Information sharing and cumulative innovation in business networks.[1]

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ABSTRACT

How can we explain the success of cooperative networks of firms which share innovations, such as the Silicon Valley or the Open Source community? This paper shows that if innovations are cumulative, making an invention publicly available to a network of firms may be valuable if the firm expects to benefit from future improvements made by other firms. A cooperative equilibrium where all innovations are made public is shown to exist under certain conditions. Furthermore, such an equilibrium does not rest on punishment strategies being followed after a deviation: it is optimal not to deviate regardless of other firm's actions following a deviation. A cooperative equilibrium is more likely to arise, the greater the number of firms in the network. When R&D effort is endogenous, cooperative equilibria are associated with strategic complementarities between firms' research effort, which may lead to multiple equilibria.

Keywords: R & D, cooperation, innovation, growth, technical progress, information sharing, Open Source, Silicon Valley, cumulative knowledge.

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1 Introduction

There are several examples of the emergence of business networks or communities where firms exchange strategic information, or even share important innovations, with competitors.

One traditional example is the Silicon Valley. Accounts of its success insist on the role of informations sharing between firms. Information sharing allows a firm to easily improve on an innovation by another firm, thus stimulating the growth process. Employee mobility and low enforcement of trade secrets by courts implied that many innovations ended being shared by firms in the network. While such process does not seem to rely on a voluntary policy of information sharing from firms, there is evidence that the culture of Silicon Valley penalizes those entrepreneurs who do not share enough information and/or sue departing employees fo violations of trade secrets. Thus, Hyde (1998) writes that "enforceability [of trade secrets] is limited because firms that litigate in defense of their trade secrets face substantial informal social and economic sanction from other firms (whose cooperation is necessary to accomplish many projects), venture capitalists, and incumbent and prospective employees". This suggests that while the instrument of information sharing was employee mobility between firms, entrepreneurs recognized that they had an interest in cooperating with other members of the network by not blocking such mobility by litigation or other means.

Another more recent example is the emergence of the "Open Source" community in the software industry. As described by its proponents (Raymond (2003)), open source software allows very fast growth for a project, as improvements that different users implement cumulate. This is especially true with respect to debugging, where users who have access to the source have an interest in finding and fixing bugs, and, with enough good will, will make their fixes public.

To an economist, a simple question is: why do such networks function at all? Why don't individual firms just free-ride by using other participants' public information while hoarding their own valuable information? To be sure, the literature on dynamic games has highighted a number of cooperation mechanisms: "trigger strategies", "reputation", etc.[1] However, these mechanisms rely on free riding being detected, which is problematic in a context where cooperation is about information revelation, since one may well play cooperative while having nothing valuable to reveal.

In this paper I describe another, far more robust, cooperation mechanism which applies to the revelation of innovations and captures part of the incentives underlying the open source movement. I assume innovations are cumulative, i.e. an innovation opens the door for further innovations. An individual firm then has an incentive to make its innovations public, because this will increase the number of firms heading toward the next step in the technological ladder. If it is expected that the next innovation will also be made public, sharing one's invention with other members in the network will shorten the time until the next innovation arrives, which benefits the individual firm. As is shown below, this benefit may outweigh the short-run cost of lower profits for the innovating firm.

This mechanism does not rely on cheating being detected nor on the sustainability of a punishment strategy. By not revealing its information the firm punishes itself since it will have to wait longer for the next innovative step. However, given that the benefits from information sharing rely on future innovations being also public, the agument does rest on the horizon being infinite and a "Nash" equilibrium where information is kept private always exists. Nevertheless, for a range of parameters, a cooperative equilibrium also exists. Another interesting result is that the cooperative equilibrium is more likely to exist, the *larger* the number of firms in the network. This contrasts with traditional arguments, which suggest that free-riding is more likely, the greater the number of agents. Here, more firms mean a much greater benefit of making one's innovation public, because a much larger number of firms will work toward the next step.

[1] See Tirole (1990) for a survey.

I also show that the cooperative equilibrium has faster growth than the non cooperative equilibrium. In some sense, when innovations are cumulative the case for public innovations is sronger than when they are not, when it can be shown that public innovations may reduce long-run growth on net because they depress the profitability of private innovation (Saint-Paul, 2003).

Finally, the model is extended to allow for an endogenous R & D effort. It is shown that the results are robust to introducing such endogeneity. Furthermore, as the cumulative benefits of an innovation depends on improvements by other firms, and thus on their R & D effort, research efforts by different firms are now complementary, which may lead to multiple equilibria.

There exists a large literature on the value of R and D cooperation between firms when there are spillovers. The value of technology sharing is well recognized by that literature. Hence, Baumol (1992) writes that "in an industry with, say, ten firms similar in output and investment in R&D, each member of a nine-firm technology cartel can expect to obtain immediate access to nine times the number of innovations that the remaining enterprise can anticipate on the average". However, to my knowledge, this is the first paper to analyze how that advantage can be turned into a cooperation mechanism when innovation is cumulative. The literature, instead, typically relies on traditional trigger strategy mechanisms, exogenously assumes cooperation, or uses a cooperative game framework.[2] Furthermore, most of it deals with cooperation in setting R&D levels rather than in the decision to make an innovation public.[3] For example, Cozzi (1999) analyzes the consequences for growth of switches between cooperative and noncooperative equilibria; but cooperation is enforced by a standard trigger-strategy mechanism and cooperation is about the R&D level. Another stand of the

[2] This is the option in Aloysius (1999). See also Brod and Shivakumar (1997). Petit and Tolwinski (1999) assume away any problem in enforcing cooperation.

[3] An interesting paper by Dutta and Seabright (2002), looks at the impact of the extent to which knowledge is explicit on growth, and at its cross-impact with competition. However, the degree of explicitness of knowmledge is exogenous, whereas it can be viewed as endogenous in the current paper. Katz and Ordover (1990) discuss informally how improvements in intellectual property rights may enhance incentives to share information by licensing. Licensing is not considered in the present paper.

literature (d'Aspremont and Jacquemin (1988), Leahy and Neary (1997)) compares outcomes with and without R and D cooperation when there are spillovers, but again does not ask the question of how cooperation is enforced and focuses on investment rather than information.

2 The model

There are N firms, competing with each other by producing differentiated goods. Each firm i is characterized by a level of technological advancement captured by an integer number n_i. The profit of firm i is given by a function

$$\pi_i(n_1, ..., n_i, ...n_N).$$

We assume that technical progress in a given firm increases its profits:

$$\frac{\partial \pi_i}{\partial n_i} > 0,$$

while technical progress in another firm decreases profits:

$$\frac{\partial \pi_i}{\partial n_j} < 0, j \neq i.$$

Finally, progression of all firms by one step in the technological ladder increases the profits of any given firm:

$$\sum_{j=1}^{N} \frac{\partial \pi_i}{\partial n_j} > 0.$$

We shall assume that if all firms other than i have the same technical level n, while firm i has technical level $\hat{n}$, then firm i's profit can be expressed in the following way:

$$\begin{aligned} \pi_i(n, ..., n, \hat{n}, n, ..., n) &= \pi(\hat{n}, n) \\ &= A^n f(\hat{n} - n), \end{aligned} \tag{1}$$

where $A > 1$, $f(.) > 0$, $f'(.)/f(.) > \ln A$.

Example: The demand curve for firm i is isoelastic and given by $y_i = Yp^{-\alpha}(p_i/p)^{-\sigma}$, where $\sigma > 1$, Y is an index of demand for the whole sector and p a sectoral price index given by

$$p = \left(\sum_{i=1}^{N} p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

Firm i's unit cost is c/B^{n_i}, with $B > 1$; thus each step in technical progress represents a geometric reduction in unit costs. Firms set their price so as to maximize their profit $\pi_i = y_i(p_i - c/B^{n_i})$. They neglect the impact of their decision on the sectoral price level and thus set $p_i = (\sigma - 1)/\sigma \cdot c/B^{n_i}$. Thus we find that the equilibrium $\pi_i(...)$ function is given by

$$\pi_i(n_1, ..., n_N) = KB^{(\sigma-1)n_i}\left[\sum_{i=1}^{N} B^{(\sigma-1)n_j}\right]^{\frac{\sigma-\alpha}{1-\sigma}},$$

where K is a constant given by $K = Y\left[\frac{\sigma c}{\sigma-1}\right]^{\frac{\sigma-\alpha}{1-\sigma}}\frac{c^{1-\sigma}}{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}$. If $n_j = n, j \neq i$, and $n_i = \hat{n}$, we get

$$\pi_i = KB^{n(\alpha-1)}\left[B^{(\sigma-1)(p-n)}\left(N - 1 + B^{(\sigma-1)(p-n)}\right)^{\frac{\sigma-\alpha}{1-\sigma}}\right];$$

thus we have $A = B^{\alpha-1}$ and $f(q) = K\left[B^{(\sigma-1)q}\left(N - 1 + B^{(\sigma-1)q}\right)^{\frac{\sigma-\alpha}{1-\sigma}}\right]$. The properties $A > 1$ and $\partial\pi_i/\partial n_j < 0, j \neq i$ hold if $1 < \alpha < \sigma$. The property $f'(.)/f(.) > \ln A$ is equivalent to

$$(\sigma - \alpha)\ln B - \frac{(\sigma - \alpha)B^{(\sigma-1)q}\ln B}{(N - 1 + B^{(\sigma-1)q})} > 0,$$

which clearly holds if $\sigma > \alpha$.

This example shows how a functional form like (1) can be derived from monopolistic competition among differentiated goods. As the whole sector climbs the technological ladder (i.e. the n_i's grow), it grows in terms of output and, as long as $\alpha > 1$, in terms of profits.

Without loss of generality we shall normalize $f(0)$ to 1 (in the previous example this amounts to pickin the right value of Y).

Time is continuous; firms maximize the expected present discounted value of their profits given by

$$V_i(t) = E_t \int_t^{+\infty} \pi_i(n_1(u), ..., n_i(u), ..., n_N(u))e^{-ru}du, \qquad (2)$$

where r is the discount rate, assumed fixed and exogenous.

The technical level available to a given firm jumps from n_i to $n_i + 1$ according to a Poisson process with arrival rate λ. Furthermore, the technical level of a given firm is transferable to other firms. Firm i has the option of revealing its level to other firms, in which case they can jump to level n_i, i.e., in some sense, adopt its technology. Therefore, with probability λ per unit of time, a given firm's technology level improves by one step, which allows it to increase it from n_i to $n_i + 1$, and it has two options:

1. Do not share its innovation with other firms, in which case they all remain at level n_j.

2. Share the innovation, in which case all firms such that $n_j < n_i + 1$ can upgrade to level $n_i + 1$.

To fix ideas we shall assume that an innovation can be released only at the time it occurs, and not after, although this is immaterial and only simplifies the analytics.

2.1 The Nash solution

Clearly, there always exists an equilibrium where firms do not share innovations. If I anticipate that other firms will never share their innovations with me, giving away my innovations to them will increase their technological level forever, which reduces my own profits, by virtue of the fact that $\partial\pi_i/\partial n_j < 0$. Thus it is a subgame perfect equilibrium to never release information. In such an equilibrium, the average speed of innovation for any given firm is λ.

3 Cooperative equilibria

We now construct an equilibrium such that it is profitable for firms to reveal their innovations to their competitors, despite the fact that it reduces their profits upon impact.

We construct an equilibrium where an innovation is shared every time it occurs. Thus, in the equilibrium path, all firms have the same technological level. Situations with a dispersion in technological levels only occur off the equilibrium path.

To construct such an equilibrium, we compute the value of a firm along the equilibrium path and an upper bound of the value of deviating by not releasing information; we then show that the former is greater than the latter for a given range of parameters. This yields the following proposition:

PROPOSITION 1 — Assume $f(.)$ satisfies

$$\sup_{k=0,1,\ldots} A^{-k} f(k) \leq \frac{r + \lambda(N - A)}{r - \lambda + \lambda N(2 - A)} \tag{3}$$

Then there exists a equilibrium where each innovation is instantaneously made public.

PROOF — Along the equilibrium path which we seek to construct, all firms have the same technological level n. Furthermore, all firms upgrade to level $n + 1$ with a flow probability equal to λN, since the day any given firm innovates, its innovation spreads to the whole sector. Consequently, the value of a firm when the whole sector is at technical level n is determined by

$$V_C(n) = \frac{\pi(n,n) + \lambda N V_C(n+1)}{r + \lambda N}.$$

The numerator is equal to the sum of the profit flow at technical level n, $\pi(n,n)$, and of the flow probability of any one firm innovating, equal to the product of the arrival rate for innovations λ and the number of firms N, times the value of the firm when all firms upgrade to the next level, $V_C(n+1)$. Iterating forward allows to compute the value of a firm along the equilibrium path:

$$V_C(n) = \sum_{i=0}^{+\infty} \left(\frac{\lambda N}{r + \lambda N}\right)^i \frac{\pi(n+i, n+i)}{r + \lambda N},$$

which, using (1), is equivalent to

$$V_C(n) = \frac{A^n}{r - \lambda N(A-1)} \tag{4}$$

For this formula to be meaningful, it must be that $\lambda N(A-1) < r$, i.e. that the growth rate of profits be lower than the interest rate; otherwise the value of a firm would be infinite.

Let $\bar{f} = \sup_{k=0,1,...} A^{-k} f(k)$. Then:

$$\begin{aligned} \pi_i(n_1, ..., n_i, ...n_N) &\leq \pi_i(0, ..., n_i, ...0) \\ &= f(n_i) \\ &\leq A^{n_i} \bar{f}. \end{aligned} \tag{5}$$

Let $\vec{n} = (n_1, ...n_N)$ denote a vector of firm's technological advancements, and $V_i(\vec{n})$ the value of firm i, if $\vec{n}$ is the current state of technology level, conditional on each firm pursuing its optimal strategy upon a shock.

Let

$$\bar{V}_i(n,p) = \max_{\substack{\vec{n}=(n_1,...,n_N) \\ n_i = n \\ n_j \leq p, j \neq i}} V_i(\vec{n})$$

We define operator T_j as follows. If $\vec{n} = (n_1, ..., n_N)$, then $T_j\vec{n} = (n'_1, ..., n'_N)$ such that

$$\begin{aligned} n'_k &= n_j, \text{ if } n_k < n_j, \\ n'_k &= n_k, \text{ if } n_k \geq n_j. \end{aligned}$$

Operator T_j tells us how the state vector is transformed if firm j releases its technical level: all firms with a lower technical level are lifted to level n_j. We also define U_j, which tells us how the state vector is transformed upon a shock hitting firm j, before it has decided whether to make it public or not:

$$U_j\vec{n} = (n_1, ..., n_{j-1}, n_j + 1, n_j, ..., n_N).$$

Next the indicator function $I_j(\vec{n})$ is defined as

$$\begin{aligned} I_j(\vec{n}) &= 1 \text{ if } V_j(T_j(\vec{n})) \geq V_j(\vec{n}), \\ &= 0 \text{ if not.} \end{aligned}$$

The value function $V_i(\vec{n})$ then obeys the following recursive condition:

$$V_i(\vec{n}) = \frac{\pi_i(\vec{n}) + \lambda \sum_{j=1}^{N} (I_j(U_j\vec{n})V_i(T_jU_j\vec{n}) + (1 - I_j(U_j\vec{n}))V_i(U_j\vec{n}))}{r + \lambda N}.$$

For any $\vec{n} = (n_1, ..., n_N)$ such that $n_j \leq p < n_i = n$, $j \neq i$, we have, for $j \neq i$, $V_i(T_jU_j\vec{n}) \leq \bar{V}_i(n, p+1)$ and $V_i(U_j\vec{n}) \leq \bar{V}_i(n, p+1)$, while $V_i(U_i(\vec{n})) \leq \bar{V}_i(n+1, p)$. This, along with (5), implies

$$V_i(\vec{n}) \leq \frac{A^{n_i}\bar{f} + \lambda(N-1)\bar{V}_i(n, p+1) + \lambda\bar{V}_i(n+1, p)}{r + \lambda N},$$

consequently:

$$\bar{V}_i(n, p) \leq \frac{A^{n}\bar{f} + \lambda(N-1)\bar{V}_i(n, p+1) + \lambda\bar{V}_i(n+1, p)}{r + \lambda N}. \tag{6}$$

To proceed, we now show that for $\vec{n} = (n_1, ..., n_N)$ such that $n_j \leq p \leq n_i = n$, it must be that

$$V_i(\vec{n}) \leq \frac{A^n \bar{f}}{r - \lambda N(A-1)}.$$

To see this, write

$$V_i(\vec{n}) = \int_0^{+\infty} \int_{\Omega_t} \pi_i(\vec{n}(\omega)) dF(\omega, t) e^{-rt} dt,$$

where ω is a state of nature representing the whole history of shocks between date 0 and date t, $\vec{n}(\omega)$ the corresponding state vector (which depends on the strategies followed by firms), and $dF(\omega, t)$ is the probability distribution of ω at date t. Clearly, regardless of which strategy is followed, the most advanced firm climbs by at most one step every time a firm is hit by a shock. Thus, if $\vec{n}(\omega) = (n_1(\omega), ..., n_N(\omega))$, then $\max_k n_k(\omega) \leq n + S(\omega)$, where $S(\omega)$ is the total number of shocks that has occured to any firm between 0 and t. $S(\omega)$ follows a Poisson distribution with arrival rate λN, (that is, $P_t(S(\omega) = k) = \frac{(\lambda N t)^k}{k!} e^{-\lambda N t}$). Consequently, we have in particular that $n_i(\omega) \leq n + S(\omega)$, so that $\pi_i(\vec{n}(\omega)) \leq A^{n+S(\omega)} \bar{f}$. Hence:

$$\begin{aligned} V_i(\vec{n}) &\leq A^n \bar{f} \int_0^{+\infty} \int_{\Omega_t} A^{S(\omega)} dF(\omega, t) e^{-rt} dt \\ &= A^n \bar{f} \int_0^{+\infty} \left(\sum_{k=0}^{+\infty} A^k \frac{(\lambda N t)^k}{k!} e^{-\lambda N t} \right) e^{-rt} dt \\ &= A^n \bar{f} \int_0^{+\infty} e^{-(r - \lambda N(A-1))t} dt \\ &= \frac{A^n \bar{f}}{r - \lambda N(A-1)}. \end{aligned}$$

Note that this inequality holds regardless of the strategies that are followed by firm i or any other firms with respect to revealing their technologies upon being hit by an innovation.

A corollary is that

$$\bar{V}_i(n,p) \leq \frac{A^n \bar{f}}{r - \lambda N(A-1)} \text{ for } p \leq n. \tag{7}$$

Substituting that inequality into the second term in the numerator of the RHS of (6) (which we can do since in (6) one assumes $p < n$), we get:

$$\bar{V}_i(n,p) \leq \frac{A^n \bar{f} + \lambda(N-1)\frac{A^n \bar{f}}{r-\lambda N(A-1)} + \lambda \bar{V}_i(n+1,p)}{r + \lambda N}. \tag{8}$$

By virtue of (7), we have $\lim_{k \longrightarrow +\infty} \left(\frac{\lambda}{r+\lambda N}\right)^k \bar{V}_i(n+k,p) = 0$, since $\lambda A/(r+\lambda N) < 1$. This allows us to integrate (8) forward and get:

$$\begin{aligned} \bar{V}_i(n,p) &\leq \sum_{k=0}^{+\infty} \left(\frac{\lambda}{r+\lambda N}\right)^k \frac{A^{n+k}\bar{f} + \lambda(N-1)\frac{A^{n+k}\bar{f}}{r-\lambda N(A-1)}}{r+\lambda N} \\ &= A^n \bar{f} \frac{r - \lambda N A + 2\lambda N - \lambda}{(r + \lambda N - A\lambda)(r - \lambda N(A-1))}. \end{aligned}$$

Now, if inequality (3) holds, then $\bar{V}_i(n,p) \leq V_i(n,...,n) = V_C(n) = \frac{A^n}{r-\lambda N(A-1)}$. Consequently, releasing innovation always dominates hoarding it when other firms are expected to follow the equilibrium path, which shows that it is indeed an equilibrium.Q.E.D.

An important aspect of Proposition 1, is that a cooperative equilibrium is more likely to exist, the greater the number of firms participating in the network. This effect runs counter to the usual analysis of free rider problems, where cooperation is made more difficult by a greater number of players. Here, the greater the number of players, the quicker one will be paid back for sharing one's innovations with others, as the next technological step is discovered by one participant.

Note also that the proof relies on using an upper-bound for the value of deviating for *any* response of other players to the deviation. Consequently, cooperation is not sustained by a trigger punishment strategy, as deviation is deterred even in the case where all other firms continue to release their innovations at all nodes following the deviation. The loss from other firms being one step backwards relative to cooperation is enough to deter opportunistic behavior. In some sense, the feedback effects of benefitting from other's future improvements on one's innovation generates gains from cooperation that are more robust – in that a firm contemplating deviations can ignore what others would do following its deviation – than those coming from punishment strategies. Thus, even if for some reason a deviation were undetected[4], incentives to cooperate would still remain.

Going back to the isoelastic example discussed above, in that special case we have

$$\begin{aligned}
\sup A^{-k} f(k) &= \lim_{k \longrightarrow +\infty} A^{-k} f(k) \\
&= \lim_{k \longrightarrow +\infty} K B^{(\sigma-\alpha)q} (N - 1 + B^{(\sigma-1)q})^{\frac{\sigma-\alpha}{1-\sigma}} \\
&= K \\
&= N^{\frac{\sigma-\alpha}{\sigma-1}},
\end{aligned}$$

where the last equality comes from the normalization $f(0) = 1$.

Thus, a sufficient condition for a cooperative equilibrium to exist is

$$N^{\frac{\sigma-\alpha}{\sigma-1}} < \frac{r + \lambda(N - A)}{r - \lambda + \lambda N(2 - A)}.$$

Note that the conclusion that greater values of N favor cooperation is now inverted. This is because N now also affects the $f(.)$ function, raising the value of $\lim_{k \longrightarrow +\infty} A^{-k} f(k)$. This captures the fact that with a larger number of differentiated firms, having an edge on the others is more valuable

[4]In the model's setting, a deviation would be detected as other firms would observe a fall in profits without an innovation being released; but that would change if profits were subject to shocks.

for a single firm, as the effect of decreasing marginal revenus at the sector level is less severe.

We can also establish a necessary condition for cooperation to hold, which has similar determinants as the sufficient one derived in Proposition 1.

PROPOSITION 2 – A necessary condition for a cooperative equilibrium to exist is

$$f(1)/A \leq \frac{r - \lambda(A-1)}{r - \lambda N(A-1)}. \tag{9}$$

PROOF – Assume a cooperative equilibrium exists and consider a firm which deviates by not releasing its innovation. Assume it releases its innovation next time it gets one. Then this strategy yields a value given by

$$\tilde{V}(n+1,n) = \frac{A^n f(1) + \lambda(N-1)\frac{A^{n+1}}{r-\lambda N(A-1)} + \lambda\frac{A^{n+2}}{r-\lambda N(A-1)}}{r+\lambda N}.$$

The middle term in the numerator reflect the fact that if another firm is hit by an innovation, it will release it, and all firms will be at technical level $n+1$.

For a cooperative equilibrium to exist, this strategy must yield a lower value than releasing the innovation, i.e. one must have

$$\tilde{V}(n+1,n) \leq \frac{A^{n+1}}{r - \lambda N(A-1)}.$$

One can straightforwardly check that it is equivalent to (9). Q.E.D.

4 Endogenous R and D effort: the role of strategic complementarities.

In the preceding analysis, the only decision made by a firm is whether or not to release its innovation. The arrival rate of innovations λ is entirely exogenous. It is relatively straightforward to extend the model to allow for an endogenous λ and to compute its equilibrium value in a cooperative equilibrium. The cost is that the sufficient condition that I am able to establish for such an equilibrium to exist is more stringent than in the previous section.

Endogenizing λ yields two key insights. First, one may believe that the previous result that an increase in the network's size boosts growth and is good for sustaining cooperation can be overturned with an endogenous λ. As N is greater and innovations arrive at a higher rate, an individual firm might be tempted to spend less on R & D and reduce λ. In fact, that intuition is incorrect: at a cooperative equilibrium[5], an increase in N increases λ locally. The reason is that the total arrival rate of innovations is additive in each firm's specific λ, so that an increase in the number of firms does not reduce the marginal gain from increasing one's λ. On the contrary, a greater value of N increases the incentives for R & D via a *capitalization effect.* The greater N, the faster the rate at which the increments in profits from a given innovation grow, and the greater the incentives to innovate. Or, to put it otherwise, the greater N, the greater the speed at which my innovation is improved by other firms, and the greater my incentive to innovate.

Second, there exist *strategic complementarities* between the R and D effort of different firms. The increase in R and D by one firm tends to increase the R and D by another firm. The reason is again the capitalization effect. If other firms increase their R and D effort, improvements on my innovation will come faster, and my incentive to innovate is larger. As a result, there may in principle be several cooperative equilibria with different arrival rates of innovation.

[5] Note that we rule out any cooperation on λ, which may be unobservable, and continue to focus on the incentive to make one's innovation public.

To endogenize the arrival rate of innovations, I assume that at each point in time a firm i can choose its own value of λ at a cost equal to $\pi_i c(\lambda)$, where π_i is its current profit. That the cost is proportional to π_i ensures that it will not become negligible relative to benefits as the economy grows. As cooperative equilibria are symmetrical, it is easy to compute the equilibrium common value of λ in such an equilibrium. Denoting again the value of a firm when all have a technical level equal to n by $V_C(n)$, the Bellman equation for an individual firm can be written as

$$V_C(n) = \max_{\lambda} \frac{\pi(n,n)(1-c(\lambda)) + \hat{\lambda}(N-1)V_C(n+1) + \lambda V_C(n+1)}{r+\lambda+\hat{\lambda}(N-1)},$$

where $\hat{\lambda}$ is the common value of λ of other firms, taken as given by the firm. We assume that $c(\lambda)$ is concave, differentiable, increasing over $[\underline{\lambda}, \bar{\lambda}]$, with $c'(\underline{\lambda}) = 0, c(\bar{\lambda}) = 1$. If there is an interior solution for λ, it is given by the first-order condition:

$$c'(\lambda)\pi(n,n) = V_C(n+1) - V_C(n). \tag{10}$$

This equation has the usual straightforward interpretation. The LHS is the marginal cost of increasing λ by one unit. The RHS is the marginal benefit, equal to the capital gain made when all firms climb one step in the technological ladder. In a symmetrical equilibrium we have $\lambda = \hat{\lambda}$, and the equivalent of eq. (4) holds, i.e.

$$V_C(n) = \frac{A^n(1-c(\lambda))}{r-\lambda N(A-1)}.$$

Substituting that along with (1) into (10), and normalizing again $f(0)$ to 1, we get an equation determining the equilibrium value of λ, denoted by λ^* :

$$c'(\lambda^*) = \frac{(A-1)(1-c(\lambda^*))}{r-\lambda^* N(A-1)}. \tag{11}$$

If $c(\frac{r}{N(A-1)}) > 1$, then this equation always has at least one solution, which is indeed interior and satisfies (10).[6]

[6] Furthermore, this restriction preents degenerate solutions where the growth rate is greater than the interest rate.

While the LHS is a increasing function of λ, the RHS may be either increasing or decreasing. This is because a rise in λ has two conflicting effects on the capital gains from an innovation. Because R and D costs are proportional to profits, a higher λ compresses the difference in income flows between two consecutive technological levels: one can call that a revenue effect. On the other hand, a higher λ increases the growth rate of that difference, which tends to increase its expected present discounted value: This is the capitalization effect. If the capitalization effect is strong enough, then multiple equilibria may arise, as illustrated on Figure 1.

In such a case, as in the general analysis of Cooper and John (1988), multiple equilibria comes from a strategic complementarity between a firm's research effort and the effort of other firms, as discussed above.

The preceding discussion does not tell us, however, whether a cooperative equilibrium exists. In fact, Proposition 1 can be extended and a sufficient condition, although by no means obvious, for a cooperative equilibrium can be established.

PROPOSITION 3 – Assume the following inequality holds:

$$\bar{f}\left[\frac{1}{r+N\underline{\lambda}}+\frac{(N-1)\bar{\lambda}}{(r-\bar{\lambda}N(A-1))(r+\bar{\lambda}(N-1)+\underline{\lambda})}\right]\frac{1}{1-\frac{A\bar{\lambda}}{r+(N-1)\underline{\lambda}+\bar{\lambda}}}\leq\frac{1-c(\lambda^{*})}{r-\lambda^{*}N(A-1)},$$

then there exists a cooperative equilibrium where all firms constantly set $\lambda = \lambda^*$ and make all innovations public.[7]

Proof – See Appendix.

5 Conclusion

We hope the present paper has shed some light on the viability of public innovation when technical progress has a cumulative dimension. Further

[7]The condition in Proposition 3 is non empty, since it collapses to that in Proposition 1 for $\bar{\lambda} = \underline{\lambda} = \lambda$ and $c(.) = 0$. By continuity one can then construct examples where both sides of that inequality are arbitrarily close to the corresponding expressions in Proposition 1.

research could focus on more specific aspects. For example, in the Silicon Valley, worker mobility has been an important vector of innovation sharing. It could be valuable to further analyze the role of the labor market in the diffusion of knowledge, a largely untouched topic until now.[8]

[8] Lerner and Tirole (2002), though, deals with the role of career concerns in the development of open source software.

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APPENDIX

Proof of proposition 3.

We follow the same steps as in the proof of Proposition 1. At each conceivable point in time there exists a vector of $\vec{n} = (n_1, ..., n_N)$ of technological levels and a vector $\vec{\lambda} = (\lambda_1, ..., \lambda_N)$ of R&D efforts. Assume we restrict λ_i to be a sole function of the state vector $\vec{n}$, $\lambda_i = \lambda_i(\vec{n})$. That is, we only consider "Markov strategies" that depend on the current vector of state variables. Clearly, if all agents consider than along any path markov strategies are followed, it is indeed optimal for them to follow such a strategy. Then the value of a firm i can be recursively written as

$$V_i(\vec{n}) = \frac{\pi_i(\vec{n})(1 - c(\lambda_i(\vec{n}))) + \sum_{j=1}^{N} \lambda_j(\vec{n}) \left[I_j(U_j\vec{n})V_i(T_jU_j\vec{n}) + (1 - I_j(U_j\vec{n}))V_i(U_j\vec{n})\right]}{r + \sum_{j=1}^{N} \lambda_j(\vec{n})}$$

Defining, as previously,

$$\bar{V}_i(n, p) = \max_{\substack{\vec{n}=(n_1,...,n_N) \\ n_i = n \\ n_j \leq p, j \neq i}} V_i(\vec{n}),$$

we again have the property that for any $\vec{n} = (n_1, ..., n_N)$ such that $n_j \leq p < n_i = n$, $j \neq i$,

$$V_i(\vec{n}) \leq \frac{A^n \bar{f}(1 - c(\lambda_i(\vec{n}))) + \sum_{j \neq i} \lambda_j(\vec{n})\bar{V}_i(n, p+1) + \lambda_i(\vec{n})\bar{V}_i(n+1, p)}{r + \sum_{j=1}^{N} \lambda_j(\vec{n})}. \tag{A1}$$

We can again prove that for $n = \max(n_i)$,

$$V_i(\vec{n}) \leq \frac{A^n \bar{f}}{r - \bar{\lambda}N(A-1)},$$

along the same lines as in the proof of Proposition 1. Plugging into (A1) yields

$$V_i(\vec{n}) \leq \frac{A^n \bar{f}(1 - c(\lambda_i(\vec{n}))) + \left(\sum_{j \neq i} \lambda_j(\vec{n})\right) \frac{A^n \bar{f}}{r - \bar{\lambda} N(A-1)} + \lambda_i(\vec{n}) \bar{V}_i(n+1, p)}{r + \sum_{j=1}^{N} \lambda_j(\vec{n})}. \tag{A2}$$

Then, observe that the following inequalities hold:

$$\frac{\sum_{j \neq i} \lambda_j(\vec{n})}{r + \sum_{j=1}^{N} \lambda_j(\vec{n})} \leq \frac{(N-1)\bar{\lambda}}{r + (N-1)\bar{\lambda} + \underline{\lambda}}.$$

$$\frac{\lambda_i(\vec{n})}{r + \sum_{j=1}^{N} \lambda_j(\vec{n})} \leq \frac{\bar{\lambda}}{r + (N-1)\underline{\lambda} + \bar{\lambda}}.$$

Therefore, we have that

$$V_i(\vec{n}) \leq \frac{A^n \bar{f}}{r + N\underline{\lambda}} + \frac{(N-1)\bar{\lambda}}{r + (N-1)\bar{\lambda} + \underline{\lambda}} \frac{A^n \bar{f}}{r - \bar{\lambda} N(A-1)} + \frac{\bar{\lambda}}{r + (N-1)\underline{\lambda} + \bar{\lambda}} \bar{V}_i(n+1, p). \tag{A3}$$

Since this holds for all vectors such that $n_j \leq p < n_i = n$, $j \neq i$,we have, for $p < n$:

$$\bar{V}_i(n, p) \leq \frac{A^n \bar{f}}{r + N\underline{\lambda}} + \frac{(N-1)\bar{\lambda}}{r + (N-1)\bar{\lambda} + \underline{\lambda}} \frac{A^n \bar{f}}{r - \bar{\lambda} N(A-1)} + \frac{\bar{\lambda}}{r + (N-1)\underline{\lambda} + \bar{\lambda}} \bar{V}_i(n+1, p).$$

Iterating, this yields

$$\bar{V}_i(n, p) \leq \sum_{k=0}^{+\infty} \alpha A^{n+k} \beta^k = \frac{\alpha A^n}{1 - A\beta}, \tag{A4}$$

where

$$\alpha = \bar{f}\left[\frac{1}{r+N\underline{\lambda}} + \frac{(N-1)\bar{\lambda}}{\left(r+(N-1)\bar{\lambda}+\underline{\lambda}\right)\left(r-\bar{\lambda}N(A-1)\right)}\right];$$

$$\beta = \frac{\bar{\lambda}}{r+(N-1)\underline{\lambda}+\bar{\lambda}}.$$

Note that if we can prove that $\bar{V}_i(n, n-1) \leq V_C(n)$ regardless of the strategies followed in a deviation, then clearly there exists a cooperative equilibrium. Given (A4) and that

$$V_C(n) = \frac{A^n(1-c(\lambda))}{r-\lambda N(A-1)},$$

Consequently, a sufficient condition is

$$\frac{\alpha}{1-A\beta} \leq \frac{1-c(\lambda^*)}{r-\lambda^* N(A-1)}, \tag{12}$$

where λ^* is the equilibrium value of λ in a cooperative equilibrium, that is any solution to (11). Substituting the expressions for α and β into yields the required condition and completes the proof. QED

Figure 1