Deadly Embrace: Sovereign and Financial Balance Sheets Doom Loops

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April 15, 2014

Abstract

The recent unravelling of the Eurozone's financial integration raised concerns about feedback loops between sovereign and banking insolvency, and provided an impetus for the European banking union. This paper provides a "double-decker bailout" theory of the feedback loop that allows for both domestic bailouts of the banking system by the domestic government and sovereign debt forgiveness by international creditors. Our theory has important implications for the re-nationalization of sovereign debt, macroprudential regulation, and the rationale for banking unions.
Keywords: feedback loop, sovereign and corporate spreads, bailouts, sovereign default, strategic complementarities, debt maturity.
JEL numbers: F34, F36, F65, G28, H63.

1 Introduction

Rarely does an economic idea gather so wide a consensus as the evilness of the "deadly embrace", also called "vicious circle" or "doom loop". The feedback loop between weak bank balance sheets and sovereign fragility now faces almost universal opprobrium, from the IMF[1] and central bankers to the entire political establishment and the European Commission, providing a major impetus to build the European banking union.

*The research leading to these results has received funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013) Grant Agreement #249429. Financial support of the research initiative "market risk and value creation" of the Chaire SCOR under the aegis of the Fondation du Risque is also acknowledged.

[1]See e.g. Lagarde (2012).

This paper seeks to analyze these developments by proposing a "double-decker" bailout theory of the doom loop that allows for both domestic bailouts of the banking system by the government and sovereign debt forgiveness by international creditors. The theory has important implications for the re-nationalization of sovereign debt, macroprudential regulation, and the rationale for a banking union.

Our model has three dates, 0, 1 and 2. At date 0, banks, which will need money for their date-1 banking activities, manage their liquidity by holding domestic sovereign bonds and foreign bonds (in the basic version of the model). Foreign bonds are safe, and so the standard diversification argument would call for holding no domestic bonds. Domestic sovereign government bonds mature at date 2 in this basic version, and so the date-2 fiscal capability determines the sovereign spread.

News accrues at date 1, that affects the banks' solvency (a financial shock) and/or the state's date-2 fiscal capability (a fiscal shock). A fiscal shock compounds the financial shock if banks hold government bonds. While the government ex ante dislikes transferring resources to the banking sector, it cannot refrain from bailing out banks when facing the fait accompli of a banking liquidity shortfall. The bailout further degrades the sovereign's ability to reimburse its debt at date 2, lowers the bond price and reduces bank solvency, etc., an amplification mechanism. The multiplier reflecting the loss in sovereign bond price when a bailout is required increases with the extent of home bias.

We investigate the banks' and the government's incentives to seek and prevent risk taking, respectively. When banks can count on government bailouts, they optimally diversify as little as supervision allows them to, so as to enjoy the maximal put on taxpayer money. Conversely, the government would like to limit risk and force diversification on the banks. We therefore study the extent to which banks are willing to incur costs so as to evade diversification regulation.

In the process, we develop a new argument[2] in favor of macro-prudential policies. The consequences of individual banks' undiversified portfolios, and therefore the desirability of intense supervision, depends crucially on the other banks' behaviors. We show that the banks' choices of opaqueness, and thereby their exposures to the sovereign, are strategic complements: incurring the cost of making one's balance sheet more opaque is more tempting if the put on taxpayer money is more attractive; in turn, this put is attractive when the sovereign bond price is more volatile, which it is when the other banks take a larger gamble. The corollaries to this insight are the existence of collective moral hazard and the necessity of macroprudential supervision: The social cost of poor monitoring of

[2] Standard arguments for going beyond the analysis of stand-alone bank solvency include the possibility of fire sales, interconnectedness and the policy response to, say, widespread maturity mismatches.

a bank's domestic exposure is higher when other financial institutions are themselves exposed. This is particularly true for institutions that the government is eager to rescue.

We then look at the incentives of foreign investors. We show that in bad states of nature, the legacy debt ends up on the wrong side of the Laffer curve once likely bailouts and debt increases are factored in; investors thus have an incentive to forgive some debt. This "double-decker bailout" in turn induces the government to turn a blind eye to undiversified bank portfolios. This however occurs only when the situation looks grim, a prediction that fits well with the recent re-nationalization of government debt in the Eurozone. It also provides a new argument in favor of a banking union. Indeed, if the ex-post leniency of domestic regulators is anticipated ex ante at the time of sovereign debt issuance, then it is priced in the form of higher spreads. The government is better off committing ex ante to a tough ex-post regulatory stance, but is tempted to relax it ex post. If the government lacks commitment, then it benefits from relinquishing its regulatory powers to a supranational supervisor by joining a banking union.

Finally, we study four interesting extensions of the basic model. The first three do not consider the possibility of debt forgiveness (either because debt is on the right side of the legacy Laffer curve, or because debt is on the wrong side of the legacy Laffer curve but investors have difficulties coordinating on a debt relief package).

First, we investigate the role of leverage by assuming that banks can seek refinancing in markets at date 1. The feedback loop is then stronger, the higher the leverage. This is especially so when sovereign defaults come together with defaults on banks' private debt contracts: As sovereign risk rises, banks have to reduce leverage because the probability of a default on the private debt that they issue also rises. This requires a larger bailout, which puts further pressure on the government budget etc. ad infinitum.

Second, there may be really adverse shocks for which the government can only undertake a partial bailout, as a full one would compromise public finances too much. We then show that banks enter a "rat race". While they wish to remain undiversified so as to enjoy the largest possible put on taxpayer money, they also try to jump ahead of the bailout queue by being a bit more solvent, and therefore cheaper to rescue in the race for bailouts in bad states of nature. Their holdings of foreign bonds are akin to "bids" in a first-price auction, but the analysis is richer than the standard first-price auction in that the focus of competition- the pot of subsidies to be distributed- depends on the distribution of "bids", namely the distribution of holdings of foreign bonds.

Third, we relax the assumption that sovereign debt maturity matches that of fiscal capability. We compare our economy with long-term sovereign bonds which are claims to coupons accruing at date 2 to an economy where sovereign bonds are short-term one-period bonds which are rolled over at date 1, assuming that the same amount is raised at date 0. We show that in the absence of, or under limited bailouts (i.e. under efficient supervision of bailout-prone entities), welfare is higher under long-term borrowing. By contrast, financial entities cannot benefit from the doom-loop under short-term government debt, provided that the latter can be rolled over. Short-term borrowing is then a (rather inefficient) substitute for supervision.

Fourth, we allow foreign banks to hold domestic debt. Because of the bailout guarantees, foreign banks also have an incentive to load up on risky domestic debt. The foreign government has an incentive to regulate foreign banks so that they do not take on too much domestic sovereign risk. The analysis then uncovers an additional rationale for a banking union. Domestic regulation has positive external effects for the foreign country. These effects are not internalized by the domestic government, and as a result, regulation is too lax in the domestic economy. By transferring regulatory decisions from the national to the international level, a banking union allows these effects to be internalized, leading to a toughening of regulation in the domestic country and an improvement of welfare.

The paper is organized as follows: Section 2 sets up the framework and defines equilibrium. Section 3 identifies the sovereign and financial balance sheets feedback loop. It derives the optimal regulation in the absence of renegotiation with investors; and it demonstrates that banks' choices with respect to diversification are strategic complements. Section 4 explores the incentives of legacy creditors to engage in debt forgiveness, how these incentives affect the regulatory stance of the government, and develops a rationale for a banking union. Section 5 presents the four extensions, concerning the role of leverage and joint defaults, limited bailouts and endogenous diversification, the maturity of sovereign debt, and foreign banks. Finally, Section 6 summarizes the main insights and concludes.

Relationship to the literature. Several papers have analyzed doom-loops and have identified a feedback loop similar to the one described in our paper. In Acharya et al (2013), the banks hold government bonds; the government's bailout of its financial sector so as to preserve the latter's lending to the non-financial sector both reduces the value of the financial sector's claims—a mitigating effect—and raises the prospect of future domestic taxation, thereby reducing the non-financial sector's investment. As the paper's title indicates, the stabilization of the financial sector is a Pyrrhic victory as it has deleterious long-term effects. The theoretical model is a closed-economy model, in which default costs are internalized by the government; it does not investigate topics such as re-nationalization, joint default, and domestic vs. international regulation of banks that feature prominently in our analysis. Acharya et al's empirical part looks at the price of European sovereign and bank CDSs over the period 2007-2011. These became negatively correlated after the first bank bailouts—pointing at a perception of risk transfer—and then exhibited a significant positive correlation, suggesting that the market was concerned about a feedback loop.

Cooper-Nikolov (2013) build a model in which sovereign defaults are the outcome of, as in Calvo (1988), self-fulfilling prophecies; similarly, banks fail because of Diamond-Dybvig (1983) runs. This model allows them to demonstrate the potential existence of a doom-loop: Worries about sovereign default generate concerns about the viability of banks holding sovereign bonds; conversely, bank failures require bailouts, increasing the volume of sovereign debt. The paper shows that equity cushions eliminate bad equilibria. Our paper differs from Cooper-Nikolov in several respects; on the technical side, crises are in our paper associated with fundamentals (although we of course find much interest in Cooper-Nikolov self-fulfilling crises as well); this allows us to identify a multiplier and to make unique predictions. Like Acharya et al, Cooper and Nikolov focus on a closed economy.

Several recent contributions look at the contagion from sovereigns to banks in an open economy, offering different hypotheses for sovereign debt re-nationalization and therefore sets of predictions and policy implications that differ from the unique ones summarized in Section 6; in this sense, our contribution is complementary with existing works. The overall picture is the richness of the economics of interactions between sovereign and bank solvency.

Broner et al (2013) consider environments in which the domestic government can default selectively on foreign investors. Selective default then makes domestic debt comparatively attractive to domestic residents in risky times, implying a re-nationalization. In turn, increased domestic holdings of sovereign debt crowd out domestic banks' investment in the real economy. The contagion channel (discrimination) differs from ours (bailouts) and is one-way[3] (from sovereign debt fragility to banks) rather than two-way; so does the rationale for a banking union, as Broner et al view a union as a reduction in discrimination between domestic and foreign investors while we focus on prudential supervision.

Uhlig (2014)'s model of financial repression, like ours, features banking supervision and no discrimination among investors. It assumes a monetary union, whose central bank is jointly backed by the member states and bails out commercial banks. Like in our

[3] A two-way feedback loop arises in an extension of their model in which the cost of default is proportional to the amount of defaulted debt, with a proportion decreasing with the capital stock.

paper, a country may allow its banks to load up on domestic sovereign debt as the adverse consequences will be shared abroad. Tolerating/ encouraging risk-shifting by banks that have access to the union's central bank's repurchase facility enables the risky country to borrow more cheaply, a mechanism which bears some resemblance to our rationale for strategic debt re-nationalization whereby governments allow domestic banks to buy up their bonds in order to extract concessions from legacy creditors.

In Gennaioli et al (2013), domestic banks find domestic bonds attractive for a different reason than in our paper: the sovereign's internal cost of default (the drying-up of domestic banks' liquidity as there is neither discrimination nor bank bailouts) is high when banking productivity is high; so sovereign repayment discipline is endogenously positively correlated with the banks' marginal utility of liquidity. This implies a re-nationalization of sovereign debt in bad times. There is no feedback loop but instead a disciplining effect of bank holdings of domestic debt on sovereign debt repayment[4], as well as a positive impact of developed financial institutions on sovereign credibility. Unlike our paper, the emphasis is not on prudential supervision and feedback loops.

An important ingredient of our analysis, as in Farhi-Tirole (2012) is that direct and indirect exposures may be hard to assess, leading to supervisory failures, and that banks will exploit the supervisory loopholes to secure cheaper financing and thereby increase their return on equity. This ingredient is also shared by Mengus (2013a, b), who shows that if furthermore banks in equilibrium (endogenously) choose heterogeneous portfolios, defaults involve an internal cost, and so a country may a) prefer not to default even in the absence of sanctions, and b) may want to rescue another country despite the subsidy to third-party lenders to the defaulting country. The focus in Mengus is thus on the impact of sovereign default on banks rather than on the doom-loop. Bolton-Jeanne (2011) also study the international contagion of sovereign debt crises through the financial sector and their international fiscal implications. The focus in Bolton-Jeanne is on the impact of sovereign default on banks and the role of banks in contagion rather than on the doom-loop.

Finally, there is a large literature on sovereign-debt renegotiations (see, e.g., section 2.5 of Eaton-Fernandez 1995 for an overview), that starts from the observation that either demanding reimbursement or punishing a country for default may be costly and therefore not time-consistent. The novelty of our analysis here lies in linking debt renegotiation with the prospect of bailouts and the deadly embrace idea.

[4]This effect is the focus of a branch of the international finance literature vaunts the accountability benefits associated with home biases. See e.g. Tirole (2003).

2 Model

2.1 Setup

We consider the following economy. There are three dates $t \in \{0, 1, 2\}$ and a single good at every date.

The economy is populated by international investors, a continuum of mass one of domestic banking entrepreneurs and a continuum of mass one of domestic consumers. In addition, there is a domestic government.

Uncertainty is gradually resolved over time. At date 1, a state of the world is realized $s \in S$, with (full support) probability distribution $d\pi(s)$, where S is an interval of $\mathbb{R}^+$. The banking entrepreneurs' balance sheets and the fiscal capacity of the government depend on the realization of the state of the world s.

Private agents: international investors, banking entrepreneurs and consumers. International investors have a large endowment in every period. Their utility $V_t^* = \mathbb{E}_t[\sum_{s=t}^{2} c_s^*]$ at date t is linear over consumption.

Consumers have a random endowment $E \in [0, \infty)$ at date 2, with probability distribution function $f(E|s)$ and cumulative distribution function $F(E|s)$. We assume that $\frac{\partial(f(E|s)/(1-F(E|s)))}{\partial s} \leq 0$ and that $\frac{\partial(f(E|s)/(1-F(E|s)))}{\partial E} > 0$. The first inequality will imply that decreases in s are bad news for the fiscal capacity of the government; the second is a monotone hazard rate condition that will imply a quasi-concave Laffer curve. The two conditions are equivalent if s shifts the distribution uniformly so that $F(E|s) = F(E - s)$. Consumers' utility $V_t^C = \mathbb{E}_t[c_2^C]$ at date t is linear over consumption at date 2. As usual, one can think of E as the consumers' disposable income beyond some incompressible level of consumption.

Banking entrepreneurs have an endowment A at date 0. At date 1, in state s, they have a fixed-size investment opportunity which pays off at date 2. They can invest $I(s)$ with a payoff $\rho_1(s)I(s)$ where $\rho_1(s) > 1$. The dependence of I and ρ_1 on s more generally stands for liquidity (or financial) shocks faced by banks. We assume that $\frac{dI(s)}{ds} \leq 0$ so that low s states are states in which banks badly need cash. The utility of banking entrepreneurs $V_t^B = \mathbb{E}_t[c_2^B]$ at date t is linear over consumption at date 2.

We assume for the moment that the return from the investment project of banking entrepreneurs cannot be pledged to outside investors, and as result, banking entrepreneurs cannot raise outside funding at date 1 (see Section 5.1 for a relaxation of this assumption). Instead, they must self-finance the investment project $I(s)$. Therefore, at date 0, banking entrepreneurs trade their endowment A for financial assets (stores of value), part or all of which they sell at date 1 to finance their investment project. We assume that $A \geq \bar{I}$ where $\bar{I} = \max_{s \in S} I(s)$ so that if banking entrepreneurs manage or decide to preserve their wealth between dates 0 and 1, they can always finance their investment project.

Assets. These financial assets are assumed to come in two forms, domestic sovereign bonds in amount B_0, and foreign bonds. Both domestic and foreign bonds are claims to a unit of good at date 2.

Except in Section 5.2, in which we will introduce competition among banks for access to bailout funds, we look for a symmetric equilibrium, in which banks all choose the same portfolio. We denote by b_0 and b_0^* the representative bank's holdings of domestic sovereign bonds and foreign bonds. We assume that there are no short sales so that $b_0^* \geq 0$ and $b_0 \geq 0$.

We assume that foreign bonds—which could be either private bonds or foreign government bonds—are safe, and hence their price is always 1. By contrast, we assume that domestic bonds are risky because the domestic government might default. We denote their price in period 0 by p_0 and their price in period 1 by $p_1(s)$. We assume that $p_0 B_0 > A$ so that the marginal holder of domestic bonds is an international investor.

Welfare. At each point in time, the government evaluates welfare according to $W_t = \mathbb{E}_t[c_2^C + \beta^B c_2^B + \beta^I(s)\mu(s)I(s)]$ net of default costs to be introduced below. Welfare gross of default costs W_t is a weighted average of consumer welfare, banking entrepreneur welfare and investment $\mu(s)I(s)$ where $\mu(s)$ is the mass of banking entrepreneurs that undertake their investment project. We assume that $\beta^B < 1$, and so pure consumption transfers to bankers are costly.

The term $\beta^I(s)\mu(s)I(s)$ in the social welfare function captures the welfare benefit for banking stakeholders from the banks' ability to invest.[5]

Government debt, bailouts, defaults. The domestic government makes decisions sequentially, without commitment. At date 1, the government decides whether or not to

[5] Imagine that, say, three categories of banking stakeholders' benefit from the banks' ability to invest. First, and most obviously the banking entrepreneurs themselves: They receive $\rho_1(s)\mu(s)I(s)$, where $\rho_1(s)$ is the banks' stake in continuation. Second, the higher $\mu(s)I(s)$, the better off their borrowers. Third, the workers working in banks and industrial companies; to the extent that they are better off employed (e.g., they receive an efficiency wage) and that preserved employment is related to $\mu(s)I(s)$, then workers' welfare grows with $\mu(s)I(s)$. Thus if $\rho_1^F(s)$ and $\rho_1^W(s)$ denote the stakes of the industrial firms and the workers, and if $\tilde{\beta}^B$, $\tilde{\beta}^F$ and $\tilde{\beta}^W$ denote the three categories of stakeholders' welfare weights or political influence, then $\beta^B = \tilde{\beta}^B$ and $\beta^I = (\tilde{\beta}^F \rho_1^F(s) + \tilde{\beta}^W \rho_1^W(s))$. This "credit crunch" interpretation can be formalized further along the lines of Holmström-Tirole (1997) (see Appendix A).

undertake a bailout of its domestic banks. At date 2, the government decides whether to repay its debt or to default.

The government has some outstanding bonds B_0 at date 0. We assume for the moment that these bonds mature at date 2. In Section 5.3, we will investigate whether conclusions are altered by a shorter maturity and whether the government optimally issues long-term bonds. The government's only fiscal resources are at date 2: the government can tax the (random) endowment E of domestic consumers. The endowment E can hence be interpreted as the fiscal capacity of the government.

At date 1, the government chooses whether or not to undertake a bailout of the financial sector. We assume that at date 1, the government inspects the balance sheets of banks that apply for a bailout and so can, if it so desires, tailor individual bailout levels to specific liquidity shortages of applying banks (which in equilibrium will end up being identical because of equilibrium symmetric portfolios).[6] We denote by $X(s)$ the total transfer to the banks. In order to finance this transfer, the government must issue new bonds $B_1(s) - B_0$. The assumption that the government sets the amount it promises to reimburse, $B_1(s)$, rather than the amount it borrows eliminates any multiplicity associated with erratic expectations as in Calvo (1988).

We assume that the weight $\beta^I(s)$ on investment is high enough so that the government always chooses to bail out the financial sector if such a bailout is needed, implying that $X(s) = \max\{I(s) - (b_0^* + p_1(s)b_0), 0\}$. Finally, we assume that the government can always raise enough funds at date 1 to finance the desired bailout (see Section 5.2 for a relaxation of this assumption).

At date 2, the government decides whether or not to default on its debt. The government cannot discriminate between foreign and domestic bond holders, and hence cannot selectively default on foreigners. The government incurs a fixed cost Φ if it defaults on its debt. We assume that the default cost is high enough, so that the government only defaults if it cannot pay its debt, that is if and only if $B_1(s) > E$.[7]

Illustrating example. We will make repeated use of the following simple example of our more general setup. We assume that $I(s) = \bar{I} = A$ and $\rho_1(s) = \rho_1$ for all s. The structure of uncertainty is as follows. With probability π, the state s is H and the endowment is

[6] Alternatively, we could have followed Farhi and Tirole (2012) or Mengus (2013a, b) in assuming that individual portfolios are imperfectly observed at the bailout date and that these portfolios are endogenously heterogeneous. This would make bailouts more costly and the analysis more complex, without altering the basic insights in our context.

[7] A sufficient condition is $\Phi > E$ with probability one. A weaker sufficient condition is $\Phi > B_1(s)$, but involves the equilibrium object $B_1(s)$.

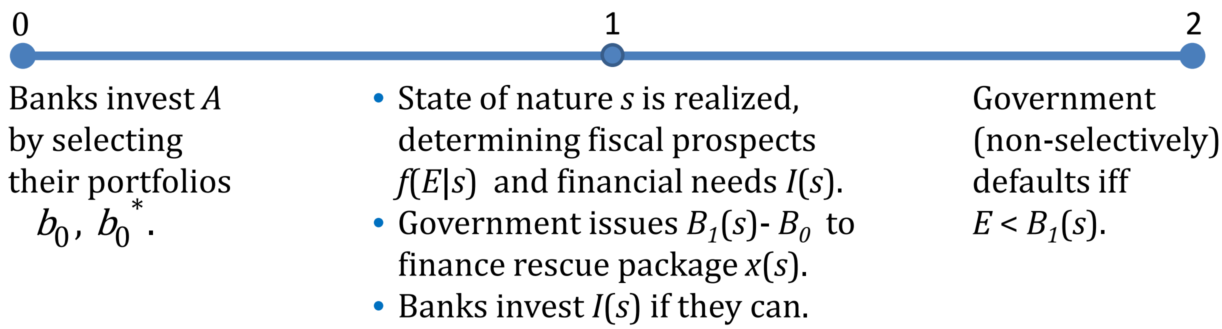

Figure 1: Timeline.

high enough at E that there is no default. With probability $1-\pi$, the state is L and the endowment is high enough at E so that there is no default with conditional probability x, intermediate e with conditional probability y, and 0 with conditional probability $1-x-y$. In addition, we assume that $e > B_0$.

For $E \geq B_1(L) > e$, we have $1 - F(B_1(L)|L) = x$ and so $p_1(L) = x$ and $p_0 = \pi + (1-\pi)x$. For $e \geq B_1(L) \geq 0$, we have $1 - F(B_1(L)|L) = x + y$ and so $p_1(L) = x + y$ and $p_0 = \pi + (1-\pi)(x+y)$. Depending on which of $(E - B_0)x$ and $(x+y)(e-B_0)$ is greater, the level of debt $B_1(L)$ that maximizes revenue in state L is either E or e.

2.2 Equilibrium

In this section, we characterize the equilibrium of the model.

Bond prices and Laffer Curve. Because the marginal investor of domestic bonds is a risk-neutral international investor, the prices of domestic bonds at dates 0 and 1 simply reflect the relevant conditional default probability:

$$p_1(s) = 1 - F(B_1(s)|s),$$
$$p_0 = \int p_1(s) d\pi(s).$$

At date 1 in state s, the government can thus collect $(B_1 - B_0)[1 - F(B_1|s)]$ by issuing $B_1 - B_0$. This revenue is strictly quasi-concave in B_1 and increasing in s from our assumptions on the distribution of date-2 endowment E. It is always optimal for the government to pick $B_1 = B_1(s)$ so as to be in the upward sloping part of the Laffer curve in state s.

Banking entrepreneurs' portfolios and regulation. Banking entrepreneurs invest their net worth into foreign bonds $b_0^* \geq 0$ and domestic bonds $b_0 \geq 0$ so that

$$A = b_0^* + p_0 b_0.$$

At date 1, their pre-bailout net worth is $b_0^* + p_1(s)b_0$. If their pre-bailout net worth falls short of the investment size $I(s)$, they receive a government bailout. In a symmetric equilibrium, they receive $X(s) = I(s) - (b_0^* + p_1(s)b_0)$. If their pre-bailout net worth exceeds the investment size $I(s)$, they simply save the difference by acquiring either domestic or international bonds (at this stage, they are indifferent between both since they are risk neutral over date-2 consumption).

Their expected utility is therefore $V_0^B = \int [\rho_1(s)I(s) + \max\{b_0^* + p_1(s)b_0 - I(s), 0\}]\, d\pi(s)$. Because $p_0 = \int p_1(s)d\pi(s)$, $I(s)$ is decreasing in s and $A \geq \bar{I}$, their utility is maximized for $b_0^* = 0$.[8] This is intuitive: Banking entrepreneurs have an incentive to take as much risk as possible to extract the biggest possible expected bailout from the government.

More generally, we introduce a minimum diversification requirement by imposing a lower bound $\underline{b}_0^*$ on b_0^* so that banking entrepreneurs must choose $b_0^* \geq \underline{b}_0^*$. Banking entrepreneurs then choose to be against the regulatory constraint $b_0^* = \underline{b}_0^*$.

The minimum diversification level $\underline{b}_0^*$ is a shortcut for the quality of supervision. It can be rationalized in multiple ways. For instance, one could imagine that the prudential supervisor imperfectly observes a bank's (direct and indirect) exposure to domestic shocks. This will allow banks to differ in their exposures; one can view the threshold level of diversification as reflecting the supervisor's ability to detect (and therefore correct) lacks of diversification. An alternative interpretation is that supervision is operated by self-interested supervisors, who enter some deal with the banks that reflects a trade-off between the supervisors' mission- forcing banks to manage their risks- and the bank's self-interest- maximizing b_0 so as to maximize their put on taxpayer money.

Occasionally we will open the black box of this minimum diversification requirement. Intuitively, the ability of an individual bank to gamble on its own sovereign depends on two factors: it is impacted negatively by the supervisor's effort to identify direct and indirect exposures, and positively by the bank's own effort to make its balance sheet opaque.

Bailouts and date-1 bond issuance. To finance the bailout at date 1 in state s

$$X(s) = \max\{I(s) - \underline{b}_0^* - (A - \underline{b}_0^*)\frac{p_1(s)}{p_0}, 0\}.$$

[8] Because $p_1(s)$ is increasing in s and $I(s)$ is decreasing in s, there exists $\tilde{s}$ such that $b_0^*(1 - \frac{p_1(s)}{p_0}) + \frac{p_1(s)}{p_0}A - I(s) \geq 0$ if and only if $s \geq \tilde{s}$. Note that if $p_1(s) \geq p_0$, then $s \geq \tilde{s}$. Now consider $b_0^{*\prime} > b_0^*$. We necessarily have $\tilde{s}' \leq \tilde{s}$. This implies that $V_0^{B\prime} - V_0^B \leq \int_{s \geq \tilde{s}'} (b_0^{*\prime} - b_0)(1 - \frac{p_1(s)}{p_0})d\pi(s) \leq 0$.

requires issuing $B_1(s) - B_0$ new bonds at date 1 with

$$p_1(s)[B_1(s) - B_0] = X(s).$$

Date-1 debt $B_1(s) \geq B_0$ is the smallest solution of the following fixed-point equation

$$[B_1(s) - B_0][1 - F(B_1(s)|s)] = \max\{I(s) - \underline{b}_0^* - (A - \underline{b}_0^*)\frac{1 - F(B_1(s)|s)}{p_0}, 0\}. \qquad (1)$$

The solution $B_1(s) \geq B_0$ is necessarily on the upward sloping part of the Laffer curve, and we assume that equation (1) has a unique solution $B_1(s) \geq B_0$ on the upward sloping part of the Laffer curve. If $B_1(s) > B_0$, this solution is then necessarily locally stable, by which we mean that the slope of the left-hand side of (1) is greater than that of the right-hand side.

There exists a cutoff $\tilde{s}$ such that $B_1(s) > B_0$ if $s < \tilde{s}$ and $B_1(s) = B_0$ for $s \geq \tilde{s}$. Furthermore, we can show that $\frac{dB_1(s)}{ds} < 0$ for $s < \tilde{s}$ and similarly that $\frac{dp_1(s)}{ds} > 0$ for $s < \tilde{s}$, and for all s if $\frac{\partial(f(E|s)/(1-F(E|s)))}{\partial s} < 0$ (strict inequality) .

3 Sovereign and Financial Balance Sheets Feedback Loops

In this section, we illustrate the amplification mechanism arising from a feedback loop between banks and sovereign balance sheets. We characterize optimal first-best frictionless regulation (when the government can perfectly enforce regulation at no cost). We show than when regulation is imperfect, banks' domestic sovereign risk loadings are strategic complements, leading to the possibility of multiple equilibria with varying degrees of banks' domestic sovereign risk exposures, and imparting a macroprudential dimension to regulation.

3.1 Amplification Mechanism

This feedback loop can be seen through the following fixed-point equation for the date-1 price of government bonds

$$p_1(s) = 1 - F(B_1(s)|s), \qquad (2)$$

where

$$B_1(s) = B_0 + \max\{\frac{I(s) - \underline{b}_0^*}{p_1(s)} - \frac{A - \underline{b}_0^*}{p_0}, 0\}. \qquad (3)$$

Using the implicit function theorem, we can then derive the following comparative static result, assuming that a bailout occurs in state s i.e. that $s < \tilde{s}$.

Proposition 1 (Feedback Loop)**.** *The sensitivity of date-*1 *bond prices* $p_1(s)$ *to the state* s *when a bailout is required is given by*

$$\frac{dp_1(s)}{ds} = \frac{-\frac{\partial F(B_1(s)|s)}{\partial s} - \frac{1}{p_1(s)} f(B_1(s)|s) \frac{dI(s)}{ds}}{1 - \frac{I(s)-\underline{b}_0^*}{p_1^2(s)} f(B_1(s)|s)}. \tag{4}$$

The numerator encapsulates the direct effect of the change in s on the debt price $p_1(s)$ if there were no change in the price at which the government issues bonds to finance the bailout and at which banking entrepreneurs liquidate their government bond holdings. The first term in the numerator captures the direct change in the probability of no-default at constant investment size $I(s)$. The second term in the numerator captures the direct impact of the change in the investment size $I(s)$.

The denominator is positive because of the local stability of the selected fixed-point solution to equations (2) and (3). It takes the form of a multiplier, which represents the indirect effect of a change in s on the debt price $p_1(s)$ through the change in the price at which the government issues bonds and at which banking entrepreneurs liquidate their government holdings. The multiplier is higher, the larger the amount of foreign-held debt $B_1(s) - (B_0 - b_0) = \frac{I(s)-\underline{b}_0^*}{p_1(s)}$ that must be issued to finance the bailout (and hence the higher the amount of domestic debt held by domestic banks, i.e. the lower is $\underline{b}_0^*$), and the larger the semi-elasticity $\frac{1}{p_1(s)} f(B_1(s)|s)$ of the debt price $p_1(s)$ to additional debt issuances. This multiplier captures the feedback loop between banks and sovereigns as an amplification mechanism: An increase in the default probability reduces the price $p_1(s)$ which increases the required bailout $X(s)$ and hence the quantity of bonds $B_1(s) - B_0$ that must be issued at date 1, which further reduces the price $p_1(s)$ etc. ad infinitum.

Consider for example the case where $\frac{dI(s)}{ds} = 0$ so that there are no variation in investment needs as we vary s, and assume that $\frac{\partial(f(E|s)/(1-F(E|s)))}{\partial s} < 0$ (strict inequality). Decreases in s are then just bad news for the fiscal capacity of the government. The effect of a bad fiscal shock $ds < 0$ on bond prices $p_1(s)$ is then amplified because some of these bonds are held by the banking system, which increases the size of the required bailout, worsening the fiscal problems etc. ad infinitum.

Similarly, consider the case where $\frac{\partial f(E|s)}{\partial s} = 0$ so that there are no variations in fiscal capacity as we vary s, and assume that $\frac{dI(s)}{ds} < 0$ (strict inequality). Decreases in s are then just increases in the liquidity needs of entrepreneurs.[9] Again, the effect of a bad financial

[9]Although this is not essential, in order for decreases in s to represent bad news, we also assume that

shock $ds < 0$ on bond prices $p_1(s)$ is amplified because some of these bonds are held by the banking system, which must then be bailed out, worsening the fiscal problems etc. ad infinitum.

3.2 First-Best Frictionless Regulation

Ex-ante welfare is given by

$$W_0 = \int \left[\int_{B_1(s)}^{\infty} [E - B_1(s)] f(E|s) dE + \int \int_0^{B_1(s)} [E - \Phi] f(E|s) dE + \beta^I(s) I(s) \right] d\pi(s)$$
$$+ \int \beta^B \left[\rho_1(s) I(s) + \max\{\underline{b}_0^* + (A - \underline{b}_0^*) \frac{p_1(s)}{p_0} - I(s), 0\} \right] d\pi(s).$$

As we argued earlier, the government may have limited ability to force banks to diversify. Nonetheless, it is instructive to investigate optimal regulation in the ideal theoretical situation where such limits to supervision are absent. We refer to this situation as first-best frictionless regulation. The government minimizes the occurrence of default by setting $\underline{b}_0^* = \bar{I}$ and hence ensuring that banking entrepreneurs can always finance their investment $I(s)$ without requiring a bailout.

The welfare of banking entrepreneurs as well as total welfare are then independent of the amount $b_0^* \geq \underline{b}_0^*$ invested in foreign bonds above the floor $\underline{b}_0^*$. Reducing $\underline{b}_0^*$ below $\bar{I}$ on the other hand would reduce welfare for two reasons. First, it would increase the occurrence of default. Second, it would have a redistributive effect from consumers to banking entrepreneurs, which, as long as $\beta^B \leq 1$, would be undesirable.

Proposition 2 (First-Best Frictionless Regulation)**.** *Setting $\underline{b}_0^* = \bar{I}$, if feasible, maximizes ex-ante welfare W_0.*

Proof. For $\underline{b}^* = \bar{I}$, ex-ante welfare is given by

$$W_0 = \int \left[\int_{B_0}^{\infty} [E - B_0] f(E|s) dE + \int \int_0^{B_0} [E - \Phi] f(E|s) dE + \beta^I(s) I(s) \right] d\pi(s)$$
$$+ \int \beta^B [\rho_1(s) I(s) + [A - I(s)]] d\pi(s).$$

$\frac{d(\rho_1(s)I(s))}{ds} > 0$, and $\frac{d(\beta^I(s)I(s))}{ds} > 0$.

For $\underline{b}_0^* < \bar{I}$, welfare can be rewritten as

$$W_0 = \int \left[\int_{B_1(s)}^{\infty} [E - B_0] f(E|s) dE + \int \int_0^{B_1(s)} [E - \Phi] f(E|s) dE + \beta^I(s) I(s) \right] d\pi(s)$$
$$+ \int \beta^B \left[\rho_1(s) I(s) + [A - I(s)] \right] d\pi(s)$$
$$+ \int (1 - \beta^B) \min\{\underline{b}_0^* + (A - \underline{b}_0^*) \frac{p_1(s)}{p_0} - I(s), 0\} d\pi(s),$$

and is clearly strictly lower as long as the set of states s with $B_1(s) > B_0$ has strictly positive measure. □

In our environment, there is a case for ex-ante regulation because of the inability of the government to commit not to bail out banks ex post. This creates a standard soft budget constraint problem. The consequences of this problem are magnified by the presence of the feedback loop between banks and sovereigns. The optimal frictionless first-best regulation actually prevents this feedback loop from occurring in the first place by preventing domestic banks from holding domestic sovereign debt to an extent that could make them illiquid.

We have already discussed in Section 2.2 some reasons why we might observe suboptimal regulation $\underline{b}_0^* < \bar{I}$, creating the possibility of the feedback loops that are the focus of this paper. These considerations lead us to adopt a pragmatic position and treat $\underline{b}_0^*$ as a parameter. In Sections 3.3 and 4.2 we flesh out two possibilities (imperfect ability to enforce regulation, and desire to extract concessions from legacy creditors) that might lead the government to let the banks take on more exposure to domestic sovereign default risk than would be required to rule out bailouts.

3.3 Collective Moral Hazard

The rationale for liquidity regulation also has a macroprudential dimension. Indeed, the benefits of liquidity regulation depend on the risk taking of the banking system as a whole. For example, if for some reason only a fraction of banks take on domestic sovereign debt, then the benefits from regulating the other banks is reduced (and might even vanish) because the government has more fiscal space, reducing the riskiness of the government bonds and hence the need for bailouts, and also weakening the feedback loop between the remaining banks and the sovereign. We now show that for a given regulatory effort, the incentives for banks to take on domestic sovereign debt are increased when other banks do so—a manifestation of the strategic complementarities in financial risk-taking at work in the model—to that the effectiveness of regulation depends on the risk taking of the banking system as a whole.

As discussed earlier, a bank's ability to engage in risk taking depends not only on supervisory policy, but also on its own ability to make its balance sheet opaque. Let us capture this idea by taking the supervisory effort as a given and assume that each bank indexed by $i \in [0,1]$ can (locally) select its individual level of foreign holdings $b_0^*(i)$ at non-monetary cost $\Psi(b_0^*(i))$, a strictly decreasing and convex function. In other words, we replace the minimum diversification requirement $\underline{b}_0^*$ by the cost Ψ, which now encodes the supervisory effort of the government. We look for a symmetric equilibrium in which all banks choose the same $b_0^*(i) = b_0^*$ for all i. For simplicity, we focus on fiscal shocks and assume that $I(s) = \bar{I}$ is independent of s. We also assume that $A = \bar{I}$.

The banks' choices of opaqueness, and thereby the exposures to the domestic government, are strategic complements: incurring the cost of making one's balance sheet more opaque is more tempting if the put on taxpayer money is more attractive; in turn, this put is attractive when the sovereign bond price is more volatile, which it is when the other banks take a larger gamble.

To show this, note that for an individual bank i, given an aggregate b_0^*, the payoff from investing $b_0^*(i)$ is

$$V_0^B(b_0^*(i); b_0^*) = \int \rho_1(s)\bar{I}d\pi(s) + \int_{\tilde{s}}^{\infty} (A - b_0^*(i))(\frac{p_1(s)}{p_0} - 1)d\pi(s) - \Psi(b_0^*(i)),$$

where we have left the dependence of p_0 and $p_1(s)$ on b_0^* implicit.

Proposition 3 (Strategic Complementarities in Banks' Domestic Exposures)**.** *Suppose that $I(s) = \bar{I}$ is independent of s, and that $A = \bar{I}$. There are strategic complementarities across banks in the choice of $b_0^*(i)$, i.e. the marginal benefit $\frac{\partial V_0^B(b_0^*(i);b_0^*)}{\partial b_0^*(i)}$ for a bank of increasing its individual investment $b_0^*(i)$ in foreign bonds is increasing in the aggregate investment b_0^* of banks in foreign bonds.*

Proof. Denote by ϵ the random variables $\frac{p_1(s)}{p_0}$. For a given aggregate b_0^*, the random variable x follows some distribution $H(\epsilon)$ such that $\int_0^1 (1-\epsilon)dH(\epsilon) = \int_1^{\infty}(\epsilon - 1)dH(\epsilon)$. For an individual bank i, the payoff from investing $b_0^*(i)$ is

$$V_0^B(b_0^*(i); b_0^*) = \int \rho_1(s)Ad\pi(s) + \int_1^{\infty} (A - b_0^*(i))(\epsilon - 1)dH(\epsilon) - \Psi(b_0^*(i)).$$

The marginal benefit of reducing $b_0^*(i)$ given by

$$-\frac{\partial V_0^B(b_0^*(i);b_0^*)}{\partial b_0^*(i)} = \int_1^\infty (\epsilon - 1) dH(\epsilon) + \Psi'(b_0^*(i)).$$

Now consider two aggregate level b_0^* and $b_0^{*\prime}$ with $b_0^* > b_0^{*\prime}$ with associated prices p_0, $p_1(s)$, p_0', $p_1'(s)$ and distributions H and H'. Let $\tilde{s}$ be such that $\frac{p_1(\tilde{s})}{p_0} = 1$ (and so bailouts occur if and only if $s < \tilde{s}$). We proceed in two steps.

In the first step, we prove that $p_0' < p_0$, $p_1'(s) = p_1(s)$ for $s \geq \tilde{s}$, and $\frac{p_1'(s)}{p_0'} > \frac{p_1(s)}{p_0}$ for $s \geq \tilde{s}$. Indeed, the price $p_1(s)$ is a locally stable solution of the following fixed-point equation

$$p_1(s) = 1 - F(B_0 + (A - b_0^*) \max\{\frac{1}{p_1(s)} - \frac{1}{p_0}, 0\} | s).$$

Towards a contradiction, suppose that $p_0' \geq p_0$. Then for any $p_1(s)$, the right-hand side of the above equation decreases when b_0^* is replaced by $b_0^{*\prime}$. Hence $p_1'(s) < p_1(s)$ decreases for all s, and strictly decreases for $s < \tilde{s}$. This contradicts the martingale property of prices. This proves that $p_0' < p_0$. For all $s \geq \tilde{s}$, $\frac{p_1(\tilde{s})}{p_0'} > \frac{p_1(\tilde{s})}{p_0} \geq 1$. Hence for all $s \geq \tilde{s}$ the pre-bailout net worth of banks satisfies $b_0^{*\prime} + (A - b_0^{*\prime})\frac{p_1(\tilde{s})}{p_0'} > b_0^* + (A - b_0^*)\frac{p_1(\tilde{s})}{p_0}$. This in turn implies that it still the case that there are no bailouts for $s > \tilde{s}$ when aggregate debt is $b_0^{*\prime}$. By implication, $p_1'(s) = p_1(s)$ is the same for $s \geq \tilde{s}$.

In the second step, we use the first step to get

$$\int_1^\infty (\epsilon - 1) dH'(\epsilon) \geq \int_1^\infty (\frac{p_0}{\tilde{p}_0}\epsilon - 1) dH(\epsilon) > \int_1^\infty (\epsilon - 1) dH(\epsilon).$$

The incentive to marginally reduce $b_0^*(i)$ is therefore higher when the aggregate foreign debt level is $b_0^{*\prime}$ than when it is b_0^*:

$$-\frac{\partial V_0^B(b_0^*(i);b_0^{*\prime})}{\partial b_0^*(i)} > -\frac{\partial V_0^B(b_0^*(i);b_0^*)}{\partial b_0^*(i)}.$$

□

As is well understood, depending on the exact shape of the cost function Ψ, these strategic complementarities can lead to multiple equilibria: equilibria with low exposure of domestic banks to domestic sovereign default risk (high b_0^*) and equilibria with high exposure of domestic banks to domestic sovereign default risk (low b_0^*). This is the manifestation of a collective moral hazard problem as in Farhi-Tirole (2012).[10] Because this is

[10]In Farhi-Tirole (2012), we study a related model where the combination of limited commitment on

not the focus of this paper, we simply illustrate this possibility with a simple example but we do not develop this theme further.

Illustrating example. We consider the simple example introduced in Section 2.1. We assume that $(E - B_0)x > (x+y)(e - B_0)$ so that the revenue maximizing level of debt $B_1(L)$ in state L is E. We assume throughout that $(-\Psi')^{-1}(\frac{\pi(1-\pi)(1-\theta)}{\pi+(1-\pi)\theta}) \in (0, A)$ for $\theta \in \{x, x+y\}$.

There are two possible equilibria depending on whether $B_1(L) \leq e$ or $B_1(L) > e$, which determines the probability θ of repayment in state L. When $B_1(L) \leq e$, we have $\theta = x + y$, and when $B_1(L) > e$, we have $\theta = x$. And prices are given by $p_1(L) = \theta$, $p_0 = \pi + (1-\pi)\theta$.

The welfare of a banker i who invests $b_0^*(i)$ is

$$\pi[\rho_1 A + (A - b_0^*(i))(\frac{1}{p_0} - 1)] + (1-\pi)\rho_1 A = \rho_1 A + \pi(A - b_0^*(i))\frac{(1-\pi)(1-\theta)}{\pi + (1-\pi)\theta} - \Psi(b_0^*(i)).$$

In order for banking entrepreneurs to choose $b_0^* \in (0, A)$, we must have

$$-\Psi'(b_0^*) = \frac{\pi(1-\pi)(1-\theta)}{\pi + (1-\pi)\theta}.$$

The debt issuance condition is then

$$B_1(L) = B_0 + \Phi(\theta),$$

where Φ is a decreasing function defined by

$$\Phi(\theta) = \frac{1}{\theta}\frac{\pi(1-\theta)}{\pi + (1-\pi)\theta}[A - (-\Psi')^{-1}(\frac{\pi(1-\pi)(1-\theta)}{\pi + (1-\pi)\theta})].$$

We have an equilibrium with $B_1(L) \leq e$ if and only if

$$\Phi(x+y) \leq e - B_0. \tag{5}$$

the part of the government, and ex-post untargeted bailouts gives rise to strategic complementarities in financial risk-taking, and provides a rationale for macroprudential regulation. The main difference here is that bailouts are perfectly targeted. Here there are also strategic complementarities in financial risk-taking, which justify macroprudential regulation, but through a different, general equilibrium effect on the pricing of debt and the occurrence of default. This is also a difference with other papers emphasizing strategic complementarities arising from bailout guarantees, such as Schneider and Tornell (2004), Acharya and Yorulmazer (2008), and Ranciere, Tornel and Westerman (2008), which assume that bailouts are extended when sufficiently many banks are in trouble.

Similarly, we have an equilibrium with $B_1(L) > e$ if and only if

$$e - B_0 < \Phi(x) \leq E - B_0. \tag{6}$$

The two equilibria coexist if and only if

$$E - B_0 \geq \Phi(x) > e - B_0 \geq \Phi(x+y). \tag{7}$$

Proposition 4 (Multiple Equilibria)**.** *In the illustrating example, there are two possible equilibria. There is an equilibrium with low diversification* $b_0^* = (-\Psi')^{-1}(\frac{\pi(1-\pi)(1-x)}{\pi+(1-\pi)x})$ *and a high probability of default* $(1-\pi)(1-x)$*, which exists if and only if condition* (5) *is verified. There is also an equilibrium with high diversification* $b_0^* = (-\Psi')^{-1}(\frac{\pi(1-\pi)(1-x-y)}{\pi+(1-\pi)(x+y)})$ *and a low probability of default* $(1-\pi)(1-x-y)$*, which exists if and only if condition* (6) *is verified. The two equilibria coexist if and only if condition* (7) *is verified.*

Because the function Φ is decreasing, we can always find values of B_0, E and e such that condition (7) is verified so that there can be multiple equilibria for a range of parameter values. These multiple equilibria are a consequence of the strategic complementarities in the banks' individual exposures to domestic sovereign default risk.

This examples also has other interesting implications.

Proposition 5 (Multiple Equilibria and Debt Renationalization)**.** *In the illustrating example, for* $B_0 \in (0, E - \Phi(x))$*, the equilibrium with low diversification and high probability of default is more likely to exist, the higher is legacy debt* B_0 *and the lower is fiscal capacity (proxied by the intermediate value of the endowment* e*). Conversely, the equilibrium with high diversification and low probability of default is more likely to exist, the lower is legacy debt and the higher is fiscal capacity.*

This shows a precise sense in which high values of legacy debt or a reduction in fiscal capacity can lead to debt re-nationalization and offers a possible explanation for the well-known fact that a re-nationalization of sovereign debt was observed in Europe as the recent crisis intensified.[11] Here this is due to the imperfect ability of the government to limit the exposure of banks to domestic sovereign default risk through regulation. The rationale for re-nationalization is based on the idea that sovereign bonds are more attractive to banks in bad times. But in bad times monitoring banks is also more attractive to the regulator. Proposition 5 nonetheless would still hold as long as the regulatory capability does not adjust rapidly with the state of nature.

[11] See Broner et al (2013), Genaioli et al (2013) and Uhlig (2014) for careful documentations.

We return to this issue in Sections 4.2 and 5.2. In Section 4.2, we propose a different mechanism for debt re-nationalization which relies on the desirability for the government to allow banks to load up on domestic sovereign default risk in order to push legacy creditors to forgive some debt, even if the government can perfectly regulate the banking system. In Section 5.2, we uncover an opposing mechanism based on limits to the capacity of the government to bail out the banking system.

4 Debt Forgiveness, Lax Regulation, and Banking Unions

In this section, we investigate the possibility of debt forgiveness at date 1. We show that this can give rise to an incentive for lax regulation whereby the domestic government allows its banks to take on domestic sovereign risk exposure in order to extract concessions from legacy creditors. If the ex-post leniency of domestic regulators is anticipated ex ante at the time of sovereign debt issuance, then it is priced in the form of higher spreads. The government is better off committing ex ante to a tough ex-post regulatory stance, but is tempted to relax it ex post. If the government lacks commitment, then it benefits from relinquishing its regulatory powers to a supranational supervisor by joining a banking union.

4.1 Debt Forgiveness

We model date-1 debt forgiveness which we model as follows. We assume that after the state of nature s is observed at date 1, international investors can forgive some of the legacy debt to $\tilde{B}_0 \leq B_0$, before the government undertakes the bailout policy.

We show that it can be in the interest of legacy creditors (international investors who hold the legacy debt $B_0 - \frac{A-b_0^*}{p_0}$) to forgive some of the debt at date 1, bringing the overall stock of legacy debt to $\tilde{B}_0 \leq B_0$, even if banks free-ride and do not forgive any debt.[12] In other words, there is a legacy Laffer curve, and it is possible for legacy debt B_0 to be on the wrong side of the legacy Laffer curve, i.e. $\frac{d(p_1(s;\tilde{B}_0)(\tilde{B}_0-b_0^*))}{d\tilde{B}_0}|_{\tilde{B}_0=B_0} < 0$ where we have made the dependence of the date-1 price of debt $p_1(s;\tilde{B}_0)$ on the post-debt forgiveness debt stock $B_0(s)$ explicit. Moreover, we show that the feedback loop between sovereign and financial balance sheets that we have characterized in Section 3 makes it more likely that the economy is on the wrong side of the Laffer curve.

[12]Of course organizing debt forgiveness requires coordination among legacy creditors to neutralize the free-riding incentives of individual creditors.

We can compute the sensitivity of the value $p_1(s;\tilde{B}_0)(\tilde{B}_0 - \frac{A-b_0^*}{p_0})$ of legacy debt with respect to (post-debt forgiveness) legacy debt $\tilde{B}_0$:

$$\frac{d[p_1(s;\tilde{B}_0)(\tilde{B}_0 - \frac{A-b_0^*}{p_0})]}{d\tilde{B}_0} = p_1(s;\tilde{B}_0) - (\tilde{B}_0 - \frac{A-b_0^*}{p_0})\frac{f(B_1(s)|s)}{1 - \frac{I(s)-b_0^*}{p_1^2(s;\tilde{B}_0)} f(B_1(s)|s)}. \tag{8}$$

The first term on the left-hand-side of equation (8) is the direct quantity-of-debt effect of debt forgiveness. Because of this effect, marginal debt forgiveness $d\tilde{B}_0 < 0$ contributes negatively to the value $p_1(s;\tilde{B}_0)[\tilde{B}_0 - \frac{A-b_0^*}{p_0}]$ of the claims of legacy creditors. The second term on the left-hand-side of equation (8) is the indirect price-of-debt effect of forgiveness. Because of this effect, debt forgiveness $d\tilde{B}_0 < 0$ contributes positively to the value $p_1(s;\tilde{B}_0)(\tilde{B}_0 - \frac{A-b_0^*}{p_0})$ of the claims of legacy creditors. The net effect of debt forgiveness depends on the relative strength of these two effects.

The indirect price-of-debt effect of debt forgiveness is stronger, the more elastic is the price $p_1(s;\tilde{B}_0)$ to the amount of legacy debt held by international investors $\tilde{B}_0 - \frac{A-b_0^*}{p_0}$. And the feedback loop between sovereign and financial balance sheets that we have characterized in Section 3 works precisely to increase this elasticity. Indeed, debt forgiveness increases the date-1 price of debt, which improves the balance sheets of banking entrepreneurs, reducing the size of the bailout, and hence reducing the need for the government to engage in additional borrowing at date 1, which reduces the probability of default and further increases the date-1 price of debt, etc. ad infinitum. The feedback loop therefore makes the price-of-debt effect more potent, without affecting the quantity-of-debt effect, therefore pushing the economy towards the decreasing part of the legacy Laffer curve $p_1(s;\tilde{B}_0)(\tilde{B}_0 - \frac{A-b_0^*}{p_0})$.

When some debt forgiveness can improve the outcome of the legacy creditors, a mutually beneficial negotiation can take place between legacy creditors and the domestic government. We assume that domestic banks free-ride on this renegotiation. The outcome of the negotiation depends on the ability of legacy creditors to coordinate and on the distribution of bargaining power between legacy creditors and the domestic government. We assume that legacy creditors are able to coordinate, and have all the bargaining power: They collectively make a take-it-or-leave-it offer to the domestic government.

We can rewrite equation (8) as

$$\frac{1}{p_1(s;\tilde{B}_0)}\frac{d[p_1(s;\tilde{B}_0)(\tilde{B}_0 - \frac{A-b_0^*}{p_0})]}{d\tilde{B}_0} = 1 - (\tilde{B}_0 - \frac{A-b_0^*}{p_0})\frac{\frac{f(B_1(s)|s)}{1-F(B_1(s)|s)}}{1 - \frac{I(s)-b_0^*}{1-F(B_1(s)|s)}\frac{f(B_1(s)|s)}{1-F(B_1(s)|s)}}. \tag{9}$$

The best outcome that legacy creditors can achieve corresponds to the peak of the legacy Laffer curve

$$\frac{d(p_1(s;\tilde{B}_0)(\tilde{B}_0 - \frac{A-\underline{b}_0^*}{p_0}))}{d\tilde{B}_0}|_{\tilde{B}_0=B_0(s)} = 0. \tag{10}$$

Proposition 6 (Legacy Laffer Curve and Debt Forgiveness)**.** *Suppose that there are only fiscal shocks so that $I(s)$ is independent of the state s. Then for every state s, the peak of the legacy Laffer curve $B_0(s) \leq B_0$ is increasing in s so that worse states are associated with more debt forgiveness.*

Proof. We can rewrite equation (10) as

$$\frac{f(\frac{I(s)-\underline{b}_0^*}{p_1(s;B_0(s))} - \frac{A-\underline{b}_0^*}{p_0} + B_0(s)|s)}{1-F(\frac{I(s)-\underline{b}_0^*}{p_1(s;B_0(s))} - \frac{A-\underline{b}_0^*}{p_0} + B_0(s)|s)}(B_0(s) - \frac{A-\underline{b}_0^*}{p_0}) =$$

$$1 - \frac{I(s)-\underline{b}_0^*}{1-F(\frac{I(s)-\underline{b}_0^*}{p_1(s;B_0(s))} - \frac{A-\underline{b}_0^*}{p_0} + B_0(s)|s)} \frac{f(\frac{I(s)-\underline{b}_0^*}{p_1(s;B_0(s))} - \frac{A-\underline{b}_0^*}{p_0} + B_0(s)|s)}{1-F(\frac{I(s)-\underline{b}_0^*}{p_1(s;B_0(s))} - \frac{A-\underline{b}_0^*}{p_0} + B_0(s)|s)}. \tag{11}$$

When $I(s)$ is independent of s, the left-hand side of this equation is increasing in $B_0(s)$ while the right-hand side is decreasing in $B_0(s)$. Hence the equation has a unique solution in $B_0(s)$ which characterizes corresponds to the global maximum of the legacy Laffer curve $p_1(s;B_0(s))(B_0(s) - \frac{A-\underline{b}_0^*}{p_0})$. The result follows easily from the monotone hazard rate assumption. □

4.2 Strategic Regulatory Leniency

The possibility of a legacy Laffer curve can make it optimal for the government to set $\underline{b}_0^* < \bar{I}$ even when regulation is frictionless, because it allows to extract larger concessions from legacy creditors. Another way to put this is that the government might have incentives to let its domestic banks load up on domestic sovereign debt in order to extract concessions from legacy creditors. We illustrate this possibility with a simple example.

Illustrating example. We consider the simple example introduced in Section 2.1. Recall that in this example, $I(s) = \bar{I}$ is independent of s and $A = \bar{I}$. We assume that $e(1+\frac{y}{x}) > B_0 > e$ and that $\beta^I(s) = \beta^I$ is independent of s. We now proceed to construct an equilibrium where it is optimal for the government to set $\underline{b}_0^*$ in order to obtain concessions from legacy creditors.

There is no debt forgiveness in state H and no default. At date 1, in state L, legacy creditors either forgive no debt so that $B_0(L) = B_0$ or forgive debt $B_0(L) < B_0$ in the following amount

$$B_0(L) + \frac{[1 - \frac{x+y}{p_0}](A - \underline{b}_0^*)}{x+y} = e, \tag{12}$$

in which case $B_1(L) = e$. There is debt forgiveness provided that when $B_0(L)$ is defined by equation (12), the following conditions is verified:[13]

$$(x+y)[B_0(L) - \frac{A - \underline{b}_0^*}{p_0}] \geq x[B_0 - \frac{A - \underline{b}_0^*}{p_0}]. \tag{13}$$

In this case, p_0 is given by

$$p_0 = \pi + (1-\pi)(x+y)\frac{B_0(L) - \frac{A-\underline{b}_0^*}{p_0}}{B_0 - \frac{A-\underline{b}_0^*}{p_0}}, \tag{14}$$

which using equation (12), can be rewritten as

$$p_0 = \pi + (1-\pi)(x+y)\frac{e - \frac{A-\underline{b}_0^*}{x+y}}{B_0 - \frac{A-\underline{b}_0^*}{p_0}}. \tag{15}$$

This equation has a unique solution (the left-hand side is increasing in p_0 while the right-hand side is decreasing in p_0), which defines a function $p_0(\underline{b}_0^*; \pi, B_0)$ which is increasing in $\underline{b}_0^*$, decreasing in B_0 and increasing in π.[14,15,16]

[13] Even though this is not crucial for our result, recall that banks are able to free ride on the renegotiation. This means that banks get a higher expected return on their holdings of domestic sovereign debt than foreigners. The minimum diversification requirement $\underline{b}_0^*$ prevents them from exploiting that advantage any further.

[14] The function $p_0(\underline{b}_0^*; \pi, B_0)$ is locally increasing in $\underline{b}_0^*$ if and only if $e(x+y) < p_0(\underline{b}_0^*; \pi, B_0)B_0$. It is easy to see that this inequality automatically holds when $\underline{b}_0^* = A$. This implies that it holds for all $\underline{b}_0^*$. Indeed, suppose that there exists $\underline{b}_0^* < A$ such that $e(x+y) > p_0(\underline{b}_0^*; \pi, B_0)B_0$. Then as we increase $\underline{b}_0^*$ from that point towards A, $p_0(\underline{b}_0^*; \pi, B_0)$ keeps decreasing and hence $e(x+y) > p_0(\underline{b}_0^*; \pi, B_0)B_0$ keeps being verified, a contradiction. Therefore $e(x+y) \leq p_0(\underline{b}_0^*; \pi, B_0)B_0$ for all $\underline{b}_0^*$. This in turn implies that $p_0(\underline{b}_0^*; \pi, B_0)$ is increasing in $\underline{b}_0^*$.

[15] That the function is decreasing in B_0 follows from the fact that the left-hand side of equation (15) is increasing in p_0 and independent of B_0, while the right-hand side is decreasing in p_0 and decreasing in B_0.

[16] That the function is increasing in π follows from the fact that the left-hand side of equation (15) is increasing in p_0 and independent of π, while the right-hand side is decreasing in p_0 and increasing in π. To see that the right-hand side of equation (15) is increasing in π, rewrite the right-hand side using equation (15) as $\pi + (1-\pi)(x+y)\frac{B_0(L) - \frac{A-\underline{b}_0^*}{p_0}}{B_0 - \frac{A-\underline{b}_0^*}{p_0}}$ where $B_0(L) \leq B_0$.

To summarize, using equations (12) and (13), there is debt forgiveness in state L if

$$\frac{x}{x+y}B_0 + \frac{[1-\frac{x}{p_0(\underline{b}_0^*;\pi,B_0)}](A-\underline{b}_0^*)}{x+y} \leq e, \tag{16}$$

which is always satisfied for $\underline{b}_0^* = A$.

It is then always optimal for the government to choose at date 0 the lowest value of $\underline{b}_0^*$ that satisfies equation (16) in order to maximize welfare

$$\begin{aligned} &W_0 = \pi(E-B_0) + (1-\pi)[x(E-e) - (1-x-y)\Phi] \\ &+\beta^I A + \beta^B[\rho_1 A + \pi(A-\underline{b}_0^*)(\frac{1}{p_0(\underline{b}_0^*;\pi,B_0)} - 1) + (1-\pi)(A-\underline{b}_0^*)(\frac{x+y}{p_0(\underline{b}_0^*;\pi,B_0)} - 1)]. \end{aligned} \tag{17}$$

Proposition 7 (Strategic Regulatory Leniency)**.** *In the illustrating example, it is optimal for the government to set $\underline{b}_0^* < A = \bar{I}$ and equal to the smallest value that satisfies equation* (16)*. The optimal value of $\underline{b}_0^*$ is decreasing in the probability $1-\pi$ of the occurrence of a bad fiscal shock (state L) where a debt renegotiation takes place.*

Proof. Consider $\pi' < \pi$, and let $\underline{b}_0^*$ satisfy equation (16) when the probability is π. Then $\underline{b}_0^*$ also satisfies equation (16) when legacy debt is π'. Hence the optimal value of $\underline{b}_0^*$ when the probability is π' is smaller than the optimal value of $\underline{b}_0^*$ when the probability is π, and so the optimal value of $\underline{b}_0^*$ is increasing in π (and hence decreasing in $1-\pi$). □

Proposition 7 shows that in the illustrating example, it is optimal for the government to set $\underline{b}_0^* < A = \bar{I}$ and allow domestic banks to take on domestic debt, and risk needing a bailout when the government experiences a bad fiscal shock. This allows the government to extract more concessions from legacy creditors. The government reduces the diversification requirement (lowers $\underline{b}_0^*$) when the probability $1-\pi$ of a bad fiscal shock where a debt renegotiation takes place because it makes it more attractive to extract concessions from legacy creditors.

This offers a possible explanation for the well-known fact that a re-nationalization of sovereign debt was observed in Europe as the recent crisis intensified. In Section 3.3, we propose a different mechanism based on the imperfect ability of the government to limit the exposure of banks to domestic sovereign default risk through regulation.

4.3 A Rationale for a Banking Union

In the illustrating example developed in Section 4.2, foreign investors are made worse off by the relaxation of regulation of domestic banks by the domestic government. Once they have lent, their welfare is maximized by a tough regulation $\underline{b}_0^* = A = \bar{I}$. Of course their welfare is adversely impacted only if this relaxation of regulation is not anticipated at the time of the debt issuance, otherwise it is fully priced in. Interestingly, in this case, domestic welfare can be increased by a tough regulation $\underline{b}_0^* = A = \bar{I}$ because of its positive effects on the issuance price of date-0 debt. But this requires commitment on the part of the domestic government not to relax regulation after the debt is issued. A banking union can help deliver the commitment outcome.

Indeed, building on the illustrating example of Section 4.2, consider the debt level $\tilde{B}_0 < B_0$ that generates the same amount of revenue at date 0 when the diversification requirement is A as the debt level B_0 when the diversification requirement is $\underline{b}_0^*$. This debt level is defined implicitly by

$$p_0(\underline{b}_0^*; \pi, B_0)B_0 = p_0(A; \pi, \tilde{B}_0)\tilde{B}_0,$$

where we assume that the solution of this equation satisfies $e(1 + \frac{t}{r}) > \tilde{B}_0 > e$. The associated level of date-0 welfare $\tilde{W}_0$ is given by

$$\tilde{W}_0 = \pi(E - \tilde{B}_0) + (1 - \pi)[x(E - e) - (1 - x - y)\Phi] + \beta^I A + \beta^B \rho_1 A,$$

which is guaranteed to be greater than W_0.[17]

This requires commitment on the part of the domestic government since once the date-0 debt $\tilde{B}_0$ has been issued at price $p_0(A, \tilde{B}_0)$, the government faces the temptation to renege and lower the diversification requirement $\underline{b}_0^*$. Foreigners are powerless to resist the re-nationalization of domestic debt unless they are able to coordinate not to sell their domestic sovereign bonds to domestic banks, which unlike debt relief negotiations, seems to have few real world counterparts. [18]

One of the important aspects of banking unions is the transfer of banking regulation

[17] This is immediate since under commitment and no commitment, all investments are financed, defaults occur in the same states, and foreigners are as well off. As a result, the sum of consumer welfare and banking entrepreneur welfare is the same under commitment and no-commitment $\tilde{V}_0^C + \tilde{V}_0^B = V_0^C + V_0^B$. However the welfare of bankers is higher and that of consumers lower under no commitment $V_0^B > \tilde{V}_0^B$ and $V_0^C < \tilde{V}_0^C$. Because $\beta^B < 1$, this implies that $\tilde{W}_0 = \tilde{V}_0^C + \beta^B \tilde{V}_0^B + \beta^I A - (1 - \pi)(1 - x - y)\Phi$ is greater than $W_0 = \tilde{V}_0^C + \beta^B \tilde{V}_0^B + \beta^I A - (1 - \pi)(1 - x - y)\Phi$.

[18] To the extent that foreign investors are located in different countries, foreign national regulators would also need to coordinate in order to facilitate this outcome.

from the national to the supranational level. Such a transfer weakens or removes the temptation of domestic governments to strategically allow their banks to load up on domestic sovereign bonds to extract larger concessions from legacy creditors. It can therefore facilitate the implementation of the commitment solution with a high diversification requirement $\underline{b}_0^* = A$. This is because the international regulator's objective function naturally puts more weight on international investors than the domestic government, making it less tempting to relax regulation ex post.[19] To make this point starkly, we study the limit where the supranational regulator puts full weight on international investors and no weight on domestic agents. In this limit, the commitment solution is implemented.

Proposition 8 (Banking Union)**.** *In the illustrating example, if the relaxation of regulation is fully priced in by international investors at the time of the issuance of date−0 debt, then the domestic government faces a time-inconsistency problem. It is made better off by promising to implement a high diversification requirement $\underline{b}_0^* = A = \bar{I}$ before issuing debt at date 0, but it is tempted to relax this requirement after the issuance. A banking union removes this temptation and improves welfare from W_0 to $\tilde{W}_0 > W_0$.*

With frictionless regulation as assumed in the illustrating example above, the banking union completely shuts down the feedback loop between banks and sovereign. We believe that the insight is likely to be more general, so that even when regulation is not frictionless, the banking union leads to toughening of regulation and a weakening of the feedback loops between banks and sovereigns.

5 Extensions

In this Section we consider a number of extensions of the basic model presented in Sections 2 and 3. We investigate in turn the role of leverage, limited bailouts, sovereign debt maturity, and foreign banks. Unless stated otherwise, we do not consider the possibility of debt forgiveness (either because debt is on the upward-sloping side of the legacy Laffer curve, or because debt is on the wrong side of the legacy Laffer curve but investors have difficulties coordinating on a debt relief package).

5.1 The Role of Leverage

In this section, we introduce leverage into the model. We assume that a fraction $\rho_0(s)I(s)$ of the return $\rho_1(s)I(s)$ is pledgeable to outside international investors at date 1. Banking

[19] Another possibility is that the international regulator has a better ability to commit to regulation than the domestic government.

entrepreneurs can now raise $\rho_0(s)$ units of funds per unit of investment at date 1. Consistent with our previous assumptions, we assume that financial claims on $\rho_0(s)$ are issued abroad. This can be accommodated by our formalization along the lines of Holmström-Tirole (1997) (see Appendix A).

Leverage and financial shocks. Because banking entrepreneurs can lever up, they only need a net worth of $I(s)(1-\rho_0(s))$ in order to invest $I(s)$. As a result, the required bailout is now

$$X(s) = \max\{I(s)(1-\rho_0(s)) - (b_0^* + p_1(s)b_0), 0\}.$$

The pricing equation (2) is unchanged, leading to the following fixed-point for the date-1 price $p_1(s)$ of government bonds

$$p_1(s) = 1 - F(B_1(s)|s),$$

where

$$B_1(s) = B_0 + \max\{\frac{I(s)(1-\rho_0(s)) - b_0^*}{p_1(s)} - \frac{A - b_0^*}{p_0}, 0\}.$$

Proposition 9 (Feedback Loop and Leverage)**.** *When a fraction $\rho_0(s)$ of the date-2 return of the investment project of banking entrepreneurs is pledgeable, the sensitivity of date-1 bond prices $p_1(s)$ to the state s when a bailout is required is given by*

$$\frac{dp_1(s)}{ds} = \frac{-\frac{\partial F(B_1(s)|s)}{\partial s} - \frac{1}{p_1(s)} f(B_1(s)|s) \frac{d[(1-\rho_0(s))I(s)]}{ds}}{1 - \frac{I(s)(1-\rho_0(s)) - b_0^*}{p_1^2(s)} f(B_1(s)|s)}.$$

Proposition 9 extends Proposition 1 to the case where leverage is positive. The main difference is that the financing needs $I(s)$ are replaced by $I(s)(1-\rho_0(s))$. This is simply because banking entrepreneurs can leverage every unit of bailout with private funds by borrowing $\rho_0(s)$ units of funds from international investors.

Joint defaults. So far we have ignored the possibility that private debt contracts of banking entrepreneurs might be defaulted upon. In other words, we have assumed that the enforcement of private debt contracts is perfect. In reality, whether or not to enforce private contracts is to a large extent a decision by the domestic government. And the decisions to enforce private debt contracts and to repay sovereign debt tend to be correlated. After all, not enforcing private debt contracts is another way for the government to default on the country's obligations.[20] We capture this idea by assuming that the costs of not enforcing debt contracts and to default on sovereign debt take the form of a single fixed cost. This feature builds in a complementarity between the two decisions. As a result, sovereign defaults come together with defaults on the private debt contracts issues by banking entrepreneurs, resulting in a positive correlation between bank and sovereign spreads.

Private debt contracts are priced fairly and reflect the probability that they will not be enforced. As a result, leverage becomes endogenous. Entrepreneurs can raise $\rho_0(s)p_1(s)$ units of funds per unit of investment. The fact that the debt that they raise bears enforcement risk limits their ability to raise funds at date 1, and increases the size of the required bailout to

$$X(s) = \max\{I(s)(1-\rho_0(s)p_1(s)) - (b_0^* + p_1(s)b_0), 0\}.$$

The pricing equation (2) is unchanged, leading to the following fixed-point for the date-1 price $p_1(s)$ of government bonds

$$p_1(s) = 1 - F(B_1(s)|s),$$

where

$$B_1(s) = B_0 + \max\{\frac{I(s)(1-\rho_0(s)p_1(s)) - \underline{b}_0^*}{p_1(s)} - \frac{A - \underline{b}_0^*}{p_0}, 0\}.$$

Proposition 10 (Feedback Loop and Joint Defaults)**.** *When a fraction $\rho_0(s)$ of the date-2 return of the investment project of banking entrepreneurs is pledgeable and private debt contracts are defaulted upon when there is a sovereign default, the sensitivity of date-1 bond prices $p_1(s)$ to the state s when a bailout is required is given by*

$$\frac{dp_1(s)}{ds} = \frac{-\frac{\partial F(B_1(s)|s)}{\partial s} - \frac{1}{p_1(s)} f(B_1(s)|s) \frac{d[(1-\rho_0(s)p_1(s))I(s)]}{ds}}{1 - \frac{I-\underline{b}_0^*}{p_1^2(s)} f(B_1(s)|s)}.$$

There are two key differences between Proposition 10 and Proposition 9. The first difference is that the second term in the numerator is now $-\frac{1}{p_1(s)} f(B_1(s)|s) \frac{d[(1-\rho_0(s)p_1(s))I(s)]}{ds}$

[20] In our model, private financial contracts are between domestic agents (banking entrepreneurs) and foreign agents (international investors). A more general model would also feature private financial contracts between domestic agents. To the extent that enforcement decisions cannot discriminate between contracts based on the identities of the parties to the contract, this introduces a potential additional costs to the decision of not enforcing private contracts. These costs are both ex-post in the form of undesirable redistribution and ex-ante in the form of a reduction in private trade between domestic agents (see e.g. Broner and Ventura 2011). We purposefully stay away from these fascinating issues, which are not the focus of this paper.

instead of $-\frac{1}{p_1(s)} f(B_1(s)|s) \frac{d[(1-\rho_0(s))I(s)]}{ds}$, reflecting the dependence of the liquidity needs of banking entrepreneurs on $p_1(s)$ through the pledgeability of returns and leverage. The second difference is in the denominator. For given values of the date-1 bond price $p_1(s)$, of the reinvestment need $I(s)$, of the bailout $X(s)$, and hence of date-1 debt $B_1(s)$, the denominator is now smaller at $1 - \frac{I(s)-b_0^*}{p_1^2(s)} f(B_1(s)|s)$ instead of $1 - \frac{I(s)(1-\rho_0(s)p_1(s))-b_0^*}{p_1^2(s)} f(B_1(s)|s)$. As a result the sensitivity $\frac{dp_1(s)}{ds}$ of the price $p_1(s)$ to the state s is larger.

The feedback loop is stronger, because of a new mechanism through the endogenous leverage of banks. As sovereign risk rises, banks have to reduce leverage. This is because banks' borrowing spreads increase, reflecting the increased probability of a default on the private debt that they issue rises. This requires a larger bailout, which puts further pressure on the government budget etc. ad infinitum.

5.2 Limited Bailouts and Endogenous Diversification

So far, we have maintained the assumption that no matter what portfolios banks hold, the government can always raise enough funds at date 1 to bail them out completely. We now relax this assumption. We show that when the government's ability to bail out the banking system is limited, banks naturally limit their exposure to domestic sovereign default risk.

To simplify, we assume that $I(s) = \bar{I}$ is independent of s so that there are no financial shocks but only fiscal shocks. Because $A \geq \bar{I}$, if banks choose $b_0^* = \bar{I}$, they do not need a bailout. But we assume that there are some states of the world where the government is not able to fully bail out banks if they choose $b_0^* = \underline{b}_0^*$. In states of the world s where funds are insufficient to bail out all the banks, the government optimally bails out as many banks as possible, saving first the banks with the highest pre-bailout net worth. This pecking order maximizes the number of banks that can be saved and hence ex-post welfare.

While banks are ex-ante identical, equilibria can be asymmetric. We therefore look for an equilibrium in which banking entrepreneurs invest different amounts in foreign bonds, according to a probability distribution with g with support contained in $[\underline{b}_0^*, I]$. This probability distribution g is an endogenous object, to be solved for as part of the equilibrium. It might be a degenerate atom, in which case the equilibrium is symmetric.

In every state s, there is an endogenous threshold $b_0^*(s)$ such that banking entrepreneurs with $b_0^* \geq b_0^*(s)$ secure enough post-bailout funds to finance their investment. This threshold is monotonically decreasing in s. There is also an endogenous threshold $\tilde{b}_0^*(s) \geq b_0^*(s)$ such that banking entrepreneurs with $b_0^* \geq \tilde{b}_0^*(s)$ can finance their investment without any bailout. This threshold is defined by $\bar{I} - \tilde{b}_0^*(s) - (A - \tilde{b}_0^*(s))\frac{p_1(s)}{p_0} = 0$, and is also monotonically decreasing in s.

In states where $b_0^*(s) > \underline{b}_0^*$ so that bailouts are partial, the following bailout equations must hold

$$\int_{b_0^* \in [b_0^*(s), \tilde{b}_0^*(s))} (A - b_0^*)\frac{1}{p_0} dg(b_0^*) = \frac{p_1(s)}{f(B_1(s)|s)} - [B_1(s) - B_0], \tag{18}$$

$$\int_{b_0^* \in [b_0^*(s), \tilde{b}_0^*(s))} [\bar{I} - b_0^* - (A - b_0^*)\frac{p_1(s)}{p_0}] dg(b_0^*) = [B_1(s) - B_0] p_1(s),$$

where

$$B_1(s) = B_0 + \int_{b_0^* \in [b_0^*(s), \tilde{b}_0^*(s))} \max\{\frac{I - b_0^*}{p_1(s)} - \frac{A - b_0^*}{p_0}, 0\} dg(b_0^*). \tag{19}$$

This simply guarantees that the government determines how much debt to issue at date 1 in order to maximize the number of banks that can be saved.[21] Note that the government necessarily issues less debt than the amount that would maximize the revenues from this issuance. This is because at the peak of the issuance Laffer curve (the value of $B_1(s)$ which maximizes $[B_1(s) - B_0][1 - F(B_1(s)|s)]$, a marginal reduction in issuance $B_1(s) - B_0$ brings about a second-order reduction in issuance revenues $[B_1(s) - B_0][1 - F(B_1(s)|s)]$ but a first-order improvement in banks' pre-bailout net worth $b_0^* + (A - b_0^*)\frac{1 - F(B_1(s)|s)}{p_0}$, and hence a first-order reduction in required bailouts and by implication a first-order increase in the number of banks that can be saved.

In addition, the following pricing equations must hold

$$p_1(s) = 1 - F(B_1(s)|s), \tag{20}$$

$$p_0 = \int p_1(s) d\pi(s). \tag{21}$$

An individual banking entrepreneur who invests b_0^* gets a bailout in states $s > s(b_0^*)$ but no bailout in states $s < s(b_0^*)$, where $s(b_0^*)$ is the inverse of $b_0^*(s)$ and is hence monotonically decreasing in b_0^*. There is another threshold $\tilde{s}(b_0^*)$ such that the entrepreneur doesn't need a bailout to finance his investment when $s > \tilde{s}(b_0^*)$, where $\tilde{s}(b_0^*)$ is the in-

[21] Indeed equation (18) is the first-order condition for the following planning problem:

$$b_0^*(s) = \min_{\{\hat{b}_0^*(s), B_1(s)\}} \hat{b}_0^*(s)$$

s.t.

$$\int_{b_0^* \geq \hat{b}_0^*(s)} \max\{I - b_0^* - (A - b_0^*)\frac{1 - F(B_1(s)|s)}{p_0}, 0\} dg(b_0^*) = [B_1(s) - B_0][1 - F(B_1(s)|s)].$$

verse of $\tilde{b}_0^*(s)$ and is hence monotonically decreasing in b_0^*. This banking entrepreneur now faces a meaningful tradeoff in his portfolio decision. By increasing his investment b_0^* in foreign bonds, he secures a bailout in some states of the world where he did not get a bailout by rising in the government bailout pecking order, but loses out in states where he does not need a bailout to fund his investment. The corresponding optimality conditions states that b_0^* maximizes his welfare[22]

$$b_0^* \in \arg\max_{b_0^*(i)} V_0^B(b_0^*(i)),$$

where

$$V_0^B(b_0^*(i)) = \int \rho_1(s)\bar{I}d\pi(s) + \int_{\{s \geq \tilde{s}(b_0^*(i))\}} [b_0^*(i) + (A - b_0^*(i))\frac{p_1(s)}{p_0} - \bar{I}]d\pi(s)$$
$$+ \int_{\{s < s(b_0^*(i))\}} [b_0^*(i) + (A - b_0^*(i))\frac{p_1(s)}{p_0} - \rho_1(s)\bar{I}]d\pi(s).$$

The determination of equilibrium resembles that of equilibria of full-information first-price auctions or wars of attrition. The complication here comes from the fact that the object that competitors vie for—here subsidies—is itself endogenous, as from equation (18), the pot of subsidies depends on the distribution of "bids", namely the holdings of foreign bonds.

An interesting feature of these equilibria is that they display a force for endogenous diversification. Banking entrepreneurs choose to hold foreign bonds even in the absence of regulation. This is because they cannot be certain to count on a government bailout. We illustrate this possibility with two simple examples. In the first example, the distribution g is a degenerate atom. In the second example, it the distribution g is non-degenerate. In both cases, we abstract away from regulation and set $\underline{b}_0^* = 0$.[23]

[22]For b_0^* in the interior of the support of g, π must be absolutely continuous with respect to the Lebesgue measure in the neighborhood of $s(b_0^*)$ with Radon-Nikodym derivative $d\pi(s) = \pi(s)ds$, and the entrepreneur must be left indifferent by marginal changes in b_0^*, which requires that the following differential equation in $s(b_0^*)$ hold on the interior of the support of g:

$$-s'(b_0^*)\pi(s(b_0^*))\left[\rho_1(s(b_0^*))\bar{I} - \left(b_0^* + (A - b_0^*)\frac{p_1(s(b_0^*))}{p_0}\right)\right] = \int_{\tilde{s}(b_0^*)}^{\infty} \left(\frac{p_1(s)}{p_0} - 1\right)\pi(s)ds.$$

The left-hand-side represents the marginal utility gain from securing bailouts in more states of the world, while the right-hand-side represents the utility loss in states where no bailout is required to fund the investment.

[23]Another form of bailout rat race is developed in Nosal-Ordonez (2014). In their paper as in ours, the government ex ante dislikes bailing out banks, but cannot help doing so when faced with the fait accompli. The innovation of their paper is that a) a bank can be rescued either by the government or (more cheaply)

Illustrating example 1. Our first example is a variant of the example in Section 2.1. We assume that $(E - B_0)x < (x+y)(e - B_0)$, so that the revenue maximizing level of $B_1(L)$ in state L is e.

Our candidate equilibrium is symmetric with $B_1(L) = e$, $p_1(L) = x+y$ and $p_0 = \pi + (1-\pi)(x+y)$.[24] The limited-bailout condition is

$$\frac{\pi(1-x-y)}{\pi + (1-\pi)(x+y)}(A - b_0^*) = (e - B_0)(x+y). \tag{22}$$

In order for bankers to prefer b_0^* to 0, we must have

$$\pi[\frac{1}{\pi + (1-\pi)(x+y)} - 1]b_0^* \leq (1-\pi)A[\rho_1 - \frac{x+y}{\pi + (1-\pi)(x+y)}]. \tag{23}$$

The solution b_0^* of equation (22) always (strictly) verifies equation (23). This guarantees that our candidate equilibrium is indeed an equilibrium as long as the solution of equation (22) verifies $0 < b_0^* < A$.

Illustrating example 2. We now consider a simple variant of the previous example. The structure of uncertainty is as follows. With probability π, the state s is H and the endowment is high enough at E that there is no default. With probability $(1-\pi)z$, the state is M, and the endowment is high enough at E so that there is no default with conditional probability x, intermediate e_M with conditional probability y, and 0 with conditional probability $1 - x - y$. With probability $(1-\pi)(1-z)$, the state is L, and the endowment is high enough at E so that there is no default with conditional probability x, intermediate e_L with conditional probability y, and 0 with conditional probability $1 - x - y$. What distinguishes states M and L is that $e_M > e_L$. We assume that $(x+y)(e_L - B_0) > (E - B_0)x$ so that the revenue maximizing level of debt is e_M in state M and e_L in state L.

Our candidate asymmetric equilibrium is such that there are full bailouts in the medium state, but limited bailouts in the low state. Bankers invest $\hat{b}_0^*(L)$ with probability ϕ and 0 with probability $1 - \phi$. Prices are $p_0 = \pi + (1-\pi)(x+y)$, $p_1(L) = p_1(M) = x+y$.

by a healthy bank and the government prefers a private takeover to a public takeover. The government however does not know whether the first distressed bank's shock is idiosyncratic or aggregate (in which case there will be no healthy bank to rescue the distressed one). In a situation in which the conditional probability of an aggregate shock is not too large, the government waits, and therefore banks prefer not to be the first distressed institution. If they can sink resources to augment the probability of not being first, they will do so, a behavior akin to a rat race.

[24] It can be shown that there are no asymmetric equilibria in this example.

The bailout conditions are

$$\phi \frac{\pi(1-x-y)}{\pi+(1-\pi)(x+y)}(A-\hat{b}_0^*(L)) = (e_L - B_0)(x+y), \tag{24}$$

$$\phi \frac{\pi(1-x-y)}{\pi+(1-\pi)(x+y)}(A-\hat{b}_0^*(L)) + (1-\phi)\frac{\pi(1-x-y)}{\pi+(1-\pi)(x+y)}A \leq (e_M - B_0)(x+y). \tag{25}$$

In order for bankers to be indifferent between $b_0^* = \hat{b}_0^*(L)$ and $b_0^* = 0$, we must have

$$\pi[\frac{1}{\pi+(1-\pi)(x+y)} - 1]\hat{b}_0^*(L) = A(1-z)(1-\pi)[\rho_1 - \frac{x+y}{\pi+(1-\pi)(x+y)}]. \tag{26}$$

We can rewrite equation (26) as

$$\hat{b}_0^*(L) = A(1-z)[(\rho_1-1)\frac{\pi+(1-\pi)(x+y)}{\pi(1-x-y)} + 1].$$

Using equation (24), we find

$$\phi = \frac{e_L - B_0}{A}\frac{(x+y)\frac{\pi+(1-\pi)(x+y)}{\pi(1-x-y)}}{1-(1-z)[(\rho_1-1)\frac{\pi+(1-\pi)(x+y)}{\pi(1-x-y)}+1]}.$$

We have an equilibrium if $\hat{b}_0^*(L) < A$, $0 < \phi < 1$, and

$$(1-\phi)\frac{\pi(1-x-y)}{\pi+(1-\pi)(x+y)}A \leq (e_M - e_L)(x+y),$$

which can always be ensured for appropriate parameter values.

Proposition 11 (Bailout Rat-Race and Incentives for Diversification)**.** *In the illustrating examples with limited bailouts and symmetric or asymmetric equilibria, it is optimal for banks to not fully load up on domestic sovereign default risk and instead choose a non-zero degree of diversification $b_0^* > 0$ with positive probability even when there is no regulation ($\underline{b}_0^* = 0$).*

5.3 Sovereign Debt Maturity

In this section, we investigate the role of sovereign debt maturity. More specifically, we compare our economy with long-term sovereign bonds which are claims to coupons accruing at date 2 with an economy where sovereign bonds are short-term one-period bonds which are rolled over at date 1.

With long-term bonds, welfare is given by

$$W_0 = \int \left[\int_{B_1(s)}^{\infty} [E - B_0] f(E|s) dE + \int \int_0^{B_1(s)} [E - \Phi] f(E|s) dE + \beta^I(s) I(s) \right] d\pi(s)$$
$$+ \int \beta^B [\rho_1(s) I(s) + [A - I(s)]] d\pi(s)$$
$$+ \int (1 - \beta^B) \min\{\underline{b}_0^* + (A - \underline{b}_0^*) \frac{p_1(s)}{p_0} - I(s), 0\} d\pi(s).$$

We now consider the economy with short-term bonds. We denote all variables with a tilde. To make the comparison with the economy with short-term bonds meaningful, we impose that the government must raise the same amount of revenues G_0 in period 0, i.e.

$$\tilde{B}_0 = B_0 \int [1 - F(B_1(s)|s)] = G_0. \tag{27}$$

In addition, the government must raise exactly enough revenues at date 1 to repay the date-0 debt that is coming due, i.e. we must have for all s[25]

$$\tilde{B}_1(s)[1 - F(\tilde{B}_1(s)|s)] = \tilde{B}_0. \tag{28}$$

Because $\bar{I} \leq A$, this means that no bailouts are required. Welfare is given by

$$\tilde{W}_0 = \int \left[\int_{\tilde{B}_1(s)}^{\infty} [E - \tilde{B}_1(s)] f(E|s) dE + \int \int_0^{\tilde{B}_1(s)} [E - \Phi] f(E|s) dE + \beta^I(s) I(s) \right] d\pi(s)$$
$$+ \int \beta^B [\rho_1(s) I(s) + [A - I(s)]] d\pi(s).$$

We can write

$$W_0 - \tilde{W}_0 = \int \Phi[F(\tilde{B}_1(s)|s) - F(B_1(s)|s)] d\pi(s)$$
$$+ \int (1 - \beta^B) \min\{\underline{b}_0^* + (A - \underline{b}_0^*) \frac{p_1(s)}{p_0} - I(s), 0\} d\pi(s). \tag{29}$$

There are two terms on the right-hand side of equation (29). The first term represents the difference in default costs, and the second term represents the welfare impact of transfers from consumers to banking entrepreneurs resulting from bailouts when domestic

[25] To give short-term debt a good shot, we assume that the government is always able to roll over its short-term debt. This is indeed the case if negative shocks s are not too catastrophic, so that the debt can be rolled over by pledging income in the good realizations at date 2.

sovereign bonds are long term. The second term is always negative and arises because by issuing short-term bonds, the government reduces the risk-taking possibilities of banks and insulates the banks from fiscal developments—there is no feedback loop between banks and sovereigns. In the proof of the proposition below, we show that under some additional assumptions on the distributions of E and s, the first term is positive.[26] As a result, the intuitive asset-liability management (ALM) principle of matching maturities of incomes and payments holds—when the minimum diversification requirement $\underline{b}_0^*$ is high enough: Long-term debt then leads to a strictly lower expected probability of default than short-term debt.

Proposition 12 (Optimal Debt Maturity)**.** *Suppose that $F(E|s) = F(E-s)$ where F is increasing and convex. Then for $\underline{b}_0^*$ high enough, welfare is higher with long-term sovereign bonds than with short-term sovereign bonds $W_0 > \tilde{W}_0$.*

Proof. Note that $B_1(s) \geq B_0$ and that $B_1(s)$ converges to B_0 for all s as $\underline{b}_0^*$ goes to $\bar{I}$, while $\tilde{B}_1(s)$ is independent of $\underline{b}_0^*$. Hence the result follows if we can show that $\int \Phi[F(\tilde{B}_1(s)|s) - F(B_0|s)]d\pi(s) < 0$. We now proceed to prove this result, which we refer to as result A. The result is a direct consequence of the following related (dual) result, which we refer to as result B. Let B_0 be defined by

$$\int B_0[1 - F(B_0|s)]d\pi(s) = G_0$$

as above, and let $\tilde{\tilde{B}}_1(s)$ and $\tilde{\tilde{G}}_0$ be defined by the system of equations

$$\tilde{\tilde{B}}_1(s)[1 - F(\tilde{\tilde{B}}_1(s)|s)] = \tilde{\tilde{G}}_0 \quad \text{for all } s,$$

$$\int F(\tilde{\tilde{B}}_1(s)|s)d\pi(s) = \int F(B_0|s)d\pi(s).$$

Result B is that $\tilde{\tilde{G}}_0 < G_0$. We now prove result B, which in turn directly implies result A.

Since

$$\tilde{\tilde{G}}_0 \int \frac{B_0}{\tilde{\tilde{B}}_1(s)} d\pi(s) = G_0,$$

we need to show that

$$\int \frac{B_0}{\tilde{\tilde{B}}_1(s)} d\pi(s) > 1.$$

[26]The additional assumptions are that $F(E|s) = F(E-s)$ where F is increasing and convex. These assumptions, which imply the monotone hazard rate properties $\frac{\partial(f(E|s)/(1-F(E|s)))}{\partial s} \leq 0$ and $\frac{\partial(f(E|s)/(1-F(E|s)))}{\partial E} > 0$, are sufficient but not necessary to prove the result.

By Jensen's inequality, result B is implied by the following result, which we refer to as result C:

$$\frac{B_0}{\int \tilde{\tilde{B}}_1(s)d\pi(s)} > 1.$$

Since $\int F(\tilde{\tilde{B}}_1(s) - s)d\pi(s) = \int F(B_0 - s)d\pi(s)$ and F is increasing, result C is equivalent to

$$\int F(\int \tilde{\tilde{B}}_1(s')d\pi(s') - s)d\pi(s) < \int F(\tilde{\tilde{B}}_1(s) - s)d\pi(s).$$

Define

$$g(\lambda) = \int F[\int \tilde{\tilde{B}}_1(s')d\pi(s') + \lambda(\tilde{\tilde{B}}_1(s) - \int \tilde{\tilde{B}}_1(s')d\pi(s')) - s]d\pi(s).$$

We have

$$g'(\lambda) = \int f[\int \tilde{\tilde{B}}_1(s')d\pi(s') + \lambda(\tilde{\tilde{B}}_1(s) - \int \tilde{\tilde{B}}_1(s')d\pi(s')) - s][\tilde{\tilde{B}}_1(s) - \int \tilde{\tilde{B}}_1(s')d\pi(s')]d\pi(s).$$

Because $f[\int \tilde{\tilde{B}}_1(s')d\pi(s') + \lambda(\tilde{\tilde{B}}_1(s) - \int \tilde{\tilde{B}}_1(s')d\pi(s')) - s]$ and $\tilde{\tilde{B}}_1(s) - \int \tilde{\tilde{B}}_1(s')d\pi(s')$ are decreasing in s for all $\lambda \geq 0$, the right-hand side is the covariance of two decreasing functions of the random variable s and is therefore positive. It follows that $g'(\lambda) > 0$ for all $\lambda \geq 0$. Since $g(0) = \int F(\int \tilde{\tilde{B}}_1(s')d\pi(s') - s)d\pi(s)$ and $g(1) = \int F(\tilde{\tilde{B}}_1(s) - s)d\pi(s)$, we get result C. Results B and A follow, concluding the proof of the Proposition.

□

There are obvious extensions of our setup that would reinforce the conclusion of Proposition 12. For example, the desirable features of short-term sovereign debt in terms of limiting the risk-taking possibilities of banks, bailouts and feedback loops between banks and sovereigns, would be mitigated in a model with a richer set of risk taking possibilities apart from domestic sovereign debt, or in an infinite horizon version of our model with overlapping generations of banking entrepreneurs, consumers and investors, where some banks hold domestic sovereign debt for liquidity in all periods.

5.4 Foreign Banks

We could introduce foreign banks (in the foreign country) as follows. Foreign banks face a similar problem to domestic banks. They have some net worth A^F at date 0, and some investment opportunities $I^F(s)$ at date 1 with private and foreign social returns given by $\rho_1^F(s)$ and $\beta^{I,F}(s)$. Foreign banks invest their net worth at date 0 in a portfolio of risky "domestic" bonds (bonds of the domestic economy) and safe "foreign" bonds (bonds of the foreign economy).[27] The return on their portfolio at date 1 determines their net worth at date 1. If it falls short of their investment need, then they are bailed out by the foreign government. But these bailouts do not endanger the ability of the foreign government to repay its debt. The domestic and foreign countries differ only in the riskiness of their sovereign bonds. Domestic sovereign bonds are risky and foreign sovereign bonds are safe.

Our analysis goes through in this extended model as long as risk-neutral international investors who do not benefit from bailout guarantees (not foreign banks) remain the marginal buyers of domestic and foreign sovereign bonds. In particular, Propositions 1-12 still hold without any modification. The key observation is that foreign banks' portfolio decisions are irrelevant for equilibrium prices, domestic bailouts and sovereign default probabilities, and domestic banks' portfolio decisions. Foreign banks' risk exposures do not give rise to any feedback loop, because the foreign government has enough fiscal capacity to bail them out without endangering its ability to repay its debt.

The extended model has additional predictions on the incentives of foreign banks and of the foreign government. Because of the bailout guarantees, foreign banks have an incentive to load up on risky domestic debt. The foreign government has an incentive to regulate foreign banks so that they do not take on too much domestic sovereign risk. We elaborate on these issues now.

Frictionless regulation of foreign banks. We first consider first optimal regulation in the foreign country as in Section 3.2. We can derive the following equivalent of Proposition 2.

Proposition 13 (Frictionless First-Best Regulation in Foreign Country)**.** *When the basic model in Section 3.2 is extended to include foreign banks, setting $\underline{b}_0^{F*} = \bar{I}^F$, if feasible, maximizes ex-ante welfare W_0^F in the foreign country.*

Just like the domestic government, the foreign government has an incentive to prevent its banks from taking on (domestic) sovereign risk. This is because when foreign banks take on more risk, they receive a bailout from the foreign government following a bad shock, which has adverse distributional effects.

[27] Of course domestic bonds are foreign bonds from the perspective of foreign banks, and similarly foreign bonds are domestic bonds from the perspective of foreign banks. To avoid confusion, we always refer to domestic bonds as the sovereign bonds of the domestic economy, independently of whether they are held by domestic or foreign agents. Similarly, we refer to foreign bonds as the bonds of the foreign economy, independently of whether they are held by domestic of foreign agents.

Next we revisit the argument for regulatory leniency in the domestic economy given in Section 4.2 in the presence of debt forgiveness. We can derive the following equivalent of Proposition 7.

Proposition 14 (No Strategic Regulatory Leniency in Foreign Country)**.** *When the illustrating example of Section 4.2 with debt forgiveness is extended to include foreign banks, it is optimal for the foreign government to set* $\underline{b}_0^{F*} = A^F = \bar{I}^F$*. In particular, optimal regulation in the foreign country* b_0^{F*} *is independent of the probability* $1-\pi$ *of the bad domestic fiscal shock.*

Propositions 7 and 14 display a sharp contrast between the regulatory incentives of the domestic government and those of the foreign government. Because foreign government debt is safe, the foreign government cannot extract any concessions from its creditors. As a result, the foreign government has no incentive to engage in strategic regulatory leniency. Instead it always seeks to strictly limit the exposure of foreign banks to domestic sovereign risk. The implication of the extended model is then that as the probability $1-\pi$ of a bad domestic fiscal shock increases, domestic regulation of domestic banks gets laxer, but foreign regulation of foreign bank does not, and as a result domestic banks tilt their portfolios towards risky domestic bonds and away from safe foreign bonds, but foreign banks do not.

Collective moral hazard and foreign banks. It is also interesting to investigate the portfolio decisions of foreign banks in the environment of Section 3.3, assuming that foreign banks face a cost of making their balance sheets opaque Ψ^F similar to that of domestic banks and that $I^F(s) = \bar{I}^F$ is independent of s and that $A^F = \bar{I}^F$. We can derive the following equivalent of Proposition 4.

Proposition 15 (Multiple Equilibria)**.** *When the illustrating example of Section 3.3 is extended to include foreign banks, the portfolio of foreign banks is given by* $b_0^{F*} = (-\Psi^{F\prime})^{-1}(\frac{\pi(1-\pi)(1-\theta)}{\pi+(1-\pi)(\theta)})$ *with* $\theta = x$ *in the low (domestic) diversification equilibrium and* $\theta = x+y$ *in the high (domestic) diversification equilibrium.*

Foreign banks' exposure to domestic sovereign risk is higher in the low (domestic) diversification equilibrium than in the high (domestic) diversification equilibrium.[28]

The key observation that underlies these results is that there are strategic complementarities running from domestic banks' to foreign banks, but no strategic complementarities running in the other direction. Indeed, when domestic banks increase their exposure

[28] Note that contrary to domestic and foreign banks, international investors have less exposure to domestic sovereign risk in the low (domestic) diversification equilibrium than in the high (domestic) diversification equilibrium.

to domestic sovereign risk, the benefits of doing so also increases for foreign banks. But when foreign banks increase their exposure to domestic sovereign risk, the benefits of doing for domestic banks remains unchanged. This is because the riskiness of domestic debt increases in the former case but not in the latter.

This also implies that there are regulatory externalities running from the domestic country to the foreign country but not vice versa. Indeed, suppose that at some cost R (respectively R^F), the domestic (respectively foreign) government can impose perfect regulation, in which case it chooses $\underline{b}_0^* = A$ (respectively $\underline{b}_0^{*F} = A^F$). Otherwise, regulation is irrelevant (Ψ and Ψ^F are both zero), so that banks can perfectly evade regulation. Assume that $B_0 + \frac{1}{x}\frac{\pi(1-x)}{\pi+(1-\pi)x}A > e > B_0$.

If the domestic government chooses to incur the regulatory cost R, we have $B_1(L) = B_0$ and $\theta = x + y$. Otherwise $B_1(L) = B_0 + \frac{1}{x}\frac{\pi(1-x)}{\pi+(1-\pi)x}A$ and $\theta = x$. In both cases, we have $p_1(L) = \theta$ and $p_0 = \pi + (1-\pi)\theta$.

The net gain $(1-\pi)(1-\beta^B)A^F\frac{\pi(1-\theta)}{\pi+(1-\pi)\theta} - R^F$ from incurring the regulatory cost for the foreign government is lower ($\theta = x$) when the domestic government incurs the regulatory cost than when it doesn't ($\theta = x + y$). By contrast, the net gain from incurring the regulatory cost for the domestic government is independent of whether or not the domestic government incurs the regulatory cost. More interestingly, we have the following proposition.

Proposition 16 (Regulatory Externalities and Banking Union)**.** *In the illustrating example with either perfect or irrelevant regulation and foreign banks, foreign welfare increases with the regulatory effort (decreases with the regulatory cost) of the domestic country, but domestic welfare is independent of the regulatory effort (independent of the regulatory cost) of the foreign country.*

Proposition 16 uncovers an additional rationale for a banking union. Domestic regulation has positive external effects for the foreign country. These effects are not internalized by the domestic government, and as a result, regulation is too lax in the domestic economy. By transferring regulatory decisions from the national to the international level, a banking union allows these effects to be internalized, leading to a toughening of regulation in the domestic country and an improvement of welfare.

6 Summing Up

We built on a relatively standard model of feedback loop, with shocks reducing the value of sovereign debt, leading to bailouts if bank portfolios exhibit a home bias, leading to further debt sustainability problems, etc. Relative to the earlier literature, we have uncovered the following insights:

- The feedback loop paradoxically stems from a prudent matching of debt maturity with the country's fiscal capability.
- The feedback loop is stronger in case of joint default on private and sovereign debt.
- As long as the country has the capability to bail out banks, the latter's exposures to their domestic government's debt are strategic complements.
- When push comes to shove and the government may run out of money to finance bailouts, banks may by contrast engage in a diversification rat race.
- There are two distinct rationales for re-nationalization. First, the banks may invest in opacity and try to evade prudential diversification rules. Second, even when government can perfectly monitor its banks, the government may strategically turn a blind eye to their lack of country diversification and count on legacy debt forgiveness to finance the rescue of its banking sector in case of difficulties. In either case, re-nationalization occurs when the legacy debt increases or prospects about the country's fiscal capability worsen.
- There are two distinct rationales for a banking union. First, if the ex-post leniency of domestic regulators is anticipated ex-ante at the time of sovereign debt issuance, then it is priced in the form of higher spreads. The government is better off committing ex ante to a tough ex-post regulatory stance, but is tempted to relax it ex post. If the government lacks commitment, then it benefits from relinquishing its regulatory powers to a supranational supervisor by joining a banking union. Second, voluntary or involuntary regulatory leniency makes the supervision of foreign banks' portfolios more complex, generating an externality that can only be internalized in a banking union.

Our research leaves open a number of fascinating questions. First, we have assumed that the bailouts take the fiscal route. As observed recently in many countries, the Central Bank may participate in the bailout, perhaps risking inflation and devaluation. Second, we have assumed that sovereign defaults are not strategic (the government defaults only if it cannot repay). If defaults are strategic, domestic exposure choices by domestic banks influence the incentives to default (the government is less likely to default if its debt is held domestically), opening up the possibility of complex strategic interactions between banks and sovereigns, and providing additional benefits (disciplining the government) to debt re-nationalization. Finally, further research should be devoted to the governance of the banking union, and in particular to the interactions between prudential and fiscal integrations: should the union be committed to solidarity? Should a country bear the first losses when one of its banks defaults? We leave this and other questions for future research.

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A Appendix: Credit Crunch Foundations of the Social Welfare Function

We sketch the foundations of the welfare function, following Homlström and Tirole (1997). At date 1, the bank can make an investment in knowledge/staff so as to be able to invest in a mass $I(s)$ of firms at date 1. These firms enter in a relationship with the bank at date 1; from then on, they share available resources in coalition with the banks. At date 2, firms succeed (return $r(s)$ per firm) or fail (return 0). Success is guaranteed if the firm managers as well as the workers in the firm do not shirk. Otherwise success accrues with probability 0. Shirking for a firm manager brings benefit $\phi^F(s)$, and shirking for a firm worker brings benefit $\phi^W(s)$. Therefore incentive payments $\rho_1^F(s) = \phi^F(s)$ and $\rho_1^W(s) = \phi^W(s)$ per firm are required to discipline the firm manager and the workers. For simplicity, we assume that workers and firms are cashless. Banking entrepreneurs can divert their share of the return $\rho_1(s) = r(s) - \phi^F(s) - \phi^W(s)$ on the project of each firm so that they cannot borrow.

In Section 5.1, we relax the assumption that banking entrepreneurs cannot borrow. This can be modeled as follows. Instead of assuming that banking entrepreneurs can divert their share of the return on the project of each firm, we assume that banks need to monitor firms. A firm succeeds if not only workers and firms do not shirk, but also banking entrepreneurs. Shirking for a banking entrepreneur brings about a benefit ϕ^B. Banks are then able to borrow $\rho_0(s) = r(s) - (\phi^B(s) + \phi^F(s) + \phi^W(s))$ per firm that they finance, and receive a share $\rho_1(s) = r(s) - (\phi^F(s) + \phi^W(s))$ of the return of each firm.