The Informational Effects of Competition and Collusion in Legislative Politics\textsuperscript{1}

David Martimort\textsuperscript{2} and Aggey Semenov\textsuperscript{3}

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Abstract: We use a mechanism design approach to study the organization of interest groups in an informational model of lobbying. Interest groups influence the legislature only by communicating private information on their preferences and not by means of monetary transfers. Interest groups have private information on their ideal points in a one-dimensional policy space and may either compete or adopt more collusive behaviors. Optimal policies result from a trade-off between imposing rules which are non-responsive to the groups’ preferences and flexibility that pleases groups better. Within a strong coalition, interest groups credibly share information which facilitates communication of their joint interests, helps screening by the legislature and induces flexible policies responsive to the groups’ joint interests (an informativeness effect). Competing interest groups better transmit information on their individual preferences (a screening effect). The socially and privately optimal organization of lobbying favors competition between groups only when their preferences are not too congruent with those of the legislature. With more congruence, a strong coalition is preferred. Finally, within a weak coalition, interest groups must design incentive compatible collusive mechanisms to share information. Such weak coalitions are always inefficient.

Keywords: Communication Mechanisms, Lobbying, Competition, Coalition, Legislative Politics.

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\textsuperscript{2}Toulouse School of Economics (IDEI, GREMAQ) and EHESS.

\textsuperscript{3}National University of Singapore.
1 Introduction

Modern legislative politics in the U.S. is characterized by two important features. First, the number of active interest groups has dramatically increased over the last four decades, from 5000 in 1955 to over 33000 at the end of the twentieth century. Political scientists like Huntington (1975), Salisbury (1990) and Wilson (1979) viewed this proliferation of interests as an indication of a more fragmented and atomistic political system. Second, coalitions of interest groups abound. By conducting a survey on data of the Congressional Information Services Index and interviews, Hula (1999) showed that respectively 81.3, 79.6 and 83.3 percent of interviewed institutional members agree on thinking that forming coalitions is the best way to be effective in politics in areas like transportation, education and civil rights issues. The field of transportation, for instance, features heavy lobbying by business interests gathered in strong coalitions, primarily trade associations such as the American Bus Association or the Air Transport Association defending long-term economic interests on regulation and deregulation of transport industries. In the case of education, although business interests play a much lesser role, two- and four-year colleges as well as universities and the “Big Six” organizations are associations acting as major lobbyists in the reforms of the education system.

The choice by interest groups to compete fiercely or to adopt collusive behaviors in legislative politics certainly reflects the huge diversity in their objectives, audiences and in the related economic issues at stake. The macro-organization of interest groups may also be related to the structure of the transaction costs which shape interactions in the political arena when informational asymmetries prevail. In spite of the fact that understanding this organization is of paramount importance to explain the design of economic policies, still, very little is known about the interest groups’ incentives either to compete head-to-head or to form an active coalition. Although this organizational issue has attracted much attention in political science, it is still being ignored, by and large, in the political economy literature.

To address these issues, we start from the well-admitted view that interest groups play an important informational role in legislative politics. To influence policy decision-

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1 An example is given by the Committee for Education Funding which is the best-known coalition in education policy. It involves more than one hundred organizations looking for higher levels of funding. Hula (1999) reported that “By organizing and speaking with a common voice, the Committee for Education Funding can define the public debate in terms of overall funding on education while simultaneously making a case for each member’s specific programs, in effect collectivizing the benefits and the risks.”

2 For instance, Hula (1999) argued that “the macroeconomic view of the interest groups community often overlooks a number of institutional links between interest groups, most notably the increasing use of long-term, recurrent, and institutionalized coalitions in many policy arenas.”

making, lobbyists spend time and resources conveying information to uninformed political decision-makers. Because their preferences may conflict with those of the policy-makers, interest groups manipulate information to promote their own interests. Public policies result from a trade-off between the benefits that the legislature finds when communicating with the privately informed interest groups to make policies more flexible, and the cost of departing from its own preferences to induce information revelation. Informational asymmetries create significant transaction costs and public policies result from a “rules versus discretion” trade-off. Rules may better reflect the legislature’s preferences but are chosen without any information held by private interests. More flexible policies are feasible provided that society chooses policies which are better aligned with those preferred by private interests. Rules are certainly more valuable when the interest groups’ preferences and those of society diverge whereas, otherwise, more flexible policies become more attractive.

The study of this trade-off is key not only to better understand the relationships of informed interest groups with the legislature, but also how interest groups interact with each other. Transaction costs minimization provides some rationale for the macro-organization of interest groups: Whether interest groups stay apart or adopt more collusive behaviors certainly reflects their private incentives towards such cost minimization and how various organizational forms of lobbying affect the congruence of private and social interests. This paper analyzes the consequences of interest groups adopting various kinds of behaviors from competitive to more collusive ones taking the perspectives of society’s and interest groups’ welfare. We also investigate how optimal policies respond to the lack of congruence between private and social interests induced by those behaviors.

Our basic insight can be summarized as follows. By remaining split apart and competing, interest groups make it possible that their own preferences end up being represented in the implemented policy with some probability: a screening effect. By the same token however, the cost of competition for a given group is that optimal policies may reflect with some probability the preferences of a competing group. On the other hand, by merging and credibly sharing information within a strong coalition, interest groups may better represent their joint interest although individual preferences are no longer represented: an informativeness effect. The best organizational form of lobbying depends on the degree of congruence between the groups and the legislature induced by those organizations. With high congruence, flexible policies become more attractive. Strong coalitions are favored both from a private and a social viewpoint. Instead, greater conflicts between private and social interests call for more rigid rules which can be somewhat relaxed by having groups compete. Indeed, favoring one group in the policy choice eases information revelation on the latter’s preferences although, at the same time, it makes impossible to reflect the preferences of competing groups. Competition implements more flexible policies although
these policies are biased towards one particular group at the time.

Key to the success of a coalition is its ability to credibly share information. *Weak coalitions* that fail to do so are never optimal. Because of its internal informational problem, a weak coalition pretends having a lowest joint interest for the policy at stake as compared to its strong counterpart. This makes a weak coalition less congruent with the rest of society and calls for rigid polices.

Our model predicts an increase in the number of active interest groups when conflicts of interests on political issues are exacerbated. More collusive behaviors are expected for minor conflicts of interests with the rest of society. In this case, since only the strong coalitions should emerge, coalitions should look for efficient means to credibly share information.\(^4\)

Our paper departs significantly from the existing literature on the informational role of interest groups in terms of modelling tools and in scope of analysis. Starting with the seminal contributions by Crawford and Sobel (1982), Austen-Smith (1990) and Krebs (1992), lobbying groups are generally viewed as informed Stackelberg leaders in the communication game played with an uninformed policy-maker.\(^5\) Although a priori attractive, this approach nevertheless faces some difficulties to reach normative implications on the overall organization of lobbying. Signalling games are generally plagued with a multiplicity of inefficient partition equilibria and, in the absence of any convincing equilibrium refinement, the comparison of alternative organizational forms is only indicative of the forces shaping the overall organization of lobbying groups. The mechanism design perspective adopted in this paper does not suffer from this weakness.\(^6\) We reverse the timing of standard lobbying games and assume that the policy-maker commits ex-ante to a mechanism which stipulates how policies respond to the lobbyists’ private information. For a given organizational form of lobbying, the Revelation Principle fully characterizes the set of feasible incentive allocations. The normative comparison between the optimal mechanisms obtained under various arrangements is then meaningful.

This mechanism design approach was initiated by Melumad and Shibano (1991) in the context of a single informed agent who cannot influence the decision-maker through transfers.\(^7\) It was pursued by Baron (2000) in a model studying the organization of the legislature. Legislators in a given committee have similar preferences (driven by the same common shock) but now monetary resources can be exchanged between the uninformed floor and an informed committee. Monetary transfers obviously help to satisfy incentive

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4 Trade associations in the field of transportation policy are typical examples in order.

5 Klevorick et al. (1984) compared information gathering with majority voting without considering the incentive problem.

6 Laffont (2000) offered a more general defense of this perspective to explain constitutional choices.

7 See also Holmström (1984), Green and Stokey (1981) and Armstrong (1994).
compatibility constraints. We depart from this set of assumptions by focusing on the case where there are no such monetary transfers (except within a coalition), thereby stressing only the informational role of interest groups.\(^8\) Also, in our model, interest groups have different pieces of private information. Finally, the issue of finding the optimal organization of lobbying is not analyzed by Baron (2000).\(^9\) In particular, our focus on the different forms of collusive behavior that interest groups may adopt is novel.

Although the mechanism design approach can be viewed as an alternative to the signalling literature, that literature has nevertheless delivered some insights on organizational issues by comparing equilibria outcomes. Austen-Smith (1993a, 1993b) analyzed communication patterns when interest groups report either sequentially or simultaneously.\(^10\) Krishna and Morgan (2000) studied a lobbying game with two informed lobbyists who share the same information on the state of nature but may have conflicting or congruent views on what should be the optimal policy. They showed that conflicting views help the policy-maker to extract information. A mechanism design approach predicts in such a context the existence of a costless, fully communicative equilibrium irrespective of bias.\(^11\) This extreme result leads us to focus on the case where interest groups have idiosyncratic private information on their ideal points.\(^12\) Finally, Battaglini and Benabou (2003) developed a signalling model with multiple interest groups entering into costly lobbying activities. They also argue that low conflict of interests may favor coalitional behavior just as we will do below.\(^13\)

At a broader theoretical level, our analysis of interest groups coalition formation also contributes to the literature on collusion under asymmetric information pioneered by Laffont and Martimort (1997, 2000). However, contrary to this literature which was de-

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\(^8\)The informational role of interest groups results in a trade-off between control and communication. Recently, Amador, Werning and Angeletos (2006) considered a similar tradeoff in a rather different context.

\(^9\)Baron and Meirowitz (2001) draw also some links between the signalling and the mechanism design approaches. They argued that with a convenient choice of out-of-equilibrium beliefs capturing the lack of commitment assumption of the signalling game, the two approaches are equivalent. On the relationship between models with or without commitment, see also Mylovanov (2004) who showed that any incentive compatible mechanism without transfers can be implemented with very little commitment using a veto-based delegation. Glazer and Rubinstein (1998) also considered the problem of designing a mechanism without transfers to achieve a given target when agents are driven by different motives.

\(^10\)Ottaviani and Sorensen (2001) showed that the order of speech can improve information revelation.

\(^11\)This would be obtained by using a revelation mechanism à la Maskin (1999).

\(^12\)Grossman and Helpman (2001, Chapter 4) also compared the informativeness of various equilibria depending on whether the lobbyists’ messages on their private information are public or private.

\(^13\)Although some of their results look similar to ours, the two models are hardly comparable because, on top of a major difference in timing coming from their analyzing a signalling model, several ingredients which are key to equilibrium characterization differ. In their paper, interest groups have discrete signals on a binary state of the world in a common values environment whereas we focus on a private values setting with a continuum of signals; lobbying is costly in their paper and costless here; and the public decision is of a zero-one nature in their model whereas it responds smoothly to communicated information in ours.
veloped in a framework where monetary transfers between the principals and the colluding agents are feasible, the analysis of weak and strong coalitions developed in this paper takes place in a framework where transfers cannot be used by the principal.

Section 2 presents the model. Section 3 derives the optimal continuous mechanism when interest groups compete.1\textsuperscript{4} Section 4 motivates our modelling assumptions for the game of coalition formation between interest groups. Section 5 analyzes the case of a strong coalition where two groups credibly share information and collectively influence a policy-maker to promote their joint interests. Finally, Section 6 does the same for a weak coalition where interest groups share information by means of an incentive compatible side-mechanism. Section 7 compares the organizational forms of lobbying using either a social or a private perspective. Section 8 proposes alleys for further research. Proofs are relegated to the Appendix.

2 The Model

Preferences and Information: We consider a legislature (the principal) that is influenced by two interest groups (the agents, indexed by $i = 1, 2$) in an otherwise standard model of informational lobbying. The sole means of influence available to the agents is the communication of their private information. The principal aggregates information privately held by those interest groups and chooses a one-dimensional policy $q$ on behalf of the society. Depending on the application, this policy can be a tax, an import tariff, a regulated price, or a number of allowed permits. The agents and the principal have all single-peaked quadratic preferences defined over the policy $q$ respectively as follows:

$$U_i(q, \theta_i) = -\frac{1}{2}(q - \theta_i)^2, \text{ for } i = 1, 2 \text{ and } V(q, \theta_1, \theta_2) = -\frac{1}{2} \left( q - \frac{1}{2} (\theta_1 + \theta_2) - \delta \right)^2.$$

The legislature not only aggregates the preferences of active interest groups but also takes into account those of the rest of society. We capture this effect by assuming that the principal has a bias $\delta > 0$ with respect to the benchmark policy which is derived by averaging the interest groups’ ideal points. Without any such bias, and if completely informed on the groups’ preferences, the principal would choose an efficient decision averaging the groups’ ideal points with an equal weight for each. More generally, the principal can be viewed as a social welfare maximizer taking into account in his objective the well-being of the rest of society. That the legislature also values the interests of the general public although it is not organized as an active lobby is also justified when policy-makers have reelection

\textsuperscript{14}The restriction to continuous mechanisms is important to get a simple characterization of optimal mechanisms which is more amenable to comparisons between different organizational forms.
concerns and want to please voters who do not belong to any organized groups.\textsuperscript{15}

Interest group \(i\) has private information on his ideal point \(\theta_i\). The preference parameters \(\theta_1\) and \(\theta_2\) are drawn identically, independently and uniformly on \(\Theta = [0, 1]\) according to the cumulative distribution \(F(\theta) = \theta\). \(E(\cdot)\) denotes the expectation operator.

To give the best opportunity to collusive behavior and still introduce some heterogeneity between groups, we assume that although both agents have the same expected conflict with the principal ex-ante, their ideal points may differ ex-post. On average both groups tend to prefer a lower policy than the principal. This is common knowledge although the precise extent by which they prefer so is their private information.\textsuperscript{16}

**Grand-Mechanisms:** For a given postulated behavior of the interest groups (either competition or collusion), the Revelation Principle states that there is indeed no loss of generality in restricting the principal to offer direct and truthful revelation grand-mechanisms. With such mechanisms, the principal commits to a (possibly random) rule stipulating which policy to follow in response to the groups’ reports on their preferences. Following Melumad and Shibano (1991), we focus on deterministic mechanisms of the form \(\{q(\hat{\theta}_1, \hat{\theta}_2)\} \) where \(\hat{\theta}_i\) is group \(i\)’s report on his ideal point.\textsuperscript{17}

The commitment assumption is attractive from a normative viewpoint because it solves the equilibrium indeterminacy that arises in the signalling environment where interest groups would move first. The mechanism design approach fully characterizes the set of incentive feasible allocations achievable at any equilibrium of a communication game. This property is quite attractive as far as one is concerned with the normative comparison between various organizational forms of lobbying.\textsuperscript{18}

**Timing:** The game unfolds as follows. First, an organizational form of lobbying is chosen

\textsuperscript{15}Grossman and Helpman (2001) developed a signalling model with two interest groups who have private information on the same preference parameter. They argued that similar analysis should be performed in the case of idiosyncratic private information. They suggested analyzing a legislature with the same objective function than here but did not perform any formal analysis.

\textsuperscript{16}We could account for differences in the directions in which interest groups want the policy to be pushed by having preferences being drawn from different supports or by having asymmetric biases. This would be at the cost of an increase in the technicalities without much new insights on the incentives to collude or not. We could also easily address the case where interest groups prefer on average a lower policy than the principal’s average ideal point (i.e., \(\delta < 0\)). The corresponding results and intuitions can be easily obtained by permuting the direction of most effects with respect to the present paper.

\textsuperscript{17}It is not known whether random mechanisms could help in our context with a continuum of types. Moreover, stochastic mechanisms are harder to enforce than deterministic ones; they require that the randomizing device used to determine allocations be publicly verifiable to be not manipulable by the principal himself if he was finding worth to do so. We leave the analysis of stochastic mechanisms which is slightly orthogonal to our main purposes for further research.

\textsuperscript{18}The commitment assumption prevails in the political economy literature on influence (see for instance the common agency models of Grossman and Helpman, 2001) as well as in the axiomatic theory of bargaining with a politician (Tauman and Zapecelnyuk, 2006).
with interest groups either competing or colluding.\footnote{We do not analyze the game of coalition formation at that stage. We will simply compare the expected payoffs of both the legislature and the interest groups under various organizational forms.} Second, each group observes only his own preferences. Third, the principal offers a grand-mechanism \( \{q(\hat{\theta}_1, \hat{\theta}_2)\} (\hat{\theta}_1, \hat{\theta}_2) \in \Theta^2 \). Fourth, if a coalition has been formed, member interest groups agree on some collusive way of transmitting information to the decision-maker. This will be more explicitly explained in Section 4. Fifth, interest groups report their preferences. Finally, the corresponding policy is implemented by the principal.

**Benchmark:** Suppose that the legislature remains uninformed on the interest groups’ preferences and commits ex-ante to a policy. The principal then chooses a rigid policy \( q^p \) which maximizes his expected payoff \( E(\theta_1, \theta_2) \left\{ -\frac{1}{2} \left( q - \frac{1}{2} (\theta_1 + \theta_2) - \delta \right)^2 \right\} \). One easily finds:

\[
q^p = \frac{1}{2} + \delta.
\]

This is the expected value of the interest groups’ ideal points augmented by the social bias \( \delta \). Of course, this policy might be improved by using an effective communication mechanism between the legislature and the interest groups. This may be valuable when \( \delta \leq \frac{1}{2} \) as we will see below.

### 3 Competing Interest Groups

To model competition between interest groups, we rely on dominant strategy implementation. The motivation for doing so is three-fold. First, we follow most of the social choice literature\footnote{See Moulin (1980) and the survey by Sprumont (1995) among others.} to characterize incentive mechanisms in a context where agents do not respond to monetary incentives. Although Bayesian implementation would obviously relax incentive constraints, dominant strategy implementation is amenable to a straightforward comparison between competition and collusion. Another (and quite standard) motivation for using dominant strategy implementation is that this concept is not sensitive to the beliefs that interest groups have on each other’s preferences.\footnote{Bergemann and Morris (2005).} Finally, focusing on dominant strategy implementation is sufficient to obtain the main result that competition between interest groups may sometimes improve the principal’s expected payoff compared to the outcome achieved with a coalition. *A fortiori*, this would also be the case had Bayesian implementation been used.

With dominant strategy, the incentive compatibility constraints for interest group \( i \) can be written as:

\[
\theta_i \in \arg \min_{\theta_i} \left( q(\hat{\theta}_i, \theta_{-i}) - \theta_i \right)^2, \quad i = 1, 2, \forall \theta_i \in \Theta.
\]
Using (1), standard revealed preference arguments show that $q(\cdot)$ is monotonically increasing in each of its arguments and thus almost everywhere differentiable in $(\theta_1, \theta_2)$.\footnote{See Laffont and Maskin (1980) for the differentiable approach of dominant strategy mechanisms in settings with monetary transfers. The revealed preferences argument used above does not depend on whether monetary transfers are available or not.} At any point of differentiability, incentive constraints can be written as:

$$\frac{\partial q}{\partial \theta_i}(\theta_i, \theta_{-i})(q(\theta_i, \theta_{-i}) - \theta_i) = 0, \quad i = 1, 2, \forall (\theta_i, \theta_{-i}) \in \Theta^2. \quad (2)$$

Hence, $q(\cdot)$ is either locally constant in a neighborhood of $\theta_i$ or equal to the ideal point of group $i$ and thus independent on $\theta_{-i}$. On the neighborhood where $q(\cdot)$ is locally constant, the policy is rigid and does not make use of interest group $i$'s report on his type. Communication has a more effective role when $q(\cdot)$ is not locally constant. The implemented policy is then type- and group-dependent. This leads to the following characterization of dominant strategy continuous schemes.

**Lemma 1**: For any symmetric dominant strategy and continuous mechanism $q(\cdot)$, there exist cut-offs $\theta^\ast$, $\theta^{**}$ and $\theta^{***}$ with $\theta^\ast \leq \theta^{**} \leq \theta^{***}$ such that:

$$q(\theta_1, \theta_2) = \min\{\theta^{***}, \max\{\theta_1, \theta^{**}\}, \max\{\theta_2, \theta^{**}\}, \max\{\theta_1, \theta_2, \theta^\ast\}\}. \quad (3)$$

These dominant strategy mechanisms can be given an interesting interpretation. For the region where a state dependent decision is implemented, the ideal point of one of the interest groups is chosen. The outcome occurs as though the interest group had residual control rights on the decision and would choose the policy on behalf of the rest of society.\footnote{Note that a fully rigid policy is obtained when $\theta^\ast = \theta^{**} = \theta^{***}$.}

The continuous mechanisms in (3) have already been described in Moulin (1980). However, his characterization was obtained by imposing dominant strategy on a larger domain including all single-peaked preferences. Our restriction to quadratic preferences could a priori leave the possibility that other continuous mechanisms might be feasible but Lemma 1 shows that this is actually not the case. Finally, the focus on continuous mechanisms allows us to maintain a tractable analysis and will facilitate comparisons between organizational structures.\footnote{In the case of a single agent, Alonso and Matouschek (2005) and Martimort and Semenov (2006) provided also conditions under which continuity is obtained at the optimal mechanism. A uniform distribution of types is enough to obtain this continuity.}

Of particular importance in the sequel are the following mechanisms which depend on only two parameters $\theta^\ast$ and $\theta^{**}$ with $\theta^{**} \leq 1$. These parameters define various areas
where the implemented policy is either rigid or flexible, in which case it depends only on the preferences of a single group:

\[
q(\theta_1, \theta_2) = \begin{cases} 
\theta^* & \text{if } \max\{\theta_1, \theta_2\} \leq \theta^* \\
\max\{\theta_1, \theta_2\} & \text{if } \theta^* \leq \max\{\theta_1, \theta_2\} \leq \theta^{**} \\
\theta^{**} & \text{if } \min\{\theta_1, \theta_2\} \leq \theta^{**} \leq \max\{\theta_1, \theta_2\} \\
\min\{\theta_1, \theta_2\} & \text{if } \theta^{**} \leq \min\{\theta_1, \theta_2\}.
\end{cases}
\] (4)

**Proposition 1**: The optimal dominant strategy incentive compatible and continuous grand-mechanism when interest groups compete has the form given in (4).

i) For \(0 \leq \delta < \frac{1}{4}\), there are two cut-offs \(\theta_d^*(\delta) = 2\delta\) and \(\theta_d^{**}(\delta) = \frac{1}{2} + 2\delta\),

ii) For \(\frac{1}{2} \geq \delta \geq \frac{1}{4}\), there is only one cut-off \(\theta_d^*(\delta) = 2\delta\) and the optimal policy is

\[
q(\theta_1, \theta_2) = \min\{\theta_d^*(\delta), \max\{\theta_1, \theta_2\}\},
\]

iii) For \(\delta \geq \frac{1}{2}\), the optimal policy is fully rigid and equal to \(q^p\).

Under competition, the optimal policy might be more flexible than the policy without any communication. It might indeed depend explicitly on the groups’ preferences, although it cannot, at the same time depend on both groups’ ideal points and has necessarily to be biased towards a single interest group. This corresponds to regions in the \((\theta_1, \theta_2)\) space where there is effective screening of the preferences of this selected interest group. This **screening effect** captures the benefits of competition.

To understand why such unilateral screening occurs, it should be reminded that there is more screening between a single group and the legislature and the implemented policy is more flexible as their conflict of interests is less pronounced.\(^{26}\) With two interest groups, the same logic applies but in a more complex way. The “virtual conflict of interests” between the first interest group and the policy-maker now depends on the preferences of the second one. This virtual conflict will typically be equal to

\[
\delta' = \delta + \frac{\theta_1 + \theta_2}{2} - \theta_1 = \delta - \frac{(\theta_1 - \theta_2)}{2}.
\]

This virtual conflict is a decreasing function of the “distance” between the interest groups’ ideal points when \(\theta_1 \geq \theta_2\). As this distance increases, the principal’s ideal point is closer to that of the interest group with the highest ideal point. Everything happens as if an endogenous bias of the decision process towards that interest group appears. Given

\(^{25}\)It will be shown in the Appendix that the optimal mechanism is necessarily in this class.

\(^{26}\)This is a point made by Melumad and Shibano (1991) in a screening context but this effect also occurs in the case of signalling games à la Crawford and Sobel (1982).
that the implemented policy cannot depend on both interest groups’ ideal points, the legislature prefers to have the ideal point of the closest’s group being selected.

The optimal mechanism with competing groups has two cut-offs only when $\delta$ is sufficiently small. Since under competition a mechanism depends at most on the preferences of only one group at the time, introducing a cut-off $\delta^{**}$ might be beneficial to avoid having only one group’s preferences being represented too often in circumstances when there is much congruence between the legislature and each group.

4 Coalitional Behavior

Different norms of collusive behavior between interest groups may emerge depending on the technology available for sharing of information within a coalition, the ability of those groups to enforce credible information sharing and their capacity to punish deviations from a collusive agreement by member groups. In the sequel, we will analyze different norms and relate them to various behaviors that interest groups may entertain.

Third-Party Mediated Collusion: Following Laffont and Martimort (1997, 2000), the collusive behavior of interest groups is modelled as being organized by a third-party. This third-party acts as a broker for the coalition and maximizes the gains he withdraws from organizing the collusive behavior.\(^{27}\)

This modelling device has two main motivations. The first motivation is related to the practical means by which interest groups collude in the political arena. Political scientists agree on the fact that interest groups are joining coalitions for information. For instance, Laumann and Knoke (1987) and Heinz et al. (1990) examined information exchanges between group dyads and argued that it is key to intergroup coordination. Hula (1999, Chapter 4) also reported that interest groups are linked by the career paths of their staff members and that a phenomenon akin to the “revolving door” occurs between the public and the private sectors also takes place across interest groups. This phenomenon certainly facilitates information flow between otherwise distinct organizations. Almost all organizations active in the U.S. politics have a board of directors, and those boards are interlocked through knitted relationships which provide efficient means of information sharing and resources for members of the staff of those organizations, as noticed by Hula (1999, p. 65). In practice, these agents act as brokers between the objectives of the different groups involved as noticed by Loomis (1986). The third-party of our modelling device can then be viewed as a metaphor for the common board members of different organizations involved

\(^{27}\)In Laffont and Martimort (1997, 2000), it is instead assumed that the third-party is benevolent and maximizes the sum of the expected payoffs of the colluding partners. Having the third-party as a budget breaker corresponds in fact to the real world institutional practices found in the case of interest groups coalitions as we argue below.
in a coalition. Relatedly, political consulting firms provide specialized services and act in the political arena as pro-active advocates for special interests. Those actors, whose importance in current politics cannot be overlooked, also provide examples of potential third-parties facilitating communication with key political decision-makers and helping interest groups organize active coalitions.

The second motivation for our modelling device is theoretical. Relying on an un-informed mediator to organize the collusive ring helps to model the bargaining procedure between colluding groups as a black-box. Those procedures are dynamic in nature but, using the approach of Myerson and Sattherthwaite (1983), they can also be viewed as static mechanisms without having to worry about details of the bargaining.

**Side-Mechanisms:** Before learning about the types of the colluding partners, the third-party commits to a side-mechanism that is offered to the agents. This side-mechanism first stipulates a manipulation of the reports $\phi(\theta_1, \theta_2) = (\phi_1(\theta_1, \theta_2), \phi_2(\theta_1, \theta_2))$ that the interest groups collectively make to their principal, and second a side-transfer $t_i(\theta_1, \theta_2)$ that interest group $i$ gives back to the third-party in exchange for organizing collusion.

Two assumptions are made which give its best chances to collusion. First, by assuming commitment to a side-mechanism, we short-cut the issue of enforcement of that collusive behavior although, in practice, interest groups might rely on their repeated relationships to enforce this agreement.

Second, we also allow for side-transfers within the side-mechanism. Two interpretations can be given for those side-transfers. First, they can also be viewed as the shares of the interest groups’ resources that are pocketed by the broker in exchange of his services in lobbying decision-makers (this will typically be the case if collusion between the interest groups is organized by a political consulting firm). Given that the third-party may act as a budget-breaker, we certainly obtain there an upper bound on what can be collectively achieved by interest groups when coordinating their behavior. Second, side-transfers can be viewed as continuation values of the relationship if we were explicitly modelling collusion between interest groups as a repeated game. Of course, this issue of enforcement is less relevant if we keep in mind the interpretation of the third-party as a political consulting firm with whom private contracts may be signed and enforced.

Several justifications can be found for allowing side-transfers within the coalition

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28 In the U.S., the American Association of Political Consultants counts more than 1,100 active members.
29 As a recent example, we might refer to the case of the health reform in the U.S. where some active lobbying has been taken place over summer 2007 under the aegis of Small Business California, a political consulting firm acting on behalf of some of the largest U.S. firms.
30 Tirole (1992) and Martimort (1999) presented models of such self-enforceable collusive behavior. The lessons of those models are quite close to those obtained by assuming enforceability of the side-mechanism.
whereas they are not allowed with the principal. First, and on practical grounds, many public policies severely limit transfers towards private interests. A typical example would be regulatory policies in the U.S. where direct transfers to regulated firms (viewed as interest groups) are banned. At the same time, there is no such limit on the private contracts that link interest groups to the political consulting firms that might represent them in the political arena. Second, and on more theoretical grounds, our modelling choices can be viewed as a metaphor for cases where it is simply easier to transfer money (or more generally utility streams if those transfers are viewed as continuation values of the relationships) between interest groups rather than between those groups and the legislature as a whole. This will typically be the case in a repeated relationship environment since, due to reelection concerns, the legislature might certainly be better modelled as a short-term principal facing long-term agents (the interest groups) better able to promise rewards and punishments among themselves than with such principal.

Distinguishing between strong and weak coalitions: We will analyze two different norms of collusive behavior which vary both in terms of the instruments available to enforce collusive behavior and in the degree of credible information sharing that they might reach. That classification, although extreme, provides some modelling for different kinds of coalitions that political scientists have highlighted. For instance, Lauman and Knake (1987) examined information exchanges between group dyads by differentiating groups sharing casual information (modelled below as weak coalitions) from those sharing more confidential information (modelled as strong coalitions). Of course, the extent of information sharing may depend on the career paths of staff members of those groups or of those political consulting firms which organize their collusion. It may also depend on the kind of political issue under scrutiny and whether it involves long-lasting interests or not.

In a strong coalition, interest groups share information perfectly regarding their preferences by the mere fact of colluding. In other words, the third-party organizing collusion is endowed with a costless technology to get access to the groups’ ideal points and to release this information within the coalition. Of course, the third-party still has incentives to manipulate information when communicating with the legislature. Using the expression coined by Baron and Besanko (1999) in an I.O. context, information is internally verifiable although it cannot be externally verified by the legislature. One possible justification for this information structure can again be found by viewing our model as a short-cut for repeated relationships. Whenever groups are long-term players facing a legislature with a shorter horizon, one may expect coalitions to be better able to credibly share information internally than with the legislature.

31 Of course, these justifications would deserve a full-fledged dynamic model. We leave it for further research.
In practice, this kind of strong collusive behavior is expected when a set of well-defined interest groups (often referred to as the coalition core in the political science literature\textsuperscript{32}) have long-lasting common interests. Take the example of transportation. This field involves long-lasting business interests gathered in trade associations such as the American Bus Association and the Air Transport Association. Core players in such a coalition have developed expertise on relevant issues, spent time and resources and repeatedly interacted in the past so that they have learned their preferences over time. These associations are certainly better modelled as strong coalitions.

Instead, in a weak coalition, interest groups must be given incentives to reveal private information on their preferences. Weak coalitions are expected on issues which arise unexpectedly at a given time and involve actors which may not have developed strong expertise on the policies at stake. An example in order is the recent lobbying on health reform in the U.S. where large business firms which were beforehand uninvolved in the health sector have formed lobbying coalitions but may have difficulties in gauging how much member groups wants to strive for a health reform.\textsuperscript{33}

**Enforcement:** If a member group deviates and refuses the collusive agreement, the two groups non-cooperatively play the grand-mechanism offered by the legislature. Of course, the equilibrium concept will depend on the kind of coalition that breaks apart.

Consider a given grand-mechanism \( \{ q(\hat{\theta}_1, \hat{\theta}_2) \}_{(\hat{\theta}_1, \hat{\theta}_2) \in \Theta^2} \) and a strong coalition where member groups know each other’s preferences. A pair of reporting strategy (not necessarily truthful) \((\hat{\theta}_1^*, \hat{\theta}_2^*)\) forms a Nash equilibrium of that mechanism when

\[
\hat{\theta}_i^* \in \arg \min_{\hat{\theta}_i} (q(\hat{\theta}_i, \hat{\theta}_i) - \theta_i)^2.
\]

The benefit of the deviation for an agent \(i\) with type \(\theta_i\) can thus be computed as:

\[
v^d_i(\theta_i, \theta_{-i}) = -\frac{1}{2}(q(\hat{\theta}_i^*, \hat{\theta}_{-i}^*) - \theta_i)^2.
\] (5)

Given the symmetry of the model, we are looking for grand-mechanisms \(q(\cdot)\) which are themselves symmetric so that \(v^d_i(\cdot)\) will not depend on the deviating agent’s identity \(i\) and we will omit indices accordingly.

Instead, for a weak coalition, there is asymmetric information within the coalition. The non-deviating agent \(-i\) must thus form out-of-equilibrium beliefs on agent \(i\)’s type when contemplating the latter’s deviation while the deviating group still holds prior beliefs

\textsuperscript{32}Hula (2000).

\textsuperscript{33}Sometimes, even long term players may be reluctant to share information. Lauman and Knoke (1987) reported the example of the petroleum industry trade associations which strongly opposed the announcement of the Federal Aviation Administration to execute new regulations requiring detailed flight plans to be filed by pilots of non-commercial aircrafts since that would make public the strategic data of their aerial explorations that are worth millions of dollars.
on agent \(-i\) since the latter has not deviated. Given that pair of beliefs, both groups now play a Bayesian-Nash equilibrium. To simplify that step of analysis, we will follow Laffont and Martimort (1997, 2000) and consider passive beliefs, i.e., the non-deviating agent does not change his beliefs on the deviating agent and still keep the uniform prior on \([0, 1]\). A Bayesian-Nash (symmetric) equilibrium strategy \(\hat{\theta}^*(\theta_i)\) satisfies thus:

\[
\hat{\theta}^*(\theta_i) \in \arg \min_{\hat{\theta}_i} E_{\theta_{-i}}(q(\hat{\theta}_i, \hat{\theta}^*(\theta_{-i})) - \theta_i)^2.
\]

The benefit of the deviation for an agent \(i\) with type \(\theta_i\) can thus be computed as:

\[
v^d(\theta_i) = E_{\theta_{-i}} \left\{ \frac{1}{2} (q(\hat{\theta}^*(\theta_i), \hat{\theta}^*(\theta_{-i})) - \theta_i)^2 \right\}.
\] (6)

5 Strong Coalitions

By assuming credible information sharing in a strong coalition, we get an upper bound of the benefits that any coalition of interest groups could withdraw. This is important in view of our comparative analysis of organizational forms. Whenever competition between interest groups increases the groups’ payoffs with respect to such strong norm of collusion, it will also be the case if the coalitional behavior of those groups is not as perfect as it is assumed in a strong coalition.

With a strong norm of collusive behavior, recall that collusion takes place under complete information between the colluding interest groups. The optimal side-mechanism must thus solve the following problem:

\[
(TP^{ac}) : \max_{\{\phi(\theta_1, \theta_2), t_1(\theta_1, \theta_2), t_2(\theta_1, \theta_2)\}} \sum_{i=1}^{2} t_i(\theta_1, \theta_2)
\]

subject to

\[
U_i(\theta_i, \theta_{-i}) \geq v^d(\theta_i, \theta_{-i}), \quad i = 1, 2, \forall (\theta_i, \theta_{-i}) \in \Theta^2,
\] (7)

where \(v^d(\theta_i, \theta_{-i})\) satisfies (6) and the interest group \(i\)'s utility can be written as:

\[
U_i(\theta_i, \theta_{-i}) = -\frac{1}{2} (q(\phi(\theta_1, \theta_2)) - \theta_i)^2 - t_i(\theta_i, \theta_{-i}).
\] (8)

Under complete information the best collusive manipulation \(\phi^*(\theta_1, \theta_2)\) that can be achieved by the interest groups does not depend on the reservation payoffs obtained following a deviation. This manipulation minimizes point-wise the following expression:

\[
\frac{1}{2} \sum_{i=1}^{2} (q(\phi) - \theta_i)^2 = \left( q(\phi) - \frac{\theta_1 + \theta_2}{2} \right)^2 + \frac{(\theta_2 - \theta_1)^2}{4}.
\] (9)

\[34\] Assuming one such symmetric equilibrium exists as it will be shown later.
Up to some terms which do not depend on the policy chosen and thus cannot be screened by the grand-mechanism, the third-party’s payoff only depends on the agents’ average ideal point. When communicating with the uninformed principal, the coalition behaves as a single consolidated interest group having a type \( \theta = \frac{1}{2}(\theta_1 + \theta_2) \) now drawn from the distribution \( G(\cdot) \) of the “average” between two independent variables uniformly distributed on \([0, 1]\). The corresponding distribution \( G(\cdot) \) is:

\[
G(\theta) = \begin{cases} 
2\theta^2 & \text{if } 0 \leq \theta \leq \frac{1}{2} \\
1 - 2(1 - \theta)^2 & \text{if } 1 \geq \theta \geq \frac{1}{2},
\end{cases}
\]

which admits the density \( g(\cdot) \)

\[
g(\theta) = \begin{cases} 
4\theta & \text{if } 0 \leq \theta \leq \frac{1}{2} \\
4 - 4\theta & \text{if } 1 \geq \theta \geq \frac{1}{2}.
\end{cases}
\]

Compared to a uniform distribution, \( G(\cdot) \) shifts more weight around the same mean. One may already guess that this certainly reduces the need for a strong coalition to communicate with the principal. In other words, a commitment by the legislature to a rigid policy close to the expected average ideal point, namely \( \frac{1}{2} \), is likely to perform relatively well given that the inverse U-shape distribution puts quite a bit of mass around that point.

To confirm that insight, let us analyze the optimal grand-mechanism for a strong coalition. Of course, a version of the Revelation Principle still holds in our context although incentive constraints for truthtelling must be adapted to take into account the interest groups’ collusive behavior. Again, focusing on deterministic mechanisms, there is no loss of generality in restricting the analysis to direct mechanisms which satisfy coalition incentive constraints, i.e., such that the third-party adopts a truthtelling strategy \((\hat{\theta}_1, \hat{\theta}_2) = (\theta_1, \theta_2)\). Such a grand-mechanism is said to be collusion-proof.\(^{35}\)

The corresponding strong coalitional incentive constraints can be written as:

\[
(\theta_1, \theta_2) \in \arg \min_{(\tilde{\theta}_1, \tilde{\theta}_2)} (q(\tilde{\theta}_1, \tilde{\theta}_2) - \theta)^2. \tag{10}
\]

We can easily prove that \( q(\cdot) \) depends only on the average ideal point \( \theta \) and we will slightly abuse notations by writing \( q(\theta) \). To do so, observe that all pairs \((\theta_1, \theta_2)\) with the same mean \( \theta = \frac{1}{2}(\theta_1 + \theta_2) \) should, from (10), correspond to the same policy. Looking at the coalition incentive compatibility off such diagonal, we immediately obtain that \( q(\cdot) \) is monotonically increasing and thus almost everywhere differentiable in \( \theta \).

\(^{35}\)Our notion of collusion-proofness is somewhat weaker than that developed in Laffont and Martimort (1997, 2000). Indeed grand-mechanisms do not specify transfers between interest groups and thus cannot replicate what can be done with side-mechanisms employing such transfers. Nevertheless grand-mechanisms can be designed in such a way that reports are not manipulable by a strong coalition.
At any point of differentiability, the strong coalition incentive constraint becomes:

\[ q'(\theta)(q(\theta) - \theta) = 0, \quad \forall \theta \in \Theta. \]

(11)

Hence, \( q(\cdot) \) is either locally constant along any diagonal \( \theta = \frac{1}{2}(\theta_1 + \theta_2) \) or equal to the third-party’s ideal point which is an average over the groups’ own ideal points. This leads to the following characterization of the strong collusion-proof mechanisms.\(^{36}\)

**Lemma 2**: Any strong collusion-proof continuous mechanism \( q(\cdot) \) depends only on the average ideal points of the interest groups, namely \( \theta = \frac{1}{2}(\theta_1 + \theta_2) \). Such mechanism is characterized by two cut-offs \( \theta^* \) and \( \theta^{**} \) with \( \theta^* \leq \theta^{**} \):

\[
q(\theta) = \min\{\theta^{**}, \max\{\theta, \theta^*\}\}.
\]

(12)

In other words, everything happens as though the legislature was dealing only with the third-party and either let it choose its most preferred point within the range \([\theta^*, \theta^{**}]\) or imposes a rigid policy either at \( \theta^* \) or \( \theta^{**} \).

**Proposition 2**: The optimal strong collusion-proof continuous mechanism is characterized by a unique cut-off \( \theta^*_{sc}(\delta) \) satisfying for \( \delta \leq \frac{1}{2} \):

\[
\int_0^{\theta^*_{sc}(\delta)} (\theta^*_{sc}(\delta) - \theta - \delta)g(\theta)d\theta = 0.
\]

(13)

i) For \( \delta \leq \frac{1}{6} \), \( \theta^*_{sc}(\delta) = 3\delta \),

ii) For \( \frac{1}{6} \leq \delta \leq \frac{1}{2} \), \( \theta^*_{sc}(\delta) \) solves

\[
(\theta^*_{sc}(\delta) - \delta)(12(\theta^*_{sc}(\delta))^2 - 24\theta^*_{sc}(\delta) + 6) + 12(\theta^*_{sc}(\delta))^2 - 8(\theta^*_{sc}(\delta))^3 - 1 = 0,
\]

(14)

iii) For \( \delta \geq \frac{1}{2} \), the optimal policy is fully rigid and equal to \( q^p \).

From (12), it should be clear that the optimal strong collusion-proof mechanism entails only a lower bound on the possible policies at a threshold \( \theta^* \). Such lower bound ensures that the incentives of the third-party for understating the average preferences of the member groups no longer matter if this average is small enough. On the other hand, introducing an upper bound on the set of possible policies can only increase the distance between the principal’s ideal point and that of a strong coalition in case \( \theta \) is large enough. This is not optimal from the legislature’s viewpoint.

\(^{36}\)See Martimort and Semenov (2006) for such characterization.
Proposition 2 already suggests a basic tension that drives the formation of a strong coalition. On one hand, by dealing with such a coalition, communication with the legislature becomes less essential and the optimal policy comes close to what would be obtained with an ex ante commitment to a rigid rule. On the other hand, and from an ex-ante viewpoint, interest groups may be reluctant to form such a coalition. Contrary to what occurs when groups compete, the respective ideal points of each group are now never chosen because either communication has less value for the legislature or, when communication leads to a flexible policy, this is only an aggregate of the preferences of both interest groups which is used to determine that policy.

6 Weak Coalitions

Interest groups in a weak coalition can no longer credibly share information among themselves. Any side-mechanism thus has also be incentive compatible. Asymmetric information may significantly undermine the efficiency of such collusive agreements. One important question is to know whether collusion might still help the principal even though a weak collusion itself suffers from asymmetric information problems.

With a weak norm of collusive behavior, collusion takes place under asymmetric information. Relying on Bayesian incentive compatibility as the implementation concept within the coalition, a side-mechanism must satisfy the following Bayesian incentive constraints:

\[
\theta_i \in \arg \max_{\hat{\theta}_i} E_{\theta_{-i}} \left\{ -\frac{1}{2} (q(\phi(\hat{\theta}_i, \theta_{-i})) - \theta_i)^2 - t_i(\hat{\theta}_i, \theta_{-i}) \right\}, \quad i = 1, 2, \quad \forall (\theta_i, \hat{\theta}_i, \theta_{-i}) \in \Theta^2. \tag{15}
\]

From (15), we can easily prove that \( E_{\theta_{-i}}(q(\phi(\theta_i, \theta_{-i})) \) is monotonically increasing in each of its arguments and thus almost everywhere differentiable in \( \theta_i \).

To stress the trade-off between efficiency of a collective manipulation of reports and rent extraction of the interest groups’ information rent within a weak coalition, we express incentive constraints in terms of the interim utility \( U_i(\theta_i) \) of a group with type \( \theta_i \):

\[
U_i(\theta_i) = \max_{\hat{\theta}_i} E_{\theta_{-i}} \left\{ -\frac{1}{2} (q(\phi(\hat{\theta}_i, \theta_{-i})) - \theta_i)^2 - t_i(\hat{\theta}_i, \theta_{-i}) \right\}. \tag{16}
\]

Using (16) and the Envelope Theorem, we obtain

\[
\dot{U}_i(\theta_i) = E_{\theta_{-i}} (q(\phi(\theta_i, \theta_{-i})) - \theta_i), \quad i = 1, 2, \quad \forall (\theta_i, \theta_{-i}) \in \Theta^2. \tag{17}
\]

\[37\]This second-order condition can easily be checked ex-post on the optimal weak collusion-proof mechanism.
To induce acceptance of the collusive agreement, it must be that

\[ U_i(\theta_i) \geq v^d(\theta_i), \quad i = 1, 2, \forall (\theta_i, \theta_{-i}) \in \Theta^2, \]  

where \( v^d(\theta_i) \) is defined in (6).

The third-party’s problem can thus be rewritten as:

\[
(TP_{wc}) : \min_{\{\phi(\cdot), U_1(\cdot), U_2(\cdot)\}} E_{\theta_1, \theta_2} \left\{ \left( q(\phi(\theta_1, \theta_2)) - \frac{\theta_1 + \theta_2}{2} \right)^2 + \frac{(\theta_2 - \theta_1)^2}{4} \right\} + \sum_{i=1}^{2} E_{\theta_i} U_i(\theta_i)
\]

subject to (17) and (18).

Problem \((TP_{wc})\) is a priori complex because of the type-dependent participation constraint (18) and the difficulty in knowing a priori where this constraint actually binds. Moreover, another difficulty is that \((TP_{wc})\) itself depends on the grand-mechanism chosen by the principal. The legislature may offer a mechanism with an eye on how information undermines the efficiency of a side-mechanism for a weak coalition. To simplify the analysis, we will focus on grand-mechanisms such that the ex-post participation constraint (18) binds at \( \theta = 0.38 \). We can then write the interest groups’ information rent as:

\[ U_i(\theta_i) = v^d(0) + \int_0^{\theta_i} E_{\theta_{-i}} (q(\phi(x, \theta_{-i})) - x) \, dx. \]  

Integrating by parts in the maximand of \((TP_{wc})\) yields an objective of the third-party which is an expectation of terms writing as:

\[
\left( q(\phi) - \frac{\theta_1 + \theta_2}{2} \right)^2 + \sum_{i=1}^{2} (1 - \theta_i)(q(\phi) - \theta_i).
\]

Of course, the optimal manipulation function \( \phi^*(\cdot) \) must be such that (18) holds, which requires:

\[
\int_0^{\theta_i} E_{\theta_{-i}} (q(\phi^*(x, \theta_{-i})) - x) \, dx \geq v^d(\theta_i) - v^d(0) \quad \forall \theta_i.
\]

We will neglect this constraint and check ex-post that it holds with the optimal weak collusion-proof mechanism found below.

As before, a version of the Revelation Principle still applies and there is no loss of generality in restricting the analysis to mechanisms (again focusing on deterministic ones) such that the optimal manipulation of reports obtained by point-wise optimization is

38 We characterize in the Appendix the polar class of mechanisms where the participation constraint (7) binds instead at \( \theta = 1 \). As we will prove there, such collusion-proof grand-mechanisms imply full pooling. This gives some motivation for focusing on those grand-mechanisms such that the participation constraint (7) binds at \( \theta = 0 \) as far as flexible policies are concerned.

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truthful. This leads us to express the following \textit{weak coalition incentive compatibility constraints}:

\[
(\theta_1, \theta_2) = \arg\min_{(\theta_1, \theta_2)} (q(\tilde{\theta}_1, \tilde{\theta}_2) - \theta)^2 + \sum_{i=1}^{2} (1 - \theta_i)(q(\tilde{\theta}_1, \tilde{\theta}_2) - \theta_i). \tag{22}
\]

From (22), we can again prove that \( q(\cdot) \) depends only on the average ideal points \( \theta \). Moreover, \( q(\cdot) \) is monotonically increasing and thus almost everywhere differentiable in \( \theta \). At any point of differentiability, the weak coalition incentive constraints can thus be written as:

\[
\dot{q}(\theta)(q(\theta) - 2\theta + 1) = 0, \quad \forall \theta \in \Theta. \tag{23}
\]

Hence, \( q(\cdot) \) is either locally constant along the diagonal \( \theta = \frac{1}{2}(\theta_1 + \theta_2) \) or equal to the third-party’s \textit{virtual ideal point} \( 2\theta - 1 \).

When the participation constraint (7) only binds at \( \theta = 0 \), interest groups have some incentives to understate their ideal points. By doing so, they signal a lower willingness to pay for the broker’s services in organizing collusion. To reduce these incentives to understate types, this third-party commits to a side-mechanism inducing a downward distortion of the optimal manipulation of reports away from the efficient one that would be chosen in a strong coalition. Instead of maximizing the sum of the interest groups’ payoffs, the optimal manipulation now trades off internal efficiency within the coalition against the costly information rents which must be left to interest groups with the highest ideal points who have less incentives to renege on collusion. Everything happens as though, within a weak coalition, the ideal points \( \theta_i \) were now replaced by \textit{virtual ideal points} \( 2\theta_i - 1 \) which are necessarily lower to capture the downward bias of a weak coalition. This leads to the following characterization of the \textit{weak collusion-proof mechanisms}.

\textbf{Lemma 3}: \textit{Any weak collusion-proof and continuous mechanism} \( q(\cdot) \) \textit{such that the participation constraint (7) binds at} \( \theta = 0 \) \textit{and satisfying}

\[
\int_0^{\theta_i} E_{\theta_{-i}}(q(x, \theta_{-i}) - x)dx \geq v_d(\theta_i) - v_d(0) \quad \forall \theta_i, \tag{24}
\]

\textit{depends only on the average ideal points of the interest groups} \( \theta = \frac{1}{2}(\theta_1 + \theta_2) \). \textit{This mechanism can be characterized by two cut-offs} \( \theta^* \) \textit{and} \( \theta^{**} \) \textit{with} \( \theta^* \leq \theta^{**} \) \textit{such that}:

\[
q(\theta) = \min\{\theta^{**}, \max\{2\theta - 1, \theta^*\}\}. \tag{25}
\]

With such a mechanism, the range of possible policies is still \([\theta^*, \theta^{**}]\) exactly as with a strong collusion-proof one having the same cut-offs as in (12). The difference comes from
the fact that, under a weak coalition, a flexible policy is more sensitive to a change in the average preferences of the member groups whereas a strong collusion-proof mechanism corresponds to a policy closer to the legislature’s ideal point. There is less congruence with a weak coalition than with its strong counterpart.

**Proposition 3**: i) The optimal weak collusion-proof continuous mechanism is characterized by an upper bound on policy \( \theta^*_{wc}(\delta) = 1 \) and a lower bound \( \theta^*_{wc}(\delta) \) satisfying for \( \delta \leq \frac{1}{2} \):

\[
\int_0^{\theta^*_{wc}(\delta)+1} \frac{1}{2} (\theta^*_{wc}(\delta) - \theta - \delta) g(\theta) d\theta = 0.
\]

The set of types corresponding to a flexible policy for a weak coalition is included into the set of types corresponding to a flexible policy for a strong coalition:

\[
\frac{\theta^*_{wc}(\delta) + 1}{2} > \theta^*_{sc}(\delta).
\]

Moreover, \( \theta^*_{wc}(0) > 0 \), i.e, even with no a priori conflict of interests, the optimal policy with a weak coalition entails some rigidity.

ii) For \( \delta \geq \frac{1}{2} \), the optimal policy is fully rigid and equal to \( q^p \).

Since virtual ideal points replace true ideal points to assess the preferences of a weak coalition, there exists an extra bias between the principal and that coalition. Even if there is a priori no conflict of interests between the principal and the coalition \( (\delta = 0) \) asymmetric information within the coalition introduces such a conflict and precludes a fully flexible policy rule. More generally, the exacerbated conflict of interests between the legislature and a weak coalition calls for more rigid rules than with a strong coalition. The difficulty of communicating information on preferences internally within the weak coalition also makes a rigid policy more likely.

### 7 Optimal Organization of Lobbying

We now compare the outcomes achieved when interest groups either compete or adopt more collusive behavior. We want to stress the costs and benefits of each organizational form. To do so, we will use two criteria. The first one is the legislature’s expected payoff under the various organizational forms. This will give us some insights on the social incentives for either having competition or collusion between groups. The second criterion consists in comparing the interest groups’ expected payoff when they compete with what they get together with the third-party organizing their coalition if a more
collusive behavior is adopted. This approach will instead stress the private incentives for competing or colluding.

As a preliminary remark, Propositions 1 and 2 show that optimal policies entail some flexibility only when \( \delta \leq \frac{1}{2} \) whether interest groups compete or instead collude in a strong coalition. Of course, this does not mean that the legislature’s expected payoffs in both cases are the same since the degree of flexibility is different. For \( \delta \geq \frac{1}{2} \), a rigid policy is chosen for any organizational form and thus all such forms are payoff-equivalent.

**Proposition 4**: The following rankings in payoffs hold:

- There exists \( \delta^* < \frac{1}{2} \) such that a strong coalition dominates competition from a social point of view if and only if \( \delta \in [\delta^*, \frac{1}{2}] \);
- There exists \( \delta^{**} \in (\delta^*, \frac{1}{2}) \) such that a strong coalition dominates competition from a private viewpoint if and only if \( \delta \in [\delta^{**}, \frac{1}{2}] \).

The main idea behind this proposition is to compare the optimal mechanism for a strong coalition with a dominant strategy mechanism of the kind described in (4) dealing with both groups separately. To do so, the following figure is useful.

![Figure 1: The case \( \delta \in [\frac{1}{5}, \frac{1}{2}] \).](image)

On Figure 1, we have drawn the downward sloping line \( \frac{\theta_1 + \theta_2}{2} = \theta_{sc}(\delta) \) which separates the areas \( A + B \) where a strong coalition receives a pooling policy from the area \( C \) where
the ideal point of this coalition is implemented. On $C$, isopolicy lines are $45^\circ$ downward sloping lines. They do not correspond to any of the interest groups’ ideal points except of course when both interest groups have the same ideal points.

Consider now a dominant strategy mechanism of type (4) having only one floor policy $\theta^* = \theta_{sc}(\delta)$. On area $A$, this mechanism is pooling and yields to the principal the same expected payoff as the optimal strong collusion-proof mechanism since the policy chosen is the same in both cases.

On area $B$, dealing separately with non-cooperating interest groups allows us to implement a policy which depends on the preferences of the group who has the highest ideal point. Screening, although imperfect and biased towards one group, is possible, whereas dealing with a coalition would still entail a fully rigid policy. Everything happens thus as though an endogenous bias of the decision process towards the interest group whose ideal point is the closest to that of the principal appears. Instead, the mechanism dealing with a strong coalition is unable to account for this endogenous bias.

On area $C$, dealing with a coalition allows a more efficient communication with the principal since only the statistic $\frac{\theta_1 + \theta_2}{2}$ is relevant for decision-making and isopolicy lines cannot depend on this statistic with dominant strategy. However, since the density of the average type $\frac{\theta_1 + \theta_2}{2}$ decreases over $[\theta_{sc}(\delta), 1]$, this potential benefit of a coalition is not sufficiently strong to offset the cost of an excessive pooling when the interest groups’ preferences are sufficiently far apart.

To give further intuition on the benefits of competition, let us consider the case of a sufficiently strong conflict of interests. Of course for $\delta = \frac{1}{2}$ there is full pooling with both organizations reaching the same welfare. Assume now that $\delta = \frac{1}{2} - \varepsilon$ for $\varepsilon$ small enough. With a strong coalition, $\theta_{sc}^*(\delta)$ is close to 1 and the optimal policy is almost full pooling. The area $C$ where the policy is flexible is then a small triangle having an area of order $\varepsilon^2$. The gain of dealing with a strong coalition rather than imposing pooling everywhere is thus of order $\varepsilon^2$. Instead, by having competing interest groups, the principal can screen preferences on areas $B + C$ and obtain a gain of order $\varepsilon$ everywhere. This dimensionality argument underscores the benefit of competition at least if the conflict of interests between the legislature and the groups is sufficiently pronounced.

For small values of $\delta$ instead, the $45^\circ$ degree line $\frac{\theta_1 + \theta_2}{2} = \theta_{sc}^*(\delta)$ lies on the very southwest of the $[0, 1] \times [0, 1]$ square. The policy with a strong coalition is very flexible and the communicated statistic perfectly fits with what is needed by the legislature to choose the policy. Clearly, dealing with such coalition now dominates. When $\delta$ is close to zero, there is full congruence between the legislature and a strong coalition with the optimal

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39 The mechanism on Figure 1 is a dominant strategy mechanism but is not optimal. The optimal dominant strategy mechanism will perform better.
policy being fully flexible. Relying on competition is clearly suboptimal. Indeed, under competition, isopolicy lines are never aligned with those of the legislature contrary to what happens with a strong coalition.

Proposition 4 shows that interest groups also gain from being split for sufficiently large $\delta$. Indeed, when they remain separated, there is a positive probability that the optimal policy fits their ideal points. This benefit is sufficiently strong to dominate the cost of having the other interest group’s ideal point being implemented with positive probability when the latter has sufficiently pronounced preferences. Instead, the average policy that would be chosen with a strong coalition never coincides with the ideal points of either agent except, with probability zero, when these ideal points are equal and above $\theta^*_c(\delta)$.

When instead $\delta$ is small enough, the strong coalition’s objectives are perfectly aligned with those of the principal and the optimal policy is fully flexible. Under competition instead, the optimal policy entails a cap $\theta^*_c(\delta)$ close to $\frac{1}{2}$ and thus much pooling. The principal and the interest groups all gain from having a strong coalition. Thus one should expect less (respectively more) interest groups and more (respectively less) collusion in environments with little (respectively significant) conflicts since communication is less costly there. To a large extent, the social and private incentives to form a strong coalition are aligned.

Our previous results showed that a strong coalition may be preferred by the principal since it may be the best way to reduce the conflict of interests with interest groups. At first glance, one might conjecture that a weak coalition could still help the principal, because it might somewhat help coordination between the interest groups. The next proposition shows that it is actually not the case.

**Proposition 5**: i) The legislature’s expected payoff with a weak coalition is always strictly lower than with a strong coalition.

   ii) Competition dominates a weak coalition both from a social and a private viewpoint.

Proposition 5 shows that asymmetric information within the coalition always hurts the principal. The principal prefers that interest groups credibly share information within a strong coalition since the distance between the principal’s ideal point and that of such coalition is thereby reduced. Under asymmetric information, this distance increases because one must add to the preferences bias $\delta$ some extra terms coming from the fact that the virtual ideal point of a weak coalition is distorted downwards and further apart from the legislature’s preferences than if a strong coalition would form.

As a result of Propositions 4 and 5, either competition dominates any coalitional form
from a social and a private viewpoint (for large conflicts of interests) or a strong coalition dominates competition (and a weak coalition) from a social and a private perspective, in the case of low conflicts of interests.

8 Conclusion and Directions for Future Research

Interest groups may adopt various kinds of behavior in day-to-day legislative politics using either very competitive or more collusive strategies. This paper has investigated the consequences of different organizations of lobbying in relation to the pattern of communication and the degree of flexibility of optimal policies that respond to those organizations. Our main message is that strong coalitions with credible information sharing among member groups are both socially and privately optimal when their preferences are congruent with the rest of society. Weak coalitions are instead always suboptimal both from a private and a social viewpoint. This points to the key role of core players who have long-term interests at stake in organizing coalitional behavior in the political arena. When conflicts are instead exacerbated, a competitive playing field is both socially and privately preferred.

Concerning the robustness of our results, it could be useful to generalize our model to account for other functional forms for the groups’ utility functions although quadratic preferences are good approximations for most single-peaked functions. The same analysis would characterize the competition case: At most one group’s ideal point might be chosen. Regarding the analysis of strong coalitions, the third-party would still represent the joint interest of member groups although isopolicy lines would be downward sloping curves but generally no longer along the diagonal. It is rather clear that our intuitive argument justifying why competition dominates a strong coalition in the case of large conflicts of interests would still go through: Starting from the minimal level of conflict inducing full pooling with a strong coalition and reducing by a small amount $\epsilon$ this level of conflict, the optimal policy with a strong coalition is flexible only for a set of size $\epsilon^2$ in the preferences space whereas screening occurs on a set of size $\epsilon$ if competition is chosen. This argument is also robust to changes in the distribution of preferences as long as densities are smooth and positive everywhere.

Our analysis might also be worth generalizing to the case where interest groups have different biases with the decision-maker. As long as groups have preferences which are at least ex-ante sufficiently close even though they may differ ex-post, we expect our results to carry over. However, we conjecture that competition dominates when groups have asymmetric biases with the legislature but the optimal mechanism is certainly no longer symmetric. Indeed, following the general thrust of the paper, the best organizational form should make the legislature’s choice more congruent with those of the groups. This can
certainly be so when one group has preferences which are close to those of the legislature by communicating only with that group in an asymmetric dominant strategy mechanism. Coalitional behavior is thus less likely with asymmetric biases. This also suggests that competition certainly dominates when groups have opposite biases, one preferring “less” policy than the socially optimal one whereas the other prefers “more”.

Other robustness checks should address the choice of the information structure by, for instance, allowing some correlation between the various interest groups’ ideal points. When the groups’ ideal points are highly correlated, there is little value of competition under dominant strategy and dealing with a strong coalition certainly dominates. When the groups’ ideal points are rather negatively correlated instead, it might be rather inefficient to deal with a coalition since not much can be learned from its reports on joint interests. Finally, a particular information structure which has received much attention in the literature has interest groups receive signals on an underlying state of nature which affects the choice of an optimal policy and those signals are conditionally independent. We feel relatively confident that our results would generalize to this case, but this of course should be investigated more thoroughly.

Our modelling of the formation of coalitions gave us a clear characterization of collusion-proof grand-mechanisms. However, it somewhat short-cuts a number of issues which arise when the collusive side-mechanism is offered by one informed interest group himself: a “core player” (in the vocabulary of the political science literature) willing to attract other member groups in his own coalition. Informational leakages towards other groups coming from the mere signalling by that core player of his incentives to form a coalition might be worth studying. More generally, our welfare comparison between competition and collusion has taken an ex-ante viewpoint but one could also be interested in determining what kinds of interest groups are more eager to coalesce or to compete and the dynamic pattern in which groups join on-going coalitions.

One might find some tension between two assumptions we made in our analysis, namely allowing side-transfers between colluding groups and not allowing side-transfers between those groups and the legislature. Although we have justified this assumption above, it might be worth investigating how our results could be modified in the case where transfers between the legislature and the groups are allowed. The same kind of issues has been investigated in an I.O. context where a downstream firm wants to build a network of its suppliers and use monetary transfers to do so. Baron and Besanko (1992, 1999), Dana (1993), Gilbert and Riordan (1995), Laffont and Martimort (1998), Mookherjee and Tsugamari (2004) and Dequiedt and Martimort (2004) (among many others) have found conditions on preferences and information under which dealing with a strong coalition of

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40 This is also a case where Bayesian implementation might be quite useful.
suppliers dominates both a decentralized mechanism where suppliers keep on competing and a weak coalition of suppliers. In the framework where transfers between the principal and agents are feasible, this literature shows that such strong coalition may help groups to internalize informational externalities. This suggests that the benefits of strong coalitions could be greater if the legislature was allowed to target groups with monetary transfers.

Returning to our initial context where transfers are not allowed between the legislature and groups, the organization of lobbying that emerges might also depend on the specification of the policy-maker’s objective function and it may be worth studying to which extent it is so. We have assumed that the principal is a social welfare maximizer choosing an average between the interest groups’ ideal points and that of the general public. Instead, they could choose a policy corresponding to the median of the public and the interest groups’ ideal points, or a policy which might result from some unspecified legislative bargaining among representatives of different constituencies. The point here is that the social choice process that leads to a particular policy might have some consequences on the macro-organization of interest groups which are worth studying.\footnote{Huntington (1975) and McCool (1989) have noticed that changes in the government structure and, more precisely, its higher level of decentralization may have led towards an increasingly atomistic organization of interest groups. This observation suggests the importance of the public choice process on the extent to which interest groups coalesce.}

We hope to investigate some of these issues in future research.

References


**Appendix**

- **Proof of Lemma 1**: From Melumad and Shibano (1991) and Martimort and Semenov (2006), we can remind the characterization of one-dimensional mechanisms in a framework with a single privately informed agent:
Lemma 4: Any uni-dimensional incentive compatible continuous deterministic mechanism \( \tilde{q}(\cdot) \) must satisfy
\[
(\tilde{q}(\theta) - \theta) \hat{q}(\theta) = 0,
\]
and is of the form
\[
\tilde{q}(\theta) = \min\{\theta^*, \max\{\theta, \theta^*\}\},
\]
where \( \theta^* \) and \( \theta^{**} \) are two thresholds such that \( \theta^* \leq \theta^{**} \).

Take now an arbitrary continuous dominant strategy mechanism \( q(\theta_1, \theta_2) \) in a setting with two interest groups acting non-cooperatively. Fix \( \theta_2 \) and consider the one-dimensional scheme \( \tilde{q}(\theta_1) = q(\theta_1, \theta_2) \). From Lemma 4, this mechanism can be written as:
\[
q(\theta_1, \theta_2) = \tilde{q}(\theta_1) = \min\{\theta^{(2)}(\theta_2), \max\{\theta_1, \theta^{(1)}(\theta_2)\}\},
\]
where \( \theta^{(2)}(\theta_2) \geq \theta^{(1)}(\theta_2) \quad \forall \theta_2 \).

Again from Lemma 4, we can compute \( \theta^{(1)}(\theta_2) \) and \( \theta^{(2)}(\theta_2) \) and get
\[
\theta^{(1)}(\theta_2) = \min\{\theta^{**}, \max\{\theta_2, \theta^*\}\}
\]
for some thresholds \( \theta_2^{**} \) and \( \theta^* \) such that \( \theta^* \leq \theta_2^{**} \), and
\[
\theta^{(2)}(\theta_2) = \min\{\theta^{***}, \max\{\theta_2, \theta_2^{**}\}\}
\]
for some thresholds \( \theta_2^{**} \) and \( \theta^{***} \) such that \( \theta_2^{**} \leq \theta^{***} \). Moreover, \( \theta^{(2)}(\theta_2) \geq \theta^{(1)}(\theta_2) \quad \forall \theta_2 \)
implies also that \( \theta^* \leq \theta_2^{**} \) and \( \theta_2^{**} \leq \theta^{***} \).

Coming back to (A3) and observing that \( \max\{x, \min\{y, z\}\} = \min\{\max\{x, y\}, \max\{x, z\}\} \), we obtain:
\[
q(\theta_1, \theta_2) = \min\{\theta^{***}, \max\{\theta_1, \theta_2^{**}\}, \max\{\theta_2, \theta_2^{**}\}, \max\{\theta_1, \theta_2, \theta^*\}\},
\]
which yields (3) for a symmetric mechanism such that \( \theta_1^{**} = \theta_2^{**} = \theta^{**} \).

Proof of Proposition 1: We first prove that the optimal grand-mechanism with non-cooperating interest groups is indeed of the form given in (4), i.e., it has only two thresholds. Denote by \( V_d(\theta^*, \theta^{**}, \theta^{***}) \) the legislature's expected payoff with a dominant strategy incentive compatible scheme of the form (3) with \( \theta^{***} \leq 1 \). Using symmetry and taking into account equation equation (3), we get:
\[
V_d(\theta^*, \theta^{**}, \theta^{***}) = \\
\left. - \int_0^{\theta^*} \left( \int_0^{\theta_1} \left( \theta^* - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 d\theta_2 \right) d\theta_1 \right|_{\tilde{q}(\theta_1, \theta_2) = \theta^*} - \int_0^{\theta^{**}} \left( \int_0^{\theta_1} \left( \theta^* - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 d\theta_2 \right) d\theta_1 \right|_{\tilde{q}(\theta_1, \theta_2) = \theta_1},
\]

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Diﬀerentiating with respect to \( V \) which is below the line us to get rid of the factor denoting then by two thresholds suﬃce to compute an optimal mechanism. Abusing slightly notations, therefore having

\[
\begin{align*}
\text{Proof of Proposition 2:} \quad & \text{Proof of Lemma 2:} \quad \\
\text{proof continuous mechanisms. Denote by } V_d(\theta^*, \theta^{**}) \text{ the principal’s expected payoff with a mechanism of the form (4) and using the symmetry along the diagonal, we have:}^{43} \\
V_d(\theta^*, \theta^{**}) &= - \int_0^{\theta^*} \int_0^{\theta_1} \left( \theta^* - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 d\theta_2 d\theta_1 - \int_0^{\theta^{**}} \int_0^{\theta_1} \left( \theta_1 - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 d\theta_2 d\theta_1 \\
& \quad - \int_0^{\theta^{**}} \int_0^{\theta_2} \left( \theta^{**} - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 d\theta_2 d\theta_1 - \int_0^{\theta^{**}} \int_0^{\theta_1} \left( \theta_2 - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 d\theta_2 d\theta_1. \quad (A5)
\end{align*}
\]

Optimizing then (A5) with respect to \( \theta^* \) and \( \theta^{**} \) yields the results in the text. For \( \delta \geq \frac{1}{2} \), it is optimal to set \( \theta^* = \theta^{**} = \theta^{***} = q^\rho \) as it can be easily seen.

\[ \begin{align*}
\partial V_d \partial \theta^{***}(\theta^*, \theta^{**}, \theta^{***}) &= \frac{1}{2} \left( 1 - \theta^{***} \right)^2 \left( 1 + 2\delta - \theta^{***} \right) \geq 0. \\
\text{Therefore, having } \theta^{***} = 1 \text{ is optimal and dominant strategy grand-mechanisms with only two thresholds suﬃce to compute an optimal mechanism. Abusing slightly notations, denoting then by } V_d(\theta^*, \theta^{**}) \text{ the principal’s expected payoff with a mechanism of the form (4) and using the symmetry along the diagonal, we have:}^{43}\\n\text{\textbullet \ Proof of Lemma 2: The proof is a straightforward consequence of Lemma 4.}^{43}\\n\text{\textbullet \ Proof of Proposition 2: From Lemma 2, we know the structure of the strong collusion-proof continuous mechanisms. Denote by } V_{sc}(\theta^*, \theta^{**}) \text{ the principal’s expected payoff with a strong collusion-proof mechanism of the form (12), we have:}^{43}\\nV_{sc}(\theta^*, \theta^{**}) &= -\frac{1}{2} \left( \int_0^{\theta^*} (\theta^* - \theta - \delta)^2 g(\theta) d\theta + \delta^2 \int_{\theta^*}^{\theta^{**}} g(\theta) d\theta + \int_{\theta^{**}}^{1} (\theta^{**} - \theta - \delta)^2 g(\theta) d\theta \right). \quad (A6)\\n\text{Differentiating with respect to } \theta^{**} \text{ yields:} \\
\partial V_{sc} \partial \theta^{***}(\theta^*, \theta^{**}) &= -\int_{\theta^{**}}^{1} (\theta^{**} - \theta - \delta) g(\theta) d\theta > 0.
\end{align*} \]

^{43}Note that in the expression for \( V_d(\theta^*, \theta^{**}) \) we use only the part of dominant strategy mechanism which is below the line \( \theta_2 = \theta_1 \). The part which is above this line is the same by symmetry. This allows us to get rid of the factor \( \frac{1}{2} \).
Therefore, it is optimal to always set $\theta_{sc}^* = 1$. Optimizing with respect to $\theta^*$ yields then the necessary and sufficient first-order condition (13). Integrating by parts, this equation can be rewritten as:

$$\int_0^{\theta_{sc}(\delta)} \frac{G(x)dx}{G(\theta_{sc}(\delta))} = \delta.$$  \hspace{1cm} (A7)

It can be easily checked that $\frac{\int_0^{\delta} G(x)dx}{G(\delta)}$ is increasing in $y$ so that this equation has a unique solution as long as $\delta \leq \frac{1}{2}$.

Two cases must be distinguished depending on whether $\theta_{sc}^*(\delta)$ lies on the increasing part of $g(\cdot)$ or not. For $\delta \leq \frac{1}{6}$, the solution is $\theta_{sc}^* = 3\delta$. For $\delta \geq \frac{1}{6}$, the threshold $\theta_{sc}^*(\delta)$ solves (14). \footnote{Equation (14) has three parametric roots: $\hat{\theta}_1(\delta), \hat{\theta}_2(\delta), \hat{\theta}_3(\delta)$. The one that is selected is such that $\hat{\theta}_1(\frac{1}{6}) = \frac{1}{2}$. Then set $\theta_{sc}^*(\delta) = \hat{\theta}_1(\delta)$.}

Note that, for $\delta \geq \frac{1}{2}$, we obtain the corner solution $\theta_{sc}^*(\delta) = 1$.

- **Proof of Lemma 3:** The proof is again a straightforward application of Lemma 4.

The only thing to check is that a weak collusion-proof mechanism defined by (25) is such that the participation constraint (7) binds indeed always at $\theta_i = 0$. Note that the symmetric Bayesian-Nash equilibrium of the game induced by $g(\theta_1, \theta_2) = \max\{\theta^*, \theta_1 + \theta_2 - 1\}$ is such that:

$$\hat{\theta}^*(\theta_i) \in \arg \max_{\theta_{-i}} E_{\theta_{-i}} \left\{ -\frac{1}{2} \left( \max\left\{ \theta^*, \hat{\theta}_i + \hat{\theta}^*(\theta_{-i}) - 1 \right\} - \theta_i \right)^2 \right\}.$$  

We conjecture that such an equilibrium is obtained when $\hat{\theta}^*(\theta_i) = \theta^*$ for all $\theta_i$. Given that agent $-i$ plays this strategy, we have indeed $\frac{1}{2} \left( \max\left\{ \theta^*, \hat{\theta}_i + \theta^* - 1 \right\} - \theta_i \right)^2 = \frac{1}{2} (\theta^* - \theta_i)^2$ and thus taking $\hat{\theta}^*(\theta_i) = \theta^*$ is in the best-response correspondence. Then, we have:

$$v_d(\theta_i) = -\frac{1}{2} (\theta^* - \theta_i)^2.$$  

Finally, using (17) for a weak collusion-proof mechanism, we obtain:

$$\hat{U}_i(\theta_i) = E_{\theta_{-i}}(\max\{\theta^*, \theta_i + \theta_{-i} - 1\}) - \theta_i, \text{ for } i = 1, 2, \forall \theta_i \in \Theta.$$  \hspace{1cm} (A8)

Hence, $\hat{U}_i(\theta_i) \geq v_d(\theta_i) = \theta^* - \theta_i$ with this inequality being an equality in a right neighborhood of $\theta_i = 0$. The participation constraint (7) binds at $\theta_i = 0$ but also on a right interval of $\theta_i = 0$ so that (21) holds for the truthful manipulation $\phi^*(\theta_1, \theta_2) = (\theta_1, \theta_2)$. \footnote{Equation (14) has three parametric roots: $\hat{\theta}_1(\delta), \hat{\theta}_2(\delta), \hat{\theta}_3(\delta)$. The one that is selected is such that $\hat{\theta}_1(\frac{1}{6}) = \frac{1}{2}$. Then set $\theta_{sc}^*(\delta) = \hat{\theta}_1(\delta)$.}

- **Proof of Proposition 3:** Denoting by $V_{wc}(\theta^*, \theta^{**})$ the principal’s expected payoff with a weak collusion-proof mechanism of the form (25), we have:

$$V_{wc}(\theta^*, \theta^{**}) = -\frac{1}{2} \int_0^{\theta^{**}} (\theta^* - \theta - \delta)^2 g(\theta) d\theta$$
\[-\frac{1}{2} \int_{\theta_{+1}^*}^{\theta_{+1}^{**}} (\theta - 1 - \delta)^2 g(\theta) d\theta - \frac{1}{2} \int_{\theta_{+1}^*}^{1} (\theta^{**} - \theta - \delta)^2 g(\theta) d\theta. \tag{A9}\]

The derivative of $V_{wc}(\theta^*, \theta^{**})$ with respect to $\theta^{**}$ is zero at $\theta^{**} = 1$ since:

\[
\frac{\partial V_{wc}(\theta^*, \theta^{**})}{\partial \theta^{**}} = -\int_{\theta_{+1}^*}^{1} (\theta^{**} - \theta - \delta) g(\theta) d\theta.
\]

Therefore, setting $\theta^{**} = 1$ is optimal.

Optimizing then (A9) with respect to $\theta^*$ yields (26) for an interior solution. Integrating by parts, this equation can be rewritten as:

\[
\frac{\int_0^{\theta_{+1}^*(\delta)+1} G(x) dx}{G\left(\frac{\theta_{+1}^*(\delta)}{2}\right)} = \delta + \frac{1 - \theta_{wc}^*(\delta)}{2}. \tag{A10}\]

From this equation, one can derive that for $\delta = \frac{1}{2}$, we have $\theta_{wc}^*(\frac{1}{2}) = 1$ and that there is full pooling for $\delta > \frac{1}{2}$.

On the other hand, for $\delta = 0$, the fact that $y + \frac{\int_y^0 G(x) dx}{G(y)}$ is increasing in $y$ implies that necessarily $\theta_{wc}^*(0) > 0$, i.e., even when there is a priori no conflict with the agents, there is always some pooling in the optimal policy.

For $\delta \leq \frac{1}{2}$, we have:

\[
\frac{\int_0^{\theta_{wc}^*(\delta)} G(x) dx}{G\left(\frac{\theta_{wc}^*(\delta)}{2}\right)} < \delta + \frac{1 - \theta_{wc}^*(\delta)}{2}.
\]

Comparing with (A10), and using again the fact that $y + \frac{\int_y^0 G(x) dx}{G(y)}$ is increasing in $y$ yields $\frac{\theta_{wc}^*(\delta)+1}{2} > \theta_{wc}^*(\delta)$.

Note that the contract $q(\theta) = \max\{\theta^*, 2\theta - 1\}$ is always dominated by the contract $\hat{q}(\theta) = \max\{\theta^*, \theta\}$ from the principal’s point of view. Therefore, a strong coalition merger is always better than a weak one from the principal’s point of view.

- **Other Weak Collusion-Proof Mechanisms:** Let us now consider grand-mechanisms such that the participation constraint (7) binds at $\theta = 1$. We can then write the interest groups’ rents as:

\[
U_i(\theta_i) = \nu^d(1) - \int_{\theta_i}^1 E_{\theta_{-i}}(q(\phi(x, \theta_{-i})) - x) dx,
\]

and the participation constraint hold at all $\theta_i$ when

\[
\nu^d(1) - \nu^d(\theta_i) \geq \int_{\theta_i}^1 (q(\phi(x, \theta_{-i})) - x) dx \tag{A11}\]

Integrating by parts in the maximand of $(TP_{wcd})$ yields now an objective for the third-party which is an expectation of terms of the form:

\[
\left(\frac{q(\phi) - \theta_1 + \theta_2}{2}\right)^2 + \sum_{i=1}^{2} \theta_i (q(\phi) - \theta_i). \tag{A12}\]
As before, there is no loss of generality in restricting the analysis to mechanisms such that the (point-wise) optimal manipulation of reports made by the third-party is truthful. This leads to express the following weak coalition incentive compatibility constraints:

\[(\theta_1, \theta_2) = \arg \min_{(\hat{\theta}_1, \hat{\theta}_2)} (q(\hat{\theta}_1, \hat{\theta}_2) - \theta)^2 + \sum_{i=1}^{2} \theta_i (q(\hat{\theta}_1, \hat{\theta}_2) - \theta_i). \quad (A13)\]

From (A13), we can again easily prove that \(q(\cdot)\) depends only on the average ideal points \(\theta\). Moreover, \(q(\cdot)\) is monotonically increasing and thus almost everywhere differentiable in \(\theta\). At any point of differentiability, the weak coalition incentive constraints can now be written as:

\[q(\theta)q(\theta) = 0, \quad \forall \theta \in \Theta. \quad (A14)\]

Hence, we get:

**Lemma 5**: Any weak collusion-proof and continuous mechanism \(q(\cdot)\) such that the participation constraint (7) binds at \(\theta = 1\) is everywhere constant.

Clearly, such mechanisms are thus dominated by a weak collusion-proof mechanism satisfying (25) from the principal’s viewpoint. Moreover, (A11) holds trivially everywhere. There is no point considering such fully pooling mechanisms to compute the optimal response to the threat of a weak collusion.\(^{45}\)

**Proof of Proposition 4**: We will begin this proof by showing some formal results in the neighborhoods of \(\delta = 0\) and of \(\delta = \frac{1}{2}\). Then, we will move to some simulations confirming our insights obtained with those local analysis.

Let us first denote by \(V_i(\delta)\) (respectively \(U_i(\delta), TP_i(\delta)\)) for \(i = d, sc\), the principal’s (respectively an interest group’s and third-party’s) expected payoff under each organizational form of lobbying.

**Welfare comparison for the principal**: In the optimal grand-mechanism with a strong coalition the principal obtains

\[V_{sc}(\delta) = -\frac{1}{2} \int_0^{\theta_1} \int_0^{\theta_2} \left( \theta_{sc}(\delta) - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 \, d\theta_2 \, d\theta_1 \]

\[-\int_0^{\theta_{sc}(\delta)} \left( \int_0^{\delta \theta_{sc}(\delta) - \theta_1} \left( \theta_{sc}(\delta) - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 \, d\theta_2 \right) \, d\theta_1 - \int_0^{\theta_{sc}(\delta)} \left( \int_0^{\theta_1} \delta^2 \, d\theta_2 \right) \, d\theta_1 \]

\[^{45}\text{More generally, one may ask whether other collusion-proof mechanisms such that the the participation constraint (7) binds on an set in the interior of } \Theta \text{ would help the principal. Although, we have not derived those optimal collusion-proof mechanisms, we conjecture that they are sub-optimal.}\]
for $\delta \geq \frac{1}{6}$, and
\[
V_{sc}(\delta) = -\frac{1}{2} \int_0^{6\delta} \int_0^{6\delta - \theta_1} \left( 3\delta - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 \theta_2 d\theta_1 - \frac{1}{2} \int_0^{6\delta} \left( \int_{6\delta - \theta_1}^{1} \delta^2 d\theta_2 \right) d\theta_1 - \frac{1}{2} \int_{6\delta}^{1} \left( \int_{6\delta}^{1} \delta^2 d\theta_2 \right) d\theta_1
\]
for $\delta < \frac{1}{6}$.

The optimal dominant strategy mechanism leads to the payoff $V_d(\delta)$ to the principal given by:
\[
V_d(\delta) = -\int_0^{2\delta} \int_0^{\theta_1} \left( 2\delta - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 \theta_2 d\theta_1 - \int_0^{2\delta} \int_0^{\theta_1} \left( \theta_1 - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 \theta_2 d\theta_1
\]
if $\delta < \frac{1}{4}$, and
\[
V_d(\delta) = -\int_0^{2\delta} \int_0^{\theta_1} \left( 2\delta - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 \theta_2 d\theta_1 - \int_0^{2\delta} \int_0^{\theta_1} \left( \theta_1 - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 \theta_2 d\theta_1
\]
\[-\int_0^{1} \int_{2\delta + \frac{1}{2}}^{2\delta + \frac{1}{2}} \left( 2\delta + \frac{1}{2} - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 d\theta_2 d\theta_1 - \int_0^{2\delta} \int_0^{\theta_1} \left( \theta_2 - \frac{\theta_1 + \theta_2}{2} - \delta \right)^2 d\theta_2 d\theta_1,
\]
if $\frac{1}{4} \leq \delta \leq \frac{1}{2}$.

**Local analysis:**

**Case $\delta = 0$:** The strong coalition is aligned with the legislature and there is no cost of private information in this case (because of no bias) whereas there is such a cost for competition, we have: $V_{sc}(0) = 0 > V_d(0)$. For the interest groups:
\[
2U_{sc}(0) + TP_{sc}(0) = -\left( \int_0^{1} \int_0^{1} \left( \frac{\theta_1 + \theta_2}{2} - \theta_1 \right)^2 d\theta_2 d\theta_1 \right) = -\frac{1}{24}.
\]
Under competition, the total expected payoff of the interest groups is:
\[
2U_d(0) = -\int_0^{\theta^*(0)} \int_0^{\theta^*(0)} (\theta_2 - \theta_1)^2 d\theta_2 d\theta_1 - \int_0^{\theta^*(0)} \int_0^{1} (\theta^*(0) - \theta_1)^2 d\theta_2 d\theta_1
\]
\[-\int_0^{1} \int_0^{\theta_1} (\theta_2 - \theta_1)^2 d\theta_2 d\theta_1 - \int_0^{\theta^*(0)} \int_0^{1} (\theta^*(0) - \theta_1)^2 d\theta_2 d\theta_1 = -\frac{1}{16},
\]
where $\theta^*(0) = \frac{1}{2}$.

Therefore, $2U_d(0) < 2U_{sc}(0) + TP_{sc}(0)$ and the strong coalition is the optimal organization from a private viewpoint.

**Case $\delta \approx \frac{1}{2}$:** Let us denote $\delta = \frac{1}{2} - \epsilon$ for $\epsilon$ small enough and non-negative. Under competition, there is only one cut-off $\theta^*_d(\delta) = 1 - 2\epsilon$. With a strong coalition, Taylor expansions in (14) yield: $\theta^*_d(\delta) = 1 - \epsilon$.
With a strong coalition, the principal’s expected payoff can be approximated as:

\[
V_{sc}\left(\frac{1}{2} - \epsilon\right) = -\frac{1}{2} \int_{0}^{1-2\epsilon} \int_{0}^{1} \left(\frac{1}{2} - \theta_1 + \theta_2\right)^2 d\theta_2 d\theta_1
- \frac{1}{2} \int_{1-2\epsilon}^{1} \left(\frac{1}{2} - \theta_1 + \theta_2\right)^2 d\theta_2 + \int_{1-2\epsilon}^{1} \left(\frac{1}{2} - \epsilon\right)^2 d\theta_2 = -\frac{1}{48} + o(\epsilon^2).
\]

Under competition, the principal’s expected payoff can be approximated as:

\[
V_d\left(\frac{1}{2} - \epsilon\right) = -\frac{1}{2} \int_{0}^{1-2\epsilon} \int_{0}^{\theta_1} \left(\frac{1}{2} - \epsilon - \theta_1 + \theta_2\right)^2 d\theta_2 d\theta_1
- \int_{1-2\epsilon}^{1} \int_{0}^{\theta_1} \left(\theta_1 - \frac{1}{2} + \epsilon - \theta_1 + \theta_2\right)^2 d\theta_2 d\theta_1 = -\frac{1}{48} (1 - 24\epsilon) + o(\epsilon^2).
\]

Hence, for \(\epsilon\) small enough, we have: \(V_{sc}\left(\frac{1}{2} - \epsilon\right) < V_d\left(\frac{1}{2} - \epsilon\right)\).

With a strong coalition, the sum of the expected payoff of the interest groups and the third-party can be approximated as:

\[
2U_{sc}\left(\frac{1}{2} - \epsilon\right) + TP_{sc}\left(\frac{1}{2} - \epsilon\right) = -\int_{0}^{1-\epsilon} \int_{0}^{1-\epsilon} (1 - \epsilon - \theta_1)^2 d\theta_2 d\theta_1
- \int_{1-\epsilon}^{1} \int_{0}^{2-2\epsilon-\theta_1} (1 - \epsilon - \theta_1)^2 d\theta_2 d\theta_1
- \int_{1-\epsilon}^{1} \int_{2-2\epsilon-\theta_1}^{1} \frac{1}{4} (\theta_2 - \theta_1)^2 d\theta_2 d\theta_1
= -\frac{1}{3} (1 - 3\epsilon) + O(\epsilon^2).
\]

Under competition and using again symmetry, the expected payoff of an interest group can be approximated as:

\[
2U_d\left(\frac{1}{2} - \epsilon\right) = -\int_{0}^{1-2\epsilon} \int_{0}^{1-2\epsilon} (1 - 2\epsilon - \theta_1)^2 d\theta_2 d\theta_1
- \int_{1-2\epsilon}^{1} \int_{0}^{2-2\epsilon-\theta_2} (\theta_2 - \theta_1)^2 d\theta_1 d\theta_2
= -\frac{1}{3} (1 - 6\epsilon) + O(\epsilon^2).
\]

Hence, for \(\epsilon\) small enough, we have: \(2U_{sc}\left(\frac{1}{2} - \epsilon\right) + TP_{sc}\left(\frac{1}{2} - \epsilon\right) < 2U_d\left(\frac{1}{2} - \epsilon\right)\).

**Global analysis:** To compare the utilities of the principal under competition \(V_d\) and strong coalition \(V_{sc}\) it is convenient to make the following one-to-one change of variables:

\[
\delta(\theta_{sc}^*) = \begin{cases} 
\frac{\theta_{sc}^*}{6} & \text{if } 0 \leq \delta < \frac{1}{5} \\
\frac{1}{5} \frac{4(\theta_{sc}^*)^3 - 12(\theta_{sc}^*)^2 + 6\theta_{sc}^* - 1}{2(\theta_{sc}^*)^2 - 4\theta_{sc}^* + 1} & \text{if } \frac{1}{5} \leq \delta \leq \frac{1}{3}, \\
3\delta & \text{if } \delta < \frac{1}{6},
\end{cases}
\]

where the unique cut-off for a strong coalition \(\theta_{sc}^*(\delta)\) is determined by (14) if \(\delta \geq \frac{1}{5}\), and \(\theta_{sc}^*(\delta) = 3\delta\) if \(\delta < \frac{1}{6}\).
Since this change is monotonically increasing we may equivalently compare the payoffs in terms of \( \theta_{sc}^* \). Let us denote again (with a slight abuse of notations) by \( V_i (\theta_{sc}^*) \) (respectively \( U_i (\theta_{sc}^*) \) and \( TP_i (\theta_{sc}^*) \)) for \( i = d, sc, wc \), the principal’s (respectively an interest group’s and third party’s) expected payoff under each organizational form of lobbying.

To compare the payoffs of the principal under competition and strong coalition we replace the bias \( \delta \) by the threshold \( \theta_{sc}^* \) using (A15).

The utility of the principal under competition \( V_d (\theta_{sc}^*) \) is depicted on Figure 2 in solid line. For strong coalition the principal’s payoff \( V_{sc} (\theta_{sc}^*) \) is in dotted line.

Figure 2: Welfare comparison for the legislature.

Comparing these payoffs we conclude that \( V_d (\delta) > V_{sc} (\delta) \) for \( \delta > \delta^* \approx 0.11 \) and for \( \delta < \delta^* \), \( V_d (\delta) < V_{sc} (\delta) \). Summarize: for \( \delta < \delta^* \approx 0.11 \) the strong coalition is the optimal organization from a social viewpoint. For \( \delta > \delta^* \), the situation is reversed.

**Welfare comparison for groups:** Consider first the case \( \frac{1}{4} \leq \delta \leq \frac{1}{2} \). With a strong coalition, the sum of expected payoff of the interest groups and the third-party is:

\[
2U_{sc} (\delta) + TP_{sc} (\delta) = - \int_0^{\theta_{sc}^* (\delta)} \int_0^{\theta_{sc}^* (\delta)} (\theta_{sc}^* (\delta) - \theta_1)^2 d\theta_2 d\theta_1 - \int_{\theta_{sc}^* (\delta)}^1 \int_0^{\theta_{sc}^* (\delta) - \theta_1} (\theta_{sc}^* (\delta) - \theta_1)^2 d\theta_2 d\theta_1
\]

\[
- \int_{\theta_{sc}^* (\delta)}^1 \int_0^{\theta_{sc}^* (\delta) - \theta_2} (\theta_{sc}^* (\delta) - \theta_1)^2 d\theta_1 d\theta_2 - \int_{\theta_{sc}^* (\delta) - \theta_1}^1 \int_{\theta_{sc}^* (\delta) - \theta_1}^1 \frac{1}{4} (\theta_2 - \theta_1)^2 d\theta_2 d\theta_1
\]

\[
= \frac{1}{3} \theta_{sc}^* (\delta) \left( \theta_{sc}^* (\delta)^2 - 40_{sc}^* (\delta)^2 + 30_{sc}^* (\delta) - 1 \right).
\]

\[\text{For dominant strategy mechanism we split the interval } \delta \in \left[ 0, \frac{1}{2} \right] \text{ into three intervals: } \delta \in \left[ 0, \frac{1}{8} \right], \delta \in \left[ \frac{1}{8}, \frac{1}{4} \right], \text{ and } \delta \in \left[ \frac{1}{4}, \frac{1}{2} \right].\]

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Under competition and using again the symmetry, the expected payoff of the interest groups is:

$$2U_d(\delta) = -\int_0^{\delta} \int_0^{\delta} (2\delta - \theta_1)^2 d\theta_2 d\theta_1 - \int_0^{\theta_2} \int_0^{\theta_2} (\theta_2 - \theta_1)^2 d\theta_1 d\theta_2$$

$$= -4\delta^4 - \frac{1}{12}. \quad (A16)$$

We make the same change of variables as for principals from (A15). This leads to the expression of $2U_d(\delta)$ in terms of $\theta_{sc}^*$:

$$2U_d(\theta_{sc}^*) = -\frac{1}{12} \left[ \frac{1}{27} \left( \frac{4(\theta_{sc}^*)^3 - 12(\theta_{sc}^*)^2 + 6\theta_{sc}^* - 1}{2(\theta_{sc}^*)^2 - 4\theta_{sc}^* + 1} \right)^4 + 1 \right].$$

For $\frac{1}{6} \leq \delta \leq \frac{1}{3}$, the expected payoff of the interest groups and the third-party under strong coalition is the same as above. Under competition, the expected payoff of the interest groups is:

$$2U_d(\delta) = -\int_0^{\delta} \int_0^{\delta} (2\delta - \theta_1)^2 d\theta_2 d\theta_1 - \int_0^{\frac{1}{2}+\delta} \int_0^{\theta_2} (\theta_2 - \theta_1)^2 d\theta_1 d\theta_2$$

$$- \int_{\frac{1}{2}+\delta}^{1} \int_0^{\theta_1} \left( \frac{1}{2} + 2\delta - \theta_1 \right)^2 d\theta_2 d\theta_1 - \int_{\frac{1}{2}+\delta}^{1} \int_0^{\theta_2} \left( \frac{1}{2} + 2\delta - \theta_1 \right)^2 d\theta_1 d\theta_2.$$  

We again express the difference $2U_d(\delta) - (2U_{sc}(\delta) + TP_{sc}(\delta))$ in terms of $\theta_{sc}^*$.

For $\frac{1}{6} \leq \delta$, the optimal policy for strong coalition can be explicitly found. The dominant strategy payoff is as in the case 2 above. Coalition payoff is

$$2U_{sc}(\delta) + TP_{sc}(\delta) = -\int_0^{\delta} \int_0^{\delta} (3\delta - \theta_1)^2 d\theta_2 d\theta_1$$

$$- \int_0^{\delta} \left( \int_0^{\delta} \left( \frac{\theta_1 + \theta_2}{2} - \theta_1 \right)^2 d\theta_2 \right) d\theta_1 - \frac{1}{2} \int_0^{\delta} \left( \int_0^{1} \left( \frac{\theta_1 + \theta_2}{2} - \theta_1 \right)^2 d\theta_2 \right) d\theta.$$  

Finally we draw the respected payoffs in terms of $\theta_{sc}^*$, for competition and for strong coalition. As for the principals, the interest groups’ expected payoffs under competition is depicted on Figure 3 in solid line. For a strong coalition the payoff is in dotted line.
Figure 3: Welfare comparison for groups.

For $\frac{1}{4} \leq \delta \leq \frac{1}{2}$, we have: $2U_{sc}(\delta) + TP_{sc}(\delta) < 2U_d(\delta)$. Competition is still preferable for the groups for $\delta > \delta^{**} \approx 0.22$. However, for smaller biases: $\delta < \delta^{**} \approx 0.22$, the preferences for groups regarding the organization are reversed. Hence, the groups preferences regarding the lobbying organization are similar to those of the principal.

**Proof of Proposition 5:** As it can be checked from Lemma 5 in the Appendix, weak collusion-proof mechanisms such that the participation constraint (7) only binds at $\theta = 1$ entail full pooling and are thus clearly dominated by the mechanisms used with a strong coalition.

For weak collusion-proof mechanisms such that the participation constraint (7) only binds at $\theta = 0$ the proof is straightforward given that the virtual ideal point of a weak coalition is always further apart from that of the principal than what arises with a strong coalition.

To compare a weak coalition with competition we first determine the cut-off of the weak collusion-proof mechanism given by the equation

$$
\int_0^{\theta_{wc}(\delta)^+} \frac{\theta_{wc}^{\star}(\delta) - \theta - \delta}{2} g(\theta)d\theta = 0,
$$

where $g(\theta)$ is the density of the average of types. This equation in our case leads to

$$
-2(\theta_{wc}^{\star}(\delta))^3 + 6(\theta_{wc}^{\star}(\delta))^2 + 3\delta(\theta_{wc}^{\star}(\delta))^2 - 6\delta\theta_{wc}^{\star}(\delta) - 3\delta - 1 = 0.
$$

Again, for convenience, we make the change of variables:

$$
\delta(\theta_{wc}^{\star}) = \frac{6(\theta_{wc}^{\star})^2 - 2(\theta_{wc}^{\star})^3 - 1}{3(1 + 2\theta_{wc}^{\star} - (\theta_{wc}^{\star})^2)}.
$$

(A17)
This change of variables is a bijection from the interval \([\theta_{wc}^*(0), 1]\) to \([0, \frac{1}{2}]\). Note that even for zero conflict \(\delta\) the cut-off is positive, \(\theta_{wc}^*(0) = 0.44 > 0\).

We already have the principal’s payoffs for competition, however, the cut-off \(\theta_{wc}^*\) is different:\(^{47}\)

\[
V_d(\theta_{wc}^*) = \begin{cases} 
\frac{2}{3} \delta^4 (\theta_{wc}^*) - \frac{1}{2} \delta^3 (\theta_{wc}^*) + \frac{1}{6} \delta (\theta_{wc}^*) - \frac{1}{48} & \text{if } 0.44 \leq \theta_{wc}^* < 0.73 \\
0 & \text{if } 0.73 \leq \theta_{wc}^* \leq 1.
\end{cases}
\] (A18)

For weak coalition the payoff of the principal is:

\[
V_{wc}(\theta_{wc}^*) = -\frac{1}{2} \int_0^{\theta_{wc}^*} \int_0^{\theta_{wc}^*} \left( \theta_{wc}^* - \frac{\theta_1 + \theta_2}{2} - \delta(\theta_{wc}^*) \right)^2 d\theta_2 d\theta_1 - \int_0^{\theta_{wc}^*} \left( \theta_{wc}^* - \frac{\theta_1 + \theta_2}{2} - \delta(\theta_{wc}^*) \right)^2 d\theta_2 \right) d\theta_1 - \\
- \int_0^{\theta_{wc}^*} \left( \theta_{wc}^* - \frac{\theta_1 + \theta_2}{2} - \delta(\theta_{wc}^*) \right)^2 d\theta_2 \right) d\theta_1 = -\frac{1}{3} \delta(\theta_{wc}^*) - \frac{1}{2} \delta^2 (\theta_{wc}^*)
\] (A19)

We plug \(\delta\) from (A17) to (A18) and (A19) and then draw the payoffs in the relevant interval \(0.44 \leq \theta_{wc}^* \leq 1\). The competition payoff (solid line) is everywhere bigger that the payoff under a weak coalition (dotted line).

\(^{47}\)\(\theta_{wc}^* \left( \frac{1}{2} \right) = 0.73\).
For groups the total payoff under weak coalition is:

$$2U_{wc}\left(\theta_{wc}^*\right) + TP_{wc}\left(\theta_{wc}^*\right) = -\int_0^{\theta_{wc}^*+1} \int_0^{\theta_{wc}^*+1} (\theta_{wc}^* - \theta_1)^2 d\theta_2 d\theta_1$$

$$- \int_0^{\theta_{wc}^*+1} \int_0^{1+\theta_{wc}^* - \theta_1} (\theta_{wc}^* - \theta_1)^2 d\theta_2 d\theta_1 - \int_0^{\theta_{wc}^*+1} \int_0^{1+\theta_{wc}^* - \theta_2} (\theta_{wc}^* - \theta_1)^2 d\theta_1 d\theta_2$$

$$- \int_0^{\theta_{wc}^*+1} \int_0^{1+\theta_{wc}^* - \theta_1} (\theta_1 + \theta_2 - 1 - \theta_1)^2 d\theta_2 d\theta_1 = \frac{1}{6} \theta_{wc}^4 - \frac{2}{3} \theta_{wc}^3 + \frac{1}{3} \theta_{wc}^2 - \frac{1}{6}.$$

For competition if $\delta \geq \frac{1}{4}$:

$$2U_d(\delta) = -\int_0^{2\delta} \int_0^{2\delta} (2\delta - \theta_1)^2 d\theta_2 d\theta_1 - \int_0^{\theta_2} \int_0^{2\delta} (\theta_2 - \theta_1)^2 d\theta_1 d\theta_2 = -4\delta^4 - \frac{1}{12},$$

and if $\delta \leq \frac{1}{4}$:

$$2U_d(\delta) = -\int_0^{2\delta} \int_0^{2\delta} (2\delta - \theta_1)^2 d\theta_2 d\theta_1 - \int_0^{\frac{1}{2}+2\delta} \int_0^{\theta_2} (\theta_2 - \theta_1)^2 d\theta_1 d\theta_2$$

$$- \int_0^{1+\theta_1} \int_0^{\frac{1}{2}+2\delta} \left(\frac{1}{2} + 2\delta - \theta_1\right)^2 d\theta_2 d\theta_1 - \int_0^{1+\theta_2} \int_0^{\frac{1}{2}+2\delta} \left(\frac{1}{2} + 2\delta - \theta_1\right)^2 d\theta_1 d\theta_2$$

$$= -\frac{13}{192} + \frac{1}{4} \delta - \frac{5}{2} \delta^2 + 4\delta^3.$$

We again change the variable $\delta$ for $\theta_{wc}^*$ and compare the payoffs of groups. Similarly to the case of principals one can see that for groups competition overweights a weak coalition.