Market Participation under Delegated and Intrinsic Common Agency Games*

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Abstract

We study how competition in nonlinear pricing between two principals (sellers) affects market participation by a privately-informed agent (consumer). When participation is restricted to all-or-nothing ("intrinsic" agency), the agent must choose between both principals' contracts and selecting her outside option. When the agent is afforded the additional possibilities of choosing only one contract ("delegated" agency), competition is more intense. The two games have distinct predictions for participation. Intrinsic agency always induces more distortion in participation relative to the monopoly outcome and equilibrium allocations are discontinuous for the marginal consumer. Under delegated agency, relative to monopoly market participation increases (resp. decreases) when contracting variables are substitutes (resp. complements) on the intensive margin. Equilibrium allocations are continuous for the marginal consumer and the range of product offerings is identical to both the first-best and the monopoly outcome.

Keywords: Common Agency, Nonlinear Pricing, Market Participation.

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1 Introduction

This paper studies common agency games between competing principals using screening contracts targeted at a distribution of privately informed agents. Many interesting economic applications fit into such a setting. For example, when two non-cooperating regulatory bodies regulate the same privately-informed firm but on different dimensions (e.g., output and pollution), the outcome can be modeled as an equilibrium to a common agency game. As a second example, when two firms sell non-homogenous goods to the same consumer using nonlinear pricing as a price discrimination strategy, the price schedules which arise can also be modeled as an equilibrium to a common agency game. There is a critical difference between these two examples which is the subject of this paper. In the first example, the regulated firm does not have a choice to be regulated by only one regulator; that is, the firm can choose to leave the industry and face no regulation, or it can choose to abide by both sets of regulations. Here, common agency and non-participation are the only potential outcomes and therefore common agency is *intrinsic* to the game. In the second example it is natural to allow the consumer the option of purchasing exclusively from one firm, and so common agency is no longer intrinsic to the game but a choice variable that is *delegated* to the agent. In this paper, we explore both the intrinsic and delegated variations of common agency games. We are especially interested in how these variations impact the familiar misallocations that arise in monopoly screening settings and, in particular, the distortions on the external participation margin.

Early efforts by Martimort (1992, 1996) and Stole (1991), as well as most subsequent applications of common agency with asymmetric information to date, have been in the context of *intrinsic*[1] common agency games – games in the form of our regulation example.[2] In the canonical form of this common agency game, an agent learns some private preference parameter regarding the margins of two economic activities, say q_1 and q_2. Both principals simultaneously and non-cooperatively offer selection contracts with the restriction that principal i's contract cannot depend upon activity q_j, $j \neq i$. Hence, the contracting variables are *private* rather than *public*. Following the offers, the agent must decide between accepting or rejecting *both* contracts; the agent is not allowed to accept one contract and reject the other. Activities are subsequently chosen and payoffs are awarded in accord with the activities and the contracts. Such a modeling is naturally appropriate

[1] Bernheim and Whinston (1986a) coined the expressions of *intrinsic* and *delegated* common agency.

[2] See Mezzetti (1997), Laffont and Tirole (1993, ch. 17), Bond and Gresik (1997), Ivaldi and Martimort (1994), Olsen and Torsvik (1993, 1995) and Olsen and Osmudsen (2001), among others.

when the agent is a regulated firm and the principals are distinct regulatory bodies, each with authority over a mutually exclusive set of activities. A firm may decide to exit the industry (i.e., "reject" the regulatory contracts), but the firm can never decide to accept one set of regulations and reject the other.

The delegated common agency game that allows the agent the extra options of accepting a subset of the principals' contract offers has received far less attention than its intrinsic counterpart.[3] The value of exclusivity for the agent, however, depends upon the agent's private information and, as a consequence, delegated agency games require the imposition of type-dependent participation constraints.[4]

To the best of our knowledge, no one has studied the economic consequences of common agency (both intrinsic and delegated) on distortions in contractual activities *and participation*. In our early papers, Martimort (1992) and Stole (1991), we studied the equilibrium outcomes of the intrinsic agency game under the assumption of full participation and argued that the analysis also applies to the case of delegation when contracting activities are complements. Moreover, when the activities are substitutes, the economics of the intrinsic agency distortions still provide considerable insight into the marginal distortions in delegated agency games. One has to pay closer attention to the agent's rents and participation constraints, however, when common agency is delegated. Calzolari and Scarpa (2004) explored non-intrinsic common agency and proved that the agent obtains greater rents in a non-intrinsic game but that otherwise the productive allocations are identical. This conclusion relies, however, on the assumption of full participation. When the market is not covered, as we show below, the participation distortions typically depend upon whether the common agency game is delegated or intrinsic. The primary contribution of the present paper is to provide an analysis of the two forms of distortions – intensive margins (activity levels) and extensive margins (participation) – and to relate the directions and magnitudes of these distortions to the underlying game form and the preferences of the agent.

Our paper also contributes to an understanding of the interactions between competition and price discrimination. Because theoretical work in multi-principal contract games has largely restricted attention to intrinsic settings, it has remained unclear how compe-

[3]Bernheim and Whinston (1986b) develops this concept in common agency games under complete information.

[4]Laussel and Lebreton (2001) have shown that delegated common agency may lead to outcomes where the agent gets a positive rent even under complete information if the number of principals is greater than two. Our focus on asymmetric information and two principals can thus be seen as complementary to theirs.

tition affects the character of nonlinear pricing between duopolists.[5] In addition, most competitive nonlinear pricing applications have assumed one-stop-shopping or exclusive-purchasing in which the consumer may buy from only one firm in equilibrium, thereby incorporating all competitive pressures in the outside option.[6] Other papers that have allowed for purchasing from multiple vendors in equilibrium have also restricted preferences such that full coverage arises in equilibrium.[7] The present paper allows for multiple vendors and exclusivity. It also allows for less-than-full coverage. In this sense, our results indicate which intuitions from nonlinear pricing are robust with respect to incomplete market coverage and the possibility of purchasing from both firms as well as just one.

Section 2 begins with a stylized example using unit demands in which only participation (extensive) distortions can arise; intensive distortions on consumption are assumed away. This allows us to focus on the relationship between the participation distortion and the nature of consumer preferences and provides a simple intuition that underlies much of the analysis which follows. Section 3 describes our more general multi-principal contracting games. While the application of our results is quite general to multi-principal games, for concreteness, we focus on the competitive setting between two firms (the principals) selling to a possibly common consumer (the agent) using nonlinear pricing.

In Section 4, we present a methodology for solving multi-principal games whenever preferences satisfy a set of practical regularity conditions. In the case of a single firm selling both product lines (the monopoly benchmark), these regularity conditions are easily satisfied for a range of preferences. In the case of multi-principal games, however, our approach is to construct an indirect utility function from a hypothetical equilibrium price schedule to study best responses. Because the preference construction is endogenous, regularity can not be imposed exogenously, but must be verified in the candidate equilibrium. This endogeneity is the unique source of technical complexities inherent in multi-principal screening games. We discuss these issues at some length, so this section should be of independent interest to those studying multiprincipal incentives contracting.[8] In this paper, we take the approach of assuming that the equilibria under study are regular, thereby simplifying the proofs and highlighting the economic content of the

[5]Stole (2007) provides a survey of the literature on price discrimination in competitive environments.

[6]Armstrong and Vickers (2001) and Rochet and Stole (2002), for example.

[7]For example, Ivaldi and Martimort (1994).

[8]These techniques for solving multiprincipal screening games are evoked or presented in parts in a few papers (see, for instance, Martimort and Stole (2003), Martimort (2007) and Stole (2007)). No single paper, however, has provided a self-contained treatment of the approach in the completeness that we undertake here.

results, and leaving the verification of regularity to settings with additional structure on preferences. In particular, we will show that when preferences are quadratic and types are uniformly distributed, regular equilibria always exist.

We turn to the study of delegated games in Section 5 and intrinsic games in Section 6. We collect the results of the paper and present our main comparative theorems in Section 7.

2 A simple example

The main theorem of the paper (Theorem 1) establishes that competition between two sellers facing a population of heterogeneous buyers with private information on their willingness to pay has different implications for market coverage, depending upon the nature of consumer preferences. In this section, we briefly develop a simple example which provides an important intuition for the more general results that follow. We obtain simplicity by assuming consumer preferences do not have any variation on the intensive margins; only an extensive decision about participation exists.

Consider a consumer desiring to buy at most two units of an homogenous good with a willingness to pay, θ, that is uniformly distributed on the interval $[0, \bar{\theta}]$. A consumer of type θ has a utility function given by

$$u(q_1, q_2, \theta) = \begin{cases} \theta(q_1 + q_2) + w & \text{if } q_1 + q_2 \leq 2 \\ 2\theta + w & \text{otherwise,} \end{cases}$$

where $q_i \in \{0, 1, 2\}$ is the quantity consumed from firm i and w is wealth. Assume also that the unit cost of production is constant at $c < \bar{\theta}$.

We begin with a multi-product monopolist who sells both product lines. Because the goods are perfect substitutes this is admittedly an odd benchmark. The monopolist can maximize its profits by offering all of its units (of both product lines) at the same price.[9] The optimal price can be thought of as the optimal cutoff point, $p^m = \theta_0^m$, such that all types above θ_0^m consume one unit of each product line and all types below θ_0^m refuse to purchase. The optimal price, p^m, maximizes $2(\bar{\theta} - p)(p - c)$ and is characterized by the familiar first-order condition

$$p^m = \theta_0^m = \frac{1}{2}(\bar{\theta} + c).$$

Suppose that instead of a monopoly, there are two duopolists, each choosing their unit prices p_i noncooperatively. Under delegated agency, it is immediate that the familiar logic

[9] The monopolist could alternatively offer to bundle the two units and reach the same payoff.

of Bertrand applies: In a pure-strategy equilibrium, each firm offers to sell up to two units at a per-unit price of $p^d = c$, profits are zero, and the marginal type is $\theta_0^d = c$. It follows that $\theta_0^d < \theta_0^m$.[10]

In an intrinsic common agency game, our setting of two firms making offers to a consumer is a contrivance. It is hard to think of a market example with substitute goods in which the consumer is required to purchase from both firms. Rather than change our example to one of regulation or construct a more elaborate story for this case, we will stay with our simple example. Given the agent only evaluates the gain from jointly purchasing, the participating types will satisfy $2\theta - p_1 - p_2 \geq 0$. Consequently, each firm i chooses p_i to maximize $\left(\bar{\theta} - \left(\frac{p_1+p_2}{2}\right)\right)(p_i - c)$, and the unique symmetric equilibrium has prices

$$p^i = \theta_0^i = \frac{1}{3}(2\bar{\theta} + c).$$

It follows that $p^i > p^m > p^d$ and $\theta_0^i > \theta_0^m > \theta_0^d$. Participation is greater under delegated agency than under intrinsic common agency.

Repeating our analysis with the assumption of Leontief complements, i.e., when the consumers' preferences are given by

$$u(q_1, q_2, \theta) = \begin{cases} 2\theta \min\{q_1, q_2\} + w & \text{if } \min\{q_1, q_2\} \leq 1 \\ 2\theta + w & \text{otherwise,} \end{cases}$$

we find that the intrinsic agency and monopoly prices are unchanged, but that the delegated agency program is now equivalent to the intrinsic agency problem. This implies that in the case of perfect complements, $\theta_0^i = \theta_0^d > \theta_0^m$. The option of refusing one of the possible contracts is inconsequential for equilibrium participation. The additional options under competition have no impact on market coverage because the options are unattractive. In the general model that follows, we explore the robustness of these simple insights.

3 The model

We now recast our analysis in a richer framework, allowing for imperfect substitutability or complementarity, multi-unit consumption and nonlinear pricing.

In what follows, we take both a general and a specific approach. We state preferences for the firms and the agent with general functions at the outset, but we will apply the

[10]There are, of course, exclusive agency equilibria in which one firm sells two units at the combined price of $2c$ and the other firm sells zero. The welfare consequences are equivalent, however.

results to the specific setting with quadratic preferences and a uniform distribution of marginal utilities to produce closed-form solutions. There are a set of technical requirements for equilibria that have less interest to us than the fundamental economic results presented in Theorem 1. Assuming the preferences and equilibria are sufficiently regular produces a more transparent argument for our main results. We then can use the specific case of the quadratic-uniform model to demonstrate that such regularity is not vacuous and is easily satisfied by the simple models typically chosen in applications.

3.1 General preferences and information structure

We model competition between two firms (sometimes referred to generically as principals) indexed by $j = 1, 2$. Each of these firms offers the privately-informed consumer (generically, the agent) a price schedule, $P_j : \mathcal{Q} \mapsto \Re_+$, defined over a compact set of available outputs, $\mathcal{Q} \equiv [0, \bar{q}]$ and continuous over the interior. We choose $\bar{q}$ sufficiently large that the any consumer's utility from consuming $\bar{q}$ is less than its cost of production.[11] We allow that P may have a left hand discontinuity at 0, exhibiting a fixed fee equal to $P(0^+) > 0$ with $P(0) = 0$. Upon observing the posted price schedules, the consumer decides whether or not to participate and, conditionally upon participation, how much to consume from each firm. When common agency is intrinsic, the consumer must choose between joint contracting and non participation. In the delegated agency game, the consumer can additionally choose to contract exclusively with either firm.

In our general setting, the consumer has a privately known type θ which is distributed on the support $[0, \bar{\theta}]$ according to a differentiable distribution function $F(\theta)$ and corresponding density $f(\theta)$.

The consumer's utility is quasi-linear in money and his preferences for consuming (q_1, q_2) are represented by a symmetric, smooth utility function, $u(q_1, q_2, \theta)$, with the properties that u is increasing in θ, has strictly increasing differences with respect to q_i and θ, is strictly concave in (q_1, q_2), and $u(0, 0, \theta) = 0$ for all θ. Moreover, goods are substitutes (respectively, complements) whenever u (resp., $-u$) has strictly increasing differences with respect to q_1 and q_2. The consumer's net utility of purchasing q_1 and q_2 is therefore

$$u(q_1, q_2, \theta) - P_1(q_1) - P_2(q_2).$$

Each firm's cost of producing q_j is given by the symmetric, smooth cost function $C(q_j)$

[11] Formally, using the notation for utility and cost that follows, $u(\bar{q}, q, \theta) - u(0, q, \theta) < C(\bar{q})$ for all (q, θ).

which is continuous through the origin, with $C(0)=0$, increasing and convex. Each firm maximizes expected profit,

$$E_\theta[P_j(q_j(\theta)) - C(q_j(\theta))],$$

where $q_j(\theta)$ is the consumer's optimal choice given the equilibrium price schedules. We further assume that full coverage is inefficient under full information:

$$u_{q_1}(0,0,0) < C'(0) < u_{q_1}(0,0,\bar{\theta}).$$

Given this assumption, it is efficient to serve only an upper interval of types when maximizing social surplus.

3.2 Special setting: quadratic-uniform preferences

In our specialized setting, we will place more structure on the problem by additionally assuming the consumer's utility function is quadratic, $C(q) = cq$, and θ is distributed uniformly. We will refer to this as the quadratic-uniform case. We choose to represent the parameters of the quadratic utility function by looking directly at the implicit demand curves of the consumer for each good. Specifically, the consumer's demand function for (q_1, q_2) is symmetric, linear in prices, and the parameter θ appears only in the demand intercepts:

$$q_j = \alpha + \theta - \beta p_j + \gamma p_{-j},$$

with $\alpha > 0$ and $\beta > |\gamma| \neq 0$. This is equivalent to assuming that the agent has a quadratic utility function for consumption of the form:[12]

$$u(q_1,q_2,\theta) = \frac{\alpha+\theta}{\beta-\gamma}(q_1+q_2) - \frac{\beta}{2(\beta^2-\gamma^2)}(q_1^2+q_2^2) - \frac{\gamma}{\beta^2-\gamma^2}q_1q_2.$$

For $\gamma \in (0,\beta)$, goods are demand substitutes in the traditional sense, while goods are demand complements if $\gamma \in (-\beta, 0)$. In many of the calculations that follow, the relationship between β and γ is homogenous and depends only on the ratio, $\tau \equiv \gamma/\beta$, so we will sometimes use τ to simplify the presentation. In this case, note that $\tau \in (-1,1)$, $\tau > 0$ represents the case of substitutes and $\tau < 0$ represents the case of complements.

[12]These conditions on preferences are restrictive to the extent that actual demand is nonlinear in prices or that θ affects the own or cross-price derivatives of the demand function. More generally, following Martimort (1992), one could instead posit that the utility function satisfies a separability restriction: $u_{q_j}(q_1,q_2,\theta) = \tilde{u}^1_{q_1}(q_1,\rho(q_2,\theta)) = \tilde{u}^2_{q_2}(q_2,\rho(q_1,\theta))$, where $\tilde{u}^j(q_i,\rho)$ satisfies the increasing-differences condition $\tilde{u}^j_{q_j\rho}(q_j,\rho) > 0$. With the restriction to quadratic preferences in consumption, this separability restriction is equivalent to the requirement that demand curves are linear in prices and θ only enters through the intercepts. Note that the chosen representation provides that $u_\theta > 0$ and the standard single-crossing property $u_{q_j\theta} > 0$ is satisfied for each good.

3.3 The first-best (full information) benchmark

Because preferences are symmetric and strictly concave over (q_1, q_2), we can define the first-best allocation by a single function, $q^{fb}(\theta)$ which is a pointwise solution to the following program:

$$\max_{q \in \mathcal{Q}} W(q,\theta) \equiv u(q,q,\theta) - 2C(q),$$

where $W(q,\theta)$ is the social surplus function. Formally, $q^{fb}(\theta)$ satisfies

$$u_{q_1}(q^{fb}(\theta), q^{fb}(\theta), \theta) = C'(q^{fb}(\theta))$$

for all θ such that $u_{q_1}(0,0,\theta) > C'(0)$ and $q^{fb}(\theta) = 0$ otherwise. Given the increasing-differences condition on u and $u_{q_1}(0,0,0) < C'(0)$, there exists a unique root, $\theta_0^{fb} \in (0,\bar{\theta})$, that solves $C'(0) = u_{q_1}(0,0,\theta_0^{fb})$ such that it is inefficient to serve any type less than θ_0^{fb} under full information. Hence, $q^{fb}(\theta) = 0$ for all types $\theta \in [0, \theta_0^{fb}]$ and $q^{fb}(\theta) > 0$ for all types $\theta > \theta_0^{fb}$.[13] We will refer to θ_0^{fb} as the marginal (participating) consumer under full information.

Define the value function of this program by

$$J^{fb}(\theta) \equiv \max_{q \in \mathcal{Q}} W(q,\theta),$$

which is continuous, strictly increasing on $(\theta_0^{fb}, \bar{\theta})$ and zero on $[0, \theta_0^{fb}]$.

In the quadratic-uniform specification, the first-best (full information) allocation is continuous and has a particularly simple representation:

$$q^{fb}(\theta) \equiv \max\{0, \theta + \alpha - (\beta - \gamma)c\}.$$

Our assumption that efficient participation is less than complete implies $\alpha - (\beta - \gamma)c < 0 < \bar{\theta} + \alpha - (\beta - \gamma)c$ and the marginal consumer is given by

$$\theta_0^{fb} \equiv (\beta - \gamma)c - \alpha > 0.^{14}$$

[13]Requiring that $\theta_0^{fb} > 0$ focuses the analysis on instances in which participation is actually modified by changes in market structure which is precisely the case we investigate presently. Otherwise, we would be obliged to describe a variety of uninteresting and tedious cases where participation could remain unchanged. If marginal costs are positive, it also seems reasonable that there exists a set of consumers with lower valuations if heterogeneity is sufficiently great.

[14]Note also that the assumptions $\theta_0^{fb} > 0$ and $\alpha > 0$ altogether put an upper bound on the degree of substitutability between goods that rules out the case of perfect substitutes.

4 A methodology for best-response contracts

We are interested in characterizing equilibria to three agency games. The first is the case in which both firms are able to collude or otherwise implement the monopoly outcome. This is a straightforward application of optimal contract design to a two-good setting. The other cases represent our two common agency games – delegated and intrinsic. In all three settings, the methodology we use to construct optima and equilibria is identical once one defines the appropriate indirect utility function for each of these games. We present that methodology here.

4.1 The bilateral screening problem

We begin by reconsidering the bilateral contracting problem between a single firm selling a single product line to a privately-informed consumer with an outside option of zero utility. This is the canonical setting of the monopoly self-selection program as explored by such papers as Mussa and Rosen (1978) and Maskin and Riley (1984), to list a few. The solution to this monopoly program, however, has much value in calculating best-response functions in a multi-principal game, so we present the details here.

The firm's program is to choose the tariff $P(q)$ to maximize expected profit subject to the consumer's choice satisfying incentive compatibility and all participating consumers receiving non-negative rents. Assuming that the consumer's utility function is increasing in θ, the set of participating consumers will be an upper interval, $[\theta_0, \bar{\theta}]$, where the marginal consumer θ_0 is implicitly determined by $P(q)$.

Because we are interested in a single product line, for the moment we replace the consumer's multi-product preference function, $u(q_1, q_2, \theta)$, with a simpler function defined over a single good, $v(q, \theta)$, and assume that $v(q, \theta)$ is nondecreasing in θ. In this case, we abuse our notation slightly and modify the social surplus function to $W(q, \theta) \equiv v(q, \theta) - C(q)$ and again denote its maximizer as $q^{fb}(\theta)$. We maintain our previous assumption that the type space can be partitioned into two non-degenerate intervals – $[0, \theta_0^{fb}]$ and $(\theta_0^{fb}, \bar{\theta}]$ – such that $q^{fb}(\theta) = 0$ on the lower interval and $q^{fb}(\theta) > 0$ on the upper interval.

The firm's program can be stated as

$$\max_{P} \int_{\theta_0}^{\bar{\theta}} (P(q(\theta)) - C(q(\theta)))\, dF(\theta),$$

subject to $q(\theta) \in \arg\max_{q \in \mathcal{Q}} v(q, \theta) - P(q)$ (incentive compatibility) and $v(q(\theta), \theta) - P(q(\theta)) \geq 0$ for all $\theta \geq \theta_0$ (participation). Alternatively, using a change of variables,

$U(\theta) \equiv v(q(\theta),\theta) - P(q(\theta))$, we can restate the program as

$$\max_P \int_{\theta_0}^{\bar{\theta}} \left(v(q(\theta),\theta) - C(q(\theta)) - U(\theta)\right) dF(\theta),$$

subject to $q(\theta) \in \arg\max_{q\in\mathcal{Q}} v(q,\theta) - P(q)$ (incentive compatibility) and $U(\theta) \geq 0$ for all $\theta \geq \theta_0$ (participation). Providing that $v(q,\theta)$ satisfies the single-crossing property $v_{q\theta}(q,\theta) \geq 0$, we can replace the incentive constraint with the equivalent requirements that $U'(\theta) = v_\theta(q(\theta),\theta)$ and $q(\theta)$ is nondecreasing. Because $v_\theta(q,\theta) \geq 0$, we can replace the participation constraints with $U(\theta_0) \geq 0$. Integrating the objective function by parts, this affords us the simplification

$$\max_{\{q,U(\theta_0),\theta_0\}} \int_{\theta_0}^{\bar{\theta}} \left(v(q(\theta),\theta) - C(q(\theta)) - \frac{1-F(\theta)}{f(\theta)} v_\theta(q(\theta),\theta) - U(\theta_0)\right) dF(\theta),$$

subject to $q(\theta)$ nondecreasing and $U(\theta_0) \geq 0$. Because $U(\theta_0)$ is optimally set to 0, we can eliminate this instrument from the program. Lastly, we can define the associated virtual surplus function for the firm as

$$\Lambda(q,\theta) \equiv v(q,\theta) - C(q) - \frac{1-F(\theta)}{f(\theta)} v_\theta(q,\theta),$$

generating the following succinct program

$$\max_{\{q,\theta_0\}} \int_{\theta_0}^{\bar{\theta}} \Lambda(q(\theta),\theta) dF(\theta),$$

subject to $q(\theta)$ nondecreasing.

As is standard in the screening literature, we consider the relaxed program in which the monotonicity constraint is absent. Here, the optimal q is determined by finding the pointwise maximum of $\Lambda(q,\theta)$ for each $\theta \in [\theta_0, \bar{\theta}]$. We denote this optimizer as $\tilde{q}(\theta)$,

$$\tilde{q}(\theta) \in \arg\max_{q\in\mathcal{Q}} \Lambda(q,\theta).$$

If Λ is differentiable and strictly quasi-concave in q, this relaxed solution satisfies

$$\Lambda_q(\tilde{q}(\theta),\theta) = 0.$$

Note that if $\Lambda(\tilde{q}(\theta),\theta)$ is negative for some range of θ, the optimal allocation $q(\theta)$ may be zero rather than $\tilde{q}(\theta)$. Formally, define

$$J(\theta) \equiv \Lambda(\tilde{q}(\theta),\theta).$$

Because $v_\theta > 0$, J is necessarily increasing in the neighborhood of $\bar{\theta}$. More generally, if $\Lambda_\theta \geq 0$,[15] J is weakly increasing everywhere in θ with $\dot{J}(\theta) = \Lambda_\theta(q(\theta), \theta)$. In this case, θ_0 is simply the root of $J(\theta)$ in $[0, \bar{\theta}]$ if it exists

$$\Lambda(\tilde{q}(\theta_0), \theta_0) = 0,$$

and $\theta_0 = 0$ otherwise. If J is not monotonic, then the candidates for the optimal cutoff are still either the corner $\theta_0 = 0$ or a root of $J(\theta) = 0$ around which J is locally nondecreasing. Given such θ_0, the optimal allocation is simply

$$q(\theta) = \begin{cases} \tilde{q}(\theta) & \text{if } \theta \geq \theta_0, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Once one obtains the optimal q allocation, if it is strictly increasing it can be inverted for $\theta^{-1}(q)$ and substituted to produce the differential equation for the price schedule:

$$P'(q) = v_q(q, \theta^{-1}(q))$$

with the initial condition that $P(q(\theta_0)) = v(q(\theta_0), \theta_0)$. If $q(\theta)$ is constant over an interval of types, the differential equation can be suitably modified to allow for a kink at such quantity.[16] The solution uniquely generates the nonlinear optimal tariff $P(q)$.

To summarize, the pointwise optimization approach above finds an optimal solution if v and Λ have increasing differences in (q, θ), v is nondecreasing in θ, and Λ is strictly quasi-concave in q. The assumptions that are frequently made in the traditional monopoly screening literature (e.g., quadratic preferences and a monotone hazard rate condition) are sufficient for these regularity requirements. We now catalogue our discussion above with the following definition of regularity of $v(q, \theta)$ and the proposition of its consequence:

Definition 1 (Regularity) *The bilateral self-selection program induced by* $v(q, \theta)$ *is* **regular** *if and only if*

1. $v(q, \theta)$ *is continuous, nondecreasing in* θ*, and has strict increasing differences in* (q, θ)*;*[17]
2. *the virtual surplus associated with* v*,*

$$\Lambda(q, \theta) \equiv v(q, \theta) - C(q) - \frac{1 - F(\theta)}{f(\theta)} v_\theta(q, \theta),$$

[15] Sufficient conditions for this are that $v_\theta > 0$, $v_{\theta\theta} \leq 0$ and the monotone hazard rate property holds.

[16] More precisely, suppose that $[\theta_1, \theta_2]$ is one such pooling interval for which $q(\theta) = q$. It follows, then, that we have $\partial P(q) = [v_q(q, \theta_1), v_q(q, \theta_2)]$ where $\partial P(q)$ is the subdifferential of P at q.

[17] A function $v(q, \theta)$ has *increasing differences* in (q, θ) if $v(q, \theta) - v(q, \theta') \geq v(q', \theta) - v(q', \theta')$ for all pairs $q > q'$ and $\theta > \theta'$; thus, at twice-differentiable points, $v_{\theta q} \geq 0$. A function has *strictly increasing differences* if it satisfies the previous condition with strict inequalities.

is strictly quasi-concave in q and has increasing differences in (q, θ).

The consequence of regularity follows.

Proposition 1 *Given that $v(q, \theta)$ is regular, the firm's optimal self-selection price schedule induces $q(\theta)$ such that $q(\theta) = 0$ for $\theta < \theta_0$ and*

$$q(\theta) \in \arg\max_{q \in \mathcal{Q}} \Lambda(q, \theta), \tag{2}$$

for $\theta \geq \theta_0$. The optimal participation cutoff is either a corner, $\theta_0 = 0$, or a root

$$\Lambda(q(\theta_0), \theta_0) = 0, \tag{3}$$

where in either case the virtual value function $J(\theta) = \Lambda(q(\theta), \theta)$ is nondecreasing.

There is a more specialized property of the optimal price schedule that emerges if the utility function also satisfies $v(0, \theta) = 0$ for all θ. We will present the result maintaining the assumption of regularity.

Lemma 1 *Suppose that $v(q, \theta)$ is regular and $v(0, \theta) = 0$ for all θ, then the optimal allocation, $q(\theta)$, is continuous and $q(\theta) = 0$ for all $\theta \leq \theta_0$. The marginal participating consumer satisfies $\theta_0 \in (\theta_0^{fb}, \bar{\theta})$ and the corresponding tariff P is right-continuous through the origin with $P(0) = 0$.*

This idea follows quite naturally from the assumption that the social surplus function, $v(q, \theta) - C(q)$, is continuous through the origin. As such, discontinuities in the price schedule at the origin are suboptimal and, provided that there is less than full coverage under monopoly, the marginal consumer consumes zero and is not charged a fixed fee for access.[18] Particularly in our analysis of the delegated common agency game, this lemma will be valuable. In the game of intrinsic common agency, the lemma is inapplicable and we will obtain discontinuous allocations.

4.2 Applying the methodology to multi-product monopoly

We can apply Proposition 1 directly to the monopoly setting to obtain our second benchmark by noting that the strict concavity and symmetry of $u(q_1, q_2, \theta)$ implies that the monopolist's optimal tariff also solves the program for $v^m(q, \theta) \equiv u(q, q, \theta)$, $C^m(q) = 2C(q)$,

[18] This remarkably simple and powerful idea – that in the absence of fixed costs tariffs should not exhibit positive access prices – is discussed at some length in Wilson (1993, section 6.7).

and $\Lambda^m(q,\theta)$ constructed accordingly. Because $u(0,0,\theta) = 0$ for all θ and $u_{q_1}(0,0,0) < C'(0) < u_{q_1}(0,0,\bar{\theta})$, Lemma 1 applies and the monopoly allocation $q^m(\theta)$ must be continuous with a price schedule passing through the origin.

An immediate comparison can be made between the full-information and monopoly allocations by simply comparing the two pointwise programs. Note that

$$\Lambda^m(q,\theta) = W(q,\theta) - \frac{1-F(\theta)}{f(\theta)} u_\theta(q,q,\theta).$$

Because W is strictly concave and $u_{q\theta}(q,\theta) > 0$, the pointwise optimum of $\Lambda^m(q,\theta)$ must exceed that which maximizes $W(q,\theta)$. Hence, for all $\theta \in (\theta_0^m, \bar{\theta})$, we have $q^m(\theta) < q^{fb}(\theta)$. Because $q^m(\theta)$ and $q^{fb}(\theta)$ are both continuous, it follows immediately from this ordering that $\theta_0^m > \theta_0^{fb}$. Collecting these results together, we have the following proposition:

Proposition 2 *If the monopoly screening program is regular, then*

$$\bar{\theta} > \theta_0^m > \theta_0^{fb}$$

and for all participating consumers $\theta \in [\theta_0^m, \bar{\theta})$

$$q^m(\theta) < q^{fb}(\theta)$$

with $q^m(\bar{\theta}) = q^{fb}(\bar{\theta})$.

In the quadratic-uniform setting regularity and monotonicity in value is easily verified. To provide a closed-form benchmark, we present the solution for this special case.

Proposition 3 *In the quadratic-uniform monopoly setting, the marginal consumer is given by*

$$\bar{\theta} > \quad \theta_0^m = \frac{1}{2}\left(\bar{\theta} + \theta_0^{fb}\right) \quad > \theta_0^{fb}. \tag{4}$$

and the firm's allocation is distorted below the first best with the continuous allocation

$$q^m(\theta) = q^{fb}(\theta) - (\bar{\theta} - \theta)$$

for all $\theta \in [\theta_0^m, \bar{\theta}]$.

As is well known, the intensive marginal distortions under the monopoly pricing arise because the firm trades off the marginal gain of increased output to any type θ against the inframarginal loss of reduced revenues on all types greater than θ. A straightforward result that is slightly less well known is that the monopolist distorts on the extensive margin for similar reasons. In the case of quadratic-uniform preferences, the monopolist chooses a participation cutoff that is the average of the first-best cutoff, θ_0^{fb}, and the highest type, $\bar{\theta}$. In the applications of delegated and intrinsic agency that follow, a weighted average of θ_0^{fb} and $\bar{\theta}$ will also describe the marginal customer, but their weights will differ from 50:50. It is also worth noting that in the monopoly setting, the range of quantities served remains the same as in the first best, namely, $\mathcal{Q}^{fb} = \mathcal{Q}^m = [0, q^{fb}(\bar{\theta})]$. As we will see, this range will also arise in the delegated game because of Lemma 1, but the range of quantities will be strictly smaller in the intrinsic game for which the assumptions of Lemma 1 are violated.

4.3 Applying the methodology to multi-principal games

The key insight in understanding multi-principal games is that in any pure-strategy equilibrium to a multi-principal game, any individual firm (say, for example, firm 1) behaves as a monopolist facing an agent with the following indirect preference function:

$$v(q,\theta) \equiv \max_{\tilde{q}\in\mathcal{Q}} \; u(q,\tilde{q},\theta) - P_2(\tilde{q}) - \phi(\theta),$$

where $\phi(\theta)$ is the reservation utility obtained if firm 1's contract is rejected. The indirect utility function v represents the net gain from contracting with firm 1 for a consumption of q. Of course ϕ takes different values depending on whether we consider an intrinsic or a delegated common agency game.

If we can establish the regularity of $v(q,\theta)$, we can apply the results from Proposition 1. v is also increasing in q. Beyond these properties, we will need to consider the specific nature of the goods and the form of the common agency game to verify if v is regular. Indeed, when the goods are substitutes, the regularity of v can only be established by first constructing candidate equilibrium tariffs and checking for regularity, ex post. This calculation, fortunately, is straightforward with the additional structure of the quadratic-uniform setting.

To be clear about our approach, in the analysis that follows we proceed by assuming regularity at the outset.[19] We apply Proposition 1 and deduce properties of the firm's best-response price schedule and find a symmetric fixed point in the best-response correspondences of the principals. We then verify that such candidate price schedules do indeed induce regularity in the indirect utility functions, implying that they constitute

[19]Because our approach is restricted to regular, symmetric equilibria, it is perhaps a refinement on the equilibrium set. We have decided not to undertake a general study of equilibria in which the single-crossing property is violated or the equilibrium is asymmetric.

an equilibrium. Because the indirect utility functions are different across delegated and indirect common agency games, we study each game in turn.

Delegated common-agency game. Given our focus on symmetric equilibria, we will construct our indirect utility function under the assumption that the equilibrium consists of each firm offering $P(q)$ to the agent. In the case of delegated agency, we take the convention that each firm is required to offer $P(0) = 0$ as a component of its tariff schedule. This is without loss of generality.[20] That said, we will reserve the term *participation* to describe a consumer's choice of positive consumption.

Recall that Proposition 1 applies to the case in which the agent's reservation utility is zero. As such, we need to construct $v^d(q, \theta)$ to capture the participating consumer's net utility of contracting with a firm for a positive amount $q > 0$, relative to the best alternative of either non-participation or exclusive contracting with the rival. Because $P(q)$ implicitly allows for non-participation, we need only subtract the consumer's outside option of exclusively contracting with the rival. That is, in the case of delegation games, $\phi^d(\theta) = \max_{q\in\mathcal{Q}}\ u(0, q, \theta) - P(q)$, and so

$$v^d(q, \theta) \equiv \left(\max_{\tilde{q}\in\mathcal{Q}}\ u(q, \tilde{q}, \theta) - P(\tilde{q})\right) - \left(\max_{\tilde{q}\in\mathcal{Q}}\ u(0, \tilde{q}, \theta) - P(\tilde{q})\right).$$

Note that in the case of delegated agency, there is no fixed effect in net utility, $v^d(0, \theta) = 0$ for all θ. Hence Lemma 1 applies and tariffs are necessarily continuous through the origin whenever there is less than full coverage and the equilibrium is symmetric and regular.

Some components of regularity can easily be verified for v^d. Because v^d is a linear combination of continuous value functions, v^d is continuous. It is immediate, as well, that v^d is increasing in q. Perhaps less obvious, if v^d has (resp., strict) increasing differences, then v^d is nondecreasing in θ (resp., increasing). To understand why, it is helpful to define the consumer's optimal purchase from firm 2, given a purchase of q from firm 1 in a symmetric equilibrium in which both firms offer $P(q)$:

$$q^*(q, \theta) \in \arg\max_{\tilde{q}\in\mathcal{Q}}\ u(q, \tilde{q}, \theta) - P(\tilde{q}).$$

Because we consider only pure-strategy equilibria, we focus on the case where $q^*(q, \theta)$ is a well-defined equilibrium function.[21] This allows us to express the derivative of v^d with

[20] We could, alternatively, allow firms to offer price schedules which are strictly positive at 0, but the agent could choose not to participate effectively choosing the $P(0) = 0$ option under our convention.

[21] Hence, we rule out cases where the agent's optimal choice is multi-valued either on or off the equilibrium path. This can be viewed as a minor refinement within a potentially larger set of equilibria but this required uniqueness of the agent's best-response is satisfied by our quadratic-uniform model.

respect to θ as

$$v_\theta^d(q,\theta) = u_\theta(q, q^*(q,\theta), \theta) - u_\theta(0, q^*(0,\theta), \theta) = \int_0^q v_{q\theta}^d(x,\theta)dx.$$

Hence, that v^d has (resp. strict) increasing differences is sufficient for weak (resp. strict) monotonicity in θ.

Determining if v^d has such increasing differences in (q,θ) is more difficult. From the previous expression, having increasing differences is equivalent to establishing that $u_\theta(q, q^*(q,\theta), \theta)$ is nondecreasing in q. Because $q^*(q,\theta)$ is monotonic, it is differentiable almost everywhere. At all points of differentiability, we have

$$v_{q\theta}^d(q,\theta) = u_{q_1\theta}(q, q^*(q,\theta), \theta) + u_{q_2\theta}(q, q^*(q,\theta), \theta)\frac{\partial q^*(q,\theta)}{\partial q}.$$

Observe that the q appears directly as an argument of the utility function and indirectly as an argument of $q^*(q,\theta)$. By assumption, $u_{q_i\theta}(q_1, q_2, \theta) > 0$, which signs the first term positively. When the goods are complements, $q^*(q,\theta)$ is nondecreasing in q; in this case, the second term reinforces the first, and v^d has strictly increasing differences. When the goods are substitutes, however, the indirect and direct effects are in opposition. Therefore, for v^d to have strictly increasing differences in (q,θ), the equilibrium construction of $\frac{\partial q^*(q,\theta)}{\partial q}$ must not be too large and negative. In the case of our quadratic-uniform setting, fortunately, equilibrium regularity is directly verifiable.[22]

In addition to the conditions on v^d, regularity also requires that the virtual surplus function, Λ^d, that is derived from v^d have increasing differences and be strictly quasi-concave. Given that v^d has strict increasing differences, it is sufficient for Λ^d to have increasing differences that $\theta - (1 - F(\theta)/f(\theta)$ is nondecreasing and $v_{q\theta\theta}^d \leq 0$. In our specific quadratic-uniform model, these conditions are easily verified. Λ^d is also strictly quasi-concave in the quadratic-uniform setting, but this verification is less straightforward. When computing $q^*(q,\theta)$, we need to account for the possibility that a critical value of q could induce the agent to choose the corner solution of $q^*(q,\theta) = 0$. Economically, $q^*(q,\theta) = 0$ corresponds to inducing the consumer to chose exclusivity rather than common agency. Technically, such a corner solution generates a discontinuity in the derivative of $q^*(q,\theta)$. Precisely at this point, $\Lambda^d(q,\theta)$ will be exhibit a kink. Thus, even if Λ^d is strictly concave on both sides of the kink (as it is in the quadratic-uniform specification), it is unclear whether or not such a kink destroys strict quasi-concavity. Fortunately, we

[22]Martimort (1992) and Stole (1991) showed that regularity holds beyond the quadratic case in models of intrinsic agency.

are able to show in Lemma 2 in the Appendix that these kinks are concavity-preserving in the general setting.

Intrinsic common-agency game. When common agency is intrinsic, the agent does not have the option to participate exclusively with one of the principals and so $\phi^i(\theta) = 0$. It follows that the corresponding indirect utility function is

$$v^i(q,\theta) \equiv \max_{\tilde{q}\in\mathcal{Q}} u(q,\tilde{q},\theta) - P(\tilde{q}).$$

It is important to note that because an active agreement requires that the rival principal's contract is also accepted by the agent, $v^i(q,\theta)$ may be negative over some subset of $\mathcal{Q}\times\Theta$. Thus, the option of non-participation may be optimal, but this option is *not embedded in* $v^i(q,\theta)$. Hence, the principal will need to compare the maximized virtual value function, $J^i(\theta) \equiv \max_{q\in\mathcal{Q}} \Lambda^i(q,\theta)$, to the option of nonparticipation with payoff 0.

Some properties of $v^i(q,\theta)$ are otherwise similar to those of $v^d(q,\theta)$: The indirect utility function is continuous and increasing in q. Moreover, it is also increasing in θ because the agent's reservation utility is type independent in the intrinsic game and $v^i_\theta(q,\theta) = u_\theta(q,q^*(q,\theta),\theta) > 0$.

Verifying regularity in the equilibria of the intrinsic game suffers from similar difficulties as discussed for the delegated game. In Proposition 7, however, we are able to prove that any regular, symmetric equilibrium of the delegated game will have a corresponding regular, symmetric equilibrium in the intrinsic game. Thus, verifying the regularity of an equilibrium in the delegated game is sufficient for our purposes of studying the intrinsic game.

5 The delegated common-agency game

Consider any equilibrium to the delegated common-agency game. We say that the equilibrium is regular if each firm's price schedule generates a regular indirect utility function vis-a-vis the rival firm. Given that $v^d(0,\theta) = 0$ for all θ, regularity and Lemma 1 implies that each consumption schedule $q_i(\theta)$ is continuous. Suppose in addition that the equilibrium is symmetric and each firm offers $P^d(q)$ which induces $v^d(q,\theta)$ as the symmetric indirect utility function and which is differentiable on $(0,\bar{q})$.[23] Let $\Lambda^d(q,\theta)$ be the

[23]If principals were asymmetric, we would have to address the possibility that those principals might have different market shares so that, for instance, exclusivity arises endogenously for a subset of the type space. We leave the analysis of those complex issues for further research.

corresponding virtual surplus function. Proposition 1 implies that

$$q^d(\theta) \in \arg\max_{q\in\mathcal{Q}} \Lambda^d(q,\theta),$$

and either $\theta_0^d = 0$ or $\Lambda^d(q(\theta_0^d),\theta_0^d) = 0$. At this point, several economic implications necessarily follow.

Proposition 4 *Suppose that $\{q^d(\theta),\theta_0^d\}$ is the allocation in a regular, symmetric equilibrium in the delegated agency game with equilibrium tariff, $P^d(q)$, differentiable on $(0,\bar{q})$. Then*

- *the equilibrium tariff, $P^d(q)$, is continuous at $q = 0$ (i.e., no fixed fee);*
- *in the case of* **substitutes***:*

$$\theta_0^{fb} < \theta_0^d < \theta_0^m,$$

  and

$$q^{fb}(\theta) \geq q^d(\theta) \geq q^m(\theta),$$

  with strict inequalities for all $\theta \in (\theta_0^d,\bar{\theta})$ and equalities at $\bar{\theta}$;

- *in the case of* **complements***:*

$$\theta_0^{fb} < \theta_0^m < \theta_0^d,$$

  and

$$q^{fb}(\theta) > q^m(\theta) \geq q^d(\theta),$$

  with strict inequalities for all $\theta \in (\theta_0^m,\bar{\theta})$ and equalities at $\bar{\theta}$.

Note that the allocation under complements is distorted below that of monopoly on both the intensive and extensive margins; the reverse being true for the case of substitutes. The result is similar in spirit to the discussion by Cournot (1838) who observed that competition in prices between complementary producers reduces both consumer surplus and profits, as each firm separately introduces a distortion that reduces the demand for the other firm's product, and hence its profitability. An integrated monopoly would introduce a smaller distortion. Remarkably, a similar intuition is present when strategy spaces are enlarged to allow nonlinear price schedules.

It is also worth noting that the requirement of differentiability, while a reasonable restriction and one that is satisfied in the equilibria of the quadratic-uniform model, is not essential for the central conclusion in Proposition 4. As the proof demonstrates, a re-statement of the proposition using weak inequalities everywhere for the ordering of allocations can be proven without recourse to differentiability.

The previous results are only necessary conditions. We now turn to our specific setting of quadratic-uniform preferences to establish that a regular, symmetric equilibrium exists. To this end, we take advantage of the homogeneity in preferences by substituting for $\tau = \frac{\gamma}{\beta} \in (-1, 1)$.[24]

Proposition 5 *In the quadratic-uniform delegated game, the following constitutes a regular symmetric equilibrium*

$$q^d(\theta) = q^{fb}(\theta) - (\bar{\theta} - \theta)\left(1 - \frac{4\tau}{1 + \sqrt{1 + 8\tau^2}}\right) = q^m(\theta) + (\bar{\theta} - \theta)\left(\frac{4\tau}{1 + \sqrt{1 + 8\tau^2}}\right) \quad (5)$$

for all $\theta \in [\theta_0^d, \bar{\theta}]$, $q^d(\theta_0^d) = 0$ *for all* $\theta \leq \theta_0^d$, *and* $q^d(\theta)$ *is continuous and increasing, where*

$$\theta_0^d = \lambda^d(\tau)\theta_0^{fb} + (1 - \lambda^d(\tau))\bar{\theta}, \quad (6)$$

and $\lambda^d(\tau) = \frac{1}{2} + \frac{\sqrt{1+8\tau^2}+2\tau-1}{4(1+\tau)}$, $\lambda^d(\tau) \in \left(\frac{1}{3}, 1\right)$.

Consistent with the findings in Martimort (1992) and Stole (1991) for the case of intrinsic common agency with complementary goods, $q^d(\theta) < q^m(\theta) < q^{fb}(\theta)$, and thus the distortion is greater with competing principals relative to the multi-product monopolist. The extreme case is obtained when goods are almost perfect complements, i.e., $\tau \to -1$; $q^d(\theta)$ involves a double distortion with respect to the monopoly outcome. When the goods are substitutes, $q^{fb}(\theta) > q^d(\theta) > q^m(\theta)$ and the consumption distortion is smaller with competing principals relative to the multi-product monopolist. As τ approaches 1, the goods become closer substitutes, and $q^d(\theta)$ approaches $q^{fb}(\theta)$.[25]

A numerical example of a quadratic-uniform equilibrium is depicted in Figure 1. For purposes of illustrating the cases of substitutes and complements within the same graph, we have fixed the value of $\beta - \gamma$ rendering the monopoly and first-best solutions invariant to offsetting changes in β and γ. In the case of substitutes, we have assumed $\gamma = 1 > 0$ and $\beta = 2$; in the case of complements we have taken $\gamma = -\frac{1}{3} < 0$ and $\beta = \frac{2}{3}$.

[24] It is worth noting that once either $q^{fb}(\theta)$ or $q^m(\theta)$ are defined, $q^d(\theta)$ can be determined knowing only τ. The calculation of $q^{fb}(\theta)$ and $q^m(\theta)$, however, depends upon the difference $\beta - \gamma$. The same is true for the calculations of θ_0^d and θ_0^{fb} or θ_0^m.

[25] Some care needs to be taken in the interpretation of the perfect substitutes limit because the assumption of $\theta_0^{fb} > 0$ puts a lower bound on $\beta - \gamma$ and an upper bound on τ when $\alpha > 0$.

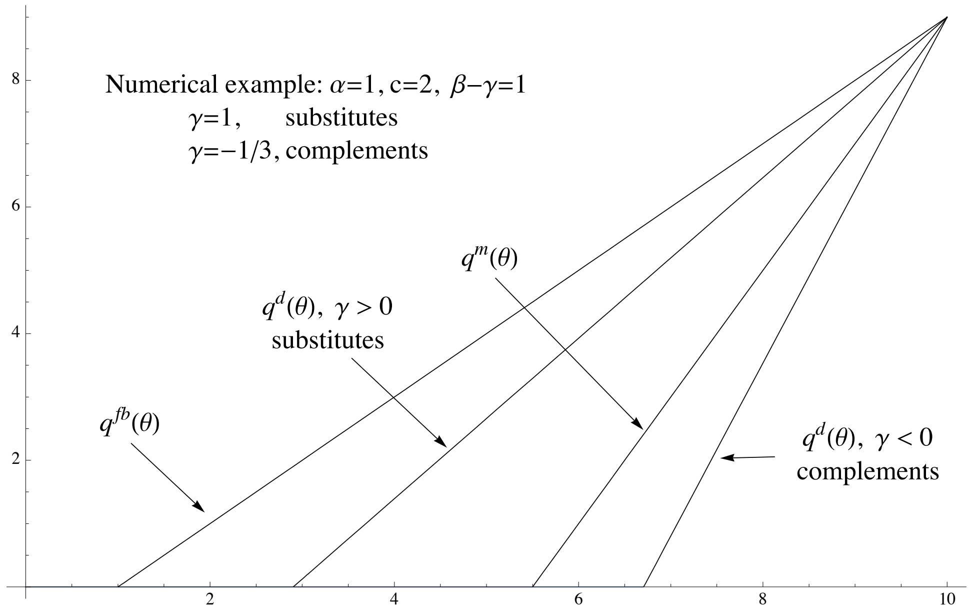

Figure 1: Quadratic-uniform preferences with $\bar{\theta} = 10$, $\alpha = 1$, $c = 2$, and $\beta - \gamma = 1$. The case of substitutes is modeled with $\gamma = 1 > 0$; the case of complements is modelled with $\gamma = -\frac{1}{3} < 0$. Because $\beta - \gamma$ is held constant, $q^{fb}(\theta)$ and $q^m(\theta)$ are invariant across the cases.

6 The intrinsic common-agency game

We now return to the case of intrinsic common agency. Recall that

$$v^i(q, \theta) \equiv \max_{\tilde{q} \in \mathcal{Q}} u(q, \tilde{q}, \theta) - P(\tilde{q}),$$

but this value is not necessarily nonnegative and generally $v^i(0, \theta) \neq 0$ for all θ. Nevertheless, with the assumption of regularity, we can deduce several properties of equilibria in intrinsic games by simply comparing them to the analogous monopoly and delegated outcomes. The first result relates the intrinsic outcome to the monopoly outcome under the assumption of regularity.

Proposition 6 *For any regular, symmetric intrinsic equilibrium*

$$\theta_0^i \geq \theta_0^m.$$

If the equilibrium tariff is differentiable on the interior of $\mathcal{Q}$, then this inequality is strict.

The monopolist introduces a smaller participation distortion than competing firms under intrinsic agency, regardless of whether the goods are substitutes or complements on the intensive margin. This result is surprising given that we normally think of competition as increasing efficiency, except when the goods are demand complements. Here, on the other hand, is a setting where inefficient exclusion is *more* pronounced under competition – even when the goods are substitutes on the intensive margin. It is perhaps less surprising once we understand that intrinsic agency is equivalent to delegated agency with goods that are perfect complements at the base level (extensive margin). Perfect complementarity on the extensive margin implies that competition generates greater extensive (participation) distortions relative to monopoly. The nature of preferences on the intensive margins is therefore irrelevant. This is the remarkable content of the proposition: Perfect complementarity on the extensive margin is the unique source of the higher participation inefficiencies.

We next turn to a more remarkable comparison between intrinsic and delegated agency participation.

Proposition 7 *Suppose that $P^d(q)$ is a symmetric equilibrium in a regular delegated agency game. Then there exists a $P_0 > 0$ such that*

$$P^i(q) \equiv P^d(q) + P_0$$

is a symmetric equilibrium in the intrinsic game.

The simple fact that for every equilibrium to the delegated game there exists a corresponding equilibrium to the intrinsic game in which the tariffs are shifted up by a fixed fee generates an immediate characterization of the equilibrium allocation.

Corollary 1 *For any regular symmetric equilibrium outcome in the delegated game, $\{q^d(\theta), \theta_0^d\}$, the corresponding symmetric intrinsic equilibrium satisfies*

$$\theta_0^i > \theta_0^d,$$

and

$$q^i(\theta) = \begin{cases} q^d(\theta) > 0, & \text{if } \theta \geq \theta_0^i, \\ 0, & \text{if } \theta < \theta_0^i. \end{cases}$$

The allocation $q^i(\theta)$ is discontinuous at θ_0^i.

Returning to our specific quadratic-uniform setting, we can use our previously established fact that there exists a regular, symmetric, linear equilibrium to the delegated game to derive the corresponding equilibrium in the intrinsic game.

Proposition 8 *In the quadratic-uniform delegated game, the following constitutes a regular symmetric equilibrium*

$$q^i(\theta) = q^{fb}(\theta) - (\bar{\theta} - \theta)\left(1 - \frac{4\tau}{1 + \sqrt{1 + 8\tau^2}}\right) = q^m(\theta) + (\bar{\theta} - \theta)\left(\frac{4\tau}{1 + \sqrt{1 + 8\tau^2}}\right) \quad (7)$$

for all $\theta \in [\theta_0^i, \bar{\theta}]$ *and* $q^i(\theta) = 0$ *for* $\theta < \theta_0^i$*, where*

$$\theta_0^i = \lambda^i(\tau)\theta_0^{fb} + (1 - \lambda^i(\tau))\bar{\theta}, \quad (8)$$

where $\lambda^i(\tau) = \frac{1+\sqrt{1+8\tau^2}}{4(1+\tau)+4\sqrt{1+8\tau^2}} < \min\left\{\frac{1}{2}, \lambda^d(\tau)\right\}$*, and* $q^i(\theta)$ *is discontinuous with* $q^i(\theta_0^i) > 0$.

It is worth emphasizing that the residual utility function under intrinsic agency is such that $v^i(0,\theta) \neq 0$ for all θ and the marginal customer consumes a positive amount regardless of whether goods are substitutes or complements. The marginal consumer pays a positive fixed fee to access even a small purchase from firm 2. Serving such a marginal type would require that firm 1 subsidizes consumption of his own good with a negative fee. This is of course viewed as costly by this firm who prefers to restrict market coverage; it follows that the marginal customer consumes a positive amount. The consumption discontinuity of the marginal consumer also implies that the equilibrium consumption set in the intrinsic game, $\mathcal{Q}^i = \{0\} \cup [q^i(\theta_0^i), q^{fb}(\bar{\theta})]$, is a strict subset of that available under full information, monopoly, and the delegated games: $\mathcal{Q}^{fb} = \mathcal{Q}^m = \mathcal{Q}^d = [0, q^{fb}(\bar{\theta})]$.

Returning to our numerical example from Section 5, we can illustrate how the allocations change in the intrinsic common agency game.

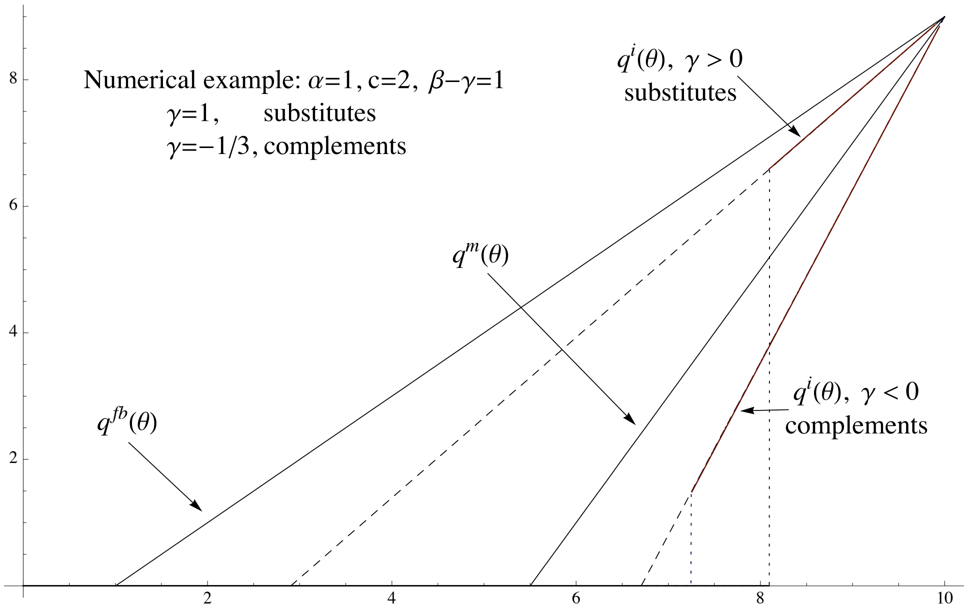

Figure 2: Quadratic-uniform preferences with $\bar{\theta} = 10$, $\alpha = 1$, $c = 2$, and $\beta - \gamma = 1$. The case of substitutes is modeled with $\gamma = 1 > 0$; the case of complements is modelled with $\gamma = -\frac{1}{3} < 0$. Because $\beta - \gamma$ is held constant, $q^{fb}(\theta)$ and $q^m(\theta)$ are invariant across the cases. The dashed lines indicate the corresponding delegated allocations. The vertical dotted lines indicate the discontinuity points at θ_0^i.

Notice that the extensive distortion in the intrinsic agency can dominate the intensive distortions in magnitude.

7 Comparison across games

Our primary motivation in this paper was to understand how variations in the agency game affect equilibrium outcomes on the extensive margin. Collecting and organizing all of our previous results, we can state our main theorem:

Theorem 1 *Suppose that there exists a symmetric, regular equilibrium to the delegated common agency game. Then there also exists a symmetric equilibrium of the intrinsic agency game with identical price margins for participating consumers and allocation $q^i(\theta)$ satisfying the following conditions:*

- *when the goods are* **substitutes***:*

$$\theta_0^{fb} < \theta_0^d < \theta_0^m < \theta_0^i < \bar{\theta}, \tag{9}$$

and for all $\theta \in [\theta_0^i, \bar{\theta})$,

$$q^{fb}(\theta) > q^d(\theta) = q^i(\theta) > q^m(\theta). \tag{10}$$

- *when the goods are* **complements***:*

$$\theta_0^{fb} < \theta_0^m < \theta_0^d < \theta_0^i < \bar{\theta}, \tag{11}$$

and for all $\theta \in [\theta_0^i, \bar{\theta})$*,*

$$q^{fb}(\theta) > q^m(\theta) > q^d(\theta) = q^i(\theta). \tag{12}$$

Furthermore, when preferences are quadratic-uniform, regular, symmetric equilibria exist.

It is also worth comparing the consumer's rents under both regimes. Given the consumer's preferences have strictly increasing differences, it follows that consumer surplus is higher if the integral of an increasing function of consumption is higher.

$$U(\theta) = \int_0^\theta u_\theta(q(t), q(t), t)dt.$$

Although output remains the same under intrinsic and delegated agencies for those consumers who participate in both games, the fact that $\theta_0^i > \theta_0^d$ in Theorem 1 leads us to an unambiguous conclusion:

Corollary 2 *For all consumer types who participate in the delegated agency game,* $\theta > \theta_0^d$*, consumer surplus is strictly higher in the delegated game than in the associated intrinsic game. For nonparticipating types,* $\theta \leq \theta_0^d$*, consumer surplus is zero in both games.*

In short, delegated common agency benefits the consumer because it unambiguously increases market coverage.

We began this paper considering two applications - regulation and competitive nonlinear pricing. The theorem provides insights into each. In settings in which intrinsic agency is institutionally imposed such as the regulation of firms by multiple governmental authorities, we can expect regulatory "competition" to reduce the number of firms participating in the industry because of the greater regulatory burden it generates. Of the firms that chose to participate and submit to regulation, as shown by Martimort (1992) and Stole (1991), indirect externalities between regulatory bodies may increase or decrease social efficiency, depending upon the nature of the regulated activities (i.e., whether the activities are substitutes or complements on the intensive margin) and the nature of the private information. For example, take the case of a public utility in which the private information is a cost-efficiency parameter for the production of output and the reduction of pollution, and the relevant agencies are the public utility commission and an environmental protection agency. If the activities are substitutes (e.g., producing greater output requires using less efficient and less green idle plant capacity), then each independent regulator will distort output less and reduce pollution less than they would if they merged and offered coordinated price-setting and pollution regulation. It follows that environmentalists who are "greener" than the environmental protection agency would prefer that the agencies are prohibited from cooperating even though the environmental protection agency would prefer to coordinate with the public utility commission. Of course, if the activities are complements (perhaps less plausible in the public utility context, but more plausible in the case of a firm being taxed simultaneously by two authorities on closely-related output measures) these findings would be reversed for the intensive margins.

We can also reinterpret our results in Theorem 1 to understand nonlinear pricing when demand preferences vary over both the intensive and extensive margins. To this end, consider a setting in which preferences on the intensive margin are captured by γ as before, but the goods on the extensive (base) margin can be either independent goods or perfect complements. For concreteness, suppose that there are two firms competing with nonlinear prices in a delegated agency market setting. If the goods are perfect complements on the base margin, then some consumption from each firm is necessary to obtain value from either good (i.e., $u(q_1, 0, \theta) = u(0, q_2, \theta) = 0$); if they are independent (the case considered in the previous sections of this paper), then valuable consumption is possible from a single firm. A few conclusions are immediate from the application of Theorem 1. First, if either the goods are complements on the intensive margin or the goods are perfect complements on the extensive margin, then $\theta_0 > \theta_0^m$. If the goods are independent on the extensive margin and substitutes on the intensive margin, then we have the outcome of the delegated agency game, $\theta_0^{fb} < \theta_0 < \theta_0^m$. And, of course, if the goods are independent on the intensive margin, then we have two unrelated monopolies. The practical import of this reinterpretation is that it allows us to think about a broader set of problems.

For example, we can now understand what would happen in a delegation setting when the goods are substitutes on the intensive margin but perfect complements on the extensive (or base) margin. Suppose that the personal computer market consists of one monopolist selling operating systems and another monopolist selling computer hardware. The goods are arguably perfect complements on the extensive margin (i.e., you need one of each to obtain any value), but the quality of the computer's CPU may be a substitute for the quality of the operating system. If both vendors practice second-degree price discrimination and offer a menu of different qualities, the equilibrium set of the delegated game is equivalent to the equilibrium set of the intrinsic game because of the perfect complementarity on the base margin. It follows that (relative to a merger of the two monopolists) competition generates higher quality software and hardware for purchasing consumers *but* fewer consumers purchase computers relative to the case of cooperating monopolists. In Figure 2, the relevant comparison is between $q^i(\theta)$ under substitutes and $q^m(\theta)$.

There are still other preferences to consider. There is the possibility that the base goods are perfect (extensive) substitutes, meaning that at most one of the two goods can generate value to a consumer. Now there is no meaning to an assumption of substitutes or complements on the intensive margin. Perfect competition with exclusive agency and marginal cost pricing emerges as a pure-strategy equilibrium when firms and preferences are symmetric.[26]

8 Concluding remarks

Our primary question has been, "How does competition (in its two possible manifestations) affect the participation region of consumers?" The short answer: competition with delegated agency and demand substitutes leads to lower participation distortion relative to monopoly; competition with either intrinsic agency or delegated agency with demand complements leads to greater participatory distortions.

In settings in which intrinsic agency is not imposed but arises as a natural characteristic of consumer demand (i.e., that base goods are perfect complements on the *extensive* margin), we again conclude that participation distortions under competition are greater than in the case of multi-product monopoly. When the goods are demand complements on the intensive margin (i.e., $\gamma > 0$), participating consumers will be inefficiently underserved relative to monopoly; when goods are demand substitutes on the intensive margin (i.e., $\gamma < 0$), participating consumers are more efficiently served relative to monopoly. It follows that when goods are perfect complements on the extensive margin but substitutes on the intensive margin, participation decisions are more distorted while marginal output or quality decisions are less distorted. In this sense, it is important to understand the nature of consumer preferences when evaluating the social impact of merger.

[26]With additional product differentiation capturing only a fixed effect for brand preference, one can enlarge this category to contain interesting examples of duopoly price discrimination that depart from marginal cost pricing, as in Rochet and Stole (2002).

Finally, our results, when reinterpreted, allow us to revisit on old question of the non-linear pricing literature which goes back at least to Mussa and Rosen (1978): when does competition increase the space of product offerings? Of course, in that interpretation of our model, the q_i are no longer quantities but indexes of the quality of each good. With this reinterpretation, the product space is then viewed as the set of qualities offered in equilibrium. There are two effects to consider. When goods are complements (substitutes) on the extensive margins, we have seen that participation is reduced (increased) and this first effect reduces (does not affect) the range of qualities. When goods are complements (substitutes) on the intensive margins, we have also documented that the quality range is enlarged (unaffected) and consumption distortions increased (reduced) compared with the monopoly outcome. Combining these effects, the outcome is unambiguous only for the case of extensive complements and intensive substitutes, and competition unambiguously lowers the range of product qualities.[27] In other cases, the effects are opposing. For example, when goods are substitutes on the extensive and intensive margins, competition reduces the product space for a given participation rate but enlarges the participation set so that, in the end, the range of equilibrium quantities under first-best and delegated agency are the same. This result stands in sharp contrast with earlier analysis of competition between vertically differentiated suppliers be it passive like in Champsaur and Rochet (1989) or active like in Stole (1995). There competition introduces a non-zero participation constraint which always limits the product space. Whether competition should lead to too many or too few products remains to be seen on a case by case basis but we hope that our taxonomy will help to clarify that issue.

[27] Given that the case of intensive complements and extensive substitutes is vacuous (as argued above), we ignore this case.

Appendix 1: Proofs

Proof of Lemma 1: Because $\Lambda(q, \theta)$ is strictly quasi-concave in q, it follows from the Maximum Theorem that $\tilde{q}(\theta) = \arg\max_{q \in \mathcal{Q}} \Lambda(q, \theta)$ is a well-defined, continuous function and the value function for this program, $J(\theta) \equiv \Lambda(\tilde{q}(\theta), \theta)$ is also continuous.

Because $v(0, \theta) = 0$ for all θ, it follows that $v_\theta(0, \theta) = 0$ and $\Lambda(0, \theta) = 0$ for all θ as well. Hence, $J(\theta) = \max_{q \in \mathcal{Q}} \Lambda(q, \theta) \geq 0$ for all θ. For this it follows that $q(\theta) = \tilde{q}(\theta)$ for all θ, and $q(\theta)$ is continuous.

The marginal participating type is $\theta_0 \equiv \max \{\theta \mid \tilde{q}(\theta) = 0\}$. Define the social surplus function as $W(q, \theta) \equiv v(q, \theta) - C(q)$. By assumption, its maximizer is positive for all $\theta \in (\theta_0^{fb}, \bar{\theta}]$. Because $\Lambda(q, \bar{\theta}) = W(q, \bar{\theta})$ for all q, it therefore follows that $q^{fb}(\bar{\theta}) = \tilde{q}(\bar{\theta}) > 0$. Hence, $\theta_0 < \bar{\theta}$. For $\theta < \bar{\theta}$, we have $\Lambda(q, \theta) \leq W(q, \theta)$, with a strict inequality for any $q > 0$. Thus, for all $\theta \in (\theta_0^{fb}, \bar{\theta})$,

$$J(\theta) = \max_{q \in \mathcal{Q}} \Lambda(q, \theta) < \max_{q \in \mathcal{Q}} W(q, \theta).$$

It follows that any root of $J(\theta) = 0$ must satisfy $\theta_0 \in (\theta_0^{fb}, \bar{\theta})$. □

Proof of Proposition 3: Inserting the quadratic-uniform preferences into the first-order equation $\Lambda^m(q^m(\theta), \theta) = 0$, we obtain:

$$\alpha + \theta - q^m(\theta) - c(\beta - \gamma) = \bar{\theta} - \theta,$$

or, after simplification, $q^m(\theta) = q^{fb}(\theta) - (\bar{\theta} - \theta)$. Finally,

$$J^m(\theta) = \Lambda^m(q^m(\theta), \theta) = \frac{q^m(\theta)^2}{\beta - \gamma} \leq J^{fb}(\theta) = \frac{q^{fb}(\theta)^2}{\beta - \gamma}$$

where the latter definitions are available also when $q^m(\theta)$ or/and $q^{fb}(\theta)$ are zero. It immediately follows from $q^m(\theta^m) = 0$ that θ^m is the mean between θ_0^{fb} and $\bar{\theta}$. □

Proof of Proposition 4: The fact that $P^d(q)$ is continuous through the origin with no fixed fee follows directly from Lemma 1 and $v^d(0, \theta) = 0$ for all θ.

Recall the consumer's best choice from the rival firm's contract:

$$q^*(q, \theta) = \arg\max_{\tilde{q} \in \mathcal{Q}} u(q, \tilde{q}, \theta) - P^d(\tilde{q}).$$

$q^*(q, \theta)$ is nondecreasing in θ and weakly increasing (resp., decreasing) in q if the goods are complements (resp., substitutes). Monotonicity also implies that $q^*(q, \theta)$ is almost everywhere differentiable. Moreover, observe that, whenever P is differentiable on $(0, \bar{q})$, we have for any q such that $q^*(q, \theta)$ is interior,

$$P^{d'}(q^*(q, \theta)) = u_q(q, q^*(q, \theta), \theta).$$

This implies that $P^{d'}(q^*(q, \theta)) < u_q(q', q^*(q, \theta), \theta)$ (resp. $>$) whenever goods are complements (resp. substitutes). Therefore, from the fact that $q^*(q, \theta)$ is uniquely defined at any q, $q^*(q, \theta)$ is strictly increasing (resp. decreasing) in q and thus $\frac{\partial q^*}{\partial q}(q, \theta) > 0$ (resp. $<$).[28]

Now, consider the virtual surplus function $\Lambda^d(q, \theta)$. Using the function $q^*(q, \theta)$ and the Envelope Theorem, we can write its derivative at all points of differentiability as

$$\Lambda_q^d(q, \theta) = u_q(q, q^*(q, \theta), \theta) - C'(q) - \frac{1 - F(\theta)}{f(\theta)} u_{q\theta}(q, q^*(q, \theta), \theta) \left(1 + \frac{\partial q^*}{\partial q}(q, \theta)\right).$$

Consider a symmetric equilibrium allocation $q^d(\theta) = q^*(q^d(\theta), \theta)$ and a type θ such that $\Lambda_q^d(q^d(\theta), \theta) = 0$. Comparing the margins of Λ^d and Λ^m, each evaluated at $q^d(\theta)$, we have an identity for all θ:

$$\Lambda_q^d(q^d(\theta), \theta) = \frac{1}{2}\Lambda_q^m(q^d(\theta), \theta) - \frac{1 - F(\theta)}{f(\theta)} u_{q\theta}(q^d(\theta), q^d(\theta), \theta) \frac{\partial q^*(q^d(\theta), \theta)}{\partial q}.$$

Thus,

$$\Lambda_q^d(q^d(\theta), \theta) = 0 > \frac{1}{2}\Lambda_q^m(q^d(\theta), \theta) \Leftrightarrow \frac{\partial q^*}{\partial q}(q^d(\theta), \theta) < 0.$$

It follows that for any $\theta \in (\theta_0^d, \bar{\theta})$, in the case of substitutes, $q^d(\theta) > q^m(\theta)$ and in the case of complements, $q^d(\theta) < q^m(\theta)$. Moreover, by Lemma 1, these allocations are all continuous at the participation boundaries so in the case of substitutes, $\theta_0^d < \theta_0^m$ and in the case of complements, $\theta_0^d > \theta_0^m$.

Comparing the margins of Λ^d and W, each evaluated at $q^d(\theta)$, we have for all θ:

$$\Lambda_q^d(q^d(\theta), \theta) = \frac{1}{2}W_q(q^d(\theta), \theta) - \frac{1 - F(\theta)}{f(\theta)} u_{q\theta}(q^d(\theta), q^d(\theta), \theta) \left(1 + \frac{\partial q^*(q^d(\theta), \theta)}{\partial q}\right).$$

Because v^d has strict increasing differences in (q, θ), along the equilibrium consumption, $q^d(\theta)$,

$$v_{q\theta}^d(q^d(\theta), \theta) = u_{q\theta}(q^d(\theta), q^d(\theta), \theta) \left(1 + \frac{\partial q^*(q^d(\theta), \theta)}{\partial q}\right) > 0.$$

$$\Lambda_q^d(q^d(\theta), \theta) = 0 \leq \frac{1}{2}W_q(q^d(\theta), \theta).$$

with a strict inequality for $\theta \neq \bar{\theta}$. Hence, for all $\theta \in (\theta_0^d, \bar{\theta})$, regardless of whether the goods are substitutes or complements, we have $q^d(\theta) < q^{fb}(\theta)$ which proves existence of

[28]Note that without differentiability, we would only have weak inequalities for the derivative of $q^*(q, \theta)$, which would only allows us to establish weak inequalities in the allocation orderings. This is the full import of the assumption.

a $\theta_0^d \in (\theta_0^{fb}, \bar{\theta})$ such that $\Lambda_q^d(q^d(\theta_0^d), \theta_0^d) = 0$. □

As we discussed in the text, one has to exercise caution in taking derivatives of $\Lambda^d(q,\theta)$ with respect to q because the derivatives of $q^*(q,\theta)$ will be discontinuous at the instant where either $q^*(q,\theta) = 0$ or $\bar{q}$. This introduces a kink in Λ^d. Fortunately, at all such points of nondifferentiability, the kinks preserve concavity (i.e., the right derivative is less than the left derivative). We state and prove this result here.

Lemma 2 *Suppose that $P(q) \geq C(q)$ for all $q \in \mathcal{Q}$ and $(\hat{q},\theta)$ is a point at which $\Lambda^d(q,\theta)$ is nondifferentiable in q. The left and right derivatives of $\Lambda^d(q,\theta)$ satisfy*

$$\lim_{q \uparrow \hat{q}} \Lambda_q^d(q,\theta) > \lim_{q \downarrow \hat{q}} \Lambda_q^d(q,\theta).$$

Proof of Lemma 2 First, note that it can never be the case that $\bar{q} = q^*(q,\theta)$ because $u(q,\bar{q},\theta) - u(q,0,\theta) < C(\bar{q}) \leq P(\bar{q})$ by assumption. Any kink must occur where $q^*(q,\theta) = 0$. We next define $\hat{q}(\theta)$ as the critical value of output such that

$$q^*(\hat{q}(\theta),\theta) \equiv 0.$$

Consider the case of complements. Note that $q^*(q^d(\theta),\theta) = q^d(\theta) \geq 0$, so it follows that $q^d(\theta) \geq \hat{q}(\theta)$. Differentiating Λ^d over the separate regions of q, we have

$$\Lambda_q(q,\theta) = \begin{cases} u_q(q, q^*(q,\theta),\theta) - C'(q) - \\ \qquad \frac{1-F(\theta)}{f(\theta)} u_{q\theta}(q, q^*(q,\theta),\theta)\left(1 + \frac{\partial q^*}{\partial q}(q,\theta)\right), \text{if } q > \hat{q}(\theta); \\ u_q(q,0,\theta) - C'(q) - \frac{1-F(\theta)}{f(\theta)} u_{q\theta}(q,0,\theta), \ \ \text{if } q < \hat{q}(\theta). \end{cases}$$

Because $q^*(q,\theta)$ is continuous in q, we know that the left and right limits are equal: $q^*(\hat{q}(\theta)^-,\theta) = q^*(\hat{q}(\theta)^+,\theta) = 0$. Thus,

$$\lim_{q \uparrow \hat{q}(\theta)} \Lambda_q^d(q,\theta) = u_q(\hat{q}(\theta),0,\theta) - C'(\hat{q}(\theta)) - \frac{1-F(\theta)}{f(\theta)} u_{q\theta}(\hat{q}(\theta),0,\theta),$$

$$\lim_{q \downarrow \hat{q}(\theta)} \Lambda_q^d(q,\theta) = u_q(\hat{q}(\theta),0,\theta) - C'(\hat{q}(\theta)) - \frac{1-F(\theta)}{f(\theta)} u_{q\theta}(\hat{q}(\theta),0,\theta)\left(1 + \frac{\partial q^*}{\partial q}(\hat{q}(\theta)^+,\theta)\right).$$

Because the goods are complements $\frac{\partial q^*}{\partial q}(\hat{q}(\theta)^+,\theta) > 0$, implying

$$\lim_{q \uparrow \hat{q}(\theta)} \Lambda_q^d(q,\theta) > \lim_{q \downarrow \hat{q}(\theta)} \Lambda_q^d(q,\theta).$$

When the goods are substitutes, $q^*(q^d(\theta),\theta) = q^d(\theta) > 0$ implies $q^d(\theta) < \hat{q}(\theta)$. Differentiating Λ^d over the separate regions of q, we have

$$\Lambda_q(q,\theta) = \begin{cases} u_q(q, q^*(q,\theta),\theta) - C'(q) - \\ \qquad \frac{1-F(\theta)}{f(\theta)} u_{q\theta}(q, q^*(q,\theta),\theta)\left(1 + \frac{\partial q^*}{\partial q}(q,\theta)\right), \text{if } q < \hat{q}(\theta); \\ u_q(q,0,\theta) - C'(q) - \frac{1-F(\theta)}{f(\theta)} u_{q\theta}(q,0,\theta), \ \ \text{if } q > \hat{q}(\theta). \end{cases}$$

Taking limits as before,

$$\lim_{q\uparrow\hat{q}(\theta)} \Lambda_q^d(q,\theta) = u_q(\hat{q}(\theta),0,\theta) - C'(\hat{q}(\theta)) - \frac{1-F(\theta)}{f(\theta)} u_{q\theta}(\hat{q}(\theta),0,\theta)\left(1 + \frac{\partial q^*}{\partial q}(\hat{q}(\theta)^-,\theta)\right),$$

$$\lim_{q\downarrow\hat{q}(\theta)} \Lambda_q^d(q,\theta) = u_q(\hat{q}(\theta),0,\theta) - C'(\hat{q}(\theta)) - \frac{1-F(\theta)}{f(\theta)} u_{q\theta}(\hat{q}(\theta),0,\theta).$$

Because the goods are substitutes $\frac{\partial q^*}{\partial q}(\hat{q}(\theta)^-,\theta) < 0$, implying

$$\lim_{q\uparrow\hat{q}(\theta)} \Lambda_q^d(q,\theta) > \lim_{q\downarrow\hat{q}(\theta)} \Lambda_q^d(q,\theta).$$

□

Proof of Proposition 5: We begin by guessing that a symmetric equilibrium allocation can be found which is linear in θ as in the case of monopoly and the first-best.[29] With this conjecture, it follows that the equilibrium tariffs are quadratic and of the form

$$P(q) = \begin{cases} a_0 + a_1 q + \frac{a_2}{2} q^2 & \text{if } q \le q^{fb}(\bar{\theta}) \\ P(q^{fb}(\bar{\theta})) + c(q - q^{fb}(\bar{\theta})) & \text{otherwise.} \end{cases}$$

Note that over the relevant range, P is quadratic and at $q = q^{fb}(\bar{\theta})$, P is extended in a linear and smooth fashion so as to remain above cq; this particular extension is convenient but could take other forms.

Lemma 1 implies that such a schedule must also satisfy right-continuity at the origin, so $a_0 = 0$. We proceed by assuming the equilibrium regular with tariff parameters a_1 and a_2, and then check ex post that the candidate equilibrium is indeed regular. In our quadratic-uniform model, a symmetric equilibrium q^d must solve at any θ such that $q^d(\theta) > 0$, $\Lambda_q(q^d(\theta),\theta) = 0$, which implies

$$\alpha + \theta - q^d(\theta) - c(\beta - \gamma) = (\bar{\theta} - \theta)\left(1 + \frac{\partial q^*}{\partial q}(q^d(\theta),\theta)\right).$$

Because $P(q) = a_1 q + \frac{a_2}{2} q^2$ on the relevant range of equilibrium outputs, in a symmetric equilibrium and $q^*(q,\theta)$ is constrained to be nonnegative, we obtain

$$q^*(q,\theta) = \max\left\{0, \frac{-\gamma q + (\beta+\gamma)\theta + (\beta+\gamma)(\alpha - a_1(\beta-\gamma))}{\beta + a_2(\beta^2-\gamma^2)}\right\}, \tag{13}$$

and for $q^*(q^d(\theta),\theta) = q^d(\theta) > 0$,

$$\frac{\partial q^*(q^d(\theta),\theta)}{\partial q} = -\frac{\gamma}{\beta + a_2(\beta^2-\gamma^2)}. \tag{14}$$

[29] We know from Martimort (1992) and Stole (1991) that, in the case of complements, other equilibria can be found that are not linear. Because our findings are expressed with only the assumption of regularity, they apply equally well to all those other less tractable equilibria.

Substituting and collecting terms yields

$$q^d(\theta) = q^{fb}(\bar{\theta}) - (\bar{\theta} - \theta) \left(1 + (\beta - \gamma) \left(\frac{1 + (\beta + \gamma)a_2}{\beta + (\beta^2 - \gamma^2)a_2}\right)\right). \tag{15}$$

Given $P(q) = a_1 q + \frac{a_2}{2} q^2$, the agent's first-order condition, $u_q(q, q, \theta) - P'(q) = 0$ can be rewritten as

$$q^d(\theta) = \frac{\alpha + \theta - a_1(\beta - \gamma)}{1 + a_2(\beta - \gamma)}. \tag{16}$$

Identifying (15) and (16), the coefficient a_2 must solve:

$$2(\beta^2 - \gamma^2)a_2^2 + 3\beta a_2 + 1 = 0$$

which has two real roots. In the text, it was established that, if $v^d(q, \theta)$ has strict increasing differences in equilibrium, then

$$1 + \frac{\partial q^*}{\partial q}(q^d(\theta), \theta) > 0. \tag{17}$$

Using (14) one can check that one root always violates (17) while the other always satisfies it. The acceptable root is defined by

$$a_2 = -\frac{2}{3\beta + \sqrt{\beta^2 + 8\gamma^2}} < 0.$$

In addition, (15) and (16) also imply that $q^d(\bar{\theta}) = q^{fb}(\bar{\theta})$, which provides a second identifying restriction. Substituting our result for a_2, we have

$$a_1 = c + \frac{2q^{fb}(\bar{\theta})}{3\beta + \sqrt{\beta^2 + 8\gamma^2}} > c.$$

Returning to (15), we can substitute in the equilibrium values for a_2 and simplify to obtain the formula for the equilibrium consumption in the text.

We now establish regularity at this solution. First, note that $v^d(q, \theta)$ is continuous because it is a maximum value function. Note that $v^d_\theta(q, \theta)$ is continuous because $q^*(q, \theta)$ is continuous. To establish that v^d has strict increasing differences, we need only establish that $v^d_{q\theta}(q, \theta) > 0$ at all points of differentiability.

$$\begin{aligned} v^d_{q\theta}(q, \theta) &= u_{q_1\theta}(q, q^*(q, \theta), \theta) + u_{q_2\theta}(q, q^*(q, \theta), \theta) \frac{\partial q^*(q, \theta)}{\partial q} \\ &= \frac{1}{\beta - \gamma} \left(1 + \frac{\partial q^*(q, \theta)}{\partial q}\right). \end{aligned}$$

This expression is possibly discontinuous, but strictly positive given our solution for a_2. Because $v^d(q, \theta)$ has strict increasing differences, it follows that v^d is increasing in θ. Hence, v^d satisfies the requisite regularity conditions.

Next consider the virtual surplus function, $\Lambda^d(q,\theta)$, that $v^d(q,\theta)$ generates. Inserting (14) into the expression above for $v^d_{q\theta}$, we observe that $v^d_{q\theta}(q,\theta)$ is independent of θ and, since the distribution of θ is uniform, it follows that $\Lambda^d(q,\theta)$ inherits the increasing differences in (q,θ) property from $v^d(q,\theta)$. Given that $P(q)$ is quadratic on the relevant range of outputs, it follows that $\Lambda^d(q,\theta)$ is quadratic in q over the region where $q^*(q,\theta) > 0$. The solution for a_2 further guarantees that $\Lambda^d(q,\theta)$ is strictly concave over this region. Over the region of the domain for which $q^*(q,\theta) = 0$, $\Lambda^d(q,\theta)$ is also quadratic and strictly concave. Thus, we need only to establish that on the boundary of these two regions, $\Lambda^d(q,\theta)$ has only an inward (i.e., concave-preserving) kink. But this is true given Lemma 2. Hence, $\Lambda^d(q,\theta)$ is strictly concave (and hence strictly quasi-concave), which establishes regularity for the symmetric equilibrium. □

Proof of Proposition 6: Suppose to the contrary that $\theta^i_0 < \theta^m_0$. If $\theta^i_0 < \theta^m_0$, then $q^i(\theta) > 0$, $q^m(\theta) = 0$ and $\Lambda^m(q^m(\theta),\theta) = 0$ for all $\theta \in (\theta^i_0, \theta^m_0)$. It follows also that over this interval of types

$$\begin{aligned} 0 = \Lambda^m(q^m(\theta),\theta) &> \Lambda^m(q^i(\theta),\theta) \\ &= u(q^i(\theta), q^i(\theta), \theta) - 2C(q^i(\theta)) - \frac{1-F(\theta)}{f(\theta)} u_\theta(q^i(\theta), q^i(\theta), \theta) \\ &= \Lambda^i(q^i(\theta),\theta) + P^i(q^i(\theta)) - C(q^i(\theta)) \\ &\geq \Lambda^i(q^i(\theta),\theta). \end{aligned} \tag{18}$$

The first inequality follows from $q^m(\theta)$ being the unique maximizer of $\Lambda^m(q,\theta)$ in tandem with the fact that $q^i(\theta) \neq q^m(\theta)$ over this interval. The middle substitution follows from the definition of $v^i(q,\theta)$; note that this step is invalid in the case of delegation. The last inequality above follows from the fact that the firms earn nonnegative profit for each served consumer type in the intrinsic agency game.[30] Because $J^i(\theta) = \Lambda^i(q^i(\theta),\theta) < 0$ for all $\theta \in (\theta^i_0, \theta^m_0)$, it cannot be that θ^i_0 is the optimal participation cutoff. A contradiction.

Suppose in addition that $P(q)$ is differentiable in the neighborhood of 0. If $\theta^i_0 = \theta^m_0 = \theta_0$ and $q^i(\theta_0) \neq q^m(\theta_0) = 0$, then a similar contradiction emerges. Thus, if $\theta^i_0 = \theta^m_0 = \theta_0$, it must be that $q^i(\theta_0) = q^m(\theta_0) = 0$, which implies $q^i(\theta)$ is continuous. In that case, it must also be that $0 = \Lambda^m_q(q^m(\theta_0),\theta_0) = \Lambda^m_q(q^i(\theta_0),\theta_0)$. Using the following identity (valid

[30]Using an argument from Jullien (2000, Lemma 3), if profits were negative for the marginal consumer, a firm could offer a new price schedule of $\tilde{P}(q) = \max\{P(q), C(q)\}$ and improve profits, yielding a contradiction. Moreover, in any symmetric equilibrium, the last inequality is also strict at θ^i_0 if $q^i(\theta^i_0) > 0$.

at any $q^i(\theta) > 0$)

$$\Lambda_q^i(q^i(\theta),\theta) = \frac{1}{2}\Lambda_q^m(q^i(\theta),\theta) - \frac{1-F(\theta)}{f(\theta)}u_{q\theta}(q^i(\theta),q^i(\theta),\theta)\frac{\partial q^*(q^i(\theta),\theta)}{\partial q}$$

and passing to the limit as θ converges towards θ_0 yields

$$\frac{1-F(\theta_0)}{f(\theta_0)}u_{q\theta}(0,0,\theta_0)\frac{\partial q^*(0^+,\theta_0)}{\partial q} = 0$$

which cannot arise since $q^m(\theta_0) = 0$ implies $\theta_0 \neq \bar{\theta}$ and $\frac{\partial q^*(0^+,\theta_0)}{\partial q} \neq 0$ when either goods are substitutes or complements and $P^i(q)$ is differentiable on the interior of its domain (using the same reasoning as in the Proof of Proposition 4). □

Proof of Proposition 7: Suppose that $P^i(q) = P^d(q) + P_0$ for some P_0 to be found below. Let $v^d(q,\theta)$ be the regular indirect utility function in the delegated game. Then the associated indirect utility function in the intrinsic game is

$$v^i(q,\theta) = v^d(q,\theta) - P_0 + \left(\max_{q_2\in\mathcal{Q}} u(0,q_2,\theta) - P^d(q_2)\right),$$

where the parentheses contain the hypothetical return to a consumer who could exclusively contract with the rival firm and pay $P^d(q)$. Using a more compact notation, we have simply

$$v^i(q,\theta) = v^d(q,\theta) - P_0 + \phi^d(\theta),$$

where $\phi^d(\theta) \geq 0 = u(0,0,\theta) - P^d(0)$ and $\phi^d(\theta)$ is nondecreasing in θ since $\dot{\phi}^d(\theta) = u_\theta(0,q^*(0,\theta),\theta) \geq 0$. Hence, if v^d is increasing in θ and has strict increasing differences in (q,θ), then so is v^i.

Now consider the intrinsic virtual surplus function:

$$\begin{aligned}
\Lambda^i(q,\theta) &= v^d(q,\theta) - P_0 + \phi^d(\theta) - C(q) - \frac{1-F(\theta)}{f(\theta)}\left(v_\theta^d(q,\theta) + \dot{\phi}^d(\theta)\right) \\
&= \Lambda^d(q,\theta) - P_0 + \left(\phi^d(\theta) - \frac{1-F(\theta)}{f(\theta)}\dot{\phi}^d(\theta)\right) \\
&= \Lambda^d(q,\theta) - P_0 + \Phi(\theta),
\end{aligned}$$

where $\Phi(\theta)$ has been defined as the bracketed term of the second line. Notice that $\Lambda^i(q,\theta)$ inherits increasing differences and strict quasi-concavity from $\Lambda^d(q,\theta)$.

It follows that if $q^d(\theta) \in \arg\max_{q\in\mathcal{Q}} \Lambda^d(q,\theta)$, then $q^d(\theta) \in \arg\max_{q\in\mathcal{Q}} \Lambda^i(q,\theta)$ for all $\theta \in [\theta_0^i,\bar{\theta}]$. This implies that the marginal price schedules are identical in each game so that $P^d(q) = P^i(q) + P_0$. It remains to verify that $P_0 > 0$ in a symmetric equilibrium.

To do so, we may first rewrite $\Lambda^i(q^i(\theta),\theta)$ as

$$\Lambda^i(q^i(\theta),\theta) = u(q^i(\theta), q^*(q^i(\theta),\theta),\theta) - P_0 - P^d(q^*(q^i(\theta),\theta)) \\ - C(q^i(\theta)) - \frac{1-F(\theta)}{f(\theta)} u(q^i(\theta), q^*(q^i(\theta),\theta),\theta)$$

and using the condition of a symmetric equilibrium that $q^*(q^i(\theta),\theta) = q^i(\theta)$,

$$\Lambda^i(q^i(\theta),\theta) = u(q^i(\theta), q^i(\theta),\theta) - P_0 - P^d(q^i(\theta)) - C(q^i(\theta)) \\ - \frac{1-F(\theta)}{f(\theta)} u_\theta(q^i(\theta), q^i(\theta),\theta).$$

Define the value function under intrinsic agency as $J^i(\theta) = \Lambda^i(q^i(\theta),\theta)$ which is implicitly a function of P_0 (we slightly abuse notations here because the true value function would be $\max\{0, \Lambda^i(q^i(\theta),\theta)\}$ to take into account the option of non-participation). We want to find a fixed-point solution (θ_0^i, P_0) such that $J^i(\theta_0^i) = 0$ given P_0 (with a second-order condition of the firm's problem requiring that $J^i(\theta_0^i)$ is nondecreasing) and $2P_0 + 2P^d(q^d(\theta_0^i)) = u(q^i(\theta_0^i), q^i(\theta_0^i), \theta_0^i)$ given θ_0^i. Hence, we must find a solution to the equation:

$$u(q^i(\theta_0^i), q^i(\theta_0^i), \theta_0^i) - 2C(q^i(\theta_0^i)) - 2\frac{1-F(\theta_0^i)}{f(\theta_0^i)} u_\theta(q^i(\theta_0^i), q^i(\theta_0^i), \theta_0^i) = 0. \tag{19}$$

A first candidate is the pair $(\theta_0^i = \theta_0^d, P_0 = 0)$ corresponding to the delegated agency solution. For such a pair, we have $J^i(\theta_0^d) = 0$ and $2P_0 = u(q^d(\theta_0^d), q^d(\theta_0^d), \theta_0^i) = 0$. We will show that J^i is strictly decreasing to the right of θ_0^d, therefore ruling out $\theta_0^i = \theta_0^d$ as a possibility. To this end, note that $J^i(\theta) = 0$ for all $\theta \le \theta_0^d$, so it is nondecreasing to the left. Using our identity that $\Lambda^i(q,\theta) = \Lambda^d(q,\theta) - P_0 + \Phi(\theta)$, consider the right derivative of J^i:

$$\dot{J}^i(\theta) = \frac{\partial \Lambda^d(q^d(\theta),\theta)}{\partial q}\dot{q}^d(\theta) + \frac{\partial \Lambda^d(q^d(\theta),\theta)}{\partial \theta} + \dot{\Phi}(\theta).$$

By the envelope theorem, the first term is zero. Because $q^d(\theta)$ is continuous at θ_0^d and $q^i(\theta_0^d) = q^d(\theta_0^d) = 0$, it follows that $v_\theta^i(0,\theta_0^d) = v_{\theta\theta}^i(0,\theta_0^d) = 0$, implying that the second term is also zero at θ_0^d. Expanding the third term, we have

$$\dot{\Phi}(\theta) = \dot{\phi}^d(\theta)\left(1 - \frac{d}{d\theta}\frac{1-F(\theta)}{f(\theta)}\right) \\ - \left(\frac{1-F(\theta)}{f(\theta)}\right)\left(u_{\theta\theta}(q^d(\theta), q^d(\theta),\theta) + u_{\theta q}(q^d(\theta), q^d(\theta),\theta)\frac{\partial q^*(q^d(\theta),\theta)}{\partial \theta}\right).$$

Again, because $q^i(\theta_0^d) = 0$, it follows that $\dot{\phi}^d(\theta_0^d) = 0$ and $u_{\theta\theta}(0,0,\theta_0^d) = 0$. We are left with the following expression for the right derivative of J^i at θ_0^d:

$$\dot{J}^i(\theta) = -\left(\frac{1-F(\theta)}{f(\theta)}\right)\left(u_{\theta q}(q^d(\theta), q^d(\theta),\theta)\frac{\partial q^*(q^d(\theta),\theta)}{\partial \theta}\right) < 0,$$

which is decidedly negative. Hence, $(\theta_0^i = \theta_0^d, P_0 = 0)$ is not an acceptable solution.

Note that the previous argument establishes that $J^i(\theta_0^d + \varepsilon) < 0$ for sufficiently small $\varepsilon > 0$. At $\bar{\theta}$ we know $J^i(\bar{\theta}) > 0$ because $u(q^{fb}(\bar{\theta}), q^{fb}(\bar{\theta}), \bar{\theta})) - 2C(q^{fb}(\bar{\theta})) = W^{fb}(q^{fb}(\bar{\theta}), \bar{\theta}) > 0$ by assumption. By continuity of J^i and the mean value theorem, there exists such $\theta_0^i \in (\theta_0^d, \bar{\theta})$ such that J^i is nondecreasing and $2P_0 = u(q^i(\theta_0^i), q^i(\theta_0^i), \theta_0^i) > 0$. □

Proof of Proposition 8: Proposition 7 implies $q^i(\theta) = q^d(\theta)$ for all $\theta \in [\theta_0^i, \bar{\theta}]$ and $q^i(\theta) = 0$ otherwise. Proposition 7 also implies that the intrinsic equilibrium is regular whenever the associated delegated equilibrium is regular. Proposition 5 establishes that the equilibrium under delegation and quadratic-uniform preferences is regular. The explicit calculations for the quadratic-uniform are thus the same as in Proposition 5. What remains is the calculation of θ_0^i.

As observed in the proof to Proposition 7, the proposed solution of $\theta_0^i = \theta_0^d$ is unacceptable because J^i is decreasing is to the right of θ_0^d. In the specific context of the quadratic-uniform model, (19) can be restated as

$$q^i(\theta_0^i)\left(\frac{q^i(\theta_0^i)}{2} + (\bar{\theta} - \theta_0^i)\left(\frac{\partial q^*}{\partial q}(q^i(\theta), \theta) - 1\right)\right) = 0 \tag{20}$$

where $\frac{\partial q^*}{\partial q}(q^i(\theta), \theta)$ is given by (14). This quadratic equation has two roots

$$q^i(\theta_0^i) = 0 \text{ and } q^i(\theta_0^i) = 2(\bar{\theta} - \theta_0^i)\left(1 + \frac{\gamma}{\beta + a_2(\beta^2 - \gamma^2)}\right) > 0,$$

with the acceptable solution is the positive root above. □

Proof of Theorem 1: The relationships in equations (9)-(12) follow from the results in Propositions 2, 4 and 6. That a regular, symmetric equilibrium exists in the quadratic-uniform model follows from Propositions 3, 5 and 8. □

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