59)	0.303	(0.58)	0.374	(0.64)	0.373	(0.64
π_q : land owned								
total land in village	0.147	(3.23)	0.147	(3.25)	0.170	(3.10)	0.171	(3.1
rainfed land in village	-0.193	(-3.09)	-0.199	(-3.20)	-0.224	(-2.95)	-0.232	(-3.0
total land out village			0.037	(4.39)			0.078	(4.6
rainfed land out village			-0.038	(-3.83)			-0.073	(-4.3
canal irrig. land out village			-0.022	(-3.05)			-0.074	(-4.2
wasted land out village			-0.039	(-4.57)			-0.081	(-4.7
π_z								
male illness days	-0.001	(-0.18)	-0.003	(-0.42)	-0.005	(-0.55)	-0.005	(-0.5
female illness days	-0.054	(-3.67)	-0.054	(-3.69)	-0.054	(-3.41)	-0.054	(-3.4
exogenous income shocks	0.23	(4.62)	0.21	(4.47)	0.141	(2.93)	0.1409	(2.94
$-\pi_0\beta$								
(log) household size	-0.363	(-0.72)	-0.367	(-0.72)	-0.394	-0.73	-0.395	(-0.7
$\tilde{\pi}_{vt}$								
44 district*time : F tests	13.87		14.10		14.40		14.37	
Observations	8939		8939		8163		8163	

According to the theoretical model (proposition 2) and the semi-parametric identification section, a testable restriction on the identified parameters is that $\pi_0 \geq 0$. As shown by the estimation results, it is not rejected by the data. It means that past consumption determines future income profit. Moreover, if one includes past income profit in this equation, ones finds a very small effect (0.01) which is completely insignificant (t-stat=0.48).

Different specifications have been tested for this income profit equation. Household preference shocks x_{it} are finally well represented by the (log) household size but other characteristics like the number of children or age of the household head or other demographic characteristics have been added in the specification and proved to be insignificant. Also, one

[7] Robust standard errors for all specifications.

could wonder about autocorrelation of unobservables affecting income profit in this panel estimation. However, this autocorrelation is in fact very low, 0.0097 and insignificant ($p = 0.38$). Higher order autocorrelation also appeared to be close to zero and insignificant. In this case, the presence of unobserved household specific effects is doubtful. For specification (2), the autocorrelation is also very low, 0.0089 and insignificant ($p = 0.42$). In the case of columns (3) and (4)[8], we have the same absence of autocorrelation.

The second structural equation given by our model consists in the following consumption dynamics equation

$$E\left(\Delta \ln c_{it} \mid w_{it}\right) = \Delta x_{it}\beta + \Delta\delta_{vt} + H_1(\underline{\phi}_{vt} + w_{it}\underline{\phi}) + H_2(\overline{\phi}_{vt} + w_{it}\overline{\phi})$$

with

$$\begin{aligned}
\phi_{it}^* &= \phi_{vt} + (\mu_0\pi_0 + 1)\ln c_{it-1} - x_{it-1}(\mu_0\pi_0 + 1)\beta + x_{it}(\mu_0\pi_x + \mu_x) + q_{it}(\mu_0\pi_q + \mu_q) + z_{it}\mu_0\pi_z \\
&= \phi_{vt} + w_{it}\phi \\
w_{it} &= (\ln c_{it-1}, x_{it-1}, \Delta x_{it}, q_{it}, z_{it})
\end{aligned}$$

and

$$\begin{aligned}
H_1(x) &= -E((x + \overline{u}_t)\mathbf{1}_{\{\overline{u}_t < -x\}} \mid w_t) \\
H_2(x) &= -E((x + \underline{u}_t)1_{\{\underline{u}_t > -x\}} \mid w_t)
\end{aligned}$$

H_1 is a positive, decreasing and convex function and H_2 is a negative, decreasing and concave function. Assuming normality of $\underline{u}$ and $\overline{u}$: we have $H_1(x) = \overline{\sigma}h(-\frac{x}{\overline{\sigma}})$ and $H_2(x) = -\underline{\sigma}h(\frac{x}{\underline{\sigma}})$ where $h(x) = x.\Phi(x) + \varphi(x)$.

Before estimating this non linear structural equation and as a first reduced form approximation, on can look at the regression of $\Delta \ln c_{it}$ on w_{it} and dummies specific to v and t. An implication of the structural model is in particular that $E\left(\Delta \ln c_{it} \mid w_{it}\right)$ should be a non linear function of $\ln c_{it-1}$. In particular, the second derivative of $E\left(\Delta \ln c_{it} \mid w_{it}\right)$ with respect

[8] Where we selected only households whose size is between 3 and 14.

to $\ln c_{it-1}$ is $(\overline{\mu_0}\pi_0 + 1)^2 H_1'' + (\underline{\mu_0}\pi_0 + 1)^2 H_2''$ which is the sum of a strictly positive and a strictly negative term as soon as $1 + \mu_0 \pi_0 \neq 0$. Table 3 shows the results of this informal reduced form regression. In columns (2) and (4), we use a two stage least squares estimation instrumenting $\ln c_{it-1}$ by $\ln c_{it-2}$ in order to take into account some possible measurement error in consumption. We actually see that the negative coefficient on lagged consumption is then much smaller in absolute value but still negative.

It is interesting to not that $E\left(\Delta \ln c_{it} \mid w_{it}\right)$ is a decreasing function of $\ln c_{it-1}$ and that it is non linear in $\ln c_{it-1}$ (this is obtained with a 3 parts linear spline function but also with 5 or more break points). This shape both convex on some interval and concave on another is consistent with the curvature properties of H_1 and H_2 of the structural model. However, one has to remain prudent since the results of Table 3 are a purely reduced form expression of the structural consumption dynamics equation which is in general not valid since $E\left(\Delta \ln c_{it} \mid w_{it}\right)$ should not be a separable function of w_{it} and dummy variables as it is assumed in the reduced form estimation. The estimation of the structural form requires the estimation of a semi-parametric multiple index model which can give very different results.

Table 3: Reduced Form of Consumption Equation[9]

[9] Robust standard errors.

	(1) OLS		(2) IV 2SLS		(3) OLS		(4) IV 2SL	
Dependent variable: $\Delta \ln c_{it}$	Coeff.	(t-stat)	Coeff.	(t-stat)	Coeff.	(t-stat)	Coeff.	(t-
$\ln c_{it-1}$	-0.62	(-32.06)	-0.136	(-2.15)				
Continuous spline function on $\ln c_{it-1}$								
1^{st} tercile					-0.64	(-13.99)	0.004	(0.
2^{nd} tercile					-0.50	(-13.45)	-0.413	(-2
3^{rd} tercile					-0.73	(-23.36)	-0.080	(-0
x_{t-1}: lag (log) household size	-0.057	(-0.92)	0.410	(5.45)	-0.054	(-0.90)	0.410	(5
x_t: (log) household size	0.33	(5.30)	-0.353	(-4.41)	0.32	(5.31)	-0.349	(-4
q_{it} : land owned								
total land in village	0.016	(3.60)	0.002	(0.28)	0.016	(3.55)	0.001	(0
rainfed land in village	-0.001	(-0.16)	0.004	(0.38)	-0.001	(-0.16)	0.005	(0
z_{it}								
male illness days	0.001	(0.27)	-0.001	(0.13)	0.0012	(0.24)	-0.001	(0
female illness days	0.016	(3.34)	0.017	(2.69)	0.0164	(3.33)	0.016	(2
exogenous income shocks	0.026	(2.63)	0.001	(0.17)	0.026	(2.56)	0.004	(0
44 district*time dummies: F tests	..		..		..		..	
Observations	8163		7327		8163		7327	

Assuming normality of $\underline{u}$ and $\overline{u}$ one can estimate the model parameters using the GMM method described previously.

Our theoretical model is actually able to explain empirically the observed pattern of consumption dynamics interaction with the household income process. Using the profit equation parameter estimates and the GMM estimates of the consumption dynamics equation, all parameters are identified and Table 4 presents the results. Consistently with the model, we get that $\overline{\mu_0}$ is significantly negative while $\underline{\mu_0}$ is not significantly different from zero. This means that the complete markets hypothesis is rejected and the self-enforcing constraint of households is actually binding when their income is too high.

Table 4: GMM Estimates of the Consumption Dynamics Equation

Parameter		Coeff.	t-stat
β			
	household size	0.0413	(0.96)
	number of children	0.049	(0.72)
$\underline{\mu_0}$		**0.007**	**(0.50)**
$\underline{\mu_x}$			
	household size	-0.0170	(-0.044)
	number of children	-0.2339	(-0.308)
$\underline{\mu_q}$			
	land owned in the village	-0.318	(-0.66)
	rainfed land owned	0.2267	(0.517)
$\overline{\mu_0}$		**-0.0048**	**(-5.01)**
$\overline{\mu_x}$			
	household size	0.0244	(0.18)
	number of children	0.1663	(0.75)
$\overline{\mu_q}$			
	land owned in the village	0.136	(5.71)
	rainfed land owned	-0.143	(-2.79)
$\underline{\sigma}$		0.167	(0.91)
$\underline{\phi_{vt}}$: All district and time dummies (not shown)			
$\overline{\sigma}$		1.94	(3.44)
$\overline{\phi_{vt}}$: All district and time dummies (not shown)			
$\Delta\delta_{vt}$: All district and time dummies (not shown)			
Observations		8906	

It is clear that the identification of parameters ρ_x,$\overline{\mu}_0$,$\overline{\mu}_x$,$\overline{\mu}_q$,$\underline{\mu}_0$,$\underline{\mu}_x$,$\underline{\mu}_q$ relies on both equations. Therefore, one needs to take it into account when estimating the variance-covariance matrix of estimated coefficients.

5 Conclusion

In conclusion, we can underline the importance of the structural modelling of alternative assumptions about risk sharing mechanisms. Since, the complete markets hypothesis is generally rejected, the modelling of risk sharing and contracting mechanisms is now necessary to better understand the household behavior in an environment of incomplete markets. Here, we have elaborated a theoretical setting which nests the case of complete markets when all risks can be insured by formal contracts (because all states of nature would be verifiable) and the case where only informal agreements are available. This theoretical model provides two important structural equations of interest, an Euler-type equation of consumption dynamics and the equation of determination of the formal contract. We show that the model is semi-parametrically identified and implement two estimation methods using either GMM with some parametric assumption or average derivatives estimation allowing to do only semi-parametric assumptions (to be completed). Estimating both equations using data of village economies in Pakistan, we found consistent results with the theoretical model developed.

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A Proof of Proposition 1

First we show that the problem is strictly concave because of the preference shock.

Lemma 3 $Q(.)$ *is strictly concave.*

Proof. Consider the set $B \subset \left[\hat{P}'(\bar{v}_2), \hat{P}'(\underline{v}_2)\right]$ of slopes b such that $b = -\hat{P}'(v)$ occurs for more than one value v. The solution of $Q(v)$ is a continuous function v_η of η with

$$\begin{aligned}
\eta\hat{P}'(v_\eta) &= Q'(v) \text{ if } \underline{v}_2 < v_\eta < \bar{v}^2, \\
v_\eta &= \bar{v}^2 \text{ if } \eta \leq \frac{Q'(v)}{\hat{P}'(\bar{v}^2)}, \\
v_\eta &= \underline{v}^2 \text{ if } \eta \geq \frac{Q'(v)}{\hat{P}'(\underline{v}^2)}.
\end{aligned}$$

This defines completely v_η as a function of $Q'(v)$ except at those points where $\frac{Q'(v)}{\eta} \in B$. But $prob\{\frac{Q'(v)}{\eta} \in B\} = 0$ because B is a countable set and η is a continuous random variable. Consider now $v' > v$ with a solution v'_η. It is impossible that $Q'(v') = Q'(v)$ because this would imply $v'_\eta = v_\eta$ with probability one and thus contradicts $E\left\{v'_\eta\right\} = v' > v$. Thus $Q'(.)$ must be decreasing. ■

The program for given μ and t_s, in the event s, is then:

$$\Phi_s(\mu, t_s) = \max_{(c^1_\sigma, c^2_\sigma, v_\sigma)} E\left[u_1\left(c^1_\sigma\right) + \beta Q(v_\sigma) \mid s\right] + \mu E\left[u_2\left(c^2_\sigma\right) + \beta v_\sigma \mid s\right]$$

s.t.

$$\left(\frac{\pi_\sigma}{\pi_s}\lambda^1_\sigma\right) \qquad u_1\left(c^1_\sigma\right) + \beta Q(v_\sigma) \geq u_1\left(z^1_\sigma + t_s\right) + \beta\underline{v}^1 \qquad \forall\sigma \in s \tag{37}$$

$$\left(\frac{\pi_\sigma}{\pi_s}\lambda^2_\sigma\right) \qquad u_2\left(c^2_\sigma\right) + \beta v_\sigma \geq u_2\left(z^2_\sigma - t_s\right) + \beta\underline{v}^2 \qquad \forall\sigma \in s \tag{38}$$

$$\left(\frac{\pi_\sigma}{\pi_s}\psi_\sigma\right) \qquad c^1_\sigma + c^2_\sigma \leq z_\sigma \qquad \forall\sigma \in s \tag{39}$$

$$\left(\frac{\pi_\sigma}{\pi_s}\beta\bar{\gamma}_\sigma\right) \qquad v_\sigma \leq \bar{v}^2 \tag{40}$$

$$\left(\frac{\pi_\sigma}{\pi_s}\beta\underline{\gamma}_\sigma\right) \qquad \underline{v}^2 \leq v_\sigma \tag{41}$$

The terms in brackets are Lagrange multipliers. The Lagrangian of the program is:

$$\sum_{\sigma\in s}\frac{\pi_\sigma}{\pi_s}\left\{\begin{array}{c} u_1\left(c^1_\sigma\right) + \beta Q(v_\sigma) + \mu\left[u_2\left(c^2_\sigma\right) + \beta v_\sigma\right] + \lambda^1_\sigma\left[u_1\left(c^1_\sigma\right) + \beta Q(v_\sigma)\right] \\ +\lambda^2_\sigma\left[u_2\left(c^2_\sigma\right) + \beta v_\sigma\right] - \psi_\sigma\left[c^1_\sigma + c^2_\sigma\right] + (\underline{\gamma}_\sigma - \bar{\gamma}_\sigma)\beta v_\sigma \end{array}\right\}$$

As the program is strictly concave, the first order conditions of this program are necessary and sufficient for optimality. After elimination of ψ_σ, $\underline{\gamma}_\sigma$, $\bar{\gamma}_\sigma$, they reduce to:

$$\begin{aligned}
\frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} &= \frac{\mu+\lambda_\sigma^2}{1+\lambda_\sigma^1} \\
-Q'\left(v_\sigma\right) &= \frac{\mu+\lambda_\sigma^2}{1+\lambda_\sigma^1} \text{ if } \underline{v}^2 < v_\sigma < \bar{v}^2 \\
v_\sigma &= \bar{v}^2 \text{ if } -Q'\left(\bar{v}^2\right) \leq \frac{\mu+\lambda_\sigma^2}{1+\lambda_\sigma^1} \\
v_\sigma &= \underline{v}^2 \text{ if } -Q'\left(\underline{v}^2\right) \geq \frac{\mu+\lambda_\sigma^2}{1+\lambda_\sigma^1}
\end{aligned}$$

along with complementary slackness conditions.

Let $\phi(.)$ be the inverse function of $-Q'(.)$ (which is increasing). Notice that $\underline{v}_2 = \phi(\underline{v}_2)$, $\bar{v}^2 = \phi(\bar{v}^2)$, and $\underline{v}_2 < \phi(\mu) < \bar{v}^2$ if $-\hat{P}'(\underline{v}_2) < \mu < -\hat{P}'(\bar{v}^2)$. Define $\psi^i(z,\mu)$ as the solution of

$$\begin{aligned}
\frac{u_1'\left(\psi^1\right)}{u_2'\left(\psi^2\right)} &= \mu, \\
\psi^1+\psi^2 &= z.
\end{aligned}$$

The solution coincide with $c_\sigma^i = \psi^i(z_\sigma,\mu)$ and $v_\sigma = \phi(\mu)$ in all states where

$$u_1\left(\psi^1\left(z_\sigma,\mu\right)\right)+\beta Q(\phi(\mu)) \geq u_1\left(z_\sigma^1+t_s\right)+\beta\underline{v}_1, \tag{42}$$

$$u_2\left(\psi^2\left(z_\sigma,\mu\right)\right)+\beta\phi(\mu) \geq u_2\left(z_\sigma-z_\sigma^1-t_s\right)+\beta\underline{v}_2. \tag{43}$$

The LHS of the first condition decreases with μ while the LHS of the second condition increases with μ. Thus there exists $\bar{\mu}\left(z_\sigma,z_\sigma^1+t_s\right)$ and $\underline{\mu}\left(z_\sigma,z_\sigma^1+t_s\right)$ such that the two conditions are verified if

$$\underline{\mu}\left(z_\sigma,z_\sigma^1+t_s\right) \leq \mu \leq \bar{\mu}\left(z_\sigma,z_\sigma^1+t_s\right).$$

Lemma 4 *$\underline{\mu}\left(z_\sigma,z_\sigma^1+t_s\right) < \bar{\mu}\left(z_\sigma,z_\sigma^1+t_s\right)$ or $\underline{\mu}\left(z_\sigma,z_\sigma^1+t_s\right) = \bar{\mu}\left(z_\sigma,z_\sigma^1+t_s\right) \in \{-\hat{P}'(\underline{v}_2), -\hat{P}'(\bar{v}^2)\}$.*

Proof. It suffices to notice that it is not possible that

$$\begin{aligned}
u_1\left(\psi^1\left(z_\sigma,\mu\right)\right)+\beta Q(\phi(\mu)) &\leq u_1\left(z_\sigma^1+t_s\right)+\beta\underline{v}_1, \\
u_2\left(\psi^2\left(z_\sigma,\mu\right)\right)+\beta\phi(\mu) &\leq u_2\left(z_\sigma-z_\sigma^1-t_s\right)+\beta\underline{v}_2,
\end{aligned}$$

given that $\psi^1+\psi^2 = z_\sigma$, $\phi(\mu)+Q(\phi(\mu)) > \underline{v}_1+\underline{v}_2$, $\phi(\mu) \geq \underline{v}_2$, $Q(\phi(\mu)) \geq \underline{v}_1$. ■

Moreover $\underline{\mu}\left(z_\sigma,z_\sigma^1+t_s\right)$ and $\bar{\mu}\left(z_\sigma,z_\sigma^1+t_s\right)$ decreases in their second argument as the RHS of (42) increases and the RHS of (43) decreases with $z_\sigma^1+t_s$.

Now suppose that $\mu \geq \bar{\mu}\left(z_\sigma,z_\sigma^1+t_s\right) > -\hat{P}'(\underline{v}_2)$. Then $\frac{u_1'(c_\sigma^1)}{u_2'(c_\sigma^2)} = -Q'(v_\sigma) = \bar{\mu}\left(z_\sigma,z_\sigma^1+t_s\right)$ verifies the first order conditions and thus is the solution.

Suppose that $\bar{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) = -\hat{P}'(\underline{v}_2)$. In this case $u_1\left(\psi^1\left(z_\sigma, \mu\right)\right) + \beta Q(\phi(\mu)) < u_1\left(z_\sigma^1 + t_s\right) + \beta \underline{v}_1$ for all μ. which implies that $c_\sigma^1 > \psi^1\left(z_\sigma, \mu\right)$. The solution verifies

$$\begin{aligned} u_1\left(c_\sigma^1\right) + \beta Q\left(v_\sigma\right) &= u_1\left(z_\sigma^1 + t_s\right) + \beta \underline{v}^1 \\ \frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} &< \mu \\ \frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} &= -Q'(v_\sigma) \text{ if } v_\sigma > \underline{v}_2 \\ \frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} &\leq -Q'\left(\underline{v}^2\right) \text{ if } v_\sigma = \underline{v}^2 \end{aligned}$$

But $\frac{u_1'(c_\sigma^1)}{u_2'(c_\sigma^2)} = -Q'(v_\sigma)$ is not possible as this would imply $u_1\left(\psi^1\left(z_\sigma, \hat{\mu}\right)\right) + \beta Q(\phi(\hat{\mu})) = u_1\left(z_\sigma^1 + t_s\right) + \beta \underline{v}_1$ for $\hat{\mu} = -Q'(v_\sigma)$. Thus we have

$$\begin{aligned} v_\sigma &= \underline{v}^2 \\ \frac{u_1'\left(c_\sigma^1\right)}{u_2'\left(c_\sigma^2\right)} &\leq -Q'\left(\underline{v}^2\right) \end{aligned}$$

The reverse holds for the threshold $\underline{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right)$.

B Proof of Proposition 2

The result follows from Milgrom and Shannon (1994), Theorem 4. Given the separability in t_s, $\Phi_s(\mu, t_s)$ is also quasi-supermodular in T. The following lemma shows that it also verifies the single crossing condition in $(T; \mu)$.

Lemma 5 $\frac{\partial \Phi_s(\mu, t_s)}{\partial t_s}$ *is non-increasing with μ, decreasing if at least one incentive constraint binds.*

Proof. From the envelop theorem, $\frac{\partial \Phi_s}{\partial t_s}$ is equal to :

$$E\left[\lambda_\sigma^2 u_2'\left(z_\sigma - z_\sigma^1 - t_s\right) - \lambda_\sigma^1 u_1'\left(z_\sigma^1 + t_s\right) \mid s\right]$$

Now

$$-\lambda_\sigma^1 = \inf\{1 - \frac{\mu}{\bar{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right)}, 0\} \text{ if } \bar{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) > -\hat{P}'(\underline{v}_2),$$

while

$$-\lambda_\sigma^1 = 1 - \mu \frac{u_2'(c_\sigma^2)}{u_1'(c_\sigma^1)} \text{ if } \bar{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) = -\hat{P}'(\underline{v}_2)$$

where $\frac{u_2'(c_\sigma^2)}{u_1'(c_\sigma^1)}$ is independent of μ (given by (37) and $v_\sigma = \underline{v}_2$).
Similarly

$$\lambda_\sigma^2 = \max\{\underline{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) - \mu, 0\} \text{ if } \underline{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) < -\hat{P}'(\bar{v}_2),$$

while

$$\lambda_2^1 = \frac{u_1'(c_\sigma^1)}{u_2'(c_\sigma^2)} - \mu \text{ if } \underline{\mu}\left(z_\sigma, z_\sigma^1 + t_s\right) = -\hat{P}'(\bar{v}_2)$$

where $\frac{u_1'(c_\sigma^1)}{u_2'(c_\sigma^2)}$ is independent of μ (given by (38) and $v_\sigma = \bar{v}_2$).
Both are non-increasing in μ, and decreasing if the constraint is binding. ■

C Extension to the Constrained Case

The extension parallels the development of Section 3 and we highlight differences only. Instead of equations (23) we assume that:

$$\begin{cases} \ln \underline{\mu}(r_{it}, y_{it}) = \underline{\mu}_0 r_{it} + y_{it}\underline{\mu}_y + \underline{\mu}_{vt} \\ \ln \overline{\mu}(r_{it}, y_{it}) = \ln \underline{\mu}(r_{it}, y_{it}) + \exp(\overline{\mu}_0 r_{it} + y_{it}\overline{\mu}_y + \overline{\mu}_{vt}) \end{cases} \tag{44}$$

and the constraint $\underline{\mu}(r_{it}, y_{it}) \leq \overline{\mu}(r_{it}, y_{it})$ is naturally satisfied.
Notice that we still have $\underline{\mu}_0 < 0$ but we lose the condition $\bar{\mu}_0 < 0$.
The arguments leading to equations (27) carry over. The lower bound is written as:

$$\underline{\phi}_{it} = \underline{\phi}_{it}^* + \underline{u}_{it}$$

while the upper bound is slightly modified and is:

$$\underline{\phi}_{it}^* + \underline{u}_{it} + \exp(\tilde{\phi}_{it} + \tilde{u}_{it})$$

where:

$$\tilde{\phi}_{it} = \overline{\mu}_0((\pi_{vt} - \pi_0\delta_{vt-1}) + \pi_0(\ln c_{it-1} - x_{it-1}\beta) + y_{it}\pi_y + z_{it}\pi_z) + y_{it}\overline{\mu}_y + \overline{\mu}_{vt}$$

$$\tilde{u}_{it} = \overline{\mu}_0\pi_0 \ln \tilde{\varepsilon}_{it} + \overline{\mu}_0\xi_{it}$$

and the consumption equation is modified accordingly.
The arguments leading to the consumption equation (30) carry over and we get:

$$\begin{aligned} E(\Delta \ln c_{it} \quad &| \quad w_{it}) = \Delta x_{it}\beta + \Delta\delta_{vt} \\ -E((\underline{\phi}_{it}^* + \underline{u}_{it})\mathbf{1}_{\{\underline{u}_{it} > -\underline{\phi}_{it}^*\}} \quad &| \quad w_{it}) \\ -E((\underline{\phi}_{it}^* + \underline{u}_{it} + \exp(\tilde{\phi}_{it} + \tilde{u}_{it}))\mathbf{1}_{\{\underline{u}_{it} < -(\underline{\phi}_{it}^* + \exp(\tilde{\phi}_{it} + \tilde{u}_{it}))\}} \quad &| \quad w_{it}) \end{aligned} \tag{45}$$

The derivatives of the last term are modified into:

$$\begin{aligned} &\int_{\underline{u}_{it} < -(\underline{\phi}_{it}^* + \exp(\tilde{\phi}_{it} + \tilde{u}_{it}))} (\frac{\partial}{\partial w_{it}} \underline{\phi}_{it}^*(w_{it}) + (\frac{\partial}{\partial w_{it}} \tilde{\phi}_{it}^*(w_{it})) \exp(\tilde{\phi}_{it} + \tilde{u}_{it})) dF(\underline{u}_{it}, \tilde{u}_{it}) \\ = \; & \frac{\partial}{\partial w_{it}} \phi_{it}^*(w_{it}). \int_{\underline{u}_{it} < -(\underline{\phi}_{it}^* + \exp(\tilde{\phi}_{it} + \tilde{u}_{it}))} dF(\underline{u}_{it}, \tilde{u}_{it}) \\ & + \frac{\partial}{\partial w_{it}} \tilde{\phi}_{it}^*(w_{it}). \int_{\underline{u}_{it} < -(\underline{\phi}_{it}^* + \exp(\tilde{\phi}_{it} + \tilde{u}_{it}))} \exp(\tilde{\phi}_{it} + \tilde{u}_{it}) dF(\underline{u}_{it}, \tilde{u}_{it}) \end{aligned}$$

As in the text, consider the derivative of (45):

$$\frac{\partial}{\partial w_{it}} E(\Delta \ln c_{it} \mid w_{it}) = \frac{\partial \Delta x_{it} \beta}{\partial w_{it}} - \frac{\partial \underline{\phi}_{it}^*}{\partial w_{it}} \underline{\Lambda} - \frac{\partial \tilde{\phi}_{it}^*}{\partial w_{it}} \tilde{\Lambda} \tag{46}$$

where:

$$\underline{\Lambda} = \int_{\underline{u}_{it} > -\underline{\phi}_{it}^*(w_{it})} dF(\underline{u}_{it}) + \int_{\underline{u}_{it} < -(\underline{\phi}_{it}^* + \exp(\tilde{\phi}_{it} + \tilde{u}_{it}))} dF(\underline{u}_{it}, \tilde{u}_{it})$$

$$\tilde{\Lambda} = \int_{\underline{u}_{it} < -(\underline{\phi}_{it}^* + \exp(\tilde{\phi}_{it} + \tilde{u}_{it}))} \exp(\tilde{\phi}_{it} + \tilde{u}_{it}) dF(\underline{u}_{it}, \tilde{u}_{it})$$

The average derivative equation then becomes:

$$D^w \equiv E(\frac{\partial}{\partial w_{it}} E(\Delta \ln c_{it} \mid w_{it})) = \frac{\partial \Delta x_{it} \beta}{\partial w_{it}} - \frac{\partial \underline{\phi}_{it}^*}{\partial w_{it}} \underline{\Lambda} - \frac{\partial \tilde{\phi}_{it}}{\partial w_{it}} \tilde{\Lambda} \tag{47}$$

and the derivatives are given in the following table ($\frac{\partial \underline{\phi}_{it}^*}{\partial w_{it}}$ remains the same):

w_{it}	$\frac{\partial \tilde{\phi}_{it}}{\partial w_{it}}$
$\ln c_{it-1}$	$\overline{\mu}_0 \pi_0$
x_{it-1}	$-\beta \overline{\mu}_0 \pi_0 + \pi_x \overline{\mu}_0 + \overline{\mu}_x$
Δx_{it}	$\pi_x \overline{\mu}_0 + \overline{\mu}_x$
q_{it}	$\pi_y \overline{\mu}_0 + \overline{\mu}_q$
z_{it}	$\pi_z \overline{\mu}_0$

Only the form of the first two average derivatives are affected by imposing constraints. The restriction:

$$D^x = -\beta D^c + D^{\Delta x} + \beta$$

remains unaffected.

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