Non-Exclusive Competition in the Market for Lemons[*]

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Abstract

We consider an exchange economy in which a seller can trade an endowment of a divisible good whose quality she privately knows. Buyers compete in menus of non-exclusive contracts, so that the seller may choose to trade with several buyers. In this context, we show that an equilibrium always exists and that aggregate equilibrium allocations are generically unique. Although the good offered by the seller is divisible, aggregate equilibrium allocations exhibit no fractional trades. In equilibrium, goods of relatively low quality are traded at the same price, while goods of higher quality may end up not being traded at all if the adverse selection problem is severe. This provides a novel strategic foundation for Akerlof's (1970) results, which contrasts with standard competitive screening models postulating enforceability of exclusive contracts. Latent contracts that are issued but not traded in equilibrium turn out to be an essential feature of our construction.

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1 Introduction

One fundamental reason that may cause markets to fail is that the quality of the goods to be traded is often privately known by the sellers. In such circumstances, buyers may be concerned by the fact that, at any given price, only sellers of low quality goods are willing to trade. Despite the growing role of institutions such as certification or rating agencies, it is widely believed that this adverse selection phenomenon still represents a major obstacle to the efficient functioning of financial, insurance and second-hand markets.

Different approaches have been proposed to represent the exchange process under such circumstances. In Akerlof (1970), non-divisible goods of uncertain quality are traded on a market where privately informed sellers and uninformed buyers act as price takers. In the spirit of standard competitive equilibrium analysis, it is assumed that all trades must take place at the same price. Equality of supply and demand determines the equilibrium price level. Since rational buyers are willing to pay only for the average quality traded, sellers of high quality goods are deterred from offering them. Adverse selection may in some cases lead to a complete market breakdown. Rothschild and Stiglitz (1976) explicitly model the strategic interactions between uninformed intermediaries who compete by offering agents contracts for different quantities of a divisible good. Contracts are exclusive: each agent can trade with at most one intermediary, which requires that all agents' trades can be perfectly monitored at no cost. Different unit prices for different quantities emerge in equilibrium, allowing agents to credibly communicate their private information. This leads to lower levels of trade compared to the situation where intermediaries perfectly observe the agents' characteristics; for instance, in the context of insurance markets, high risk agents obtain full insurance, while low risk agents only purchase partial coverage.

Following Rothschild and Stiglitz (1976), most theoretical and applied contributions to the literature on competition under adverse selection have considered frameworks in which contracts are exclusive. This assumption is sometimes appropriate: for instance, in the case of car insurance, law typically forbids to take out multiple policies on a single vehicle. However, there are also many markets where exclusivity is not enforceable, mainly because little information is available about the agents' trades: for instance, competition on financial markets is typically non-exclusive, as each agent can trade with multiple partners who cannot monitor each others' trades with the agent.[1] Moreover, there are important examples of

[1]Besides stock and bond markets, examples of this phenomenon abound in the financial sector. In the banking industry, Detragiache, Garella and Guiso (2000), using a sample of small and medium-sized Italian firms, document that multiple banking relationships are very common. In the credit card industry, Rysman (2007) shows that US consumers typically hold multiple credit cards from different networks (although they

such markets where all trades are not restricted to take place at the same unit price.[2] This suggests that a theory of non-exclusive competition should allow for arbitrary trades, and avoid a priori restrictions such as linear pricing. Besides, to represent interactions in markets with a fixed number of intermediaries, such a theory should also be of a strategic nature.[3] Consistent with these features, this paper is an attempt to understand the impact of adverse selection in a strategic setting where buyers compete through non-exclusive contracts for the purchase of a divisible good.

Specifically, we shall consider the following simple model of trade. A seller endowed with a given quantity of a good attempts to trade it with a finite number of buyers. The seller and the buyers have linear preferences over quantities and transfers exchanged. In line with Akerlof (1970), the quality of the good is the seller's private information. Unlike in his model, and in line with Rothschild and Stiglitz (1976), the good is assumed to be perfectly divisible, so that the seller can trade any fraction of her endowment. Buyers are strategic and compete by simultaneously offering menus of bilateral contracts, or, equivalently, price schedules: in particular, there is no presumption that all trades should take place at a single unit price. After observing the menus offered, and conditional on her private information, the seller decides which contracts to trade. Unlike in Rothschild and Stiglitz (1976), competition is non-exclusive: the seller can trade with several buyers, subject only to the constraint that the aggregate quantities traded do not exceed her endowment. For pedagogical purposes, we first conduct our analysis in the context of a simple example with a binary distribution of quality; this notably affords a geometrical illustration of our arguments. We then generalize our results to a continuous distribution of quality. This serves the dual purpose of checking the robustness of our analysis, and of offering a more flexible framework for applications. In this context, we aim at answering the following questions. Does an equilibrium always exist? Are equilibrium allocations uniquely determined? Do different types of the seller end up trading different allocations? At which prices do trades take place? What menus of contracts are required to sustain an equilibrium?

A natural benchmark for our analysis is that of exclusive competition. In this benchmark, our results parallel those of Rothschild and Stiglitz (1976). First, whenever they exist,

tend to concentrate their spending on a single network). Cawley and Philipson (1999) and Finkelstein and Poterba (2004) report similar findings for the US life insurance market and the UK annuity market.

[2] Such is the case in dealer markets or in over-the-counter markets, where brokers/dealers negotiate directly with one another.

[3] For instance, the underwriting industry features a limited number of intermediaries. Brealey and Myers (2000, Section 15.2, Table 15.1) report that, in 1997, 68% of the securities issues were managed by the six largest underwriters (Merrill Lynch, Salomon Smith Barney, Morgan Stanley, Goldman Sachs, Lehman Brothers, and JP Morgan).

equilibria are separating: the seller can credibly signal the quality of the good she offers by trading only part of her endowment. Therefore fractional trades are a necessary feature of equilibrium despite the linearity of preferences. Second, the very existence of an equilibrium is problematic. In a simple version of the model with two possible levels of quality, pure strategy equilibria exist if and only if it is likely enough that the good is of low quality. When quality is continuously distributed, pure strategy equilibria fail to exist under very weak assumptions on the buyers' preferences.[4]

The analysis of the non-exclusive competition game yields strikingly different results. First, pure strategy equilibria always exist, both in our binary example and for any continuous distribution of quality. Next, aggregate equilibrium allocations are generically unique and feature no fractional trades: depending on whether quality is low or high, the seller either trades her whole endowment or does not trade at all. In particular, when quality is continuously distributed, the equilibrium typically exhibits partial pooling, and is characterized by a threshold level of quality that separates the two trading regimes. These allocations can be supported by simple menu offers. For instance, there always exists an equilibrium in linear price schedules whereby each buyer offers to buy any quantity at the same unit price. This price is equal to the expectation of the buyers' valuation of the good conditional on the seller accepting to trade at that price. While many other menu offers are consistent with equilibrium, corresponding to non-linear price schedules, an important insight of our analysis is that this price is also the unit price at which all trades take place in any equilibrium. That all trades take place at a single unit price is thus not an assumption, but rather a consequence of the equilibrium analysis. Consistent with this, all equilibria have the Bertrand-like feature that, on average, all buyers earn zero profit, regardless of how many they are.

These results are of course in line with Akerlof's (1970) classic study of the market for lemons, for which they provide a novel strategic foundation. It is therefore worth stressing the distinctive features of our model. First, the seller can trade any fraction of her endowment (divisibility). Second, contracting between the buyers and the seller is bilateral, and the seller can simultaneously trade with several buyers (non-exclusivity). Third, there is a finite number of strategic buyers (imperfect competition). Fourth, buyers can offer arbitrary menus of contracts (price schedules). Along with the simplicity of its predictions, these assumptions make the model applicable to a rich variety of situations.

Another insight of our analysis is that non-exclusivity has two consequences on the set of

[4] The fact that the non-existence problem is particularly severe when the agents' private information is continuously distributed is in line with Riley (1985, 2001).

deviations that are available to any given buyer. On the one hand, non-exclusivity tends to expand this set, as the buyer may choose to complement the other buyers' offers by proposing the seller to trade an additional quantity. We call this behavior *pivoting*, and paradoxically it allows the buyer to benefit from the aggressive offers of his competitors. Compared to the exclusive case, in which pivoting is prohibited by definition, this tends to mitigate competition. For instance, such deviations prevent one from supporting the usual Rothschild and Stiglitz's (1976) allocations in equilibrium. On the other hand, non-exclusivity also gives the other buyers more instruments to block potential deviations. This makes it difficult to design one's menu offer so as to attract the seller precisely when the quality of his good lies in some target set. Suppose for instance that the equilibrium price is low, so that high quality goods are not traded, and that some buyer attempts to deviate and purchase only such goods. To be successful, this cream-skimming deviation must involve trading a relatively small quantity at a relatively high price. However, this contract becomes also attractive to the seller when quality is low if, along with it, she can also trade the remaining part of her endowment with the other buyers at the equilibrium price. Thus cream-skimming deviations can be blocked by *latent* contracts, that is, contracts that are not traded in equilibrium but which the seller finds it profitable to trade at the deviation stage. As the above example suggests, these latent contracts need not be complex nor exotic: for instance, in the linear price equilibrium, all the latent contracts are issued at the equilibrium price.

An important property of any equilibrium is that infinitely many contracts need to be issued to support the equilibrium allocations. Specifically, there are infinitely many aggregate allocations that must remain available off the equilibrium path if any buyer withdraws his menu offer. This is particularly striking when the distribution of quality is discrete, since then only finitely many contracts end up being traded in a pure strategy equilibrium. As a result, an infinite number of latent contracts are issued but not traded in equilibrium. In particular, no equilibrium can in this case be sustained through direct mechanisms, which, as we discuss below, makes it difficult to apply standard tools to characterize equilibria.

Related Literature Pauly (1974) and Jaynes (1978) are the first authors to analyze competition through non-exclusive contracts in markets subject to adverse selection. Pauly (1974) stresses that Akerlof-like outcomes will typically prevail in insurance markets where intermediaries are restricted to post linear price schedules. Jaynes (1978) suggests that the separating equilibria characterized by Rothschild and Stiglitz (1976) are vulnerable to entry by an intermediary proposing additional trades that could be concealed from the rest of the industry. In addition, he argues that the non-existence problem identified by Rothschild and Stiglitz (1976) can be overcome if the sharing of information about agents is explicitly modeled as part of the game among intermediaries.[5]

This paper is also closely related to the literature on common agency between competing principals dealing with a privately informed agent. Following Stole (1990) and Martimort (1992), a number of recent contributions have used standard mechanism design techniques to construct equilibrium allocations in common agency games with incomplete information.[6] The basic idea is that, given a profile of menus offered by his competitors, the best response of any single principal can be computed by focusing on simple menu offers that correspond to direct mechanisms. In practice, however, this best response can be effectively characterized only to the extent that the agent's indirect utility function that represents her preferences in her relationship with this principal satisfies certain regularity conditions. These conditions, such as continuity and single-crossing, are robustly violated in our model, because we impose no a priori structure on the menus offered by the buyers, and because the seller faces a capacity constraint. As a result, the standard methodology does not apply to our model. Instead, we derive restrictions on equilibrium allocations by testing them against a set of well chosen deviations. Remarkably, this procedure allows us to obtain a full characterization of aggregate equilibrium allocations.

Biais and Mariotti (2005) construct a linear price schedule equilibrium for a version of our non-exclusive trading game in which gains from trade arise because the seller is more impatient than the buyers. They focus on the particular case where the unconditional average value of the good for the buyers is equal to the highest possible value of the good for the seller. This non-generic situation arises endogenously in a model where the seller is the issuer of a security, which she can optimally design ex-ante. By contrast, our analysis is general, in that we allow for a large class of quality distributions, and offer a full characterization of aggregate equilibrium allocations, which are shown to be generically unique.

Another related paper in the common agency literature is Biais, Martimort and Rochet (2000), who study a financial market in which uninformed market-makers compete in a non-exclusive way by supplying liquidity to an informed insider. Unlike the seller in our model, the insider has strictly convex preferences and faces no capacity constraint. Using the methodology outlined above, Biais, Martimort and Rochet (2000) construct an equilibrium in which market-makers post convex price schedules, and that is unique within that class.[7]

[5]See Hellwig (1988) for a discussion of the relevant extensive form for the inter-firm communication game.

[6]See for instance Biais, Martimort and Rochet (2000), Martimort and Stole (2003), Calzolari (2004), Laffont and Pouyet (2004), Khalil, Martimort and Parigi (2007) or Martimort and Stole (2009).

[7]Piaser (2006) shows that, given these restrictions, this equilibrium can actually be sustained through direct mechanisms.

One of the main features of this equilibrium is that each market-maker is indispensable in providing utility to the insider; as a result, market-makers end up earning strictly positive profits. This makes this equilibrium rather different from those we characterize in our setting: indeed, using a pivoting argument, we show that no buyer is ever indispensable, as the aggregate equilibrium allocation would still remain available to the seller in the hypothetical case where some buyer would withdraw his menu offer. Hence our results hold regardless of the number of competing buyers. Another difference is that all trades take place at the same unit price in any equilibrium of our model, while unit prices vary with the insider's private information in the equilibrium constructed by Biais, Martimort and Rochet (2000). It would be interesting, in future research, to investigate in greater detail the relationships between these two trading environments.

The importance of latent contracts as a strategic device to sustain equilibria has been so far emphasized in moral hazard environments. Hellwig (1983) and Arnott and Stiglitz (1993) argued that latent contracts play the role of threats to deter entry in insurance markets where agents' effort decisions are non-contractible. As a result, positive profits for active intermediaries typically arise in equilibrium. These intuitions have been extended by Bizer and DeMarzo (1992) and Kahn and Mookherjee (1998) to situations where intermediaries act sequentially, while the equilibrium features of latent contracts and the corresponding welfare implications have been further examined by Bisin and Guaitoli (2004) and Attar and Chassagnon (2009). A key insight of our analysis is that latent contracts also play a central role in adverse selection environments by deterring cream-skimming deviations. It should be noted that standard arguments against the use of latent contracts do not apply in our setup. For instance, latent contracts are often criticized for allowing one to support multiple equilibrium allocations, and even for inducing an indeterminacy of equilibrium.[8] This is not the case in our model, since aggregate equilibrium allocations are generically unique. Another common criticism is that latent contracts may in fact make losses off the equilibrium path in the hypothetical case where they would be traded, and constitute as such non-credible threats.[9] Again, this need not be the case in our model: actually, we construct examples of equilibria in which latent contracts if traded would be strictly profitable to the buyers that issue them.

An alternative approach to the study of non-exclusive competition under adverse selection

[8] In a complete information setting, Martimort and Stole (2003) show that latent contracts can be used to support any level of trade between the perfectly competitive outcome and the Cournot outcome.

[9] Attar and Chassagnon (2009) provide an example of a moral hazard insurance economy in which latent contracts with negative virtual profits are a necessary feature of any equilibrium.

has been suggested by Bisin and Gottardi (1999, 2003) in the context of general equilibrium analysis. They focus on situations where none of the agents' trades can be monitored. As a consequence, the terms of each contract must be independent of the exchanges made in every single market, which forces prices to be linear. It should be noted that when this restriction is postulated, competitive equilibria may fail to exist in robust circumstances (Bisin and Gottardi (1999, 2003)). To restore existence, some non-linearity in prices, or, equivalently, some observability of agents' trades must be reintroduced in the model. This can for instance be achieved through bid-ask spreads (Bisin and Gottardi (1999)) or entry fees (Bisin and Gottardi (2003)). By contrast, the present paper starts from the alternative assumption that buyers can commit to arbitrary menu offers, which we see as a natural feature of competition in contracts.

The paper is organized as follows. In Section 2, we describe the model. Section 3 focuses on the case of a binary distribution of quality. In Section 4, we analyze the general framework with a continuous distribution of quality. Section 5 concludes.

2 Non-Exclusive Trading under Adverse Selection

2.1 The Model

There are two kinds of agents: a single seller, and a finite number of buyers indexed by $i = 1, \ldots, n$, where $n \geq 2$. The seller has an endowment consisting of one unit of a perfectly divisible good that she can trade with one or several buyers. Let q^i be the quantity of the good purchased by buyer i, and t^i the transfer he makes in return. Feasible trade vectors $((q^1, t^1), \ldots, (q^n, t^n))$ are such that $q^i \geq 0$ and $t^i \geq 0$ for all i, with $\sum_i q^i \leq 1$. Thus the quantity of the good purchased by each buyer must be at least zero, and the sum of these quantities cannot exceed the seller's endowment. We take the latter as a basic technological constraint that seller's choices are subject to.

Our specification of the agents' preferences follows Samuelson (1984). The seller has preferences represented by

$$T - \theta Q,$$

where $Q = \sum_i q^i$ and $T = \sum_i t^i$ denote aggregate quantities and transfers. Here θ is a random variable that stands for the quality of the good as perceived by the seller. Each buyer i has preferences represented by

$$v(\theta) q^i - t^i.$$

Here $v(\theta)$ is a deterministic function of θ that stands for the quality of the good as perceived by the buyers. Observe that there are no externalities across buyers beyond the fact that the quantities they trade cannot in the aggregate exceed the seller's endowment. In particular, there are no efficiency gains from trading with several buyers.

We will typically assume that $v(\theta)$ is not a constant function of θ, so that both the seller and the buyers care about θ. Gains from trade arise in this common value environment if $v(\theta) > \theta$ for some realization of θ. However, in line with Akerlof (1970), mutually beneficial trades are potentially impeded because the seller is privately informed of the quality of the good at the trading stage. Following standard terminology, we shall hereafter refer to θ as to the *type* of the seller.

Trading is non-exclusive in the sense that no buyer can contract on the trades that the seller makes with his competitors.[10] Thus, as in Biais, Martimort and Rochet (2000) or Segal and Whinston (2003), a *contract* describes a bilateral trade between the seller and a particular buyer; a *menu* is a set of such contracts. Buyers compete in menus for the good offered by the seller.[11] The seller can simultaneously trade with several buyers, and optimally combine the offers made to her, subject to her endowment constraint. The following timing of events characterizes our non-exclusive competition game:

1. Each buyer i proposes a menu of contracts, that is, a set C^i of quantity-transfer pairs $(q^i, t^i) \in [0,1] \times \mathbb{R}_+$ that contains at least the no-trade contract $(0,0)$.[12]
2. After privately learning the quality θ, the seller selects one contract (q^i, t^i) from each of the menus C^i's offered by the buyers, subject to the constraint that $\sum_i q^i \leq 1$.

A pure strategy for the seller is a function that maps each type θ and each menu profile $(C^1, \ldots, C^n)$ into a vector of contracts $((q^1, t^1), \ldots, (q^n, t^n)) \in ([0,1] \times \mathbb{R}_+)^n$ such that $(q^i, t^i) \in C^i$ for all i and $\sum_i q^i \leq 1$. To ensure that the seller's problem

$$\max \left\{ \sum_i t^i - \theta \sum_i q^i : (q^i, t^i) \in C^i \text{ for all } i \text{ and } \sum_i q^i \leq 1 \right\}$$

has a solution for any type θ and menu profile $(C^1, \ldots, C^n)$, we require the buyers' menus to be compact sets. Throughout the paper, and unless stated otherwise, the equilibrium concept is pure strategy perfect Bayesian equilibrium.

[10] In particular, buyers cannot make transfers contingent on the whole profile of quantities $(q^1, \ldots, q^n)$ traded by the seller. This distinguishes our trading environment from a menu auction à la Bernheim and Whinston (1986a).

[11] As established by Peters (2001) and Martimort and Stole (2002), there is no need to consider more general mechanisms in this multiple-principal single-agent setting, see Subsection 3.3 below.

[12] The assumption that each menu must contain the no-trade contract allows one to deal with participation in a simple way. It reflects the fact that the seller cannot be forced to trade with any particular buyer.

2.2 Applications

Our model is basically a model of trade, with the following features: the good is divisible; its quality is the seller's private information; and the seller may trade with several buyers. As such it can be applied to many markets. The following examples illustrate some possible applications.

Financial Markets In line with DeMarzo and Duffie (1999) or Biais and Mariotti (2005), one can think of the seller as an issuer attempting to raise cash by selling a security backed by some of her assets, and of the buyers as underwriters managing the issue. Under risk-neutrality, gains from trade arise in this context if the issuer discounts future cash-flows at a higher rate than the market; this may for instance reflect credit constraints or, in the financial services industry, binding minimum-capital requirements. The marginal cost of the security for the issuer, that is, its value to the issuer if retained, is then only a fraction of the value of the security to the underwriters: formally, one has $\theta = \delta v(\theta)$ for some constant $\delta \in (0, 1)$. Here Q is the total fraction of the security sold by the issuer, while $1 - Q$ is the residual fraction of the security that the issuer retains. It is natural to assume that, at the issuing stage, the issuer has better information than the underwriters about the value of her assets, and hence about the value of the security she issues.

Labor Market In an alternative interpretation of the model, the seller is a worker, and the buyers are firms. The worker can work for several firms, and divide her time endowment accordingly. This is for instance the case in legal or financial services, where a consultant typically works on behalf of several customers; similarly, a salesman can represent different companies. The worker's type θ is her opportunity cost of selling one unit of her time to any given firm, while $v(\theta)$ is the productivity of a worker of type θ. Here Q is the total fraction of time spent working, while $1 - Q$ is the residual fraction of time that the worker can spend on leisure. This interpretation differs from the labor market model of Mas-Colell, Whinston and Green (1995, Chapter 13, Section B) in that labor is assumed to be divisible, and competition for the worker's services is non-exclusive.

Insurance Markets A final interpretation of our setup is as a model of insurance provision, where the insured's preference are modeled using Yaari's (1987) dual theory of choice under risk, so that her utility is linear in wealth but non linear in probabilities. Here the roles of the seller and of the buyers are reversed. There is a single insured, who can purchase insurance from several insurance companies. The insured has wealth W, and can incur a loss L with privately known probability x. An insurance contract consists of a reimbursement r^i and of a premium p^i. The utility that the insured derives from aggregate reimbursements $R = \sum_i r^i$ and aggregate premia $P = \sum_i p^i$ is

$$W - P - f(x)(L - R),$$

while the profit of insurance company i is

$$p^i - xr^i.$$

One assumes that overinsurance is prohibited, so that R is at most equal to L. Letting $t^i = -p^i$, $q^i = r^i$, $\theta = -f(x)$ and $v(\theta) = -x$ leads back to our model. Gains from trade arise in this context if some type of the issuer puts more weight on the occurrence of a loss than the insurance company does, that is, if $f(x) > x$ for some realization of x.

3 The Two-Type Case

In this section, we consider the binary version of our model in which the seller's type can be either low, $\theta = \underline{\theta}$, or high, $\theta = \overline{\theta}$, for some $\overline{\theta} > \underline{\theta} > 0$. Denote by $\nu \in (0, 1)$ the probability that $\theta = \overline{\theta}$ and by $\mathbf{E}$ the corresponding expectation operator. In order to focus on the most interesting case, we assume that the seller's and the buyers' perceptions of the quality of the good move together, that is, $v(\overline{\theta}) > v(\underline{\theta})$, and that it would be efficient to trade no matter the quality of the good, that is, $v(\underline{\theta}) > \underline{\theta}$ and $v(\overline{\theta}) > \overline{\theta}$.

3.1 The Exclusive Competition Benchmark

As a benchmark, it is helpful to characterize the equilibrium outcomes under exclusive competition, that is, when the seller can trade with at most one buyer, as in standard models of competition under adverse selection. The timing of the exclusive competition game is similar to that of the non-exclusive competition game, except that the second stage is replaced by

2'. After privately learning the quality θ, the seller selects one contract (q^i, t^i) from one of the menus C^i's offered by the buyers.

Given a menu profile $(C^1, \ldots, C^n)$, the seller's problem then becomes

$$\max\{t^i - \theta q^i : (q^i, t^i) \in C^i \text{ for some } i\}.$$

Let $(\underline{q}^e, \underline{t}^e)$ and $(\overline{q}^e, \overline{t}^e)$ be the contracts traded by each type of the seller in an equilibrium of the exclusive competition game. One has the following result.

Proposition 1 *The following holds*:

(i) Any equilibrium of the exclusive competition game is separating, with

$$(\underline{q}^e, \underline{t}^e) = (1, v(\underline{\theta})) \text{ and } (\overline{q}^e, \overline{t}^e) = \frac{v(\underline{\theta}) - \underline{\theta}}{v(\overline{\theta}) - \underline{\theta}}\,(1, v(\overline{\theta})).$$

(ii) The exclusive competition game has an equilibrium if and only if $\nu \leq \nu^e$, where

$$\nu^e = \frac{\overline{\theta} - \underline{\theta}}{v(\overline{\theta}) - \underline{\theta}}.$$

Hence, when the rules of the competition game are such that the seller can trade with at most one buyer, the structure of market equilibria is formally analogous to that obtaining in the competitive insurance model of Rothschild and Stiglitz (1976). First, any pure strategy equilibrium is separating, with type $\underline{\theta}$ selling her whole endowment, $\underline{q}^e = 1$, and type $\overline{\theta}$ only selling a fraction of her endowment, $0 < \overline{q}^e < 1$. The corresponding contracts are traded at unit prices $v(\underline{\theta})$ and $v(\overline{\theta})$ respectively, yielding each buyer a zero payoff. Second, type $\underline{\theta}$ is indifferent between her equilibrium contract and that of type $\overline{\theta}$, implying

$$\overline{q}^e = \frac{v(\underline{\theta}) - \underline{\theta}}{v(\overline{\theta}) - \underline{\theta}}$$

as stated in Proposition 1(i). The equilibrium is depicted on Figure 1. Point $\underline{A}^e$ corresponds to the equilibrium contract of type $\underline{\theta}$, while point $\overline{A}^e$ corresponds to the equilibrium contract of type $\overline{\theta}$. The two solid lines passing through these points are the equilibrium indifference curves of type $\underline{\theta}$ and type $\overline{\theta}$. The dotted line passing through the origin are indifference curves for the buyers, with slopes $v(\underline{\theta})$ and $v(\overline{\theta})$.

—Insert Figure 1 here—

As in Rothschild and Stiglitz (1976), a pure strategy equilibrium exists under exclusivity only under certain parameter restrictions. Specifically, the equilibrium indifference curve of type $\overline{\theta}$ must lie above the indifference curve for the buyers with slope $\mathbf{E}[v(\theta)]$ passing through the origin, for otherwise there would exist a profitable deviation attracting both types of the seller. As stated in Proposition 1(ii), this is the case if and only if the probability ν that the good is of high quality is low enough.

3.2 Equilibrium Outcomes under Non-Exclusive Competition

We now turn to the analysis of the non-exclusive competition model. We first characterize the restrictions that equilibrium behavior implies for the outcomes of the non-exclusive competition game. Next, we show that this game always has an equilibrium in which buyers post linear prices. Finally, we contrast the equilibrium outcomes with those arising in the exclusive competition model.

3.2.1 Aggregate Equilibrium Allocations

From a methodological viewpoint, a standard insight for the analysis of common agency games with incomplete information is that in any pure strategy equilibrium of such a game, each principal i acts like a monopolist facing an agent whose preferences are represented by an indirect utility function of (θ, q^i) that depends on the menus offered by principals $j \neq i$.[13] Whenever this function is well behaved, which is the case under restrictive assumptions over the menus offered by principals $j \neq i$, one can apply standard mechanism design techniques to characterize the best response of principal i. This, however, is typically not the case in our model. The first reason is that we do not impose any conditions over the menus offered by the buyers besides that they consist of compact sets of contracts. The second reason is that the seller makes choice under a capacity constraint. Taken together, these two key features of our model imply that the seller's indirect utility function, viewed from the perspective of buyer i, might be discontinuous, and furthermore need not satisfy a single-crossing condition in (θ, q^i).[14] This in turn makes it difficult to apply the standard methodology for common agency games to our non-exclusive competition game. Instead, we fully characterize candidate aggregate equilibrium allocations by requesting that they survive well chosen deviations.

Let $\underline{c}^i = (\underline{q}^i, \underline{t}^i)$ and $\overline{c}^i = (\overline{q}^i, \overline{t}^i)$ be the contracts traded by the two types of the seller with buyer i in equilibrium, and let $(\underline{Q}, \underline{T}) = \sum_i \underline{c}^i$ and $(\overline{Q}, \overline{T}) = \sum_i \overline{c}^i$ be the corresponding aggregate equilibrium allocations. To characterize these allocations, one only needs to require that three types of deviations by a buyer be blocked in equilibrium. In each case, the deviating buyer uses the offers of his competitors as a support for his own deviation. This intuitively amounts to pivoting around the aggregate equilibrium allocation points $(\underline{Q}, \underline{T})$ and $(\overline{Q}, \overline{T})$ in the (Q, T) space. We now consider each deviation in turn.

Attracting Type $\underline{\theta}$ by Pivoting Around $(\underline{Q}, \underline{T})$ The first type of deviations allows one

[13] See for instance Martimort and Stole (2009) for a recent exposition of this methodology.

[14] This can be checked by considering the quantity $z^{-i}(\theta, 1 - q^i)$, that represents the highest payoff a seller of type θ can get from trading with buyers $j \neq i$ while selling quantity q^i to buyer i, see (3) and (4). Because the menus C^j's are only requested to be compact, and may therefore correspond to discontinuous price schedules, the maximum theorem does not apply to (4), and an increase in q^i may generate a downward jump in $z^{-i}(\theta, 1 - q^i)$. As a result, the seller's indirect utility function $(\theta, q^i) \mapsto -\theta q^i + z^{-i}(\theta, 1 - q^i)$ may fail to exhibit decreasing differences, unlike the seller's utility function over aggregate trades.

to prove that type $\underline{\theta}$ always trades efficiently in equilibrium.

Lemma 1 $\underline{Q} = 1$ *in any equilibrium.*

One can illustrate the deviation used in Lemma 1 as follows. Observe first that a basic implication of incentive compatibility is that, in any equilibrium, $\overline{Q}$ cannot be higher than $\underline{Q}$. Suppose then that $\underline{Q} < 1$ in a candidate equilibrium. This situation is depicted on Figure 2. Point $\underline{A}$ corresponds to the aggregate equilibrium allocation $(\underline{Q}, \underline{T})$ traded by type $\underline{\theta}$, while point $\overline{A}$ corresponds to the aggregate equilibrium allocation $(\overline{Q}, \overline{T})$ traded by type $\overline{\theta}$. The two solid lines passing through these points are the equilibrium indifference curves of type $\underline{\theta}$ and type $\overline{\theta}$, with slopes $\underline{\theta}$ and $\overline{\theta}$. The dotted line passing through $\underline{A}$ is an indifference curve for the buyers, with slope $v(\underline{\theta})$.

—Insert Figure 2 here—

Suppose now that some buyer deviates and includes in his menu an additional contract that makes available the further trade $\underline{AA}'$. This leaves type $\underline{\theta}$ indifferent, since she obtains the same payoff as in equilibrium. Type $\overline{\theta}$, by contrast, cannot gain by trading this new contract. Assuming that the deviating buyer can break the indifference of type $\underline{\theta}$ in his favor, he strictly gains from trading the new contract with type $\underline{\theta}$, as the slope $\underline{\theta}$ of the line segment $\underline{AA}'$ is strictly less than $v(\underline{\theta})$. This contradiction shows that one must have $\underline{Q} = 1$ in equilibrium. The assumption on indifference breaking is relaxed in the proof of Lemma 1.

Attracting Type $\underline{\theta}$ by Pivoting Around $(\overline{Q}, \overline{T})$ Having established that $\underline{Q} = 1$, we now investigate the aggregate quantity $\overline{Q}$ traded by type $\overline{\theta}$ in equilibrium. The second type of deviations allows one to partially characterize the circumstances in which the two types of the seller trade different aggregate allocations in equilibrium. We say in this case that the equilibrium is *separating*. An immediate implication of Lemma 1 is that $\overline{Q} < 1$ in any separating equilibrium. Let then $p = \frac{\underline{T} - \overline{T}}{1 - \overline{Q}}$ be the slope of the line connecting the points $(\overline{Q}, \overline{T})$ and $(1, \underline{T})$ in the (Q, T) space. Therefore p is the implicit unit price at which the quantity $1 - \overline{Q}$ can be sold to move from $(\overline{Q}, \overline{T})$ to $(1, \underline{T})$. By incentive compatibility, p must lie between $\underline{\theta}$ and $\overline{\theta}$ in any separating equilibrium. The strategic analysis of the buyers' behavior induces further restrictions on p.

Lemma 2 *In a separating equilibrium,* $p < \overline{\theta}$ *implies that* $p \geq v(\underline{\theta})$*.*

In the proof of Lemma 1, we showed that, if $\underline{Q} < 1$, then each buyer has an incentive to deviate. By contrast, in the proof of Lemma 2, we only show that if $p < \min\{v(\underline{\theta}), \overline{\theta}\}$ in a candidate separating equilibrium, then at least one buyer has an incentive to deviate. This makes it more difficult to graphically illustrate why the deviation used in Lemma 2 might be profitable. It is however easy to see why this deviation would be profitable to an entrant or, equivalently, to an inactive buyer that would not trade in equilibrium. This situation is depicted on Figure 3. The dotted line passing through $\overline{A}$ is an indifference curve for the buyers, with slope $v(\underline{\theta})$. Contrary to the conclusion of Lemma 2, the figure is drawn in such a way that this indifference curve is strictly steeper than the line segment $\overline{A}\underline{A}$.

—Insert Figure 3 here—

Suppose now that the entrant offers a contract that makes available the trade $\overline{A}\underline{A}$. This leaves type $\underline{\theta}$ indifferent, since she obtains the same payoff as in equilibrium by trading the aggregate allocation $(\overline{Q}, \overline{T})$ together with the new contract. Type $\overline{\theta}$, by contrast, cannot gain by trading this new contract. Assuming that the entrant can break the indifference of type $\underline{\theta}$ in his favor, he earns a strictly positive payoff from trading the new contract with type $\underline{\theta}$, as the slope p of the line segment $\overline{A}\underline{A}$ is strictly less than $v(\underline{\theta})$. This shows that, unless $p \geq v(\underline{\theta})$, the candidate separating equilibrium is not robust to entry. The assumption on indifference breaking is relaxed in the proof of Lemma 2, which further shows that the proposed deviation is profitable to at least one buyer.

Attracting both Types by Pivoting Around $(\overline{Q}, \overline{T})$ A separating equilibrium must be robust to deviations that attract both types of the seller. This third type of deviations allows one to find a necessary condition for the existence of a separating equilibrium. When this condition fails, both types of the seller must trade the same aggregate allocations in equilibrium. We say in this case that the equilibrium is *pooling*.

Lemma 3 *If* $\mathbf{E}[v(\theta)] > \overline{\theta}$, *any equilibrium is pooling, with*

$$(\underline{Q}, \underline{T}) = (\overline{Q}, \overline{T}) = (1, \mathbf{E}[v(\theta)]).$$

The proof of Lemma 3 consists in showing that if $\mathbf{E}[v(\theta)] > \overline{\theta}$ in a candidate separating equilibrium, then at least one buyer has an incentive to deviate. As for Lemma 2, this makes it difficult to graphically illustrate why this deviation might be profitable. It is however easy to see why this deviation would be profitable to an entrant or, equivalently, to an inactive buyer that would not trade in equilibrium. This situation is depicted on Figure 4. The dotted line passing through $\overline{A}$ is an indifference curve for the buyers, with slope $\mathbf{E}[v(\theta)]$. Contrary to the conclusion of Lemma 3, the figure is drawn in such a way that this indifference curve is strictly steeper than the indifference curves of type $\overline{\theta}$.

—Insert Figure 4 here—

Suppose now that the entrant offers a contract that makes available the trade $\overline{A}\underline{A}'$. This leaves type $\overline{\theta}$ indifferent, since she obtains the same payoff as in equilibrium by trading the aggregate allocation $(\overline{Q}, \overline{T})$ together with the new contract. Type $\underline{\theta}$ strictly gains by trading this new contract. Assuming that the entrant can break the indifference of type $\overline{\theta}$ in his favor, he earns a strictly positive payoff from trading the new contract with both types as the slope $\overline{\theta}$ of the line segment $\overline{A}\underline{A}'$ is strictly less than $\mathbf{E}[v(\theta)]$. This shows that, unless $\mathbf{E}[v(\theta)] \leq \overline{\theta}$, the candidate equilibrium is not robust to entry. Once again, the assumption on indifference breaking is relaxed in the proof of Lemma 3, which further shows that the proposed deviation is profitable to at least one buyer.

The following result provides a partial converse to Lemma 3.

Lemma 4 *If $\mathbf{E}[v(\theta)] < \overline{\theta}$, any equilibrium is separating, with*

$$(\underline{Q}, \underline{T}) = (1, v(\underline{\theta})) \text{ and } (\overline{Q}, \overline{T}) = (0, 0).$$

The following is an important corollary of our analysis.

Corollary 1 *Each buyer's payoff is zero in any equilibrium.*

Lemmas 1 to 4 provide a full characterization of the aggregate trades that can be sustained in an equilibrium of the non-exclusive competition game. A key implication of Lemmas 3 and 4 is that the aggregate equilibrium allocation traded by the seller is generically unique.[15] While each buyer always obtains a zero payoff in equilibrium, the structure of equilibrium allocations is directly affected by the severity of the adverse selection problem:

- Whenever $\mathbf{E}[v(\theta)] > \overline{\theta}$, adverse section is mild, which rules out separating equilibria. Indeed, as shown in the proof of Lemma 3, if the aggregate allocation $(\overline{Q}, \overline{T})$ traded by type $\overline{\theta}$ were such that $\overline{Q} < 1$, some buyer would have an incentive to induce both types of the seller to trade this allocation, together with the additional quantity $1 - \overline{Q}$ at a unit price between $\overline{\theta}$ and $\mathbf{E}[v(\theta)]$. Competition among buyers then bids up the price of the seller's endowment to its average value $\mathbf{E}[v(\theta)]$ for the buyers, a price at which both types of the seller are ready to trade. This situation is depicted on Figure 5. The dotted line passing through the origin is the equilibrium indifference curve of the buyers, with slope $\mathbf{E}[v(\theta)]$.

[15]The non-generic case where $\mathbf{E}[v(\theta)] = \overline{\theta}$ is discussed after Proposition 2.

—Insert Figure 5 here—

- Whenever $\mathbf{E}[v(\theta)] < \overline{\theta}$, adverse selection is severe, which rules out pooling equilibria. This reflects that type $\overline{\theta}$ is no longer ready to trade her endowment at the maximal price $\mathbf{E}[v(\theta)]$ at which buyers would break even in such an equilibrium. More interestingly, our analysis shows that non-exclusive competition induces a specific cost of screening the seller's type in equilibrium. Indeed, any separating equilibrium must be such that no buyer has an incentive to deviate and induce type $\underline{\theta}$ to trade the aggregate allocation $(\overline{Q}, \overline{T})$, together with the additional quantity $1 - \overline{Q}$ at some mutually advantageous price. Lemma 2 shows that to eliminate any incentive for buyers to engage in such trades with type $\underline{\theta}$, the implicit unit price at which this additional quantity $1 - \overline{Q}$ can be sold in equilibrium must be at least $v(\underline{\theta})$. As shown in Lemma 4, this implies at most an aggregate payoff $\{\mathbf{E}[v(\theta)] - \overline{\theta}\}\overline{Q}$ for the buyers. Hence type $\overline{\theta}$ can trade actively in a separating equilibrium only in the non-generic case where $\mathbf{E}[v(\theta)] = \overline{\theta}$, while type $\overline{\theta}$ does not trade at all if $\mathbf{E}[v(\theta)] < \overline{\theta}$. This situation is depicted on Figure 6. The dotted line passing through the origin is the equilibrium indifference curve of the buyers, with slope $v(\underline{\theta})$.

—Insert Figure 6 here—

3.2.2 Equilibrium Existence

We now establish that, in contrast with the exclusive competition game of Subsection 3.1, the non-exclusive competition game always has an equilibrium. Specifically, we show that there always exists an equilibrium in which all buyers post linear price schedules. In such an equilibrium, the unit price at which any quantity can be traded is equal to the expected quality of the goods that are actively traded. Specifically, define

$$p^* = \begin{cases} \mathbf{E}[v(\theta)] & \text{if } \mathbf{E}[v(\theta)] \geq \overline{\theta}, \\ v(\underline{\theta}) & \text{if } \mathbf{E}[v(\theta)] < \overline{\theta}. \end{cases} \tag{1}$$

One then has the following result.

Proposition 2 *The non-exclusive competition game always has an equilibrium in which each buyer offers the menu*

$$\{(q,t) \in [0,1] \times \mathbb{R}_+ : t = p^* q\},$$

and thus stands ready to buy any quantity of the good at the constant unit price p^*.

In the non-generic case where $\mathbf{E}[v(\theta)] = \overline{\theta}$, it is easy to check that there exist two linear price equilibria, a pooling equilibrium with constant unit price $\mathbf{E}[v(\theta)]$ and a separating equilibrium with constant unit price $v(\underline{\theta})$. In addition, there exists in this case a continuum of separating equilibria in which type $\overline{\theta}$ trades actively. Indeed, to support an equilibrium trade level $\overline{Q} \in (0,1)$ for type $\overline{\theta}$, it is enough that all buyers offer to buy any quantity of the good at unit price $v(\underline{\theta})$, and that one buyer offers in addition to buy any quantity of the good up to $\overline{Q}$ at unit price $\mathbf{E}[v(\theta)]$. Both types $\underline{\theta}$ and $\overline{\theta}$ then sell a fraction $\overline{Q}$ of their endowment at unit price $\mathbf{E}[v(\theta)]$, while type $\underline{\theta}$ sells the remaining fraction of her endowment at unit price $v(\underline{\theta})$. To avoid this non-generic multiplicity issue and therefore simplify the exposition, we shall assume that $\mathbf{E}[v(\theta)] \neq \overline{\theta}$ in the remainder of this section.

3.2.3 Comparison with the Exclusive Competition Model

Our analysis provides a strategic foundation for Akerlof's (1970) original intuition. First, if adverse selection is severe enough, only goods of low quality are traded in equilibrium. Second, as can be seen from (1), the price p^* at which the seller can sell her endowment in equilibrium is the expectation of the value of the good to the buyers, conditional on the seller being willing to trade at this price:

$$p^* = \mathbf{E}[v(\theta) \,|\, \theta \leq p^*].$$

These results contrasts sharply with the predictions of standard models of competition under adverse selection, in which, as in the exclusive competition game of Subsection 3.1, exclusivity clauses are assumed to be enforceable at no cost. Specifically, the equilibrium outcomes of the non-exclusive competition game differ in three crucial ways from that of the exclusive competition game:

- First, the exclusive competition game has an equilibrium only if the probability that the good is of high quality is low enough. By contrast, the non-exclusive competition game always has an equilibrium.

- Second, when it exists, the equilibrium of the exclusive competition game is always separating, while for certain parameter values all the equilibria of the non-exclusive competition game are pooling.

- Third, even when all equilibria of the non-exclusive competition game are separating, their structure is very different from that of the exclusive competition game. In the latter case, type $\underline{\theta}$ is indifferent between her equilibrium contract and that of type $\overline{\theta}$, who trades a strictly positive fraction of her endowment. By contrast, in the former case, type $\underline{\theta}$ strictly prefers her aggregate equilibrium allocation to that of type $\overline{\theta}$, who does not trade in equilibrium.

With regard to the last point, simple computations show that the threshold $\nu^e = \frac{\overline{\theta}-\underline{\theta}}{v(\overline{\theta})-\underline{\theta}}$ for ν below which the exclusive competition game has an equilibrium is strictly greater than the threshold $\nu^{ne} = \max\left\{0, \frac{\overline{\theta}-v(\underline{\theta})}{v(\overline{\theta})-v(\underline{\theta})}\right\}$ for ν below which all equilibria of the non-exclusive competition game are separating. Thus if one assumes that $\nu \leq \nu^e$, so that equilibria exist under both exclusivity and non-exclusivity, two situations can arise. When $0 < \nu < \nu^{ne}$, the equilibrium is separating under both exclusivity and non-exclusivity, and more trade takes place in the former case. By contrast, when $\nu^{ne} < \nu \leq \nu^e$, the equilibrium is separating under exclusivity and pooling under non-exclusivity, and more trade takes place in the latter case. Therefore, from an ex-ante viewpoint, exclusive competition leads to a more efficient outcome under severe adverse selection, while non-exclusive competition leads to a more efficient outcome under mild adverse selection.

3.3 Equilibrium Menus and Latent Contracts

We now explore in more depth the structure of the menus offered by the buyers in equilibrium. We first provide equilibrium restrictions for the price of issued and traded contracts. Next, we show that a large number of latent contracts needs to be issued in equilibrium. Then, we relate our analysis to the literature on communication in common agency games. Finally, we show that the aggregate equilibrium allocations can also be sustained through non-linear price schedules.

3.3.1 Price Restrictions

Our first result provides equilibrium restrictions on the price of all issued contracts.

Proposition 3 *The unit price of any contract issued in an equilibrium of the non-exclusive competition game is at most p^*.*

The intuition for this result is as follows. First, if $\mathbf{E}[v(\theta)] > \overline{\theta}$ and some buyer offered to purchase some quantity at a unit price above $\mathbf{E}[v(\theta)]$, any other buyer would have an incentive to induce both types of the seller to trade this contract and to sell him the remaining fraction of their endowment at a unit price slightly below $\mathbf{E}[v(\theta)]$. Second, if $\mathbf{E}[v(\theta)] < \overline{\theta}$ and some buyer offered to purchase some quantity at a unit price above $v(\underline{\theta})$, then any other buyer would have an incentive to induce type $\underline{\theta}$ to trade this contract and to sell him the remaining fraction of her endowment at a unit price slightly below $v(\underline{\theta})$. As a corollary, one obtains a straightforward characterization of the price of traded contracts.

Corollary 2 *The unit price of any contract traded in an equilibrium of the non-exclusive competition game is* p^*.

3.3.2 Latent Contracts

With these preliminaries at hand, we can investigate which contracts need to be issued to support the aggregate equilibrium allocations. From a strategic viewpoint, what matters for each buyer is the outside option of the seller, that is, what aggregate allocations she can achieve by trading with the other buyers only. For each buyer i, and for each menu profile $(C^1, \ldots, C^n)$, this is described by the set of aggregate allocations that remain available if buyer i withdraws his menu offer C^i. One first has the following result.

Proposition 4 *In any equilibrium of the non-exclusive competition game, the aggregate allocation* $(1, p^*)$ *remains available if any buyer withdraws his menu offer.*

The aggregate equilibrium allocation must therefore remain available even if a buyer deviates from his equilibrium menu offer. The reason is that this buyer would otherwise have an incentive to offer both types to sell their whole endowment at a price slightly below $\mathbf{E}[v(\theta)]$ (if $\mathbf{E}[v(\theta)] > \overline{\theta}$), or to offer type $\underline{\theta}$ to sell her whole endowment at price $v(\underline{\theta})$ while offering type $\overline{\theta}$ to sell a smaller fraction of her endowment on more advantageous terms (if $\mathbf{E}[v(\theta)] < \overline{\theta}$). The flip side of this observation is that no buyer is essential in providing the seller with her aggregate equilibrium allocation. This rules out standard Cournot outcomes in which the buyers would simply share the market and in which all issued contracts would actively be traded by some type of the seller, as in Biais, Martimort and Rochet (2000). As an illustration, when there are two buyers, there is no equilibrium in which each buyer would only offer to purchase half of the seller's endowment.

Because of the non-exclusivity of competition, equilibrium in fact involves much more restrictions on menus offers than those prescribed by Propositions 3 and 4. For instance, if $\mathbf{E}[v(\theta)] > \overline{\theta}$, there is no equilibrium in which each buyer only offers the allocation $(1, \mathbf{E}[v(\theta)])$ besides the no-trade contract. Indeed, any buyer could otherwise deviate by offering to purchase a quantity $\overline{q} < 1$ at some price $\overline{t} \in (\mathbf{E}[v(\theta)] - \overline{\theta}(1-\overline{q}), \mathbf{E}[v(\theta)] - \underline{\theta}(1-\overline{q}))$. By construction, this is a cream-skimming deviation that attracts only type $\overline{\theta}$, and that yields the deviating buyer a payoff

$$\nu[v(\overline{\theta})\overline{q} - \overline{t}] > \nu\{v(\overline{\theta})\overline{q} - \mathbf{E}[v(\theta)] + \underline{\theta}(1-\overline{q})\},$$

which is strictly positive for $\overline{q}$ close enough to one. To block such deviations, latent contracts must be issued that are not actively traded in equilibrium but which the seller has an incentive to trade if some buyer attempts to break the equilibrium. In order to play this deterrence role, the corresponding latent allocations must remain available if any buyer withdraws his menu offer. For instance, in the case $\mathbf{E}[v(\theta)] > \overline{\theta}$, the cream-skimming deviation described above is blocked if the quantity $1 - \overline{q}$ can always be sold at unit price $\mathbf{E}[v(\theta)]$ at the deviation stage, since both types of the seller then have the same incentives to trade the contract proposed by the deviating buyer. This corresponds to the linear price equilibrium described in Proposition 2. In this equilibrium, the number of latent contracts is large; indeed, the menus offered by the buyers are infinite collections of contracts. The following result shows that this is a robust feature of any equilibrium.

Proposition 5 *In any equilibrium of the non-exclusive competition game, there are infinitely many aggregate allocations that remain available if any buyer withdraws his menu offer.*

The intuition for this result is as follows. As suggested by the above discussion, one of the roles of latent contracts is to prevent cream-skimming deviations that only attract type $\overline{\theta}$. Each buyer issues these contracts anticipating that type $\underline{\theta}$ will have an incentive to trade them following a cream-skimming deviation by any of the other buyers. Now, there are infinitely many such deviations. Consistent with this, the proof of Proposition 5 proceeds by showing that if only finitely many latent contracts were offered in equilibrium by buyers $j \neq i$, it would be possible to construct a cream-skimming deviation for buyer i that would yield him a strictly positive payoff.

3.3.3 Menus, Communication, and the Failure of the Revelation Principle

Our results on the necessary role played by latent contracts to support equilibrium allocations have a natural interpretation in the language of the common agency literature, whose aim is to analyze situations where several principals compete through mechanisms for the services of a single agent.[16] In our context, given a set $\mathfrak{M}^i$ of messages from the seller to buyer i, a (deterministic) mechanism for buyer i is a mapping $\pi^i : \mathfrak{M}^i \to [0,1] \times \mathbb{R}_+$ that associates to each message sent by the seller to buyer i a quantity-transfer pair or contract. Let $\Pi^i(\mathfrak{M}^i)$ be the set of mechanisms available to buyer i and $\Pi(\mathfrak{M}^1, \ldots, \mathfrak{M}^n) = \prod_{i=1}^n \Pi^i(\mathfrak{M}^i)$.

[16] To use the terminology of Bernheim and Whinston (1986b), our non-exclusive competition game is a *delegated* common agency game, as the seller can choose a strict subset of buyers with whom she wants to trade. Thus common agency is a choice variable that is delegated to the seller. See for instance Martimort (2007) for a recent overview of the common agency literature.

In the common agency game relative to $\Pi(\mathfrak{M}^1, \ldots, \mathfrak{M}^n)$, the seller takes her participation and communication decisions after having observed the profile of mechanisms $(\pi^1, \ldots, \pi^n)$ offered by the different buyers. Peters (2001) and Martimort and Stole (2002) have proven the following result, often referred to as the *Delegation Principle*: for any pure strategy equilibrium outcome relative to the space of mechanisms $\Pi(\mathfrak{M}^1, \ldots, \mathfrak{M}^n)$, there exists a pure strategy equilibrium that induces the same outcome in the game where buyers offer menus of contracts, provided any size restrictions on the original message spaces $\mathfrak{M}^i$'s are translated into corresponding restrictions on the allowed menus.

In our setting, buyers compete over menus of contracts for the trade of a divisible good. From Proposition 5, we know that equilibrium menus should contain an infinite number of contracts. In view of the Delegation Principle, this suggests that to support our Akerlof-like equilibrium outcomes when competition over mechanisms is considered, a rich structure of communication has to be postulated. That is, an infinite number of messages should be available to the seller, allowing her to effectively act as a coordinating device among buyers, so as to guarantee the existence of an equilibrium. In particular, these allocations cannot be supported if buyers are restricted to compete through simple direct mechanisms of the form $\hat{\pi}^i : \{\underline{\theta}, \overline{\theta}\} \to [0, 1] \times \mathbb{R}_+$ through which the seller can only communicate her type to the buyers. Indeed, if the buyers are restricted to direct mechanisms, only a finite set of offers will be available to the seller, which, as we have seen, makes it impossible to support our equilibrium allocations. Critically, direct mechanisms do not provide enough flexibility to buyers to make a strategic use of the seller in deterring cream-skimming deviations.[17]

The possibility to support equilibrium allocations relative to an arbitrary set of indirect mechanisms, but not in the corresponding direct mechanism game, has been acknowledged as a failure of the Revelation Principle in common agency games, and documented in purely abstract game-theoretic examples.[18] One of the contribution of our analysis is to exhibit a natural and relevant economic setting that exhibits this feature. Note furthermore that, in contrast with the exclusive competition context, where market equilibria can without any loss of generality be characterized through direct mechanisms, the restriction to such mechanisms turns out to be devastating under non-exclusivity: indeed, in this context, an immediate implication of our analysis is that no allocation can be supported in an equilibrium

[17] This difficulty would remain intact even if stochastic direct mechanisms $\tilde{\pi}^i : \mathfrak{M}^i \to \Delta([0, 1] \times \mathbb{R}_+)$ were allowed. Indeed, in any pure strategy equilibrium of a game where buyers use such mechanisms, the seller will send messages before observing the realization of uncertainty. In equilibrium, only a finite number of lotteries over allocations will be offered. Bilateral risk-neutrality then makes this situation equivalent to one in which only deterministic allocations are proposed. One should however observe that it is problematic to interpret stochastic mechanisms in our model, because the seller operates under a capacity constraint.

[18] See for instance Peck (1997), Peters (2001) and Martimort and Stole (2002).

of the direct mechanism game.

3.3.4 Non-Linear Equilibria

We now show that one can also construct non-linear equilibria in which latent contracts are issued at a unit price different from that of the aggregate allocation that is traded in equilibrium.

Proposition 6 *The following holds*:

(i) If $\mathbf{E}[v(\theta)] > \overline{\theta}$, *then, for each* $\phi \in [\overline{\theta}, \mathbf{E}[v(\theta)])$, *the non-exclusive competition game has an equilibrium in which each buyer offers the menu*

$$\left\{(q,t) \in \left[0, \frac{v(\overline{\theta}) - \mathbf{E}[v(\theta)]}{v(\overline{\theta}) - \phi}\right] \times \mathbb{R}_+ : t = \phi q\right\} \cup \{(1, \mathbf{E}[v(\theta)])\}.$$

(ii) If $\mathbf{E}[v(\theta)] < \overline{\theta}$, *then, for each* $\psi \in \left(v(\underline{\theta}), v(\underline{\theta}) + \frac{\overline{\theta} - \mathbf{E}[v(\theta)]}{1-\nu}\right]$, *the non-exclusive competition game has an equilibrium in which each buyer offers the menu*

$$\{(0,0)\} \cup \left\{(q,t) \in \left[\frac{\psi - v(\underline{\theta})}{\psi}, 1\right] \times \mathbb{R}_+ : t = \psi q - \psi + v(\underline{\theta})\right\}.$$

This results shows that the unique aggregate equilibrium allocation can also be supported through non-linear prices. In such equilibria, the price each buyer is willing to pay for an additional unit of the good is not the same for all quantities purchased. For instance, in the equilibrium for the severe adverse selection case described in Proposition 6(ii), buyers are not ready to pay anything for all quantities up to the level $\frac{\psi - v(\underline{\theta})}{\psi}$, while they are ready to pay ψ for each additional unit of the good above this level. The price schedule posted by each buyer is such that, for any $q < 1$, the unit price $\max\left\{0, \psi - \frac{\psi - v(\underline{\theta})}{q}\right\}$ at which he offers to purchase the quantity q is strictly below $\underline{\theta}$, while the marginal price ψ at which he offers to purchase an additional unit given that he has already purchased a quantity $q \geq \frac{\psi - v(\underline{\theta})}{\psi}$ is strictly above $\underline{\theta}$. Therefore the equilibrium budget set of the seller

$$\left\{(Q,T) \in [0,1] \times \mathbb{R}_+ : Q = \sum_i q^i \text{ and } T \leq \sum_i t^i \text{ where } (q^i, t^i) \in C^i \text{ for all } i\right\}$$

is not convex in this equilibrium. As a result, the seller has a strict incentive to deal with a single buyer: market equilibria can be supported with a single active buyer, provided that the other buyers coordinate by offering appropriate latent contracts. It follows in particular that non-exclusive competition does not necessarily entail that the seller enters into multiple contracting relationships.

This result contrasts with recent work on competition in non-exclusive mechanisms under incomplete information, where attention is typically restricted to equilibria in which the informed agent has a convex budget set in equilibrium, or, what amounts to the same thing, where the set of allocations available to her is the frontier of a convex budget set.[19] In our model, this would for instance arise if all buyers posted concave price schedules. It is therefore interesting to notice that, as a matter of fact, our non-exclusive competition game has no equilibrium in which each buyer i posts a strictly concave price schedule $\mathfrak{T}^i$. The reason is that the aggregate price schedule $\mathfrak{T}$ defined by $\mathfrak{T}(Q) = \max\{\sum_i \mathfrak{T}^i(q^i) : \sum_i q^i = Q\}$ would otherwise be strictly concave in the aggregate quantity traded Q. This would in turn imply that contracts are issued at a unit price strictly above $\mathfrak{T}(1)$, which, as shown by Proposition 3, is impossible in equilibrium.

A further implication of Proposition 6 is that latent contracts supporting the equilibrium allocations can be issued at a profitable price for the issuer. For instance, in the equilibrium described in Proposition 6(ii), any contract in the set $\left\{\left[\frac{\psi - v(\theta)}{\psi}, 1\right) \times \mathbb{R}_+ : t = \psi q - \psi + v(\underline{\theta})\right\}$ would yield its issuer a strictly positive payoff, even if it were traded by type $\underline{\theta}$ only. In equilibrium, no mistakes occur, and buyers correctly anticipate that none of these contracts will be traded. Nonetheless, removing them would break the equilibrium.

4 The Continuous-Type Case

In this section, we show that the results derived so far extend to the case where the seller's type is continuously distributed. The model remains the same as in Section 2, but from now on we assume that the seller's type θ has a continuously differentiable distribution F with strictly positive density f over a compact interval $[\underline{\theta}, \bar{\theta}]$ of $\mathbb{R}_{++}$. The valuation function v is assumed to be continuous; we will sometimes assume that v is strictly increasing, as is natural when the seller's private information bears on the quality of the good. We shall look for equilibria that verify a simple refinement called conservativeness. Specifically, a Perfect Bayesian Equilibrium is *conservative* if a buyer cannot profitably deviate by adding one contract to his equilibrium menu, assuming that those types of the seller that would strictly lose from trading the new contract do not change their behavior compared to the equilibrium path. Hence conservativeness requires that the seller does not play an active role

[19] See for instance Biais, Martimort and Rochet (2000), Khalil, Martimort and Parigi (2007) or Martimort and Stole (2009). Piaser (2007) offers a general discussion of the role of latent contracts in incomplete information settings.

in deterring deviations by a buyer if she does not benefit from doing so.[20] This requirement was not needed in the study of the two-type case, because we were able to perfectly control the trades of each type following a deviation. This is more difficult with a continuum of types, and for the sake of simplicity we choose to reinforce the equilibrium concept.[21]

4.1 Monopsony

As a preliminary, it is useful to consider the monopsony case with a single buyer. Suppose first that the monopsony simply offers to buy the seller's whole endowment at price p. Because only types below p accept this offer, the monopsony's payoff is then

$$w(p) = \int_{\underline{\theta}}^{p} [v(\theta) - p] \, dF(\theta). \tag{2}$$

The function w is continuous, vanishes at $\underline{\theta}$, and is strictly decreasing beyond $\overline{\theta}$. It thus has a maximum $w^m \geq 0$ that is attained at some point in $[\underline{\theta}, \overline{\theta}]$. To avoid ambiguities, define the monopsony price p^m as the highest such point. Now, assume that the monopsony can offer arbitrary menus of contracts, with quantities in $[0,1]$. From the Revelation Principle, there is no loss of generality in focusing on direct mechanisms $(Q,T) : [\underline{\theta}, \overline{\theta}] \to [0,1] \times \mathbb{R}_+$ that stipulate a quantity and a transfer as a function of the seller's report of her type.[22] The monopsony maximizes his payoff

$$\int_{\underline{\theta}}^{\overline{\theta}} [v(\theta)Q(\theta) - T(\theta)] \, dF(\theta),$$

subject to the seller's incentive compatibility and individual rationality constraints

[20] Observe that this refinement does not restrict in any way the behavior of the seller following a deviation by a buyer who withdraws some or all of his equilibrium offers. By contrast, in any subgame where the refinement has bite, the equilibrium utility of the seller remains available no matter her type.

[21] Applications of common agency games with incomplete information typically postulate restrictions on the menus offered by the principals that guarantee that each agent's type has a unique best response (see Martimort and Stole (2009) for a discussion of this point). When a notion of equilibrium refinement is introduced, attention is usually restricted to *strongly robust* equilibria (Peters (2001), Han (2008)). An equilibrium of a common agency game is strongly robust if the agent's choice is optimal from the point of view of each principal i both on and off the equilibrium path, following a unilateral deviation of principal i. In our non-exclusive competition game, however, the aggregate equilibrium allocations described below cannot in general be supported in a strongly robust equilibrium. Indeed, since any equilibrium typically involves that different types of the seller pool and trade their whole endowment, each buyer could always gain if his equilibrium offer were only traded by those types of the seller that yield him a strictly positive payoff. To get a clearer intuition of this point, consider again the two-type environment discussed in Section 3.2, and suppose that $\mathbf{E}[v(\theta)] > \overline{\theta}$, so that $(1, \mathbf{E}[v(\theta)])$ is the aggregate allocation traded by both types of the seller in equilibrium. From the point of view of any active buyer, it would be optimal to have type $\underline{\theta}$ trade with the other buyers only. As we have seen, however, such a behavior is not consistent with equilibrium.

[22] It is easy to check that, because of linear preferences, the monopsony cannot improve his payoff by offering a stochastic mechanism.

$$T(\theta) - \theta Q(\theta) \geq T(\theta') - \theta Q(\theta'),$$

$$T(\theta) - \theta Q(\theta) \geq 0,$$

for all $(\theta, \theta') \in [\underline{\theta}, \overline{\theta}]^2$. In line with Samuelson (1984), we have the following result.

Lemma 5 *Even when allowed to trade quantities in* $[0, 1]$, *the monopsony cannot do better than offering to buy the seller's whole endowment at the price* p^m.

Hence allowing to trade any fraction of the seller's endowment has no impact on the solution to the monopsony problem. This may seem intuitive, as preferences are linear. One should however be cautious; as we now show, this option does impact equilibria when buyers compete in exclusive contracts, despite the linearity of preferences.

4.2 Exclusive Competition

Suppose first that buyers are restricted to bid for the seller's whole endowment. Define p^* as the supremum of those p such that $w(p) > 0$, setting $p^* = \underline{\theta}$ if there are none. Thus p^* is the highest price at which the seller's whole endowment can be profitably bought. Since w is continuous, we know that $w(p^*) = 0$, which whenever $p^* > \underline{\theta}$ can be rewritten under the more familiar form:

$$p^* = \mathbf{E}[v(\theta) \,|\, \theta \leq p^*].$$

That is, p^* satisfies the property put forward by Akerlof (1970): at price p^* competitive supply equals competitive demand, all seller's types below p^* sell their whole endowment, while seller's types above p^* do not trade at all. To avoid discussing non-generic cases we assume that $w(p) < 0$ whenever $p > p^*$. Arguing as in Mas-Colell, Whinston and Green (1995, Proposition 13.B.1), one can then show that p^* is the price that prevails in equilibrium when buyers can only bid for the seller's whole endowment.[23]

Let us now allow for arbitrary trades, but restrict the seller to trade with a single buyer, as in the exclusive competition game of Subsection 3.1. Recall that in the two-type case studied then, equilibria under exclusive competition were similar to those derived by Rothschild and Stiglitz (1976): equilibria are separating, and exist only under restrictive assumptions on the seller's type distribution. In the continuous-type case, non-existence of equilibria turns out to be the rule, as we now explain. The intuition for this result is that exclusive contracting

[23] It should be observed that the equilibrium is generically unique here because the buyers set prices strategically. By contrast, when buyers are price-takers, as in Akerlof (1970), multiple equilibria may occur in a robust way, see Mas-Colell, Whinston and Green (1995, Chapter 13, Section B).

allows buyers to design very precisely their offers, so as to target the seller's types whose trades are profitable to them. In particular, when a quantity strictly between zero and one is traded, buyers can deviate by proposing to trade a lower or a higher quantity. This flexibility in turn implies the very strong zero-profit condition that, in equilibrium, the buyers' aggregate payoff must be zero on any type who trades such a quantity.[24] Along with the seller's incentive compatibility condition, this greatly reduces the set of possible equilibrium outcomes. When v is strictly increasing, all such allocations can be shown to be vulnerable to a pooling offer to buy the whole endowment from an interval of types.

Proposition 7 *Suppose that v is strictly increasing. Then all conservative equilibria of the exclusive competition game feature no trade. In particular, no equilibrium exists if the monopsony payoff w^m is strictly positive.*

4.3 Non-Exclusive Competition

By contrast, our first result in this section is that an equilibrium always exists under non-exclusive competition.

Proposition 8 *The non-exclusive competition game always has a conservative equilibrium in which each buyer offers the menu*

$$\{(q,t) \in [0,1] \times \mathbb{R}_+ : t = p^* q\},$$

and thus stands ready to buy any quantity of the good at the constant unit price p^*.

Hence equilibria always exists, even when v is not monotonic. Observe that the outcome induced by this linear price equilibrium is in line with Akerlof (1970): all seller's types strictly below p^* sell their whole endowment, while seller's types strictly above p^* do not trade at all. Our second result is that this must be the case in any conservative equilibrium.

Proposition 9 *In any conservative equilibrium of the non-exclusive competition game, the aggregate equilibrium allocations satisfy*

$$(Q(\theta), T(\theta)) = (1, p^*) \text{ if } \theta < p^* \text{ and } (Q(\theta), T(\theta)) = (0, 0) \text{ if } \theta > p^*.$$

In particular, each buyer's payoff is zero in any conservative equilibrium.

The intuition for this result can be easily understood in the context of a free-entry

[24] On the set of types who trade their whole endowment, one can only show that the buyers' aggregate payoff must be on average zero.

equilibrium. Suppose that some type $\theta_1 < p^*$ sells a quantity $Q_1 < 1$. Since incentive compatibility implies that the aggregate quantity traded by the seller must be a decreasing function of her type, it follows from the definition of p^* that one can moreover choose θ_1 such that $w(\theta_1) > 0$. Then an entrant could offer to buy $1 - Q_1$ at a unit price θ_1. Clearly all types above θ_1 would reject this new offer. By contrast, type θ_1 is indifferent: if she accepts the offer, she sells $1 - Q_1$ units to the entrant, and she sells as before the remaining fraction Q_1 of her endowment to the other buyers. Because types below θ_1 are more eager to sell, they must also choose to sell their whole endowment, and therefore all accept the new offer. The entrant's payoff would then be $(1 - Q_1)w(\theta_1) > 0$, meaning that entry would be profitable. In the proof of Proposition 9, we show that a deviation that makes the trade $((1 - Q_1), \theta_1(1 - Q_1))$ available, in addition to the trades already offered, is profitable to at least one buyer. As in our analysis of the two-type case, this buyer offers a further trading opportunity by pivoting on the trades already offered by the other buyers.

Proposition 9 implies that aggregate quantities and transfers are uniquely determined in equilibrium, and correspond to those that would obtain in the classical Akerlof (1970) model.[25] Yet a distinctive feature of our model is that buyers are strategic and compete for the divisible good offered by the seller by proposing non-exclusive menus of contracts to her. Our results thus provide a firm game-theoretic foundation to Akerlof's (1970) predictions. Observe in particular that if $p^* = \underline{\theta}$, and thus $w(p) < 0$ for all $p > p^*$, there is essentially no trade in equilibrium, in the sense that all types $\theta > \underline{\theta}$ do not trade, while all contracts featuring strictly positive quantities must have unit price $\underline{\theta}$, so that type $\underline{\theta}$ is indifferent between trading them or not. Thus complete market breakdown is consistent with our model. Finally, it should be noted that since $p^* \geq p^m$, there is more trade under non-exclusive competition than in the monopsony case, which does not come as a surprise.

4.4 Equilibrium Menus

We now explore the structure of the menus offered by the buyers in equilibrium, and in particular the role and necessity of latent contracts. Our first results parallel Proposition 3 and Corollary 2 and provide equilibrium restrictions on the price of all issued and traded contracts.

Proposition 10 *The unit price of any contract issued in a conservative equilibrium of the non-exclusive competition game is at most p^*.*

[25]More precisely, since p^* is defined as the highest price at which the seller's endowment can be profitably bought, the aggregate quantities and transfers characterized in Proposition 9 correspond to the highest price competitive equilibrium in Akerlof (1970), see footnote 23.

Corollary 3 *The unit price of any contract traded in a conservative equilibrium of the non-exclusive competition game is* p^*.

As in the two-type case, these results illustrate how competition disciplines the buyers in our model: even though they are allowed to propose arbitrary menus of contracts, in equilibrium they end up trading at the same price. Even non-traded contracts must be issued at a unit price at most equal to p^*: otherwise one of the buyers could strategically use such a contract and pivot on it so as to increase his payoff. It should be noted that if $p^* \leq \overline{\theta}$, this last result can be proven without relying on a pivoting argument: indeed, if a contract with unit price strictly above p^* were issued, then type p^* would have a strict incentive to trade this contract instead of those that she trades in equilibrium, and so would all types slightly below p^* by continuity of the seller's preferences with respect to her type.

We now investigate which contracts need to be issued in order to support the aggregate equilibrium allocations. In line with Proposition 4, one first has the following result.

Proposition 11 *Suppose that* $p^* > \underline{\theta}$. *Then, in any conservative equilibrium of the non-exclusive competition game, the aggregate allocation* $(1, p^*)$ *remains available if any buyer withdraws his menu offer.*

When $\mathbf{E}[v(\theta)] > \overline{\theta}$, the proof of this result is identical to that of its two-type counterpart. However, when $\mathbf{E}[v(\theta)] < \overline{\theta}$, the proof is more involved in the continuous-type case. Indeed, unlike in the two-type case, where there was a wedge between the type $\underline{\theta}$ of the active seller and the equilibrium price $v(\underline{\theta})$ at which all trades take place, in the continuous-type case the equilibrium price is equal to the type p^* of the marginal seller. This makes it impossible for a buyer to screen types $\theta > p^*$ from types $\theta \leq p^*$ at the deviation stage. Instead, we show that if the allocation $(1, p^*)$ did not remain available if a buyer removed his equilibrium offer, then for $\varepsilon > 0$ small enough this buyer could pivot on the aggregate allocation that type $p^* - \varepsilon < p^*$ would optimally trade with buyers $j \neq i$ only, and secure a strictly positive payoff by trading with types $\theta < p^* - \varepsilon$.

We now argue that many contracts need to be issued to sustain equilibria, even though each of these contracts has at most unit price p^*. Suppose for simplicity that the function v is strictly increasing, and consider types close to but below p^*. Because these types are less eager to sell than type p^*, it is possible to deviate by offering to buy a quantity slightly below one at a unit price slightly above p^*. The fact that v is strictly increasing ensures that the deviating buyer would obtain a positive payoff from trading such a contract with the types in question. However, the distinctive feature of non-exclusive competition is that other types may also be attracted by the deviating buyer's offer. Indeed, these types could accept the deviation, and sell the remaining part of their endowment to non-deviating buyers if the latter offer contracts that allow to trade small quantities at a price close enough to p^*. This in turn proves necessary to sustain equilibria, as we now show.

Proposition 12 *Suppose that $p^* > \underline{\theta}$ and v is strictly increasing. Then, in any conservative equilibrium of the non-exclusive competition game, there exists $\overline{Q}_0 > 0$ such that it remains possible to trade any quantity below $\overline{Q}_0$ if any buyer withdraws his menu offer.*

Proposition 12 implies that in equilibrium many contracts, in fact a continuum of them, must be available. A similar conclusion was derived in the two-type case, though in the case where $\mathbf{E}[v(\theta)] < \overline{\theta}$, we only established the necessity of a countably infinite number of contracts. A closer examination of the proof however reveals that the result depends on whether there are at least two types that trade in equilibrium. In that respect, the two-type case with $\mathbf{E}[v(\theta)] < \overline{\theta}$ is somewhat special, because only one type of the seller is trading in equilibrium.

An important question is whether latent contracts are necessary to sustain equilibria. Observe that all types below p^* end up trading the same aggregate quantity for the same aggregate transfer. One natural refinement would then recommend to focus on equilibria in which these types behave identically, by trading the same quantity with each buyer. Then each buyer would only trade one contract, and Proposition 12 proves that a continuum of latent contracts is needed to sustain such equilibria.

One could however argue that the seller may randomize across different buyers, while still preserving the refinement just introduced. For example, the seller may draw uniformly a vector in the set $\{(q_1, \ldots, q_n) \in \mathbb{R}^n_+ : \sum_i q_i = 1\}$, and trade quantity q_i with buyer i. Then all contracts offered would be traded, so that strictly speaking no latent contracts would be needed to sustain the equilibrium. Observe nevertheless that such equilibria with randomization cannot be implemented through simple direct mechanisms, as the quantity sold by any given type to any given buyer does not depend only on her type, but also on the result of the randomization.

One could alternatively reject the above refinement and allow different types to behave differently, while they in fact sell the same aggregate quantity for the same aggregate transfer. Then, unlike in the two-type case, one can even build equilibria that only rely on direct mechanisms, and are thus equilibria of the direct mechanism game. To see this, suppose that each buyer i proposes the seller to trade a quantity $q^i(\hat{\theta})$ at unit price p^* if she reports type $\hat{\theta}$ to him, where the functions $(q^1, \ldots, q^n)$ satisfy

(i) $\sum_i q^i(\theta) = 1$ for all $\theta < p^*$ and $\sum_i q^i(\theta) = 0$ for all $\theta > p^*$;

(ii) $\int_{\underline{\theta}}^{p^*} [v(\theta) - p^*] q^i(\theta)\, dF(\theta) = 0$ for all i;

(iii) $q^i([\underline{\theta}, \overline{\theta}]) = [0, 1]$ for all i.

Property (i) ensures that each type of the seller trades her whole endowment or refrain from trading altogether, as in the Akerlof (1970) outcome characterized in Proposition 9. Property (ii) ensures that each buyer obtains a zero payoff. Property (iii) ensures that all contracts in $\{(q, t) \in [0, 1] \times \mathbb{R}_+ : t = p^* q\}$ are traded in equilibrium by at least one type, so that there are no latent contracts. From Proposition 8, these offers form an equilibrium of the direct mechanism game, which essentially amounts to a purification of an equilibrium in which the seller would randomize across buyers. To speak frankly, we think that this construction is rather artificial, though worth mentioning for completeness. Finally, even if equilibria in direct mechanisms do exist in the continuous-type case, it should be emphasized that there still exist outcomes that can be supported through equilibrium menus in the non-exclusive competition game, but cannot be replicated in equilibrium through direct mechanisms. The simplest example of this phenomenon arises when $\mathbf{E}[v(\theta)] > \overline{\theta}$, so that $p^* = \mathbf{E}[v(\theta)]$ and each buyer stands ready to buy any quantity of the good at unit price p^*, as in the equilibrium described in Proposition 8, while each type of the seller trades her whole endowment with a single buyer, say buyer i. The corresponding direct mechanism for buyer i is the constant function $\hat{\gamma}^i : [\underline{\theta}, \overline{\theta}] \to [0, 1] \times \mathbb{R}_+ : \hat{\theta} \mapsto (1, p^*)$, and the corresponding direct mechanism for any buyer $j \neq i$ is the constant function $\hat{\gamma}^j : [\underline{\theta}, \overline{\theta}] \to [0, 1] \times \mathbb{R}_+ : \hat{\theta} \mapsto (0, 0)$. But then buyer i could deviate by mimicking the monopsony and thereby secure a payoff $w^m > 0$. The failure of the Revelation Principle thus carries over to the continuous-type case.

5 Conclusion

In this paper, we have studied a model of trade under adverse selection in which buyers compete for a good whose quality is privately observed by the seller. The most distinctive features of the model are that the good to be traded is divisible, and that buyers compete in a non-exclusive way, so that the seller may choose to trade with several of them. Contracting between the seller and each buyer is bilateral, reflecting that buyers cannot monitor each others' trades with the seller. Besides this, we impose very little restrictions on instruments, as buyers can essentially offer arbitrary menus of contracts, or price schedules. In this setting, we show that equilibria always exist, unlike in standard competitive screening models where competition among uninformed parties is exclusive. Aggregate quantities and transfers are generically unique, and correspond to the highest price competitive equilibrium in Akerlof's (1970) model. Linear price equilibria exist in which buyers stand ready to purchase any quantity at a constant unit price, but one can also construct equilibria in which buyers post non-linear price schedules and only one of them actively trades with the seller. While buyers act strategically, our results hold regardless of their number. In addition, a large number of contracts is shown to be necessary to support the equilibrium allocations, although only a tiny fraction of them end up being traded in equilibrium. The wide applicability of our assumptions along with the simplicity of the equilibrium predictions suggest that our model could easily been used as a building block in applications, for instance in finance or macroeconomics.

The fact that possible market outcomes tightly depend on the nature of competition suggests that the testable implications of competitive models of adverse selection should be evaluated with care. Indeed, these implications are typically derived from the study of exclusive competition models, such as Rothschild and Stiglitz's (1976). By contrast, our analysis shows that partial pooling is an important feature of market equilibria under non-exclusive competition. Our results offer new insights into the empirical literature on adverse selection. For instance, several studies have taken to the data the predictions of theoretical models of insurance provision, without reaching clear conclusions.[26] Cawley and Philipson (1999) argue that there is little empirical support for the adverse selection hypothesis in life insurance. In particular, they find no evidence that marginal prices raise with coverage. Similarly, Finkelstein and Poterba (2004) find that marginal prices do not significantly differ across annuities with different initial annual payments. The theoretical predictions tested by these authors are however derived from models of exclusive competition, while our results clearly indicate that they do not hold when competition is non-exclusive, as in the case of life insurance or annuities.[27] Indeed, non-exclusive competition might be one explanation for the limited evidence of screening and the prevalence of nearly linear pricing schemes on these markets. As a result, more sophisticated procedures need to be designed in order to test for the presence of adverse selection in markets where competition is non-exclusive.

[26] See Chiappori and Salanié (2003) for a survey of this literature.

[27] Chiappori, Jullien, Salanié and Salanié (2006) have derived general tests based on a model of exclusive competition, that they apply to the case of car insurance.

Appendix

Proof of Proposition 1. The proof follows more or less standard lines (see for instance Mas-Colell, Whinston and Green (1995, Chapter 13, Section D)) and goes through a series of steps.

Step 1 Denote by $(\underline{q}, \underline{t})$ and $(\overline{q}, \overline{t})$ the contracts traded by the two types of the seller in equilibrium. These contracts must satisfy the following incentive constraints:

$$\underline{t} - \underline{\theta}\underline{q} \geq \overline{t} - \underline{\theta}\overline{q},$$

$$\overline{t} - \overline{\theta}\overline{q} \geq \underline{t} - \overline{\theta}\underline{q}.$$

Since the buyers always have the option not to trade, each of them must obtain at least a zero payoff in equilibrium. Suppose that some buyer's equilibrium payoff is strictly positive. Then the buyers' aggregate equilibrium payoff is strictly positive,

$$\nu[v(\overline{\theta})\overline{q} - \overline{t}] + (1 - \nu)[v(\underline{\theta})\underline{q} - \underline{t}] > 0.$$

Any buyer i obtaining less than half of this amount in equilibrium can deviate by offering a menu consisting of the no-trade contract and of two new contracts. The first one is

$$\underline{c}^i(\underline{\varepsilon}) = (\underline{q}, \underline{t} + \underline{\varepsilon}),$$

for some strictly positive number $\underline{\varepsilon}$, and is designed to attract type $\underline{\theta}$. The second one is

$$\overline{c}^i(\overline{\varepsilon}) = (\overline{q}, \overline{t} + \overline{\varepsilon}),$$

for some strictly positive number $\overline{\varepsilon}$, and is designed to attract type $\underline{\theta}$. To ensure that type $\underline{\theta}$ trades $\underline{c}^i(\underline{\varepsilon})$ and type $\overline{\theta}$ trades $\overline{c}^i(\overline{\varepsilon})$ with him, buyer i can choose $\underline{\varepsilon}$ to be equal to $\overline{\varepsilon}$ when both types' equilibrium incentive constraints are simultaneously binding or slack, and choose $\underline{\varepsilon}$ and $\overline{\varepsilon}$ to be different but close enough to each other when one of these constraints is binding and the other is slack. The change in buyer i's payoff induced by this deviation is at least

$$\frac{1}{2}\left\{\nu[v(\overline{\theta})\overline{q} - \overline{t}] + (1 - \nu)[v(\underline{\theta})\underline{q} - \underline{t}]\right\} - \nu\overline{\varepsilon} - (1 - \nu)\underline{\varepsilon},$$

which is strictly positive for $\underline{\varepsilon}$ and $\overline{\varepsilon}$ close enough to zero. Thus each buyer's payoff is zero in any equilibrium.

Step 2 Suppose that there exists a pooling equilibrium with both types of the seller trading the same contract (q^p, t^p). It follows from Step 1 that $t^p = \mathbf{E}[v(\theta)]q^p$ and that both types of the seller must trade with the same buyer j. Any buyer $i \neq j$ can deviate by offering a menu consisting of the no-trade contract and of the contract

$$\overline{c}^i(\varepsilon) = (q^p - \varepsilon, t^p - \underline{\theta}\varepsilon(1+\varepsilon)),$$

for some strictly positive number ε. Trading $\overline{c}^i(\varepsilon)$ decreases type $\underline{\theta}$'s payoff by $\underline{\theta}\varepsilon^2$ compared to what she obtains by trading (q^p, t^p) with buyer j. Hence type $\underline{\theta}$ does not trade $\overline{c}^i(\varepsilon)$ following buyer i's deviation. By contrast, if $\varepsilon < \frac{\overline{\theta}}{\underline{\theta}} - 1$, trading $\overline{c}^i(\varepsilon)$ allows type $\overline{\theta}$ to increase her payoff by $[\overline{\theta} - (1+\varepsilon)\underline{\theta}]\varepsilon$ compared to what she obtains by trading (q^p, t^p) with buyer j. Hence type $\overline{\theta}$ trades $\overline{c}^i(\varepsilon)$ following buyer i's deviation. The payoff for buyer i induced by this deviation is

$$\nu\{v(\overline{\theta})q^p - t^p - [v(\overline{\theta}) - \underline{\theta}(1+\varepsilon)]\varepsilon\},$$

which is strictly positive for ε close enough to zero since $t^p = \mathbf{E}[v(\theta)]q^p$ and $v(\overline{\theta}) > \mathbf{E}[v(\theta)]$. This, however, is impossible by Step 1. Thus any equilibrium must be separating, with the two types of the seller trading different contracts.

Step 3 Suppose that $v(\underline{\theta})\underline{q} > \underline{t}$, so that the contract $(\underline{q}, \underline{t})$ yields the buyer who trades it with type $\underline{\theta}$ a strictly positive payoff. Any buyer i can deviate by offering a menu consisting of the no-trade contract and of the contract

$$\underline{c}^i(\varepsilon) = (\underline{q}, \underline{t} + \varepsilon),$$

for some strictly positive number ε. Type $\underline{\theta}$ trades $\underline{c}^i(\varepsilon)$ following buyer i's deviation, and also possibly type $\overline{\theta}$. The payoff for buyer i induced by this deviation is thus at least

$$(1-\nu)[v(\underline{\theta})\underline{q} - \underline{t} - \varepsilon],$$

which is strictly positive for ε close enough to zero if $v(\underline{\theta})\underline{q} > \underline{t}$. Since this is impossible by Step 1, it must be that $\underline{t} \geq v(\underline{\theta})\underline{q}$. Suppose next that $v(\overline{\theta})\overline{q} > \overline{t}$, so that the contract $(\overline{q}, \overline{t})$ yields the buyer j who trades it with type $\overline{\theta}$ a strictly positive payoff. Any buyer $i \neq j$ can deviate by offering a menu consisting of the no-trade contract and of the contract

$$\overline{c}^i(\varepsilon) = (\overline{q} - \varepsilon, \overline{t} - \underline{\theta}\varepsilon(1+\varepsilon)),$$

for some strictly positive number ε. As in Step 2, it is easy to check that type $\underline{\theta}$ does not trade $\overline{c}^i(\varepsilon)$ following buyer i's deviation, while type $\overline{\theta}$ does so provided $\varepsilon < \frac{\overline{\theta}}{\underline{\theta}} - 1$. The payoff for buyer i induced by this deviation is

$$\nu\{v(\overline{\theta})\overline{q} - \overline{t} - [v(\overline{\theta}) - \underline{\theta}(1+\varepsilon)]\varepsilon\},$$

which is strictly positive for ε close enough to zero if $v(\overline{\theta})\overline{q} > \overline{t}$. Since this is impossible by Step 1, it must be that $\overline{t} \geq v(\overline{\theta})\overline{q}$. This, along with the facts that $\underline{t} \geq v(\underline{\theta})\underline{q}$ and that the buyers' aggregate equilibrium payoff is zero, implies that $\underline{t} = v(\underline{\theta})\underline{q}$ and $\overline{t} = v(\overline{\theta})\overline{q}$. Thus the contracts $(\underline{q}, \underline{t})$ and $(\overline{q}, \overline{t})$ are traded at unit prices $v(\underline{\theta})$ and $v(\overline{\theta})$, and no cross-subsidization across types can take place in equilibrium.

Step 4 Suppose that type $\underline{\theta}$ sells a quantity $\underline{q} < 1$ in equilibrium. Any buyer i can deviate by offering a menu consisting of the no-trade contract and of the contract

$$\underline{c}^i(\varepsilon) = (1, \underline{t} + [v(\underline{\theta}) - \varepsilon](1 - \underline{q})),$$

for some strictly positive number ε. As long as $\varepsilon < v(\underline{\theta}) - \underline{\theta}$, trading $\underline{c}^i(\varepsilon)$ allows type $\underline{\theta}$ to increase her payoff by $[v(\underline{\theta}) - \underline{\theta} - \varepsilon](1 - \underline{q})$ compared to what she obtains by trading $(\underline{q}, \underline{t})$. Hence type $\underline{\theta}$ trades $\underline{c}^i(\varepsilon)$ following buyer i's deviation, and also possibly type $\overline{\theta}$. The payoff for buyer i induced by this deviation is thus at least

$$(1 - \nu)\{v(\underline{\theta}) - \underline{t} - [v(\underline{\theta}) - \varepsilon](1 - \underline{q})\} = (1 - \nu)(1 - \underline{q})\varepsilon,$$

where use was made of the fact that $\underline{t} = v(\underline{\theta})\underline{q}$ by Step 3. Since $\varepsilon > 0$, this payoff is strictly positive, which is impossible by Step 1. Thus type $\underline{\theta}$ sells her whole endowment in any equilibrium, and $(\underline{q}, \underline{t}) = (\underline{q}^e, \underline{t}^e)$ as defined in Proposition 1.

Step 5 The contract $(\overline{q}^e, \overline{t}^e)$ is characterized by two properties: it has a unit price $v(\overline{\theta})$ and type $\underline{\theta}$ is indifferent between $(\underline{q}^e, \underline{t}^e)$ and $(\overline{q}^e, \overline{t}^e)$. One cannot have $\overline{q} > \overline{q}^e$, for $(\overline{q}, \overline{t})$ is traded at unit price $v(\overline{\theta})$ by Step 3, and any contract in which a quantity strictly higher than $\overline{q}^e$ is traded at unit price $v(\overline{\theta})$ is strictly preferred by type $\underline{\theta}$ to $(\underline{q}^e, \underline{t}^e)$. Now, suppose that type $\underline{\theta}$ trades $(\underline{q}^e, \underline{t}^e)$ with buyer j in equilibrium and that $\overline{q} < \overline{q}^e$. Then type $\underline{\theta}$ strictly prefers $(\underline{q}^e, \underline{t}^e)$ to $(\overline{q}, \overline{t})$, that is, $\underline{t}^e - \underline{\theta}\underline{q}^e > \overline{t} - \underline{\theta}\overline{q}$. Any buyer $i \neq j$ can deviate by offering a menu consisting of the no-trade contract and of the contract

$$\overline{c}^i(\varepsilon) = (\overline{q} + \varepsilon, \overline{t} + \overline{\theta}\varepsilon(1 + \varepsilon)),$$

for some strictly positive number ε. Trading $\overline{c}^i(\varepsilon)$ decreases type $\underline{\theta}$'s payoff by

$$\underline{t}^e - \underline{\theta}\underline{q}^e - \overline{t} + \underline{\theta}\overline{q} - [\overline{\theta}(1 + \varepsilon) - \underline{\theta}]\varepsilon$$

compared to what she obtains by trading $(\underline{q}^e, \underline{t}^e)$ with buyer j. Since $\underline{t}^e - \underline{\theta}\underline{q}^e > \overline{t} - \underline{\theta}\overline{q}$, type $\underline{\theta}$ does not trade $\overline{c}^i(\varepsilon)$ following buyer i's deviation if ε is close enough to zero. By contrast, trading $\overline{c}^i(\varepsilon)$ allows type $\overline{\theta}$ to increase her payoff by $\overline{\theta}\varepsilon^2$ compared to what she obtains in equilibrium. Hence type $\overline{\theta}$ trades $\overline{c}^i(\varepsilon)$ following buyer i's deviation. The payoff for buyer i induced by this deviation is

$$\nu[v(\overline{\theta})(\overline{q}+\varepsilon)-\overline{t}-\overline{\theta}\varepsilon(1+\varepsilon)]=\nu[v(\overline{\theta})-\overline{\theta}(1+\varepsilon)],$$

where use was made of the fact that $\overline{t}=v(\overline{\theta})\overline{q}$ by Step 3. When $\varepsilon<\frac{v(\overline{\theta})}{\overline{\theta}}-1$, this payoff is strictly positive, which is impossible by Step 1. Thus type $\underline{\theta}$ sells a fraction $\overline{q}^e$ of her endowment in any equilibrium, and $(\overline{q},\overline{t})=(\overline{q}^e,\overline{t}^e)$ as defined in Proposition 1.

Step 6 It follows from Steps 4 and 5 that if an equilibrium exists, the contracts that are traded in this equilibrium are $(\underline{q}^e,\underline{t}^e)$ and $(\overline{q}^e,\overline{t}^e)$. To conclude the proof, one only needs to determine under which circumstances it is possible to support this allocation in equilibrium. Suppose first that $\nu>\nu^e$. Any buyer i can deviate by offering a menu consisting of the no-trade contract and of the contract

$$\tilde{c}^i(\varepsilon)=(1,v(\overline{\theta})\overline{q}^e+\overline{\theta}(1-\overline{q}^e)+\varepsilon),$$

for some strictly positive number ε. Using the fact that type $\underline{\theta}$ is indifferent between $(\underline{q}^e,\underline{t}^e)$ and $(\overline{q}^e,\overline{t}^e)$, one can check that trading $\tilde{c}^i(\varepsilon)$ allows type $\underline{\theta}$ to increase her payoff by

$$v(\overline{\theta})\overline{q}^e+\overline{\theta}(1-\overline{q}^e)+\varepsilon-v(\underline{\theta})=(\overline{\theta}-\underline{\theta})(1-\overline{q}^e)+\varepsilon$$

compared to what she obtains by trading $(\underline{q}^e,\underline{t}^e)$. Hence type $\underline{\theta}$ trades $\tilde{c}^i(\varepsilon)$ following buyer i's deviation. Similarly, trading $\tilde{c}^i(\varepsilon)$ allows type $\overline{\theta}$ to increase her payoff by ε compared to what she obtains by trading $(\overline{q}^e,\overline{t}^e)$. Hence type $\overline{\theta}$ trades $\tilde{c}^i(\varepsilon)$ following buyer i's deviation. Simple computations show that the payoff for buyer i induced by this deviation is

$$\mathbf{E}[v(\theta)]-v(\overline{\theta})\overline{q}^e-\overline{\theta}(1-\overline{q}^e)-\varepsilon=[v(\overline{\theta})-v(\underline{\theta})](\nu-\nu^e)-\varepsilon,$$

which is strictly positive for ε close enough to zero. Since this is impossible by Step 1, it follows that no equilibrium exists when $\nu>\nu^e$. Suppose then that $\nu\leq\nu^e$. Consider a candidate equilibrium in which each buyer proposes the menu consisting of the no-trade contract and of the contracts $(\underline{q}^e,\underline{t}^e)$ and $(\overline{q}^e,\overline{t}^e)$. Then, on the equilibrium path, it is a best response for type $\underline{\theta}$ to trade $(\underline{q}^e,\underline{t}^e)$ and for type $\underline{\theta}$ to trade $(\overline{q}^e,\overline{t}^e)$. By Step 3, this yields each buyer a zero payoff. To verify that this constitutes an equilibrium, one first needs to check that no buyer can strictly increase his payoff by proposing a single contract besides the no-trade contract. By Steps 3, 4 and 5, there is no profitable deviation that would attract only one type of the seller. Moreover, a profitable pooling deviation exists if and only if, given the menus offered in equilibrium, both types of the seller would have a strict incentive to sell their whole endowment at price $\mathbf{E}[v(\theta)]$. This is the case if and only if $\mathbf{E}[v(\theta)] > v(\overline{\theta})\overline{q}^e + \overline{\theta}(1-\overline{q}^e)$, or equivalently $\nu > \nu^e$. Thus when $\nu \leq \nu^e$, no menu consisting of a single contract besides the no-trade contract can constitute a profitable deviation. To conclude the proof, one only needs to check that no buyer can strictly increase his payoff by offering two contracts besides the no-trade contract, that attract both types of the seller. The maximum payoff that any buyer can achieve in this way is given by

$$\max_{(\underline{q},\underline{t},\overline{q},\overline{t})} \{\nu[v(\overline{\theta})\overline{q} - \overline{t}] + (1-\nu)[v(\underline{\theta})\underline{q} - \underline{t}]\}$$

subject to the following incentive and participation constraints:

$$\underline{t} - \underline{\theta}\underline{q} \geq \overline{t} - \underline{\theta}\overline{q},$$

$$\overline{t} - \overline{\theta}\overline{q} \geq \underline{t} - \overline{\theta}\underline{q},$$

$$\underline{t} - \underline{\theta}\underline{q} \geq \underline{t}^e - \underline{\theta}\underline{q}^e,$$

$$\overline{t} - \overline{\theta}\overline{q} \geq \overline{t}^e - \overline{\theta}\overline{q}^e.$$

Note from the incentive constraints that $\overline{q} \leq \underline{q}$. It is clear that at least one of the participation constraints must be binding. Suppose first that type $\underline{\theta}$'s participation constraint is binding. If $\underline{q} \leq \overline{q}^e$, then the relevant constraint for type $\overline{\theta}$ is her incentive constraint. It is then optimal to let type $\overline{\theta}$ be indifferent between $(\underline{q}, \underline{t})$ and $(\overline{q}, \overline{t})$. Since $v(\underline{\theta}) > \underline{\theta}$, $v(\overline{\theta}) > \overline{\theta}$ and $\underline{q} \leq \overline{q}^e$, the maximum payoff that the deviating buyer can achieve in this way is obtained by offering $(\overline{q}, \overline{t}) = (\underline{q}, \underline{t}) = (\overline{q}^e, \overline{t}^e)$, and is therefore strictly negative. If $\underline{q} > \overline{q}^e$, then the relevant constraint for type $\overline{\theta}$ is her participation constraint. It is then optimal to let type $\overline{\theta}$ be indifferent between $(\overline{q}, \overline{t})$ and $(\overline{q}^e, \overline{t}^e)$. One cannot have $\overline{q} > \overline{q}^e$, for otherwise type $\underline{\theta}$ would strictly prefer $(\overline{q}, \overline{t})$ to $(\underline{q}, \underline{t})$. Since $v(\underline{\theta}) > \underline{\theta}$, $v(\overline{\theta}) > \overline{\theta}$ and $\overline{q} \leq \overline{q}^e$, the maximum payoff that the deviating buyer can achieve in this way is obtained by offering the equilibrium contracts $(\underline{q}^e, \underline{t}^e)$ and $(\overline{q}^e, \overline{t}^e)$. Suppose finally that type $\overline{\theta}$'s participation constraint is binding. If $\overline{q} \leq \overline{q}^e$, then the relevant constraint for type $\underline{\theta}$ is her participation constraint. It is then optimal to let type $\underline{\theta}$ be indifferent between $(\underline{q}, \underline{t})$ and $(\underline{q}^e, \underline{t}^e)$. Again, since $v(\underline{\theta}) > \underline{\theta}$, $v(\overline{\theta}) > \overline{\theta}$ and $\overline{q} \leq \overline{q}^e$, the maximum payoff that the deviating buyer can achieve in this way is obtained by offering the equilibrium contracts $(\underline{q}^e, \underline{t}^e)$ and $(\overline{q}^e, \overline{t}^e)$. If $\overline{q} > \overline{q}^e$, then the relevant constraint for type $\underline{\theta}$ is her incentive constraint. It is then optimal to let type $\underline{\theta}$ be indifferent between $(\underline{q}, \underline{t})$ and $(\overline{q}, \overline{t})$. Simple computations show that the payoff for the deviating buyer is

$$\{\nu[v(\overline{\theta}) - \underline{\theta}] - \overline{\theta} + \underline{\theta}\}\overline{q} + (1-\nu)[v(\underline{\theta}) - \underline{\theta}]\underline{q} - \overline{t}^e + \overline{\theta}\overline{q}^e.$$

Since $\nu \leq \nu^e$, $v(\underline{\theta}) > \underline{\theta}$ and $\overline{q} > \overline{q}^e$, this is at most equal to the payoff that the deviating buyer would obtain by offering the equilibrium contracts $(\underline{q}^e, \underline{t}^e)$ and $(\overline{q}^e, \overline{t}^e)$. The result follows. ■

Proof of Lemma 1. Suppose instead that $\underline{Q} < 1$. Any buyer i can deviate by offering a menu consisting of the no-trade contract and of two new contracts. The first one is

$$\underline{c}^i(\varepsilon) = (\underline{q}^i + 1 - \underline{Q}, \underline{t}^i + (\underline{\theta} + \varepsilon)(1 - \underline{Q})),$$

for some strictly positive number ε, and is designed to attract type $\underline{\theta}$. The second one is

$$\overline{c}^i(\varepsilon) = (\overline{q}^i, \overline{t}^i + \varepsilon^2),$$

and is designed to attract type $\overline{\theta}$. The key feature of this deviation is that type $\underline{\theta}$ can sell her whole endowment by trading $\underline{c}^i(\varepsilon)$ together with the contracts $\underline{c}^j$, $j \neq i$. Since the unit price at which buyer i offers to purchase the quantity increment $1 - \underline{Q}$ in $\underline{c}^i(\varepsilon)$ is $\underline{\theta} + \varepsilon$, this guarantees her a payoff increase $(1 - \underline{Q})\varepsilon$ compared to what she obtains in equilibrium. When ε is close enough to zero, she cannot obtain as much by trading $\overline{c}^i(\varepsilon)$ instead. Indeed, even if this were to increase her payoff compared to what she obtains in equilibrium, the corresponding increase would at most be $\varepsilon^2 < (1 - \underline{Q})\varepsilon$. Hence type $\underline{\theta}$ trades $\underline{c}^i(\varepsilon)$ following buyer i's deviation. Consider now type $\overline{\theta}$. By trading $\overline{c}^i(\varepsilon)$ together with the contracts $\overline{c}^j$, $j \neq i$, she can increase her payoff by ε^2 compared to what she obtains in equilibrium. By trading $\underline{c}^i(\varepsilon)$ instead, the most she can obtain is her equilibrium payoff, plus the payoff from selling the quantity increment $1 - \underline{Q}$ at unit price $\underline{\theta} + \varepsilon$. For ε close enough to zero, $\underline{\theta} + \varepsilon < \overline{\theta}$ so that this unit price is too low from the point of view of type $\overline{\theta}$. Hence type $\overline{\theta}$ trades $\overline{c}^i(\varepsilon)$ following buyer i's deviation. The change in buyer i's payoff induced by this deviation is

$$-\nu\varepsilon^2 + (1 - \nu)[v(\underline{\theta}) - \underline{\theta} - \varepsilon](1 - \underline{Q})$$

which is strictly positive for ε close enough to zero if $\underline{Q} < 1$. Thus $\underline{Q} = 1$, as claimed. ■

Proof of Lemma 2. Suppose that $p < \overline{\theta}$ in a separating equilibrium. Any buyer i can deviate by offering a menu consisting of the no-trade contract and of two new contracts. The first one is

$$\underline{c}^i(\varepsilon) = (\overline{q}^i + 1 - \overline{Q}, \overline{t}^i + (p + \varepsilon)(1 - \overline{Q})),$$

for some strictly positive number ε, and is designed to attract type $\underline{\theta}$. The second one is

$$\overline{c}^i(\varepsilon) = (\overline{q}^i, \overline{t}^i + \varepsilon^2),$$

and is designed to attract type $\overline{\theta}$. The key feature of this deviation is that type $\underline{\theta}$ can sell her whole endowment by trading $\underline{c}^i(\varepsilon)$ together with the contracts $\overline{c}^j$, $j \neq i$. Since the unit price at which buyer i offers to purchase the quantity increment $1 - \overline{Q}$ in $\underline{c}^i(\varepsilon)$ is $p + \varepsilon$, this guarantees her a payoff increase $(1 - \overline{Q})\varepsilon$ compared to what she obtains in equilibrium. As in the proof of Lemma 1, it is easy to check that when ε is close enough to zero, she cannot obtain as much by trading $\overline{c}^i(\varepsilon)$ instead. Hence type $\underline{\theta}$ trades $\underline{c}^i(\varepsilon)$ following buyer i's deviation. Consider now type $\overline{\theta}$. By trading $\overline{c}^i(\varepsilon)$ together with the contracts $\overline{c}^j$, $j \neq i$, she can increase her payoff by ε^2 compared to what she obtains in equilibrium. As in the proof of Lemma 1, it is easy to check that when $p + \varepsilon < \overline{\theta}$, she cannot obtain as much by trading $\underline{c}^i(\varepsilon)$ instead. Hence type $\overline{\theta}$ trades $\overline{c}^i(\varepsilon)$ following buyer i's deviation. The change in buyer i's payoff induced by this deviation is

$$-\nu\varepsilon^2 + (1 - \nu)\{v(\underline{\theta})(\overline{q}^i - \underline{q}^i) - \overline{t}^i + \underline{t}^i + [v(\underline{\theta}) - p - \varepsilon](1 - \overline{Q})\},$$

which must at most be zero for any ε close enough to zero. Since $\underline{Q} = 1$ by Lemma 1, summing over the i's and letting ε go to zero then yields

$$v(\underline{\theta})(\overline{Q} - 1) - \overline{T} + \underline{T} + n[v(\underline{\theta}) - p](1 - \overline{Q}) \leq 0,$$

which, from the definition of p and the fact that $\overline{Q} < 1$, implies that

$$(n - 1)[v(\underline{\theta}) - p] \leq 0.$$

Since $n \geq 2$, it follows that $p \geq v(\underline{\theta})$, as claimed. ■

Proof of Lemma 3. Suppose that a separating equilibrium exists. Any buyer i can deviate by offering a menu consisting of the no-trade contract and of the contract

$$\tilde{c}^i(\varepsilon) = (\overline{q}^i + 1 - \overline{Q}, \overline{t}^i + (\overline{\theta} + \varepsilon)(1 - \overline{Q})),$$

for some strictly positive number ε, that is designed to attract both types of the seller. The key feature of this deviation is that both types can sell their whole endowment by trading $\tilde{c}^i(\varepsilon)$ together with the contracts $\overline{c}^j$, $j \neq i$. Since the unit price at which buyer i offers to purchase the quantity increment $1 - \overline{Q}$ in $\tilde{c}^i(\varepsilon)$ is $\overline{\theta} + \varepsilon$, and since $\overline{\theta} \geq p$, this guarantees both types of the seller a payoff increase $(1 - \overline{Q})\varepsilon$ compared to what they obtain in equilibrium. Hence both types trade $\tilde{c}^i(\varepsilon)$ following buyer i's deviation. The change in buyer i's payoff induced by this deviation is

$$\{\mathbf{E}[v(\theta)] - \overline{\theta} - \varepsilon\}(1 - \overline{Q}) + (1 - \nu)[v(\underline{\theta})(\overline{q}^i - \underline{q}^i) - \overline{t}^i + \underline{t}^i],$$

which must at most be zero for any ε. Since $\underline{Q} = 1$ by Lemma 1, summing over the i's and letting ε go to zero then yields

$$n\{\mathbf{E}[v(\theta)] - \overline{\theta}\}(1 - \overline{Q}) + (1 - \nu)[v(\underline{\theta})(\overline{Q} - 1) - \overline{T} + \underline{T}] \leq 0,$$

which, from the definition of p and the fact that $\overline{Q} < 1$, implies that

$$n\{\mathbf{E}[v(\theta)] - \overline{\theta}\} + (1 - \nu)[p - v(\underline{\theta})] \leq 0.$$

Starting from this inequality, two cases must be distinguished. If $p < \overline{\theta}$, then Lemma 2 applies, and therefore $p \geq v(\underline{\theta})$. It then follows that $\mathbf{E}[v(\theta)] \leq \overline{\theta}$. If $p = \overline{\theta}$, the inequality can be rearranged so as to yield

$$(n - 1)\{\mathbf{E}[v(\theta)] - \overline{\theta}\} + \nu[v(\overline{\theta}) - \overline{\theta}] \leq 0.$$

Since $n \geq 2$ and $v(\overline{\theta}) > \overline{\theta}$, it follows again that $\mathbf{E}[v(\theta)] \leq \overline{\theta}$, which shows the first part of the result. Consider next some pooling equilibrium, and denote by $(1, T)$ the corresponding aggregate equilibrium allocation. To show that $T = \mathbf{E}[v(\theta)]$, one needs to establish that the buyers' aggregate payoff is zero in equilibrium. Let B^i be buyer i's equilibrium payoff, which must be at least zero since each buyer always has the option not to trade. Buyer i can deviate by offering a menu consisting of the no-trade contract and of the contract

$$\hat{c}^i(\varepsilon) = (1, T + \varepsilon),$$

for some strictly positive number ε. It is immediate that both types trade $\hat{c}^i(\varepsilon)$ following buyer i's deviation. The change in payoff for buyer i induced by this deviation is

$$\mathbf{E}[v(\theta)] - T - \varepsilon - B^i,$$

which must at most be zero for any ε. Letting ε go to zero yields

$$B^i \geq \mathbf{E}[v(\theta)] - T = \sum_j B^j$$

where the equality follows from the fact that each type of the seller sells her whole endowment in a pooling equilibrium. Since this inequality holds for each i and all the B^i's are at least zero, they must all in fact be equal to zero. Hence $T = \mathbf{E}[v(\theta)]$, as claimed. ∎

Proof of Lemma 4. Suppose first that a pooling equilibrium exists, and denote by $(1, T)$ the aggregate allocation traded by both types in this equilibrium. Then the buyers' aggregate payoff is $\mathbf{E}[v(\theta)] - T$. One must have $T - \overline{\theta} \geq 0$ otherwise type $\overline{\theta}$ would not trade. Since the buyers' aggregate payoff must be at least zero in equilibrium, it follows that $\mathbf{E}[v(\theta)] \geq \overline{\theta}$, which shows the first part of the result. Next, observe that in any separating equilibrium, the buyers' aggregate payoff is equal to

$$(1-\nu)[v(\underline{\theta}) - \underline{T}] + \nu[v(\overline{\theta})\overline{Q} - \overline{T}] = (1-\nu)[v(\underline{\theta}) - p(1-\overline{Q})] + \nu v(\overline{\theta})\overline{Q} - \overline{T}$$

by definition of p. One shows that $p \geq v(\underline{\theta})$ in any such equilibrium. If $p < \overline{\theta}$, this follows from Lemma 2. If $p = \overline{\theta}$, this follows from Lemma 3, which implies that $\overline{\theta} \geq \mathbf{E}[v(\theta)] > v(\underline{\theta})$ whenever a separating equilibrium exists. Using this claim along with the fact that $\overline{T} \geq \overline{\theta Q}$, one obtains that the buyers' aggregate payoff is at most $\{\mathbf{E}[v(\theta)] - \overline{\theta}\}\overline{Q}$. Since this must be at least zero, one necessarily has $(\overline{Q}, \overline{T}) = (0, 0)$ whenever $\mathbf{E}[v(\theta)] < \overline{\theta}$. In particular, the buyers' aggregate payoff $(1-\nu)[v(\underline{\theta}) - p]$ is then equal to zero. It follows that $p = v(\underline{\theta})$ and thus $\underline{T} = v(\underline{\theta})$, which shows the second part of the result. ■

Proof of Corollary 1. In the case of a pooling equilibrium, the result has been established in the proof of Lemma 3. In the case of a separating equilibrium, it has been shown in the proof of Lemma 4 that the buyers' aggregate payoff is at most $\{\mathbf{E}[v(\theta)] - \overline{\theta}\}\overline{Q}$. As a separating equilibrium exists only if $\mathbf{E}[v(\theta)] \leq \overline{\theta}$, it follows that the buyers' aggregate payoff is at most zero in any such equilibrium. Since each buyer always has the option not to trade, the result follows. ■

Proof of Proposition 2. Assume first that $\mathbf{E}[v(\theta)] \geq \overline{\theta}$, so that $p^* = \mathbf{E}[v(\theta)]$. The proof goes through a series of steps.

Step 1 Given the menus offered, any best response of the seller leads to an aggregate trade $(1, \mathbf{E}[v(\theta)])$ irrespective of her type. Assuming that each buyer trades the same quantity with both types of the seller, all buyers obtain a zero payoff.

Step 2 No buyer can profitably deviate in such a way that both types of the seller trade the same contract (q, t) with him. Indeed, such a deviation is profitable only if $\mathbf{E}[v(\theta)]q > t$. However, given the menus offered by the other buyers, the seller always has the option to trade quantity q at unit price $\mathbf{E}[v(\theta)]$. She would therefore be strictly worse off trading the contract (q, t) no matter her type. Such a deviation is thus infeasible.

Step 3 No buyer can deviate in such a way that he obtains a strictly positive payoff from trading with type $\underline{\theta}$. Indeed, an additional contract $(\underline{q}, \underline{t})$ attracts type $\underline{\theta}$ only if $\underline{t} \geq \mathbf{E}[v(\theta)]\underline{q}$, since she has the option to trade any quantity at unit price $\mathbf{E}[v(\theta)]$. The corresponding payoff for the deviating buyer is then at most $\{v(\underline{\theta}) - \mathbf{E}[v(\theta)]\}\underline{q}$ which is at most zero.

Step 4 By Step 3, a profitable deviation must attract type $\overline{\theta}$. An additional contract $(\overline{q},\overline{t})$ attracts type $\overline{\theta}$ only if $\overline{t} \geq \mathbf{E}[v(\theta)]\overline{q}$, since she has the option to trade any quantity at unit price $\mathbf{E}[v(\theta)]$. However, type $\underline{\theta}$ can then also weakly increase her payoff by mimicking type $\overline{\theta}$'s behavior. One can therefore construct the seller's strategy in such a way that it is impossible for any buyer to deviate by trading with type $\overline{\theta}$ only.

Step 5 By Steps 3 and 4, a profitable deviation must involve trading with both types. Whatever the contract traded by the seller with the deviating buyer, and no matter her type, she can sell to the other buyers the remaining fraction of her endowment at unit price $\mathbf{E}[v(\theta)]$. Hence each type of the seller faces the same problem, namely to optimally use the deviating buyer's and the other buyers' offers to sell her whole endowment at the maximum price. One can therefore construct the seller's strategy in such a way that each type selects the same contract from the deviating buyer's menu. By Step 2, this makes such a deviation non profitable. Hence the result.

Assume next that $\mathbf{E}[v(\theta)] < \overline{\theta}$, so that $p^* = v(\underline{\theta})$. Again, the proof goes through a series of steps.

Step 1 Given the menus offered, any best response of the seller leads to aggregate trades $(1, v(\underline{\theta}))$ for type $\underline{\theta}$ and $(0,0)$ for type $\overline{\theta}$, and all buyers obtain a zero payoff.

Step 2 No buyer can profitably deviate in such a way that both types of the seller trade the same contract (q,t) with him. Indeed, such a deviation is profitable only if $\mathbf{E}[v(\theta)]q > t$. Since $\overline{\theta} > \mathbf{E}[v(\theta)]$, this however implies that $t - \overline{\theta}q < 0$, so that type $\overline{\theta}$ would be strictly worse off trading the contract (q,t). Such a deviation is thus infeasible.

Step 3 No buyer can deviate in such a way that he obtains a strictly positive payoff from trading with type $\underline{\theta}$. Indeed, an additional contract $(\underline{q},\underline{t})$ attracts type $\underline{\theta}$ only if $\underline{t} \geq v(\underline{\theta})\underline{q}$, since she always has the option to trade quantity $\underline{q}$ at unit price $v(\underline{\theta})$. The corresponding payoff for the deviating buyer is then at most zero.

Step 4 By Step 3, a profitable deviation must attract type $\overline{\theta}$. An additional contract $(\overline{q},\overline{t})$ attracts type $\overline{\theta}$ only if $\overline{t} \geq \overline{\theta}\overline{q}$. However, since $\overline{\theta} > \mathbf{E}[v(\theta)] > v(\underline{\theta})$, type $\underline{\theta}$ can then strictly increase her payoff by trading the contract $(\overline{q},\overline{t})$ and selling to the other buyers the remaining fraction of her endowment at unit price $v(\underline{\theta})$. It is thus impossible for any buyer to deviate by trading with type $\overline{\theta}$ only.

Step 5 By Steps 3 and 4, a profitable deviation must involve trading with both types.

Given the menus offered, the most profitable deviations involve trading some quantity $\overline{q}$ at unit price $\overline{\theta}$ with type $\overline{\theta}$, and trading a quantity 1 at unit price $\overline{\theta}\overline{q}+v(\underline{\theta})(1-\overline{q})$ with type $\underline{\theta}$. By construction, type $\underline{\theta}$ is indifferent between trading the contract $(1,\overline{\theta}\overline{q}+v(\underline{\theta})(1-\overline{q}))$ and trading the contract $(\overline{q},\overline{\theta}\overline{q})$ while selling to the other buyers the remaining fraction of her endowment at unit price $v(\underline{\theta})$. As for type $\overline{\theta}$, she is indifferent between trading the contract $(\overline{q},\overline{\theta}\overline{q})$ and not trading at all. The corresponding payoff for the deviating buyer is then

$$\nu[v(\overline{\theta})-\overline{\theta}]\overline{q}+(1-\nu)[v(\underline{\theta})-\overline{\theta}\overline{q}-v(\underline{\theta})(1-\overline{q})]=\{\mathbf{E}[v(\theta)]-\overline{\theta}\}\overline{q},$$

which is at most zero when $\mathbf{E}[v(\theta)]<\overline{\theta}$. Hence the result. ■

Proof of Proposition 3. Assume first that $\mathbf{E}[v(\theta)]>\overline{\theta}$, so that $p^*=\mathbf{E}[v(\theta)]$. Suppose an equilibrium exists in which some buyer i offers a contract $c^i=(q^i,t^i)$ at unit price $\frac{t^i}{q^i}>\mathbf{E}[v(\theta)]$. Notice that one must have $\mathbf{E}[v(\theta)]-t^i\geq\overline{\theta}(1-q^i)$ otherwise c^i would give type $\overline{\theta}$ more than her equilibrium payoff. Similarly, one must have $q^i<1$ otherwise c^i would give both types more than their equilibrium payoff. Any other buyer j could offer a menu consisting of the no-trade contract and of the contract

$$c^j(\varepsilon)=(1-q^i,\mathbf{E}[v(\theta)]-t^i+\varepsilon),$$

with $0<\varepsilon<t^i-q^i\mathbf{E}[v(\theta)]$. If both c^i and $c^j(\varepsilon)$ were available, both types of the seller would sell their whole endowment at price $\mathbf{E}[v(\theta)]+\varepsilon$ by trading c^i with buyer i and $c^j(\varepsilon)$ with buyer j, thereby increasing their payoff by ε compared to what they obtain in equilibrium. Buyer j's equilibrium payoff is thus at least

$$\mathbf{E}[v(\theta)](1-q^i)-\{\mathbf{E}[v(\theta)]-t^i+\varepsilon\}=t^i-q^i\mathbf{E}[v(\theta)]-\varepsilon>0,$$

which is impossible since each buyer's payoff is zero in any equilibrium by Corollary 1. Hence no contract can be issued at a price strictly above $\mathbf{E}[v(\theta)]$. The result follows.

Assume next that $\mathbf{E}[v(\theta)]<\overline{\theta}$, so that $p^*=v(\underline{\theta})$. Suppose an equilibrium exists in which some buyer i offers a contract $c^i=(q^i,t^i)$ at unit price $\frac{t^i}{q^i}>v(\underline{\theta})$. Notice that one must have $t^i\leq\overline{\theta}q^i$ otherwise c^i would give type $\overline{\theta}$ more than her equilibrium payoff. Similarly, one must have $v(\underline{\theta})-t^i\geq\underline{\theta}(1-q^i)$ and $q^i<1$ otherwise c^i would give type $\underline{\theta}$ more than her equilibrium payoff. Any other buyer j could offer a menu consisting of the no-trade contract and of the contract

$$c^j(\varepsilon)=(1-q^i,v(\underline{\theta})-t^i+\varepsilon),$$

where $0<\varepsilon<\min\{t^i-q^iv(\underline{\theta}),\overline{\theta}-v(\underline{\theta})\}$. If both c^i and $c^j(\varepsilon)$ were available, type $\underline{\theta}$ would sell her whole endowment at price $v(\underline{\theta})+\varepsilon$ by trading c^i with buyer i and $c^j(\varepsilon)$ with buyer j, thereby increasing her payoff by ε compared to what she obtains in equilibrium. Moreover, since $v(\underline{\theta})+\varepsilon<\overline{\theta}$, type $\overline{\theta}$ would strictly lose from trading $c^j(\varepsilon)$ with buyer j. Buyer j's equilibrium payoff is thus at least

$$(1-\nu)\{v(\underline{\theta})(1-q^i)-[v(\underline{\theta})-t^i+\varepsilon]\}=(1-\nu)[t^i-q^iv(\underline{\theta})-\varepsilon]>0,$$

which is impossible since each buyer's payoff is zero in any equilibrium by Corollary 1. Hence no contract can be issued at a price strictly above $v(\underline{\theta})$. The result follows. ■

Proof of Corollary 2. Assume first that $\mathbf{E}[v(\theta)]>\overline{\theta}$, so that $p^*=\mathbf{E}[v(\theta)]$. From Proposition 3, no contract is issued, and a fortiori traded, at a unit price strictly above $\mathbf{E}[v(\theta)]$ in equilibrium. Suppose now that a contract with unit price strictly below $\mathbf{E}[v(\theta)]$ is traded in equilibrium. Then, since the aggregate allocation traded by both types is $(1,\mathbf{E}[v(\theta)])$, a contract with unit price strictly above $\mathbf{E}[v(\theta)]$ must be traded in equilibrium, a contradiction. Hence the result.

Assume next that $\mathbf{E}[v(\theta)]<\overline{\theta}$, so that $p^*=v(\underline{\theta})$. From Proposition 3, no contract is issued, and a fortiori traded, at a unit price strictly above $v(\underline{\theta})$ in equilibrium. Suppose now that a contract with unit price strictly below $v(\underline{\theta})$ is traded in equilibrium. Then, since the aggregate allocation traded by type $\underline{\theta}$ is $(1,v(\underline{\theta}))$, a contract with unit price strictly above $v(\underline{\theta})$ must be traded in equilibrium, a contradiction. Hence the result. ■

Proof of Proposition 4. Fix some equilibrium with menu offers $(C^1,\ldots,C^n)$, and let

$$\mathfrak{A}^{-i}=\left\{\sum_{j\neq i}(q^j,t^j):(q^j,t^j)\in C^j \text{ for all } j\neq i \text{ and } \sum_{j\neq i}q^j\leq 1\right\} \tag{3}$$

be the set of aggregate allocations that remain available if buyer i withdraws his menu offer C^i. By construction, $\mathfrak{A}^{-i}$ is a compact set. One must show that $(1,p^*)\in\mathfrak{A}^{-i}$.

Assume first that $\mathbf{E}[v(\theta)]>\overline{\theta}$, so that $p^*=\mathbf{E}[v(\theta)]$. Suppose the aggregate allocation $(1,\mathbf{E}[v(\theta)])$ traded by both types does not belong to $\mathfrak{A}^{-i}$. Since $\mathfrak{A}^{-i}$ is compact, there exists some open set of $[0,1]\times\mathbb{R}_+$ that contains $(1,\mathbf{E}[v(\theta)])$ and that does not intersect $\mathfrak{A}^{-i}$. Moreover, any allocation $(Q^{-i},T^{-i})\in\mathfrak{A}^{-i}$ is such that $T^{-i}\leq\mathbf{E}[v(\theta)]Q^{-i}$ by Proposition 3. Since $\mathbf{E}[v(\theta)]>\overline{\theta}$, this implies that $\mathfrak{A}^{-i}$ does not intersect the set of allocations that are weakly preferred by both types to $(1,\mathbf{E}[v(\theta)])$. By continuity of the seller's preferences, it follows that there exists some strictly positive number ε such that the contract $(1,\mathbf{E}[v(\theta)]-\varepsilon)$ is strictly preferred by each type to any allocation in $\mathfrak{A}^{-i}$. Thus, if this contract were available, both types would trade it. This implies that buyer i's equilibrium payoff is at least ε, which is impossible since each buyer's payoff is zero in any equilibrium by Corollary 1. Hence $(1, \mathbf{E}[v(\theta)]) \in \mathfrak{A}^{-i}$. The result follows.

Assume next that $\mathbf{E}[v(\theta)] < \overline{\theta}$, so that $p^* = v(\underline{\theta})$. Suppose the aggregate allocation $(1, v(\underline{\theta}))$ traded by type $\underline{\theta}$ does not belong to $\mathfrak{A}^{-i}$. Since $\mathfrak{A}^{-i}$ is compact, there exists an open set of $[0,1] \times \mathbb{R}_+$ that contains $(1, v(\underline{\theta}))$ and that does not intersect $\mathfrak{A}^{-i}$. Moreover, any allocation $(Q^{-i}, T^{-i}) \in \mathfrak{A}^{-i}$ is such that $T^{-i} \leq v(\underline{\theta})Q^{-i}$ by Proposition 3. Since $\underline{\theta} < v(\underline{\theta})$, this implies that $\mathfrak{A}^{-i}$ does not intersect the set of allocations that are weakly preferred by type $\underline{\theta}$ to $(1, v(\underline{\theta}))$. Since the latter set is closed and $\mathfrak{A}^{-i}$ is compact, it follows that there exists a contract $(\overline{q}^i, \overline{t}^i)$ with unit price $\frac{\overline{t}^i}{\overline{q}^i} \in (\overline{\theta}, v(\overline{\theta}))$ such that the allocation $(1, v(\underline{\theta}))$ is strictly preferred by type $\underline{\theta}$ to any allocation obtained by trading the contract $(\overline{q}^i, \overline{t}^i)$ together with some allocation in $\mathfrak{A}^{-i}$.[28] Moreover, since $\frac{\overline{t}^i}{\overline{q}^i} > \overline{\theta}$, the contract $(\overline{q}^i, \overline{t}^i)$ guarantees a strictly positive payoff to type $\overline{\theta}$. Thus, if both $(1, v(\underline{\theta}))$ and $(\overline{q}^i, \overline{t}^i)$ were available, type $\underline{\theta}$ would trade $(1, \underline{\theta})$ and type $\overline{\theta}$ would trade $(\overline{q}^i, \overline{t}^i)$. This implies that buyer i's equilibrium payoff is at least $\nu[v(\overline{\theta})\overline{q}^i - \overline{t}^i] > 0$, which is impossible since each buyer's payoff is zero in any equilibrium by Corollary 1. Hence $(1, v(\underline{\theta})) \in \mathfrak{A}^{-i}$. The result follows. ■

Proof of Proposition 5. Fix some equilibrium and some buyer i, and define the set $\mathfrak{A}^{-i}$ as in (3). One must show that $\mathfrak{A}^{-i}$ is infinite. Define

$$z^{-i}(\theta, Q) = \max\{T^{-i} - \theta Q^{-i} : (Q^{-i}, T^{-i}) \in \mathfrak{A}^{-i} \text{ and } Q^{-i} \leq Q\} \tag{4}$$

to be the highest payoff that a seller of type θ can get from trading with buyers $j \neq i$, when her remaining stock is Q. Notice that $z^{-i}(\theta, Q)$ is positive and increasing in Q. Observe that

$$T^{-i} - \overline{\theta}Q^{-i} = T^{-i} - \underline{\theta}Q^{-i} + (\underline{\theta} - \overline{\theta})Q^{-i} \geq T^{-i} - \underline{\theta}Q^{-i} + (\underline{\theta} - \overline{\theta})Q$$

as long as $Q^{-i} \leq Q$. Taking maximums on both sides of this inequality yields

$$z^{-i}(\overline{\theta}, Q) \geq z^{-i}(\underline{\theta}, Q) + (\underline{\theta} - \overline{\theta})Q \tag{5}$$

for all $Q \in [0,1]$. Now, let $U(\theta)$ be the equilibrium payoff of type θ. It follows from Proposition 4 that this payoff remains available to type θ if buyer i withdraws his menu offer. Suppose that buyer i deviates by offering a menu consisting of the no-trade contract and of a contract $(\overline{q}, \overline{t})$ that is designed to attract type $\overline{\theta}$ only. To ensure that this is so, one

[28]This follows directly from the fact that if K is compact and F is closed in some normed vector space X, and if $K \cap F = \emptyset$, then for any vector u in X, $(K + \lambda u) \cap F = \emptyset$ for any sufficiently small scalar λ.

imposes the following incentive compatibility constraints:

$$U(\underline{\theta}) > \bar{t} - \underline{\theta}\bar{q} + z^{-i}(\underline{\theta}, 1 - \bar{q}),$$

$$\bar{t} - \bar{\theta}\bar{q} + z^{-i}(\bar{\theta}, 1 - \bar{q}) > U(\bar{\theta}).$$

Clearly these constraints together require that

$$\underline{\theta}\bar{q} - z^{-i}(\underline{\theta}, 1 - \bar{q}) + U(\underline{\theta}) > \bar{\theta}\bar{q} - z^{-i}(\bar{\theta}, 1 - \bar{q}) + U(\bar{\theta}). \tag{6}$$

The resulting payoff for buyer i is then $v(\bar{\theta})\bar{q} - \bar{t}$, which must at most be zero by Corollary 1. Since $\bar{t}$ can be as close as one wishes to $\bar{\theta}\bar{q} - z^{-i}(\bar{\theta}, 1 - \bar{q}) + U(\bar{\theta})$, one thus obtains the following implication: if $\bar{q}$ satisfies (6), then

$$[v(\bar{\theta}) - \bar{\theta}]\bar{q} \leq U(\bar{\theta}) - z^{-i}(\bar{\theta}, 1 - \bar{q}). \tag{7}$$

Two cases must now be distinguished.

Assume first that $\mathbf{E}[v(\theta)] > \bar{\theta}$, so that $U(\underline{\theta}) = \mathbf{E}[v(\theta)] - \underline{\theta}$ and $U(\bar{\theta}) = \mathbf{E}[v(\theta)] - \bar{\theta}$ by Lemma 3. Then (7) is false if and only if

$$z^{-i}(\bar{\theta}, 1 - \bar{q}) > \mathbf{E}[v(\theta)] - \bar{\theta} - [v(\bar{\theta}) - \bar{\theta}]\bar{q}. \tag{8}$$

Define $\bar{q}^* = \frac{\mathbf{E}[v(\theta)] - \bar{\theta}}{v(\bar{\theta}) - \bar{\theta}}$, and observe that $0 < \bar{q}^* < 1$. For $\bar{q} > \bar{q}^*$, the right-hand side of (8) is negative, and thus (8) holds. Hence (7) is false, and therefore (6) is false as well:

$$z^{-i}(\bar{\theta}, 1 - \bar{q}) \leq z^{-i}(\underline{\theta}, 1 - \bar{q}) + (\underline{\theta} - \bar{\theta})(1 - \bar{q}).$$

Letting $Q = 1 - \bar{q}$ and combining this inequality with (5), one obtains that

$$z^{-i}(\bar{\theta}, Q) = z^{-i}(\underline{\theta}, Q) + (\underline{\theta} - \bar{\theta})Q \tag{9}$$

for all $Q < 1 - \bar{q}^*$. One now shows that (9) implies that for any such Q, and for any solution $(Q^{-i}(\underline{\theta}, Q), T^{-i}(\underline{\theta}, Q))$ to the maximization problem that defines $z^{-i}(\underline{\theta}, Q)$, one has $Q^{-i}(\underline{\theta}, Q) = Q$. To see this, observe that the trade $(Q^{-i}(\underline{\theta}, Q), T^{-i}(\underline{\theta}, Q))$ is also feasible for type $\bar{\theta}$ in the maximization problem that defines $z^{-i}(\bar{\theta}, Q)$. Thus one must have

$$z^{-i}(\bar{\theta}, Q) \geq T^{-i}(\underline{\theta}, Q) - \bar{\theta}Q^{-i}(\underline{\theta}, Q) = z^{-i}(\bar{\theta}, Q) + (\underline{\theta} - \bar{\theta})Q^{-i}(\underline{\theta}, Q). \tag{10}$$

The inequality in (10) cannot be strict, for otherwise $z^{-i}(\bar{\theta}, Q) > z^{-i}(\bar{\theta}, Q) + (\underline{\theta} - \bar{\theta})Q$ as $Q^{-i}(\underline{\theta}, Q) \leq Q$, which would contradict (9). It follows that (10) holds as an equality, which implies that $Q^{-i}(\underline{\theta}, Q) = Q$ by (9). Since this equality is true for all $Q \in [0, 1 - \bar{q}^*)$, it follows from the definition of $z^{-i}(\underline{\theta}, \cdot)$ that there exists a continuum of distinct points in $\mathfrak{A}^{-i}$. Hence the result.

Assume next that $\mathbf{E}[v(\theta)] < \overline{\theta}$, so that $U(\underline{\theta}) = v(\underline{\theta}) - \underline{\theta}$, $U(\overline{\theta}) = 0$ and $z^{-i}(\overline{\theta}, \cdot) = 0$ by Lemma 4. Then the right-hand side of (7) is zero, while the left-hand side is strictly positive as long as $\overline{q}$ is strictly positive. Therefore (6) cannot hold for any such $\overline{q}$, which implies that

$$v(\underline{\theta}) - \underline{\theta} - (\overline{\theta} - \underline{\theta})\overline{q} \leq z^{-i}(\underline{\theta}, 1 - \overline{q})$$

for all $\overline{q} \in (0, 1]$. Moreover, by Proposition 3, no contract can be issued at a price strictly above $p^* = v(\underline{\theta})$. Thus

$$z^{-i}(\underline{\theta}, 1 - \overline{q}) \leq [v(\underline{\theta}) - \underline{\theta}](1 - \overline{q})$$

for all $\overline{q} \in (0, 1]$. Letting $Q = 1 - \overline{q}$ and combining these two inequalities, one obtains the following lower and upper bounds for $z^{-i}(\underline{\theta}, Q)$:

$$v(\underline{\theta}) - \overline{\theta} + (\overline{\theta} - \underline{\theta})Q \leq z^{-i}(\underline{\theta}, Q) \leq [v(\underline{\theta}) - \underline{\theta}]Q$$

for all $Q \in [0, 1)$. Since these bounds are strictly increasing in Q and coincide at $Q = 1$, it follows from the definition of $z^{-i}(\underline{\theta}, \cdot)$ that there exists a sequence in $\mathfrak{A}^{-i}$ composed of distinct points that converges to $(1, v(\underline{\theta}))$. Hence the result. ■

Proof of Proposition 6. (i) The proof goes through a series of steps.

Step 1 Given the menus offered, any best response of the seller leads to an aggregate trade $(1, \mathbf{E}[v(\theta)])$ irrespective of her type. Since $\phi < \mathbf{E}[v(\theta)]$, it is optimal for each type of the seller to trade her whole endowment with a single buyer. Assuming that each type of the seller trades with the same buyer, all buyers obtain a zero payoff. Note also that if any buyer withdraws his menu offer, the most the seller can achieve by trading with the other buyers consists in trading with a single buyer.

Step 2 No buyer can profitably deviate in such a way that both types of the seller trade the same contract (q, t) with him. Indeed, such a deviation is profitable only if $\mathbf{E}[v(\theta)]q > t$. Since $\phi < \mathbf{E}[v(\theta)]$, the highest payoff the seller can achieve by purchasing the contract (q, t) together with some contract in the menu offered by the other buyers is less than the payoff from trading the contract $(1, \mathbf{E}[v(\theta)])$, which remains available at the deviation stage. She would therefore be strictly worse off trading the contract (q, t) no matter her type. Such a deviation is thus infeasible.

Step 3 No buyer can deviate in such a way that he obtains a strictly positive payoff from trading with type $\underline{\theta}$. Indeed, trading an additional contract $(\underline{q}, \underline{t})$ with type $\underline{\theta}$ is profitable only if $v(\underline{\theta})\underline{q} > \underline{t}$. The same argument as in Step 2 then shows that type $\underline{\theta}$ would be strictly worse off trading the contract $(\underline{q}, \underline{t})$ rather than the contract $(1, \mathbf{E}[v(\theta)])$, which remains available at the deviation stage. Such a deviation is thus infeasible.

Step 4 By Step 3, a profitable deviation must attract type $\overline{\theta}$. An additional contract $(\overline{q}, \overline{t})$ that is profitable when traded with type $\overline{\theta}$ attracts her only if $\overline{t} + \phi(1 - \overline{q}) \geq \mathbf{E}[v(\theta)]$, that is, only if she can weakly increase her payoff by trading the contract $(\overline{q}, \overline{t})$ and selling to the other buyers the remaining fraction of her endowment at unit price ϕ. That this is feasible follows from the fact that, when $\overline{t} + \phi(1 - \overline{q}) \geq \mathbf{E}[v(\theta)]$ and $v(\overline{\theta})\overline{q} > \overline{t}$, the quantity $1 - \overline{q}$ is less than the maximal quantity $\frac{v(\overline{\theta}) - \mathbf{E}[v(\theta)]}{v(\overline{\theta}) - \phi}$ that can be traded at unit price ϕ with the other buyers. Moreover, the fact that $\phi \geq \overline{\theta}$ guarantees that it is indeed optimal for type $\overline{\theta}$ to behave in this way at the deviation stage. However, type $\underline{\theta}$ can then also weakly increase her payoff by mimicking type $\overline{\theta}$'s behavior. One can therefore construct the seller's strategy in such a way that it is impossible for any buyer to deviate by trading with type $\overline{\theta}$ only.

Step 5 By Steps 3 and 4, a profitable deviation must involve trading with both types. Whatever the contract traded by the seller with the deviating buyer, and no matter her type, she will sell to the other buyers the remaining fraction of her endowment at unit price ϕ. Hence, each type of the seller faces the same problem, namely to use optimally the deviating buyer's and the other buyers' offers to sell her whole endowment at the maximum price. One can therefore construct the seller's strategy in such a way that each type selects the same contract from the deviating buyer's menu. By Step 2, this makes such a deviation non profitable. The result follows.

(ii) The proof goes through a series of steps.

Step 1 Given the menus offered, any best response of the seller leads to an aggregate trade $(1, v(\underline{\theta}))$ for type $\underline{\theta}$ and $(0, 0)$ for type $\overline{\theta}$. Since each buyer is not ready to pay anything for quantities up to $\frac{\psi - \theta}{\psi}$ and offers to purchase each additional unit at a constant marginal price ψ above this level, it is optimal for type $\underline{\theta}$ to trade her whole endowment with a single buyer, and all buyers obtain a zero payoff. Note also that if any buyer withdraws his menu offer, the most the seller can achieve by trading with the other buyers consists in trading with a single buyer.

Step 2 No buyer can profitably deviate in such a way that both types of the seller trade the same contract (q,t) with him. This can be shown as in Step 2 of the first part of the proof of Proposition 2.

Step 3 No buyer can deviate in such a way that he obtains a strictly positive payoff from trading with type $\underline{\theta}$. Indeed, trading an additional contract $(\underline{q},\underline{t})$ with type $\underline{\theta}$ is profitable only if $v(\underline{\theta})\underline{q} > \underline{t}$. Since $\psi > v(\underline{\theta})$, the highest payoff type $\underline{\theta}$ can achieve by purchasing the contract (q,t) together with some contract in the menu offered by the other buyers is less than the payoff from trading the contract $(1, v(\underline{\theta}))$, which remains available at the deviation stage. She would therefore be strictly worse off trading the contract (q,t). Such a deviation is thus infeasible.

Step 4 By Step 3, a profitable deviation must attract type $\overline{\theta}$. An additional contract $(\overline{q},\overline{t})$ attracts type $\overline{\theta}$ only if $\overline{t} \geq \overline{\theta}\overline{q}$. Two cases must be distinguished. If $\overline{q} \leq \frac{v(\underline{\theta})}{\psi}$, then type $\underline{\theta}$ can trade the contract $(\overline{q},\overline{t})$ and sell to some other buyer the remaining fraction of her endowment at price $\psi(1-\overline{q}) - \psi + v(\underline{\theta})$. The price at which she can sell her whole endowment is therefore at least $(\overline{\theta}-\psi)\overline{q} + v(\underline{\theta})$, which is strictly higher than the price $\underline{\theta}$ that she obtains in equilibrium since $\overline{\theta} > v(\underline{\theta}) + \frac{\overline{\theta}-\mathbf{E}[v(\theta)]}{1-\nu} \geq \psi$. If $\overline{q} > \frac{v(\underline{\theta})}{\psi}$, then by trading the contract $(\overline{q},\overline{t})$, type $\underline{\theta}$ obtains at least a payoff $\frac{(\overline{\theta}-\underline{\theta})v(\underline{\theta})}{\psi}$, which, since $\overline{\theta} > \psi > v(\underline{\theta})$, is more than her equilibrium payoff $v(\underline{\theta}) - \underline{\theta}$. Thus type $\underline{\theta}$ can always strictly increase her payoff by trading the contract $(\overline{q},\overline{t})$. It is therefore impossible for any buyer to deviate by trading with type $\overline{\theta}$ only.

Step 5. By Steps 3 and 4, a profitable deviation must involve trading with both types. Given the menus offered, the most profitable deviations lead to trading some quantity $\overline{q} \leq \frac{v(\underline{\theta})}{\psi}$ at unit price $\overline{\theta}$ with type $\overline{\theta}$, and trading a quantity 1 at unit price $\overline{\theta}\overline{q} + v(\underline{\theta}) - \psi\overline{q}$ with type $\underline{\theta}$. By construction, type $\underline{\theta}$ is indifferent between trading the contract $(1, \overline{\theta}\overline{q} + v(\underline{\theta}) - \psi\overline{q})$ and trading the contract $(\overline{q}, \overline{\theta}\overline{q})$ while selling to the other buyers the remaining fraction of her endowment at price $\psi(1-\overline{q}) - \psi + v(\underline{\theta})$. As for type $\overline{\theta}$, she is indifferent between trading the contract $(\overline{q}, \overline{\theta}\overline{q})$ and not trading at all. The corresponding payoff for the deviating buyer is then

$$\nu[v(\overline{\theta}) - \overline{\theta}]\overline{q} + (1-\nu)\{v(\underline{\theta}) - [\overline{\theta}\overline{q} + v(\underline{\theta}) - \psi\overline{q}]\} = [\nu v(\overline{\theta}) + (1-\nu)\psi - \overline{\theta}]\overline{q},$$

which is at most zero since $\psi \leq v(\underline{\theta}) + \frac{\overline{\theta}-\mathbf{E}[v(\theta)]}{1-\nu}$. The result follows. ■

Proof of Lemma 5. For further reference, one solves here a slightly more general problem, that is parameterized by $(\theta_0, \theta_1, Q_0, Q_1)$, where $\underline{\theta} \leq \theta_0 \leq \theta_1 \leq \overline{\theta}$ and $0 \leq Q_1 \leq Q_0 \leq 1$. This problem consists in maximizing

$$\int_{\underline{\theta}}^{\theta_1} [v(\theta)Q(\theta) - T(\theta)]\, dF(\theta),$$

subject to the seller's incentive compatibility and individual rationality constraints

$$T(\theta) - \theta Q(\theta) \geq T(\theta') - \theta Q(\theta'),$$

$$T(\theta) - \theta Q(\theta) \geq 0,$$

for all $(\theta, \theta') \in [\underline{\theta}, \theta_1]^2$, and to the two additional constraints that

$$Q(\theta) = Q_0$$

for all $\theta \in [\underline{\theta}, \theta_0]$, and that

$$Q(\theta) \geq Q_1$$

for all $\theta \in [\underline{\theta}, \theta_1]$. The monopsony problem corresponds to $(\theta_0, \theta_1, Q_0, Q_1) = (\underline{\theta}, \overline{\theta}, 1, 0)$. Letting $U(\theta) = T(\theta) - \theta Q(\theta)$, standard techniques imply that the incentive compatibility constraints are equivalent to the two conditions that $U(\theta) = \int_{\theta}^{\theta_1} Q(\vartheta)\, d\vartheta + U(\theta_1)$ for all $\theta \in [\underline{\theta}, \theta_1]$ and that the function Q be decreasing over $[\underline{\theta}, \theta_1]$ (Rochet (1985)). Clearly, the participation constraint of the seller must be binding at θ_1, $U(\theta_1) = 0$. Substituting for $U(\theta)$ in the objective function and integrating by parts, the problem reduces to maximizing

$$\int_{\underline{\theta}}^{\theta_1} [v(\theta) - \theta]Q(\theta)\, dF(\theta) - \int_{\underline{\theta}}^{\theta_1} F(\theta)Q(\theta)\, d\theta \tag{11}$$

subject to the constraint that Q be decreasing, and to the two additional constraints stated above. Observe that, for each $p \in [\underline{\theta}, \theta_1]$,

$$\int_{\underline{\theta}}^{p} [v(\theta) - \theta]\, dF(\theta) - \int_{\underline{\theta}}^{p} F(\theta)\, d\theta = w(p),$$

with $w(p)$ defined as in (2). Thus the objective in (11) can be more compactly rewritten as

$$\int_{\underline{\theta}}^{\theta_1} Q(\theta)\, dw(\theta),$$

which, from the integration by parts formula for functions of bounded variation (Dellacherie and Meyer (1982, Chapter VI, Theorem 90)), is in turn equal to

$$Q_1 w(\theta_1) + \int_{\underline{\theta}}^{\theta_1} w(\theta)\, d(Q_0 - Q^+)(\theta), \tag{12}$$

where Q^+ is the right-continuous regularization of Q such that $Q^+(\theta_1) = Q_1$.[29] Since Q is decreasing and bounded below by Q_1, $d(Q_0 - Q^+)$ is a positive measure of mass $Q_0 - Q_1$ over $[\underline{\theta}, \theta_1]$. Moreover, since $Q = Q_0$ over $[\underline{\theta}, \theta_0]$, $d(Q_0 - Q^+)$ does not charge $[\underline{\theta}, \theta_0)$. Thus the maximum in (12) is reached by putting all the weight of the measure $d(Q_0 - Q^+)$ on a maximum point of the function w over $[\theta_0, \theta_1]$, yielding a payoff

$$Q_1 w(\theta_1) + (Q_0 - Q_1) \max_{\theta \in [\theta_0, \theta_1]} \{w(\theta)\}. \tag{13}$$

In the case of the monopsony, $(\theta_0, \theta_1, Q_0, Q_1) = (\underline{\theta}, \overline{\theta}, 1, 0)$. It then follows from (13) and from the definition of p^m that the maximum payoff that the monopsony can obtain is $w^m = w(p^m)$. Hence the result. ■

Proof of Proposition 7. Consider a conservative equilibrium in which each type θ sells a quantity $Q(\theta)$ and obtains a payoff $U(\theta)$. Define $B^i(\theta)$ as the payoff obtained by buyer i from trading with type θ. For the purpose of this proof, it is convenient to extend these functions to $(\overline{\theta}, \infty)$, which raises no difficulty.[30] Consistent with this, a type hereafter refers to an arbitrary element of $[\underline{\theta}, \infty)$. Note that $Q(\theta)$ goes to 0 as θ goes to infinity. Observe that $U(\theta) = \int_\theta^{\overline{\theta}} Q(\vartheta)\, d\vartheta + U(\overline{\theta})$ by the envelope theorem; thus U is affine over an interval of types if and only if Q is constant over the interior of this interval; moreover U is convex as Q is decreasing by incentive compatibility. The following result will be used repeatedly.

Lemma 6 *Suppose that U is not affine over $[\theta_a, \theta_b]$, where $\underline{\theta} \leq \theta_a < \theta_b$. Define*

$$q_0 = \frac{U(\theta_a) - U(\theta_b)}{\theta_b - \theta_a} \text{ and } t_0 = \frac{\theta_b U(\theta_a) - \theta_a U(\theta_b)}{\theta_b - \theta_a}. \tag{14}$$

Then one must have

$$n \int_{\theta_a}^{\theta_b} [v(\theta) q_0 - t_0]\, dF(\theta) \leq \int_{\theta_a}^{\theta_b} [(v(\theta) - \theta) Q(\theta) - U(\theta)]\, dF(\theta). \tag{15}$$

Proof. Since $U'(\theta) = -Q(\theta)$ except at most for a countable number of types, q_0 is an average of the quantities sold by types in $[\theta_a, \theta_b]$. Because U is not affine over this interval, it must be that these quantities take at least two different values. Therefore $Q(\theta_b) < q_0 < Q(\theta_a)$. Any

[29] To apply the integration by parts formula, observe that one can assume without loss of generality that Q is left-continuous.

[30] That is, for each $\theta \in (\overline{\theta}, \infty)$, simply set $U(\theta) = \max\{t^i - \theta q^i : (q^i, t^i) \in C^i \text{ for some } i\}$ and arbitrarily select some $Q(\theta)$ in $\arg\max\{t^i - \theta q^i : (q^i, t^i) \in C^i \text{ for some } i\}$. As for the $B^i(\theta)$'s, it is immaterial how they are defined outside of the support of the seller's type distribution. For consistency we shall nevertheless assume that they add up to $[v(\theta) - \theta)]Q(\theta) - U(\theta)$, as for types belonging to the support of the seller's type distribution.

buyer i can deviate by adding to his equilibrium menu the contract (q_0, t_0). By definition of q_0 and t_0 one has

$$U(\theta_a) = t_0 - \theta_a q_0 \text{ and } U(\theta_b) = t_0 - \theta_b q_0, \tag{16}$$

so that types θ_a and θ_b are indifferent to this new offer. Consider a type $\theta \in (\theta_a, \theta_b)$. If this type were also indifferent, then the convex function U would have to be equal to the affine mapping $\theta \mapsto t_0 - \theta q_0$ over the interval $[\theta_a, \theta_b]$, contradicting the assumption. Thus type θ cannot be indifferent, and because U is convex it must be that $U(\theta) < t_0 - \theta q_0$. Therefore all types in (θ_a, θ_b) are strictly better off trading the contract (q_0, t_0). Consider now types $\theta > \theta_b$. Convexity of U implies that for these types $U(\theta) \geq U(\theta_b) - Q(\theta_b)(\theta - \theta_b)$, and using (16) along with the fact that $q_0 > Q(\theta_b)$ yields

$$U(\theta) \geq t_0 - \theta q_0 + [q_0 - Q(\theta_b)](\theta - \theta_b) > t_0 - \theta q_0$$

for all $\theta > \theta_b$. Therefore all types $\theta > \theta_b$ are strictly worse off trading the contract (q_0, t_0). As the equilibrium under scrutiny is assumed to be conservative, such types do not change their behavior following buyer i's deviation. The same properties can similarly be established for all types $\theta < \theta_a$. The change in buyer i's payoff induced by this deviation is thus

$$\int_{\theta_a}^{\theta_b} [v(\theta) q_0 - t_0 - B^i(\theta)] \, dF(\theta),$$

which must at most be zero. Summing over the i's and using the fact that the buyers' aggregate payoff is $\sum_i B^i(\theta) = [v(\theta) - \theta]Q(\theta) - U(\theta)$ for any type θ then yields (15). ■

Define $\|Q\|_\infty = \inf\{q > 0 : \int 1_{\{Q(\theta) \leq q\}} \, dF(\theta) = 1\}$ to be the essential supremum of the set of quantities traded in equilibrium. Define $\hat{\theta} = \sup\{\theta \in [\underline{\theta}, \overline{\theta}] : Q(\theta) = \|Q\|_\infty\}$, letting $\hat{\theta} = \underline{\theta}$ if this set is empty. If $\|Q\|_\infty = 0$ then the equilibrium essentially features no trade, which implies that even a monopsony could not extract any rent from the seller, that is $w^m = 0$. One now proves that any equilibrium must indeed be such that $\|Q\|_\infty = 0$, and therefore that no equilibrium exists whenever $w^m > 0$. The following result holds.

Lemma 7 *If $\|Q\|_\infty > 0$, the buyers' aggregate payoff is zero when a quantity at most equal to $\|Q\|_\infty$ is sold by some type in $[\underline{\theta}, \overline{\theta}]$. Moreover, if $\hat{\theta} < \theta < \overline{\theta}$,*

$$U(\theta) = [v(\theta) - \theta]Q(\theta), \tag{17}$$

so that the buyers' aggregate payoff is zero when the seller's type is θ.

Proof. The proof goes through a series of steps.

Step 1 Let $\theta_0 \in [\underline{\theta}, \overline{\theta}]$ be a type who sells a quantity $Q(\theta_0) \leq \|Q\|_\infty$, and suppose that θ_0 is the only type in $[\underline{\theta}, \overline{\theta}]$ who sells $Q(\theta_0)$ and that Q is continuous at θ_0. One can then choose θ_a and θ_b such that $\underline{\theta} \leq \theta_a \leq \theta_0 < \theta_b$ and apply Lemma 6. As $t_0 = U(\theta_a) + \theta_a q_0$,

$$n \int_{\theta_a}^{\theta_b} [v(\theta) q_0 - U(\theta_a) - \theta_a q_0] \, dF(\theta) \leq \int_{\theta_a}^{\theta_b} [(v(\theta) - \theta) Q(\theta) - U(\theta)] \, dF(\theta). \tag{18}$$

Because Q is continuous at θ_0 and $U(\theta) = \int_\theta^{\overline{\theta}} Q(\vartheta) \, d\vartheta + U(\overline{\theta})$, U is differentiable at θ_0 and $U'(\theta_0) = -Q(\theta_0)$. It thus follows from the definition (14) of q_0 that q_0 goes to $Q(\theta_0)$ as θ_a and θ_b go to θ_0. Using the fact that v, U and Q are continuous at θ_0, one can then divide (18) by $F(\theta_b) - F(\theta_a)$ and take limits as θ_a and θ_b go to θ_0 to obtain

$$n[v(\theta_0) Q(\theta_0) - U(\theta_0) - \theta_0 Q(\theta_0)] \leq [v(\theta_0) - \theta_0] Q_0 - U(\theta_0)$$

so that $[v(\theta_0) - \theta_0] Q(\theta_0) - U(\theta_0) \leq 0$ as $n \geq 2$. Observe that since Q is decreasing, it has at most a countable number of discontinuity points. Thus, with the exception of such points, this inequality holds for any type θ_0 who is the only type in $[\underline{\theta}, \overline{\theta}]$ who sells $Q(\theta_0) \leq \|Q\|_\infty$.

Step 2 Let $\theta_0 \in [\underline{\theta}, \overline{\theta}]$ be a type who sells a quantity $Q(\theta_0) \leq \|Q\|_\infty$, and suppose now that there exists a maximal interval of types in $[\underline{\theta}, \overline{\theta}]$ containing θ_0, with lower bound θ_1 and upper bound $\theta_2 > \theta_1$, and such that any type in this interval sells $Q(\theta_0)$. Observe that one may have $Q(\theta_0) = \|Q\|_\infty$ and thus $\theta_1 = \underline{\theta}$, and that one may also have $\theta_2 = \overline{\theta}$. In any case, since $\|Q\|_\infty > 0$ and $Q(\theta)$ goes to 0 as θ goes to infinity, one can choose θ_a and θ_b such that $\underline{\theta} \leq \theta_a \leq \theta_1 < \theta_2 < \theta_b$ and apply Lemma 6. Observe that if $\theta_2 = \overline{\theta}$ and thus $\theta_b > \overline{\theta}$, the integrals on each side of (15) can be taken over the range $[\theta_a, \overline{\theta}]$. Taking limits as θ_a goes to θ_1 and θ_b goes to θ_2 yields

$$n \int_{\theta_1}^{\theta_2} [v(\theta) Q(\theta_0) - U(\theta) - \theta Q(\theta_0)] \, dF(\theta) \leq \int_{\theta_1}^{\theta_2} [(v(\theta) - \theta) Q(\theta_0) - U(\theta)] \, dF(\theta)$$

so that $\int_{\theta_1}^{\theta_2} [(v(\theta) - \theta) Q(\theta_0) - U(\theta)] \, dF(\theta) \leq 0$ as $n \geq 2$.

Step 3 It follows from Steps 1 and 2 that, with the possible exception of quantities traded by at most a countable number of types in $[\underline{\theta}, \overline{\theta}]$, the buyers' aggregate payoff is at most zero when any quantity in $Q([\underline{\theta}, \overline{\theta}])) \cap [0, \|Q\|_\infty]$ is sold. Because the buyers' aggregate payoff must be at least zero since each buyer always has the option not to trade, it follows that the buyers' aggregate payoff is exactly zero when any quantity in $Q([\underline{\theta}, \overline{\theta}])) \cap [0, \|Q\|_\infty]$ is sold, with the possible exception of quantities traded by a set of types of measure zero under the distribution F. This also shows that each buyer's payoff is exactly equal to zero.

Step 4 Now, let $\theta_0 \in (\underline{\theta}, \overline{\theta}]$ be a type who sells a quantity $Q(\theta_0) \in (0, \|Q\|_\infty)$, and suppose that there exists a maximal interval of types in $(\underline{\theta}, \overline{\theta}]$ containing θ_0, with lower bound θ_1 and upper bound $\theta_2 > \theta_1$, and such that any type in this interval sells $Q(\theta_0)$. The difference with Step 2 is that one must have $\theta_1 > \underline{\theta}$ as $Q(\theta_0) < \|Q\|_\infty$. One can therefore choose $\theta_a < \theta_1 < \theta_b < \theta_2$, and apply Lemma 6. Taking the limit as θ_a goes to θ_1 then yields

$$\int_{\theta_1}^{\theta_b} [(v(\theta) - \theta)Q(\theta_0) - U(\theta)]\, dF(\theta) \leq 0 \tag{19}$$

for all $\theta_b \in (\theta_1, \theta_2)$. Similarly, since $Q(\theta_0) > 0$ and $Q(\theta)$ goes to 0 as θ goes to infinity, one can choose θ_a and θ_b such that $\theta_1 < \theta_a < \theta_2 < \theta_b$ and apply Lemma 6. As in Step 2, observe that if $\theta_2 = \overline{\theta}$ and thus $\theta_b > \overline{\theta}$, the integrals on each side of (15) can be taken over the range $[\theta_a, \overline{\theta}]$. Taking limits as θ_b goes to θ_2 then yields

$$\int_{\theta_a}^{\theta_2} [(v(\theta) - \theta)Q(\theta_0) - U(\theta)]\, dF(\theta) \leq 0 \tag{20}$$

for all $\theta_a \in (\theta_1, \theta_2)$. Since $\int_{\theta_1}^{\theta_2} [(v(\theta) - \theta)Q(\theta_0) - U(\theta)]\, dF(\theta) = 0$ by Step 3, it follows from (19) and (20) that the mapping $\tilde{\theta} \mapsto \int_{\theta_1}^{\tilde{\theta}} [(v(\theta) - \theta)Q(\theta_0) - U(\theta)]\, dF(\theta)$ is identically zero over (θ_1, θ_2). Since v is continuous and $Q(\theta) = Q(\theta_0)$ for all $\theta \in (\theta_1, \theta_2)$, it follows by differentiation that (17) holds for all $\theta \in (\theta_1, \theta_2)$.

Step 5 If there exists a maximal interval of types in $[\underline{\theta}, \overline{\theta}]$ with lower bound θ_1 and upper bound $\overline{\theta} > \theta_1$, and such that any type in this interval sells zero, then clearly all these types must obtain a zero payoff, for otherwise the buyers' aggregate payoff when a quantity zero is sold would be strictly negative, contradicting Step 3. It follows that (17) holds for all $\theta \in (\theta_1, \overline{\theta}]$.

Step 6 By Steps 4 and 5, (17) holds for any type in the interior of a pooling interval contained in $[\underline{\theta}, \overline{\theta}]$, as long as the quantity sold by all types in this interval is strictly below $\|Q\|_\infty$. By Steps 1 and 3, (17) also holds for any type who is the only type in $[\underline{\theta}, \overline{\theta}]$ who sells a quantity at most $\|Q\|_\infty$, except perhaps for a set of set of types of measure zero under the distribution F. Thus (17) holds for any type in $(\hat{\theta}, \overline{\theta})$, except perhaps for a set of set of types of measure zero under the distribution F. Now let $\theta_0 \in (\hat{\theta}, \overline{\theta})$ be one of these possibly problematic types. If $v(\theta_0) \neq \theta_0$, one can deduce from the fact that v and U are continuous and that (17) holds along sequences of types converging to θ_0 from below and from above that Q is continuous at θ_0 and that (17) also holds at θ_0. If $v(\theta_0) = \theta_0$, one can deduce from the continuity of the functions v and U and from the fact that (17) holds along a sequence of types converging to θ_0 that $U(\theta_0) = 0$, so that (17) also holds at θ_0 since $v(\theta_0) = \theta_0$, no matter the value of $Q(\theta_0)$.

Step 7 By Step 6, (17) holds for any type in $(\hat{\theta}, \overline{\theta})$. To conclude, one need only to check that the buyers' aggregate payoff is zero when a quantity $\|Q\|_\infty$ is sold. One knows from Steps 2 and 3 that the buyers' aggregate payoff is zero when the quantity $\|Q\|_\infty$ is sold by a non trivial interval of types in $[\underline{\theta}, \overline{\theta}]$. Now, if $\|Q\|_\infty$ is sold by $\underline{\theta}$ only, so that $\hat{\theta} = \underline{\theta}$, then Q must be continuous at $\underline{\theta}$, by definition of $\|Q\|_\infty$. Since, by Step 6, (17) holds along a sequence of types converging to $\underline{\theta}$ and since v, U and Q are continuous at $\underline{\theta}$, (17) also holds at $\underline{\theta}$ in this case. The result follows. ■

To complete the proof of Proposition 7, we show that $\|Q\|_\infty = 0$. Supposing by way of contradiction that $\|Q\|_\infty > 0$, three cases need to be distinguished.

Case 1 Suppose first that $\hat{\theta} = \underline{\theta}$, so that $Q(\theta) < \|Q\|_\infty \leq 1$ for all $\theta > \underline{\theta}$. By Lemma 7, it follows that (17) holds everywhere over $(\underline{\theta}, \overline{\theta})$. Moreover, since $U'(\theta) = -Q(\theta)$ except at most for a countable number of types, the mapping $\theta \mapsto U(\theta) + \theta$ is strictly increasing. Any buyer i can deviate by adding to his equilibrium menu the contract $(1, U(\theta_0) + \theta_0)$, for some $\theta_0 > \underline{\theta}$. All types $\theta < \theta_0$ are strictly better off trading this contract, while all types $\theta > \theta_0$ are strictly worse off trading it. As the equilibrium under scrutiny is assumed to be conservative, the latter do not change their behavior following buyer i's deviation. The change in buyer i's payoff induced by this deviation is thus

$$\int_{\underline{\theta}}^{\theta_0} [v(\theta) - U(\theta_0) - \theta_0 - B^i(\theta)]\, dF(\theta),$$

which must at most be zero. Summing over the i's and using the fact that the buyers' aggregate payoff is $\sum_i B^i(\theta) = [v(\theta) - \theta)]Q(\theta) - U(\theta) = 0$ for any type $\theta \in (\underline{\theta}, \overline{\theta})$ then yields

$$g(\theta_0) = \int_{\underline{\theta}}^{\theta_0} [v(\theta) - U(\theta_0) - \theta_0]\, dF(\theta) \leq 0$$

for all $\theta_0 > \underline{\theta}$. Observe that g is absolutely continuous and differentiable except at most for a countable number of types, with a derivative that satisfies

$$g'(\theta_0) = [v(\theta_0) - U(\theta_0) - \theta_0]f(\theta_0) - [1 - Q(\theta_0)]F(\theta_0) = [1 - Q(\theta_0)]\{[v(\theta_0) - \theta_0]f(\theta_0) - F(\theta_0)\},$$

where the second equality follows from the fact that (17) holds everywhere over $(\underline{\theta}, \overline{\theta})$. One now proves that g' whenever defined is strictly positive in a right-neighborhood of $\underline{\theta}$, which implies that $g(\theta_0) > 0$ for θ_0 close enough to $\underline{\theta}$, a contradiction. To prove this, observe first that $1 - Q(\theta_0) > 1 - \|Q\|_\infty \geq 0$ as $\theta_0 > \underline{\theta}$. Second, since $Q(\theta)$ goes to $\|Q\|_\infty > 0$ as θ goes to $\underline{\theta}$, and since $U'(\theta) = -Q(\theta)$ except at most for a countable number of types, one has $U(\underline{\theta}) > 0$. As v and U are continuous, this in turn implies by (17) that $v(\underline{\theta}) > \underline{\theta}$. Since by assumption f is bounded away from zero over $[\underline{\theta}, \overline{\theta}]$ and F vanishes at $\underline{\theta}$, this implies that $[v(\theta_0) - \theta_0]f(\theta_0) - F(\theta_0) > 0$ in a right-neighborhood of $\underline{\theta}$. The claim then follows from the above expression for $g'(\theta_0)$.

Case 2 Suppose next that $\hat{\theta} = \overline{\theta}$, so that all types in $(\underline{\theta}, \overline{\theta})$ exactly sell $\|Q\|_\infty$. Since by Lemma 7 the buyers' aggregate payoff is zero when the quantity $\|Q\|_\infty$ is sold, this must be against a transfer $\mathbf{E}[v(\theta)]\|Q\|_\infty$. Any buyer i can deviate by adding to his equilibrium menu the contract $(q_0, U(\theta_0) + \theta_0 q_0)$, for some $q_0 < \|Q\|_\infty$ and $\theta_0 \in (\underline{\theta}, \overline{\theta})$. All types $\theta > \theta_0$ are strictly better off trading this contract, while all types $\theta < \theta_0$ are strictly worse off trading it. As the equilibrium under scrutiny is assumed to be conservative, the latter do not change their behavior following buyer i's deviation. Observe that $U(\theta_0) = \{\mathbf{E}[v(\theta)] - \theta_0\}\|Q\|_\infty$. The change in buyer i's payoff induced by this deviation is thus

$$\int_{\theta_0}^{\overline{\theta}} [v(\theta)q_0 - \{\mathbf{E}[v(\theta)] - \theta_0\}\|Q\|_\infty - \theta_0 q_0 - B^i(\theta)]\, dF(\theta)$$

which must at most be zero. Summing over the i's and using the fact that the buyers' aggregate payoff is $\sum_i B^i(\theta) = [v(\theta) - \theta)]Q(\theta) - U(\theta) = \{v(\theta) - \mathbf{E}[v(\theta)]\}\|Q\|_\infty$ for any type $\theta \in (\underline{\theta}, \overline{\theta})$ then yields, when one lets q_0 go to $\|Q\|_\infty$,

$$(n-1)\|Q\|_\infty \int_{\theta_0}^{\overline{\theta}} \{v(\theta) - \mathbf{E}[v(\theta)]\}\, dF(\theta) \leq 0,$$

so that $\int_{\theta_0}^{\overline{\theta}} \{v(\theta) - \mathbf{E}[v(\theta)]\}\, dF(\theta) \leq 0$ as $n \geq 2$ and $\|Q\|_\infty > 0$. This, however, is impossible as $\theta_0 > \underline{\theta}$ and v is strictly increasing.

Case 3 Suppose finally that $\underline{\theta} < \hat{\theta} < \bar{\theta}$, so that all types in $(\underline{\theta}, \hat{\theta})$ exactly sell $\|Q\|_\infty$. Since by Lemma 7 the buyers' aggregate payoff is zero when the quantity $\|Q\|_\infty$ is sold, this must be against a transfer $\mathbf{E}[v(\theta)\,|\,\theta \leq \hat{\theta}]\|Q\|_\infty$. One can then choose θ_a and θ_b such that $\underline{\theta} < \theta_a < \hat{\theta} < \theta_b$ and apply Lemma 6 to get (18). As θ_b goes to $\hat{\theta}$, q_0 goes to $\|Q\|_\infty$. Since $U(\theta) = \{\mathbf{E}[v(\theta)\,|\,\theta \leq \hat{\theta}] - \theta\}\|Q\|_\infty$ for all $\theta \in (\underline{\theta}, \hat{\theta})$, one then obtains

$$n \int_{\theta_a}^{\hat{\theta}} \{v(\theta) - \mathbf{E}[v(\theta)\,|\,\theta \leq \hat{\theta}]\}\|Q\|_\infty\, dF(\theta) \leq \int_{\theta_a}^{\hat{\theta}} \{v(\theta) - \mathbf{E}[v(\theta)\,|\,\theta \leq \hat{\theta}]\}\|Q\|_\infty\, dF(\theta)$$

so that $\int_{\theta_a}^{\hat{\theta}} \{v(\theta) - \mathbf{E}[v(\theta)\,|\,\theta \leq \hat{\theta}]\}\, dF(\theta) \leq 0$ as $n \geq 2$ and $\|Q\|_\infty > 0$. Using the fact that v is continuous, one can then divide this inequality by $F(\hat{\theta}) - F(\theta_a)$ and take the limit as θ_a goes to $\hat{\theta}$ to obtain $v(\hat{\theta}) \leq \mathbf{E}[v(\theta)\,|\,\theta \leq \hat{\theta}]$. This, however, is impossible as v is strictly increasing. Hence the result. ■

Proof of Proposition 8. The result is obvious whenever $p^* = \underline{\theta}$, as even a monopsony could not extract any rent from the seller; only type $\underline{\theta}$ can then trade actively, and any contract featuring a strictly positive quantity must have unit price $p^* = \underline{\theta}$ so as not to attract other types. Suppose now that $p^* > \underline{\theta}$. The proof goes through a series of steps.

Step 1 Given the menus offered, any best response of the seller leads to aggregate trades $(1, p^*)$ for types $\theta < p^*$ and $(0, 0)$ for types $\theta > p^*$. Assuming that each buyer trades the same quantity with each type of the seller, all buyers obtain a zero payoff as $p^* = \mathbf{E}[v(\theta) \,|\, \theta \leq p^*]$.

Step 2 An additional contract (q, t) attracts a type $\theta \leq p^*$ only if $t \geq p^* q$, since she has the option to trade any quantity at unit price p^*. Hence each type $\theta \leq p^*$ faces the same problem, namely to optimally use the deviating buyer's and the other buyers' offers to sell her whole endowment at the maximum price. One can therefore construct the seller's strategy in such a way that each type $\theta \leq p^*$ selects the same contract $(\underline{q}, \underline{t})$ from the deviating buyer's menu. Since $\underline{t} \geq p^* \underline{q}$ and $p^* = \mathbf{E}[v(\theta) \,|\, \theta \leq p^*]$, this implies that no deviation can be profitable over types $\theta \leq p^*$. Observe that since each type $\theta \leq p^*$ attempts to maximize

$$t - \theta q + (p^* - \theta)(1 - q) = t - p^* q + p^* - \theta$$

over the menu of contracts (q, t) offered by the deviating buyer, one has $\underline{t} - p^* \underline{q} \geq t - p^* q$ for any such contract.

Step 3 If $\overline{\theta} > p^*$, a deviating buyer may also attempt to attract some types $\theta > p^*$. Over this set of types, he effectively acts as a monopsony, since none of them has an incentive to sell to the other buyers at unit price p^*. Now, take any contract (q, t) in the deviating buyer's menu, and suppose that $q > \underline{q}$. Then, since $\underline{t} - p^* \underline{q} \geq t - p^* q$ by Step 2, one a fortiori has $\underline{t} - \theta \underline{q} > t - \theta q$ for all $\theta > p^*$, so that each type $\theta > p^*$ would rather trade $(\underline{q}, \underline{t})$ than (q, t). It follows that the types $\theta > p^*$ sell at most $\underline{q}$ to the deviating buyer. For any fixed contract $(\underline{q}, \underline{t})$ such that $\underline{t} \geq p^* \underline{q}$, the problem of the deviating buyer is to maximize

$$\int_{\underline{\theta}}^{\overline{\theta}} [v(\theta) q(\theta) - t(\theta)] \, dF(\theta),$$

subject to the seller's incentive compatibility and individual rationality constraints

$$\underline{t} - p^* \underline{q} \geq t(\theta) - p^* q(\theta),$$

$$t(\theta) - \theta q(\theta) \geq t(\theta') - \theta q(\theta'),$$

$$t(\theta) - \theta q(\theta) \geq 0,$$

for all $(\theta, \theta') \in (p^*, \overline{\theta}] \times [\underline{\theta}, \overline{\theta}]$, and to the two additional constraints that

$$(q(\theta'), t(\theta')) = (\underline{q}, \underline{t})$$

for all $\theta' \in [\underline{\theta}, p^*]$ and that

$$q(\theta) \leq \underline{q}$$

for all $\theta \in (p^*, \overline{\theta}]$. This last constraint along with the constraint that $\underline{t} - p^*\underline{q} \geq t(\theta) - p^*q(\theta)$ implies that $\underline{t} - \theta'\underline{q} \geq t(\theta) - \theta'q(\theta)$ for all $(\theta, \theta') \in (p^*, \overline{\theta}] \times [\underline{\theta}, p^*]$. Thus the deviating buyer's payoff is at most equal to the value of the problem studied in the proof of Lemma 5, with $(\theta_0, \theta_1, Q_0, Q_1) = (p^*, \overline{\theta}, \underline{q}, 0)$, that is, by (13), $\underline{q} \max_{\theta \in [p^*, \overline{\theta}]} \{w(\theta)\} = 0$. The result follows. ■

Proof of Proposition 9. Consider a conservative equilibrium in which each type θ sells an aggregate quantity $Q(\theta)$ and obtains a payoff $U(\theta)$. Define $B^i(\theta)$ as the payoff obtained by buyer i from trading with type θ. Define also θ_0 as the supremum of those types that sell their whole endowment, setting $\theta_0 = \underline{\theta}$ if there are none. By the maximum theorem, one can without loss of generality assume that type θ_0 sells her whole endowment. If $\theta_0 = \overline{\theta}$, the result follows, as Q is decreasing by incentive compatibility. Otherwise, take some $\theta_1 \in (\theta_0, \overline{\theta}]$, and let $(q^i(\theta_1), t^i(\theta_1))$ be the contract traded by type θ_1 with buyer i, so that

$$Q(\theta_1) = \sum_i q^i(\theta_1) \text{ and } U(\theta_1) = \sum_i t^i(\theta_1) - \theta_1 \sum_i q^i(\theta_1). \tag{21}$$

Any buyer i can deviate by adding to his equilibrium menu the contract

$$\tilde{c}^i = (q^i(\theta_1) + 1 - Q(\theta_1), t^i(\theta_1) + \theta_1[1 - Q(\theta_1)]).$$

The seller reacts to this deviation depending on her type θ. Each type $\theta > \theta_1$ strictly prefers $(q^i(\theta_1), t^i(\theta_1))$ to $\tilde{c}^i$, because the unit price θ_1 at which $\tilde{c}^i$ allows her to sell the quantity increment $1 - Q(\theta_1)$ is too low from her point of view. As the equilibrium under scrutiny is assumed to be conservative, type θ does not change her behavior following buyer i's deviation. Each type $\theta < \theta_1$ can sell her whole endowment by trading $\tilde{c}^i$ together with the contracts $(q^j(\theta_1), t^j(\theta_1))$, $j \neq i$, thereby obtaining a payoff

$$t^i(\theta_1) + \theta_1[1 - Q(\theta_1)] + \sum_{j \neq i} t^j(\theta_1) - \theta = U(\theta_1) + \theta_1 - \theta > U(\theta),$$

where the strict inequality follows from the fact that $U(\theta) = \int_\theta^{\theta_1} Q(\vartheta)\, d\vartheta + U(\theta_1)$ by the envelope theorem, and that $Q < 1$ over $(\theta_0, \theta_1]$. Since $U(\theta)$ is the highest payoff type θ can obtain by rejecting $\tilde{c}^i$, it follows that she trades $\tilde{c}^i$ following buyer i's deviation. The change in buyer i's payoff induced by this deviation is thus

$$\int_{\underline{\theta}}^{\theta_1} \{[q^i(\theta_1) + 1 - Q(\theta_1)]v(\theta) - t^i(\theta_1) - \theta_1[1 - Q(\theta_1)] - B^i(\theta)\}\, dF(\theta),$$

which must at most be zero. Using the definition of w, we obtain

$$[q^i(\theta_1) + 1 - Q(\theta_1)]w(\theta_1) \le \int_{\underline{\theta}}^{\theta_1} [t^i(\theta_1) - \theta_1 q^i(\theta_1) + B^i(\theta)]\, dF(\theta).$$

Summing over the i's and using (21) and the fact that the buyers' aggregate payoff is $\sum_i B^i(\theta) = [v(\theta) - \theta)]Q(\theta) - U(\theta)$ for any type θ then yields

$$\begin{aligned} \{Q(\theta_1) + n[1 - Q(\theta_1)]\}w(\theta_1) &\le \int_{\underline{\theta}}^{\theta_1} \{[v(\theta) - \theta]Q(\theta) - [U(\theta) - U(\theta_1)]\}\, dF(\theta) \\ &= \int_{\underline{\theta}}^{\theta_1} [v(\theta) - \theta]Q(\theta)\, dF(\theta) - \int_{\underline{\theta}}^{\theta_1} F(\theta)Q(\theta)\, d\theta, \end{aligned} \tag{22}$$

where the equality follows from an integration by parts. Note that the right-hand side of (22) is (11). By incentive compatibility, Q is decreasing, which in particular implies that $Q(\theta) \ge Q(\theta_1)$ for all $\theta \in [\underline{\theta}, \theta_1]$; moreover, $Q(\theta) = Q_0$ for all $\theta \in [\underline{\theta}, \theta_0]$. It follows that the buyers' aggregate payoff on the right-hand side of (22) is at most equal to the value of the problem studied in the proof of Lemma 5, with $(\theta_0, \theta_1, Q_0, Q_1) = (\theta_0, \theta_1, 1, Q(\theta_1))$, that is, by (13), $Q(\theta_1)w(\theta_1) + [1 - Q(\theta_1)] \max_{\theta \in [\theta_0, \theta_1]} \{w(\theta)\}$. Substituting in (22) and simplifying as $Q(\theta_1) < 1$, one finally obtains that

$$nw(\theta_1) \le \max_{\theta \in [\theta_0, \theta_1]} \{w(\theta)\}.$$

Since this inequality holds for all $\theta_1 \in (\theta_0, \overline{\theta}]$, one can take suprema to get

$$n \sup_{\theta_1 \in (\theta_0, \overline{\theta}]} \{w(\theta_1)\} \le \sup_{\theta_1 \in (\theta_0, \overline{\theta}]} \left\{ \max_{\theta \in [\theta_0, \theta_1]} \{w(\theta)\} \right\} = \max_{\theta \in [\theta_0, \overline{\theta}]} \{w(\theta)\},$$

which, by continuity of w, and because $n \ge 2$, implies that

$$\max_{\theta \in [\theta_0, \overline{\theta}]} \{w(\theta)\} \le 0.$$

Using the definition of p^* along with the fact that w is strictly decreasing beyond $\overline{\theta}$, this implies that $\theta_0 \ge p^*$, so that $Q(\theta) = 1$ for $\theta < p^*$. It follows that the buyers' aggregate payoff is at most equal to the value of the problem studied in the proof of Lemma 5, with

$(\theta_0, \theta_1, Q_0, Q_1) = (p^*, \overline{\theta}, 1, 0)$, that is, by (13), $\max_{\theta \in [p^*, \overline{\theta}]} \{w(\theta)\} = 0$. Proceeding as for (12), it is easy to check that the buyers' aggregate payoff is

$$\int_{\underline{\theta}}^{\overline{\theta}} w(\theta)\, d(1 - Q^+)(\theta) = \int_{p^*}^{\overline{\theta}} w(\theta)\, d(1 - Q^+)(\theta), \tag{23}$$

where the equality reflects the fact that the measure $d(1 - Q^+)$ does not charge $[\underline{\theta}, p^*)$ since $Q = 1$ over $[\underline{\theta}, p^*]$. Since by assumption $w < 0$ over $(p^*, \overline{\theta}]$, and since the buyers' aggregate payoff must be at least zero in equilibrium, it follows from (23) that $d(1-Q^+)$ is a unit mass at p^*, so that $Q = 0$ over $(p^*, \overline{\theta}]$. Hence the result. ■

Proof of Proposition 10. The result is obvious whenever $p^* = \underline{\theta}$. Suppose then that $p^* > \underline{\theta}$ and that a conservative equilibrium exists in which some buyer i offers a contract $c^i = (q^i, t^i)$ at unit price $\frac{t^i}{q^i} > p^*$. One must have $q^i < 1$ otherwise c^i would give types $\theta < \frac{t^i}{q^i}$ more than their equilibrium payoff. Any other buyer j could offer a menu consisting of the no-trade contract and of the contract

$$c^j(\varepsilon) = (1 - q^i, (p^* - \varepsilon)(1 - q^i)),$$

where $0 < \varepsilon < \frac{t^i - p^* q^i}{1 - q^i}$. If both c^i and $c^j(\varepsilon)$ were available, each type $\theta < p^* - \varepsilon$ would sell her whole endowment at price $t^i + (p^* - \varepsilon)(1 - q^i)$ by trading c^i with buyer i and $c^j(\varepsilon)$ with buyer j, thereby increasing her payoff by $t^i - p^* q^i - \varepsilon(1 - q^i)$ compared to what she obtains in equilibrium. By contrast, types $\theta > p^* - \varepsilon$ do not gain by trading $c^j(\varepsilon)$ with buyer j, since the unit price at which this contract is issued is too low from their point of view. Buyer j's equilibrium payoff is thus at least

$$\int_{\underline{\theta}}^{p^* - \varepsilon} [v(\theta) - p^* + \varepsilon](1 - q^i)\, dF(\theta) = (1 - q^i) w(p^* - \varepsilon),$$

which by definition of p^* is strictly positive for some well chosen $\varepsilon \in \left(0, \frac{t^i - p^* q^i}{1 - q^i}\right)$. This, however, is impossible, since each buyer's payoff is zero in any conservative equilibrium by Proposition 8. Hence no contract can be issued at a price strictly above p^* in such an equilibrium. The result follows. Observe that if $p^* \le \overline{\theta}$, so that p^* is in the support of the seller's type distribution, a much simpler proof goes as follows: if $\frac{t^i}{q^i} > p^*$, then $\frac{p^* - t^i}{1 - q^i} < p^*$. But then $t^i - \theta q^i > p^* - \theta$ for all types $\theta \in \left[\max\left\{\underline{\theta}, \frac{p^* - t^i}{1 - q^i}\right\}, p^*\right)$, so that c^i would give any such type more than her equilibrium payoff, a contradiction. This argument breaks down whenever $p^* > \overline{\theta}$, so that p^* does not correspond to a possible type for the seller. ■

Proof of Corollary 3. Again, the result is obvious whenever $p^* = \underline{\theta}$. Suppose then that $p^* > \underline{\theta}$. From Proposition 10, no contract is issued, and a fortiori traded, at a unit price strictly above p^* in a conservative equilibrium. Suppose first that a contract with unit price strictly below p^* is traded by some type $\theta < p^*$ in a conservative equilibrium. Then, since the aggregate allocation traded by type θ is $(1, p^*)$, a contract with unit price strictly above p^* must be traded in this equilibrium, a contradiction. Suppose next that $p^* \leq \overline{\theta}$ and that a contract with unit price strictly below p^* is traded by type p^* in a conservative equilibrium. Then, since type p^*'s payoff is zero, a contract with unit price strictly above p^* must be traded in this equilibrium, a contradiction. Hence the result. ■

Proof of Proposition 11. Fix some conservative equilibrium and some buyer i, and define the set $\mathfrak{A}^{-i}$ as in (3). One must show that $(1, p^*) \in \mathfrak{A}^{-i}$.

Assume first that $\mathbf{E}[v(\theta)] > \overline{\theta}$, so that $p^* = \mathbf{E}[v(\theta)]$. Then the argument is exactly the same as in the first case examined in the proof of Proposition 4.

Assume next that $\mathbf{E}[v(\theta)] \leq \overline{\theta}$, so that $p^* \leq \overline{\theta}$. Suppose the aggregate allocation $(1, p^*)$ traded by types $\theta < p^*$ does not belong to $\mathfrak{A}^{-i}$. Since $\mathfrak{A}^{-i}$ is compact, there exists an open set of $[0, 1] \times \mathbb{R}_+$ that contains $(1, p^*)$ and that does not intersect $\mathfrak{A}^{-i}$. Moreover, any allocation $(Q^{-i}, T^{-i}) \in \mathfrak{A}^{-i}$ is such that $T^{-i} \leq p^* Q^{-i}$ by Proposition 10. For ε close enough to zero, any solution $(Q^{-i}(p^* - \varepsilon, 1), T^{-i}(p^* - \varepsilon, 1))$ to the maximization problem that defines $z^{-i}(p^* - \varepsilon, 1)$ must be such that $Q^{-i}(p^* - \varepsilon, 1)$ is bounded away from one: otherwise, there would exist a sequence $\{\varepsilon_n\}_{n\geq 1}$ converging to zero and a sequence $\{(Q^{-i}(p^* - \varepsilon_n, 1), T^{-i}(p^* - \varepsilon_n, 1))\}_{n\geq 1}$ in $\mathfrak{A}^{-i}$ such that the sequence $\{Q^{-i}(p^* - \varepsilon_n, 1)\}_{n\geq 1}$ converges to one and

$$T^{-i}(p^* - \varepsilon_n, 1) - (p^* - \varepsilon_n)Q^{-i}(p^* - \varepsilon_n, 1) \geq 0$$

for all $n \geq 1$. Taking limits as n goes to infinity and using the fact $\mathfrak{A}^{-i}$ is compact, this would imply that the quantity one can be traded in an aggregate allocation in $\mathfrak{A}^{-i}$ at a price at least p^*, a contradiction. Now let $(\overline{Q}^{-i}(p^* - \varepsilon, 1), \overline{T}^{-i}(p^* - \varepsilon, 1))$ be the solution to the maximization problem that defines $z^{-i}(p^* - \varepsilon, 1)$ with highest quantity traded. From the above argument, one can choose ε in such a way that $\overline{Q}^{-i}(p^* - \varepsilon, 1) < 1$. By definition of p^*, one can further choose ε in such a way that $w(p^* - \varepsilon) > 0$. Buyer i could offer a menu consisting of the no-trade contract and of the contract

$$c^i(\varepsilon) = (1 - \overline{Q}^{-i}(p^* - \varepsilon, 1), (p^* - \varepsilon)[1 - \overline{Q}^{-i}(p^* - \varepsilon, 1)]).$$

Consider any type $\theta < p^* - \varepsilon$, and let $(Q^{-i}(\theta, 1), T^{-i}(\theta, 1))$ be a solution to the maximization problem that defines $z^{-i}(\theta, 1)$. By incentive compatibility, $Q^{-i}(\theta, 1) \geq \overline{Q}^{-i}(p^* - \varepsilon, 1)$. If $Q^{-i}(\theta, 1) = \overline{Q}^{-i}(p^* - \varepsilon, 1)$ and thus $T^{-i}(\theta, 1) = \overline{T}^{-i}(p^* - \varepsilon, 1)$, type θ could sell her whole endowment at price $T^{-i}(\theta,1)+(p^*-\varepsilon)[1-\overline{Q}^{-i}(p^*-\varepsilon,1)]$ by trading the aggregate allocation $(Q^{-i}(\theta,1),T^{-i}(\theta,1))$ with buyer $j\neq i$ and the contract $c^i(\varepsilon)$ with buyer i, thereby increasing her payoff by $(p^*-\varepsilon-\theta)[1-\overline{Q}^{-i}(p^*-\varepsilon,1)]$ compared to what she could obtain from trading with buyers $j\neq i$ only. If $Q^{-i}(\theta,1)>\overline{Q}^{-i}(p^*-\varepsilon,1)$, one has

$$\overline{T}^{-i}(p^*-\varepsilon,1)-(p^*-\varepsilon)\overline{Q}^{-i}(p^*-\varepsilon,1)>T^{-i}(\theta,1)-(p^*-\varepsilon)Q^{-i}(\theta,1)$$

by definition of $\overline{Q}^{-i}(p^*-\varepsilon,1)$, from which it follows that

$$\begin{aligned}\overline{T}^{-i}(p^*-\varepsilon,1)+(p^*-\varepsilon)[1-\overline{Q}^{-i}(p^*-\varepsilon,1)] &> T^{-i}(\theta,1)+(p^*-\varepsilon)[1-Q^{-i}(\theta,1)]\\ &\geq T^{-i}(\theta,1)+\theta[1-Q^{-i}(\theta,1)]\end{aligned}$$

and finally that

$$\overline{T}^{-i}(p^*-\varepsilon,1)+(p^*-\varepsilon)[1-\overline{Q}^{-i}(p^*-\varepsilon,1)]-\theta>T^{-i}(\theta,1)-\theta Q^{-i}(\theta,1).$$

Thus, by trading the aggregate allocation $(\overline{Q}^{-i}(p^*-\varepsilon,1),\overline{T}^{-i}(p^*-\varepsilon,1))$ with buyer $j\neq i$ and the contract $c^i(\varepsilon)$ with buyer i, type θ would strictly increase her payoff compared to what she could obtain from trading with buyers $j\neq i$ only. Thus, in any case, all types $\theta<p^*-\varepsilon$ would trade $c^i(\varepsilon)$ if this contract were offered by buyer i. By contrast, types $\theta>p^*-\varepsilon$ do not gain by trading $c^i(\varepsilon)$ with buyer i, since the unit price at which this contract is issued is too low from their point of view. Buyer i's equilibrium payoff is thus at least

$$\int_{\underline{\theta}}^{p^*-\varepsilon}[v(\theta)-p^*+\varepsilon][1-\overline{Q}^{-i}(p^*-\varepsilon,1)]\,dF(\theta)=[1-\overline{Q}^{-i}(p^*-\varepsilon,1)]w(p^*-\varepsilon),$$

which is strictly positive by assumption. This, however, is impossible, since each buyer's payoff is zero in any conservative equilibrium by Proposition 8. The result follows. ■

Proof of Proposition 12. Fix some conservative equilibrium and some buyer i, and define the function z^{-i} as in (4). In line with (5), one can show that

$$z^{-i}(\theta,Q)\geq z^{-i}(\theta',Q)+(\theta'-\theta)Q$$

for all $(Q,\theta,\theta')\in[0,1]\times[\underline{\theta},\overline{\theta}]^2$ such that $\theta\geq\theta'$, so that the mapping $\theta\mapsto z^{-i}(\theta,Q)+\theta Q$ is increasing over $[\underline{\theta},\overline{\theta}]$ for all $Q\in[0,1]$. Proceeding as for (9), one can further show that if this function is constant over some interval of types, then, for any type θ in this interval, and for any solution $(Q^{-i}(\theta,Q),T^{-i}(\theta,Q))$ to the maximization problem that defines $z^{-i}(\theta,Q)$, one has $Q^{-i}(\theta,Q)=Q$, so that there is an aggregate allocation in $\mathfrak{A}^{-i}$ that allows the seller to exactly trade the quantity Q. One now shows that this is the case for any quantity Q close enough to zero, which implies the result. To see this, fix some $\theta_0 \in (\underline{\theta}, \min\{p^*, \bar{\theta}\})$ and some $Q_0 \in (0,1)$, and suppose that for each $(\theta', \theta'') \in [\underline{\theta}, \overline{\theta}]^2$ such that $\theta' < \theta_0 < \theta''$, one has

$$z^{-i}(\theta', Q_0) + \theta' Q_0 < z^{-i}(\theta_0, Q_0) + \theta_0 Q_0 < z^{-i}(\theta'', Q_0) + \theta'' Q_0. \tag{24}$$

Then buyer i could offer a menu consisting of the no-trade contract and of a contract $(1 - Q_0, t_0)$ such that θ_0 is indifferent between trading the contract $(1 - Q_0, t_0)$ with buyer i along with some aggregate allocation in $\mathfrak{A}^{-i}$ with buyers $j \neq i$, and trading with buyers $j \neq i$ only, and therefore getting her equilibrium utility as shown in Proposition 11:

$$t_0 - \theta_0(1 - Q_0) + z^{-i}(\theta_0, Q_0) = p^* - \theta_0.$$

Now, from (24), all types $\theta > \theta_0$ strictly prefer accepting buyer i's offer to selling their whole endowment at price p^*, while all types $\theta < \theta_0$ strictly prefer to their whole endowment at price p^*. As for types $\theta > p^*$, they satisfy $z^{-i}(\theta, Q_0) = 0$ since they obtain a zero payoff in equilibrium. Hence any such type accepts buyer i's offer if $t_0 > \theta(1 - Q_0)$, or equivalently $\theta < \theta_1$, where

$$t_0 = \theta_1(1 - Q_0) = \theta_0(1 - Q_0) + p^* - \theta_0 - z^{-i}(\theta_0, Q_0).$$

It is easily checked that $\theta_1 \geq p^*$ if and only if $(p^* - \theta_0)Q_0 \geq z^{-i}(\theta_0, Q_0)$, which is indeed the case since, by Proposition 10, no contract is issued at a price strictly above p^* in a conservative equilibrium. It thus follows that the contract $(1 - Q_0, t_0)$ offered by buyer i attracts all types in some interval (θ_0, θ_1), with $\theta_0 < p^* \leq \theta_1$, that types θ_0 and θ_1 are indifferent, and that all other types reject buyer i's offer. Buyer i's equilibrium payoff is thus at least

$$\int_{\theta_0}^{\theta_1} [v(\theta)(1 - Q_0) - t_0]\, dF(\theta). \tag{25}$$

Now let Q_0 go to zero. Then $z^{-i}(\theta_0, Q_0)$ goes to zero as $(p^* - \theta_0)Q_0 \geq z^{-i}(\theta_0, Q_0) \geq 0$, so that t_0 and θ_1 go to p^*. Hence the limit of (25) is $\int_{\theta_0}^{p^*} [v(\theta) - p^*]\, dF(\theta)$, which is strictly positive since $v - p^*$ is strictly increasing, $\theta_0 \in (\underline{\theta}, \min\{p^*, \bar{\theta}\})$, and $\int_{\underline{\theta}}^{p^*} [v(\theta) - p^*]\, dF(\theta) = w(p^*) = 0$. This, however, is impossible, since each buyer's payoff is zero in any conservative equilibrium by Proposition 9. The result follows. ■

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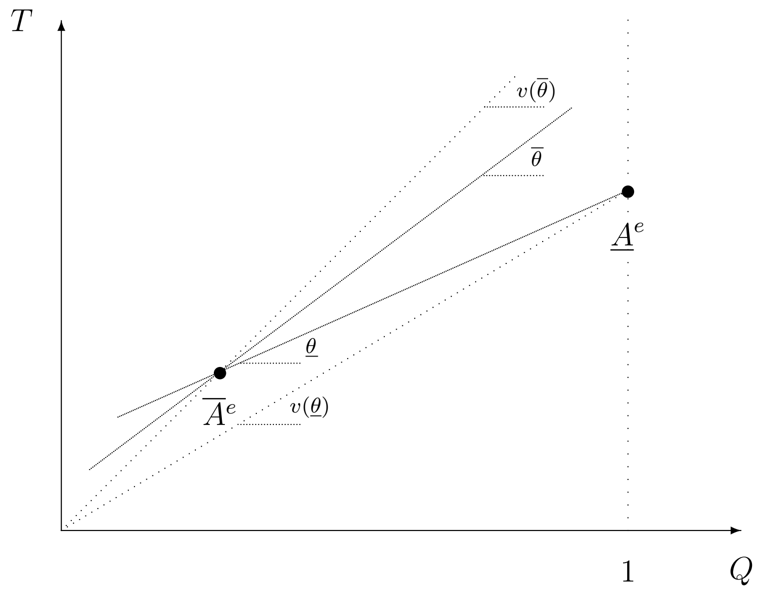

Figure 1 Equilibrium allocations under exclusive competition

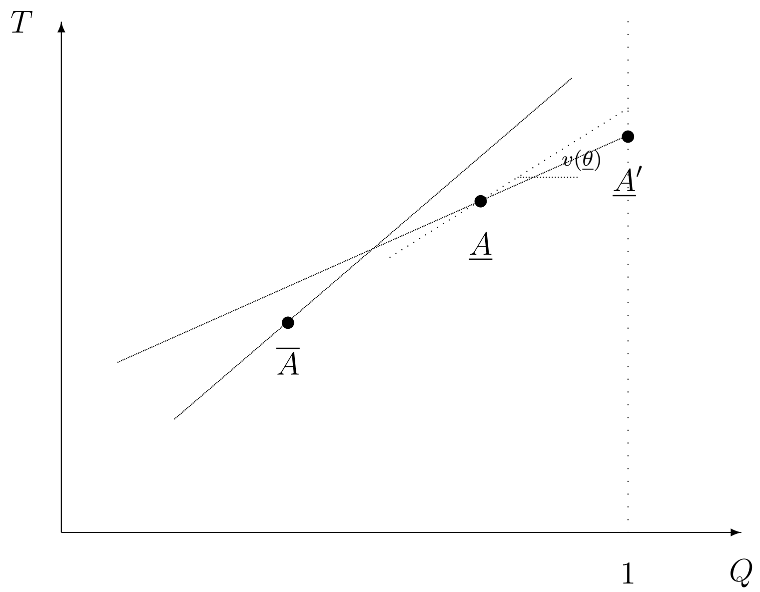

Figure 2 Attracting type $\underline{\theta}$ by pivoting around $\underline{A} = (\underline{Q}, \underline{T})$

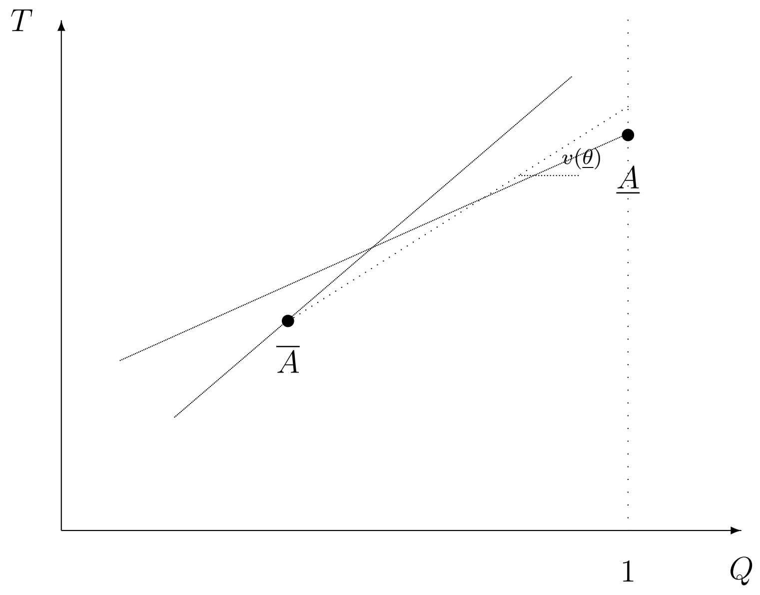

Figure 3 Attracting type $\underline{\theta}$ by pivoting around $\overline{A} = (\overline{Q}, \overline{T})$

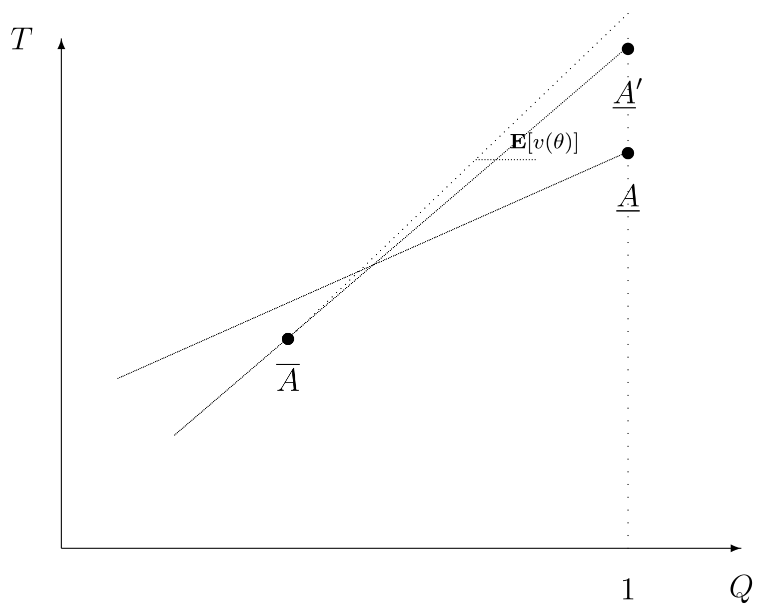

Figure 4 Attracting both types by pivoting around $\overline{A} = (\overline{Q}, \overline{T})$

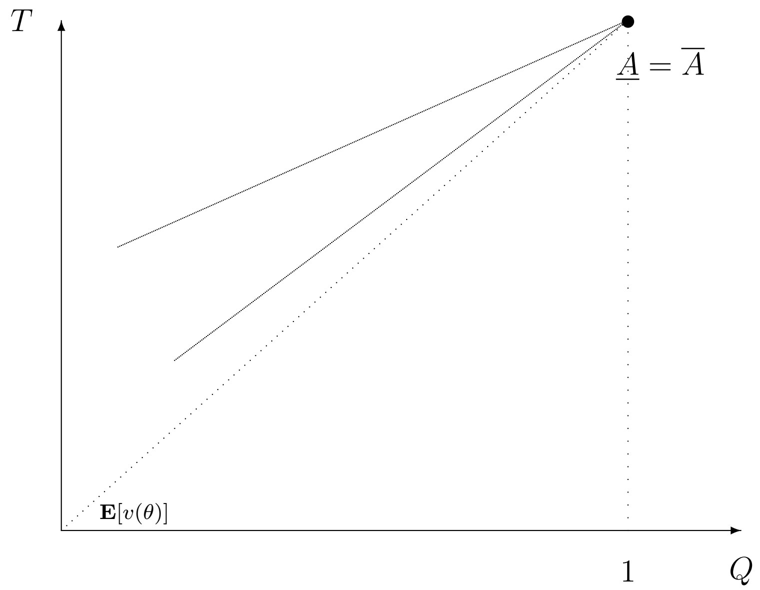

Figure 5 Aggregate equilibrium allocations when $\mathbf{E}[v(\theta)] > \overline{\theta}$

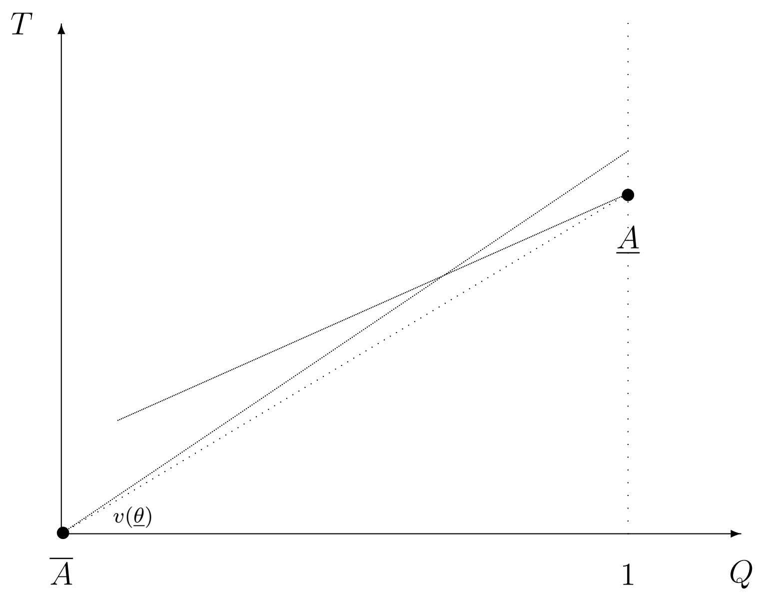

Figure 6 Aggregate equilibrium allocations when $\mathbf{E}[v(\theta)] < \overline{\theta}$