Optimal Risk-sharing under Adverse Selection and Imperfect Risk Perception*

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Abstract

The present paper thoroughly explores second-best efficient allocations in an insurance economy with adverse selection. We start with a natural extension of the classical model, assuming less than perfect risk perception. We characterize the constraints on efficient redistribution, and we summarize the incidence of incentive constraints on the economy with the notions of weak and strong adverse selection. Finally, we show in what sense improving risk perception enhances welfare.

Keywords : Principal-agent, adverse selection, twofold asymmetric information, welfare theorems, second-best optimum.

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1 Introduction

A typical situation where the risk perception of the insurer and that of policyholders differ is one in which each party knows something that the other does not. The insurer may correctly assess the impact on risk of an individual's characteristics without observing them all, whereas policyholders may know all their characteristics without relating them correctly to their risks. These simple and realistic assumptions inevitably introduce imperfect risk perception and adverse selection.

In articles combining adverse selection with nonexpected utility, Young and Browne (2001) and Jeleva and Villeneuve (2004) study, respectively, the Rothschild-Stiglitz equilibrium and the monopolist's problem. Though the modeling is similar, our objective is wider: rather than focusing on particular allocations, we study the whole set of optimal risk sharing in a context that we interpret as twofold asymmetric information.

Redistribution possibilities and insurance quality under the informational constraints are first characterized. We show that the set of feasible redistributions is a convex subset of the redistributions that a first best economy would allow, and that the set of second-best efficient redistributions is a convex subset of all feasible redistributions. In other terms, the social planner should limit itself to moderate redistribution. Due to imperfect risk perception, in general none of the types gets full insurance. More precisely, we show that the type which values most (respectively least) insurance is increasingly overcovered (respectively undercovered) as wealth transfers in his favor increase. This generalizes results by Crocker and Snow (1985) and Dionne and Fombaron (1996) to the context of imperfect risk perception.

Risk perception biases are the critical factors in determining the nature of risk sharing. We distinguish between *weak* and *strong* adverse selection. The former occurs when agents overstate the difference between types: they tend not to envy the others' optimal insurance and the economy admits a continuum of undistorted incentive compatible allocations. The latter occurs when agents underestimate the differences between types: the weight of incentive constraints is maximal and there is a continuum of distorted pooling allocations. In a model where policyholders differ by their risk aversions and costs of effort, de Meza and Webb (2001) find inefficient equilibrium pooling. They can solve partially this problem with appropriate taxation, but the constrained efficient allocations and the means of implementing them are not explored.

Finally, we question the effects of improving risk perception. In the context of *pure* adverse selection à la Rothschild-Stiglitz (1976), reducing (statistically) the existing asymmetries of information through categorical discrimination enhances welfare (Crocker and Snow, 1986). The intuition is that a fraction of the costs that incentive compatibility constraints impose on the mechanism can be economized. Our message is quite different. In a Bayesian setting, an improved understanding of risk on the part of policyholders typically corresponds to polarization in some segments, and to depolarization in others. But only the polarization of beliefs facilitates redistribution and guarantees welfare gains for all the segment, the argument being that, for given risks, more different tastes soften the impact of self-selection constraints. We discuss the ambiguity of the impact of public information campaigns when they are not accompanied with appropriate transfers.

Our departure from the assumption that policyholders are better informed raises two issues. The first is their resistance to learning their risk. The second is the objective of the social planner: should it maximize utilities as actually perceived by the consumers or maximize utilities calculated with true probabilities? These issues are addressed in turn.

Resistance to learning. The causes of adverse selection are well known, but why doesn't the consumer infer his type from the contracts he is offered? The sophisticated consumer would think: "if I prefer an offer which is seen by the insurer as appropriate to a certain type, I should infer that I have this type and therefore improve my probability assessment" (Villeneuve, forthcoming).

We assume away this possibility. The objective of this paper is to analyze the situation where policyholders are not able to reconstruct the reasoning of the insurer. Deducting one's unknown characteristics requires an unlikely knowledge of the composition (types and proportions) of the pool one belongs with. For example, a menu may redistribution wealth between policyholders; in that case, the insurance premium of the contract one prefers is not actuarially fair and its interpretation is ambiguous. Moreover, if the consumer fails to observe which offers are taken by some other consumers, he may attach importance to contracts that are never chosen in actuality. In sum, the policyholder lacks the key parameters that meaningful inference demands.

Welfare. In front of consumers that somewhat err in their risk assessment, the social planner faces a dilemma: should consumers' preferences be taken as they are or as they should be? Whatever the choice, some insurance can be provided, though coverage may be less than perfect. The ex ante Pareto optimum (EA) amounts to taking consumers' preferences as they are at the moment of choice, i.e. based on subjective probabilities. The ex post Pareto optimum (EP) is evaluated with the true distribution of loss ex post, which amounts to considering consumers' preferences as they should be. These two concepts disagree in general.

An EA is decentralizable (after appropriate redistribution) since competitive insurers base their strategies on the actual (not the ideal) preferences of the consumers. By contrast, an EP program is implementable only via centralized provision, which is a political and practical disadvantage. For this reason, EA is privileged in the paper. A comparison between EA and EP will be given in the case of strong adverse selection.

Organization of the paper. Section 2 sets up the insurance model with subjective belief and adverse selection. Section 3 explores the whole set of constrained Pareto optima for given objective probabilities and risk perceptions. Section 4 presents the comparative statics with respect to risk perception.

2 Model

2.1 Consumers, risk and insurance

Throughout the paper, we consider a unique benevolent insurer (hereafter "the insurer") in charge of implementing the constrained Pareto optima that the social planner chooses. The insurer is assumed to be risk neutral and is constrained to make no losses.

There is a continuum of two types of consumers i and j in proportions λ_i and λ_j respectively ($\lambda_i + \lambda_j = 1$), and one commodity in the economy. Each consumer faces an individual risk, with two individual states $s = 1$ (no loss) and $s = 2$ (loss).

The objective probability p_i of being in state 2 (loss) for a type-i individual is statistically known to the insurer ($p_i \neq p_j$). Individual risks are assumed to be independently distributed, and both types have the same initial contingent endowment $\omega = (\omega_1, \omega_2) \in \mathbb{R}_+^{*2}$. We suppose that consumers evaluate "expected" utility with the same VNM utility function u defined over $\mathbb{R}_+^*$. However, they use different subjective probabilities (q_i and q_j, respectively). We do not assume that q_i and q_j are ranked like p_i and p_j.

We propose a Bayesian interpretation of the discrepancy between risk assessments. There are two risk factors: one is privately observed by policyholders and takes one of two possible values, i and j, the other is privately observed by the insurer and takes one of two possible values, a and b. In a given insurance segment (say policyholders bearing marker a), there are two "types" (i, a) and (j, a), whose loss probabilities are perceived differently by the parties. If we drop the segment marker, we retrieve p_i, p_j for the insurer and q_i, q_j for the policyholders, all these parameters being conditional probabilities.

This is not restrictive for the understanding of optimal risk-sharing. Indeed, optimal risk-sharing is decomposable into two dimensions: within a segment (a or b), and between segments (a to b or the other way around). For the social planner, the latter is trivial, since, by definition, segments are based on the insurer's information. This paper develops the former dimension.

With the Bayesian interpretation, there are overall restrictions on the subjective probabilities but to integrate interpretations other than the Bayesian one, we have chosen to keep the four parameters (p_i, p_j, q_i, q_j) free. The assumption that policyholders do not revise their beliefs as they see the contracts they are offered denies common knowledge. Our arguments are in the Introduction.

2.2 Contracts and type-efficiency

Insurance contracts consist in an exchange, by the policyholder, of risk ω for a conditional consumption plan $x = (x_1, x_2)$. As in Prescott and Townsend (1984), x_1 and x_2 might be lotteries. This approach is more general and many proofs are simplified. Indeed, the decision variables of the insurer are now a finite number of *probability* *distributions* over the consumption set (a pair of contracts here is a quadruple of distributions). The objective, the choice sets and the feasibility constraints (incentive, profit) are linear with respect to these variables. Linear programming results, like uniqueness or continuity with respect to exogenous parameters, can be invoked (see also Landsberger and Meilijson, 1999). Lotteries do not seem to be observed empirically. Accordingly, Proposition 1 shows that for optimal allocations (constrained or not), contracts are always "degenerate".

Given a contract x, the insurer's net profit $\pi_k(x)$ depends on the consumer's type:

$$\pi_k(x) = (1 - p_k)\,(\omega_1 - Ex_1) + p_k\;(\omega_2 - Ex_2), \forall k = i, j, \tag{1}$$

and the consumer's utility is

$$u_k(x) = (1 - q_k)\,Eu(x_1) + q_k\;Eu(x_2), \forall k = i, j. \tag{2}$$

The expectation operator E only recalls that lotteries are allowed.

We define the *coverage rate* of a deterministic contract x by

$$c(x) = \frac{u'(x_1)}{u'(x_2)}. \tag{3}$$

Full insurance means a coverage rate of 1, underinsurance a coverage rate of less than 1 and overinsurance a coverage rate of more than 1. The curve of contracts ensuring a constant coverage is an income expansion path.

In any unconstrained Pareto optimal allocation $(\overline{x}_i, \overline{x}_j)$, no lotteries are used and each type's marginal rate of substitution is equal to that of the insurer:

$$c(\overline{x}_k) = \overline{c}_k = \frac{q_k}{1 - q_k} \cdot \frac{1 - p_k}{p_k}. \tag{4}$$

A contract $\overline{x}_k$ satisfying the above condition is said to be *k-efficient*, or simply type-efficient in the absence of ambiguity. The related coverage rate is denoted by $\overline{c}_k$.

Type efficiency does not mean full insurance when objective probability and beliefs differ. An *optimistic* consumer ($q_k < p_k$) has an optimal coverage strictly lower than the full coverage rate ($\overline{c}_k < 1$), and the rate of coverage is higher than 1 for a *pessimistic* consumer ($\overline{c}_k > 1$).

2.3 Feasible allocations and redistribution profile

In the situation of adverse selection that we assume, implementing Pareto optimal allocations (type-efficient contracts and no profit overall) is impossible in general, though there are important exceptions. Indeed, a pair of efficient contracts $x_\bullet = (\overline{x}_i, \overline{x}_j)$ is likely to violate one (or more) incentive compatibility constraints (Rothschild and Stiglitz, 1976).

We apply the revelation principle to reason directly on *menus.* We have indeed a classical principal-agent structure. Any allocation that can be implemented by some mechanism can also be implemented via a direct mechanism in which consumers are offered a menu of two contracts. We denote by $\mathcal{F}$ the set of *feasible menus,* i.e. menus that are incentive compatible and that satisfy the resource constraint:

$$x_\bullet \in \mathcal{F} \Leftrightarrow \begin{cases} \lambda_i \ \pi_i(x_i) + \lambda_j \ \pi_j(x_j) \ \geq 0, \\ \ u_i(x_i) \ \ \geq \ u_i(x_j), \\ \ u_j(x_j) \ \ \geq \ u_j(x_i). \end{cases} \tag{5}$$

The redistribution of expected wealth is parametrized by the profit profile $\pi_\bullet = (\pi_i, \pi_j)$. For any $\pi_\bullet$, we define

$$\mathcal{F}_{\pi_\bullet} = \{(x_i, x_j) | \pi_i(x_i) \geq \pi_i \ ; \ \pi_j(x_j) \geq \pi_j \ ; \ u_i(x_i) \geq u_i(x_j) \ ; \ u_j(x_j) \geq u_j(x_i)\} \tag{6}$$

as the set of menus for which profit profile $\pi_\bullet$ is feasible. All sets $\mathcal{F}_{\pi_\bullet}$ or $\mathcal{F}$ comprise quadruples of probability distributions. Constraints being linear, these sets are linear and convex. By linear of u with respect to probabilities, the set of feasible payoffs $u(\mathcal{F})$ is convex.

A profit profile making zero profit is called a *redistribution* profile. The set of feasible redistribution profiles, which is denoted by Π, is a segment (a convex, bounded, one-dimensional set in $\mathbb{R}^2$).

3 Welfare analysis of transfers

3.1 Redistribution constrained optima

The second fundamental theorem of welfare states that any redistribution is compatible with efficiency, provided that the Walrasian market mechanism determines the allocation. The following definition will serve to show how second-best economies depart from first-best economies. Pareto dominance is envisaged in terms of ex ante welfare (EA in the Introduction).

Definition 1 (RCO) $x_\bullet$ *is a redistribution constrained optimum (RCO) relative to profit profile* $\pi_\bullet$ *if it is not Pareto-dominated in* $\mathcal{F}_{\pi_\bullet}$.

Under *symmetric* information, an RCO is always a Pareto optimum. Under asymmetric information, this concept of efficiency is weaker than second-best optimality, since we ignore for the moment whether the profit profile we consider is compatible or not with second-best efficiency.

The proposition shows the relationships between the redistribution profiles, the set of RCOs and the frontier of the set of implementable payoffs.

Proposition 1 *Under adverse selection,*

1. *The RCO related to some feasible profit profile* $\pi_\bullet$ *is unique; the contracts supporting it are degenerate, they Pareto-dominate all the menus in* $\mathcal{F}_{\pi_\bullet}$; *the budget constraints by type are both binding;*

2. *The application which associates to any feasible redistribution profile the unique related RCO,* $\Pi \to \mathcal{F}, \pi_\bullet \mapsto \widehat{x}_\bullet = (\widehat{x}_i, \widehat{x}_j)$, *is continuous;*

3. *The application which associates to any feasible redistribution profile the utilities of the types at the related RCO,* $\Pi \to u(\mathcal{F}), \pi_\bullet \mapsto (u(\widehat{x}_i), u(\widehat{x}_j))$, *is one-to-one and its image is a continuous portion of the frontier of* $u(\mathcal{F})$.

INSERT FIGURE 1 ABOUT HERE

It is never socially desirable that the insurer retains positive profit (first point). The rest of the paper works with redistribution profiles and their unique RCOs (second point). Efficient RCOs are on the North-East frontier of $u(\mathcal{F})$. The corresponding redistribution profiles are said to be *efficient* and they form a convex subset of Π (third point). When RCOs are sought via weighted sums of the types' utilities, the greater the weight assigned to a type, the greater the expected wealth this type receives. Inefficient RCOs correspond to negative weights given to one of the types and extreme transfers.

The Rothschild-Stiglitz allocation, that is the unique candidate equilibrium in the standard model, is in fact the RCO associated with the no-redistribution profile. For the very reason that an implementable redistribution profile may not be efficient, the Rothschild-Stiglitz allocation may not be a second-best optimum.

3.2 Redistribution and coverage

The critical question with second-best optima is whether or not types are efficiently insured. Next proposition shows that the type whose expected wealth is low gets a type-efficient contract at the RCO.

Proposition 2 *Consider a redistribution profile in* Π *and the corresponding RCO* $(\widehat{x}_i, \widehat{x}_j)$*,*

1. *If type j's incentive constraint is not binding,* $\widehat{x}_i$ *is i-efficient.*
2. *If* $\widehat{x}_i$ *is i-efficient, then type i's contract remains i-efficient at the RCO when more wealth is transferred from type i to type j.*

A direct corollary is that there are two thresholds in redistribution levels, each separating, for a given type, RCOs assigning type-efficient contracts from RCOs assigning type-inefficient contracts.

In the standard model ($q_i = p_i$ and $q_j = p_j$), the two thresholds are identical, and for this particular redistribution, all types are fully insured at the average price. For any other redistribution profile, the type that receives low transfers is assigned an efficient contract, not the other.

In our more general setting, we retrieve this idea for relatively low and relatively high transfers. However, for intermediate transfers (i.e. between the two thresholds), RCOs assign type-efficient contracts either to both types or to neither. This important difference with the Rothschild-Stiglitz model that we find is explored in more detail in Section 4. We show there how it relates with the biases of risk perception.

Coverage varies with the expected wealth allocated to a type. The simplest fact is that any pair of contracts that satisfies incentive constraints is such that the type that values coverage more (i.e. with the highest subjective loss probability) gets more coverage. The proposition goes further.

Proposition 3 *At the RCO,*

1. *The coverage rate of the type which values coverage more (resp. less) is greater (resp. smaller) than this type's optimal coverage rate.*

2. *The coverage rate of the type which values coverage more (resp. less) increases (resp. decreases) with the expected wealth this type receives.*

In the Rothschild-Stiglitz model, the first point means that the high risk is fully insured for a small expected wealth, but that this type receives overinsurance if transfers overpass those implicit in the average actuarially fair full insurance (Dionne and Fombaron, 1996). The only way by which one can implement such high transfers is by providing overinsurance that low risk policyholders value less.

The second point goes further in the comparative statics. The intuition is simple but requires a careful proof. Increasing transfers increases the weight of incentive constraints: it becomes increasingly difficult to discourage the disadvantaged type from choosing the advantaged type's contract. The increasingly generous contract has to be increasingly distorted away from the coverage quality the envious type likes most. This causes the inefficiency of extreme transfers: at some point, the marginal distortion (degraded quality) becomes too costly compared with the benefit of the marginal increase of expected consumption.

4 The effects of risk perception

4.1 Weak and strong adverse selection

Under adverse selection, significant redistribution causes envy and therefore distortions are necessary to circumvent it. What happens with moderate redistribution? What it the minimal distortion one can expect in an economy? The economy is said to exhibit *weak adverse selection* when the intersection of first-best efficient allocations and second-best allocations is non-empty. It exhibits *strong adverse selection* when the intersection is empty, i.e. when envy always restricts efficiency. These qualitative properties critically depend on risk perception.

Proposition 4 *The economy exhibits weak (strong) adverse selection if and only if subjective accident probability and optimal coverage are positively (negatively) correlated* $(q_i - q_j) \cdot (\bar{c}_i - \bar{c}_j) \geq (<) 0$.

Strong adverse selection corresponds to the situation in which there is a contradiction between first-best requirements (e.g. $\bar{c}_i > \bar{c}_j$: type i *should* be more covered than type j) and feasibility constraints (e.g. $q_i < q_j$: type i *will* be less covered than type j). This excludes that both types receive type-efficient contracts at the same time. This is an instance of the phenomenon that Guesnerie and Laffont (1984) name nonresponsiveness.

Fix the objective probabilities with $p_i > p_j$. Figure 2 is the phase diagram of the model when q_i and q_j vary from 0 to 1. Between the frontiers $q_i = q_j$ and $\bar{c}_i = \bar{c}_j$,[1] the economy exhibits strong adverse selection, outside, it exhibits weak adverse selection. Strong adverse selection is met under two conditions: risk perceptions are relatively close, and they are positively correlated with true probabilities. The Rothschild-Stiglitz model is represented by the unique point $q_i = p_i$ and $q_j = p_j$. In any neighborhood of that economy, strong and weak adverse selection are possible.

INSERT FIGURE 2 ABOUT HERE

For intermediate redistribution, contracts are either both type-efficient or type-inefficient (a corollary of Proposition 2). Under weak adverse selection, intermediate RCOs assign type-efficient contracts (no adverse effects of adverse selection), while under strong adverse selection, intermediate RCOs assign a pooling, i.e. a unique contract which is type-inefficient for both types.

In the particular but significant case where types do not perceive their difference ($q_i = q_j$), there is a unique second-best optimum.[2] Each type is assigned a type-efficient contract, since the economy exhibiting weak adverse selection, this unique allocation is necessarily a first-best optimum. The two contracts in the menu are

[1]Notice that $\bar{c}_i(q_i, q_j) = \bar{c}_j(q_i, q_j)$ is a section of an ellipse passing through (0,0), (p_i, p_j) and (1,1). In factorized (non-polynomial) form, the equation is indeed

$$\left(\frac{q_i}{1-q_i}\right)\left(\frac{1-p_i}{p_i}\right) - \left(\frac{q_j}{1-q_j}\right)\left(\frac{1-p_j}{p_j}\right) = 0. \tag{7}$$

[2]Incentive compatibility imposes that the two types receive the same utility in a given RCO. The RCO that provides maximum utility is the unique second-best optimum.

equivalent for both types but, in equilibrium, the right type must choose the right contract. The implementability of this allocation depends on the ability of the insurer to coordinate policyholders on the appropriate choices.[3]

The two objectives (EA and EP) discussed in the Introduction are sometimes reconciled. With strong adverse selection and intermediate redistribution, incentive constraints command and types are pooled: the objective of the social planner is locally irrelevant. By contrast, under weak adverse selection, EA and EP always disagree.

4.2 Feasibility and efficiency with polarized beliefs

Intuitively, differences in tastes facilitate the implementation of different contracts since envy-free conditions are easier to satisfy. Interpreted in terms of risk perception, this idea suggests that, other things equal, increasing the *disparity* between beliefs alleviates incentive constraints. In this section, we consider changes of the consumers beliefs, without affecting the objective parameters p_i and p_j.

Definition 2 *Consider beliefs $\mathcal{Q} = (q_i, q_j)$. Beliefs $\mathcal{Q}^e = (q_i^e, q_j^e)$ are a* polarization *of $\mathcal{Q}$ if, when $q_i > q_j$ then $q_i^e \geq q_i$ and $q_j \geq q_j^e$ (with at least one strict inequality).*

The contrary of polarization is depolarization.

Theorem 1 *Let beliefs $\mathcal{Q}^e$ be a polarization of beliefs $\mathcal{Q}$.*

1. *The set of* feasible *menus associated with $\mathcal{Q}^e$ is greater than the one associated with $\mathcal{Q}$;*

2. *The set of transfers associated with $\mathcal{Q}^e$ such that type i gets an i-efficient contract at the RCO is greater than the one associated with $\mathcal{Q}$;*

3. *The set of* efficient *transfers associated with $\mathcal{Q}^e$ is greater than the one associated with $\mathcal{Q}$.*

We come back to the Bayesian interpretation of the model and show the ambiguous effects of information sharing. Assume that segments a and b are such that $p_{ja} = p_{jb} =$

[3] When beliefs differ, the issue is less disturbing since, for all $\varepsilon > 0$, an ε-optimum, with strong preference for their contracts on the part of the types, always exists.

$q_j = p_j$ (type j is not affected by the factor the insurer observes) but $p_{ia} > q_i > p_{ib}$ (being a is bad news and being b is good news for type-i).

Should the insurer disclose the risk factor? In segment a, this implies passing from risk perceptions (q_i, q_j) to risk perceptions (p_{ia}, p_j). This is a polarization only if $q_i > q_j$. Conversely, disclosing the risk factor in segment b is a polarization only if $q_i < q_j$. In other words, disclosing the information cannot improve welfare, in the sense of the Theorem, in both groups.

Theoretically, limiting the transmission of information to the well chosen segment could be welfare improving: if $q_i > q_j$, "say bad news, never say goods news" (tell a, not b); if $q_i < q_j$, "say good news, never say bad news" (tell b, not a). In practice, targeting a or b might be unfeasible and the open question now is whether a public information campaign associated with compensatory transfers between segments enhances welfare.

5 Conclusion

The possible inefficiency in the Rothschild and Stiglitz model hinges on the market's inability to perform transfers between types. To overcome this failure, the simplest policy is to choose the optimum one wants to implement, then to impose the basic uniform coverage performing the desired redistribution, and finally to leave the market reach the Rothschild-Stiglitz equilibrium (Crocker and Snow, 1985).

Previous results on redistribution remained unclear as to the degrees of freedom left for public choice (Dahlby 1981, Crocker and Snow 1985). The first contribution of this paper is to prove that second-best allocations are confined to a convex set of redistribution profiles. If redistribution goes further, the allocation becomes inefficient, and if it goes even further, it becomes unimplementable. We indicate how incentive constraints, through risk perception and derived tastes, distort the quality of insurance: the greater the expected wealth a type receives, the lesser the quality of coverage this type is assigned.

The second contribution is to find that for a large set of parameters, pooling types is second-best efficient. In the case of strong adverse selection, for a convex set of transfers, *none* of the types obtains a type-efficient contract. This contrasts with the original Rothschild-Stiglitz economy, in which the only efficient pooling is the average fair full insurance. Our model permits degrees of freedom in risk perception. We show that if for some parameters a redistribution profile is efficient, then it remains so as risk perception polarizes. Said differently, if for some initial endowment, the Rothschild-Stiglitz equilibrium exists, existence is not lost by polarization.

Last but not least, this paper proposes a Bayesian interpretation of the disparity between risk perception and true probabilities in terms of two-sided asymmetric information. We show that when the insurer cannot observe the information policyholders possess, information transmission from the insurer to policyholders has ambiguous effects. We propose an important example in which, in one market segment, efficiency requires risk perception improvement, while in the other segment, information sharing hardens the incentive constraints. The original criterion we propose to evaluate efficiency gains (more efficient redistribution, more contracts) deserves development.

A Appendix

Technical note. In the proofs, we adopt the weak topology for lotteries, but to simplify, we never write the restriction "almost surely". Two lotteries are considered as equal if their consequences differ only for events of null probability.

A.1 Proof of Proposition 1

The maximum element in $\mathcal{F}_{\pi_\bullet}$. Fix $\pi_\bullet$ and suppose that $\mathcal{F}_{\pi_\bullet}$ is non-empty. Define $\mathcal{C}_{\pi_\bullet}$ as the set of contracts appearing in some menu of $\mathcal{F}_{\pi_\bullet}$ ($x_\bullet = (x_i, x_j) \in \mathcal{F}_{\pi_\bullet} \Rightarrow x_i \in \mathcal{C}_{\pi_\bullet};\ x_j \in \mathcal{C}_{\pi_\bullet}$). Define $x_k^M \in \arg\max_{x \in \mathcal{C}_{\pi_\bullet}} u_k(x)$ for $k = i, j$. By continuity of u, $\mathcal{C}_{\pi_\bullet}$ is closed, therefore x_i^M and x_j^M are in $\mathcal{C}_{\pi_\bullet}$. There is a contract $X_i \in \mathcal{C}_{\pi_\bullet}$ such that $u_i(X_i) \geq u_i(x_i^M)$ and $\pi_i(X_i) \geq \pi_i$ (possibly, $X_i = x_i^M$). Similarly, there is a contract $X_j \in \mathcal{C}_{\pi_\bullet}$ such that $u_j(X_j) \geq u_j(x_j^M)$ and $\pi_j(X_j) \geq \pi_j$ (possibly, $X_j = x_j^M$). Moreover, x_i^M and x_j^M are such that $u_i(x_i^M) \geq u_i(X_j)$ and $u_j(x_j^M) \geq u_j(X_i)$. The preceding conditions imply that menu $(X_i, X_j) \in \mathcal{F}_{\pi_\bullet}$ dominates (weakly) any other menu of $\mathcal{F}_{\pi_\bullet}$, and $u_k(X_k) = u_k(x_k^M)$. This implies that there is at least one maximum element of $\mathcal{F}_{\pi_\bullet}$ which is, necessarily, an RCO.

Binding constraints. We prove that for RCO (X_i, X_j), profit constraints by type are binding. We reason by contradiction. Suppose that $\pi_i(X_i) > \pi_i$. The components of X_i are denoted by $\tilde{x}_1$ and $\tilde{x}_2$, which are lotteries a priori ($X_i = (\tilde{x}_1, \tilde{x}_2)$). As u is concave, the degenerate lottery $(u^{-1}(Eu(\tilde{x}_1)), u^{-1}(Eu(\tilde{x}_2)))$, instead of X_i, implements the same payoffs for the types, but yields a larger profit than π_i. There is an open ball $\mathcal{B}$ around $(u^{-1}(Eu(x_1)), u^{-1}(Eu(x_2)))$ in which $\pi_i(\cdot) > \pi_i$. Now we define the (degenerate) contract $X_{\varepsilon,\eta} = (x_1, x_2)$ by the following equations:

$$u(x_1) = Eu(\tilde{x}_1) + \varepsilon, \tag{8}$$

$$u(x_2) = Eu(\tilde{x}_2) + \eta. \tag{9}$$

Profit functions being continuous, ε and η exist such that $X_{\varepsilon,\eta}$ is in $\mathcal{B}$ and verifies

$$(1 - q_i)\,\varepsilon + q_i\ \eta > 0, \tag{10}$$

$$(1 - q_j)\,\varepsilon + q_j\ \eta < 0. \tag{11}$$

It follows that $(X_{\varepsilon,\eta}, X_j)$ satisfies incentive constraints, belongs to $\mathcal{F}_{\pi_\bullet}$, and $u_i(X_{\varepsilon,\eta}) > u_i(X_i)$, a contradiction. Moreover, X_i is composed of degenerate lotteries, else $(X_{0,0}, X_j)$ would be a menu belonging to $\mathcal{F}_{\pi_\bullet}$, yielding the same utility as (X_i, X_j), which would verify $\pi_i(X_{0,0}) > \pi_i$, a contradiction.

Uniqueness of the RCO and continuity of the mapping. For a given redistribution profile $\pi_\bullet$, RCOs are unique in terms of utilities implemented, since they are all maximum elements of $\mathcal{F}_{\pi_\bullet}$. Therefore that they are all solution of the following program (the objective could be any other function increasing in u_i and u_j):

$$\begin{aligned}&\max_{x_i,x_j} u_i(x_i) + u_j(x_j) \\ &\text{s.t.} \quad u_i(x_i) \geq u_i(x_j); u_j(x_j) \geq u_j(x_i); \pi_i(x_i) \geq \pi_i; \pi_j(x_j) \geq \pi_j.\end{aligned} \tag{12}$$

The constraints and the objective are continuous with respect to $\pi_\bullet$ and the objective is never collinear to a constraint,[4] therefore the solution is necessarily at a corner. This implies that the solution is unique, and that the mapping which associates that solution to any feasible redistribution is continuous.

The application $\Pi \to \mathcal{U}, \pi_\bullet \mapsto u_\bullet$. Consider an RCO $\widehat{x}_\bullet$, associated with a redistribution profile in Π, whose payoffs are $\widehat{u}_\bullet = (\widehat{u}_i, \widehat{u}_j)$. We prove that $\widehat{u}_\bullet$ cannot be in the interior of $u(\mathcal{F})$. We reason by contradiction: assume that $\widehat{u}_\bullet$ has a neighborhood v in the interior of $u(\mathcal{F})$. Choose two points $(\widehat{u}_i, \widehat{u}_j + \varepsilon)$ and $(\widehat{u}_i + \eta, \widehat{u}_j)$ in v with $\varepsilon > 0$ and $\eta > 0$. We denote by $y_\bullet = (y_i, y_j)$ (resp. $z_\bullet = (z_i, z_j)$) a menu implementing $(\widehat{u}_i, \widehat{u}_j + \varepsilon)$ (resp. $(\widehat{u}_i + \eta, \widehat{u}_j)$).

One can readily see that $(\widehat{x}_i, y_j)$ and $(z_i, \widehat{x}_j)$ satisfy incentive constraints. The Pareto optimality of $(\widehat{x}_i, \widehat{x}_j)$ in $\mathcal{F}_{\pi_\bullet}$ implies that these pairs of contracts cannot belong to $\mathcal{F}_{\pi_\bullet}$, and we must conclude that:

$$\pi_j(y_j) < \pi_j(\widehat{x}_j), \tag{13}$$

$$\pi_i(z_i) < \pi_i(\widehat{x}_i). \tag{14}$$

[4]For the profit conditions, remark that, u being concave, expected value and expected utility are not collinear. For the incentive constraints, note that the two independent operators u_i and u_j are combined independently to generate the objective and the constraints.

Profit on $(\widehat{x}_i, \widehat{x}_j)$ being zero, this implies in turn that

$$\pi_i(y_i) \quad > \quad \pi_i(\widehat{x}_i), \tag{15}$$

$$\pi_j(z_j) \quad > \quad \pi_j(\widehat{x}_j). \tag{16}$$

Now consider the menu of lotteries (l_i^α, l_j^α) where, for $k = i, j$, l_k^α pays y_k with probability α and z_k with probability $1-\alpha$. Menu (l_i^α, l_j^α) belongs to $\mathcal{F}$ and Pareto dominates $(\widehat{x}_i, \widehat{x}_j)$; moreover, by continuity of profit functions, α_0 exists such that

$$\pi_i(l_i^{\alpha_0}) = \pi_i(\widehat{x}_i), \tag{17}$$

which implies that

$$\pi_j(l_j^{\alpha_0}) \geq \pi_j(\widehat{x}_j), \tag{18}$$

in contradiction with the fact that $(\widehat{x}_i, \widehat{x}_j)$ is a maximum element in $\mathcal{F}_{\pi_\bullet}$.

Finally we prove that the mapping $\Pi \to u(\mathcal{F}), \pi_\bullet \mapsto u_\bullet$ is one-to-one. Each redistribution profile corresponds to a unique RCO, and a unique element of $u(\mathcal{F})$. Assume that two RCOs $(\widehat{x}_i, \widehat{x}_j)$ and $(\widehat{y}_i, \widehat{y}_j)$ implement the same payoffs $(\widehat{u}_i, \widehat{u}_j)$. Without loss of generality, suppose that $\pi_i(\widehat{x}_i) > \pi_i(\widehat{y}_i)$. This condition implies that $(\widehat{x}_i, \widehat{y}_j)$ implements the same utility as the RCOs, is feasible and makes *strictly* positive profits. This is impossible (Proposition 1/1).

A.2 Proof of Proposition 2

Point 1. When type j's incentive constraint is not binding, any possibility to improve type i's coverage is exploitable (Proposition 1/1), thus type i gets an i-efficient contract.

Point 2. Denote by $\pi_\bullet^1$ and $\pi_\bullet^2$ two redistribution profiles such that $\pi_i^1 < \pi_i^2$ ($\pi_\bullet^1$ is more favorable to i than $\pi_\bullet^2$). Assume that $\widehat{x}_i(\pi_\bullet^1)$ is i-efficient (we denote it $\overline{x}_i^1$). We prove that $\overline{x}_i(\pi_\bullet^2)$ (or $\overline{x}_i^2$) is implementable, which, with Proposition 1, implies that $\widehat{x}_i(\pi_\bullet^2) = \overline{x}_i^2$.

Assume that type i's incentive constraint is binding at $\widehat{x}_\bullet(\pi_\bullet^1)$. Denote by c_j^1 the coverage rate of $\widehat{x}_j(\pi_\bullet^1)$. Denote by x_j^2 the contract whose coverage rate is c_j^1 and which gives the same utility to i as $\overline{x}_i^2$. The single crossing condition imposes that since type j prefers $\widehat{x}_j(\pi^1_\bullet)$ to $\widehat{x}_i(\pi^1_\bullet)$, then type j also prefers x^2_j to $\overline{x}^2_i$. Menu $(\overline{x}^2_i, x^2_j)$ is incentive compatible. Remark also that this menu offers more profitable contracts to both types than $\widehat{x}_\bullet(\pi^1_\bullet)$ (smaller value, for the same coverage rates). We conclude that $\overline{x}^2_i$ is implementable.

As long as type i's incentive constraint is not binding at $\widehat{x}_\bullet(\pi^1_\bullet)$, then one can increase profits on that type without losing i-efficiency (which is what the proposition says). Once the incentive constraint starts to be binding, the paragraph above can be applied.

A.3 Proof of Proposition 3

To fix ideas, we suppose in this proof that $q_j \geq q_i$.

Point 1. We reason by contradiction. Suppose that, given the redistribution profile $\pi_\bullet$, the RCO $(\widehat{x}_i, \widehat{x}_j)$ is such that type j's coverage c_j is strictly smaller than $\overline{c}_j$. Clearly, type j strictly prefers the corresponding j-efficient contract $\overline{x}_j$ to $\widehat{x}_j$. Consider then contract y_j which coverage equals $\overline{c}_j$ and that gives to type j the same utility as $\widehat{x}_j$. Obviously, this contract is less expensive than $\overline{x}_j$ and $\widehat{x}_j$. Moreover, as type j weakly prefers $\widehat{x}_j$ to y_j whenever $c(\widehat{x}_j) < c(y_j)$, single crossing conditions implies that it is also the case for type i $(u_i(\widehat{x}_j) \geq u_i(y_j))$. It follows that menu $(\widehat{x}_i, y_j)$ is feasible; however, it gives the same utility as the RCO and belongs to $\mathcal{F}_{\pi_\bullet}$; this is in contradiction with the uniqueness result of Proposition 1.

Point 2. The following remark is instrumental. Call OI (for overinsurance) the set of contracts whose coverage rates are greater or equal to $\overline{c}_j$. Take two contracts in OI, if the one with the greatest expected wealth for j has the lowest coverage rate, then it is the one preferred by i and j.

Consider two RCOs $(\widehat{x}_i, \widehat{x}_j)$ and $(\widehat{z}_i, \widehat{z}_j)$ such that type j's expected wealth is greater with $\widehat{z}_j$. We reason by contradiction. Suppose that $c(\widehat{z}_j) < c(\widehat{x}_j)$.

1. Proposition 3/1 implies that $c(\widehat{z}_j) \geq \overline{c}_j$; but $c(\widehat{x}_j) > c(\widehat{z}_j)$ and thus $c(\widehat{x}_j) > \overline{c}_j$. We conclude that $\widehat{x}_j$ is not j-efficient and $u_i(\widehat{x}_i) = u_i(\widehat{x}_j)$.

2. Remark also that $\widehat{z}_j$ gives more expected wealth to type j than $\widehat{x}_j$, therefore Proposition 2/2 implies that $\widehat{z}_j$ is not j-efficient, and $u_i(\widehat{z}_j) = u_i(\widehat{z}_i)$.

3. $\widehat{z}_j$ is preferred to $\widehat{x}_j$ by both types, because contracts $\widehat{x}_j$ and $\widehat{z}_j$ belong to OI (Proposition 3/1) and the remark on OI above applies. We thus have

$$u_i(\widehat{x}_i) = u_i(\widehat{x}_j) < u_i(\widehat{z}_j) = u_i(\widehat{z}_i), \tag{19}$$

meaning that $\widehat{z}_i$ is preferred to contract $\widehat{x}_i$ by type i.

4. $\widehat{z}_i$, though less expensive, is preferred to $\widehat{x}_i$ by type i (equation 19). This implies that $\widehat{x}_i$ is not i-efficient.

5. $\widehat{x}_\bullet$ is a pooling ($\widehat{x}_i = \widehat{x}_j$), the two incentive constraints being binding (points 1 and 4).

6. $\overline{c}_i > \overline{c}_j$, because from Proposition 3/1, and from point 5, one knows that $\overline{c}_i \geq c(\widehat{x}_i) = c(\widehat{x}_j) > \overline{c}_j$.

7. $c(\widehat{z}_i) \leq c(\widehat{z}_j)$ (property of any menu), $c(\widehat{z}_j) < c(\widehat{x}_j)$ (by assumption), $c(\widehat{x}_j) = c(\widehat{x}_i)$ (point 5) and $c(\widehat{x}_i) \leq \overline{c}_i$ (Proposition 3/1). Consequently, $\widehat{z}_i$ is not i-efficient, and $u_j(\widehat{z}_j) = u_j(\widehat{z}_i)$.

8. $\widehat{z}_\bullet$ is a pooling ($\widehat{z}_i = \widehat{z}_j$), the two incentive constraints being binding (points 2 and 7).

9. $p_j < p_i$. Indeed, pooling $\widehat{x}_\bullet$ covers more than pooling $\widehat{z}_\bullet$, and type j's (type i's) expected wealth is smaller (resp. larger) with $\widehat{x}_\bullet$ than with $\widehat{z}_\bullet$;

10. $\overline{c}_j > \overline{c}_i$, since $p_j < p_i$ and $q_j > q_i$.

There is a contradiction between points 6 and 10.

A.4 Proof of Proposition 4

Under weak adverse selection, there is at least one transfer system such that both types get an efficient contract at the RCO, and then $c(\widehat{x}_i) = \overline{c}_i$ and $c(\widehat{x}_j) = \overline{c}_j$. This implies that $(q_i - q_j) \cdot (\overline{c}_i - \overline{c}_j) \geq 0$. To prove the reciprocal, there are two cases to be considered once, to fix ideas, we assume that $p_i > p_j$.

$q_i > q_j$ **and** $\overline{c}_i > \overline{c}_j$. Denote by x_I the contract at the intersection of the two curves of equations $c(x) = \overline{c}_i$ and $\lambda_i\ \pi_i(x) + \lambda_j\ \pi_j(x) = 0$. Consider redistribution profile $(\pi_i(x_I), \pi_j(x_I))$ and the RCO for this profile. Clearly $x_I = \overline{x}_i$ therefore $\widehat{x}_i = \overline{x}_i$ is i-efficient.

We apply the same argument for x_J, the contract at the intersection of the two curves of equations $c(x) = \overline{c}_j$ and $\lambda_i\ \pi_i(x) + \lambda_j\ \pi_j(x) = 0$. The corresponding redistribution profile assigns a j-efficient contract at the CPO to type-j policyholders. However, since $\overline{c}_i > \overline{c}_j$, the transfers implicitly defined by x_J are more favorable to type j than $(\pi_i(x_I), \pi_j(x_I))$. So from Proposition 2/2, we deduce that type-j policyholders also get a j-efficient contract at the RCO associated with $(\pi_i(x_I), \pi_j(x_I))$. Consequently $(\overline{x}_i, \overline{x}_j)$ is the RCO associated with this transfer. We are in a situation of weak adverse selection.

$q_i < q_j$ **and** $\overline{c}_i < \overline{c}_j$. We apply the intermediate value theorem to define implicitly a redistribution such that the associated i- and j-efficient contracts verify $u_i(\overline{x}_i) = u_i(\overline{x}_j)$. Given that $c(\overline{x}_i) < c(\overline{x}_j)$, it follows that $\overline{x}_{i1} > \overline{x}_{j1}$, and the single crossing property of the indifference curves with $q_i < q_j$ implies that $u_j(\overline{x}_i) < u_j(\overline{x}_j)$, which proves that $(\overline{x}_i, \overline{x}_j)$ is feasible. We are in a situation of weak adverse selection.

A.5 Proof of Theorem 1

Let $u_k^e(x)$ denote the indirect utility that type k with beliefs q_k^e draws from contract x.

A.5.1 Point 1

Lemma 1 *Let x and y be two contracts. If $u_i^e(y) \geq u_i^e(x)$ and $u_j^e(y) \geq u_j^e(x)$, with at least one strict inequality, then $u_i(y) \geq u_i(x)$ and $u_j(y) \geq u_j(x)$ with at least one strict inequality.*

Proof. This is a direct consequence of the single-crossing property.■

Suppose that (x_i, x_j) is a menu for parameters $(\mathcal{Q}, \pi_\bullet)$. Incentive constraints are satisfied $u_i(x_i) \geq u_i(x_j)$ and $u_j(x_i) \leq u_j(x_j)$. With Lemma 1, this implies that $u_i^e(x_i) \geq u_i^e(x_j)$ and $u_j^e(x_i) \leq u_j^e(x_j)$, meaning that incentive constraints relative to beliefs $\mathcal{Q}^e$ are verified. We conclude that (x_i, x_j) is a menu relative to parameters $(\mathcal{Q}^e, \pi_\bullet)$.

A.5.2 Point 2

Let $\pi_\bullet$ be a redistribution profile such that type i gets an i-efficient contract, $\overline{x}_i$, at the RCO with beliefs $\mathcal{Q}$ (the RCO is $(\overline{x}_i, \widehat{x}_j)$). We denote by $\overline{x}_i^e$ the i-efficient contract relative to parameters $(\mathcal{Q}^e, \pi_\bullet)$ (the RCO is $(\widehat{x}_i^e, \widehat{x}_j^e)$). We treat separately cases $q_i = q_i^e$ and $q_i \neq q_i^e$.

Case $q_i = q_i^e$. Point 1 of this theorem implies that $(\overline{x}_i, \widehat{x}_j)$ is a also menu for parameters $(\mathcal{Q}^e, \pi_\bullet)$. This menu is Pareto dominated by the RCO $(\widehat{x}_i^e, \widehat{x}_j^e)$, in particular $u_i^e(\widehat{x}_i^e) \geq u_i^e(\overline{x}_i)$. However, as $q_i = q_i^e$, $\overline{x}_i = \overline{x}_i^e$. We conclude that type i gets an i-efficient contract at the RCO relative to parameters $(\mathcal{Q}^e, \pi_\bullet)$.

Case $q_i \neq q_i^e$. Type-i agents with beliefs q_i and q_i^e prefer their own i-efficient contracts:

$$u_i^e(\overline{x}_i^e) \;>\; u_i^e(\overline{x}_i), \tag{20}$$

$$u_i(\overline{x}_i^e) \;<\; u_i(\overline{x}_i). \tag{21}$$

Then, it follows from Lemma 1 that

$$u_j^e(\overline{x}_i^e) < u_j^e(\overline{x}_i). \tag{22}$$

We know from point 1 of this theorem that menu $(\overline{x}_i, \widehat{x}_j)$ is feasible for parameters $(\mathcal{Q}^e, \pi_\bullet)$, hence:

$$u_i^e(\overline{x}_i) \;\geq\; u_i^e(\widehat{x}_j), \tag{23}$$

$$u_j^e(\overline{x}_i) \;\leq\; u_j^e(\widehat{x}_j). \tag{24}$$

$(\overline{x}_i^e, \widehat{x}_j)$ is a menu for parameters $(\mathcal{Q}^e, \pi_\bullet)$. Indeed, feasibility is immediate, and

$$u_i^e(\overline{x}_i^e) \;>\; u_i^e(\widehat{x}_j), \tag{25}$$

$$u_j^e(\overline{x}_i^e) \;<\; u_j^e(\widehat{x}_j), \tag{26}$$

where (25) is deduced from (20) and (23) while (26) is deduced from (22) and (24).

However, the RCO $(\widehat{x}_i^e, \widehat{x}_j^e)$ relative to parameters $(\mathcal{Q}^e, \pi_\bullet)$ Pareto dominates any feasible menu for parameters $(\mathcal{Q}^e, \pi_\bullet)$, and particularly $(\overline{x}_i^e, \widehat{x}_j)$. This means:

$$u_i^e(\widehat{x}_i^e) \geq u_i^e(\overline{x}_i^e), \tag{27}$$

which implies in turn that $\widehat{x}_i^e = \overline{x}_i^e$. Type i gets an i-efficient contract for the RCO relative to parameters $(\mathcal{Q}^e, \pi_\bullet)$.

A.5.3 Point 3

We know from Proposition 1 (/2 and /3) that the set of *efficient* redistribution profiles is an interval, so we have to check that this interval is bigger with beliefs $\mathcal{Q}^e$ than with beliefs $\mathcal{Q}$. We focus, without loss of generality, on the RCO (x_i, x_j) that maximizes type j's utility for beliefs $\mathcal{Q}$, the associated profit being denoted by $\pi_\bullet$. We check that the RCO relative to parameters $(\mathcal{Q}^e, \pi_\bullet)$, (x_i^e, x_j^e), is also a second-best allocation.

An RCO is of one of the following three types: (a) the two contracts are type-efficient, (b) one of the contracts only is type-efficient, (c) no contract is type-efficient.

Point 2 of this theorem implies that the set of type-efficient contracts cannot decrease when beliefs are polarized. This implies that if (x_i, x_j) if of type (a), then so is (x_i^e, x_j^e), and we are done, as for any case where (x_i^e, x_j^e) is of type (a). If (x_i, x_j) if of type (b), the only case which is possible and nontrivial is (x_i^e, x_j^e) of type (b); this is treated in "(b) to (b)". If (x_i, x_j) if of type (c), the case (x_i^e, x_j^e) of type (c) is treated in "(c) to (c)" and (x_i^e, x_j^e) of type (b) is treated in "(c) to (b)".

The following lemma will be used on several occasions.

Lemma 2 *Let $(\overline{x}_i, \widehat{x}_j)$ be an RCO for beliefs $\mathcal{Q}$ in which $\overline{x}_i$ is i-efficient, type i's incentive constraint is binding and type j's incentive constraint is not binding. $(\overline{x}_i, \widehat{x}_j)$ is a second-best Pareto optimum if and only if $\tau(q_i, q_j) \geq 0$ with*

$$\tau(q_i, q_j) = \frac{\lambda_i(1-p_i)}{(1-q_i)\,u'(\overline{x}_{i1})} + \frac{\lambda_j}{q_j - q_i}\left(\frac{q_j\,(1-p_j)}{u'(x_{j1})} - \frac{(1-q_j)\,p_j}{u'(x_{j2})}\right). \tag{28}$$

Proof. Let $(\overline{x}_i, \widehat{x}_j)$ be an RCO in which type i is assigned an i-efficient contract, type i's incentive constraint is binding and type j's incentive constraint is not binding. Notice that type j's contract is fully determined by type i's utility and expected wealth. Given that $\overline{x}_i$ is i-efficient, $\widehat{x}_j$ is fully determined by type i's utility.

Modify $\widehat{x}_j$ so that it gives the same utility to type j and it gives utility $u_i(\widehat{x}_j) + d\varepsilon$ to type i. This contract is unique (single-crossing condition). Meanwhile, we assign to type i the i-efficient contract that gives utility $u_i(\widehat{x}_j) + d\varepsilon$. By continuity, for a small $d\varepsilon$, type j prefers the modified $\widehat{x}_j$ to the modified $\overline{x}_i$. Thus, by construction, the new pair of contract satisfies the incentive constraint for a small $d\varepsilon$ and type i is indifferent between the two offers.

The original RCO is a second-best allocation if and only if the new menu cannot be financed, which is what we see now by analyzing the case $d\varepsilon > 0$.

We denote by (dx_{j1}, dx_{j2}) the variation, component by component, of type j's contract and we denote $u'(x_{j1})$ and $u'(x_{j2})$ by u'_1 and u'_2 respectively. By construction

$$\begin{aligned} (1-q_j)\, u'_1 dx_{j1} + q_j\ u'_2 dx_{j2} &= 0, \qquad (29) \\ (1-q_i)\, u'_1 dx_{j1} + q_i\ u'_2 dx_{j2} &= d\varepsilon, \qquad (30) \end{aligned}$$

that is

$$dx_{j1} = \frac{q_j}{q_j - q_i} \frac{d\varepsilon}{u'_1} \text{ and } dx_{j2} = -\frac{1-q_j}{q_j - q_i} \frac{d\varepsilon}{u'_2}. \qquad (31)$$

The variation of type i's expected wealth (for a utility increase of $d\varepsilon$) is

$$\frac{1-p_i}{(1-q_i)\, u'(\overline{x}_{i1})}\ d\varepsilon. \qquad (32)$$

As for type j, the variation of expected wealth is $(1-p_j)\ dx_{j1} + p_j\, dx_{j2}$; using (31) and (32), we find that the change cannot be financed iff $\tau(q_i, q_j) \geq 0$ where

$$\tau(q_i, q_j) = \frac{\lambda_i (1-p_i)}{(1-q_i)\, u'(\overline{x}_{i1})} + \frac{\lambda_j}{q_j - q_i} \left(\frac{q_j(1-p_j)}{u'_1} - \frac{(1-q_j)p_j}{u'_2} \right). \blacksquare \qquad (33)$$

(b) to (b). We apply Lemma 2 for beliefs $\mathcal{Q}$ and $\mathcal{Q}^e$. To determine the sign of τ, we study separately changes of type j's and type i's beliefs.

We first check that, the RCO of interest maximizing type j's utility, the characteristics of the contracts are exactly those required by the lemma. If the type-efficient contract were type j's, then $u_j(x_i) = u_j(x_j)$ (to explain that the other contract is inefficient). We also know that $u_i(x_i) = u_i(x_j)$: indeed, if type i's incentive constraint were not binding, the RCO being continuous with respect to redistribution, type j's contract would remain j-efficient with a (slightly) more favorable redistribution, but the new contract to j would be better for this type than the optimum, a contradiction. As a consequence of these two equalities, $x_i = x_j$: the RCO is a pooling. We find that x_j is at the same time j-efficient and optimal for j among pooling allocations, an impossibility because this supposes that two different marginal rates of substitution are equal. The lemma is applicable.

Type j's beliefs are modified. For a given redistribution, the menu with polarized beliefs (q_i, q_j^e) is the same as before since it depends on p_i, p_j and q_i but not on q_j. We can now calculate the variations the τ with respect to q_j. Remark that $\tau(q_i, q_j) = 0$. It follows that

$$\begin{aligned}
\tau(q_i, q_j^e) &= \tau(q_i, q_j^e) - \frac{q_j - q_i}{q_j^e - q_i}\, \tau(q_i, q_j) \\
&= \frac{\lambda_i(1-p_i)}{(1-q_i)\, u'(\overline{x}_{i1})} \left[1 - \frac{q_j - q_i}{q_j^e - q_i}\right] + \frac{\lambda_j}{q_j^e - q_i} \left[\frac{(q_j^e - q_j)(1-p_j)}{u'_1} + \frac{(q_j^e - q_j)p_j}{u'_2}\right] \\
&= \frac{q_j^e - q_j}{q_j^e - q_i} \left[\frac{\lambda_i(1-p_i)}{(1-q_i)\, u'(\overline{x}_{i1})} + \lambda_j \left(\frac{1-p_j}{u'_1} + \frac{p_j}{u'_2}\right)\right].
\end{aligned} \tag{34}$$

This expression being always positive, the considered redistribution remains efficient for the polarized beliefs.

Type i's beliefs are modified. We parameterize the effects on the menu of changing q_i. Point 1 of this theorem states that type j's utility increases when beliefs are polarized; the increase of type j's utility is a monotonic function denoted by $\eta(q_i^e)$. We calculate dx_{j1} and dx_{j2} as a function of $d\eta$ by solving

$$\begin{cases} (1-p_j)\, dx_{j1} + p_j\ dx_{j2} &= 0, \\ (1-q_j)\, u'_1 dx_{j1} + q_j\ u'_2 dx_{j2} &= d\eta. \end{cases} \tag{35}$$

We find

$$\begin{cases} dx_{j1} &= -\dfrac{p_j}{\Delta} d\eta, \\ dx_{j2} &= \dfrac{1-p_j}{\Delta} d\eta. \end{cases} \tag{36}$$

where $\Delta = (1-p_j)\, q_j\ u'_2 - p_j\ (1-q_j)\, u'_1$ ($\Delta \neq 0$ since type j coverage is inefficient). Given that $\tau(q_i, q_j) = 0$, simple algebra shows that $\Delta \cdot (q_i - q_j) > 0$.

We distinguish two cases, $A : q_i^e < q_i < q_j$ and $B : q_i^e > q_i > q_j$. We show that $\tau(\cdot, q_j)$ multiplied by a well-chosen positive function increases when we pass from q_i to q_i^e, which is sufficient to establish that $\tau(q_i^e, q_j) > 0$. We can then conclude that the redistribution profile considered remains efficient for beliefs (q_i^e, q_j).

Case $A : q_i^e < q_i < q_j$. Define

$$\tau_A(q_i, q_j) = (1-q_i)\, \tau(q_i, q_j) = \frac{\lambda_i(1-p_i)}{u'(\overline{x}_{i1})} + \lambda_j\ f_A(q_i)\ g_A(x_j(\eta)) \tag{37}$$

where

$$f_A(q_i) = \frac{1-q_i}{q_j-q_i}, \tag{38}$$

$$g_A(x_j(\eta)) = \frac{q_j\,(1-p_j)}{u'(\widehat{x}_{j1})} - \frac{(1-q_j)\,p_j}{u'(\widehat{x}_{j2})}. \tag{39}$$

We can now collect the arguments.

1. When we pass from q_i to q_i^e, the i-efficient contract offers less coverage to type i, meaning that $\overline{x}_{i1}$ increases as well as the first term of $\tau_A(q_i, q_j)$;

2. f_A and $\partial f_A/\partial q_i$ are positive;

3. g_A is negative at $q_i^e = q_i$; indeed, at this point $\tau(q_i, q_j) = 0$ implying that $f_A \cdot g_A = -\frac{\lambda_i}{\lambda_j}\frac{1-p_i}{u'(\overline{x}_{i1})} < 0$. The derivative $\partial g_A/\partial\eta$ at the same point is calculated from (36). We find

$$\frac{\partial g_A}{\partial \eta} = \frac{q_j\,(1-p_j)\,p_j\;u''(x_{j1})}{\Delta\;(u'(\widehat{x}_{j1}))^2} + \frac{(1-q_j)\,p_j\;(1-p_j)\,u''(x_{j2})}{\Delta\;(u'(\widehat{x}_{j2}))^2}, \tag{40}$$

   which is positive ($\Delta < 0$ since $q_i - q_j < 0$).

4. Type j's utility increases when q_i^e diminishes ($\partial\eta/\partial q_i < 0$).

This implies that the derivative of the second term of $\tau_A(q_i, q_j)$,

$$\lambda_j \left(\frac{\partial f_A}{\partial q_i}\; g_A + f_A\; \frac{\partial g_A}{\partial \eta}\frac{\partial \eta}{\partial q_i} \right), \tag{41}$$

is unambiguously negative, and we conclude that $\tau_A(q_i, q_j)$ increases when the first variable decreases.

Case $B : q_i^e > q_i > q_j$. Define

$$\tau_B(q_i, q_j) = q_i\;\tau(q_i, q_j) = \frac{\lambda_i p_i}{u'(\overline{x}_{i2})} + \lambda_j\; f_B(q_i)\; g_B(x_j(\eta)) \tag{42}$$

where

$$f_B(q_i) = \frac{q_i}{q_j - qi}, \tag{43}$$

$$g_B(x_j(\eta)) = g_A(x_j(\eta)). \tag{44}$$

We use the fact that, type i's contract being i-efficient,

$$q_i = \frac{p_i}{1-p_i}\;\frac{(1-q_i)\;u'(x_{i1})}{u'(x_{i2})}. \tag{45}$$

The useful arguments are the following.

1. When we pass from q_i to q_i^e, the i-efficient contract offers more coverage to type i, meaning that $\overline{x}_{i2}$ increases as well as the first term in $\tau_B(q_i, q_j)$;

2. f_B is negative and its derivative $\partial f_B/\partial q_i$ is positive;

3. g_B is positive at $q_i^e = q_i$; indeed, $f_B \cdot g_B < 0$. The derivative $\partial g_B/\partial \eta$ is negative (see (40) with $\Delta > 0$ since $q_i - q_j > 0$).

4. Type j's utility increases when q_j^e increases ($\partial \eta/\partial q_i > 0$).

This implies that the derivative of the second term in $\tau_B(q_i, q_j)$,

$$\lambda_j \left(\frac{\partial f_B}{\partial q_i}\, g_B + f_B\, \frac{\partial g_B}{\partial \eta} \frac{\partial \eta}{\partial q_i} \right), \tag{46}$$

is unambiguously negative, and we conclude that $\tau_B(q_i, q_j)$ increases when the first variable increases.

(c) to (c). Denote the RCO by (z, z). By continuity of the RCO with respect to redistribution, the RCO for beliefs $\mathcal{Q}^e$ remains of type (c) in a open neighborhood of $\pi_\bullet$. If the RCO for parameters $(\mathcal{Q}^e, \pi_\bullet)$ were not efficient, then there would be another redistribution profile associated with a pooling RCO (Z, Z) such that

$$u_i^e(Z) \geq u_i^e(z) \text{ and } u_j^e(Z) \geq u_j^e(z). \tag{47}$$

with a least one strict inequality. Lemma 1 implies then that:

$$u_i(Z) \geq u_i(z) \text{ and } u_j(Z) \geq u_j(z). \tag{48}$$

with at least one strict inequality, which implies that (z, z) is not a second-best menu relative to beliefs $\mathcal{Q}$, a contradiction.

(c) to (b). Define $\mathcal{Q}(\lambda) = (1 - \lambda)\mathcal{Q} + \lambda\mathcal{Q}^e$. Beliefs are increasingly polarized as λ goes from 0 to 1. Define $\pi_\bullet(\lambda)$ as the redistribution that maximizes type j's utility for beliefs $\mathcal{Q}(\lambda)$. It suffices to show that $\pi_j(\lambda)$ is smaller than π_j (more transfers to type j).

We reason by contradiction. Assume that for some λ, $\pi_j(\lambda) > \pi_j$.

1. The RCO associated with $(\mathcal{Q}(\lambda), \pi_\bullet)$ is not a second-best allocation since it gives more expected wealth to type j than $\pi_\bullet(\lambda)$.

2. The RCO associated with $(\mathcal{Q}(\lambda), \pi_\bullet)$ is of type (b), since it cannot be of type (a) without contradicting 1 and it cannot be of type (c) ((c) to (b) would be applicable but it contradicts 1).

3. At the RCO associated with $(\mathcal{Q}(\lambda), \pi_\bullet)$, only one type gets a type-efficient contract. If it were type j, then type j would also obtain a j-efficient contract at the RCO associated with $(\mathcal{Q}(\lambda), \pi_\bullet(\lambda))$, since $\pi_j(\lambda) > \pi_j$ (see Proposition 2). This configuration would contradict the beginning of (b) to (b). We conclude that type i gets an i-efficient contract for the RCO associated with $(\mathcal{Q}(\lambda), \pi_\bullet)$ and also for the RCO associated with $(\mathcal{Q}(\lambda), \pi_\bullet(\lambda))$

Denote by λ_∞ the largest λ in $[0, 1]$ such that for all $\mu \in [0, \lambda)$, the RCO associated with $(\mathcal{Q}(\mu), \pi_\bullet)$ is a second-best allocation.

1. By continuity, the RCO associated with $(\mathcal{Q}(\lambda_\infty), \pi_\bullet)$ is a second-best allocation. This implies that $\pi_j(\lambda_\infty) \leq \pi_j$.

2. In any interval $[\lambda_\infty, \lambda_\infty + \varepsilon)$, there is at least some μ such that the RCO associated with $(\mathcal{Q}(\mu), \pi_\bullet)$ is not a second-best allocation. This implies that (i) $\pi_j(\mu) > \pi_j$ and that (ii) the RCO associated with $(\mathcal{Q}(\mu), \pi_\bullet(\mu))$ is of type (b) (see 1-3 above). From (i), we draw that by continuity, $\pi_j(\lambda_\infty) \geq \pi_j$.

We conclude from 1-2 that $\pi_j(\lambda_\infty) = \pi_j$ i.e. $\pi_\bullet(\lambda_\infty) = \pi_\bullet$, and that the RCO associated with $(\mathcal{Q}(\lambda_\infty), \pi_\bullet)$ is a second-best allocation of type (b). Paragraph (b) to (b) is now applicable with $(\mathcal{Q}(\lambda_\infty), \pi_\bullet)$ as starting point: for all $\lambda \geq \lambda_\infty$, the RCO associated with $(\mathcal{Q}(\lambda), \pi_\bullet)$ is a second-best allocation. Consequently, $\lambda_\infty = 1$, implying that the RCO associated with $(\mathcal{Q}^e, \pi_\bullet)$ is a second-best allocation.

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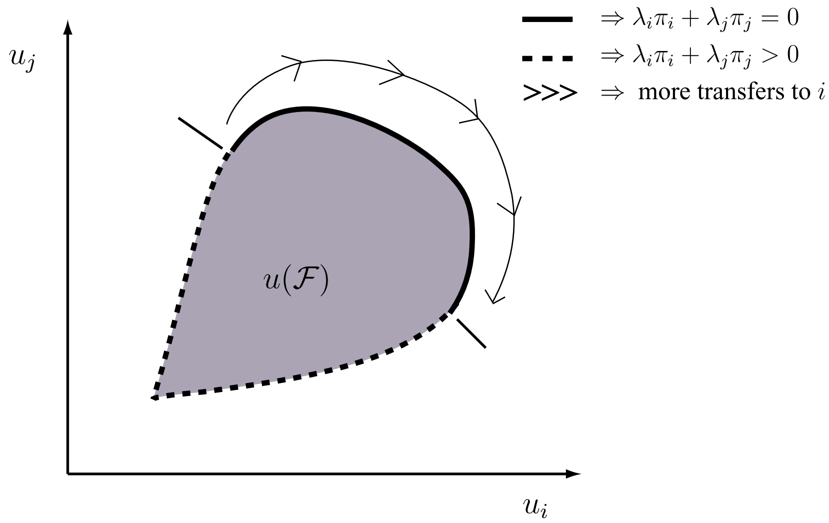

Figure 1 : Feasible utility set and RCOs

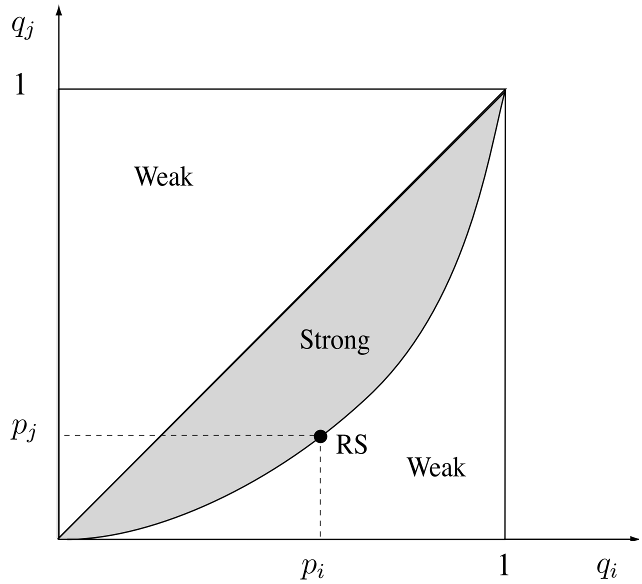

Figure 2 : The effect of risk perceptions on adverse selection