Non-Exclusive Competition under Adverse Selection*

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Abstract

A seller of a divisible good faces several identical buyers. The quality of the good may be low or high, and is the seller's private information. The seller has strictly convex preferences that satisfy a single-crossing condition. Buyers compete by posting menus of non-exclusive contracts, so that the seller can simultaneously and privately trade with several buyers. We provide a necessary and sufficient condition for the existence of a pure-strategy equilibrium. Aggregate equilibrium allocations are unique. Any traded contract must yield zero profit. If a quality is indeed traded, then it is traded efficiently. Depending on parameters, both qualities may be traded, or only one of them, or the market may break down to a no-trade equilibrium.

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1 Introduction

The recent financial crisis has spectacularly recalled that the liquidity of financial markets cannot be taken for granted, even for markets that usually attract many traders and on which exchanged volumes tend to be very high. For instance, Adrian and Shin (2010) document that the issuance of asset-backed securities declined from over three hundred billion dollars in 2007 to only a few billion in 2009. Similarly, Brunnermeier (2009) emphasizes the severe liquidity dry-up of the interbank market over the 2007–2009 period, when many banks chose to keep their liquidity idle instead of lending it even at short maturities. It is tempting to associate these difficulties with asymmetries in the allocation of information among traders. Indeed, during the crisis, one of the banks' main concern was the unknown exposure to risk of their counterparties.[1] Moreover, structured financial products such as mortgage-backed securities, collateralized debt obligations, and credit default swaps often involve many different underlying assets, and their designers clearly have more information about their quality; as pointed out by Gorton (2009), this may create an adverse selection problem and reduce liquidity provision.[2] Finally, most of these securities were traded outside of organized exchanges on over-the-counter markets, with poor information on the trading volumes or on the net positions of traders. Hence agents were able to interact secretly with multiple partners, at the expense of information release. These two features, adverse selection and non-exclusivity, are at the heart of the present paper.

Theoretical studies of adverse selection in competitive environments have been mainly developed in the context of two alternative paradigms. Akerlof (1970) studies an economy where privately informed sellers and uninformed buyers act as price takers. All trades are assumed to take place at the same price. Competitive equilibria typically exist, but feature a form of market failure: because the market-clearing price must be equal to the average quality of the goods offered by the sellers, the highest qualities are generally not traded in equilibrium. It seems therefore natural to investigate whether such a drastic outcome can be avoided by allowing buyers to screen goods of different qualities. In this spirit, Rothschild and Stiglitz (1976) consider a strategic model in which buyers offer to trade different quantities at different unit prices, thereby allowing sellers to credibly communicate their private information. They show that low-quality sellers trade efficiently, while high-quality sellers end up trading a suboptimal, but nonzero quantity. For instance, on insurance

[1] See, among others, Taylor and Williams (2009), and Philippon and Skreta (2011).

[2] In addition, there is some evidence that lending standards and the intensity of screening have been progressively deteriorating with the expansion of the securitization industry in the pre–2007 years. See, for instance, Keys, Mukherjee, Seru, and Vig (2010), and Demyanyk and van Hemert (2011).

markets, high-risk agents are fully insured, while low-risk agents only obtain partial coverage; no pure-strategy equilibrium exists if the proportion of low-risk agents is too high.

The present paper revisits these classical approaches by relaxing the assumption of exclusive competition, which states that each seller is allowed to trade with at most one buyer. This assumption plays a central role in Rothschild and Stiglitz's (1976) model, and it is also satisfied in the simplest versions of Akerlof's (1970) model, in which sellers can only trade one or zero unit of an indivisible good. However, situations where sellers can simultaneously and secretly trade with several buyers naturally arise on many markets—one may even say that non-exclusivity is the rule rather than the exception. In addition to the contexts we have already mentioned, well-known examples include the European banking industry, the US credit card market, and the life insurance and annuity markets of several OECD countries.[3] The structure of annuity markets is of particular interest because some legislations explicitly rule out the possibility of designing exclusive contracts: for instance, on September 1, 2002, the UK Financial Services Authority ruled in favor of the consumers' right to purchase annuities from suppliers other than their current pension provider (Open Market Option).

Our aim is to study the impact of adverse selection in markets with such non-exclusive trading relationships. To do so, we allow for non-exclusive trading in a generalized version of Rothschild and Stiglitz's (1976) model. This exercise is interesting per se: as we shall see, the reasonings that lead to the characterization of equilibria are quite different from those put forward by these authors. The results are also different: the equilibria we construct typically involve linear pricing, possibly with a bid-ask spread, and trading is efficient whenever it occurs. On the other hand, pure-strategy equilibria may fail to exist, as in Rothschild and Stiglitz (1976), and some types may be excluded from trade, as in Akerlof (1970). It might even be that the only equilibrium is a no-trade equilibrium. This variety of outcomes may help to better understand how financial markets react to informational asymmetries.

Our analysis builds on the following simple model of trade. There is a finite number of buyers, who compete for a divisible good offered by a single seller. The seller is privately informed of the quality of the good, which may be low or high. The seller's preferences are strictly convex, but otherwise arbitrary, provided they satisfy a single-crossing condition. Buyers compete by simultaneously posting menus of contracts, where a contract specifies

[3]Ongena and Smith (2000) and Detragiache, Garella, and Guiso (2000) document that multiple banking relationships have become very widespread in Europe. Rysman (2007) provides recent evidence of multi-homing in the US credit card industry. Cawley and Philipson (1999) and Finkelstein and Poterba (2004) report similar findings for the US life insurance market and the UK annuity market.

both a quantity and a transfer. After observing the menus offered, and taking into account her private information, or type, the seller chooses which contracts to trade. Our model encompasses pure trade and insurance environments as special cases.[4]

In this context, we provide a full characterization of the seller's aggregate trades in any pure-strategy equilibrium. First, we provide a necessary and sufficient condition for such an equilibrium to exist. This condition can be stated as follows: let v be the average quality of the good. Then, a pure-strategy equilibrium exists if and only if, at the no-trade point, the low-quality type would be willing to *sell* a small quantity of the good at price v, whereas the high-quality type would be willing to *buy* a small quantity of the good at price v. Second, we show that there exists a unique aggregate equilibrium allocation. Any contract traded in equilibrium yields zero profit, so that there are no cross-subsidies across types. In addition, if the willingness to trade at the no-trade point varies enough across types, equilibria are first-best efficient: the low-quality type *sells* the efficient quantity, while the high-quality type *buys* the efficient quantity. By contrast, if the two types have similar willingness to trade at the no-trade point, there is no trade in equilibrium. Finally, in intermediate cases, one type of the seller trades efficiently, while the other type does not trade at all.

These results suggest that, under non-exclusivity, the seller may only signal her type through the sign of the quantity she proposes to trade with a buyer. This is however a very rough signalling device, and it is only effective when one type acts as a seller, while the other one acts as a buyer. In particular, there is no equilibrium in which both types of the seller trade nonzero quantities on the same side of the market. Overall, non-exclusive competition exacerbates the adverse selection problem: if the first-best outcome cannot be achieved, a nonzero level of trade for one type of the seller can be sustained in equilibrium only if the other type of the seller is left out of the market. That is, the market breakdown originally conjectured by Akerlof (1970) also arises when buyers compete in arbitrary non-exclusive menu offers.

From a methodological viewpoint, the analysis of non-exclusive competition under adverse selection gives rise to interesting strategic insights. On the one hand, each buyer can build on his competitors' offers by proposing additional trades that are attractive to the seller. Thus new deviations become available to the buyers compared to the exclusive competition case. On the other hand, the fact that competition is non-exclusive also implies that each buyer gets access to a rich set of devices to block such deviations and discipline his competitors. In particular, he can issue *latent* contracts, that is, contracts that are not

[4] The labels *seller* and *buyers* are only used for expositional purposes. On financial markets, one may sell as well as buy assets. This translates in our model into allowing for negative as well as positive quantities.

traded by the seller on the equilibrium path, but which she finds it profitable to trade in case a buyer deviates from equilibrium play, in such a way that this deviating buyer is punished. Such latent contracts are in particular useful to deter cream-skimming deviations designed to attract one specific type of the seller.

In principle, the best response correspondence of any buyer could be determined by considering a situation where he would act as a monopsonist facing a seller with preferences represented by an indirect utility function that would depend on the menus offered by his competitors. However, because we impose very little structure on the menus that can be offered by the buyers, we cannot assume from the outset that this indirect utility function satisfies useful properties such as, for instance, a single-crossing condition. Moreover, we do not assume that, if the seller has multiple best responses in the continuation game, she necessarily chooses one that is best from the buyer's viewpoint. This rules out using standard mechanism-design techniques to characterize each buyer's best response.

To develop our characterization, we consider instead a series of deviations by a single buyer who designs his menu offer in such a way that a specific type of the seller will select a particular contract from this menu, along with some other contracts offered by the other buyers. We refer to this technique as *pivoting*, as the deviating buyer makes strategic use of his competitors' offers to propose attractive trades to the seller. For instance, consider the equilibrium allocation characterized by Rothschild and Stiglitz (1976). Because the low-risk agent pays a low unit price to obtain partial coverage, an insurance company can propose a high-risk agent to benefit from this low unit price offered by its competitors, while providing additional coverage at a mutually beneficial price. Our analysis thus shows that this allocation cannot be supported in equilibrium when competition is non-exclusive. Although this intuition dates back to Jaynes (1978), our paper generalizes this pivoting technique to derive a full characterization of the set of aggregate equilibrium trades.

Related Literature The implications of non-exclusive competition have been extensively studied in moral-hazard contexts. Following the seminal contributions of Hellwig (1983) and Arnott and Stiglitz (1993), many recent works emphasize that, in financial markets where agents can take non-contractible effort decisions, the impossibility of enforcing exclusive contracts can induce positive profits for financial intermediaries and a reduction in trades. Positive profits arise in equilibrium because none of the intermediaries can profitably deviate without inducing the agents to trade several contracts and select inefficient levels of effort.[5]

[5]See, for instance, Parlour and Rajan (2001), Bisin and Guaitoli (2004), and Attar and Chassagnon (2009) for applications to loan and insurance markets.

The present paper rules out moral-hazard effects and argues that non-exclusive competition under adverse selection drives intermediaries' profits to zero.

Pauly (1974), Jaynes (1978), and Hellwig (1988) pioneered the analysis of non-exclusive competition under adverse selection. Pauly (1974) suggests that Akerlof-like outcomes can be supported in equilibrium when buyers are restricted to offer linear price schedules. Jaynes (1978) points out that the separating equilibrium characterized by Rothschild and Stiglitz (1976) is vulnerable to entry by an insurance company proposing additional trades that can be concealed from the other companies. He further argues that the non-existence problem identified by Rothschild and Stiglitz (1976) can be overcome if insurance companies can share the information they have about the agents' trades. Hellwig (1988) discusses the relevant extensive form for the inter-firm communication game.

Biais, Martimort, and Rochet (2000) study a model of non-exclusive competition among uninformed market-makers who supply liquidity to an informed insider whose preferences are quasilinear, and quadratic in the quantities she trades. Although our model encompasses this specification of preferences, we develop our analysis in the two-type case, whereas Biais, Martimort, and Rochet (2000) consider a continuum of types. Despite the similarities between the two setups, their results stand in stark contrast with ours. Indeed, restricting attention to equilibria where market-makers post convex price schedules, they argue that non-exclusivity leads to a Cournot-like equilibrium outcome, in which each market-maker earns a positive profit. This is very different from our Bertrand-like equilibrium outcome, in which each traded contract yields zero profit.

Attar, Mariotti, and Salanié (2011) consider a situation where a seller is endowed with one unit of a good, the quality of which she privately knows. The good is divisible, so that the seller may trade any quantity of it with any of the buyers, as long as she does not trade more than her endowment in the aggregate. Both the buyers' and the seller's preferences are linear in quantities and transfers. Using pivoting arguments, it is shown that pure-strategy equilibria always exist, and that the corresponding aggregate allocations are generically unique. Pivoting is made simpler than in the present paper by the fact that agents have linear preferences and that the seller faces a capacity constraint that is independent of her type. Depending on whether quality is low or high, and on the probability with which quality is high, the seller may either trade her whole endowment, or abstain from trading altogether. Buyers earn zero profit in any equilibrium. These results offer a fully strategic foundation for Akerlof's (1970) classic study of the market for lemons, based on non-exclusive competition. Besides equilibrium existence, a key difference with our setting is that equilibria in Attar, Mariotti, and Salanié (2011) may exhibit non-trivial pooling and hence cross-subsidies across types. This reflects that trades are subject to an aggregate capacity constraint. By contrast, the present paper considers a situation where the seller's trades are unrestricted, as in a financial market where agents can take arbitrarily long or short positions. Another key feature of our model is that we consider general preferences for the seller, provided that they are strictly convex and satisfy a single-crossing condition. Thus the range of applications of the present paper is wider than in Attar, Mariotti, and Salanié (2011).

Ales and Maziero (2011) study non-exclusive competition in an insurance context similar to the one analyzed by Rothschild and Stiglitz (1976). Relying on free-entry arguments, they argue that only the high-risk agent can obtain a positive coverage in equilibrium. This is consistent with the results derived in the present paper; however, a distinguishing feature of our analysis is that it is fully strategic and avoids free-entry arguments. Our results are also more general in that we do not rely on a particular parametric representation of the seller's preferences, which allows us to uncover the common logical structure of a large class of potential applications.

This paper also contributes to the common-agency literature that analyzes situations where several principals compete through mechanisms to influence the decisions of a common agent. In our bilateral-contracting setting, the trades between the seller and the buyers are not public, and the seller may choose to trade with any subset of buyers. Moreover, in line with our focus on competitive markets, the profit of each buyer only depends on the trade he makes with the seller, and not on the other trades his competitors may make with her. In the standard terminology of common agency, our model is thus a *private* and *delegated* common-agency game with *no direct externalities* between principals.[6] In contrast with most of the common-agency literature, our analysis yields a unique prediction for aggregate equilibrium trades and equilibrium payoffs. In our view, this uniqueness result is tied to three key ingredients of our model. First, there are no direct externalities between principals.[7] Second, each buyer's profit is linear in the allocation he trades; whereas if some convexity

[6]The distinction between *delegated* common-agency games, in which the agent can trade with any subset of principals, and *intrinsic* common-agency games, in which the agent must either trade with all principals or with none of them, has been introduced by Bernheim and Whinston (1986). Martimort (2006) formulates the distinction between *public-agency* settings, in which each principal's transfer can be made contingent on all the agent's decisions, and *private-agency* settings, in which the transfer made by each principal is only contingent on the trades that the agent makes with him. Finally, the role of *direct externalities* between principals has been emphasized by Martimort and Stole (2003) and Peters (2003).

[7]Direct externalities between principals typically lead to multiple equilibrium outcomes even in complete-information environments, as shown by Martimort and Stole (2003) and Segal and Whinston (2003).

were introduced in the buyers' preferences, then multiple equilibrium outcomes would arise even in a complete-information version of our model.[8] Finally, each type of the seller cares only about the aggregate quantity she sells to the buyers and the aggregate transfer she receives in return; whereas if the buyers' offers were not perfectly substitutable from the seller's viewpoint, then one would again expect multiple equilibrium outcomes to arise even under complete information.[9]

Finally, it should be stressed that our uniqueness result obtains despite the fact that very few restrictions are imposed on the set of instruments available to the buyers, who are basically free to propose arbitrary menus of contracts. In this respect, our results contrast with the literature on supply-function equilibria, which considers oligopolistic industries where firms compete in supply schedules instead of simple price or quantity offers. Wilson (1979) and Grossmann (1981) were the first to observe that this additional degree of freedom may significantly expand the set of equilibrium outcomes. Klemperer and Meyer (1989) and Kyle (1989) suggest that the introduction of some uncertainty, either in the form of imperfect information over market demand or in the form of noise traders, may limit the multiplicity of equilibria. Vives (2011) develops these intuitions in a general setting where rational traders interact in the presence of idiosyncratic shocks; he shows that there exists a unique symmetric equilibrium in which supply functions are linear.

The paper is organized as follows. Section 2 describes the model. Section 3 characterizes pure-strategy equilibria. Section 4 derives necessary and sufficient conditions under which such equilibria exist. Section 5 concludes.

2 The Model

Our model features a seller who can simultaneously trade with several identical buyers. We put restrictions neither on the sign of the quantities of the good traded by the seller, nor on the sign of the transfers she receives in return. The labels *seller* and *buyers*, although useful, are therefore conventional.

[8]This setting is analyzed by Chiesa and De Nicolò (2009), who show that, although the aggregate quantity traded in equilibrium always coincides with the first-best quantity, equilibrium transfers and payoffs are not uniquely determined.

[9]Examples in this direction are provided by d'Aspremont and Dos Santos Ferreira (2010), who analyze the strategic competition between firms selling differentiated goods to a representative consumer under complete information, both in the cases of intrinsic and delegated agency.

2.1 The Seller

The seller is privately informed of her preferences. She may be of two types, L or H, with positive probabilities m_L and m_H such that $m_L + m_H = 1$. Subscripts i and j are used to index these types, with the convention that $i \neq j$. Each type cares only about the aggregate quantity Q she sells to the buyers and the aggregate transfer T she receives in return. Type i's preferences over aggregate quantity-transfer bundles (Q, T) are represented by a utility function u_i defined over $\mathbb{R}^2$. For each i, we assume that u_i is continuously differentiable, with $\partial u_i / \partial T > 0$, and that u_i is strictly quasiconcave. Hence type i's marginal rate of substitution of the good for money

$$\tau_i \equiv -\frac{\partial u_i / \partial Q}{\partial u_i / \partial T}$$

is everywhere well defined and strictly increasing along her indifference curves. Note that $\tau_i(Q, T)$ can be interpreted as type i's marginal cost of supplying a higher quantity, given that she already trades (Q, T). We do not impose any a priori restriction on the sign of $\tau_i(Q, T)$. The following assumption is key to our results.

Assumption SC *For each* (Q, T), $\tau_H(Q, T) > \tau_L(Q, T)$.

Assumption SC expresses a strict single-crossing condition: type H is less eager to sell a higher quantity than type L is. As a result, in the (Q, T) plane, a type-H indifference curve crosses a type-L indifference curve only once, from below.

2.2 The Buyers

There are $n \geq 2$ identical buyers. There are no direct externalities across them: each buyer cares only about the quantity q he purchases from the seller and the transfer t he makes in return. Each buyer's preferences over individual quantity-transfer bundles (q, t) are represented by a linear profit function: if a buyer receives from type i a quantity q and makes a transfer t in return, he earns a profit $v_i q - t$. We do not impose any a priori restriction on the sign of v_i. The following assumption will be maintained throughout the analysis.

Assumption CV $v_H > v_L$.

We let $v \equiv m_L v_L + m_H v_H$ be the average quality of the good, so that $v_H > v > v_L$. Assumption CV reflects common values: the seller's type has a direct impact on the buyers' profits. Together with Assumption SC, Assumption CV captures a fundamental tradeoff of our model: type H provides a more valuable good to the buyers than type L, but at a higher marginal cost. These assumptions are natural if we interpret the seller's type as the quality of the good she offers. Together, they create a tension that will be exploited later on: Assumption SC leads type H to offer less of the good, but Assumption CV would induce buyers to demand more of the good offered by type H, if only they could observe quality.

2.3 The Non-Exclusive Trading Game

Trading is non-exclusive in that no buyer can control, and a fortiori contract on, the trades that the seller makes with other buyers. The timing of events is as follows. First, buyers compete in menus of contracts for the good offered by the seller.[10] Next, the seller can simultaneously trade with several buyers. Formally:

1. Each buyer k proposes a menu of contracts, that is, a set $C^k \subset \mathbb{R}^2$ of quantity-transfer bundles that contains at least the no-trade contract $(0, 0)$.[11]

2. After privately learning her type, the seller selects one contract from each of the menus C^k offered by the buyers.

A pure strategy for type i is a function that maps each menu profile $(C^1, \ldots, C^n)$ into a vector of contracts $((q^1, t^1), \ldots, (q^n, t^n)) \in C^1 \times \ldots \times C^n$. To ensure that type i's utility-maximization problem

$$\max \left\{ u_i \left(\sum_k q^k, \sum_k t^k \right) : (q^k, t^k) \in C^k \text{ for each } k \right\}$$

always has a solution, we require the buyers' menus C^k to be compact sets. This allows us to use perfect Bayesian equilibrium as our equilibrium concept. Throughout the paper, we focus on pure-strategy equilibria.

2.4 Applications

The following examples illustrate the range of our model.

[10] As shown by Peters (2001) and Martimort and Stole (2002), there is no need to consider more general mechanisms in this multiple-principal single-agent setting.

[11] This requirement allows one to deal with participation in a simple way. It reflects the fact that the seller cannot be forced to trade with any particular buyer.

Pure Trade In the pure trade model, each type i has quasilinear preferences:

$$u_i(Q,T) = T - c_i(Q),$$

where the cost function c_i is continuously differentiable and strictly convex. Assumption SC requires that $c'_H(Q) > c'_L(Q)$ for all Q. In line with the continuous-type model of Biais, Martimort, and Rochet (2000), one may for instance consider a quadratic cost function $c_i(Q) = \theta_i Q + \frac{\gamma}{2} Q^2$, for some positive constant γ. Assumption SC then reduces to $\theta_H > \theta_L$. Biais, Martimort, and Rochet (2000) moreover assume that the first-best quantities are implementable, a situation sometimes called *responsiveness* in the literature (Caillaud, Guesnerie, Rey, and Tirole (1988)). In our two-type specification, this would amount to assuming that $v_H - \theta_H < v_L - \theta_L$. Our analysis does not rely on such an assumption.

Insurance In the insurance model, an agent can sell a risk to several insurance companies. As in Rothschild and Stiglitz (1976), the agent faces a binomial risk on her wealth, which can take two values W_G and W_B, with probabilities π_i and $1-\pi_i$ that define her type. Here $W_G - W_B$ is the positive monetary loss that the agent incurs in the bad state. A contract specifies a reimbursement r to be paid in the bad state, and an insurance premium p. Let R be the sum of the reimbursements, and let P be the sum of the insurance premia. We assume that the agent's preferences have an expected utility representation

$$\pi_i u(W_G - P) + (1-\pi_i) u(W_B - P + R),$$

for some von Neumann–Morgenstern utility function u which is assumed to be continuously differentiable, increasing, and strictly concave. An insurance company's profit from selling the contract (r,p) to type i is $p - (1-\pi_i)r$, which can be written as $v_i q - t$ if we set $v_i \equiv -(1-\pi_i)$, $q \equiv r$, and $t \equiv -p$, so that $Q = R$ and $T = -P$. Hence the agent purchases for a transfer $-T$ a reimbursement Q in the bad state, and her expected utility writes as

$$u_i(Q,T) = \pi_i u(W_G + T) + (1-\pi_i) u(W_B + Q + T).$$

Then

$$\tau_i(Q,T) = -\frac{(1-\pi_i)u'(W_B + T + Q)}{\pi_i u'(W_G + T) + (1-\pi_i)u'(W_B + T + Q)},$$

so that Assumption SC requires that type H has a lower probability of incurring a loss, $\pi_H > \pi_L$. Given our parametrization, this implies that $v_H > v_L$, so that Assumption CV is satisfied. Therefore, our model encompasses the non-exclusive version of Rothschild and Stiglitz's (1976) model considered by Ales and Maziero (2011). Note that we could also allow for non-expected utility in the modelling of the agent's preferences.

3 Equilibrium Characterization

An equilibrium specifies individual trades (q_i^k, t_i^k) between each type i and each buyer k, and corresponding aggregate trades $(Q_i, T_i) = \left(\sum_k q_i^k, \sum_k t_i^k\right)$. In the present section, we characterize these equilibrium trades, assuming that an equilibrium exists, and we provide a simple necessary condition for the existence of an equilibrium.

3.1 Pivoting

In line with Rothschild and Stiglitz (1976), we shall examine well-chosen deviations by a buyer, and use the fact that in equilibrium deviations cannot be profitable. A key difference, however, is that in Rothschild and Stiglitz (1976) competition is exclusive, whereas in our setting competition is non-exclusive.

Under exclusive competition, what matters from the viewpoint of any given buyer k is simply the maximum utility level U_i^{-k} that each type i can get by trading with some other buyer. A deviation by buyer k targeted at type i is then a contract (q_i^k, t_i^k) that gives type i a strictly higher utility, $u_i(q_i^k, t_i^k) > U_i^{-k}$. Type j may be attracted or not by this contract; in each case, one can compute the deviating buyer's profit.

By contrast, under non-exclusive competition, all the contracts offered by the other buyers matter from the viewpoint of buyer k. Suppose indeed that the seller can trade a bundle (Q^{-k}, T^{-k}) with the buyers other than k. Then buyer k can use this as an opportunity to build more attractive deviations. For instance, to attract type i, buyer k can propose the contract $(Q_i - Q^{-k}, T_i - T^{-k} + \varepsilon)$, for some positive number ε: combined with (Q^{-k}, T^{-k}), this contract gives type i a strictly higher utility than her aggregate equilibrium trade (Q_i, T_i). In that case, we say that buyer k *pivots on* (Q^{-k}, T^{-k}) *to attract type* i. Type j may be attracted or not by this contract; in each case, one can provide a condition on profits that ensures that the deviation is not profitable.

The key difference between exclusive and non-exclusive competition is thus that, in the latter case, each buyer k faces a single seller whose type is unknown, but whose preferences are defined by an *indirect* utility function, rather than by the primitive utility function u_i as in the former case. Formally, type i's indirect utility of trading a contract (q, t) with buyer k is given by

$$z_i^{-k}(q, t) = \max\left\{u_i\left(q + \sum_{l \neq k} q^l, t + \sum_{l \neq k} t^l\right) : (q^l, t^l) \in C^l \text{ for each } l \neq k\right\},$$

so that in equilibrium $U_i \equiv u_i(Q_i, T_i) = z_i^{-k}(q_i^k, t_i^k)$ for all i and k. Notice that $z_i^{-k}(q, t)$ is strictly increasing in t. Moreover, because u_i is continuous and menus are assumed to be compact, it follows from Berge's maximum theorem that z_i^{-k} is continuous.[12]

What makes the analysis difficult is that the functions z_i^{-k} are endogenous, because they depend on the menus offered by the buyers other than k, on which we impose no restriction besides compactness. As a result, there is no a priori guarantee that the functions z_i^{-k} are well behaved, which prevents us from using mechanism-design techniques to determine each buyer's best response to the other buyers' menus. Instead, we only rely on pivoting arguments to fully characterize aggregate equilibrium allocations and individual equilibrium payoffs, as in Attar, Mariotti, and Salanié (2011).

Remark. The idea of determining each principal's equilibrium behavior by considering his interaction with an agent endowed with an indirect utility function that incorporates the optimal choices she makes with the other principals is a standard device in the common-agency literature.[13] In the context of private agency, this methodology has been applied to games of complete information (Chiesa and De Nicolò (2009), d'Aspremont and Dos Santos Ferreira (2010)), as well as to games of incomplete information (Biais, Martimort, and Rochet (2000), Martimort and Stole (2003, 2009), Calzolari (2004), Laffont and Pouyet (2004), or Khalil, Martimort, and Parigi (2007)). Although this approach has been used to derive a full characterization of equilibrium payoffs under complete information, the analysis of incomplete-information environments typically involves additional restrictions. Indeed, attention is usually restricted to equilibria in which the screening problem faced by each principal is regular enough, which amounts to considering well-behaved z_i^{-k} functions that are concave in quantities and satisfy a single-crossing condition.[14] A distinguishing feature of our analysis is that we provide a full characterization of aggregate equilibrium allocations and individual equilibrium payoffs by exploiting only the continuity of the z_i^{-k} functions and the fact that each of them is strictly increasing in transfers.

Denote type-by-type individual profits by $b_i^k \equiv v_i q_i^k - t_i^k$, and expected individual profits by $b^k \equiv m_L b_L^k + m_H b_H^k$. The following lemma encapsulates our pivoting technique.

[12] This differs from Attar, Mariotti, and Salanié (2011), where the presence of a capacity constraint may induce discontinuities in the seller's indirect utility function.

[13] A similar approach has been followed in the literature on supply-function equilibria, in which each supplier's equilibrium behavior is determined by taking into account the residual demand he faces given the supply functions offered by his competitors (see Wilson (1979), Grossman (1981), Klemperer and Meyer (1989), Kyle (1989), and Vives (2011)).

[14] See Martimort and Stole (2009) for a general exposition of this methodology, and for a detailed analysis of the conditions that need to be imposed on the agent's preferences and on the corresponding virtual surplus function to guarantee the regularity of each principal's program.

Lemma 1 *Let i, k, q, and t be such that, in equilibrium, the quantity $Q_i - q$ can be traded with the buyers other than k, in exchange for a transfer $T_i - t$. Then*

$$v_i q - t > b_i^k \quad \textit{implies} \quad vq - t \leq b^k. \tag{1}$$

The intuition for this result is as follows. If the bundle $(Q_i - q, T_i - t)$ can be traded with the buyers other than k, then buyer k can pivot on it to attract type i, while still offering the contract (q_j^k, t_j^k). If the contract (q, t) allows buyer k to increase the profit he earns with type i, then it must be that type j also selects it instead of (q_j^k, t_j^k) following buyer k's deviation; moreover, this contract cannot increase buyer k's average profit if traded by both types, for, otherwise, we would have constructed a profitable deviation.

We are now ready to use our pivoting technique to gain insights into the structure of aggregate equilibrium allocations. Because each type only cares about her aggregate trade, and buyers only care about their individual trades and have identical linear profit functions, in equilibrium aggregate trades and aggregate profits can be computed as if both types were trading (Q_j, T_j), with type i trading in addition $(Q_i - Q_j, T_i - T_j)$. What can be said about this additional trade? A first information comes from Assumption SC, which implies that both $Q_L - Q_H$ and $T_L - T_H$ are nonnegative. More interestingly, Lemma 1 allows us to show that, in the aggregate, buyers cannot make a profit by trading $(Q_i - Q_j, T_i - T_j)$ with type i. Formally, denote by $S_i \equiv v_i(Q_i - Q_j) - (T_i - T_j)$ the corresponding aggregate profit. Then the following result obtains.

Proposition 1 *In any equilibrium, $S_i \leq 0$ for each i.*

Proof. Choose i and k and set $q \equiv q_j^k + Q_i - Q_j$ and $t \equiv t_j^k + T_i - T_j$. Then the quantity $Q_i - q = \sum_{l \neq k} q_j^l$ can be traded with the buyers other than k in exchange for a transfer $T_i - t = \sum_{l \neq k} t_j^l$. We can thus apply Lemma 1. One has

$$\begin{aligned} v_i q - t - b_i^k &= v_i(q_j^k + Q_i - Q_j) - (t_j^k + T_i - T_j) - b_i^k \\ &= v_i(Q_i - Q_j) - (T_i - T_j) - [v_i(q_i^k - q_j^k) - (t_i^k - t_j^k)] \\ &= S_i - s_i^k, \end{aligned}$$

where $s_i^k \equiv v_i(q_i^k - q_j^k) - (t_i^k - t_j^k)$, and

$$\begin{aligned} v_j q - t - b_j^k &= v_j(q_j^k + Q_i - Q_j) - (t_j^k + T_i - T_j) - b_j^k \\ &= -[v_j(Q_j - Q_i) - (T_j - T_i)] \\ &= -S_j. \end{aligned}$$

Therefore, according to (1),

$$S_i > s_i^k \text{ implies } m_i(S_i - s_i^k) \leq m_j S_j. \tag{2}$$

Suppose by way of contradiction that $S_i > 0$. Because $S_i = \sum_k s_i^k$ by construction, one must have $S_i > s_i^k$ for some k. From (2), we obtain that $S_j > 0$, and thus that $S_i + S_j > 0$. As $S_i + S_j = (v_i - v_j)(Q_i - Q_j)$ and $v_H > v_L$, this implies that $Q_L < Q_H$, a contradiction. Hence the result. Note for future reference that, because $S_j \leq 0$, it actually follows from (2) that $S_i \leq s_i^k$ for all i and k. ■

The intuition for Proposition 1 can be easily understood in the context of a free-entry equilibrium. Indeed, under free entry, the seller can trade (Q_j, T_j) with the existing buyers, so that an entrant can pivot on (Q_j, T_j) to attract type i. That is, an entrant could simply propose to buy a quantity $Q_i - Q_j$ in exchange for a transfer slightly above $T_i - T_j$. This contract would certainly attract type i; besides, if it also attracted type j, this would be good news for the entrant, because $v_j(Q_i - Q_j) \geq v_i(Q_i - Q_j)$ as $v_H > v_L$ and $Q_L \geq Q_H$. In a free-entry equilibrium, it must therefore be that $v_i(Q_i - Q_j) \leq T_i - T_j$. Proposition 1 shows that the same result holds when the number of buyers is fixed, although the argument is more involved.

As simple as it is, this result is powerful enough to rule out equilibrium outcomes that have been emphasized in the literature. Consider first the separating equilibrium of Rothschild and Stiglitz's (1976) exclusive-competition model of insurance provision. In this equilibrium, insurance companies earn zero profit, and no cross-subsidization takes place. Using the parametrization of Section 2.4, this means that the equilibrium contract (Q_i, T_i) of each type i lies on the line with negative slope $v_i = -(1 - \pi_i)$ going through the origin. Moreover, the high-risk agent, that is, in our parametrization, type L, is indifferent between the contracts (Q_L, T_L) and (Q_H, T_H). Hence, as $Q_L > Q_H > 0$, the line connecting these two contracts has a negative slope strictly lower than v_L. That is, $T_L - T_H < v_L(Q_L - Q_H)$, in contradiction with Proposition 1. Therefore, the Rothschild and Stiglitz's (1976) equilibrium is not robust to non-exclusive competition.

Proposition 1 also rules out equilibria with linear prices in which both types trade nonzero quantities on the same side of the market. To see this, suppose for instance that there exists an equilibrium in which each buyer stands ready to buy any quantity at a unit price p, and that in this equilibrium $Q_L > Q_H \geq 0$. Because the expected aggregate profit $B \equiv \sum_k b^k$ must be nonnegative, one must have $v > p$. Moreover, according to Proposition 1 and the definition of S_L, one must have $p \geq v_L$. In particular, buyers cannot make profits by trading with type L. Now, any buyer k can deviate by offering a contract $(Q_H, T_H + \varepsilon_H)$, for some positive number ε_H. This contract certainly attracts type H. At worst, it also attracts type L, and therefore one must have $b^k \geq (v - p)Q_H$ by letting ε_H go to zero. Summing these inequalities over k yields

$$B \geq n(v - p)Q_H. \tag{3}$$

Because one can compute the aggregate profit as if both types were trading (Q_H, T_H), with type L trading in addition $(Q_L - Q_H, T_L - T_H)$, one has

$$B = vQ_H - T_H + m_L S_L = (v - p)Q_H + m_L S_L. \tag{4}$$

Merging (3) and (4) yields $m_L S_L \geq (n-1)(v-p)Q_H$. Because $n \geq 2$, $v > p$, and $S_L \leq 0$ by Proposition 1, one must thus have $Q_H \leq 0$. Because $Q_H \geq 0$ by assumption, it follows that $Q_H = S_L = 0$: type H does not trade at all, while type L trades at the fair price v_L. Hence there is no equilibrium with linear prices in which both types trade nonzero quantities on the same side of the market. This runs contrary to the presumption that non-exclusive competition entails linear pricing, with different types of sellers actively trading (see, for instance, Chiappori (2000)).

3.2 The Zero-Profit Result

In any Bertrand-like setting, the usual argument consists in making buyers compete for any profit that may result from serving the whole demand. This also applies to our setting, although the logic is different. Specifically, the following zero-profit result obtains.

Proposition 2 *In any equilibrium,* $B = 0$, *so that* $b^k = 0$ *for each* k.

Proof. Denote type-by-type aggregate profits by $B_i \equiv \sum_k b_i^k$, and recall that the expected aggregate profit is denoted by B. We first prove that, for each j and k,

$$B_j > b_j^k \ \text{ implies } \ B - b^k \leq m_i S_i. \tag{5}$$

Indeed, if $B_j > b_j^k$, buyer k can deviate by proposing a menu consisting of the no-trade contract and of the contracts $c_i^k = (q_i^k, t_i^k + \varepsilon_i)$ and $c_j^k = (Q_j, T_j + \varepsilon_j)$, for some positive numbers ε_i and ε_j. Because $U_j \geq z_j^{-k}(q_i^k, t_i^k)$ and the function z_j^{-k} is continuous, it is possible, given the value of ε_j, to choose ε_i small enough so that type j trades c_j^k following buyer k's deviation. Turning now to type i, observe that she must trade either c_i^k or c_j^k following buyer k's deviation: indeed, because ε_i is positive, type i strictly prefers c_i^k to any contract she could have traded with buyer k before the deviation. If type i selects c_i^k, then buyer k's profit from this deviation is $m_i(b_i^k - \varepsilon_i) + m_j(B_j - \varepsilon_j)$, which, because $B_j > b_j^k$ by assumption, is strictly higher than b^k when ε_i and ε_j are small enough, a contradiction. Therefore, type i must select c_j^k following buyer k's deviation, and for this deviation not to be profitable one must have $vQ_j - T_j - \varepsilon_j \leq b^k$. In line with (4), this may be rewritten as $B - m_i S_i - \varepsilon_j \leq b^k$, from which (5) follows by letting ε_j go to zero.

Now, if $B > 0$, then $B > b^k$ for some k. Because $S_i \leq 0$ and $S_j \leq 0$ by Proposition 1, it follows from (5) that $B_i \leq b_i^k$ and $B_j \leq b_j^k$ for each k. Averaging over types yields $B \leq b^k$ for each k, a contradiction. Hence the result. ■

The intuition for Proposition 2 can be easily understood in the context of a free-entry equilibrium. Indeed, suppose for instance that the aggregate profit from trading with type j is positive, $B_j > 0$. Then an entrant could propose to buy Q_j in exchange for a transfer slightly above T_j. This contract would certainly attract type j, which would benefit the entrant; in equilibrium, it must therefore be that this trade also attracts type i, and that $vQ_j - T_j \leq 0$. Now, recall that the aggregate profit may be written as $B = vQ_j - T_j + m_i S_i$. Our first result in Proposition 1 was that $S_i \leq 0$, and we have just argued that $vQ_j - T_j \leq 0$ when $B_j > 0$. Hence the aggregate profit must be zero. Proposition 2 shows that the same result holds when the number of buyers is fixed, which is not a priori obvious.

Remark An inspection of their proofs reveal that Propositions 1 and 2 only require weak assumptions on feasible trades, namely that if the quantities q and q' are tradable, then so are the quantities $q + q'$ and $q - q'$. Hence we allow for negative and positive trades, but we may for instance have integer constraints on quantities. Finally, we did use in Lemma 1 the fact that the functions u_i, and thus the functions z_i^{-k}, are continuous with respect to transfers, but, for instance, we did not use the fact that the seller's preferences are convex.

3.3 Pooling versus Separating Equilibria

We say that an equilibrium is *pooling* if both types of the seller make the same aggregate trade, that is, $Q_L = Q_H$, and that it is *separating* if they make different aggregate trades, that is, $Q_L > Q_H$. We now investigate the basic price structure of these two kinds of candidate equilibria.

Proposition 3 *The following holds*:

- *In any pooling equilibrium*, $T_L = vQ_L = T_H = vQ_H$.

- *In any separating equilibrium,*

  (i) If $Q_L > 0 > Q_H$, then $T_L = v_L Q_L$ and $T_H = v_H Q_H$.

  (ii) If $Q_L > Q_H \geq 0$, then $T_H = v Q_H$ and $T_L - T_H = v_L(Q_L - Q_H)$.

  (iii) If $0 \geq Q_L > Q_H$, then $T_L = v Q_L$ and $T_H - T_L = v_H(Q_H - Q_L)$.

Proof. In the case of a pooling equilibrium, the conclusion follows immediately from the zero-profit result. Consider next a separating equilibrium, and let us start with case (ii): $Q_L > Q_H \geq 0$. We know from Proposition 1 that $S_L \leq 0$. Suppose that $S_L < 0$. From (5) and the zero-profit result, we get $B_H \leq b_H^k$ for each k, which implies that $B_H \leq 0$. Now, notice from (4) that

$$B = vQ_H - T_H + m_L S_L = B_H + m_L[S_L - (v_H - v_L)Q_H].$$

Because $B_H \leq 0$, $S_L < 0$ and $Q_H \geq 0$, we obtain that $B < 0$, a contradiction. Therefore, it must be that $S_L = 0$, so that $T_L - T_H = v_L(Q_L - Q_H)$. It follows that $B = vQ_H - T_H$, so that $T_H = vQ_H$ as $B = 0$. Hence the result. Case (iii) follows in a similar manner, exchanging the roles of L and H. Consider finally case (i): $Q_L > 0 > Q_H$. As above, $B = B_H + m_L[S_L - (v_H - v_L)Q_H] = 0$. Suppose that $B_H > 0$ and thus $B_H > b_H^k$ for some k. Again, from (5), this implies that $S_L = 0$ and thus that $B_H - m_L(v_H - v_L)Q_H = B = 0$. Because $v_H > v_L$ and $B_H > 0$, one must have $Q_H > 0$, a contradiction. Hence $B_H = 0$, and therefore $B_L = 0$ as $B = 0$. It follows that $T_L = vQ_L$ and $T_H = v_H Q_H$. Hence the result. ■

The first statement of Proposition 3 is a direct consequence of the zero-profit result. Otherwise, the equilibrium is separating, and three cases may arise. In case (i), type L *sells* a positive quantity Q_L, while type H *buys* a positive quantity $|Q_H|$. There are no cross-subsidies in equilibrium, as each type i trades at the fair price v_i. In case (ii), everything happens as if, in the aggregate, both types were selling a quantity Q_H at the fair price v, with type L selling an additional quantity $Q_L - Q_H$ at the fair price v_L. When $Q_H > 0$, there are cross-subsidies in equilibrium, with $B_L < 0 < B_H$. In that case, the structure of aggregate equilibrium allocations is similar to that obtained by Jaynes (1978) and Hellwig (1988) in a non-exclusive version of Rothschild and Stiglitz's (1976) model where insurance companies can share information about their clients. It is also reminiscent of the equilibrium of the limit-order book analyzed by Glosten (1994). When $Q_H = 0$, the structure of aggregate equilibrium allocations is similar to that which prevails in Akerlof (1970), or, in a model of non-exclusive competition, in Attar, Mariotti, and Salanié (2011). Finally, case (iii) is the mirror image of case (ii).

3.4 The No-Cross-Subsidization Result

In this section, we prove that our non-exclusive competition game has no equilibrium with cross-subsidies. We first establish that the aggregate profit earned on each type must be zero in equilibrium. As discussed below, this drastically reduces the set of candidate equilibria. We then refine this result by showing that any traded contract must actually yield zero profit in equilibrium.

The first step of the analysis consists in showing that, if buyers make profits in the aggregate when trading with type j, then type j must trade inefficiently in equilibrium. Specifically, her marginal rate of substitution at her aggregate equilibrium trade is not equal to the quality of the good she sells, but rather to the *average* quality of the good.

Lemma 2 *If $B_j > 0$ in equilibrium, then $\tau_j(Q_j, T_j) = v$.*

The intuition for Lemma 2 is as follows. If $\tau_j(Q_j, T_j)$ were different from v, then any buyer could propose a contract in the neighborhood of (Q_j, T_j) that would attract type j, thereby generating a positive profit close to B_j, and that would generate a small positive profit even if it were traded by both types. This, however, is impossible according to the zero-profit result.

The second step of the analysis consists in showing that, if buyers make profits in the aggregate when trading with type j, then the aggregate trade made by type j in equilibrium must remain available if any buyer withdraws his menu offer. In our oligopsony model, this rules out Cournot-like outcomes in which the buyers would share the market in such a way that each of them would be needed to provide type j with her aggregate equilibrium trade, as is the case in the equilibrium described in Biais, Martimort, and Rochet (2000). This is more in the spirit of Bertrand competition, where cross-subsidies are harder to sustain.

Lemma 3 *If $B_j > 0$ in equilibrium, then, for each k, the quantity Q_j can be traded with the buyers other than k in exchange for a transfer T_j.*

The proof of Lemma 3 proceeds as follows. First, we show that if B_j is positive, then the equilibrium utility of type j must remain available following any buyer's deviation; the reason for this is that, otherwise, a buyer could deviate and reap the aggregate profit on type j. As a result, for any buyer k, there exists an aggregate trade (Q^{-k}, T^{-k}) with the buyers other than k that allows buyer j to achieve the same level of utility as in equilibrium, $u_j(Q^{-k}, T^{-k}) = U_j$. From the strict quasiconcavity of u_i and Lemma 2, we obtain that if

$Q^{-k} \neq Q_j$, then $T^{-k} > vQ^{-k}$. We finally show that this would allow buyer k to profitably deviate by pivoting on (Q^{-k}, T^{-k}).

We are now ready to state and prove the main result of this section.

Proposition 4 *In any equilibrium, $B_j = 0$ for each j.*

Proof. Suppose by way of contradiction that $B_j > 0$ for some j. Then any buyer k such that $b_j^k > 0$ can deviate by proposing a menu consisting of the no-trade contract and of the contracts $c_i^k = (Q_i - Q_j + \delta_i, v_i(Q_i - Q_j) + \varepsilon_i)$ and $c_j^k = (q_j^k, t_j^k + \varepsilon_j)$, for some numbers δ_i, ε_i, and ε_j. Choose δ_i and ε_i such that $\tau_i(Q_i, T_i)\delta_i < \varepsilon_i$. This ensures that, when δ_i and ε_i are small enough, type i can strictly increase her utility by trading c_i^k with buyer k and (Q_j, T_j) with the buyers other than k; according to Lemma 3, this is feasible as $B_j > 0$. Because $U_i \geq z_i^{-k}(q_j^k, t_j^k)$ and the function z_i^{-k} is continuous, it is possible, given the values of δ_i and ε_i, to choose ε_j positive and small enough so that type i trades c_i^k following buyer k's deviation. Turning now to type j, observe that she must trade either c_i^k or c_j^k following buyer k's deviation: indeed, because ε_j is positive, type j strictly prefers c_j^k to any contract she could have traded with buyer k before the deviation. If type j selects c_j^k, then buyer k's profit from this deviation is $m_i(v_i\delta_i - \varepsilon_i) + m_j(v_j q_j^k - t_j^k - \varepsilon_j)$, which, because $v_j q_j^k - t_j^k = b_j^k > 0$ by assumption, is positive when δ_i, ε_i, and ε_j are small enough, in contradiction with the zero-profit result. Therefore, type j must select c_i^k following buyer k's deviation, and for this deviation not to be profitable one must have

$$v(Q_i - Q_j + \delta_i) - v_i(Q_i - Q_j) - \varepsilon_i \leq 0. \tag{6}$$

Now, recall that, as a consequence of Assumption SC, $(v - v_i)(Q_i - Q_j) \geq 0$. Therefore, letting δ_i and ε_i go to zero in (6), we get $Q_i = Q_j$, and hence the equilibrium must be pooling. Replacing in (6), what we have shown is that for any small enough δ_i and ε_i such that $\tau_i(Q_i, T_i)\delta_i < \varepsilon_i$, one has $v\delta_i \leq \varepsilon_i$. As δ_i can be positive or negative, it follows that $\tau_i(Q_i, T_i) = v$. However, according to Lemma 2, one also has $\tau_j(Q_j, T_j) = v$ as $B_j > 0$. Because $(Q_i, T_i) = (Q_j, T_j)$, this contradicts Assumption SC. Hence the result. ■

Along with Proposition 3, this no-cross-subsidization result leads to the conclusion that one must have $Q_H \leq 0 \leq Q_L$ in any equilibrium. This excludes two types of equilibrium outcomes that have been emphasized in the literature: first, pooling outcomes such as the one described in Attar, Mariotti, and Salanié (2011), in which both types would trade the same nonzero quantity at a price equal to the average quality of the good; second, separating outcomes such as the one described by Jaynes (1978), Hellwig (1988), and Glosten (1994), and illustrated on Figure 1 below. If one leaves aside the case in which both types trade nonzero quantities on opposite sides of the market, the remaining possibilities for equilibrium outcomes have a structure reminiscent of Akerlof (1970): either there is no trade in the aggregate, or only one type actively trades at a fair price in the aggregate.

To illustrate the logic of the no-cross-subsidization result, consider a candidate separating equilibrium with positive quantities $Q_L > Q_H > 0$, as illustrated on Figure 1. The basic price structure of such an equilibrium is delineated in Proposition 3(ii).

—Insert Figure 1 Here—

Let k be a buyer whose profit b_H^k from trading with type H is positive. According to Lemma 3, the bundle (Q_H, T_H) remains available if buyer k removes his menu offer. He can thus attempt to pivot on (Q_H, T_H) to attract type L, which amounts to offer a contract $c_L^k = (Q_L - Q_H, T_L - T_H + \varepsilon_L)$, for some positive number ε_L. When ε_L is small enough, the loss for buyer k from trading c_L^k with type L is negligible, as the slope of the line segment connecting (Q_H, T_H) and (Q_L, T_L) is the fair price v_L. For buyer k's deviation to be profitable, he must make a profit when trading with type H. To do so, he can offer an additional contract $c_H^k = (q_H^k, t_H^k + \varepsilon_H)$, for some positive number ε_H. Because (q_H^k, t_H^k) was available for trade in equilibrium, c_L^k is more attractive than c_H^k for type L as long as ε_L is large enough relative to ε_H. Now, if type H trades c_H^k, the deviation is profitable, because, when ε_H is small enough, c_H^k yields a profit close to $b_H^k > 0$ when traded by type H, whereas the loss from trading c_L^k with type L is negligible. If type H trades c_L^k instead, the deviation is still profitable, because c_L^k yields a positive profit when traded by both types. This shows that there exists no separating equilibrium with positive quantities. The reasoning for a pooling equilibrium is slightly more involved, but reaches the same conclusion.

Remark. The proof of Proposition 4 shows that the reason why cross-subsidies are not sustainable in equilibrium is that it is possible for some buyer to neutralize the type on which he makes a loss by proposing her to mimic the behavior of the other type when facing the other buyers. A key feature of this deviation is that it can only be performed by a buyer who is actively and profitably trading with one type in equilibrium; indeed, an entrant would not be able to upset the above candidate equilibrium. Moreover, it is crucial for the argument that this buyer deviates to a menu including two non-trivial contracts targeted at the two types of the seller. Observe that this class of deviations was not considered in the early contributions of Jaynes (1978) and Glosten (1994). Jaynes (1978), who studies strategic competition between insurance providers under non-exclusivity, indeed restricts firms to use simple insurance policies. That is, each firm can propose at most one contract different from the no-trade contract.[15] As a consequence, an incumbent firm cannot profitably deviate by simultaneously making a loss when trading with the high-risk agent and compensating this loss when trading with the low-risk agent. Glosten (1994) characterizes an aggregate price-quantity schedule that is robust to entry. In our setting, this schedule would be as depicted on Figure 1. By contrast, we do not take the aggregate price-quantity schedule as given, but we derive it from the individual menus offered by the buyers.

So far, we have focused on the aggregate equilibrium implications of our model. We now briefly sketch a few implications for individual equilibrium trades. The following result shows that each traded contract yields zero profit, and that aggregate and individual equilibrium trades have the same sign.

Proposition 5 *In any equilibrium,* $b_j^k = 0$ *and* $q_L^k \geq 0 \geq q_H^k$ *for all* j *and* k*.*

Proposition 5 reinforces the basic insight of our model, according to which, in equilibrium, the seller can signal her type only through the sign of the quantities she trades. It follows that if a type does not trade in the aggregate, then she does not trade at all. Hence a pooling equilibrium, when it exists, is actually a no-trade equilibrium. Observe also that, when a type trades a nonzero quantity in the aggregate, there need not be more than one active buyer, as will be clear from considering the equilibria we will construct in Section 4.

3.5 Aggregate Equilibrium Trades

In this section, we fully characterize the candidate aggregate equilibrium trades, and we provide necessary conditions for the existence of an equilibrium. Given the price structure of equilibria delineated in Section 3.4, all that remains to be done is to give restrictions on each type's equilibrium marginal rate of substitution. Two cases need to be distinguished, according to whether or not a type's aggregate trade is zero in equilibrium.

Our first result is that, if type j does not trade in the aggregate, then her equilibrium marginal rate of substitution must lie between v and v_j. This is why an equilibrium may fail to exist for some parameters.

Lemma 4 *If* $Q_j = 0$*, then* $v_j - \tau_j(0,0)$ *and* $\tau_j(0,0) - v$ *have the same sign.*

The intuition for Lemma 4 is as follows. Suppose for instance that $Q_H = 0$. If $v_H > \tau_H(0,0)$, then any buyer could attract type H by proposing a contract offering to buy a small

[15] This assumption is maintained in the reformulation of Jaynes (1978) provided by Hellwig (1988).

positive quantity at a unit price lower than v_H. For this deviation not to be profitable, type L must also trade this contract, and one must have $\tau_H(0,0) \geq v$, so that the deviator makes a loss when both types trade this contract. The same reasoning applies if $v_H < \tau_H(0,0)$, by considering a contract offering to sell a small positive quantity at a unit price higher than v_H. The case $Q_L = 0$ can be handled in a symmetric way.

Our second result is that, if type i trades a nonzero quantity in the aggregate, then she must trade efficiently in equilibrium.

Lemma 5 *If $Q_i \neq 0$, then $\tau_i(Q_i, T_i) = v_i$.*

The intuition for Lemma 5 is as follows. Suppose for instance that $Q_L > 0$. As cross-subsidization cannot occur in equilibrium, $T_L = v_L Q_L$. If type L were trading inefficiently in equilibrium, that is, if $\tau_L(Q_L, T_L) \neq v_L$, then there would exist a contract offering to buy a positive quantity at a unit price lower than v_L, and that would give type L a strictly higher utility than (Q_L, T_L). Any of the buyers could profitably attract type L by proposing this contract, which would be even more profitable for the deviating buyer if traded by type H. Hence type L must trade efficiently in equilibrium. The case $Q_H < 0$ can be handled in a symmetric way.[16]

To state our characterization result, it is necessary to define first-best quantities. The following assumption ensures that these quantities are well defined.

Assumption FB *For each i, there exists Q_i^* such that $\tau_i(Q_i^*, v_i Q_i^*) = v_i$.*

Assumption FB states that Q_i^* is the efficient quantity for type i to trade at a unit price v_i that gives an aggregate zero profit for the buyers. In the pure trade model, Q_i^* is defined by $c_i'(Q_i^*) = v_i$. In the insurance model, because or the seller's risk aversion, efficiency requires full insurance for each type, so that $Q_i^* = W_G - W_B$.[17] An important consequence of the strict quasiconcavity of u_i is that $Q_i^* \geq 0$ if and only if $\tau_i(0,0) \leq v_i$, and that $Q_i^* = 0$ if and only if $\tau_i(0,0) = v_i$. We can now state our main characterization result.

Theorem 1 *If an equilibrium exists, then $\tau_L(0,0) \leq v \leq \tau_H(0,0)$. Moreover,*

- *If $v_L \leq \tau_L(0,0) \leq v \leq \tau_H(0,0) \leq v_H$, all equilibria are pooling, with $Q_L = Q_H = 0$.*

[16] It should be noted that the proofs of Lemmas 4 and 5 involve no pivoting arguments—or, what amounts to the same thing, only pivoting on the no-trade contract—and would therefore also go through in an exclusive competition context.

[17] A special feature of these two examples is that efficient quantities depend on the type of the seller, but not on the buyers' aggregate profit.

- *Otherwise, all equilibria are separating, and*

  (i) If $\tau_L(0,0) < v_L < v < v_H < \tau_H(0,0)$*, then* $Q_L = Q_L^* > 0$ *and* $Q_H = Q_H^* < 0$*.*

  (ii) If $\tau_L(0,0) < v_L < v \leq \tau_H(0,0) \leq v_H$*, then* $Q_L = Q_L^* > 0$ *and* $Q_H = 0$*.*

  (iii) If $v_L \leq \tau_L(0,0) \leq v < v_H < \tau_H(0,0)$*, then* $Q_L = 0$ *and* $Q_H = Q_H^* < 0$*.*

The first message of Theorem 1 is a negative one: the non-exclusive competition game need not have an equilibrium. A necessary condition for an equilibrium to exist is that, at a price equal to the average quality v, type L would like to *sell* some of the good, whereas type H would like to *buy* some of it. In the pure trade model, no equilibrium exists if the cost function of type L is such that $c_L'(0) > v$, or if the cost function of type H is such that $c_H'(0) < v$; that is, if the low-cost type L is not eager enough to sell, or if the high-cost type H is too eager to sell. In the insurance model, no equilibrium exists if $[\pi_H/(1-\pi_H)]u'(W_G)/u'(W_B) < \pi/(1-\pi)$, where $\pi \equiv m_L\pi_L + m_H\pi_H$, that is, if the low-risk agent H is too eager to buy insurance.[18] Overall, Theorem 1 reinforces the insight of the no-cross-subsidization result: an equilibrium exists only if the adverse selection problem is severe enough, so that both types' incentives to trade are not too closely aligned. On a more positive note, we show below in Theorem 2 that the necessary condition $\tau_L(0,0) \leq v \leq \tau_H(0,0)$ also turns out to be sufficient for the existence of an equilibrium. Thus Theorem 1 provides a complete description of the structure of aggregate equilibrium outcomes, which is summarized on Figure 2.

—Insert Figure 2 Here—

Second, Theorem 1 shows that pooling requires $v_L \leq \tau_L(0,0)$ and $v_H \geq \tau_H(0,0)$; by the no-cross-subsidization result, we already know that a pooling equilibrium involves no trade for both types. The conditions $v_L \leq \tau_L(0,0)$ and $v_H \geq \tau_H(0,0)$ together imply that $Q_L^* \leq 0 \leq Q_H^*$. When one of these inequalities is strict, the first-best quantities are not implementable. Thus pooling requires a strong form of nonresponsiveness: namely, in the first-best scenario, type L would like to buy, and type H to sell. This cannot arise in the insurance model, for in that case $Q_L^* = Q_H^* = W_G - W_B$. Therefore, the insurance model admits no pooling equilibrium. In the pure trade model, a pooling equilibrium exists only if $c_L'(0) \geq v_L$ and $c_H'(0) \leq v_H$.[19]

[18] This result is also obtained in Ales and Maziero (2011) assuming free entry. The second condition $\tau_L(0,0) \leq v$, or, equivalently, $[\pi_L/(1-\pi_L)]u'(W_G)/u'(W_B) \leq \pi/(1-\pi)$, is automatically satisfied in the insurance model as $\pi > \pi_L$ and $u'(W_B) > u'(W_G)$.

[19] This is for instance the case in the Biais, Martimort, and Rochet (2000) setting if $\theta_L \geq v_L$ and $\theta_H \leq v_H$. It should however be noted that they explicitly rule out this parameter configuration.

Third, Theorem 1 states that in a separating equilibrium, at least one of the types trades efficiently. In case (i), types L and H's preferences are sufficiently far apart from each other, in the sense that $Q_L^* > 0 > Q_H^*$: in the first-best scenario, type L would like to sell, and type H to buy. In that case, both types end up trading their first-best quantities in equilibrium. Clearly, the insurance model admits no equilibrium of this kind. In the pure trade model, a first-best equilibrium may exist if $c_L'(0) < v_L$ and $c_H'(0) > v_H$. In case (ii), both Q_L^* and Q_H^* are nonnegative: in the first-best scenario, both types would like to sell. The unique candidate equilibrium outcome is then similar to the one which prevails in Akerlof (1970): type L trades efficiently, while type H does not trade at all. This is the situation that prevails in the insurance model when an equilibrium exists: in that case, the high-risk agent L obtains full insurance at an actuarially fair price, while the low-risk agent H purchases no insurance. In the pure trade model, this type of equilibrium may exist if $c_L'(0) < v_L$ and $c_H'(0) \leq v_H$. Finally, case (iii) is symmetric to case (ii), exchanging the roles of types L and H. Note that, in any separating equilibrium, each type strictly prefers her aggregate equilibrium trade to that of the other type. This contrasts with the predictions of models of exclusive competition under adverse selection, such as Rothschild and Stiglitz's (1976), in which the high-risk agent L is indifferent between her equilibrium contract and that of the low-risk agent H.

Remark It is interesting to compare the conclusions of Theorem 1 with those reached by Attar, Mariotti, and Salanié (2011). Compared to the present setup, the two distinguishing features of their model is that the seller has linear preferences, $u_i(Q,T) = T - \theta_i Q$, and makes choices under an aggregate capacity constraint, $Q \leq 1$. Observe that, in this context, type i's marginal rate of substitution is constant and equal to θ_i up to capacity. In a two-type version of their model in which there are potential gains from trade for each type, that is, $v_L > \theta_L$ and $v_H > \theta_H$, Attar, Mariotti, and Salanié (2011) show that the non-exclusive competition game always admits an equilibrium, that the buyers earn zero profits, and that the aggregate equilibrium allocation is generically unique. If $\theta_H > v$, the equilibrium is similar to the separating equilibrium found in case (ii) of Theorem 1: type L trades efficiently, $Q_L = 1$ and $T_L = v_L$, while type H does not trade at all, $Q_H = T_H = 0$. By contrast, if $\theta_H < v$, the situation is markedly different from that described in Theorem 1. First, an equilibrium exists, whereas, in the analogous situation where $\tau_H(0,0) < v$, no equilibrium exists in our model. Second, any equilibrium is pooling and efficient, that is, $Q_L = Q_H = 1$ and $T_L = T_H = v$, whereas cross-subsidies and, therefore, non-trivial pooling equilibria are ruled out in our model. The key difference between the two setups that explains these discrepancies is that, in the present paper, we do not require the seller's choices to satisfy an aggregate capacity constraint. This implies that some deviations that are crucial for our characterization result are not available in Attar, Mariotti, and Salanié (2011). A case in point is the no cross-subsidization result: key to the proof of Proposition 4 is the possibility, for a deviator that makes profit when trading with type j, to pivot on (Q_j, T_j) to attract type i, while preserving the profit he makes with type j. However, for the argument to go through, there must be no restrictions on the quantities traded in such deviations; in particular, it is crucial that the deviator be able to induce type i to consume more than Q_i in the aggregate.[20] This, however, is precisely what is impossible to do in the presence of a capacity constraint when both types trade up to capacity, as in the pooling equilibrium described in Attar, Mariotti, and Salanié (2011).

4 Equilibrium Existence

To establish the existence of an equilibrium, we impose the following technical assumption on preferences.

Assumption T *There exist* $\underline{Q}_H$ *and* $\overline{Q}_L$ *such that*

$$\tau_H(Q,T) < v_H \text{ if } Q < \underline{Q}_H, \quad \text{and} \quad \tau_L(Q,T) > v_L \text{ if } Q > \overline{Q}_L,$$

uniformly in T.

Assumption T ensures that equilibrium menus can be constructed as compact sets of contracts. It should be emphasized that the restrictions it imposes on preferences are rather mild. In the pure trade model, because of the quasilinearity of preferences, Assumption T follows from Assumption FB, and one can take $\underline{Q}_H = Q^*_H$ and $\overline{Q}_L = Q^*_L$. In the insurance model, Assumption T follows from the seller's risk aversion, and one can take $\underline{Q}_H = \overline{Q}_L = W_G - W_B = Q^*_H = Q^*_L$.

Theorem 2 *An equilibrium exists if and only if* $\tau_L(0,0) \leq v \leq \tau_H(0,0)$. *Moreover there exists* $\overline{Q} > 0 > \underline{Q}$ *such that any equilibrium can be supported by at least two buyers posting the same tariff*

$$t(q) \equiv \min\{v_L q, v_H q\}, \qquad \underline{Q} \leq q \leq \overline{Q},$$

[20] Formally, it follows from the proof of Proposition 4 that, if $B_H > 0$ in a pooling equilibrium where each type trades a positive aggregate quantity Q, then, for any small enough additional trade $(\delta_L, \varepsilon_L)$ such that $\tau_L(Q,T)\delta_L < \varepsilon_L$, and that would thus attract type L, one must have $v\delta_L \leq \varepsilon_L$. If there are no restrictions on δ_L, this implies that $\tau_L(Q,T) = v$, from which a contradiction can be derived using Lemma 2. Yet if, for some reason, only nonpositive δ_L were admissible, say, because the seller could not trade more than Q in the aggregate, then one could only conclude that $\tau_L(Q,T) \leq v$, from which no contradiction would follow.

while the other buyers stay inactive.

Theorem 2 shows that the necessary condition for the existence of an equilibrium given in Theorem 1 is also sufficient. These two results together provide a complete description of the aggregate equilibrium outcomes of our game. As for individual strategies, the tariffs chosen here to support equilibria entail linear pricing for both positive and negative quantities, with a kink at zero that one may interpret as a bid-ask spread. Another noteworthy feature of these strategies is that in no case can a buyer make a loss. Hence, even if these strategies involve contracts that are not traded in equilibrium, these latent contracts cannot turn out to be costly for the buyers.

The lower and upper bounds $\underline{Q}$ and $\overline{Q}$ were introduced only to make sure that the corresponding menus of contracts are compact, but the intuition of the result is easier to grasp when one eliminates these bounds. Suppose that one of the buyers were to deviate, for instance in the hope of making profits from trading with type H. Because no buyer can make a loss, this implies that, following the deviation, the aggregate trade $(\tilde{Q}_H, \tilde{T}_H)$ chosen by type H should verify $v_H\tilde{Q}_H > \tilde{T}_H$. Let T^- be the tariff posted in the aggregate by the deviator's competitors. As the trade $(\tilde{Q}_H, T^-(\tilde{Q}_H))$ is available anyway, we get $\tilde{T}_H \geq T^-(\tilde{Q}_H)$, which implies $\tilde{Q}_H > 0$. Because we have $\tau_H(0,0) \geq v$ by assumption, we also get that the final transfer $\tilde{T}_H$ cannot be less than $v\tilde{Q}_H$.

Similarly define $(\tilde{Q}_L, \tilde{T}_L)$ as the aggregate trade of type L following the deviation. Type L could trade as type H does, and sell in addition a quantity $\tilde{Q}_L - \tilde{Q}_H$ in exchange for a transfer $T^-(\tilde{Q}_L - \tilde{Q}_H)$. By the single-crossing condition, $\tilde{Q}_L \geq \tilde{Q}_H$. Hence type L can end up selling an aggregate quantity $\tilde{Q}_L$ in exchange for a transfer $\tilde{T}_H + v_L(\tilde{Q}_L - \tilde{Q}_H)$. As she chooses to trade $(\tilde{Q}_L, \tilde{T}_L)$ instead, this shows that $\tilde{T}_L \geq \tilde{T}_H + v_L(\tilde{Q}_L - \tilde{Q}_H)$. But we already know that $\tilde{T}_H \geq v\tilde{Q}_H$. In line with (4), we obtain that aggregate profits cannot be positive. Hence, because the other buyers cannot make losses, the deviation cannot be profitable.

The fact that buyers cannot make losses should not be interpreted as an extreme aversion to the hazard of trading under adverse selection. Recall indeed from Proposition 5 that, in equilibrium, the seller credibly signals her information by the sign of the trade she proposes to make with each buyer. The buyers then become perfectly informed of the seller's type, and Bertrand competition reduces prices down to their willingness-to-pay. Hence buyers cannot make losses, but they do not make any profits neither. The fact that only two active buyers are needed to sustain an equilibrium confirms the Bertrand-like nature of non-exclusive competition in our setting.

Finally, we made no attempt at minimizing the size of equilibrium menus. The proof of Theorem 2 provides such an implementation in the efficient case (i) of Theorem 1, for which it is sufficient that at least two buyers propose the efficient trades $(Q^*_L, v_L Q^*_L)$ and $(Q^*_H, v_H Q^*_H)$; but for the other more complex cases, we only got partial results. The question of minimum implementation thus remains open.

5 Conclusion

In this paper, we analyzed the impact of adverse selection on markets where competition is non-exclusive. We fully characterized aggregate equilibrium allocations, which are uniquely determined, and we provided a necessary and sufficient condition for the existence of a pure-strategy equilibrium. Our results show that, under non-exclusivity, market breakdown may arise in a competitive environment where buyers compete through arbitrary menu offers: specifically, whenever first-best allocations cannot be achieved, equilibria when they exist involve no trade for at least one type of the seller.

These predictions contrast with those that obtain under exclusive competition, namely, that one type of the seller trades efficiently, while the other type signals the quality of the good she offers by trading a suboptimal, but nonzero quantity of this good. When competition is non-exclusive, each buyer's inability to control the seller's trades with his opponents creates additional deviation opportunities. This makes screening more costly, and implies that the seller either trades efficiently, or does not trade at all.

Our results may explain why some markets are underdeveloped. For instance, theory predicts that individual should find it in their best interest to annuitize a large part of their lifetime savings (Yaari (1965)), yet in practice the demand for annuities remains low. Although several demand-side explanations, such as bequest motives, have been proposed to solve this puzzle, our analysis points at an alternative supply-side explanation based on non-exclusivity and adverse selection. As mentioned in the introduction, non-exclusivity is a common feature of annuity markets. Adverse selection may arise because individuals have private information about their survival prospects. In this context, our analysis predicts that market participation should be limited to individuals with the best survival prospects, who have more to gain from purchasing annuities. This severely limits the size of the market, unless participation is made mandatory.

There has been so far few investigations of the welfare implications of adverse selection in markets where competition is non-exclusive. A natural development of our analysis would be to study the decision problem faced by a planner seeking to implement an efficient allocation, subject to informational constraints and to the constraint that exclusivity be non-enforceable. It is unclear that such a planner may improve on the market allocations characterized in this paper. If he could, this would provide new theoretical insights in favor of welfare-based regulatory interventions, in particular in the context of financial or insurance markets.

Appendix

Proof of Lemma 1. Let i, k, q, and t be as in the assumption of the lemma, and suppose that $v_i q - t > b_i^k$. Buyer k can deviate by proposing a menu consisting of the no-trade contract and of the contracts $c_i^k = (q, t + \varepsilon_i)$ and $c_j^k = (q_j^k, t_j^k + \varepsilon_j)$, for some positive numbers ε_i and ε_j. Given the assumption of the lemma, by trading c_i^k with buyer k and $(Q_i - q, T_i - t)$ with the buyers other than k, type i gets a utility $u_i(Q_i, T_i + \varepsilon_i) > U_i$. In equilibrium one has $U_i \geq z_i^{-k}(q_j^k, t_j^k)$, and the function z_i^{-k} is continuous. Thus $u_i(Q_i, T_i + \varepsilon_i) > z_i^{-k}(q_j^k, t_j^k + \varepsilon_j)$ for all small enough ε_j, so that, for any such ε_j, type i must select c_i^k following buyer k's deviation. Consider now type j's behavior. By trading c_j^k, type j can get a utility $u_j(Q_j, T_j + \varepsilon_j) > U_j$, so that she must select either c_i^k or c_j^k following buyer k's deviation. If type j selects c_j^k, then, by deviating, buyer k earns a profit

$$m_i(v_i q - t - \varepsilon_i) + m_j(v_j q_j^k - t_j^k - \varepsilon_j) = m_i(v_i q - t) + m_j b_j^k - (m_i \varepsilon_i + m_j \varepsilon_j).$$

However, from the assumption that $v_i q - t > b_i^k$, this is strictly higher than b^k when ε_i and ε_j are small enough, a contradiction. Hence type j must select c_i^k following buyer k's deviation. In equilibrium this deviation cannot be profitable, so that $vq - t - \varepsilon_i \leq b^k$. Letting ε_i go to zero yields the desired implication. The result follows. ■

Proof of Lemma 2. If $B_j > 0$, then one must have $T_j = vQ_j$ by Proposition 3. Any buyer k can deviate by proposing a menu consisting of the no-trade contract and of the contract $c_j^k = (Q_j + \delta_j, T_j + \varepsilon_j)$, for some numbers δ_j and ε_j. Suppose by way of contradiction that $\tau_j(Q_j, T_j) \neq v$. Then one can choose δ_j and ε_j such that $\tau_j(Q_j, T_j)\delta_j < \varepsilon_j < v\delta_j$. When δ_j and ε_j are small enough, the first inequality guarantees that type j can strictly increase her utility by trading c_j^k with buyer k. If type i trades c_j^k, then buyer k's profit from this deviation is $v(Q_j + \delta_j) - (T_j + \varepsilon_j) = v\delta_j - \varepsilon_j > 0$, in contradiction with the zero-profit result. Therefore, type i must not trade with buyer k, and for this deviation not to be profitable one must have $m_j[v_j(Q_j + \delta_j) - (T_j + \varepsilon_j)] = m_j(B_j + v_j\delta_j - \varepsilon_j) \leq 0$. Letting δ_j and ε_j go to zero yields $B_j \leq 0$, a contradiction. The result follows. ■

Proof of Lemma 3. Suppose first that $U_j > z_j^{-k}(0, 0)$ for some k. Then buyer k can deviate by proposing a menu consisting of the no-trade contract and of the contract $c_j^k = (Q_j, T_j - \varepsilon)$, for some positive number ε. When ε is small enough, one has $u_j(Q_j, T_j - \varepsilon) > z_j^{-k}(0,0)$, so that type j trades the contract c_j^k following buyer k's deviation. If type i does not trade the contract c_j^k, buyer k's profit from this deviation is $m_j(v_j Q_j - T_j + \varepsilon) = m_j(B_j + \varepsilon) > 0$, in contradiction with the zero-profit result. If type i trades the contract c_j^k, then, because $T_j = vQ_j$ by Proposition 3, buyer k's profit from this deviation is $vQ_j - T_j + \varepsilon = \varepsilon > 0$, again in contradiction with the zero-profit result. As in any case $U_j \geq z_j^{-k}(0,0)$, it must be that $U_j = z_j^{-k}(0,0)$ for each k. It follows that, for any buyer k, there exists an aggregate trade (Q^{-k}, T^{-k}) with the buyers other than k such that $u_j(Q^{-k}, T^{-k}) = U_j$.

Suppose now that $Q^{-k} \neq Q_j$. Then, from the strict quasiconcavity of u_i and Lemma 2, one must have $T^{-k} > vQ^{-k}$. We now examine two deviations for buyer k that pivot on (Q^{-k}, T^{-k}). First, define (q_1, t_1) such that $(q_1, t_1) + (Q^{-k}, T^{-k}) = (Q_j, T_j)$. Then the quantity $Q_j - q_1$ can be traded with the buyers other than k in exchange for a transfer $T_j - t_1$. Moreover, using the fact that $T_j = vQ_j$ by Proposition 3, and that $T^{-k} > vQ^{-k}$, one gets

$$vq_1 - t_1 = v(Q_j - Q^{-k}) - (T_j - T^{-k}) = T^{-k} - vQ^{-k} > 0.$$

Therefore, by Lemma 1, one must have $v_j q_1 - t_1 \leq b_j^k$, that is, using again $T_j = vQ_j$, $T^{-k} - v_j Q^{-k} + (v_j - v)Q_j \leq b_j^k$. Because $T^{-k} > vQ^{-k}$, this implies that

$$(v_j - v)(Q_j - Q^{-k}) < b_j^k. \tag{7}$$

Second, define (q_2, t_2) such that $(q_2, t_2) + (Q^{-k}, T^{-k}) = (Q_i, T_i)$. Then the quantity $Q_i - q_1$ can be traded with the buyers other than k in exchange for a transfer $T_i - t_1$. Moreover, using the fact that $S_i = 0$ and $T_j = vQ_j$ by Proposition 3, that $T^{-k} > vQ^{-k}$, and that $(v - v_i)(Q_i - Q_j) \geq 0$ by Assumption SC, one gets

$$\begin{aligned} vq_2 - t_2 &= v(Q_i - Q^{-k}) - (T_i - T^{-k}) \\ &= T^{-k} - vQ^{-k} + vQ_i - [T_j + v_i(Q_i - Q_j) - S_i] \\ &= T^{-k} - vQ^{-k} + (v - v_i)(Q_i - Q_j) \\ &> 0. \end{aligned}$$

Therefore, by Lemma 1, one must have $v_i q_2 - t_2 \leq b_i^k$, that is, using again $S_i = 0$ and $T_j = vQ_j$, $T^{-k} - v_i Q^{-k} + (v_i - v)Q_j \leq b_i^k$. As $T^{-k} > vQ^{-k}$, this implies that

$$(v_i - v)(Q_j - Q^{-k}) < b_i^k. \tag{8}$$

Because $v = m_i v_i + m_j v_j$, and $m_i b_i^k + m_j b_j^k = 0$ by the zero-profit result, averaging (7) and

(8) yields $0 < 0$, a contradiction. Therefore, one must have $Q^{-k} = Q_j$, and thus $T^{-k} = T_j$ as $u_j(Q^{-k}, T^{-k}) = U_j = u_j(Q_j, T_j)$. The result follows. ■

Proof of Proposition 5. We first prove that $b_j^k = 0$ for all j and k. Suppose by way of contradiction that $b_j^k > 0$ for some j and k. We first show that $S_i = S_j = 0$. To prove that $S_i = 0$, observe that, by the no-cross-subsidization result, one has $b_j^l < 0 = B_j$ for some $l \neq k$. From (5), this implies that $m_i S_i \geq B - b^l$. Because $B - b^l = 0$ by the zero-profit result, and because $S_i \leq 0$ by Proposition 1, it follows that $S_i = 0$. To prove that $S_j = 0$, observe that if $b_j^k > 0$, then $b_i^k < 0 = B_i$ by the zero-profit result and the no cross-subsidization result. Arguing as for S_i, it follows that $S_j = 0$. Hence $S_i = S_j = 0$, as claimed. As $S_i + S_j = (v_i - v_j)(Q_i - Q_j)$, one must have $Q_i = Q_j$, and the equilibrium is pooling, with $(Q_i, T_i) = (Q_j, T_j) = (0, 0)$. Now, because $b_j^k > 0$, and because $(Q_j, T_j) = (0, 0)$ can obviously be traded with the buyers other than k, one can show as in the proof of Proposition 4 that $\tau_i(0, 0) = v$. Finally, consider buyer l as above. As $b_j^l < 0$, one has $b_i^l > 0$ by the zero-profit result. Because $(Q_i, T_i) = (0, 0)$ can obviously be traded with the buyers other than l, it follows along the same lines that $\tau_j(0, 0) = v$ as well, which contradicts Assumption SC. Hence the result.

We next prove that $q_L^k \geq 0 \geq q_H^k$ for each k. Because $v_H > v_L$ and

$$s_i^k = v_i(q_i^k - q_j^k) - (t_i^k - t_j^k) = b_i^k - b_j^k - (v_i - v_j)q_j^k = (v_j - v_i)q_j^k$$

as $b_i^k = b_j^k = 0$, we only need to show that $s_i^k \leq 0$ for all i and k. Choose i, k, and $l \neq k$, and set $q \equiv q_i^k + q_i^l - q_j^k$ and $t \equiv t_i^k + t_i^l - t_j^k$. Then the quantity $Q_i - q = q_j^k + \sum_{m \neq k,l} q_i^m$ can be traded with the buyers other than l, in exchange for a transfer $T_i - t = t_j^k + \sum_{m \neq k,l} t_i^m$. We can thus apply Lemma 1. One has

$$v_i q - t - b_i^l = v_i(q_i^k + q_i^l - q_j^k) - (t_i^k + t_i^l - t_j^k) - b_i^l = s_i^k$$

and

$$v_j q - t - b_j^l = v_j(q_i^k + q_i^l - q_j^k) - (t_i^k + t_i^l - t_j^k) - b_j^l = -(s_j^k + s_j^l).$$

Therefore, according to (1),

$$s_i^k > 0 \ \text{ implies } \ m_i s_i^k \leq m_j(s_j^k + s_j^l). \tag{9}$$

Now, suppose by way of contradiction that $s_i^k > 0$ for some i and k. Then, by (9),

$$m_i s_i^k \leq m_j(s_j^k + s_j^l) \tag{10}$$

for each $l \neq k$. Summing on $l \neq k$ yields

$$(n-1)m_i s_i^k \leq m_j[S_j + (n-2)s_j^k].$$

From Proposition 1, we know that $S_j \leq 0$. Hence, if $s_i^k > 0$, one must also have $s_j^k > 0$. Exchanging the roles of i and j in (9) yields

$$m_j s_j^k \leq m_i(s_i^k + s_i^l) \tag{11}$$

for each $l \neq k$. Combining (10) and (11) leads to $m_i s_i^k \leq m_j s_j^l + m_i(s_i^k + s_i^l)$, or, equivalently, $m_i s_i^l + m_j s_j^l \geq 0$ for each $l \neq k$. Note that we also have $m_i s_i^k + m_j s_j^k > 0$ as both s_i^k and s_j^k are strictly positive. Summing all these inequalities yields $m_i S_i + m_j S_j > 0$, in contradiction with Proposition 1. Hence the result. ∎

Proof of Lemma 4. Suppose that $Q_j = 0$. If $\tau_j(0,0) = v_j$, the result is immediate. Suppose then that $\tau_j(0,0) \neq v_j$. Any buyer k can deviate by proposing a menu consisting of the no-trade contract and of the contract $c_j^k = (\delta_j, \varepsilon_j)$, for some numbers δ_j and ε_j. Choose δ_j and ε_j such that $\tau_j(0,0)\delta_j < \varepsilon_j$. This ensures that, when δ_j and ε_j are small enough, type j can strictly increase her utility by trading c_j^k with buyer k. If moreover $v_j \delta_j > \varepsilon_j$, then type i must also trade c_j^k following buyer k's deviation, and one must have $\varepsilon_j \geq v\delta_j$, for, otherwise, this deviation would be profitable. Thus we have shown that for any small enough δ_j and ε_j, $\tau_j(0,0)\delta_j < \varepsilon_j < v_j\delta_j$ implies that $\varepsilon_j \geq v\delta_j$, which is equivalent to the statement of the lemma. The result follows. ∎

Proof of Lemma 5. By the no-cross-subsidization result, if $Q_i \neq 0$, the equilibrium must be separating. Moreover, from Proposition 3, one must have $T_i = v_i Q_i$. Suppose by way of contradiction that $\tau_i(Q_i, T_i) \neq v_i$. Then any buyer k can deviate by proposing a menu consisting of the no-trade contract and of the contract $c_i^k = (q_i, t_i)$, for some numbers q_i and t_i. As $\tau_i(Q_i, T_i) \neq v_i$, it follows from the strict quasiconcavity of u_i that one can choose (q_i, t_i) close to (Q_i, T_i) such that $U_i < u_i(q_i, t_i)$ and $t_i < v_i q_i$, where q_i is positive if $i = L$, and negative if $i = H$. The first inequality guarantees that type i trades c_i^k following buyer k's deviation. As $v_i q_i > t_i$, type j must also trade c_i^k following buyer k's deviation, and one must have $t_i \geq v q_i$, for, otherwise, this deviation would be profitable. Overall, we have shown that $v_i q_i > v q_i$. Because q_i is positive if $i = L$ and negative if $i = H$, and because $v_H > v > v_L$, we obtain a contradiction in both cases. The result follows. ∎

Proof of Theorem 1. Suppose first that a pooling equilibrium exists. Then, according to the no-cross-subsidization result, $Q_L = Q_H = 0$. Lemma 4 then implies that

$$v_L \leq \tau_L(0,0) \leq v \leq \tau_H(0,0) \leq v_H. \tag{12}$$

Suppose next that a separating equilibrium exists. Then, according again to the no-cross-subsidization result, only three scenarios are possible.

(i) In the first case, $Q_H < 0 < Q_L$. Then, by Proposition 3, $T_L = v_L Q_L$ and $T_H = v_H Q_H$. Moreover, by Lemma 5, $\tau_L(Q_L, T_L) = v_L$ and $\tau_L(Q_H, T_H) = v_H$. As a result, $Q_L = Q_L^*$ and $Q_H = Q_H^*$, so that $Q_H^* < 0 < Q_L^*$. The strict quasiconcavity of u_i then implies that

$$\tau_L(0,0) < v_L \quad \text{and} \quad \tau_H(0,0) > v_H. \tag{13}$$

(ii) In the second case, $Q_H = 0 < Q_L$. Then, by Lemma 4, $v \le \tau_H(0,0) \le v_H$. Moreover, by Proposition 3, $T_L = v_L Q_L$. Finally, by Lemma 5, $\tau_L(Q_L, T_L) = v_L$. As a result $Q_L = Q_L^*$, so that $Q_L^* > 0$. The strict quasiconcavity of u_i then implies that

$$\tau_L(0,0) < v_L \quad \text{and} \quad v \le \tau_H(0,0) \le v_H. \tag{14}$$

(iii) In the third case, $Q_H < 0 = Q_L$. Then, by Lemma 4, $v_L \le \tau_L(0,0) \le v$. Moreover, by Proposition 3, $T_H = v_H Q_H$. Finally, by Lemma 5, $\tau_H(Q_H, T_H) = v_H$. As a result $Q_H = Q_H^*$, so that $Q_H^* < 0$. The strict quasiconcavity of u_i then implies that

$$v_L \le \tau_L(0,0) \le v \quad \text{and} \quad \tau_H(0,0) > v_H. \tag{15}$$

To conclude the proof, observe that, from (12) to (15), an equilibrium exists only if $\tau_L(0,0) \le v \le \tau_H(0,0)$. As conditions (12) to (15) are mutually exclusive, the characterization of the candidate aggregate equilibrium trades is complete. Hence the result. ■

Proof of Theorem 2. Choose an integer m, $2 \le m \le n$, and fix $\underline{Q}$ and $\overline{Q}$ such that $\underline{Q} < \min\{0, \underline{Q}_H\}/(m-1)$ and $\overline{Q} > \max\{0, \overline{Q}_L\}/(m-1)$. Suppose that m buyers post the tariff t defined as in the theorem, while the other buyers stay inactive and only propose to trade $(0,0)$. Consider any buyer. In the aggregate his competitors post the tariff

$$T^-(Q^-) \equiv \min\{v_L Q^-, v_H Q^-\}, \qquad \underline{Q}_1 \le Q^- \le \overline{Q}_1,$$

where Q^- refers to the aggregate quantity traded by these competitors. Here $\underline{Q}_1$ is either $m\underline{Q}_1$ or $(m-1)\underline{Q}_1$, and thus is no greater than $\underline{Q}_H$; and similarly for $\overline{Q}_1$, which cannot be smaller than $\overline{Q}_L$. Note also that if the efficient trade Q_H^* is negative, then $\underline{Q}_1 \le Q_H^* \le \overline{Q}_1$; and symmetrically for Q_L^*.

Suppose that our buyer deviates, and ends up trading (q_L, t_L) with type L and (q_H, t_H) with type H. For the deviation to be profitable, he must make a positive profit with at least one type, say type H (the proof for type L is symmetrical). Hence $v_H q_H > t_H$. Define

$Q_i^- \in [\underline{Q}_1, \overline{Q}_1]$ as the quantity traded by type i with the deviator's competitors, following his deviation. Define also $\tilde{Q}_i$ as the total quantity traded by type i, so that $\tilde{Q}_i = q_i + Q_i^-$, and $\tilde{T}_i$ as the total transfer obtained by type i, so that $\tilde{T}_i = t_i + T^-(Q_i^-)$. Notice that the tariff T^- is such that other buyers cannot make losses following the deviation. Therefore, as $v_H q_H > t_H$, one must have $v_H \tilde{Q}_H > \tilde{T}_H$. Because the null trade is available, we get

$$u_H(\tilde{Q}_H, v_H \tilde{Q}_H) > u_H(\tilde{Q}_H, \tilde{T}_H) \geq u_H(0,0). \tag{16}$$

If $\tilde{Q}_H < 0$, then (16) implies that $\tau_H(0,0) > v_H$, so that $Q_H^* < 0$. By construction of the tariff T^-, type H can then trade $(Q_H^*,\, v_H Q_H^*)$ with the deviator's competitors, thereby getting a utility $u_H(Q_H^*, v_H Q_H^*) = \max_Q \{u_H(Q, v_H Q)\} > u_H(\tilde{Q}_H, \tilde{T}_H)$ by (16), a contradiction. The case $\tilde{Q}_H = 0$ is also easily excluded, as the deviator cannot attract type H by proposing her to pay a positive transfer $-\tilde{T}_H$ for a zero quantity. Therefore, it must be that $\tilde{Q}_H > 0$. From (16), we now get $\tau_H(0,0) < v_H$.[21] Because $\tau_H(0,0) \geq v$ by assumption, from (16) again we get $\tilde{T}_H \geq v\tilde{Q}_H$. Finally, notice that $T^-(Q^-) \leq vQ^-$ for all $Q^- \in [\underline{Q}_1, \overline{Q}_1]$. Thus $v\tilde{Q}_H = vq_H + vQ_H^- \leq \tilde{T}_H = t_H + T^-(Q_H^-) < v_H q_H + vQ_H^-$, and hence $q_H > 0$.

Type L may also choose to trade (q_H, t_H) with the deviator. He would then have to choose some Q^- to maximize $u_L(q_H + Q^-, t_H + T^-(Q^-))$, subject to $\underline{Q}_1 \leq Q^- \leq \overline{Q}_1$. Notice first from the definition of $\overline{Q}_L$ that the constraint $Q^- \leq \overline{Q}_1$ does not play any role: indeed $q_H > 0$, so that, when Q^- reaches its upper bound $\overline{Q}_1$, the total quantity traded $q_H + Q^-$ is higher than $\overline{Q}_L$, and therefore type L's marginal rate of substitution is higher than v_L by Assumption T. We can thus eliminate the constraint $Q^- \leq \overline{Q}_1$, taking care of extending the tariff T^- beyond $\overline{Q}_1$ by setting $T^-(Q^-) \equiv v_L Q^-$ for all $Q^- > \overline{Q}_1$. Now, $\tilde{Q}_L - q_H$ satisfies the remaining constraint $\underline{Q}_1 \leq Q^-$; indeed, thanks to Assumption SC, we have $\tilde{Q}_L \geq \tilde{Q}_H$, so that $\tilde{Q}_L - q_H \geq Q_H^- \geq \underline{Q}_1$. We thus have shown that type L can get at least a utility $u_L(\tilde{Q}_L, t_H + T^-(\tilde{Q}_L - q_H))$. Observe that the transfer in this expression can be rewritten as $\tilde{T}_H + T^-(\tilde{Q}_L - q_H) - T^-(Q_H^-)$, which is no less than $\tilde{T}_H + v_L(\tilde{Q}_L - \tilde{Q}_H)$ by concavity of T^-. Because type L is supposed to end up with the utility $u_L(\tilde{Q}_L, \tilde{T}_L)$ following the deviation, it follows that $\tilde{T}_L \geq \tilde{T}_H + v_L(\tilde{Q}_L - \tilde{Q}_H)$. Moreover, as shown above, $\tilde{T}_H \geq v\tilde{Q}_H$. Therefore, the aggregate profit, which may as usual be written as $v\tilde{Q}_H - \tilde{T}_H + m_L[v_L(\tilde{Q}_L - \tilde{Q}_H) - (\tilde{T}_L - \tilde{T}_H)]$, is at most zero. Because the tariff T^- is such that the deviator's competitors cannot make losses, the deviation cannot be profitable. Hence the result. ■.

[21] Note that this condition excludes the efficient case (i) of Theorem 1. In that simple case, an inspection of the above lines reveals that we have only used the fact that $(Q_H^*, v_H Q_H^*)$ is offered by the deviator's competitors. We have thus shown that in the efficient case (i) of Theorem 1, any equilibrium can be sustained by having at least two players posting the two trades $(Q_H^*, v_H Q_H^*)$ and $(Q_L^*, v_L Q_L^*)$.

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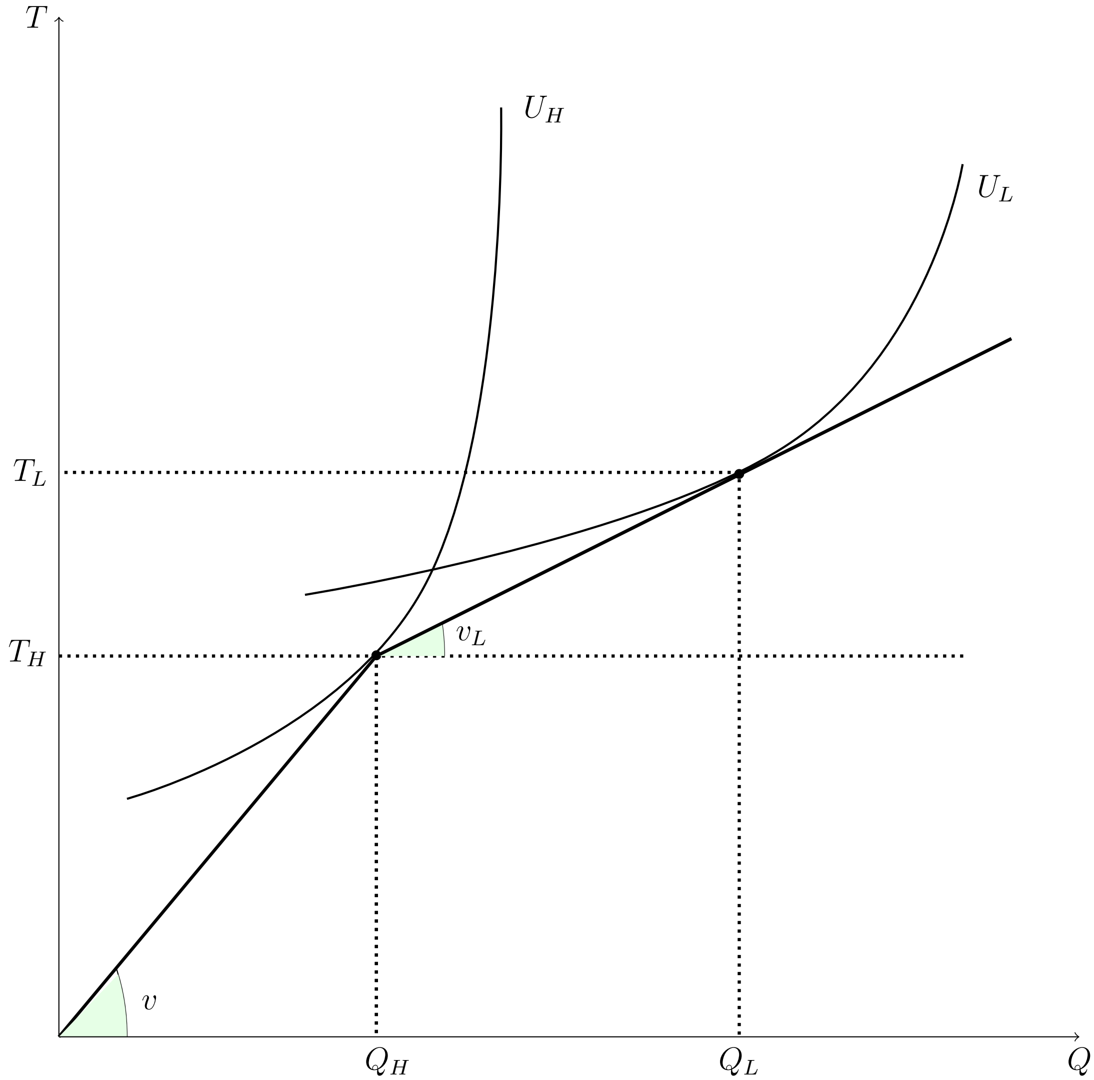

Figure 1 This figure depicts a candidate separating equilibrium with $Q_L > Q_H > 0$.

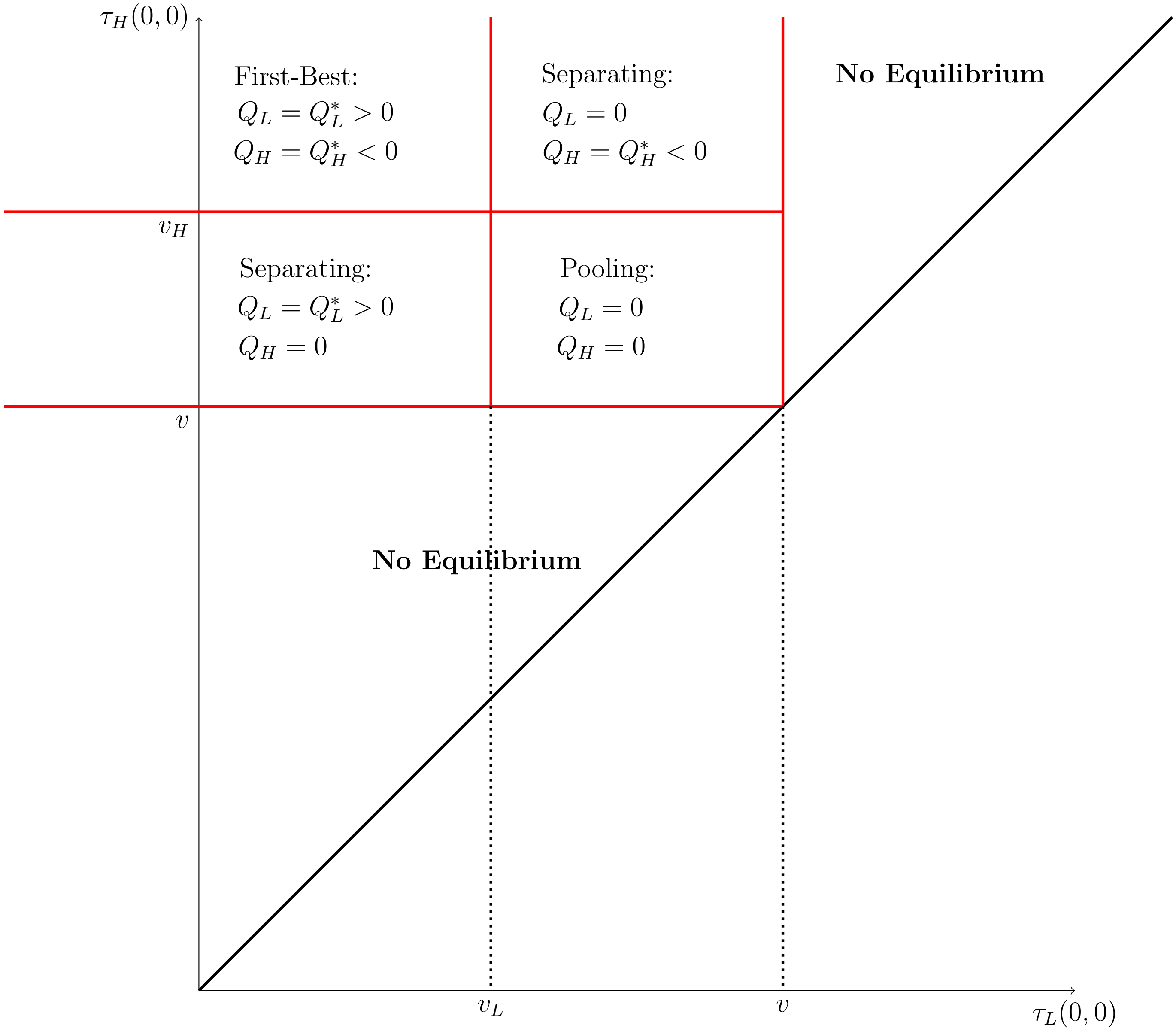

Figure 2 This figure depicts the structure of equilibrium aggregate trades as a function of $\tau_L(0,0)$ and $\tau_H(0,0) > \tau_L(0,0)$, for fixed parameters v_L, v_H, and v.