Tying in Two-Sided Markets and the Honor All Cards Rule*

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Abstract

Payment card associations offer both debit and credit cards and, until recently, engaged in a tie-in on the merchant side through the so-called honor-all-cards (HAC) rule. The HAC rule came under attack on the grounds that the credit and debit card markets are separate markets and that the associations lever their market power in the “credit card market” to exclude on-line debit cards and thereby monopolize the “debit card market”. This article analyzes the impact of the HAC rule, using a simple model with two types of transactions (debit and credit) and two platforms.

In the benchmark model, in the absence of HAC rule, the interchange fee (IF, the transfer from the merchant’s bank to the cardholder’s bank) on debit is socially too low, and that on credit is either optimal or too high (depending on downstream members’ market power). In either case, the HAC rule not only benefits the multi-card platform but also raises social welfare, due to a rebalancing effect: The HAC rule allows the multi-card platform to better perform the balancing act by raising the IF on debit and lowering it on credit, ultimately raising volume.

The paper then investigates a number of extensions of the benchmark model, including varying degrees of substitutability between debit and credit; merchant heterogeneity; and platform differentiation. While the HAC rule may no longer raise social welfare under all values of the parameters, the basic and socially beneficial rebalancing effect unveiled in the benchmark model is robust.

Keywords: Tie-ins, two-sided markets, payment cards, price rebalancing.

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1 Introduction

Buyers use both credit and debit cards. The credit facility brings about substantial benefits to some consumers, for some types of purchases or at specific moments of time.

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In other circumstances, credit is not needed.

Payment card associations, Visa and MasterCard,1 offer both debit and credit cards and, until recently, engaged in a tie-in on the merchant side through the honor-all-cards (HAC) rule. In a class action initiated by WalMart (and involving more than five million U.S. merchants), this rule came under attack on the grounds that the credit and debit card markets are separate markets and that the associations lever their market power in the “credit card market” (the tying market) to exclude on-line debit cards and thereby monopolize the “debit card market” (the tied market). In 2003, Visa and Master Card agreed to abandon their HAC rules in the U.S. and to pay over $ 3 billion in damages to the merchants. By contrast, the HAC rule is still in place in the rest of the world. Moreover, even in the U.S., the ban does not apply to incumbent proprietary payment systems such as American Express or new ones such as Paypal. Finally, card associations and proprietary systems increasingly segment the cardholder market by offering a variety of brands, with different services and benefits offered to cardholders, or targeted to different groups (e.g. corporate vs non-corporate cards). Because cards differ in their benefits to the merchants as well, the issue of bundling will most likely come up again in the future.

This article analyzes the impact of the HAC rule. For illustrative purposes, it assumes that systems are run by not-for-profit associations, but some of the general points apply as well to proprietary systems such as American Express. Section 2 constructs a simple model of the payment card industry in which there are two types of transactions, debit and credit, which may be either unbundled or tied on the merchant side. This benchmark model makes a number of strong, simplifying assumptions (which are relaxed in Section 5): independent demands for debit and credit, merchant homogeneity, perfect consumer information about merchant card acceptance policies, and undifferentiated payment systems.

To understand the impact of a tie-in, Section 3 analyzes the factors that are relevant to the determination of merchant and cardholder surpluses in the case of a single card (or under unbundling when multiple cards co-exist).

With a single card, an association’s choice of interchange fee, the payment made by the merchant’s bank (the acquirer) to the cardholder’s bank (the issuer), is constrained in two ways:

- Even if the association faces no competition from another system, it must get both sides (consumers, merchants) on board. The interchange fee must be high enough so as to induce consumers to use the card, but low enough so as not to meet merchant

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1MasterCard is now for profit, but it was until recently an association.
resistance.2

- When competing with other payment systems, the association is further constrained. In particular, when consumers hold multiple cards, each system tries to de-stabilize its rivals’ balancing act and to “steer” merchants by lowering the interchange fee so as to incentivize them to turn down the rivals’ card.

A not-for-profit association cannot exercise its market power by inflating the overall price level. By contrast, the association has discretion in the allocation of cost between cardholders and merchants and, like ordinary firms, it may or may not get the price structure (i.e., the relative prices for different end users) “socially right”. On the one hand, both a social planner and an association ought to design the price structure so as to account for the elasticities on both sides of the market and thereby get both sides on board. On the other hand, the literature has identified factors, such as downstream (issuer, acquirer) market power or merchants’ competition for market share, that may tilt an association’s (or, for that matter, a proprietary system’s) price structure away, in either direction, from the socially optimal one.

System competition is one such factor. Leaving aside the standard benefits of competition on managerial incentives,3 system competition has here an ambiguous impact on welfare because it influences only the price structure and not the price level (which must track cost, due to the not-for-profit status).

As noted above, when consumers hold multiple cards (multi-home4), system competition tilts the price structure toward lower merchant discounts and higher cardholder fees, because merchants then have an incentive to turn down the card that is most expensive to them. This is most clearly the case when cardholders are unaware of the merchants’ card acceptance policies before purchasing and so card acceptance generates no competitive edge over rival merchants (the “Baxter5 case”). Interestingly, we show that steering is effective even when consumers are informed of the merchants’ card acceptance policies and card acceptance therefore buys merchants a strategic edge (as is assumed in the most of the treatment below). In the polar case of a monopoly system, the association, in order

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2Such inducements are indirect to the extent that an increase in the interchange fee is partly or fully passed through by issuing banks to cardholders and by acquirers to merchants.

3E.g., through the owners’ ability to benchmark their management’s performance.

4The term “multi-homing” is borrowed from the Internet literature. It refers to a situation where some users are members of several platforms. In the context of payment cards it means either that consumers hold several cards (buyers’ multi-homing) or that merchants accept several cards (sellers’ multihoming).

5This refers to Baxter (1983), the first formal analysis of the determination of interchange fees in payment card networks. Baxter assumes that merchants’ acceptance decision are driven only by their convenience benefits from card payments and so card acceptance does not help them attract customers.

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to maximize volume, chooses an interchange fee equal to the highest value that merchants will bear. In the opposite polar case of perfect system competition, the equilibrium interchange fee is the (lower) one that maximizes total user surplus. Intuitively, merchants prefer the card that gives them the highest sum of their own surplus (convenience benefit minus merchant discount) and of the cardholder’s average surplus (convenience benefit minus cardholder fee), since they internalize the latter when trying to attract customers. Under monopoly, the system tries to please consumers in order to maximize volume; under competition, merchants have an important say on the usage of a specific card, because rejecting a card is much less costly to them if cardholders multi-home; and so merchants receive a better deal (and the cardholders a worse deal).

System competition focuses an association’s attention on the system’s own elasticities rather than on the socially more relevant end-user elasticities. And so, whether competition improves social welfare depends on the price-structure bias of a monopoly system. If the merchant discount (or the interchange fee) is initially too high, then competition forces it down and may improve welfare. By contrast, competition reduces welfare if the merchant discount (or the interchange fee) was initially too low.

The issue at stake, however, is not whether competition improves welfare, but whether, given competition, tying (the HAC rule) increases or decreases welfare. A first intuition might be that “bundling reduces competition, and so, if competition is socially desirable, bundling reduces welfare as well”. This intuition turns out to be incorrect. Section 4 shows that, when merchants are homogenous, regardless of the desirability of system competition, the HAC rule always improves welfare.

To see why this is so, suppose that consumers are informed of the merchants’ card acceptance policies, and that a system issues both credit and debit cards, and faces more intense competition on one segment. To simplify the exposition, the system is a monopoly on credit cards and faces an on-line competitor for debit cards that is a perfect substitute. Then in the absence of the HAC rule, the outcome is the monopoly outcome for credit and the competitive outcome for debit. From our previous analysis, the interchange fee is higher on credit than on debit. This interchange fee structure is not the one predicated by the demand specificities of the two-sided debit and credit markets, but rather reflects the difference in the merchants’ “bypass opportunities”.

Suppose now that a merchant has to accept the system’s two cards or none. The merchants accept the two cards only if the corresponding total user surplus (which is also the merchants’ willingness to pay for accepting the system’s cards) exceeds, as in the absence of the HAC rule, the highest total user surplus that can be offered by the on-line network. The system gains *flexibility to rebalance its interchange fee structure* as
the competitive constraint binds over the set of cards, rather than over each card. The
two-card system can therefore increase volume by raising the interchange fee on debit and
lowering interchange fee on credit. Social welfare always increases.

Section 5 investigates a number of extensions of the benchmark model to allow for
varying degrees of substitutability between debit and credit; different structures of card-
holders’ information about merchants’ card acceptance policies; merchant heterogeneity
with respect to their customers debit/credit mix; and platform differentiation. While
the HAC rule may no longer raise social welfare under all values of the parameters, the
basic and socially beneficial rebalancing effect unveiled in the benchmark model is robust.
Finally, Section 6 concludes.

Contribution to the literature on tie-ins.

There are two key differences with the literature on tying.\footnote{For recent views on tying in one-sided markets, see, e.g., Carlton-Waldman (2002), Choi-Stefanadis (2001), Nalebuff (2003), and Rey et al (2003).}

First, we analyze tying by an association. Hence, anticompetitive motives, like entry
deterrence, cannot be associated with the standard purpose of raising the price level (an
association can only affect the price structure). In Whinston (1990) for example, tying is
totally motivated by entry deterrence: By lowering its opportunity cost of selling in the
competitive market (losing a sale in that market implies losing a sale in the monopoly
market), the tie, provided that it is technologically irreversible, commits the tying firm
to be aggressive and thereby may deter entry. In our model, like in Whinston’s, tying
may deter entry of a more efficient rival. But the consequences are different here because
a not-for-profit firm cannot exercise its market power by raising the price level. So, for
example, deterrence of entry of a slightly more efficient rival through tying always raises
welfare in our model.\footnote{We ignore “corporate governance” or “benchmarking” benefits of product market competition.}

Second, tying occurs in a two-sided market.\footnote{For overviews of the economics of two-sided markets, see Armstrong (2004) and Rochet-Tirole (2004).} There is then a natural benefit of tying
in terms of a greater flexibility to rebalance charges between the two sides. We view
this insight as the main contribution of the paper. We do not argue that tying is not
innocuous in two-sided markets; indeed much research is needed to understand its impact
in such markets. The much more limited goal of this analysis is to point at a robust
efficiency-enhancing effect of tying by two-sided platforms.
2 The Model

We extend the model of the payment card industry developed in Rochet and Tirole (2002), by introducing two types of cards (debit cards and credit cards, respectively indexed by superscripts $k = d$ and $c$), and two competing networks (indexed by subscripts $i = 1$ and 2). Both networks are not-for-profit associations run by their members.\(^9\) Network 1 offers only a debit card, while network 2 offers both a debit card and a credit card. The two debit cards are perfect substitutes for both cardholders and merchants.

Figure 1 shows the costs and benefits attached to a card transaction through a given system.\(^{10}\) The total cost of this service is the sum of the issuer’s cost $c_I$ and the acquirer’s cost $c_A$. The benefit accruing to the cardholder (buyer) for the marginal use of a payment card is denoted $b_B$. Similarly, the benefit to the merchant (seller) of this marginal use of a payment card is $b_S$. The benefits $b_i$ and costs $c_i$ referred to above are net benefits and costs: The cardholder and the merchant must compare the utilities they get by using payment cards with those associated with alternative payment methods (cash, checks, etc.). At the social optimum, the total benefit of the marginal transaction, $b_B + b_S$, is equal to its total cost, $c_I + c_A \equiv \gamma$. Figure 1 also features the payments from end users to intermediaries: cardholders pay $f$ to issuers and merchants pay merchant discount $m$ to acquirers. These two fees are market determined given the association’s choice of interchange fee $a$.

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\(^9\)We later examine the case where one (or both) network(s) is(are) for-profit.

\(^{10}\)For more on the payment card industry and an economic analysis thereof, see Evans-Schmalensee (2004).
As in Rochet and Tirole (2002) we focus on the choice of interchange fees by the two networks. Final prices for both types of users (merchants and cardholders) are determined by the extent of competition in downstream (issuing and acquiring) markets. For simplicity we assume that margins on the issuing and acquiring sides are constant, that is users’ prices react one for one to variations of issuers’ and acquirers’ net costs. Constant mark-up by issuers and acquirers offer the convenient simplification that all members of each association are congruent (issuers and acquirers want to maximize network volume), and so the modeling of the governance structure of the associations is a no-brainer. Formally, for all three cards, cardholders’ per transaction fees $f^k_i$ (recall that $k = d, c$ represents the type of card, debit or credit, while $i = 1, 2$ stands for the network offering the card) and merchant discounts $m^k_i$ are related to interchange fees $a^k_i$ by the following formulas:

$$f^k_i = f^k_0 - a^k_i$$  \hspace{1cm} (1)$$
$$m^k_i = m^k_0 + a^k_i,$$  \hspace{1cm} (2)

where $f^k_0$ and $m^k_0$ are given. Note that total user price is independent of the interchange fee:

$$f^k_i + m^k_i = f^k_0 + m^k_0 = \gamma^k + \pi^k,$$  \hspace{1cm} (3)

where $\gamma^k$ is the (total) unit cost of payment services with card $k$ and $\pi^k$ is the sum of the per-transaction margins levied by acquirers and issuers. These costs and margins are assumed identical for the two debit cards.

Following Wright (2004), we assume that cardholders are ex-ante identical. Ex-post, i.e., for a specific purchase in a given store, by contrast, their transactional benefit $b^k_B$ from using card $k$ ($k = d, c$) rather than check (or cash) is drawn from a distribution\(^\text{11}\) with a positive density $h^k(b^k_B)$. Merchants are for the moment homogenous: they derive the same net benefit $b^k_S$ from card $k$, and have the same fraction of debit and credit card consumers.

In the simplest version of our model, we neglect any substitutability between credit and debit cards. In other words we assume for the moment that there are two distinct subsets of transactions: $N^d$ transactions for which payment can be made by either debit or cash/check, and $N^c$ transactions for which payment can be made by either credit or cash/check. Thus there are two markets: market $c$ is the market for the “credit good” and market $d$ is the market for the “debit good”. We assume that the two markets are

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\(^{11}\)The assumption that consumers are ex-ante identical (so that transactional benefits $b^k_B$ are only drawn once the consumer is in the store) simplifies the analysis of merchants’ acceptance of cards. In Rochet and Tirole (2002), by contrast, the assumption of ex-ante heterogeneity of consumers introduces complementarity in merchants’ acceptance decisions.
independent\textsuperscript{12} and that there is no possible substitutability between cards.\textsuperscript{13} We also assume on that the total demand for final goods by consumers is inelastic, so that the total numbers of transactions $N^d$ and $N^c$ for each type of good are fixed.\textsuperscript{14} Introducing price elasticities would not fundamentally change the analysis, but would complicate the exposition.

A key ingredient of our analysis is cardholder multi-homing (in membership; they do not need to multi-home in usage). To simplify the exposition, we assume a strong form of multi-homing: All debit cardholders hold the two debit cards. When given the choice by the merchant, a debit cardholder therefore uses the card with the lowest cardholder usage fee.

### 3 Merchants’ Acceptance Decisions

Rational merchants anticipate that accepting payment cards increases patronage.\textsuperscript{15} By accepting card $k$ (with user prices $f^k$ and $m^k$), a merchant indeed increases the (per transaction) expected utility of his future customers by

$$u^k = \int_{f^k}^{+\infty} (b^k_B - f^k) h^k(b^k_B) db^k_B.$$  

However it also increases his (per transaction) net expected cost by

$$c^k = \int_{f^k}^{+\infty} (m^k - b^k_S) h^k(b^k_B) db^k_B.$$  

Because consumers decide on their purchases before knowing $b^k_B$, total demand for final goods by consumers using card $k$ depends on the “effective” (or hedonic) price $\hat{p}^k = p - u^k$, where $p$ is the price charged by the merchant. Consumer surplus and merchants’ profits also depend on this “effective” price. Thus if $C$ denotes the unit production cost, accepting card $k$ amounts to charging an effective price $p - u^k$ and to incurring an effective cost $C + c^k$. It is as if the expected profit margin of the merchant were increased by an amount:

$$u^k - c^k = \phi^k(f^k) = \int_{f^k}^{+\infty} \left( b^k_B + b^k_S - f^k - m^k \right) h^k(b^k_B) db^k_B.$$  

\textsuperscript{12}Alternatively, if the “credit good” and the “debit good” are identical, the merchant charges the same price ($p^d = p^c$) and the formula defining $b_j(\phi^d, \phi^c)$ has to be modified accordingly. The rest of the reasoning is unchanged.

\textsuperscript{13}This latter assumption is relaxed in Section 5.

\textsuperscript{14}Of course, residual demands faced by individual merchants depend on retail prices set by each merchant.

\textsuperscript{15}We assume that price setting and card acceptance decisions are made simultaneously by merchants. In fact, thanks to our assumption that consumers are ex-ante identical, the timing of merchants’ decisions is irrelevant.
\( \phi^k \) is equal to the (per transaction) expected total user surplus derived from card \( k \). Given relation (3), \( \phi^k \) can also be written:

\[
\phi^k(f^k) = \int_0^{+\infty} \left( b^k_B + b^k_S - \gamma^k - \pi^k \right) h^k(b^k_B) db^k_B.
\]

Total user surplus is a quasi-concave function of \( f^k \) (see Figure 2 below). It reaches its maximum when the sum of the cardholder fee (and thus the willingness to pay for the marginal transaction) plus the seller’s net benefit is equal to the total cost \( \gamma^k + \pi^k \) perceived by end-users and thus embodying issuer and acquirer markups:

\[
f^k = f^k_b \equiv \gamma^k + \pi^k - b^k_S.
\]

[For reasons that will become clear shortly, the subscript “\( b \)” refers to the situation where cards are provided by competing networks. We call it the competitive or Bertrand outcome.] This value is greater than the value \( f^k_* \) that maximizes social welfare per transaction, i.e., the sum of user surplus and total profit of issuers and acquirers:

\[
f^k = f^k_* \equiv \gamma^k - b^k_S.
\]

This is due to the fact that issuers’ and acquirers’ profits increase with the volume of cards transaction, which itself decreases with cardholder usage fee \( f^k \).

![Figure 2: User surplus for card k, as a function of cardholders fee f^k. The dotted curve represents social welfare. f^k_m is the minimum fee such that total user surplus is non-negative.](image)

Since volume decreases with \( f^k \), it is a dominated strategy for networks to choose \( f^k \) above \( f^k_b = \gamma^k + \pi^k - b^k_S \). If merchant \( j \) accepts card \( k \), he obtains a profit equal to \( \max_p (p - c^k - C) d^k_j(p - u^k) \), where \( d^k_j \) is the residual demand curve faced by merchant.
among holders of card $k$. Reasoning in terms of the effective price $\hat{p} = p - u^k$ faced by buyers, this profit can also be written: $\max_{p}(\hat{p} + \phi^k - C)d_j^k(\hat{p})$, where $\phi^k$ is the total user surplus defined above. Since this profit is clearly an increasing function of $\phi^k$, the equilibrium behavior of merchants consists in accepting only the card (or set of cards, taking into account possible redundancies) that maximize total user surplus, and no card at all if this maximum surplus is negative.\(^{16}\)

We are now in a position to characterize the merchants’ acceptance decisions. \(\text{Unbundling:}\) Consider, first, the case of debit cards under unbundling. Total user surplus is $\phi^d(f^d_i)$ if the merchant accepts card $i$ alone, and $\phi^d(\min(f^d_1, f^d_2))$ if the merchant accepts both, since in the latter case, only the least expensive card (from the cardholders’ viewpoint) is used. Because networks never set fees above $f^d_b$, we can restrict attention to the interval \(^{17}\) $[f^d_m, f^d_b]$, where $\phi^d$ is increasing and thus $\phi^d[\min(f^d_1, f^d_2)] \leq \phi^d(f^d_i)$. Thus merchants accept card 1 if and only if $f^d_1 > f^d_2$.

To avoid technical problems \(^{18}\) and without impact on the real allocation, we will assume that merchants accept card 1 only if accepting card 1 strictly increases their profit. Thus, we obtain:

**Proposition 1**: If the Honor All Cards (HAC) rule is not imposed by network 2, merchants accept debit card 1 alone if $\phi^d(f^d_1) > \max(0, \phi^d(f^d_2))$ and debit card 2 alone if $\phi^d(f^d_2) \geq \max(0, \phi^d(f^d_1))$. The credit card is accepted whenever $\phi^c(f^c) \geq 0$.

**Tying**: To see how the Honor All Cards rule modifies merchants’ acceptance decisions, assume now that network 2 forces merchants to accept its debit card whenever they take its credit card. The total profit obtained by merchant $j$ if he does so is:

\[
\begin{align*}
\hat{b}_j(\phi^d, \phi^c) &= \max_{\hat{p}^d, \hat{p}^c} \left\{ (\hat{p}^d + \phi^d - C^d)d^d_j(\hat{p}^d) + (\hat{p}^c + \phi^c - C^c)d^c_j(\hat{p}^c) \right\}
\end{align*}
\]

where, like before, $\hat{p}^k = p^k - u^k$ represents the effective price faced by consumers on market $k$ (market $c$ is the market for the “credit-good” and market $d$ is the market for the “debit-good”) and $d^k_j$ are residual demand curves faced by merchant $j$ on market $k$.

Given that networks charge interchange fees such that $f^d_i \leq f^d_b$, a merchant who accepts network 2’s cards cannot increase his profit by accepting network 1’s debit card: Either $f^d_1 < f^d_2$ and then card 1 yields a lower total users’ surplus, and so the merchant

\(^{16}\)This result is derived in Wright (2004) in the context of a Hotelling model and in Wright (2003) in a Cournot model, but, as we just showed, the reasoning is fully general.

\(^{17}\)Recall that $f^d_m = \min\{f | \phi^d(f) \geq 0\}$.

\(^{18}\)Technically, this assumption is made in order to avoid an “openess problem” (similar to the standard openess problem encountered under Bertrand competition with unequal costs) in the case of bundling.
strictly prefers his customers not to use card 1 (which they would do if the merchant took card 1); or $f_2^d \leq f_1^d$ and card 1 is not used by buyers, and so accepting or refusing card 1 is a matter of indifference to the merchant. The equilibrium behavior of merchant $j$ consists in accepting the network 2 cards if and only if

$$b_j(\phi^d(f_2^d), \phi^c(f^c)) \geq \max\{b_j(0,0), b_j(\phi^d(f_1^d), 0)\}.$$ 

For simplicity, we assume that merchants are identical and that the equilibrium is symmetric. We therefore drop the index $j$. 

**Proposition 2**: Under the HAC rule, merchants accept debit card 1 alone if

$$b(\phi^d(f_1^d), 0) > \max\{b(0,0), b(\phi^d(f_2^d), \phi^c(f^c))\}$$

and both cards of network 2 if

$$b(\phi^d(f_2^d), \phi^c(f^c)) \geq \max\{(b(0,0), b(\phi^d(f_1^d), 0))\}.$$ 

Note that these new conditions are more restrictive than those characterizing card acceptance in the unbundling case.\textsuperscript{19}

### 4 Network Competition: Equilibrium and Welfare Analysis

**Unbundling**: Networks select interchange fees (or equivalently cardholder fees) so as to maximize their members’ profit.\textsuperscript{20} In order to attract merchants, network 2 has to offer

\textsuperscript{19}This is because a merchant can adjust his prices after deciding, say, to change his acceptance decision from the bundle to debit card 1:

$$b(\phi^d(f_1^d), 0) \geq (\hat{p}_B^d + \phi^d(f_1^d) - C^d)d^d(\hat{p}_B^d) + (\hat{p}_B^c - C^c)d^c(\hat{p}_B^c),$$

where $(\hat{p}_B^d, \hat{p}_B^c)$ denote the effective prices chosen by the merchant when he accepts the bundle (debit card 2, credit card). Therefore the equilibrium condition implies that

$$(\hat{p}_B^d + \phi^d(f_2^d) - C^d)d^d(\hat{p}_B^d) + (\hat{p}_B^c + \phi^c(f^c) - C^c)d^c(\hat{p}_B^c)\geq (\hat{p}_B^d + \phi^d(f_1^d) - C^d)d^d(\hat{p}_B^d) + (\hat{p}_B^c - C^c)d^c(\hat{p}_B^c).$$

The left-hand side of this inequality represents by definition $b(\phi^d(f_2^d), \phi^c(f^c))$. After simplification we obtain that

$$\phi^d(f_2^d)d^d(\hat{p}_B^d) + \phi^c(f^c)\hat{p}_B^c \geq \phi^d(f_1^d)d^d(\hat{p}_B^d),$$

with equality only in the unlikely case where merchant $j$ does not change his price after changing his card acceptance policy. By adding up these inequalities for all merchants, we see that a set of cards will be accepted by merchants at equilibrium only when they maximize total user surplus, the exact condition stated in Proposition 2 being more restrictive.

\textsuperscript{20}This is one simplification afforded by the assumption that issuers and acquirers levy a constant margin. Their interests (the maximization of volume) are then congruent. Otherwise, we would need to investigate the relative bargaining power of members on either side, as in Schmalensee (2002).
a total user surplus at least equal to the one offered by network 1. The equilibrium strategy of network 1 is to choose \( f^d_1 \) that maximizes \( \phi^d \), namely \( f^d_1 = \gamma^d + \pi^d - b^d_S \).

When the HAC rule is not enforced, this leaves no choice for network 2 but also setting \( f^d_2 = \gamma^d + \pi^d - b^d_S \). As for credit cards, network 2 maximizes its members’ profit by choosing \( f^c_m = \{ \min f | \phi^c(f) \geq 0 \} \).

**Proposition 3:** In the absence of HAC rule, network competition results in identical fees for debit cards

\[
 f^d_1 = f^d_2 = f^d_b = \gamma^d + \pi^d - b^d_S.
\]

The credit card fee is equal to the minimum cardholder fee that is compatible with merchants’ acceptance:

\[
 f^c_m = \min \{ f | \phi^c(f) \geq 0 \}.
\]

The outcome of network competition in the absence of HAC rule is represented by Figure 3 below. For simplicity, this figure depicts the “symmetric” case, that is, the case where total user surplus functions \( \phi^c \) and \( \phi^d \) and welfare functions \( w^c \) and \( w^d \) are identical for credit and debit cards.

![Figure 3: The outcome of network competition in the absence of HAC rule in the symmetric case. The dotted curve represents social welfare.](image)

More generally, since cardholder fees are decreasing functions of interchange fees, Proposition 3 implies that, in the absence of HAC rule, the interchange fee can be too high or too low for credit \( f^c_m \geq f^c_* \) but is always too low for debit \( f^d_b > f^d_* \).

**Tying:** When the HAC rule is enforced, network 2 has one more degree of freedom. It can choose any combination of cardholder fees that provides merchants with at least the
profit they obtain by accepting card 1:

\[ b(\phi^d(f^d_2), \phi^c(f^c)) \geq b(\phi^d(f^d_1), 0). \] (4)

Under this constraint, network 2 chooses \((f^d_2, f^c)\) so as to maximize its members’ profit:

\[ \Pi = \pi^d D^d(f^d_2) + \pi^c D^c(f^c), \] (5)

where

\[ D^k(f^k) \equiv N^k \int_{f^k}^{+\infty} h^k (b^k_B) db^k_B \]

represents the demand functions for card transactions by each type of cardholders \((k = c, d)\).

**Proposition 4** The HAC rule raises the interchange fee for debit and lowers the interchange fee for credit.\(^{21}\)

**Proof:** Without the HAC rule, network competition forces network 2 to choose a high cardholder fee \(f^d_b = \arg \max \phi^d\) for debit. On the other hand, cardholder fees for credit are as low as possible (i.e. the minimum fees that are compatible with merchant acceptance). With the HAC rule, the same combination of cardholder fees is still feasible. However by slightly increasing cardholder fees for credit cards, the network can substantially decrease cardholder fees for debit cards (while maintaining total user surplus constant). This is because the marginal user surplus is zero at \(f^d_b\). Such a change increases total volume and is thus profitable for network 2.

The outcome of network competition with the HAC rule is represented in Figure 4 below, again in the “symmetric” case. The interchange fee for debit card 1 is unchanged. On the other hand, the HAC rule allows network 2 to increase interchange fees for debit card 2, and thus to increase volume. This is compensated by a decrease in the interchange fee for credit. Total user surplus remains unchanged, but total volume (and thus social welfare) increases.

---

\(^{21}\)These predictions are compatible with empirical evidence in the USA. After the removal of the HAC rule in 2003, Visa and MasterCard reduced their debit IFs (by roughly 30%) and increased their credit IFs (see Nelson Report 2003).
Proposition 4 shows that the HAC rule allows network 2 to lower cardholder fees for debit and increase cardholder fees for credit. Total user surplus is increased, and total volume (and total profit) is higher with the HAC rule, since network 2 has more degrees of freedom. Therefore social welfare, taken here to be the sum of total user surplus and members’ profits (or more generally any weighted average of the two surpluses), is enhanced by the HAC rule.

**Proposition 5** Under our assumptions (competing platforms, informed consumers, no substitution between debit and credit, and homogenous merchants), social welfare is higher under the HAC rule.

## 5 Robustness

For simplicity, we have worked so far under several specific assumptions. The purpose of this section is to explore the robustness of our results to alternative assumptions.

### 5.1 Monopoly system

Our results do not rely on the existence of system competition. With a single platform, tying again improves social welfare. Indeed, in the absence of HAC rule, a monopoly platform would separately choose the maximum interchange fees (or equivalently minimum cardholder fees) that are acceptable to merchants. Namely: $f^d_m = (\phi^d)^{-1}(0)$ and $f^c_m = (\phi^c)^{-1}(0)$. With the HAC rule, the monopoly would select the combination of cardholder fees $(f^d, f^c)$ that maximizes its members’ total profit $\pi^d D_d(f^d) + \pi^c D_c(f^c)$ under
the (single) constraint that merchants accept cards:

\[ b(\phi^d(f^d), \phi^c(f^c)) \geq b(0,0). \] (6)

The previous combination of fees \((f^d_m, f^c_m)\) satisfies this constraint because it has to secure acceptance of each card individually. However, the network now has more flexibility and generically chooses a different combination, that gives the same total user surplus (constraint (6) is binding) but a larger volume of transactions. Once again, social welfare is increased by the HAC rule.

**Proposition 6**: When there is a single platform, social welfare is higher under the HAC rule.

### 5.2 Uninformed consumers (Baxter’s case)

One may object to our assumption that all consumers are aware of merchants’ card acceptance policies before they choose which store to patronize. In order to check the robustness of our results to this assumption, we now consider the opposite polar case of Baxter (1983) where consumers are totally uninformed. In this case, merchants only consider their net expected convenience benefit \(B^k(f^k) = (b^k_S + f^k - \gamma^k - \pi^k)D^k(f^k)\) when deciding whether to accept card \(k\). Propositions 1 through 4 carry through by replacing total user surpluses \(\phi^k(f^k)\) by these convenience benefits \(B^k(f^k)\). When the HAC rule is not enforced, competition forces both networks to choose the cardholder fee that maximizes merchants’ convenience benefit for debit:

\[ f^d_1 = f^d_2 = \arg \max B^d(f), \]

while the cardholder fee for credit is the minimum acceptable to merchants:

\[ f^c = (B^c)^{-1}(0) = \gamma^c + \pi^c - b^c_S. \]

When the HAC rule is enforced, network 1 does not change its interchange fee, while network 2 selects \((f^d_2, f^c)\) so as to maximize total profit under the (global) acceptance decision of merchants:

\[
\begin{align*}
\max \{ & \pi^d D^d(f^d) + \pi^c D^c(f^c) \} \\
\text{s.t.} & \quad b(B^d(f^d), B^c(f^c)) \geq b(\max_f B^d(f), 0).
\end{align*}
\]

Once again the interchange fee increases for debit and decreases for credit and total volume is increased. Moreover the (total) convenience benefit of merchants remains the
same. But the impact on social welfare is a priori ambiguous since cardholder usage fees for debit and credit move in opposite directions, and therefore the net surplus of cardholders (which is no longer internalized by merchants in their acceptance decision) may decrease or increase. However in the symmetric case (identical costs, identical margins and identical demand functions for debit and credit) tying still improves social welfare under a mild technical assumption (log concave demand function).

**Proposition 7**: In the symmetric Baxter case, tying improves social welfare when the demand function of cardholders is log-concave and surpluses generated by payment instruments are small.

In the rest of the paper, we return to the case in which consumers are informed of card-acceptance policies before shopping.

### 5.3 For-profit networks

Our results immediately carry over to the case where network 1 is for-profit: Independently of whether the HAC rule is enforced by network 2, network 1 always chooses the combination of user fees that maximizes total user surplus under network 1’s breakeven condition. Similarly, when network 2 is for-profit, it selects the two combinations of user fees (on both markets) that maximize its profit under the constraint that merchants accept its cards. The fundamental rebalancing effect, that we identified in this paper, still operates. When the HAC rule is enforced, the merchant acceptance condition forces network 2 to increase (weakly) total user surplus (with respect to the case without the HAC rule), while it gains flexibility in its pricing decisions. Both user surplus and network profit (weakly) increase, leading to an unambiguous welfare increase.

**Proposition 8**: When platform 1 and/or 2 is for-profit, social welfare is higher under the HAC rule.

### 5.4 Debit-credit substitution

A simple way to introduce substitution is to assume that credit cards can also be used for debit transactions (while the opposite is not true). However this substitution is imperfect: the utility obtained by the cardholder is only $b^d_B - \Delta$ (with $\Delta \geq 0$), when using a credit card
for a debit transaction. The possibility of substitution constrains the price differential:

\[ f^d - f^c \leq \Delta. \] (7)

For simplicity we stick to the symmetric case and therefore drop the index \( k = d \) or \( c \) when feasible. When the HAC rule is enforced, the equilibrium is not altered by the possibility of substitution, since \( f^c = f^d \), and thus constraint (7) does not bind. By contrast, when the HAC rule is not enforced, the previous equilibrium no longer obtains when \( \Delta < f_b - f_m \).

**Proposition 9** : Consider the symmetric case and introduce the possibility of using credit cards for debit transactions (for a utility loss \( \Delta < f_b - f_m \)). Then:

a) When the HAC rule is enforced, the equilibrium fees remain the same as in the absence of potential substitution.

b) When the HAC rule is not enforced, the interchange fee for credit is unchanged, the interchange fee for debit increases, and user surplus decreases relative to the no-substitution case.

c) The HAC rule may decrease welfare when \( \pi \), the total profit margin of banks, is large and \( \Delta \) is small. Otherwise it increases welfare.

### 5.5 Heterogeneous merchants

Suppose that merchants differ in the numbers \( N^c \) and \( N^d \) of (potential) credit and debit transactions. For simplicity, we assume that merchants are local monopolies and we parametrize their heterogeneity by a single number \( x = \frac{N^c(x)}{N^d(x)} \). \( x \) is distributed on \((0, \infty)\) according to a c.d.f. \( G \) (and a density \( g \)). We assume that networks cannot use discriminatory prices for merchants (i.e. merchant discounts \( m \) that depend on \( x \)). It is easy to see that the equilibrium without the HAC rule is not altered: One still has \( f_1^d = f_2^d = f_b^d \) and \( f^c = (\phi^c)^{-1}(0) \). Since \( \phi^c(f^c) = 0 \), merchants’ acceptance decisions are independent of \( x \).

By contrast, when the HAC rule is enforced, things are more complex, since the profit functions of merchants vary with \( x \). Define:

\[
b(\phi^d, \phi^c, x) = \max_{\hat{p}^d, \hat{p}^c} \{ (\hat{p}^d + \phi^d - C^d) d^d(\hat{p}^d) + x(\hat{p}^c + \phi^c - C^c) d^c(\hat{p}^c) \}.
\]

\(22\Delta \), which can be small, may be strictly positive for several reasons: the cardholder may be stressed by the need to pay his bill on time so as to avoid paying penalties for delay and high interest rates, or may lose track of the magnitude of his liquidity position. It would also be interesting to relate this deadweight loss to time inconsistent behavior as in the work of Angeletos et al. (2001).
\( b(\phi^d, \phi^c, x) \) is the profit obtained by a merchant of type \( x \) when accepting the cards of network 2, divided by the size \( N^d(x) \) of its debit market. \( b \) being increasing in \( x \), there exists a critical threshold \( \hat{x} \) above which merchants accept the cards of network 2. \( \hat{x} \) is defined implicitly (assuming an interior solution) by:

\[
b(\phi^d(f^d_2), \phi^c(f^c), \hat{x}) = b(\phi^d(f^d_1), 0, \hat{x}).
\]

Thus:

- merchants with \( x < \hat{x} \) accept only debit card 1,
- merchants with \( x \geq \hat{x} \) accept only the two cards of network 2.

Network 1 solves

\[
\max_{f^d_1} \left[ \int_0^{\hat{x}} N^d(x) dG(x) \right] \left[ 1 - H^d(f^d_1) \right]
\]

where \( \hat{x} \) is defined by the above condition, while network 2 solves

\[
\max_{f^d_2, f^c} \int_{\hat{x}}^{+\infty} \left[ N^d(x) \{1 - H^d(f^d_2)\} \pi^d + N^c(x) \{1 - H^c(f^c)\} \pi^c \right] dG(x).
\]

It is interesting to compare this equilibrium (when merchants are heterogenous) to the one we have characterized earlier in Proposition 4 (when merchants are homogenous). In the latter case network 1 solves

\[
\max_{f^d_1} N^d \left[ 1 - H^d(f^d_1) \right] I \{ \frac{N^c}{N^d} < \hat{x} \},
\]

while network 2 solves

\[
\max_{f^d_2, f^c} \left[ N^d \left\{1 - H^d(f^d_2)\right\} \pi^d + N^c \left\{1 - H^c(f^c)\right\} \pi^c \right] I \{ \frac{N^c}{N^d} \geq \hat{x} \},
\]

where \( \hat{x} \) is defined as above, and for any statement \( A \), \( I_A \) denotes the indicator function of \( A \): \( I_A = 0 \) if \( A \) is false, and \( I_A = 1 \) if \( A \) is true.

When the distribution of the heterogeneity parameter \( x \) converges to a spike at \( \frac{N^c}{N^d} \), the equilibrium threshold \( \hat{x} \) also converges to \( \frac{N^c}{N^d} \), and the formulas characterizing the equilibrium when merchants are heterogenous converge to the above conditions (characterizing equilibrium in the case of homogenous merchants). When merchants’ heterogeneity is small, the equilibrium interchange fees are thus close to the equilibrium interchange fees when merchants are identical. This continuity property will be used below.
Given that network 1 now has a positive volume of transactions, it optimally trades off volume and probability of acceptance and selects a lower cardholder fee than in the homogenous-merchant equilibrium:

\[ f^d_1 < \arg \max \phi^d. \]

Merchants therefore derive a positive surplus from accepting debit card 1 only.

This implies that the user surplus generated by card 1 is smaller than under unbundling:

\[ \phi^d(f^d_1) < \max \phi^d. \]

On the other hand, for a given threshold \( \hat{x} \), network 2 selects the combination \( (f^c, f^d_2) \) that maximizes profit under the constraint

\[ b(\phi^d(f^d_2), \phi^c(f^c), \hat{x}) = b(\phi^d(f^d_1), 0, \hat{x}). \]

Since this constraint binds at equilibrium, the marginal merchant \( \hat{x} \) obtains a lower user surplus when the HAC rule is enforced (this is because \( \phi^d(f^d_1) < \max \phi^d \)).

Therefore the consequences of the HAC rule are less clearcut when merchants are heterogenous:

- Merchants with \( x < \hat{x} \) refuse the two cards of network 2, and in particular the credit card. This generates a welfare loss since some of the customers of these merchants cannot use their credit card.\(^{23}\)
- For merchants with intermediate \( x \) (above \( \hat{x} \) but close to it) \( f^d_1 \) and therefore total user surplus decrease.
- Merchants with large \( x \) benefit from the HAC rule since it induces network 2 to decrease its interchange fee on credit.

Even though we cannot exclude that the HAC rule decreases social welfare when merchants’ heterogeneity is substantial, a simple continuity\(^{24}\) argument shows that this is not true when the distribution of \( x \) is fairly concentrated around a given value: In this case the equilibrium under the HAC rule is very close to the one that obtains when merchants are identical; and the HAC rule then unambiguously increases welfare.

\(^{23}\) Notice that only network 2 (not final users) loses from these foregone credit transactions since, in the unbundling case, network 2 charges the monopoly fee and leaves no user surplus.

\(^{24}\) More precisely, we use the fact, demonstrated above, that \( f^d_1 \) converges to \( \arg \max \phi^d \) when the distribution of \( x \) converges to a spike at some \( x_0 = \frac{N^c}{N^d} \).
5.6 Competition between differentiated platforms

Our basic model considers the extreme case of a single credit card and two undifferentiated debit cards. Suppose, more generally, that the multi-card platform faces competition by differentiated single-card platforms on each segment \( k = c, d \). A simple way to introduce such a differentiation is to assume that the competing card on segment \( k \) generates a net transaction benefit \( b_k^B + b_k^S - \gamma_k - \pi_k - \delta_k \) (instead of \( b_k^B + b_k^S - \gamma_k - \pi_k \) for the \( k \)-card offered by the multi-card platform). Our basic model corresponds to the extreme case \( \delta_c = +\infty \) (no competitor on credit), and \( \delta_d = 0 \) (perfect competition on debit). We now consider general values of \((\delta^c, \delta^d)\). \( \delta^k \) can be associated with differences in marginal cost or in convenience benefit. It can be positive (in which case the multi-card platform is more efficient on segment \( k \)) or negative (in which case the competing network is more efficient).

Without the HAC rule, only the more efficient card is accepted on each segment. When \(|\delta^d|\) and \(|\delta^c|\) are small, fees are then determined by “limit pricing” conditions on each segment: the most efficient card is priced at the minimum level that induces merchants to reject the competing card. The cardholder usage fee is thus such that it generates total user surplus equal to that provided by the alternative, less efficient card (provided of course that the latter be positive, so competition has some bite).

By contrast, when the HAC rule is enforced, competition between platforms is globalized. Consider for instance the case where \( \delta^c > 0 \) (the multi-card platform is more efficient for credit) and \( \delta^d \) is either positive or slightly negative. In this case, the multi-card platform is in a position to exclude the two rival cards. It chooses a combination of fees that maximizes volume under the constraint that total user surplus (on both cards) is higher than the sum of maximum user surplus that can provided by competing systems. Total user surplus (on both cards) is thus unchanged if \( \delta^d \geq 0 \) (i.e. when the multi-card platform is more efficient on both segments) and slightly increased if \( \delta^d \) is slightly negative (if \( \delta^d \) is largely negative, entry deterrence is impossible).

In the first case \( (\delta^d \geq 0) \), social welfare is clearly increased by the HAC rule. Our usual argument remains valid: the HAC rule does not alter total user surplus, but increases volume and thus social welfare. When \( \delta^d \) is negative, the HAC rule has two opposing effects on welfare:

- total user surplus is decreased, since the efficient debit card platform is excluded,
- the price structure is “rebalanced” and total volume is increased.

Again the net effect of the HAC rule on social welfare becomes ambiguous, but by
continuity our conclusion still holds: for $\delta^a$ negative but small, the rebalancing effect dominates the inefficiency effect, and social welfare is increased by the HAC rule.

6 Concluding comments

This paper is a first pass at studying the impact of tying in two-sided markets. The analysis has been conducted under assumptions, such as the linearity of payment flows through the platform, meant to reflect the specificities of the payment system industry. Much work remains to be done in order to develop a broader vision of tying in other two-sided markets.

One important lesson that emerges from the analysis, though, is that a platform may use a tie in order to restructure its rates. In the absence of a tie, the platform must pander to the end-user side (here the merchant in the debit card market) who has attractive bypass opportunities, and this to the detriment of the other side. Thus, when one side of the market faces different bypass opportunities for two different goods, a tie on that side allows the platform to “equalize” the competitive pressure and to rebalance its rates on that side, up for the good facing the most intense competitive pressure and down for the other; the rate rebalancing on the other side of the market then operates in the opposite direction.

Furthermore, rate rebalancing may have a beneficial welfare impact; for, a Ramsey price structure should depend on the distributions of end-users’ valuations, not on the bypass opportunities. The aggregation afforded by a tie works toward allowing this basic Ramsey principle to operate. Assessing the impact of this “rebalancing effect” in other two-sided markets, such as software or media, should be the subject of further work.
APPENDIX:

**Proof of Proposition 7:**
When surpluses generated by payment instruments are small, the condition for merchants to accept cards at equilibrium becomes:

\[ N^d B^d(f^d) + N^c B^c(f^c) \geq N^d \max_f B^d(f). \]

In the symmetric case \( B^d = B^c = B \).

With the HAC rule, we have thus:

\[ f^d = f^c = B^{-1}\left(\frac{N^d b^*_f}{N^d + N^c}\right), \quad \text{with} \quad b^*_f = \max_f B(f). \]

Let us define the auxiliary function

\[ v(b) = w(B^{-1}(b)), \quad \text{for} \quad b \leq \max_f B(f). \]

Welfare is thus equal to \((N^d + N^c)v\left(\frac{N^d b^*_f}{N^d + N^c}\right)\).

By contrast, when the HAC is not enforced

\[ f^d = B^{-1}(b^*_f) \quad \text{and} \quad f^c = B^{-1}(0). \]

Welfare is thus equal to \(N^d v(b^*_f) + N^c v(0)\). Thus social welfare is increased by tying if and only if

\[ v\left(\frac{N^d b^*_f}{N^d + N^c}\right) \geq \frac{N^d v(b^*_f) + N^c v(0)}{N^d + N^c}, \]

which is true whenever \( v \) is concave.

Proposition 7 then results from the following lemma:

**Lemma 6.1** : If \( \log D \) is concave then \( v \) is concave.

**Proof:** Since \( v(b) = w(B^{-1}(b)) \), we have that:

\[ \dot{v}(b) = \frac{\dot{w}[B^{-1}(b)]}{B[B^{-1}(b)]}. \]

Since \( B^{-1} \) is increasing in the relevant range, we have to establish that \( \frac{\dot{B}}{\dot{w}}(f) \) increases with \( f \).

Now

\[ B(f) = (f - \gamma + b_S - \pi)D(f) \]
\[ B(f) = (f - \gamma + b_S - \pi)D(f) + D(f). \]

Moreover,

\[ w(f) = -\int_{f}^{+\infty} (b_B - \gamma + b_S)D(b_B)db_B. \]

\[ \Rightarrow \dot{w}(f) = (f - \gamma + b_S)D(f). \]

Thus

\[ \frac{B}{w}(f) = 1 - \frac{\pi - D}{f - \gamma + b_S}. \]

By log concavity of \( D \), \(-\frac{D}{D} \) is a decreasing function of \( f \) (and it is \( >0 \)) therefore \(-\frac{B}{w} \) increases with \( f \) and the lemma is established.

**Proof of Proposition 9:**

a) Since \( f^d = f^c \) at equilibrium, constraint (7) is not binding. Network could set \( f^d_1 < f^c + \Delta \) to trigger substitution, but debit card 1 would then be refused by merchants. So the equilibrium is unchanged.

b) If \( \Delta < f^b - f_m \) the previous equilibrium (when the HAC rule is not enforced) no longer obtains. We now establish that

\[ f^d_1 = f^d_2 = \phi^{-1}(0) + \Delta, \quad f^c = \phi^{-1}(0) \]

is a new equilibrium, by looking at possible deviations of the two networks.

**Network 1:**

- \( f^d_1 < \phi^{-1}(0) + \Delta \) is not accepted by merchants.
- \( f^d_1 > \phi^{-1}(0) + \Delta \) generates no transactions for network 1, since cardholders prefer to use their credit card, or network 2’s debit card.

**Network 2:**

- \( f^c = \phi^{-1}(0) \) is optimal given \( f^d_2 = \phi^{-1}(0) + \Delta \). Similarly \( f^d_2 = \phi^{-1}(0) + \Delta \) is optimal given \( f^c \) (for the same reason as for network 1).
- So, to benefit, network 2 must move both interchange fees, and in such a way that \( f^c = f^d_2 - \Delta \).
- If \( f^d_2 > f^d_1 \), volume is reduced on debit and also on credit.
• If $f_2 < f_1^d$ (and thus $f^c < \phi^{-1}(0)$) merchants no longer accept the credit card (nor do they accept network 2’s debit card).

So either way, a deviation by network 2 is not profitable. This completes the proof that $f^c = \phi^{-1}(0) + \Delta$, $f^c = \phi^{-1}(0)$ is the new equilibrium. $f^c$ is thus unchanged, while $f^d$ is lower (which implies that interchange fee for debit increases and total user surplus decreases).

c) To see that the HAC rule may decrease welfare, consider the case where $\pi$ is so large that $\gamma - b_S < \phi^{-1}(0)$, and $\Delta$ is so small that $\phi^{-1}(0) + \Delta < \phi^{-1}\left(\frac{N^d}{N^s+N^c}\max \phi\right)$. In this case social welfare is a decreasing function of $f$ on the interval $[\phi^{-1}(0), +\infty)$ and both $\phi^{-1}(0)$ and $\phi^{-1}(0) + \Delta$ are smaller than the equilibrium fee $\phi^{-1}\left(\frac{N^d}{N^s+N^c}\max \phi\right)$ under the HAC rule. In this case the HAC rule decreases welfare. ■
References


