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Comments welcome

LONG-TERM, SHORT-TERM AND RENEGOTIATION: ON THE VALUE OF COMMITMENT WITH ASYMMETRIC INFORMATION

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I. Introduction

While some relationships are governed by contracts that cover their whole duration, most of them rely on contracts of shorter duration that are frequently renegotiated. Understanding the determinants of contracts duration and renegotiation processes clearly is a major challenge in contract theory. On the one hand, transaction costs, the impossibility of foreseeing all relevant contingencies, etc., may make long-term contracts infeasible or prohibitively costly. On the other hand, short-term contracting is often associated with inappropriate intertemporal smoothing and/or opportunistic behavior.

In a recent paper (Rey and Salanié (1990)), we argued that short-term contracts may in fact be efficient. In a multi-period principal-agent framework, we proved that renegotiable short-term contracts that cover several periods will implement the long-term optimum when transfers are not too limited, objectives are conflicting, and there is no asymmetric information at any recontracting date. Similarly, Malcomson and Spinnewyn (1988) and Fudenberg, Holmstrom and Milgrom (1990) show that under related conditions, even spot contracting may be efficient in the absence of a need for intertemporal smoothing. The first two conditions (unlimited transfers and conflicting objectives) may be stringent in some specific relationships ; the third one is however the most problematic since it excludes all relationships where one of the parties has some relevant hidden information at some point. Such is the case in adverse selection problems, but also in moral hazard problems when the repetition of the relationship gradually gives the agent hidden information[1].

The purpose of the present paper is to examine whether the equivalence of long-term contracts and shorter-term contracts in some sense extends to situations with asymmetric information. The answer is obviously negative if "long-term" is understood as "full commitment", including the ability to commit never to renegotiate the initial agreement. Since Townsend (1982), many papers have demonstrated that no sequence of shorter-term contracts may implement the full commitment solution in adverse selection models. As stressed by Dewatripont (1986), in such situations it is generally optimal to commit *ex ante* to *ex post* inefficiencies. Since all parties would *ex post* agree to move to a Pareto-superior agreement, the full commitment optimum cannot be achieved via future negotiation of short-term contracts. This, however, also renders the full commitment solution somewhat fragile.

Full commitment, in the sense just described, refers to situations where negotiation costs are very high or where the parties rely on their reputation. It may be difficult, however, to bind oneself not to agree *ex post* on a better contract. We will therefore assume in this paper that

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(1) See Milgrom (1987) for a comprehensive discussion of that point.

renegotiation is always possible, and we will focus on renegotiation-proof contracts (hereafter "long-term contracts").

In our previous paper, we hinted that short-term renegotiated contracts might implement the renegotiation-proof optimum in multi-period principal-agent adverse selection models. We here provide a positive answer. We first detail the reasons why spot contracting will usually fail to implement the long-term optimum. We show that a long-term optimal contract can be implemented via spot contracts if and only if : (i) there is no need for intertemporal smoothing ; (ii) the optimal long-term contracts are dynamically consistent, in a sense we define precisely ; (iii) the sequence of spot contracts which might implement the optimal long-term contract is not subject to "hit-and-run" strategies from the "bad" agents, in a sense already defined by Laffont and Tirole (1990). We then show that short-term contracts can implement the optimal long-term contract even if it satisfies none of the above properties.

Section II introduces the model we will use throughout and defines precisely the long-term solution concept. Section III examines the case of spot contracting. Finally, section IV analyzes the equivalence between renegotiation-proof and short-term contracting and section V offers some concluding remarks.

II. The model

We consider a T-period principal-agent model of an intertemporal price-discrimination problem (for simplicity, we will assume that T is finite). At each period, the principal sells to the agent a perishable good, which he produces at unit cost c. Let q_t denote the quantity sold to the agent, which we will sometimes refer to as the agent's consumption, and which can take any value in a given set $Q \subset \mathbb{R}$, and let p_t be the payment to the principal in period t, which can take any value (including negative ones). The utility functions are respectively given by :

$$\sum_{t=1}^{T} \delta^{t-1} v(p_t - cq_t) \qquad \text{for the principal} \qquad \text{(II.1)}$$

$$\sum_{t=1}^{T} \delta^{t-1} (u(q_t,\theta) - p_t) \qquad \text{for the agent} \qquad \text{(II.2)}$$

The parameter θ measures the agent's valuation for the good and can take any value in a finite set $\Theta = \{\theta_1,\dots,\theta_n\} \subset \mathbb{R}$; more precisely, we will assume :

Assumption II.1:

- $\forall\ \theta \in \Theta,\ u(0,\theta) = 0$
- $u(q,\theta)$ is C^2 and non-decreasing in q,
- $\forall\ (q,\theta) \in Q \times \Theta,\ u''_{\theta q}(q,\theta) > 0$
- v(.) is an increasing concave function.

Note that the first two parts of Assumption II.1 imply that u increases w.r.t. θ if and only if q is positive : $u(q,\theta) = \theta\ q$ is a canonical example.

We will assume that the value of θ is known to the agent, but not to the principal. The latter holds prior beliefs over θ given by μ_1 at the beginning of the first period (for $i = 1,\dots,n$, $\mu_1(\theta_i)$ is the prior probability that θ is θ_i). For short, we will call "agent θ" an agent whose parameter value is θ.

We will moreover assume throughout that renegotiation is always possible. That is, at the beginning of each period, the principal can offer a new contract to the agent ; if it is accepted, the relationship becomes monitored by the new contract, otherwise it goes on according to the previously agreed contract. It should be clear that the possibility of renegotiation introduces additional constraints on the contracts that can effectively be implemented.

As is now well-known, what can be achieved via arbitrary contracts and renegotiation in this "complete contracts" context[2] can also be achieved through the use of renegotiation-proof contracts. Accordingly, all contracts that will be mentioned will be renegotiation-proof contracts, unless otherwise expressly stated (we will sometimes refer to "full commitment" optimal contracts, to designate the contracts that would be optimal if renegotiation could be prevented).

Let us now be more precise about contracts, renegotiation and implementation.

For the sake of notational simplicity, we will rule out in the following the possibility of public lotteries over levels of consumption. The reader will convince himself that their introduction would not modify the analysis presented below, replacing the words "level of consumption" with "lottery over levels of consumption" whenever needed.

We do not rule out the possibility of private lotteries, however, and in particular we allow the agent to randomize over several levels of

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(2) The terms "complete contracts" refer to the fact that only technological and informational constraints are taken into account ; in particular, we do not introduce any bound on the complexity of the contract.

consumption. An <u>allocation</u> thus is an application from Θ to the set of (private) lotteries over the set of feasible consumption levels and payments, $Q^T \times \mathbb{R}^T$.

A <u>contract</u> may include the rules of a revelation game to be played at each period, and specify consumption and payments according to the results of the successive games ; together with the possibility of future proposals from the principal and acceptances or refusals from the agent, a contract thus defines a "global game" to be played in the periods which follow its acceptance.

A <u>long-term</u> <u>contract</u> will be said to <u>implement</u> a given allocation if there exist (possibly mixed) strategies of the corresponding global game which : (i) form a perfect bayesian equilibrium (PBE for short in the following) of this game ; (ii) yield the desired allocation. A long-term contract implementing a given allocation will moreover be called <u>renegotiation-proof</u> if it is part of the corresponding PBE strategy for the principal to keep on proposing (the truncation of) it.

Contracts may *a priori* be quite complex, involving several communication stages in each period before and after the quantity q_t and the payment p_t are realized. However, without loss of generality, we can restrict attention to the following class of pure "tariff" long-term contracts :

Definition II.1 : A (T-t+1) long-term contract covering periods t,...,T is a tariff contract if and only if it takes the following form :

"In each period $\tau = t,\dots,T$, the agent pays a given amount, $p_\tau(q_t,\dots,q_\tau)$, which only depends upon his past and current levels of consumption".

It is easy to convince oneself that any sophisticated contract can be replaced with a renegotiation-proof tariff contract which implements the same allocation :[3]

Take a given contract that covers periods t,...,T. To construct the corresponding tariff contract, first consider an equilibrium consumption path $(q_t,\dots,q_T)$ (i.e., according to the original contract, the sequence $(q_t,\dots,q_T)$ is consumed in equilibrium with positive probability). Rationality then ensures that all sequences of payments associated with this equilibrium consumption path are equivalent for the agent (i.e., they yield

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(3) If we had not assumed away the possibility of proposing public lotteries over several levels of consumption, a tariff contract would associate payments to lotteries, and not to levels of consumption.

the same discounted sum), and optimality ensures that they must be equivalent also for the principal.[4] Therefore the sequence of payments to be associated with this sequence of consumption levels can be defined unambiguously.[5] It then suffices to take $p_t = +\infty$ to complete the tariff for all sequences of consumption levels which are not part of an equilibrium path.

Now assume that this tariff contract is proposed in lieu of the initial contract, and that it is accepted by the agent. Keeping on proposing and accepting this contract and, for the agent, choosing his consumption according to the initial allocation (randomizing if necessary between different levels of consumption), then clearly constitutes a PBE (no deviation is profitable with this new contract if no deviation was profitable with the initial contract). Therefore the new tariff contract implements the

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(4) The only motive for having several payment sequences which differ from the principal's point of view would be to signal the agent's type. But this could be done using a "pure" signal.

(5) Several sequences may indeed be associated with the same consumption path, particularly if v(.) is not strictly concave (if v(.) is linear, then these payment sequences are only required to yield the same discounted sum). Any of these sequences can be chosen for the tariff contract.

same allocation and is moreover renegotiation-proof.

Note that the above "tariff principle" is weaker than the standard revelation principle (which holds in the case of full commitment). In the present context, the revelation principle would state that attention can be restricted to tariff contracts where, at the beginning of the first period, the agent commits to a whole consumption path (there would usually be a unique equilibrium consumption path per type of agent) ; with such contracts all revelation takes place in the first period, which gives much bite to the renegotiation constraint. The tariff contracts considered here allow the agent to reveal his type progressively. The principal can thus resort to allocative inefficiencies to reduce the costs of information revelation in the late periods.

We can now describe more precisely the game that characterizes the optimal long-term contract. At the beginning of period t, the principal has some beliefs μ_t and there is a currently valid contract that covers the remaining (T-t+1) periods (at the beginning of the first period, the currently valid contract is the "null" contract : $q_t = p_t = 0$ in all periods t). The principal offers a new tariff contract for periods $t,\dots,T$, which can be accepted by the agent, in which case it becomes the currently valid contract, or refused, in which case the previous contract remains valid. Then the agent chooses (possibly randomly) a t^{th}-period level of consumption q_t and pays some transfer $p_t(q_1,\dots,q_t)$ as determined by the currently valid contract. The principal revises accordingly his beliefs from μ_t to μ_{t+1}.

Several perfect bayesian equilibria may be associated with the same initial contract. We will in this case focus on the equilibrium which is the most favorable to the principal. An optimal (renegotiation-proof) contract will thus be a T-period tariff contract and an associated allocation such that : (i) the allocation gives all agents a non-negative level of utility ; (ii) the tariff contract, once accepted by the agent in the first period, implements the allocation ; (iii) there exists a corresponding PBE for which the tariff contract is renegotiation-proof ; (iv) no other contract implements an allocation that gives the agent non-negative levels of utility and that is better for the principal.

Characterizing the optimal renegotiation-proof contract is a difficult question which has not yet received a general answer.[6] We are not primarily interested in that question, however, but rather in the following one : do we need long-term contracts to achieve the long-term optimum (subject to renegotiation constraints) ? In the following, we will

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[6] Optimal contracts have however been characterized in particular models that only allow for two types of agents (i.e. $\theta = \theta_1$ or θ_2) : Hart and Tirole (1988) analyze a T-period model where q can only take two values (q = 0 or q = 1) ; Laffont and Tirole (1990) relax the latter assumption in a two-period model.

assume the existence of a long-term optimum and shall provide some conditions for implementing this long-term optimum via short-term contracts and discuss the reasons why they might not be satisfied in our model.

III. Spot contracting

We assume in this section that long-term contracting is not possible, and that at the beginning of each period, the principal can only offer the agent a "spot" contract, that covers no more than the current period. The question we address then is : is it possible to achieve long-run efficiency in this context ?

Spot contracts do not allow for any intertemporal smoothing, and thus cannot generally be efficient. We briefly discuss this question in a first subsection ; we then focus on situations where intertemporal smoothing plays no role. From the results of Hart and Tirole (1988) (hereafter denoted HT) for instance, we know that spot contracting may not be efficient in such situations either ; however, it is interesting to see why this is so. We will state two necessary conditions for spot-implementability, which taken together are also sufficient. The first condition, which is derived in subsection 2, refers to the dynamic consistency of the long-term optimum, in a sense we will make precise ; this condition seems to have received little attention yet. The second condition, analyzed in subsection 3, refers to a kind of "reverse" incentive constraint (namely, one has to prevent "bad types" to hide behind "good types"), and has already been emphasized by Laffont and Tirole (1988) and (1990) (hereafter LTa and LTb).

1. Intertemporal smoothing

When intertemporal smoothing is needed, e.g. here when the timing of the payments is relevant, then in general spot contracts cannot attain long-run efficiency in the absence of asymmetric information ; introducing asymmetric information clearly will not help. Moreover, we emphasize below that even in those specific situations (stationary context, identical discount factors for both parties, etc.) where spot contracts would be efficient in the absence of asymmetric information, they may not be efficient any longer when asymmetric information is introduced.

Suppose that the principal's utility function v(.) is strictly concave ; it is then efficient to spread all payments over time. That is, an overall payment of P is best used when it comes as $p_t = P/\sum_{t=1}^{T} \delta^t$ in each period.(7) In the absence of asymmetric information, spot contracting would lead to $q_t = \bar{q}$ and $p_t = u(\bar{q},\theta)$ at each period, and would therefore be efficient.

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(7) Of course, this ideal smoothing generally conflicts with the requirement that p_t must be measurable with respect to $(q_1,\ldots q_t)$, and thus it can only be

Spot contracts are no longer efficient when there is asymmetric information, however, since it is necessary to give an agent his whole informational rent in exactly one period (once an agent has revealed his type, he gets zero utility in the remaining periods).

2. Dynamic consistency

We will assume away the need for intertemporal smoothing in the following two subsections. We thus change the principal's objective to [8]:

$$\sum_{t=1}^{T} \delta^{t-1}(p_t - cq_t) \tag{III.1}$$

In this context, only the discounted sum of the payments matters for both the principal and the agent, so that we can restrict attention to those long-term tariff contracts where all payments are made in the last period.

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done if P depends on q_1 alone.

(9) As usual with time-separable utility functions, this also implies that we forgo risk-sharing considerations.

(9) The proof of this statement is available from the authors.

Let us introduce some notation. First, for any beliefs μ about the agent's type, we will denote by $L_t(\mu)$ an optimal renegotiation-proof contract, starting from scratch at period t with beliefs μ for the principal, and taking into account a zero utility agent's participation constraint for the remaining (T-t+1) periods.

For the sake of simplicity, we will ignore the possibility of multiple optimal contracts, and thus refer in the following to the optimal contract $L_t(\mu)$; it should be clear, however, that all the statements which will appear could easily be rephrased to account for the possibility of multiplicity without altering their general meanings.

For any θ and beliefs μ such that $\mu(\theta) > 0$, we will denote by $r_t(\theta;\mu)$ the expected payoff of agent θ under the contract $L_t(\mu)$.

We will adopt a recursive strategy in this and the following sections : assuming that spot contracting does implement the long-term optimum for the (T-t)-period model that starts from period t+1 with any principal's beliefs μ, we will examine the conditions for this conclusion to extend to the (T-t+1)-period model. Without loss of generality, we will focus in the following on the first two periods under $L_1(\mu_1)$, and on the contracts $L_2(\mu)$.

We will denote by $Q_1(\theta)$ the set of those quantities q_1 such that, under $L_1(\mu_1)$, agent θ consumes q_1 in the first period with positiveprobability, and by Q_1 the set of all quantities chosen with positiveprobability by some type of agent (i.e. $Q_1 = Q_1(\theta_1)\cup\ldots\cup Q_1(\theta_n)$). Reciprocally, for any q_1 in Q_1, $\Theta_1(q_1)$ will denote the set of those θ's who choose q_1 with positive probability and $\underline{\theta}_1(q_1)$ will denote the lowest θ choosing q_1 with positive probability (i.e., $\underline{\theta}_1(q_1) = \text{Min}\ \{\theta \mid \theta \in \Theta_1(q_1)\}$). Lastly, for any $q_1 \in Q_1$ we will denote by $\mu_2(.|\mu_1,q_1)$ the principal's posterior probability distribution about the agent's type, at the beginning of period 2, given that under $L_1(\mu_1)$, q_1 has been consumed in the first period (the support of this distribution thus is $\Theta_1(q_1)$).

Now assume that spot contracting does implement the long-term optimum for the (T-1)-period model that starts from period 2 with any principal's beliefs μ (that is, the long-run efficiency associated with contract $L_2(\mu)$ can be achieved via contracting "on the spot" in periods 2,3,....,T). To extend this conclusion to the T-period game, it is necessary to find a first-period transfer function $p_1(.)$ such that for all θ and all q_1 in $Q_1(\theta)$:

$$u(q_1,\theta) - p_1(q_1) + \delta\ r_2(\theta;\mu_2(.|\mu_1,q_1)) = r_1(\theta;\mu_1) \tag{III.1}$$

The left-hand side of (III.1) is precisely agent θ's payoff under the sequence of spot contracts that starts with $p_1(.)$, assuming all agents choose the same consumption strategies as under $L_1(\mu_1)$.

Equation (III.1) only is a necessary condition for spot implementation, since it does not take the first-period incentive constraints into account. However, it is not obvious that equation (III.1) admits a solution in $p_1(.)$. If the optimal long-term contract were fully separating, the answer would be straightforward ; simply define $p_1(.)$ by :

$$p_1(q_1) = u(q_1,\theta) - r_1(\theta;\mu_1) \qquad \text{if } q_1 \in Q_1(\theta) \qquad \text{(III.2)}$$
$$p_1(q_1) = +\infty \qquad \text{if } q_1 \notin Q_1$$

which makes sense since all $\Theta_1(q_1)$'s are singletons in that case (in defining p_1, we used $r_2(\theta;\mu_2(.|\mu_1,q_1)) = 0$ since information is perfect at the beginning of the second period).

In the general case, the same q_1 may be chosen by different agents with positive probability, and we have to prove that the definition of $p_1(q_1)$ is not ambiguous ; more precisely, we must have :

$$\forall\, q_1 \in Q_1,\ \forall\, \theta,\theta' \in \Theta_1(q_1),$$

$$u(q_1,\theta') - u(q_1,\theta) + \delta[r_2(\theta';\mu_2(.|\mu_1,q_1)) - r_2(\theta;\mu_2(.|\mu_1,q_1))]$$
$$= r_1(\theta';\mu_1)-r_1(\theta;\mu_1) \qquad \text{(III.3)}$$

This points to a strong relationship between informational rents under $L_2(\mu_2(.|\mu_1,q_1))$ and under the continuation of $L_1(\mu_1)$ given q_1. More precisely, for all θ and all q_1 in $Q_1(\theta)$, let us define $R_2(\theta;\mu_1,q_1)$ by the following equality :

$$r_1(\theta;\mu_1) = u(q_1,\theta) + \delta R_2(\theta;\mu_1,q_1) \qquad \text{(III.4)}$$

$R_2(\theta;\mu_1,q_1)$ can be interpreted as the rent that the long-term optimal contract guarantees to agent θ over the last (T-1) periods if he chooses q_1 in the first period and all payments are made in the last period.

Using the definition of $R_2(\theta;\mu_1,q_1)$, equation (III.3) can be rewritten as :

$\forall\ q_1 \in Q_1,\ \forall\ \theta,\theta' \in \Theta_1(q_1),$

$$r_2(\theta';\mu_2(.\,|\mu_1,q_1)) - r_2(\theta;\mu_2(.\,|\mu_1,q_1)) = R_2(\theta';\mu_1,q_1) - R_2(\theta;\mu_1,q_1) \qquad \text{(III.5)}$$

or, equivalently, as :

$\forall\ q_1 \in Q_1,\ \forall\ \theta \in \Theta_1(q_1),$

$$r_2(\theta;\mu_2(.\,|\mu_1,q_1)) - r_2(\underline{\theta}_1(q_1);\mu_2(.\,|\mu_1,q_1)) = R_2(\theta;\mu_1,q_1) - R_2(\underline{\theta}_1(q_1);\mu_1,q_1) \qquad \text{(III.6)}$$

The informational rent $r_2(\underline{\theta}_1(q_1);\mu_2(.\,|\mu_1,q_1))$ is necessarily zero (it is always possible to uniformly increase all payments without disturbing the incentive and renegotiation constraints ; moreover, incentive compatibility and Assumption II.1 imply that the agent with the lowest θ gets the lowest rent, so that the participation constraint is binding at the optimum for the agent with the lowest θ). Condition (III.6) can thus be rewritten as :

$$\forall\, q_1 \in Q_1,\ \forall\, \theta \in \Theta_1(q_1),$$

$$r_2(\theta;\mu_2(.\,|\mu_1,q_1)) = R_2(\theta;\mu_1,q_1) - R_2(\underline{\theta}_1(q_1);\mu_1,q_1) \qquad \text{(III.7)}$$

In other words, the informational rents granted by the initial contract to the different types of agent after q_1 has been chosen in the first period must coincide, up to a certain constant, with the rents which would be granted by the optimal contract, starting from scratch at the beginning of the second period.

It can actually be proved that this property of the buyer's consumptions carries on to the contracts themselves (that is, the tariffs used by the seller have the same property)[10]. For this reason, we call it "dynamic consistency".

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(10) The proof of this statement is available from the authors.

Applying the above analysis to all renegotiation dates, we have :

<u>Theorem</u> <u>III.1</u> : If the long-term optimal contract $L_1(\mu_1)$ is implementable via spot contracting, then it is necessarily dynamically consistent at all renegotiation dates.

Dynamic consistency *a priori* seems to be a reasonable requirement ; in particular, it holds in HT's and LTb's two-types-of-agent models. However, this property only explicitly appears in LTb, where it is proved constructively (i.e., by exhibiting the long-term optimum and checking it is dynamically consistent). No direct argument has yet been provided for these or more general models. We conjecture that the long-term optimum is always dynamically consistent in models with only two types[(11)] ; however models with more than two types appear to be much more complicated. In the following we will pay more attention to the other necessary conditions for implementability.

[(11)] If this property could be proven to hold, then it should clearly make the characterization of renegotiation-proof equilibria much easier.

If the optimal long-term contract is dynamically consistent, then condition (III.1), which determines the payment in the first-period spot contract, can be rewritten as (taking $\theta = \underline{\theta}_1(q_1)$ in (III.1), and using $r_2(\underline{\theta}_1(q_1);\mu_2(.|\mu_1,q_1)) = 0$) :

$$u(q_1,\underline{\theta}_1(q_1)) - p_1(q_1) = r_1(\underline{\theta}_1(q_1);\mu_1) \qquad \text{(III.8)}$$

or, equivalently :

$$p_1(q_1) = - \delta R_2(\underline{\theta}_1(q_1);\mu_1,q_1) \qquad \text{(III.9)}$$

The interpretation of $p_1(.)$ is most simple when agent θ reveals himself in the first period by consuming q_1 ; $p_1(q_1)$ then equals $u(q_1,\theta) - r_1(\theta;\mu_1)$, which is just enough to compensate agent θ for the fact that he will get zero informational rent from period 2 onwards. When several types of agent choose the same level of consumption q_1 in the first period, $p_1(q_1)$ is chosen so as to compensate fully in the first period the agent with the lowest θ, $\underline{\theta}_1(q_1)$. Dynamic consistency then ensures that agents with a higher θ will overall get exactly the same rents as under $L_1(\mu_1)$: they obtain $u(q_1,\theta) - p_1(q_1) = u(q_1,\theta) + \delta R_2(\underline{\theta}_1(q_1);\mu_1,q_1)$ in the first period and $\delta r_2(\theta;\mu_2(.|\mu_1,q_1)) = \delta R_2(\theta;\mu_1,q_1) - \delta R_2(\underline{\theta}_1(q_1);\mu_1,q_1)$ in the remaining periods.

3. Reverse incentive compatibility constraints

The above choice of $p_1(.)$ gives the agents who (partially) reveal themselves their corresponding informational rents in the first period. As emphasized by LTa and LTb, however, this may create opportunities for lower θ's to adopt "hit-and-run" strategies, thus capturing a one-period surplus and disturbing the incentive constraints.

Assume that in period 1 agent θ_i chooses a first period consumption that mimics that of another θ_j. Should he stop playing, or reveal that he deviated, he would then get zero utility afterwards (assuming for instance that if the principal detects a deviation, he believes that the agent has the highest valuation). He will therefore go on mimicking θ_j's consumption path until some period τ and get zero utility in the remaining periods, if any. More precisely, if he chooses agent θ_j's equilibrium consumption path $(q_1,\dots q_T)$, agent θ_i gets, with straightforward notation :

$$\sum_{t=1}^{\tau} \delta^{t-1} [u(q_t,\theta_i) - p_t(q_1,\dots,q_t)]$$

$$= \sum_{t=1}^{\tau} \delta^{t-1} [u(q_t,\theta_j) - p_t(q_1,\dots,q_t)] + \sum_{t=1}^{\tau} \delta^{t-1} [u(q_t,\theta_i) - u(q_t,\theta_j)]$$

$$= \sum_{t=1}^{\tau} \delta^{t-1} [r_t(\underline{\theta}_t(q_1,\dots,q_t);\mu_t) + u(q_t,\theta_i) - u(q_t,\underline{\theta}_t(q_1,\dots,q_t))] \qquad \text{(III.10)}$$

where the last equality results from the definition of the price sequence, as given by (III.8). Note that all terms in the above summation are positive if $\theta_i > \theta_j$, since the r_t's are all non-negative. Therefore, any agent θ_i who chooses to mimick a lower θ_j's consumptions will continue to do so until the last period ; by so doing, he gets the utility level he would obtain by pretending to be θ_j under $L_1(\mu_1)$; the incentive compatibility of the latter contract therefore ensures that such a deviation is not profitable.

Alternatively, an agent θ may well find it profitable to pretend to have a higher valuation than his own. Assume for instance that for some $\theta_j > \theta_i$, $Q_1(\theta_j) = \{q_1\}$, so that θ_j reveals himself in the first period ; consuming q_1 in the first period and leaving the game then yields θ_i a payoff of :

$$u(q_1,\theta_i) - u(q_1,\theta_j) + r_1(\theta_j;\mu_1), \qquad \text{(III.11)}$$

which may be greater than $r_1(\theta_i;\mu_1)$: agent θ_i's loss of utility, $u(q_1,\theta_j) - u(q_1,\theta_i)$, may be dominated by the capture of the informational rent $r_1(\theta_j;\mu_1) - r_1(\theta_i;\mu_1)$.

The underlying idea is that agent θ_i can grab agent θ_j's informational rent in period 1 and then "run away" (the deviator may in fact prefer to go on pretending to be agent θ_j for some number of periods, before "running away" : keeping on mimicking agent θ_j's behavior brings additional losses of the form $u(q_t,\theta_i) - u(q_t,\theta_j)$, but also allows to capture the extra rent $r_t(\underline{\theta}_t(q_1,\ldots,q_t);\mu_t)$).

Consider the following example adapted from HT : there are two types of agents, $\underline{\theta}$ and $\overline{\theta}$, q is restricted to be 0 or 1, and $u(\theta,q) = \theta q$. For some range of μ_1, the optimal renegotiation-proof contract is such that $\overline{\theta}$ consumes in all periods and makes a total payment $\overline{\theta} + \sum_{t=2}^{T} \delta^{t-1}\underline{\theta}$, whereas $\underline{\theta}$ consumes in all periods but the first and pays $\sum_{t=2}^{T} \delta^{t-1}\underline{\theta}$. Thus $r_1(\overline{\theta};\mu_1) = \sum_{t=2}^{T} \delta^{t-1}(\overline{\theta}-\underline{\theta})$ and $r_1(\underline{\theta};\mu_1)$ and the r_2's are all zero. Spot implementation requires a first-period transfer given by $p_1(0) = 0$ and $p_1(1) = \overline{\theta} - \sum_{t=2}^{T} \delta^{t-1}(\overline{\theta}-\underline{\theta})$. If $p_1(1) < \underline{\theta}$, agent $\underline{\theta}$ will actually choose to deviate and consume in the first period, making spot implementation impossible.

Note that $p_1(1) < \underline{\theta}$ is equivalent to $\sum_{t=2}^{T} \delta^{t-1} > 1$; thus spot implementation will only be possible when δ is low enough ($\delta \le 1/2$ for T infinite) and/or when T is small enough (the long-term optimum is for instance always implementable via spot contracting for T=2). Intuitively, the total discounted value of the informational rent rises when the agents are less impatient and when the number of periods increases, making a "hit-and-run" strategy more attractive. Such will also be the case when the first-period move reveals the higher θ's, as in the example above.

It thus appears that spot implementation requires a sort of "reverse incentive compatibility" to hold : whereas the difficulty is usually to prevent the high-valuation buyers from pretending to have a lower valuation, here the situation is the opposite.

<u>Definition</u> <u>III.2</u> : We will say that a dynamically consistent optimal long-term contract is <u>reverse</u> <u>incentive</u> <u>compatible</u> if the above "hit-and-run" strategies are not effective for bad types.

Since we have examined all possible deviations, we can now state our second result :

<u>Theorem</u> <u>III.2</u> : If the principal is indifferent about the timing of payments, and if the optimal long-term contract is dynamically consistent and reverse incentive compatible, then the long-term optimal allocation can be implemented through spot contracting.

This result makes it easy to understand why renegotiation-proof contracts do not achieve more than spot contracts in the "durable good" model in HT, where the game is over as soon as the durable good has been bought. If agent $\underline{\theta}$ were to pretend to be $\bar{\theta}$, he would have to buy the good in the first period and to keep it in all remaining periods. He would then have to pay a price strictly higher than $\sum_{t=1}^{T} \delta^{t-1}\underline{\theta}$ (otherwise the optimal renegotiation-proof contract would have him buy in the first period, which implies that both types buy and there is no opportunity for cheating), which cannot net him a gain in utility. The arguments developed above thus allow us to obtain a very simple proof of HT's Theorem 2.

IV. Short-term contracting

The previous section emphasizes that spot contracts will be as efficient as long-term contracts only when : (i) there is no need for intertemporal smoothing (the timing of the payments thus is irrelevant) ; (ii) the optimal long-term contract is dynamically consistent, in the sense that no "extra" rent is ever promised to any type of agent ; (iii) giving informational rents to "good types" does not induce "take the money and run" strategies from the "bad types". We now show that even when these conditions are not satisfied, short-term contracts that cover a limited number of periods, may still be as efficient as long-term contracts.

To introduce short-term contracts, we modify the model as follows. At the beginning of each period t, the principal can now offer the agent a renegotiable two-period contract, covering both the current period t and the next to come. Short-term contracts allow the principal to "promise" the agent to pay him some rent in the future, and to spread this promise over time through ulterior renegotiation. This ability to design flexible promises is a powerful tool for implementing the long-term optimum in situations where spot contracting would be inefficient.

1. <u>Intertemporal smoothing</u>

Let us assume in this subsection that the principal's utility function v(.) is strictly concave. Then the timing of payments will generally be uniquely defined, and spot contracting will be inefficient. Short-term contracting may however be efficient.

For the sake of clarity, we will assume in this subsection that the optimal long-term contract is dynamically consistent and that "take the money and run" strategies are not effective for the bad types. We will relax these two assumptions in the following subsections.

Let us focus on the first period, and assume that short-term contracting implements the long-term optimum from period 2 onwards (that is, for any μ_2, the long-term optimum $L_2(\mu_2)$ can be implemented via short-term contracting). For every feasible q_1, let us denote by $p_1^*(q_1)$ the price associated with q_1 in the first period, according to the long-term optimum (with the convention $p_1^*(q_1) = +\infty$ if $q_1 \notin Q_1$, i.e. if q_1 does not belong to any equilibrium consumption path) and recall that $\underline{\theta}_1(q_1)$ denote the lowest θ such that agent θ chooses q_1 with positive probability according to the long-term optimum.

First, choose any $q_2^*(q_1)$ small enough that

$$\forall\ \theta \in \Theta_1(q_1),\quad u(q_2^*(q_1),\theta) - u(q_2^*(q_1),\underline{\theta}_1(q_1)) \leq r_2(\theta;\mu_2(.|\mu_1,q_1)) \qquad \text{(IV.1)}$$

($q_2^*(q_1)=0$ is one obvious solution)

and define $p_2^*(q_1)$ as follows :

$$u(q_1,\underline{\theta}_1(q_1)) - p_1^*(q_1) + \delta\ (u(q_2^*(q_1),\underline{\theta}_1(q_1)) - p_2^*(q_1))$$
$$= r_1(\underline{\theta}_1(q_1);\mu_1) \qquad \text{(IV.2)}$$

Now, consider the following two-period contract, to be proposed at the beginning of the first period :

"in the first period, the agent can choose any quantity q_1 in Q_1 and then pays the corresponding transfer $p_1^*(q_1)$; in the second period, he consumes $q_2 = q_2^*(q_1)$ and pays $p_2(q_1,q_2^*(q_1)) = p_2^*(q_1)$" (12)

The left-hand side of (IV.2) is precisely agent $\underline{\theta}_1(q_1)$'s payoff

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(12) I.e., $p_2(q_1,q_2) = +\infty$ if $q_2 \neq q_2^*(q_1)$.

under the sequence of renegotiable two-period contracts that starts with this contract, assuming all agents choose the same consumption strategies as under $L_1(\mu_1)$: using dynamic consistency, agent $\underline{\theta}_1(q_1)$ gets zero rent in the remaining periods, since he then has the lowest θ among those agents who choose q_1 in the first period. But, using dynamic consistency again, this implies that <u>all</u> agents choosing q_1 in the first period will obtain exactly the same level of utility under this sequence of contracts as under the long-term optimal contract : because of (IV.1), every agent is "promised" a rent lower than what $L_2(\mu_2(.|\mu_1,q_1))$ guarantees he will get.

Thus short-term contracting implements the long-term optimum if the latter is dynamically consistent and reverse incentive compatible, even if the principal's objective function is strictly concave..

2. <u>Dynamic consistency</u>

As emphasized in section II, dynamic consistency is a necessary condition for attaining long-run efficiency through spot contracting, because contracting "on the spot" does not allow for the monitoring of future rents. We now show that long-term optimal contracts need not be dynamically consistent to be implemented via short-term contracting.

Consider a long-term optimal contract, and assume for instance that it is not dynamically consistent in period 2 after some q_1 has been chosen in the first period. Let us fix such a q_1. The failure of dynamic consistency to hold implies that the rent differential between an agent θ in $\Theta_1(q_1)$ and $\underline{\theta}_1(q_1)$, given by :

$$R_2(\theta;\mu_1,q_1) - R_2(\underline{\theta}_1(q_1);\mu_1,q_1) \tag{IV.16}$$

differs from the rent which would be guaranteed "spontaneously", starting the relationship in period 2 with the same beliefs over $\Theta_1(q_1)$, i.e.:

$$r_2(\theta;\mu_2(.|\mu_1,q_1)) \tag{IV.17}$$

With short-term contracts, however, it is possible through adequately chosen "promises" to control the agent's participation constraint in the renegotiation that may take place at the beginning of the second period. Assume for instance that the principal and the agent agree in the first period on some tariff $p_2(q_1,q_2)$ for the second period. Then the agent can refuse any further renegotiation and choose in the second period the quantity q_2 which maximizes $u(q_2,\theta) - p_2(q_1,q_2)$. What the agent eventually gets also depends on the outcome of negotiations in the remaining periods. If the principal interprets refusals to renegotiate as a sign that the agent has a high valuation, however, then the agent will gain nothing from the negotiations taking place in the remaining periods. Since perfect bayesian equilibria do not place any restriction of out-of-equilibrium beliefs, there is no loss in generality in making this assumption. Therefore one way to implement a long-term optimal contract, dynamically consistent or not, is to find a tariff function $p_2(q_1,q_2)$ such that the net utility agent θ in $\Theta_1(q_1)$ gets in the second period under $p_2(q_1,q_2)$, that is :

$$\max_{q_2 \in Q} \{u(q_2,\theta) - p_2(q_1,q_2)\} \tag{IV.18}$$

coincides with what he would get under the continuation of $L_1(\mu_1)$ given q_1, that is :

$$U(q_1,\theta) = R_2(\theta;\mu_1,q_1) + p_1^*(q_1)/\delta \tag{IV.19}$$

Let us call (DC) the resulting, rather complicated system of equations in p_2. We will now see that the existence of a solution to (DC) requires no more than a surjectivity assumption much like that in our earlier paper (Rey-Salanié (1990)) and a monotonicity assumption.

To see this, we denote $\Theta_1(q_1)$ by $\{\theta^1,\ldots,\theta^p\}$, with $\theta^1=\underline{\theta}_1(q_1) < \ldots < \theta^p$ and for $i = 1,\ldots,p-1$, we define $\tilde{q}_i$ by:

$$u(\tilde{q}_i,\theta^{i+1}) - u(\tilde{q}_i,\theta^i) = U(q_1,\theta^{i+1}) - U(q_1,\theta^i). \tag{IV.20}$$

We will use the convention : $\tilde{q}_0=\inf(Q)$ and $\tilde{q}_p=\sup(Q)$.

We now prove that a necessary condition for a solution p_2 to exist is that the $\tilde{q}_i$'s exist in Q and increase w.r.t. i, and that this condition is also necessary if Q is a closed interval in $\mathbb{R}$.

Theorem IV.1 :

1) Assume that the $\tilde{q}_i$'s exist in Q and are non-decreasing in i :

$$\tilde{q}_1 \le \dots \le \tilde{q}_{p-1}$$

Then

i) the function $p_2^*(q_1,q_2)$ defined by:

$$p_2^*(q_1,q_2) = u(q_2,\theta^i) - U(q_1,\theta^i) \text{ if } \tilde{q}_{i-1} \le q_2 \le \tilde{q}_i$$
$$\text{for some } i=1,\dots,p$$

is a solution to (DC).

ii) Under p_2^*, $[\tilde{q}_{i-1},\tilde{q}_i] \cap Q$ is the preferred set of θ^i, for $i=1,\dots,p$.

iii) All other solutions of (DC) take the form

$$p_2(q_1,q_2) = p_2^*(q_1,q_2) + v(q_1,q_2),$$

where v is a non-negative function such that for all $i=1,\dots,p$, $v(q_1,q_2) = 0$ for at least one of the quantities q_2 in $[\tilde{q}_{i-1},\tilde{q}_i] \cap Q$.

2) If Q is a closed interval in $\mathbb{R}$ and (DC) has a solution, then the $\tilde{q}_i$'s exist in Q and are non-decreasing in i.

Proof :

Let us define, for any θ in $\Theta_1(q_1)$, D_θ as the set of quantities q_2 in Q for which $u(q_2,\tilde{\theta}) - U(q_1,\tilde{\theta})$ reaches its maximum when $\tilde{\theta} = \theta$.

The proof uses the following lemma :

Lemma IV.1 :

1) A necessary and sufficient condition for the existence of a solution to (DC) is that

$$\text{for all } \theta \text{ in } \Theta_1(q_1),\ D_\theta \neq \emptyset \qquad \text{(IV.21)}$$

2) If (IV.21) holds, then

i) $$p_2^*(q_1,q_2) = \underset{\tilde{\theta}\in\Theta_1(q_1)}{\text{Max}} \{u(q_2,\tilde{\theta}) - U(q_1,\tilde{\theta})\}$$

solves (DC)

ii) for all θ in $\Theta_1(q_1)$, D_θ is the preferred set of θ under p_2^*.

iii) all other solutions $p_2(.,.)$ take the form

$$p_2(q_1,q_2) = p_2^*(q_1,q_2) + v(q_1,q_2),$$

where v is a non-negative function such that

for any θ in $\Theta_1(q_1)$,

$v(q_1,q_2) = 0$ for at least one q_2 in D_θ.

Proof of the lemma :

1) Assume that there exists a solution $p_2(q_1,q_2)$ to (DC); this

implies that:

$$\forall\theta\in\Theta_1(q_1),\ \forall q_2\in Q,\quad u(q_2,\theta) - p_2(q_1,q_2) \le U(q_1,\theta),$$

and therefore

$$\forall q_2\in Q,\quad p_2(q_1,q_2) \ge \max_{\theta\in\Theta_1(q_1)} \{u(q_2,\theta) - U(q_1,\theta)\}. \tag{IV.22}$$

Define $p_2^*(q_1,q_2)$ by:

$$p_2^*(q_1,q_2) = \max_{\theta\in\Theta_1(q_1)} \{u(q_2,\theta) - U(q_1,\theta)\}. \tag{IV.23}$$

(IV.22) implies that any solution of (DC) must take the form:

$$p_2(q_1,q_2) = p_2^*(q_1,q_2) + v(q_1,q_2), \tag{IV.24}$$

where v(.,.) is a non-negative function. Moreover, (DC) implies:

$$\forall\theta\in\Theta_1(q_1),\ \max_{q_2\in Q} \{u(q_2,\theta) - U(q_1,\theta) - p_2^*(q_1,q_2) - v(q_1,q_2)\} = 0 \tag{IV.25}$$

By the definition of p_2^*,

$$\forall\theta\in\Theta_1(q_1),\ \forall q_2\in Q,\quad u(q_2,\theta) - U(q_1,\theta) \le p_2^*(q_1,q_2) \tag{IV.26}$$

Therefore (IV.25) can only hold if:

$\forall\theta\in\Theta_1(q_1)$, $\exists q_2\in Q$ s.t.:

$$\begin{cases} u(q_2,\theta) - U(q_1,\theta) = p_2^*(q_1,q_2) = \underset{\tilde\theta\in\Theta_1(q_1)}{\text{Max}} \{u(q_2,\tilde\theta) - U(q_1,\tilde\theta)\} \\ \text{and} \\ v(q_1,q_2) = 0 \end{cases} \tag{IV.27}$$

This yields the necessary condition in part 1 of the Lemma.

2) Moreover, since by construction $p_2^*(q_1,q_2)$ is no smaller than $u(q_2,\theta) - U(q_1,\theta)$ and since $U(q_1,\theta)$ does not depend on q_2, (IV.27) is equivalent to:

$\forall\theta\in\Theta_1(q_1)$, $\exists q_2\in Q$ s.t.:

$$\begin{cases} q_2 \text{ maximizes } \{ u(q_2,\theta) - p_2^*(q_1,q_2) \} \\ \text{and} \\ v(q_1,q_2) = 0 \end{cases} \tag{IV.28}$$

We now solve for agent θ's preferred set under p_2^* :

$$\underset{q_2\in Q}{\text{Max}} \{u(q_2,\theta) - p_2^*(q_1,q_2)\}$$

$$= \underset{q_2 \in Q}{\text{Max}} \{u(q_2,\theta) - \underset{\tilde{\theta}\in\Theta_1(q_1)}{\text{Max}} [u(q_2,\tilde{\theta}) - U(q_1,\tilde{\theta})]\}$$

$$= \underset{q_2 \in Q}{\text{Max}} \{u(q_2,\theta) - U(q_1,\theta) - \underset{\tilde{\theta}\in\Theta_1(q_1)}{\text{Max}} [u(q_2,\tilde{\theta}) - U(q_1,\tilde{\theta})]\} + U(q_1,\theta)$$

(IV.29)

The expression to be maximized in q_2 clearly is non-positive. Moreover, it is zero if and only if q_2 is in D_θ, which is non-empty by condition (IV.21). Thus D_θ is the preferred set of θ, and :

$$\forall\theta\in\Theta_1(q_1), \quad \underset{q_2 \in Q}{\text{Max}} \{u(q_2,\theta) - p_2^*(q_1,q_2)\} = U(q_1,\theta) \qquad \text{(IV.30)}$$

which yields part 2 of the Lemma and completes the proof.

Q.E.D.

We now go back to the proof of Theorem IV.1.

1) Assume that the $\tilde{q}_i$'s exist and are non-decreasing. We will now prove that (IV.21) must hold. Recall that q_2 is in D_θ if and only if, for all θ' in $\Theta_1(q_1)$,

$$u(q_2,\theta) - U(q_1,\theta) \geq u(q_2,\theta') - U(q_1,\theta')$$

Let $\theta=\theta^i$ and choose q_2 in $[\tilde{q}_{i-1},\tilde{q}_i] \cap Q$. We have for all $j < i$:

$$U(q_1,\theta^i) - U(q_1,\theta^j) = \sum_{l=j}^{i-1} [u(\tilde{q}_l,\theta^{l+1}) - u(\tilde{q}_l,\theta^l)]$$

Since the $\tilde{q}_l$'s are non-decreasing, $q_2 \geq \tilde{q}_l$ for all $l=j,\ldots,i-1$ and $\partial^2_{\theta q} u > 0$ implies that

$$U(q_1,\theta^i) - U(q_1,\theta^j) \leq \sum_{l=j}^{i-1} [u(q_2,\theta^{l+1}) - u(q_2,\theta^l)]$$

$$= u(q_2,\theta^i) - u(q_1,\theta^j)$$

Moreover, the equality holds if and only if $q_2 = \tilde{q}_l$ for all $l=j,\ldots,i-1$, i.e. if $i=j+1$ and $q_2 = \tilde{q}_j$.

A similar argument proves the corresponding results for the case when $i < j$. We thus have shown that

for all $i=1,\ldots,p$, $D_{\theta^i} = [\tilde{q}_{i-1},\tilde{q}_i] \cap Q$.

Because the $\tilde{q}_i$'s are in Q, the D_{θ^i}'s are non-empty and we can apply the lemma. Since the maximum in

$$p_2^*(q_1,q_2) = \underset{\tilde{\theta}\in\Theta_1(q_1)}{\text{Max}} \{u(q_2,\tilde{\theta}) - U(q_1,\tilde{\theta})\}$$

is attained in θ^i if and only if q_2 is in D_{θ^i} , p_2^* indeed takes the form stated in the theorem, and the proof of part 1 is complete.

2) Now assume that (DC) has a solution and Q is a closed interval. By construction, and using the continuity of u(.,.) w.r.t. q, each set D_θ is a closed subset of Q. Under condition (IV.21), no set D_θ is empty. Moreover, the union of the D_θ's covers Q.

Consider $(\theta,\theta')\in\Theta^2$ such that $\theta > \theta'$, and $(q_2,q_2')\in D_\theta \times D_{\theta'}$. We have:

$$\begin{cases} u(q_2,\theta) - U(q_1,\theta) \geq u(q_2,\theta') - U(q_1,\theta') \\ \\ u(q'_2,\theta') - U(q_1,\theta') \geq u(q'_2,\theta) - U(q_1,\theta) \end{cases} \qquad \text{(IV.31)}$$

Adding these two inequalities and using the assumption $\partial^2_{\theta q}u > 0$, we obtain: $q_2 \geq q'_2$. Since $\bigcup_{\theta\in\Theta_1(q_1)} D_\theta = Q$ and Q is a closed interval, this implies that for all i=1,...,p-1, $D_{\theta^{i+1}}$ lies immediately to the right of D_{θ^i}. Since these sets are non-empty and closed, their intersection is necessarily a singleton. For i = 1,...,p-1, define $\hat{q}_i$ by $D_{\theta^{i+1}} \cap D_{\theta^i} = \{\hat{q}_i\}$. By construction, $\hat{q}_i$ is such that:

$$u(\hat{q}_i,\theta^{i+1}) - u(\hat{q}_i,\theta^i) = U(q_1,\theta^{i+1}) - U(q_1,\theta^i). \qquad \text{(IV.32)}$$

Moreover, suppose that q_2 is such that

$$u(q_2,\theta^{i+1}) - u(q_2,\theta^i) = U(q_1,\theta^{i+1}) - U(q_1,\theta^i).$$

Then $u''_{\theta q} > 0$ implies: $q_2 = \hat{q}_i$. It thus suffices to take $\tilde{q}_i = \hat{q}_i$ to establish part 2 of the theorem.

Q.E.D.

The above analysis shows that dynamic consistency problems can be solved if both a surjectivity condition (namely, there exists a (unique) $\tilde{q}_i$ satisfying $u(\tilde{q}_i,\theta^{i+1}) - u(\tilde{q}_i,\theta^i) = U(q_1,\theta^{i+1}) - U(q_1,\theta^i)$) and a monotonicity condition (namely, $\tilde{q}_{i+1} \geq \tilde{q}_i$) hold. These assumptions have a familiar flavor ; however, since they refer to endogenous variables (the agent's rents under the optimal long-term contract), one may wonder how likely they are to hold. Although we do not have a complete characterization theorem, we can give sufficient conditions.

These conditions are most easily stated if we assume that θ can take a continuum of values. For that reason, assume until the end of this subsection that Θ is a real interval ; fix q_1 in Q_1 and define $\tilde{q}(\theta)$, for every θ in $\Theta_1(q_1)$, by:

$$\partial_\theta u(\tilde{q}(\theta),\theta) = \partial_\theta U(q_1,\theta) \qquad \text{(IV.33)}$$

This definition is the exact analog of the definition of the $\tilde{q}_i$'s in (IV.20) for the case when there is a continuum of values for θ. One could adapt the proof of Theorem IV.1 to show that there exists a solution p_2 to (DC) if and only if $\tilde{q}(\theta)$ exists and increases w.r.t. θ. The following proposition gives sufficient conditions for $\tilde{q}(.)$ to exist and to be non-decreasing. It relies on the following technical condition :

$$u'''_{\theta\theta q}/u''_{\theta q} \leq u''_{\theta\theta}/u'_{\theta} \qquad \text{for all } q \text{ and } \theta \qquad \text{(IV.34)}$$

In particular, all utility functions in the "generalized separable" class $u(q,\theta)=g(q)h(\theta)$ satisfy (IV.34) at equality when both g and h are increasing (note that (IV.34) does not depend on the particular valuation scale chosen for the θ's).

Proposition IV.1:

Assume that Θ is a real interval and that $\forall\theta\in\Theta$, $\partial_\theta u(Q,\theta) = \mathbb{R}$[(13)]. Assume moreover that (IV.34) holds and that consumptions are non-negative on the long-term equilibrium path. Then the function $\tilde{q}(.)$ exists and is non-decreasing.

Proof: The surjectivity assumption $\partial_\theta u(Q,\theta) = \mathbb{R}$ ensures the existence of the $\tilde{q}$'s. It remains to be shown that they increase w.r.t. θ.

First note that the underlying economic problem is unchanged if we reparametrize the valuation scale through an increasing transformation $\theta'=f(\theta)$. Easy calculations show that if (IV.34) holds, then there exists such a transformation that maps the utility function into one that satisfies, in addition to Assumption II.1 :

$$u''_{\theta\theta} \geq 0 \geq u'''_{\theta\theta q} \quad \text{and} \quad u''_{\theta q} > 0 \quad \text{for all } q \text{ and } \theta. \qquad \text{(IV.35)}$$

Under (IV.34), we can thus assume that (IV.35) holds without loss of generality.

Using (IV.33) and the enveloppe theorem yields:

$$\partial_\theta u(\tilde{q}(\theta),\theta) = \partial_\theta U(q_1,\theta) = \sum_{t=2}^{T}\delta^{t-2}\partial_\theta u(q_t(\theta),\theta) \qquad \text{(IV.36)}$$

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(13) Obviously, this implies that Q must be an interval of $\mathbb{R}$.

where $(q_2(\theta),\ldots,q_T(\theta))$ is an equilibrium consumption path of θ in $\Theta_1(q_1)$ under the optimal long-term contract.

Differentiating this equation gives:

$$\partial^2_{\theta q}u(\tilde{q}(\theta),\theta)\ \tilde{q}'(\theta) + \partial^2_{\theta\theta}u(\tilde{q}(\theta),\theta)$$
$$= \sum_{t=2}^{T}\delta^{t-2}[\partial^2_{\theta q}u(q_t(\theta),\theta)\ \tilde{q}'(\theta)] + \sum_{t=2}^{T}\delta^{t-2}[\partial^2_{\theta\theta}u(q_t(\theta),\theta)] \qquad \text{(IV.37)}$$

The second-order incentive conditions imply that the first term in the right-hand side is non-negative. Because of the assumed non-negativity of the $q_t(\theta)$'s, $\partial_\theta u(q_t(\theta),\theta) \geq 0$ for $t=3,\ldots,T$, and (IV.36) implies

$$\partial_\theta u(\tilde{q}(\theta),\theta) \geq \partial_\theta u(q_2(\theta),\theta)$$

which in turn, using $\partial^2_{\theta q}u > 0$, implies:

$$\tilde{q}(\theta) \geq q_2(\theta) \qquad \text{(IV.38)}$$

Using $\partial^2_{\theta\theta}u \geq 0$ and $\partial^3_{\theta\theta q}u \leq 0$, the above condition yields:

$$\partial^2_{\theta\theta}u(\tilde{q}(\theta),\theta) \leq \partial^2_{\theta\theta}u(q_2(\theta),\theta)$$
$$\leq \sum_{t=2}^{T}\delta^{t-2}[\partial^2_{\theta\theta}u(q_t(\theta),\theta)]$$

Therefore (IV.37) implies: $\partial^2_{\theta q}u(\tilde{q}(\theta),\theta)\ \tilde{q}'(\theta) \geq 0$, which, using again the assumption $\partial^2_{\theta q}u > 0$, yields the conclusion.

Q.E.D

3. Reverse incentive compatibility constraints

In the previous subsection, we proved that short-term contracts can bypass the dynamic consistency problem if a surjectivity and a monotonicity assumption hold. We also characterized the class of "promises" $p_2(q_1,q_2)$ that guarantee the agent the exact equivalent of his informational rent while preserving intertemporal smoothing (i.e., while paying p_1^* in the first period according to the long-term optimum). We now show that an adequate choice of the promise $p_2(q_1,q_2)$ helps prevent "hit-and-run" strategies from agents with low valuation.

To do this, assume the $\tilde{q}_i$'s have been defined as in the previous subsection, and that the surjectivity and monotonicity assumptions hold : the $\tilde{q}_i$'s exist in Q and they are non-decreasing w.r.t. i. We now pick a particular member of the class of promises that solve (DC). Let p_2 be defined by :

$$p_2(q_1,q_2) = p_2^*(q_1,\tilde{q}_i) \qquad \text{if } q_2 = \tilde{q}_i \text{ for some } i=1,\dots,p-1$$

$= +\infty$ otherwise[14]

The promise p_2 clearly solves (DC), since it is nowhere smaller than p_2^* and it equals p_2^* at one point on each preferred set D_{θ^i}. Now let some agent θ, $\theta < \underline{\theta}_1(q_1)$, attempt a "hit and run" strategy (i.e., he chooses q_1 in the first period and then leaves the relationship at the end of the second period).[15] This deviation yields him a net gain of :

$$G(\theta) = u(q_1,\theta) - p_1^*(q_1) + \delta \max_{q_2 \in Q} [u(q_2,\theta) - p_2(q_1,q_2)] - r_1(\theta;\mu_1)$$

By the definition of p_2,

$$\max_{q_2 \in Q} [u(q_2,\theta) - p_2(q_1,q_2)] = \max_{i=1,\dots,p-1} [u(\tilde{q}_1,\theta) - p_2^*(q_1,\tilde{q}_1)]$$

Let $A^i(\theta) = u(\tilde{q}_1,\theta) - p_2^*(q_1,\tilde{q}_1) = u(\tilde{q}_1,\theta) - u(\tilde{q}_1,\theta^i) + U(q_1,\theta^i)$

---

(14) This is not the only possible choice ; however, forcing p_2 to be infinite except at some well-chosen points helps restrict optimizing strategies by the agents and thus makes things simpler.

(15) We only present the simplest case of "one-period" hit-and-run strategies in the text. However, it is easy to check that preventing such deviations suffices to also deter deviations in which the agent mimicks a higher type for several periods before "running".

By the definition of $\tilde{q}_i$,

$$A^{i+1}(\theta) = A^i(\theta) + (u(\tilde{q}_{i+1},\theta) - u(\tilde{q}_i,\theta)) - (u(\tilde{q}_{i+1},\theta^{i+1}) - u(\tilde{q}_i,\theta^{i+1}))$$

and since $\partial^2_{\theta q}u > 0$ and the $\tilde{q}_i$'s are non-decreasing,

$$A^{i+1}(\theta) \le A^i(\theta) \quad \text{for all } i=1,\dots,p-1$$

Therefore $\tilde{q}_1$ is in θ's preferred set under p_2, and

$$\underset{q_2 \in Q}{\text{Max}}\ [u(q_2,\theta) - p_2(q_1,q_2)] = u(\tilde{q}_1,\theta) - u(\tilde{q}_1,\theta^1) + U(q_1,\theta)$$

Moreover, since no deviation is profitable under the optimal long-term contract, we have, for any consumption path $(q_1,\dots q_T)$ chosen by θ^1 under $L_1(\mu_1)$:

$$r_1(\theta;\mu_1) - r_1(\theta^1;\mu_1) \ge u(q_1,\theta) - u(q_1,\theta^1) + \sum_{t=2}^{T} \delta^{t-1}[u(q_t,\theta) - u(q_t,\theta^1)]$$

And finally, by the definition of $U(.,.)$:

$$r_1(\theta^1;\mu_1) = u(q_1,\theta^1) - p_1^*(q_1) + \delta\, U(q_1,\theta^1)$$

Collecting all this into the expression of $G(\theta)$ yields

$$G(\theta) \le \delta \; (V(\theta) - V(\theta^1))$$

where $V(\theta) = u(\tilde{q}_1,\theta) - \sum_{t=2}^{T} \delta^{t-2} \, u(q_t,\theta)$

Incentives on the continuation of the optimal long-term contract $L_1(\mu_1)$ imply that

$$U(q_1,\theta^2) - U(q_1,\theta^1) \ge \sum_{t=2}^{T} \delta^{t-2}[u(q_t,\theta^2) - u(q_t,\theta^1)]$$

which, by the definition of $\tilde{q}$, yields $V(\theta^2) \ge V(\theta^1)$. Assuming that all $(q_2,\ldots,q_T)$ are non-negative, the same argument and the positivity of $\partial^2_{\theta q} u$ also yield $\tilde{q}_1 \ge q_2$.

We will be done if we can prove that V(.) is concave ; since $V(\theta^2) \ge V(\theta^1)$, we will then know that $V(\theta)$ must be non-increasing for $\theta \le \theta^1$, and thus that $G(\theta) \le 0$. To complete this final step, me must resort to the technical condition we used in the previous subsection, i.e. (IV.34). Recall that under (IV.34), we can assume

$$u''_{\theta\theta} \ge 0 \ge u'''_{\theta\theta q} \quad \text{and } u''_{\theta q} > 0 \quad \text{for all q and } \theta$$

without loss of generality.

Easy calculations then give

$$V''(\theta) = u''_{\theta\theta}(\tilde{q}_1,\theta) - \sum_{t=2}^{T} \delta^{t-2} \, u''_{\theta\theta}(q_t,\theta)$$

$$\le u''_{\theta\theta}(\tilde{q}_1,\theta) - u''_{\theta\theta}(q_2,\theta)$$

$$\le 0 \text{ , as desired.}$$

(where the first inequality results from $u''_{\theta\theta} \geq 0$ and the second one results from $u'''_{\theta\theta q} \leq 0$ and from $\tilde{q}_1 \geq q_2$).

We therefore have the following result :

<u>Theorem</u> <u>IV.2</u> : assume surjectivity, monotonicity and (IV.34). Then short-term contracting implements the long-term optimum.

<u>Remark</u> : this result should be compared with those in our 1990 paper. There, we assumed surjectivity, conflicting objectives and the absence of asymmetric information at all recontracting dates . Here, conflicting objectives are built into the model ; Theorem IV.2 proves that the presence of asymmetric information may not endanger the implementability result.

The intuition behind this result is simple enough ; the "promise" forces any agent who has consumed q_1 in the first period to consume in the second period the whole discounted consumption of agent $\underline{\theta}_1(q_1)$ over the whole (T-1) remaining periods. Therefore if an agent decides to "hit" in the first period (i.e., if he chooses to mimic the behavior of an agent with a higher valuation), he can no longer "run" afterwards, as he is forced to consume as much in the second period as one of the agents whose behavior he mimicks does over all the remaining periods under the optimal long-term contract.

Consider for instance a modified version of HT's rental model (in which there are two types of agents, $\underline{\theta}$ and $\bar{\theta}$), where the principal can offer at no cost any level of consumption between $q = 0$ and $q = 1$, but also, at some high cost, any higher level. Introducing very costly levels higher than 1 clearly does not modify the optimal long-term contract ; the latter is dynamically consistent, but may be subject to "hit-and-run" strategies from agents with low valuation when $\sum_{t=2}^{T} \delta^{t-1} > 1$. This contract, however, can now be implemented through short-term contracting.

To see this, consider some μ_1 for which the optimal renegotiation-proof contract is such that $\bar{\theta}$ consumes in all periods and pays $\bar{\theta} + \sum_{t=2}^{T} \delta^{t-1}\underline{\theta}$, and $\underline{\theta}$ consumes in all periods but the first and pays $\sum_{t=2}^{T} \delta^{t-1}\underline{\theta}$. (As explained on p. 27, this type of optimal contract is the most sensitive to reverse incentive problems.) Thus $r_1(\bar{\theta};\mu_1) = \sum_{t=2}^{T} \delta^{t-1}(\bar{\theta}-\underline{\theta})$ and $r_1(\underline{\theta};\mu_1)$ and the r_2's are all zero. Clearly, short-term contracting implements the long-term optimum, starting from period 2 (information is symmetric at that date). It is easy to compute that $\tilde{q}_1 = \sum_{t=2}^{T} \delta^{t-1}$, so that in the first period, the following short-term contract completes the job :

$$
\begin{aligned}
p_1(q_1) &= 0 && \text{if } q_1 = 0, \\
&= \overline{\theta} && \text{if } q_1 = 1, \\
&= +\infty && \text{otherwise}
\end{aligned}
$$

$$
\begin{aligned}
p_2(q_1,q_2) &= 0 && \text{if } (q_1,q_2) = (0,0) \\
&= \sum_{t=2}^{T} \delta^{t-2}\underline{\theta} && \text{if } (q_1,q_2) = (1, \sum_{t=2}^{T} \delta^{t-2}), \\
&= +\infty && \text{otherwise}
\end{aligned}
\tag{IV.39}
$$

This in fact mimicks the sale of a durable good in the first period, and ensures that the high valuation agent gets his rent in such a way that the low valuation agent is not tempted to "hit", since he can no longer "run away" afterwards : If he chooses q = 1 in the first period, everything is as if he then were forced to consume q = 1 in all following periods.

While condition (IV.34) guarantees short-term implementability of the long-term optimum under monotonicity and surjectivity, it is by no means necessary. We have indeed assumed very little on the structure of the optimal long-term contract, beyond the fact that it is incentive-compatible and renegotiation-proof.

V. Concluding remarks

The preceding sections should make clear that renegotiated short-term contracts are a much more powerful tool than spot contracts when information is asymmetric. As mentioned in the introduction to this paper, intertemporal consumption-smoothing is the only reason why short-term contracting achieves more than spot contracting in symmetric information contexts. With asymmetric information however, our analysis shows that short-term contracts also allow to bypass dynamic consistency and reverse incentive compatibility, both of which are necessary conditions for spot contracts to implement the long-term optimum.

Moreover, the equivalence of short-term contracts and of the type of loan contracts used by Malcolmson-Spinnewyn (1988) no longer holds : since loan contracts only allow to spread the *payments* for any period's consumption, they cannot give any incentives as to the choice of later-periods consumptions. They will thus generally fail to bypass dynamic consistency and reverse incentive compatibility. Loan contracts therefore may not implement the long-term optimum.

Let us conclude with an application to the case where the principal exogenously learns something about the agent's characteristic before the end of the relationship. Assume for example that there are three periods, two types ($\theta = \underline{\theta}$ or $\bar{\theta}$), no discounting, and that the principal learns the value of θ at the beginning of the third period. The optimal long-term contract then can implement the complete information optimum (thanks to the threat of huge penalties if the agent is caught lying), whereas spot contracts, which cannot resort to delayed punishments, will in general be less efficient.

Short-term (two-period) contracting, however, can implement the complete information optimum, provided a strong form of surjectivity holds. Short-term contracts clearly are fully efficient from the second period on; it thus suffices to focus on the first short-teerm contract. One possiblity is to give the choice in the first period between $\{(q_1,p_1),(q_2,p_2)\}$ = $\{(\bar{q},\bar{p}),(\bar{q},\bar{p})\}$ and $\{(q_1,p_1),(q_2,p_2)\} = \{(\underline{q},\underline{p}),(-\underline{q},-\underline{p})\}$; each contract gives zero utility to the agent for whom it is designed, and a non-positive utility to the other agent. This of course supposes that negative levels of consumption are allowed.

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