Tax design with endogenous earning abilities and consumption and production externalities (with applications to France)

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Abstract

This paper develops an optimal tax system à la Mirrlees with two novel features. First, earning abilities are determined endogenously; second, energy, a polluting good, is used both as a factor of production and a final consumption good. The model is calibrated for the French economy. The results are: (i) Replacing the current system with an optimal general income tax is redistributive towards the poor and increases social welfare by an equivalent of 500 to 1,200 euro per household; (ii) optimal tax on energy as an input is always equal to its marginal social damage; (iii) optimal tax on energy as a consumption good is less than its marginal social damage; (iv) the tax will turn to an outright subsidy when the inequality aversion index is high; (v) environmental taxes per se, increase social welfare modestly (anywhere between one to six euro per household); (vi) making polluting good taxes nonlinear further increases social welfare by 5 to 16 euro per household; (vii) taxation of energy benefits the rich and its subsidization benefits the poor.

JEL classification: H21; H23; D62

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1 Introduction

The existing empirical studies of environmental taxes are almost exclusively in the Ramsey tradition. They typically allow only for linear tax instruments and adhere to a "representative consumer" framework.[1] In Cremer *et al.* (2003), we made an initial attempt to break loose from this tradition and examine the efficiency and redistributive properties of optimal environmental taxes for the French economy within the context of the modern optimal tax theory à la Mirrlees (1971). This theory justifies the use of distortionary taxes on the basis of informational asymmetries between tax authorities and taxpayers, allows for heterogeneity among individuals, and includes nonlinear tax instruments. The present paper attempts to correct and address two major shortcomings of our earlier work.

First, when there are other factors of production besides labor in the economy, the workers' pre-tax earning abilities change. The endogeneity of the wage adds another dimension to the traditional formulation of the optimal general income tax problem. With one exception [Naito (1999)], the literature on optimal general income tax has assumed that earning abilities are constant. The endogeneity of wage complicates the optimal tax problem by quite a bit. Not only do we consider this problem at a theoretical level, we also compute an optimal general income tax schedule numerically while allowing for this endogeneity.

Second, Cremer *et al.* (2003) consider only polluting goods ignoring polluting inputs. This is a serious omission; lumping final goods and inputs together may lead to incorrect policy recommendation. On the one hand, it is not difficult to find intermediate goods that are polluting; energy being an obvious example. On the other, we know from Diamond and Mirrlees (1971) that the tax treatment of intermediate and final goods

[1] See, e.g., Bovenberg and Goulder (1996) and the references contained therein. Most recently, Mayeres and Proost (2001) and Schob () have introduced consumer heterogeneity and distributional aims. However, these papers remains within the Ramsey tradition considering only linear tax instruments.

should in general be different. Applying their production efficiency result to economies with a consumption externality, leads to the conclusion that polluting intermediate goods should be taxed only in so far as they correct externalities. In contrast, we know from Cremer and Gahvari (2001) that polluting final goods are taxed for Pigouvian considerations as well as for redistributive concerns.[2]

We model the French economy as an open economy with three factors of production and two categories of consumption goods. The factors of production are labor, capital and energy. Labor is heterogeneous with different groups of individuals having different productivity levels. All types of workers are immobile so that no labor is either exported or imported. All capital and energy inputs are rented from outside.[3] There are two sources of emissions in the economy: the use of energy input on the production side and the consumption of one category of final goods (which we call polluting goods) on the consumption side. The specific emissions that we are concerned with are CO_2 emissions. The production process consists of two stages. First, a constant returns to scale production technology uses the three inputs to produce a "general-purpose" output. Second, a linear technology transforms the output into the two categories of consumption goods at constant marginal (equal to average) costs. The first-stage production function is "nested CES". Consumers' preferences are also nested CES, being a function of labor and the two final goods.

The model is calibrated for the French economy on the basis of the data from the "Institut National de la Statistique et des Etudes Economiques" (INSEE), France. We identify four groups of individuals who differ not only in earning abilities but also in tastes. They are identified as "managerial staff", "intermediate-salaried employees", "white-collar workers" and "blue-collar workers". The polluting and non-polluting goods are

[2]Additionally, for every considered policy, we compute a value for the change it induces in *social welfare*. The social welfare change calculations are in addition to calculating *individual* welfare changes that Cremer *et al.* (2003) also undertook.

[3]There are two reasons for this assumption. One, we do not have data on holdings of capital by different types of workers. Second, taxation of capital in a static setting is not an interesting question.

constructed from 117 consumption goods according to whether they are energy related or not. We use two values for the marginal social damage of carbon emissions. A French Government Commission (Groupe Interministériel sur l'effet de Serre) recommended a figure of 130 euro as the cost per ton of carbon (equivalent to 35 euro per ton of CO_2 emissions). This is the basis for the benchmark figure we use. However, this figure is rather on the high side given the published values for the social damage of a ton of CO_2 emissions. We then use a second value based on a cost of 52 euro per ton of carbon (equivalent to 14 euro per ton of CO_2 emissions).[4]

We model the behavior of the government as one of setting *optimal* tax policies in light of the constraints that it faces. We consider a general income tax system while allowing for the endogeneity of wage. Regarding emission taxes, we differentiate between two cases: Once when nonlinear *commodity* taxes are not feasible and once when they are. Finally, to assess how far nonlinear taxes go in improving the optimality of the tax system, we provide a second benchmark in terms of differential lump-sum taxes. We thus assume that the tax administration has the capability of observing the types which enables it to levy differential lump-sum taxes. This case shows the best that is hypothetically possible and sets a yardstick for judging the effectiveness of second-best taxes in attaining the government's efficiency and redistributive goals.

The design of optimal tax structures, and their redistributive implications, must be based on some underlying social welfare function. We use an iso-elastic social welfare function for this purpose. Moreover, in choosing the value of the inequality aversion index for our optimal tax calculations, we will be guided by the observed degree of redistribution in the existing French tax system. Specifically, based on a recent study by Bourguignon and Spadaro (2000), we shall use 0.1 and 1.9 to be the limiting values for the inequality aversion index.

[4]See, "Marginal damage estimates for air pollutants", U.S. Environmental Protection Agency, http://www.epa.gov/oppt/epp/guidance/top20faqexterchart.htm, which gives a figure of 1.50 to 51 dollars for the social damage of a ton of CO_2 emissions.

2 The private sector

Consider an open economy which uses three factors of production to produce two categories of consumption goods. The factors of production are labor, capital and energy. Labor is heterogeneous with different groups of individuals having different productivity levels. All types of workers are immobile so that labor is neither exported nor imported. All capital and energy inputs are rented from outside. There are two sources of emissions in the economy. On the production side, the use of labor and capital entail no emissions but the use of energy inputs does. On the consumption side, consuming one category of goods is non-polluting, but consuming the other category (energy) generates emissions.

Specifically, we assume that there are four groups of individuals with differing productivity levels and tastes. All persons, regardless of their type, are endowed with one unit of time. Denote a person's type by j $(j = 1, 2, 3, 4)$, his productivity factor by n^j, and the proportion of people of type j in the economy by π^j. Normalize the population size at one, and define the Preferences of j-type person over his labor supply, L^j, consumption of a "non-polluting" good, x^j, a "polluting good", y^j, *and* total level of emissions in the atmosphere, E.

2.1 Production

The production process consists of two stages. First, a constant returns to scale production technology uses three inputs to produce a "general-purpose" output, O. Second, a linear technology transforms the output into the two categories of consumption goods, x and y, at constant marginal (equal to average) costs. The inputs to the first stage of production are: labor, L, capital, K, and energy, D. The production function $\boldsymbol{F}(L, K, D)$ is assumed to be "nested CES". It is written as

$$O = \boldsymbol{F}\left(L, K, D\right) = B\left[\left(1-\beta\right) L^{\frac{\sigma-1}{\sigma}} + \beta\Gamma^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

with

$$\Gamma = A\left[\alpha K^{\frac{\delta-1}{\delta}} + (1-\alpha)\, D^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta}{\delta-1}}, \tag{2}$$

where B and A are constants, and σ and δ are the (Allen) elasticities of substitution between L and Γ, and between K and D. Substituting for Γ from (2) into (1), we have

$$O = B\left[(1-\beta)\, L^{\frac{\sigma-1}{\sigma}} + \beta A^{\frac{\sigma-1}{\sigma}}\left[\alpha K^{\frac{\delta-1}{\delta}} + (1-\alpha)\, D^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta(\sigma-1)}{\sigma(\delta-1)}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{3}$$

Aggregate output, O, is the numeraire and the units of x and y are chosen such that their producer prices are equal to one.

Capital services and energy inputs are imported at constant world prices of r and p_D where the units of D is chosen such that $p_D = 1$. Let w denotes the price of one unit of *effective labor*, τ_D denotes the tax on energy input, and assume that there are no *producer taxes* on labor and capital.[5] The first-order conditions for the firms' input-hiring decisions are, assuming competitive markets,

$$\boldsymbol{O}_L(L,K,D) = w, \tag{4}$$

$$\boldsymbol{O}_K(L,K,D) = r, \tag{5}$$

$$\boldsymbol{O}_D(L,K,D) = p_D(1+\tau_D). \tag{6}$$

Equations (3)–(6) determine the equilibrium values of O, L, K and D as functions of w, r and $p_D(1+\tau_D)$ [where r and $p_D(1+\tau_D)$ are given exogenously].

2.2 Consumption

The consumer side is modeled à la Cremer *et al.* (2003). Consumers' preferences are nested CES, first in goods and labor supply and then in the two categories of consumer goods. All consumer types have identical elasticities of substitution between leisure and

[5] All labor taxes are assumed to be levied on consumers. In competitive markets, this assumption is of no significance. Taxation of capital in a setting like ours will serve no purpose except to violate production efficiency.

non-leisure goods, ρ, and between polluting and non-polluting goods, ω. Differences in tastes are captured by differences in other parameter values of the posited utility function (a^j and b^j in equations (8)–(9) below). Assume further that emissions enter the utility function linearly. The preferences for a person of type j can then be represented by

$$\mho^j = \boldsymbol{U}(x, y, L^j; \theta^j) - \phi E, \quad j = 1, 2, 3, 4, \tag{7}$$

where θ^j reflects the "taste parameter" and

$$\boldsymbol{U}(x, y, L^j, \theta^j) = \left(b^j Q^{j\,\frac{\rho-1}{\rho}} + (1 - b^j)(1 - L^j)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \tag{8}$$

$$Q^j = \left(a^j x^{\frac{\omega-1}{\omega}} + (1 - a^j) y^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}}. \tag{9}$$

With emissions emanating from production as well as consumption, total level of emissions is given by

$$E = \sum_{j=1}^{4} \pi^j y^j + D. \tag{10}$$

Consumers choose their consumption bundles by maximizing (7)–(9) subject to their budget constraints. These will be nonlinear functions when the income tax schedule is nonlinear. However, for the purpose of uniformity in exposition, we characterize the consumers' choices, even when they face a nonlinear constraint, as the solution to an optimization problem in which each person faces a (type-specific) *linearized and possibly truncated* budget constraint. To do this, introduce a "virtual income," G^j, into each type's budget constraint. Denote the j-type's net of tax wage by w_n^j. We can then write j's budget constraint as

$$px^j + qy^j = G^j + M^j + w_n^j L^j, \tag{11}$$

where p and q are the consumer prices of x and y, G^j is the income adjustment term (virtual income) needed for linearizing the budget constraint (or the lump-sum rebate if the tax function is linear), and M^j is the individual's exogenous income. The first-order conditions for a j-type's optimization problem are

$$\frac{1-a^j}{a^j}\left(\frac{x^j}{y^j}\right)^{\frac{1}{\omega}} = \frac{q}{p}, \tag{12}$$

$$\frac{(1-b^j)\left(x^j/(1-L^j)\right)^{\frac{1}{\rho}}}{a^j b^j\left[a^j+(1-a^j)(x^j/y^j)^{\frac{1-\omega}{\omega}}\right]^{\frac{\omega-\rho}{\rho(1-\omega)}}} = \frac{w_n^j}{p}. \tag{13}$$

Equations (11)–(13) determine x^j, y^j and L^j as functions of p, q, w_n^j and $G^j + M^j$.

Finally, observe that w (the price of one unit of effective labor) from the production side, and w_n^j (the net of tax wage of a j-type person) from the consumption side, are related. Denote the productivity of a j-type worker by n^j. Then, j's gross of tax wage will be $w^j = wn^j$. Denoting j's marginal income tax rate by t^j, his net of tax wage is $w_n^j = w^j(1-t^j)$. Determining w thus determines the general equilibrium solution for the economy as whole [equations (4)–(13)]. This is done by equating aggregate demand for, and aggregate supply of effective labor. Now, when j works for L^j hours, his effective labor is $n^j L^j$ resulting in aggregate supply of $\sum_{j=1}^{4} \pi^j n^j L^j$. This then should be equated with aggregate demand, L, as given by equation (4):

$$L = \sum_{j=1}^{4} \pi^j n^j L^j. \tag{14}$$

2.3 Data and the calibration

To determine the general equilibrium solution of the economy numerically, one must know different workers' productivity rates and their respective shares in total labor force (n^j, π^j), the parameters of the production function $(\sigma, \delta, \alpha, \beta, A, B)$, the parameters of the utility function $(\rho, \omega, a^j, b^j, \phi)$, world prices (r, p_D), the values of the tax parameters $(p-1, q-1, \tau_D, t^j, G^j)$, and exogenous income M^j. We calibrate the values of all non-tax parameters based on the available statistics for France. In doing this, we use the values of the tax parameters as they currently are in France. Later, we calculate the tax values

Table 1. Data Summary:1989
(monetary figures in euro)

	Managerial Staff (Type 1)	Intermediary Level (Type 2)	White Collars (Type 3)	Blue Collars (Type 4)
π	15.41 %	24.77 %	20.00 %	39.82 %
I	32,753	18,071	11,795	11,650
px	38,739	26,558	19,510	19,571
qy	2,378	2,056	1,495	1,796
n	1.18735	0.72134	0.49174	0.45180
L	0.51750	0.47000	0.45000	0.48375
t	28.8 %	19.2 %	14.4 %	9.6 %
G	3,468	1,617	977	702
M	14,329	12,394	9,931	10,134
a	0.51750	0.47000	0.45000	0.48375
b	0.51750	0.47000	0.45000	0.48375
	Type-independent figures			
	$\sum_j \pi^j n^j L^j = 0.41298$	$K = 220{,}664$	$D = 2{,}388$	$O = 42{,}055$
	$p_o \equiv 1.0$	$w = 53{,}304$	$r = 8.0$ %	$p_D = 1.0$
	$p = 1.09610$	$q = 1.53807$	$\sigma = 0.8$	$\delta = 0.32345$
	$\rho = 0.6649$	$\omega = 0.2689$	$\alpha = 0.99999$	$\beta = 0.68955$
	$A = 1.0713$	$B = 0.82647$		

* Aggregate labor includes labor supplied by the four types and other residual groups.

endogenously as the solution to various optimal tax problems. All the data come from the "Institut National de la Statistique et des Etudes Economiques" (INSEE), France. We use 1989 as our base year.[6]

On the production side, σ and δ are calculated on the basis of current estimates in the literature.[7] We set $r = 0.08$. This is the commonly rate used in France for public

[6]This is the most recent year for which there exist comprehensive consumption surveys for eight different household types ("Budget des familles") as well as surveys on employment and wages classified by household types ("Enquête sur l'emploi"). The data covers 117 consumption goods which we aggregate into: (i) non-energy consumption representing non-polluting goods (x), and (ii) energy consumption representing polluting goods (y).

Observe also that all published data are in French francs. We convert these into euro using the official conversion rate of 1 euro = 6.55957 French francs.

[7]These are based on the estimates of elasticities of substitution between various factors of production in Berndt and Wood (1975, 1985), Griffin and Gregory (1976), and Devezeaux de Lavergne, Ivaldi and

investment decisions.[8] Given r, σ, δ, and the data for L, K, D and w, we calibrate α, β, A and B.

We identify *four* types of households: “managerial staff” (Type 1), “intermediate-salaried employees” (type 2), “white-collar workers” (Type 3) and “blue-collar workers” (Type 4). The data also give the number of households in each type. Their productivity rates are determined from their hourly wages in relation to the hourly wage for all workers: $hw^j = n^j hw, (j = 1, 2, 3, 4)$. The marginal tax rates faced by the four types, t^j's, and the corresponding virtual incomes, G^j's, are reported in the French official tax publications for 1989 (Ministere de l'Economie et des Finances, 1989). Turning to consumption taxes, we note that the consumption of nonpolluting goods are taxed at an average rate of 9.6% ($p - 1 = .096$), and consumption of polluting goods at 53.8% ($q - 1 = .538$); see INSEE Résultats (1998). There does not exist a reliable estimate of τ_D, the energy input tax averaged over all consumption goods. In lieu of this, we set $\tau_D = 0$.

On the consumption side, the values of ρ and ω are from Cremer *et al.* (2003).[9] We calculate a^j's and b^j's such that the observed data for households' expenditures and labor incomes are reconciled. This procedure is also used to calculate M^j's; we do not have direct estimates for them.[10]

Finally, the value of ϕ is chosen such that the marginal social damage of a unit of polluting good (or input) would be 10% of its cost of production.[11] The basis for this

Ladoux (1990).

[8] See, e.g., DGEMP-DIGEC, Ministère de lEconomie et des Finances, 1997.

[9] Cruz and Goulder (1992) and Goulder *et al.*(1999) use a higher value for ω (0.85), and Bourguignon (1999) reports a range of 0.1 to 0.5 for the existing estimates of *wage elasticity of labor supply* which translate into a range of estimates for ρ from to 0.61 to 1.39. We thus also set $\omega = 0.99$, $\rho = 0.61$ and $\rho = 1.39$ in our optimal tax calculations for sensitivity analysis.

[10] Details of the data and calibration can be obtained from the authors directly.

[11] Because at the first-best, the optimal tax on a polluting good is its marginal social damage, we calculate ϕ in such a way as it would give rise to a first-best tax of 10%. Specifying the social welfare function as $\sum_j \pi^j W(\mho^j)$, the marginal social damage of emissions is defined as $\left[\sum_j \pi^j W'(\mho^j)\right] \phi/\mu$, where μ is the shadow cost of public funds (the Lagrange multiplier associated with the government's budget constraint). This is the formula for the first-best Pigouvian tax.

is a 1990 recommendation of the "Groupe Interministériel sur l'effet de Serre". This was a French Government Commission set up to undertake an economics study of the greenhouse effect. The recommendation called for a carbon tax of 130 euro (850 French Francs) per ton of carbon. Assuming that 130 euro measures the social damage of a ton of carbon (equivalent to 35 euro per ton of CO_2 emissions), this translates into a marginal social damage for a unit of polluting good/input that would be 10% of its cost of production. We then fix the value of ϕ at this estimated value for all the second-best tax calculations. However, there is quite a bit of disagreement in the literature over the size of the marginal social damage of carbon emissions. The published values for the social damage of a ton of carbon dioxide emissions ranges between 1.5 to 51 dollars. The 35 euro figure thus seems to be rather on the high side. We thus use a second figure corresponding to the social damage of a ton of CO_2 emissions equal to 14 euro. This being 40% of the 35 euro figure, we calculate a second value for ϕ such that the first-best tax will be 4% instead of 10%.

2.4 The French benchmark tax system

In order to compare the current French tax system with the alternative tax policies we study, the current system must be simplified so that it satisfies the assumptions of our model. We thus construct a simplified version of the French economy which we call "the French benchmark tax system". This differs from the "real" French tax system in three key assumptions. First, the population is comprised of only four types of households; second all households work; and third all capital is imported so that labor is the only source of domestic income.[12] Specifically, we solve the model of Section 2 using the observed values for the tax rates in France previously mentioned and the calibrated parameter values of subsection 2.3. This differs from the specifications of the

Observe also that we will have a different value for ϕ for each set of parameters ρ, ω, a^j, b^j.

[12]As observed earlier, these simplifications are necessitated by the limitation of the existing data and the fact that we are interested only in labor income taxes. Optimal taxation of capital in a static model is not an interesting question.

Table 2. The benchmark system
(monetary figures in euro)

	Managerial Staff (Type 1)	Intermediary Level (Type 2)	White Collars (Type 3)	Blue Collars (Type 4)
I	39,714	23,674	16,089	15,514
px	29,908	19,256	13,700	13,488
qy	1,836	1,490	1,050	1,238
L	0.62749	0.61572	0.61380	0.64419
M	0.0	0.0	0.0	0.0
	Type-independent figures			
	$\sum_j \pi^j n^j L^j = 0.40110$	$K = 214{,}688$	$D = 2{,}290$	$O = 40{,}845$

French economy in that it sets all calculated exogenous incomes (M^j's) equal to zero. Table 2 reports the solution values whenever they differ from the actual system. Note that the macro (type-independent) variables are extremely close to the actual observed values given in Table 1. The solutions for the household types differ in two important respects. First, the figures for the four types' labor supplies are somewhat higher than their actual observed values. This is easy to explain. Given that the *aggregate* labor supply in the benchmark system is close to its actual observed value, the benchmark attributes the excluded groups' labor supplies to the included four. The second difference appears in consumption levels. The assumption of no domestic capital income results in expenditure levels for the benchmark system that are lower than the observed ones.

3 The government

The government is interested in designing an optimal tax system consisting of a general income tax, and taxes on energy as a consumption good and as an intermediate good. We are particularly interested in the welfare implications of these taxes for different groups of households. With the energy taxes, we differentiate between two cases: Once when commodity taxes must be taxed linearly and once when they can be levied at different rates on different groups. Finally, we consider a situation where the tax administration has the capability of observing the types and is thus able to levy differential lump-sum taxes. This case shows the best that is hypothetically possible. It thus sets a yardstick for judging the effectiveness of second-best taxes in attaining the government's efficiency and redistributive goals.

The design of an optimal tax structures must be based on some underlying social welfare function. For this purpose, we will use an iso-elastic social welfare function of the form

$$W = \frac{1}{1-\eta}\sum_{j=1}^{4}\pi^j(\mho^j)^{1-\eta} \qquad \eta \neq 1 \quad \text{and} \quad 0 \leq \eta < \infty, \tag{15}$$

where η is the "inequality aversion index". The higher is η the more the society values equality.[13]

In choosing a value for η (the inequality aversion index) for our optimal tax calculations, we will be guided by the observed degree of redistribution in the existing French tax system. Bourguignon and Spadaro (2000) have recently studied France's social preferences as revealed through its tax system. They find that, if the uncompensated wage elasticity of labor supply is 0.1, the marginal social welfare falls from 110 to 90 percent of the mean as income increases from the lowest to the highest level. The fall would be from 150 percent to 50 percent if the uncompensated labor elasticity is 0.5.

With the social welfare function (15), the marginal social utility of income for a j-type person is given by

$$\frac{\partial \mho^j}{\partial M^j}(\mho^j)^{-\eta}.$$

This implies that the ratio of the marginal social utility of the Managerial Staff's (type 1)income to Blue Collars's (type 4) income is

$$\frac{\partial \mho^1/\partial M^1}{\partial \mho^4/\partial M^4}\left(\frac{\mho^4}{\mho^1}\right)^{\eta}.$$

[13] As is well-known, $\eta = 0$ implies a utilitarian social welfare function and $\eta \to \infty$ a Rawlsian. When $\eta = 1$, the social welfare function is given by $W = \sum_{j=1}^{4}\pi^j \ln(\mho^j)$.

Calculating the values for $\partial \mho^j/\partial M^j$ and $\mho^j$ $(j = 1, 4)$ based on the French data summarized in Table 1, setting the above expression equal to 9/11, we derive a value for η equal to 0.1. This is the implied value for the inequality aversion index in France (if the uncompensated was elasticity of labor supply is 0.1). Similarly, setting the above expression equal to 5/15, we derive a value for η equal to 1.9 for the implied value of the inequality aversion index in France (if the uncompensated was elasticity of labor supply is 0.5).

3.1 Measuring welfare changes

A change in the government's tax policy, environmental or otherwise, changes the welfare different households differently. We shall measure these using the Hicksian "equivalent variation" concept of a welfare change, EV. The changes are calculated relative to the benchmark system. Denote a j-type's indirect utility function by $\mathbf{v}(.;\boldsymbol{\theta}^{\mathbf{j}})$. Then, in going from the benchmark to some other tax system (a different value for $\tau_D, p-1, q-1, t^j, M^j$), we can calculate an EV_i^j (for each type $j = 1, 2, 3, 4$), from the following relationship

$$\begin{aligned} \mathbf{v}(p_B, q_B, w_{n,B}^j, G_B^j + M_B^j + EV_i^j; \theta^j) \quad &- \quad \phi\Big(\sum_{j=1}^{4} \pi^j y_B^j + D_B\Big) = \\ \mathbf{v}(p_i, q_i, w_{n,i}^j, G_i^j + M_i^j; \theta^j) \quad &- \quad \phi\Big(\sum_{j=1}^{4} \pi^j y_i^j + D_i\Big), \end{aligned}$$

where subscript B denotes the benchmark and subscript i refers to an alternative tax option.[14] We will then measure the "welfare change" in going from policy i to k for individual j by $EV_k^j - EV_i^j$.[15]

[14]The notation assumes individuals face the same prices for consumption goods. If they do not, prices will also be indexed by j.

[15]This is of course different from calculating the EV in going "directly" from i to k. Whereas this latter concept measures the "monetary equivalent" of the utility change in terms of prices in i, $EV_k^j - EV_i^j$ is calculated in terms of p_B (regardless of what k and i are). This way, one can compare the welfare change in going from i to k versus, say, s to l, in a meaningful manner.

Similarly, we can associate a measure of "aggregate welfare change" to any tax policy by calculating how much one has to *uniformly* compensate each individual under the benchmark system, to bring social welfare under the benchmark to parity with that under the considered tax policy. Formally, the aggregate welfare measure associated with going from the benchmark B to the alternative tax system i, EV_i^S, is found from

$$\sum_{j=1}^{4} \pi^j \Big[\mathbf{v}(p_B, q_B, w_{n,B}^j, G_B^j + M_B^j + EV_i^S; \theta^j) \quad - \quad \phi\big(\sum_{j=1}^{4} \pi^j y_B^j + D_B\big)\Big]^{1-\eta} =$$

$$\sum_{j=1}^{4} \pi^j \Big[\mathbf{v}(p_i, q_i, w_{n,i}^j, G_i^j + M_i^j; \theta^j) \quad - \quad \phi\big(\sum_{j=1}^{4} \pi^j y_i^j + D_i\big)\Big]^{1-\eta}.$$

4 The optimal general income tax with linear energy taxes

Denote $G^j + w_n^j L^j \equiv c^j$. From equations (11) and (12), determine the demand functions for x^j and y^j as $x^j = \boldsymbol{x}(p, q, c^j; \theta^j)$ and $y^j = \boldsymbol{y}(p, q, c^j; \theta^j)$. Substituting these equations in the j-type's utility function, we have

$$\boldsymbol{V}\left(p, q, c^j, \frac{I^j}{wn^j}; \theta^j\right) \equiv \boldsymbol{U}\left(\boldsymbol{x}(p, q, c^j; \theta^j), \boldsymbol{y}(p, q, c^j; \theta^j), \frac{I^j}{wn^j}; \theta^j\right).$$

Next, derive q and the second-best allocations as the solution to

$$\max_{q, c^j, I^j, K, D, w} \frac{1}{1-\eta} \sum_{j=1}^{4} \pi^j \left[\boldsymbol{V}\left(p, q, c^j; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \boldsymbol{y}\left(p, q, c^j; \theta^j\right) - \phi D\right]^{1-\eta} \tag{16}$$

under the resource constraint,

$$\boldsymbol{O}(L, K, D) - \sum_{j=1}^{4} \pi^j \left[\boldsymbol{x}\left(p, q, c^j, \frac{I^j}{wn^j}; \theta^j\right) + \boldsymbol{y}\left(p, q, c^j; \theta^j\right)\right] - rK - D - \overline{R} \geq 0, \tag{17}$$

the incentive compatibility constraints,

$$\boldsymbol{V}\left(p, q, c^j, \frac{I^j}{wn^j}; \theta^j\right) \geq \boldsymbol{V}\left(p, q, c^k, \frac{I^k}{wn^j}; \theta^j\right) \quad j \neq k = 1, 2, 3, 4, \tag{18}$$

and the endogeneity of wage condition

$$w - \boldsymbol{O}_L\left(L, K, D\right) = 0, \tag{19}$$

with

$$L = \sum_{j=1}^{4} \pi^j n^j L^j = \sum_{j=1}^{4} \pi^j \frac{I^j}{w}. \tag{20}$$

Four aspects of this procedure are worth highlighting. First, the problem is more complicated than the traditional optimal income tax problems à la Mirrlees. The endogeneity of w adds another dimension to the problem which is generally missing in the formulation of optimal income tax problems.[16] Second, production efficiency implies that condition $w - \boldsymbol{O}_L(L, K, D) = 0$ must be non-binding.[17] Third, the procedure determines the optimal (linear) commodity taxes right from the outset. The design of a general income tax function follows in the usual manner and is based on the first-order conditions of the optimization problem. Fourth, with one extra degree in setting the commodity tax rates, one can set the tax rate on the nonpolluting goods equal to zero so that $p = 1$.

The first row of Table 3 reports four values for the optimal tax on the polluting good based on two values for the marginal social damage of emissions and two values for the inequality aversion index. The second row reports the values for the optimal tax on the polluting input. Additionally, the third row reports the values for the so-called "Pigouvian tax". This is defined as the marginal social damage of pollution. It is measured by $\left[\sum_j \pi^j (\mho^j)^{-\eta}\right] \phi/\mu$, given our specification of the social welfare function and preferences.[18]

The interesting feature of our result is that the optimal tax on the polluting input is always equal to the Pigouvian tax, but the optimal tax on the polluting good is always *less* than the Pigouvian tax. Indeed, in three out of four cases, the polluting good

[16]Naito (1999) is an exception.

[17]The formal proof of this proposition is also included in the Appendix.

[18]This is Cremer *et al.*'s (1998) definition of the Pigouvian tax. Bovenberg and van der Ploeg (1994), Bovenberg and de Mooij (1994) and others use the Samuelson's rule for optimal provision of public goods to define the "Pigouvian tax". They term a tax Pigouvian if it is equal to the sum of the private dollar costs of the environmental damage per unit of the polluting good across all households. In our notation, their Pigouvian tax is $\sum_j \pi^j \phi/\alpha^j$, where α^j is the j-type's private marginal utility of income.

Table 3. Optimal linear environmental and "Pigouvian" taxes and the optimal marginal income tax rates

	$\phi = 0.016$		$\phi = 0.040$	
	$\eta = 0.1$	$\eta = 1.9$	$\eta = 0.1$	$\eta = 1.9$
Optimal polluting good tax	-2.04 %	-13.11 %	3.63 %	-8.07 %
Optimal polluting input tax	4.00 %	4.01 %	10.00 %	10.03 %
Pigouvian tax	4.00 %	4.01 %	10.00 %	10.03 %
t^1	0.28 %	0.81 %	0.29 %	0.84 %
t^2	12.47 %	30.91 %	12.42 %	30.89 %
t^3	16.54 %	26.89 %	16.41 %	26.79 %
t^4	4.03 %	17.75 %	4.04 %	17.76 %

must be subsidized rather than taxed. The reason for this is in the roles that input and output taxes play. The tax on the polluting input serves one purpose only: It is imposed to correct the social damage of emissions. The tax on the polluting good, on the other hand, serves two purposes: One is, as with the polluting inputs, externality correcting; the second is redistributive. Whereas the externality correction calls for the taxation of the polluting good, the redistributive objective calls for its *subsidization* (relative to nonpolluting goods). This is because the poor spend proportionally more of their incomes on the polluting goods. The optimal tax on polluting goods balances these two objectives.

Note also that the higher is the inequality aversion index, the higher will be the deviation of the optimal tax relative to the Pigouvian tax. The intuition is found in the two roles that output taxes embody. The Pigouvian element of output taxes is invariant to redistributive ends. This is obviously not the case for their redistributive element. The more we care about the poor, the higher we want to subsidize their consumption of polluting goods (relative to non-polluting goods). Thus, with a low value for ϕ, when η increases from 0.1 to 1.9, the optimal subsidy on energy consumption increases from 2.04% to 13.11%; but the Pigouvian tax remains very much unchanged at 4%. Similarly, with a high value for ϕ, when η increases from 0.1 to 1.9, the optimal tax

Table 4. Emission and welfare changes in going to an optimal environmental-cum-general-income-tax system
(monetary figures in euro)

	$\phi = 0.016$		$\phi = 0.040$	
	$\eta = 0.1$	$\eta = 1.9$	$\eta = 0.1$	$\eta = 1.9$
E	7.31 %	5.06 %	4.69 %	2.48 %
EV^1	-5,758	-7,430	-5,822	-7,489
EV^2	-167	-360	-191	-383
EV^3	2,375	2,648	2,382	2,654
EV^4	1,937	2,305	1,940	2,306
EV^S	494	1,224	483	1,220

on the consumption decreases from 3.63% to an outright subsidy of 8.07%; with the Pigouvian tax remaining unchanged at 10%.[19]

The last four rows in Table 3 report the marginal income tax rates on the four types. These rates are radically different from the four marginal tax rates that are currently in place in France (28.8% on type 1, 19.2% on Type 2, 14.4% on Type 3, and 9.6% on Type 4). Note that in every case the highest ability persons face an extremely low, but nevertheless a non-zero, marginal tax rate. That this rate is not strictly zero is due to the linearity of commodity taxes imposed as an additional constraint on the second-best; see Cremer *et al.* (1998). Note also that the marginal income tax rates increase for all types as the inequality aversion index increases.

The first row in Table 4 reports the changes in aggregate emissions. It indicates, rather surprisingly, that the optimal policy entails an *increase* in aggregate emissions. On reflection, this makes perfect sense. The higher level of aggregate emissions is a direct result of the increase in aggregate output due to the increased efficiency of the tax system.

[19]The alternative definition of the Pigouvian tax yields the following values for $\phi \sum_j \pi^j/\alpha^j$: 3.86% ($\phi = 0.016, \eta = 0.1$), 3.66% ($\phi = 0.016, \eta = 1.9$), 9.66% ($\phi = 0.40, \eta = 0.1$), and 9.13% ($\phi = 0.040, \eta = 1.9$). Note that these values also exceed the values for the optimal polluting good taxes. However, they are smaller than the optimal input taxes.

On the redistributive front, the tax system becomes much more progressive. Whereas the average tax payments (income plus consumption taxes) were 15.49% for Type 4, 18.08% for Type 3, , 21.70% for Type 2 and 28.29% for Type 1, they are now 3.63%, 3.65%, 21.97% and 39.13% (when $\phi = .016$ and $\eta = 0.1$), and 0.98%, 1.13%, 22.73% and 42.35% (when $\phi = .016$ and $\eta = 1.9$). The same picture emerges when $\phi = 0.040$. The implied EV figures, and the associated social welfare changes, are reported in Table 4. The magnitude of the changes are extremely large. Moreover, as one might expect, the gains to the poor and the losses of the rich increase with η. The increase in social welfare is also more pronounced for the higher value of η.

These changes come about, as a result of the change in the whole structure of the tax system. To isolate the effects of environmental taxes per se, we also find the tax equilibrium of the economy in the absence of environmental taxes. The procedure for the determination of the equilibrium is the same as problem (16)–(20) above, with some adjustments. This requires one to impose two additional constraints on the problem. They are

$$q = 1, \tag{21}$$

$$\boldsymbol{O}_D\left(L, K, D\right) = 1. \tag{22}$$

The first constraint implies that we no longer optimize with respect to q; the second constraint enters as an additional term in the Lagrangian expression. The interesting implication of this latter constraint is that it implies $\boldsymbol{O}_K(L, K, D)$ should no longer be set equal to r. Put differently, it calls for a producer tax on r. On the other hand, the condition $w = \boldsymbol{O}_L\left(L, K, D\right)$ is not affected and continues to be optimal.[20]

With the exception of the values for polluting input and aggregate emissions, the equilibrium of this economy looks very much the same as when the environmental taxes were unrestricted. This again suggests that the drastic changes to the benchmark system

[20]These claims are proved in the Appendix.

Table 5. Emission and welfare changes of introducing linear environmental taxes into a general income tax system
(monetary figures in euro)

	$\phi = 0.016$		$\phi = 0.040$	
	$\eta = 0.1$	$\eta = 1.9$	$\eta = 0.1$	$\eta = 1.9$
E	-0.81 %	0.13 %	-2.59 %	-1.68 %
EV^1	-7	-48	20	-25
EV^2	0	-6	6	-0
EV^3	0	-4	4	0
EV^4	4	19	-2	14
EV^S	1	3	4	5

are essentially due to a switch to a general income tax system. Table 5 reports the redistributive effects of environmental taxes per se by comparing the equilibria of the general income tax structure with and without emission taxes. It also reports the resulting changes in aggregate emissions. As with linear income taxes (see Table 7), observe that when polluting goods are subsidized, the very poor (type 4) gains and the very rich (Type 1) loses. The reverse occurs when polluting goods are taxed. The effects on Types 2 and 3 are very marginal. Observe also that for the same marginal social damage of emissions, a higher value for the inequality aversion index translates into more gains for type 4 and more losses for Type 1. The obvious lesson here is that the *additional* redistributive impacts of environmental taxes, and the resulting reductions in emissions, would be the same whether environmental taxes are added to an optimal linear income tax or an optimal general income tax system. On the other hand, whether the income tax system is linear or general has very strong implications for the redistributive properties of tax systems.

5 Second-best with nonlinear commodity taxes

This section examines the significance of differentiating environmental taxes amongst the individual types. The feasibility of nonlinear commodity taxes (environmental or otherwise) depends on the structure of public information in the economy. If the available public information is only on aggregate sales (anonymous transactions), we can only levy linear commodity taxes. On the other hand, if consumption levels are known at the individual level (i.e. who buys how much), nonlinear commodity taxes are feasible. While the former possibility is more realistic for the majority of goods, there exist real examples where individual consumption levels of a polluting good are observable (e.g. electricity). Nonlinear taxation of such goods must be considered as a relevant policy consideration.

The availability of both a general income and a general commodity tax allows us to derive the optimal allocations directly. This requires finding the solution to the following problem. Maximize

$$\frac{1}{1-\eta}\sum_{j=1}^{4}\pi^j\left[\boldsymbol{U}\left(x^j, y^j, \frac{I^j}{wn^j};\theta^j\right)-\phi\sum_{j=1}^{4}\pi^j y^j-\phi D\right]^{1-\eta} \tag{23}$$

with respect to x^j, y^j, I^j, D, K and w; subject to the resource constraint,

$$\boldsymbol{O}(L,K,D)-\left[\sum_{j=1}^{4}\pi^j\left(x^j+y^j\right)+rK+D+\overline{R}\right]\geq 0, \tag{24}$$

the incentive compatibility constraints

$$\boldsymbol{U}\left(x^j, y^j, \frac{I^j}{wn^j};\theta^j\right)\geq \boldsymbol{U}\left(x^k, y^k, \frac{I^k}{wn^j};\theta^j\right)\quad j\neq k=1,2,3,4, \tag{25}$$

and the endogeneity of wage condition

$$w-\boldsymbol{O}_L(L,K,D)=0,$$

Table 6. Optimal nonlinear environmental and "Pigouvian" taxes

	$\phi = 0.016$		$\phi = 0.040$	
	$\eta = 0.1$	$\eta = 1.9$	$\eta = 0.1$	$\eta = 1.9$
θ_y^1	4.00 %	4.01 %	10.00 %	10.03 %
θ_y^2	-5.71 %	-21.73 %	-0.24 %	-17.20 %
θ_y^3	5.17 %	6.19 %	11.22 %	12.32 %
θ_y^4	-1.85 %	-12.35 %	3.84 %	-7.27 %
Optimal polluting input tax	4.00 %	4.01 %	10.00 %	10.03 %
Pigouvian tax	4.00 %	4.01 %	10.00 %	10.03 %

where, again,

$$L = \sum_{j=1}^{4} \pi^j n^j L^j = \sum_{j=1}^{4} \pi^j \frac{I^j}{w}.$$

The optimal nonlinear (marginal) environmental taxes are reported in Table 6. We again note that optimal input taxes are Pigouvian.[21] Type 1 now faces purely Pigouvian commodity taxes. This is a manifestation of the "no distortion at the top" result: When commodity taxes are unrestricted, the high-ability people face no income taxes and no (non-Pigouvian) commodity taxes. Type 2 always faces substantial marginal subsidies. The same is true for Type 4 with one exception where they face a tax which is smaller than the Pigouvian (marginal) tax rate. Type 3, on the other hand, faces a tax which is slightly higher than the Pigouvian tax.

The interesting question here is the extra benefits that one may get by allowing for nonlinear environmental taxes. Comparing this case with the linear environmental tax case reveals that, with the exception of emissions, the two equilibria are indistinguishable in pre-tax terms.[22] However, post-tax welfare of the four groups are different. The welfare changes are reported in Table 7. They are as expected; the nonlinearity allows

[21] The alternative definition of the Pigouvian tax yields the following values for $\phi \sum_j \pi^j/\alpha^j$: 3.87% ($\phi = 0.016, \eta = 0.1$), 3.66% ($\phi = 0.016, \eta = 1.9$), 9.66% ($\phi = 0.040, \eta = 0.1$), and 9.13% ($\phi = 0.040, \eta = 1.9$).

[22] Type 1 persons now face a zero marginal income tax rate, and the other groups' marginal income tax rates also change very slightly.

Table 7. Emission and welfare changes of making environmental taxes nonlinear

(monetary figures in euro)

	$\phi = 0.016$		$\phi = 0.040$	
	$\eta = 0.1$	$\eta = 1.9$	$\eta = 0.1$	$\eta = 1.9$
E	-0.09 %	-0.28 %	-0.09 %	-0.28 %
EV^1	-131	-115	-136	-119
EV^2	0	5	0	5
EV^3	5	5	5	5
EV^4	53	48	55	50
EV^S	5	16	6	16

further redistribution from the rich to the poor. Observe also that changing linear environmental taxes into nonlinear taxes, increases social welfare by more than the initial gain due to introducing (linear) environmental taxes into the system.

6 First-best

Finally, we consider a first-best environment where differential lump-sum taxation is feasible. This case illustrates the best possible outcome in light of which one can judge the efficacy of other tax structures. To solve this problem, one may proceed in the manner of the second-best problem with nonlinear commodity taxes but without the incentive compatibility constraints. The only constraint on the welfare maximization problem in this case is the resource constraint. This is shown by the problem (23)–(24).

The optimal environmental taxes on inputs and goods are now equal and Pigouvian. Specifically, regardless of the value of η, the optimal tax is 4% if $\phi = 0.016$, and 10% if $\phi = 0.040$. The first-best solution also calls for huge taxes on Types 1 and 2 coupled with substantial grants to Types 3 and 4. These results in EV values that are extremely huge, dwarfing even those obtained as a result of a switch to an optimal general income tax structure. While this is as expected, because of the superiority of the differential lump-

Table 8. Emission and welfare changes due to environmental taxes in a first-best environment
(monetary figures in euro)

	$\phi = 0.016$		$\phi = 0.040$	
	$\eta = 0.1$	$\eta = 1.9$	$\eta = 0.1$	$\eta = 1.9$
E	-1.26 %	-1.26 %	-3.01 %	-3.03 %
EV^1	0	1	0	5
EV^2	0	1	1	6
EV^3	0	1	3	5
EV^4	2	1	12	6
EV^S	1	1	6	5

sum taxes to the general income taxes, the magnitude of the changes are extreme. For example, the gain in social welfare runs between 12,350 and 14,450 euro per household. Moreover, the increased efficiency of the tax system increases aggregate output and in consequence causes aggregate emissions to increase by as much as 13%.

To isolate the impact of environmental taxes per se, one must compare the solution under differential lump-sum taxes with environmental taxes to that without. Thus solve the above problem anew with the additional restriction that there will be no environmental taxes. Formally, the solution to this problem is found in the manner of the second-best problem with general income tax and linear commodity taxes but without the incentive compatibility constraints. The constraints are then the resource constraint and the no environmental tax constraints: $q = \boldsymbol{O}_D(L, K, D) = 1$. This is, problem (16)–(17) plus constraints (21)–(22).

With the exception of the emissions, the before-tax equilibrium of this system is very close to that of the first-best. Table 8 reports the resulting welfare gains as well as the impact on emissions. Note that the gains are concentrated on Type 4 when inequality aversion index is low, but are more evenly distributed when the index is high. This looks counter-intuitive. The explanation lies in the fact that, with the imposition of differential lump-sum taxes, and in terms of *after-tax income and consumption*, type 4 now constitutes “the rich”.

7 Summary and conclusion

This paper has explored the design of an optimal general income tax system when earning abilities are endogenous, and when energy is used both as a polluting consumption good and a polluting input. It has shown that the optimal marginal income tax rates are radically different from the rates that are currently in place in France. The existing rates of 28.8% on type 1, 19.2% on Type 2, 14.4% on Type 3, and 9.6% on Type 4 are replaced by approximately 0.3%, 12.5%, 16.5% and 4.0% if the inequality aversion index is 0.1, and by 0.8%, 30.9%, 26.9% and 17.8% if the inequality aversion index is 1.9. In both cases, the tax system becomes fare more redistributive towards the poor and more efficient. The gains and losses run in thousands of euro per household. Social welfare also increases markedly by nearly 500 euro per household if the inequality aversion index is 0.1 and over 1,200 euro when the index is 1.9.

In the case of environmental taxes, the paper has shown that the optimal tax on energy inputs is Pigouvian and equal to its marginal social damage. The optimal tax on the consumption of energy, on the other hand, is less than its marginal social damage. The reason for this is the fact that the poor spend proportionally more of their income on energy consumption than the rich. The tax will turn to an outright subsidy when the inequality aversion index is high. Aggregate emissions also always increase as aggregate output increases. To gauge the importance of environmental taxes per se, we have compared the equilibria of the general income tax structures with and without the mission taxes. We observe that, again as a rule, when polluting goods are subsidized, the very poor (type 4) gains and the very rich (Type 1) loses. The reverse occurs when polluting goods are taxed. The effects on Types 2 and 3 are marginal.

Within the context of a general income tax framework, the paper has also explored the additional benefits due to making environmental taxes nonlinear. It has shown that the nonlinearity allows further redistribution from the rich to the poor. The gains in going from linear to nonlinear pollution taxes are rather substantial. They even exceed the initial gains due to introducing (linear) environmental taxes into a general income tax system.

Finally, the paper has considered a first-best environment where differential lump-sum taxation is feasible. It has found redistributive gains that are extremely large, dwarfing the gains obtained as result of a switch to an optimal general income tax structure. Specifically, the gain in social welfare runs between 12,350 and 14,450 euro per household. Moreover, the increased efficiency of the tax system increases aggregate output and causes aggregate emissions to increase by as much as 13%. As far as environmental taxation is concerned, it has shown that optimal environmental taxes on inputs and goods are now equal and Pigouvian. Theses values are also independent of the value of the inequality aversion index. Isolating the effects of environmental taxes per se, it has shown that they never make any groups worse off; and that they increase social welfare by as much as 6 euro per household.

We conclude with one final observation. We believe strongly that optimal tax calculations must be carried out in light of modern optimal tax theory. In particular, they must allow for a general income tax schedule. The methods of this paper can be used to compute optimal tax structures for other countries. Better data may allow for a greater number of types. It should also allow for a more disaggregated set of goods and better parameter estimates. Another extension would be to consider non-homothetic preferences. The current paper should be viewed more for its methodological contribution to this endeavor, rather than the exactness of the reported numbers. Nevertheless the numbers are very interesting even if only indicative.

Appendix

The general income plus linear commodity taxes: The Lagrangian for this second-best problem is (where p is set equal to 1),

$$\begin{aligned}
\pounds &= \frac{1}{1-\eta}\sum_{j=1}^{4}\pi^j\left[\boldsymbol{V}\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)-\phi\sum_{j=1}^{4}\pi^j\boldsymbol{y}\left(q,c^j;\theta^j\right)-\phi D\right]^{1-\eta}\\
&+ \mu\left\{\boldsymbol{O}(K,L,D)-\sum_{j=1}^{4}\pi^j\left[\boldsymbol{x}\left(q,c^j;\theta^j\right)+\boldsymbol{y}\left(q,c^j;\theta^j\right)\right]-rK-D-\overline{R}\right\}\\
&+ \sum_j\sum_{k\neq j}\lambda^{jk}\left[\boldsymbol{V}\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)-\boldsymbol{V}\left(q,c^k,\frac{I^k}{wn^j};\theta^j\right)\right]+\gamma\left[w-O_L(K,D,L)\right].
\end{aligned}$$

The first-order conditions are, for $j=1,2,3,4$,

$$\begin{aligned}
\frac{\partial\pounds}{\partial q} &= \sum_{j=1}^{4}\pi^j\left[\boldsymbol{V}\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)-\phi\sum_{j=1}^{4}\pi^j\boldsymbol{y}\left(q,c^j;\theta^j\right)-\phi D\right]^{-\eta}\\
&\times \left[\boldsymbol{V}_q\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)-\phi\sum_{j=1}^{4}\pi^j\boldsymbol{y}_q\left(q,c^j;\theta^j\right)\right]-\mu\sum_{j=1}^{4}\pi^j\left[\boldsymbol{x}_q\left(q,c^j;\theta^j\right)+\boldsymbol{y}_q\left(q,c^j;\theta^j\right)\right]\\
&+ \sum_{j=1}^{4}\sum_{k\neq j}\lambda^{jk}\left[\mathrm{V}_q\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)-\boldsymbol{V}_q\left(q,c^k,\frac{I^k}{wn^j};\theta^j\right)\right]=0, \qquad \text{(A1)}\\
\frac{\partial\pounds}{\partial c^j} &= \pi^j\left[\boldsymbol{V}\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)-\phi\sum_{j=1}^{4}\pi^j\boldsymbol{y}\left(q,c^j;\theta^j\right)-\phi D\right]^{-\eta}\boldsymbol{V}_c\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)\\
&- \phi\pi^j\boldsymbol{y}_c\left(q,c^j;\theta^j\right)\sum_{j=1}^{4}\pi^j\left[\boldsymbol{V}\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)-\phi\sum_{j=1}^{4}\pi^j\boldsymbol{y}\left(q,c^j;\theta^j\right)-\phi D\right]^{-\eta}\\
&+ \mu\pi^j\left[\boldsymbol{x}_c\left(q,c^j;\theta^j\right)+\boldsymbol{y}_c\left(q,c^j;\theta^j\right)\right]+\sum_{k\neq j}\lambda^{jk}\boldsymbol{V}_c\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)\\
&- \sum_{k\neq j}\lambda^{kj}\boldsymbol{V}_c\left(q,c^j,\frac{I^j}{wn^k};\theta^k\right)=0, \qquad \text{(A2)}
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial \boldsymbol{\pounds}}{\partial I^j} &= \pi^j \left[\boldsymbol{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \boldsymbol{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \frac{1}{wn^j} \boldsymbol{V}_L \left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) \\
&+ \mu \boldsymbol{O}_L(L, K, D) \frac{\pi^j}{w} + \sum_{k \neq j} \lambda^{jk} \frac{1}{wn^j} \boldsymbol{V}_L \left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) \\
&- \sum_{k \neq j} \lambda^{kj} \frac{1}{wn^k} \boldsymbol{V}_L \left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) - \gamma \frac{\pi^j}{w} \boldsymbol{O}_{LL}(L, K, D) = 0, \qquad \text{(A3)} \\
\frac{\partial \boldsymbol{\pounds}}{\partial D} &= -\sum_{j=1}^{4} \pi^j \left[\boldsymbol{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \boldsymbol{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \phi \\
&+ \mu \left(\boldsymbol{O}_D(L, K, D) - 1\right) - \gamma \boldsymbol{O}_{LD}(L, K, D) = 0, \qquad \text{(A4)} \\
\frac{\partial \boldsymbol{\pounds}}{\partial K} &= \mu \left(\boldsymbol{O}_K(L, K, D) - r\right) - \gamma \boldsymbol{O}_{LK}(L, K, D) = 0, \qquad \text{(A5)} \\
\frac{\partial \boldsymbol{\pounds}}{\partial w} &= \sum_{j=1}^{4} \pi^j \left[\boldsymbol{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \boldsymbol{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \left(\frac{-I^j}{n^j w^2}\right) \boldsymbol{V}_L \left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) \\
&+ \mu \boldsymbol{O}_L(L, K, D) \frac{-1}{w^2} \sum_{j=1}^{4} \pi^j I^j + \sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left(\frac{-I^j}{n^j w^2}\right) \mathrm{V}_L \left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) \\
&+ \sum_{j=1}^{4} \sum_{k \neq j} \lambda^{jk} \left(\frac{I^k}{n^j w^2}\right) \boldsymbol{V}_L \left(q, c^k, \frac{I^k}{wn^j}; \theta^j\right) + \gamma \left[1 + \frac{1}{w^2} \sum_{j=1}^{4} \pi^j I^j \boldsymbol{O}_{LL}(L, K, D) \right] = 0. \qquad \text{(A6)}
\end{aligned}
$$

Next, note that production efficiency implies that the condition $w = \boldsymbol{O}_L(L, K, D)$ is a non-binding constraint so that $\gamma = 0$ (we show this below). This simplifies equations (A3)–(A6) into

$$
\begin{aligned}
\frac{\partial \boldsymbol{\pounds}}{\partial I^j} &= \pi^j \left[\boldsymbol{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \boldsymbol{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \frac{1}{wn^j} \boldsymbol{V}_L \left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) + \mu \pi^j \\
&+ \sum_{k \neq j} \lambda^{jk} \frac{1}{wn^j} \boldsymbol{V}_L \left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \sum_{k \neq j} \lambda^{kj} \frac{1}{wn^k} \boldsymbol{V}_L \left(q, c^j, \frac{I^j}{wn^k}; \theta^k\right) = 0, \qquad \text{(A7)} \\
\frac{\partial \boldsymbol{\pounds}}{\partial D} &= -\sum_{j=1}^{4} \pi^j \left[\boldsymbol{V}\left(q, c^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j \boldsymbol{y}\left(q, c^j; \theta^j\right) - \phi D \right]^{-\eta} \phi \\
&+ \mu \left(\boldsymbol{O}_D(L, K, D) - 1\right) = 0, \qquad \text{(A8)}
\end{aligned}
$$

$$\frac{\partial \pounds}{\partial K} = \mu\left(\boldsymbol{O}_K(L,K,D) - r\right) = 0, \tag{A9}$$

$$\frac{\partial \pounds}{\partial w} = -\sum_{j=1}^{4}(\frac{I^j}{w})\frac{\partial \pounds}{\partial I^j} = 0. \tag{A10}$$

Proof of $\gamma = 0$: Multiply equation (A3) through by I^j/w, sum over j and simplify to get

$$\begin{aligned}
&- \frac{1}{w^2}\sum_{j=1}^{4}\frac{\pi^j I^j}{n^j}\left[\boldsymbol{V}\left(q, c^j, \frac{I^j}{wn^j};\theta^j\right) - \phi\sum_{j=1}^{4}\pi^j \boldsymbol{y}\left(q, c^j;\theta^j\right) - \phi D\right]^{-\eta}\boldsymbol{V}_L\left(q, c^j, \frac{I^j}{wn^j};\theta^j\right)\\
&= \mu L + \frac{1}{w^2}\sum_{j}\sum_{k\neq j}\left[(\frac{I^j}{n^j})\lambda^{jk}\boldsymbol{V}_L\left(q, c^j, \frac{I^j}{wn^j};\theta^j\right) - (\frac{I^j}{n^k})\lambda^{kj}\boldsymbol{V}_L\left(q, c^j, \frac{I^j}{wn^k};\theta^k\right)\right]\\
&- \frac{1}{w^2}\gamma\boldsymbol{O}_{LL}(L,K,D)(wL).
\end{aligned} \tag{A11}$$

Substituting (A11) into (A6) and simplifying, one gets

$$\sum_{j}\sum_{k\neq j}(\frac{I^j}{n^k})\lambda^{kj}\boldsymbol{V}_L\left(q, c^j, \frac{I^j}{wn^k};\theta^k\right) = \sum_{j}\sum_{k\neq j}(\frac{I^k}{n^j})\lambda^{jk}\boldsymbol{V}_L\left(q, c^k, \frac{I^k}{wn^j};\theta^j\right) + \gamma w^2. \tag{A12}$$

But one can rewrite the left-hand side of (A12) as

$$\sum_{j}\sum_{k\neq j}(\frac{I^j}{n^k})\lambda^{kj}\boldsymbol{V}_L\left(q, c^j, \frac{I^j}{wn^k};\theta^k\right) = \sum_{j}\sum_{k\neq j}(\frac{I^k}{n^j})\lambda^{jk}\boldsymbol{V}_L\left(q, c^k, \frac{I^k}{wn^j};\theta^j\right). \tag{A13}$$

Substituting from (A13) into (A12) then implies

$$\gamma = 0.$$

Constraints $q = 1$ and $\boldsymbol{O}_D(L,K,D) = 1$: The implications of these constraints for the optimization problem of the government are twofold. First, we no longer optimize with respect to q. Consequently, the first-order conditions do not include equation (A1). Second, denote the Lagrange multiplier associated with the constraint $1 - \boldsymbol{O}_D(L,K,D) = 0$ by ζ. This brings about the following changes in the first-order conditions (A2)–(A6): The expression for $\partial\pounds/\partial c^j$ remain unaffected; the expression for $\partial\pounds/\partial I^j$ now includes an additional term $-\zeta\boldsymbol{O}_{DL}(\pi^j/w)$; the expression for $\partial\pounds/\partial D$ now includes an additional term $-\zeta\boldsymbol{O}_{DD}$; the expression for $\partial\pounds/\partial K$ now includes an additional term $-\zeta\boldsymbol{O}_{DK}$; and the expression for $\partial\pounds/\partial w$ now includes an additional term $-\zeta\boldsymbol{O}_{DL}(\sum pi^j I^j/w^2)$.

Using the same method as previously, one can again show that the Lagrange multiplier associated with the constraint $\boldsymbol{O}_L(L,K,D) = w$ continues to be zero so that this condition remains non-binding in the presence of the additional constraint on $\boldsymbol{O}_D(L,K,D)$. On the other hand, setting $\boldsymbol{O}_D(L,K,D) = 1$ in (A4)–(A5), these will change to

$$\begin{aligned} -\sum_{j=1}^{4}\pi^j\left[\boldsymbol{V}\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)\right. &- \left.\phi\sum_{j=1}^{4}\pi^j\boldsymbol{y}\left(q,c^j;\theta^j\right)-\phi D\,\right]^{-\eta}\phi \\ &- \zeta\boldsymbol{O}_{DD}(L,K,D)=0, \qquad \text{(A14)} \\ \mu\left(\boldsymbol{O}_K(L,K,D)-r\right) &- \zeta\boldsymbol{O}_{DK}(L,K,D)=0. \qquad \text{(A15)} \end{aligned}$$

Conditions (A14)–(A15) then imply that

$$\begin{aligned} &\boldsymbol{O}_K(L,K,D) = r - \frac{\boldsymbol{O}_{DK}(L,K,D)}{\mu\boldsymbol{O}_{DD}(L,K,D)}\times \\ &\sum_{j=1}^{4}\pi^j\left[\boldsymbol{V}\left(q,c^j,\frac{I^j}{wn^j};\theta^j\right)-\phi\sum_{j=1}^{4}\pi^j\boldsymbol{y}\left(q,c^j;\theta^j\right)-\phi D\right]^{-\eta}\phi. \qquad \text{(A16)} \end{aligned}$$

Thus, the constraint $\boldsymbol{O}_D(L,K,D) = 1$ implies that $\boldsymbol{O}_K(L,K,D)$ should no longer be set equal to r. Put differently, a producer tax on r is now optimal.

The general income plus nonlinear commodity taxes: The Lagrangian for the second-best problem is

$$\begin{aligned} \boldsymbol{\pounds} &= \frac{1}{1-\eta}\sum_{j=1}^{4}\pi^j\left[\boldsymbol{U}\left(x^j,y^j,\frac{I^j}{wn^j};\theta^j\right)-\phi\sum_{j=1}^{4}\pi^j y^j-\phi D\right]^{1-\eta} \\ &+ \mu\left[\boldsymbol{O}\left(\sum_{j=1}^{4}\pi^j\frac{I^j}{w},K,D\right)-\sum_{j=1}^{4}\pi^j\left(x^j+y^j\right)-rK-D-\overline{R}\right] \\ &+ \sum_j\sum_{k\neq j}\lambda^{jk}\left[\boldsymbol{U}\left(x^j,y^j,\frac{I^j}{wn^j};\theta^j\right)-\boldsymbol{U}\left(x^k,y^k,\frac{I^k}{wn^j};\theta^j\right)\right] \\ &+ \gamma\left[w-\boldsymbol{O}_L(L,K,D)\right]. \end{aligned}$$

The first-order conditions are, for all $j = 1, 2, 3, 4$,

$$
\begin{aligned}
\frac{\partial \pounds}{\partial x^j} &= \pi^j \left[\boldsymbol{U}\left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j \right) - \phi \sum_{j=1}^{4} \pi^j y^j - \phi D \right]^{-\eta} \boldsymbol{U}_x^j - \mu \pi^j \\
&+ \sum_{k \neq j} \lambda^{jk} \boldsymbol{U}_x \left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j \right) - \sum_{k \neq j} \lambda^{kj} \boldsymbol{U}_x \left(x^j, y^j, \frac{I^j}{wn^k}; \theta^k \right) = 0, \qquad \text{(A17)} \\
\frac{\partial \pounds}{\partial y^j} &= \pi^j \left[\boldsymbol{U}\left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j \right) - \phi \sum_{j=1}^{4} \pi^j y^j - \phi D \right]^{-\eta} \boldsymbol{U}_y^j - \mu \pi^j \\
&- \phi \pi^j \sum_{j=1}^{4} \pi^j \left[\boldsymbol{U}\left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j \right) - \phi \sum_{j=1}^{4} \pi^j y^j - \phi D \right]^{-\eta} \\
&+ \sum_{k \neq j} \lambda^{jk} \boldsymbol{U}_y \left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j \right) - \sum_{k \neq j} \lambda^{kj} \boldsymbol{U}_y \left(x^j, y^j, \frac{I^j}{wn^k}; \theta^k \right) = 0, \qquad \text{(A18)} \\
\frac{\partial \pounds}{\partial I^j} &= \pi^j \left[\boldsymbol{U}\left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j \right) - \phi \sum_{j=1}^{4} \pi^j y^j - \phi D \right]^{-\eta} \boldsymbol{U}_I^j + \mu \frac{\pi^j}{w} \mathbf{O}_L(L, K, D) \\
&+ \sum_{k \neq j} \lambda^{jk} \boldsymbol{U}_I \left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j \right) - \sum_{k \neq j} \lambda^{kj} \boldsymbol{U}_I \left(x^j, y^j, \frac{I^j}{wn^k}; \theta^k \right) \\
&- \gamma (\frac{\pi^j}{w}) \mathbf{O}_{LL}(L, K, D) = 0, \qquad \text{(A19)} \\
\frac{\partial \pounds}{\partial D} &= -\phi \sum_{j=1}^{4} \pi^j \left[\boldsymbol{U}\left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j \right) - \phi \sum_{j=1}^{4} \pi^j y^j - \phi D \right]^{-\eta} \\
&+ \mu \left[\mathbf{O}_D(L, K, D) - 1 \right] - \gamma \mathbf{O}_{LD}(L, K, D) = 0, \qquad \text{(A20)} \\
\frac{\partial \pounds}{\partial K} &= \mu \left[\mathbf{O}_K(L, K, D) - r \right] - \gamma \mathbf{O}_{LK}(L, K, D) = 0, \qquad \text{(A21)} \\
\frac{\partial \pounds}{\partial w} &= \sum_{j=1}^{4} \pi^j \left[\boldsymbol{U}\left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j \right) - \phi \sum_{j=1}^{4} \pi^j y^j - \phi D \right]^{-\eta} \boldsymbol{U}_L \left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j \right) (\frac{-I^j}{n^j w^2}) \\
&+ \sum_{j} \sum_{k \neq j} \lambda^{jk} \left[(\frac{-I^j}{n^j w^2}) \boldsymbol{U}_L \left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j \right) - (\frac{-I^k}{n^j w^2}) \boldsymbol{U}_L \left(x^k, y^k, \frac{I^k}{wn^j}; \theta^j \right) \right] \\
&+ \mu \mathbf{O}_L(L, K, D) (\frac{-1}{w^2}) \sum_{j=1}^{4} \pi^j I^j + \gamma \left[1 - \mathbf{O}_{LL}(L, K, D) (\frac{-1}{w^2}) \sum_{j=1}^{4} \pi^j I^j \right] = 0. \qquad \text{(A22)}
\end{aligned}
$$

Production efficiency again implies that the condition $w = \boldsymbol{O}_L(L, K, D)$ is nonbinding so that $\gamma = 0$ (the proof is similar to that for the second best with linear commodity taxes). Setting $\gamma = 0$ then simplifies equations (A19)–(A22) into

$$\begin{aligned}
\frac{\partial \pounds}{\partial I^j} &= \pi^j \left[\boldsymbol{U}\left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j y^j - \phi D \right]^{-\eta} \boldsymbol{U}_L^j\left(\frac{1}{wn^j}\right) + \mu \pi^j \\
&+ \sum_{k \neq j} \left[\frac{\lambda^{jk}}{wn^j} \boldsymbol{U}_L\left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j\right) - \frac{\lambda^{kj}}{wn^k} \boldsymbol{U}_L\left(x^j, y^j, \frac{I^j}{wn^k}; \theta^k\right) \right] = 0, && \text{(A23)} \\
\frac{\partial \pounds}{\partial D} &= -\phi \sum_{j=1}^{4} \pi^j \left[\boldsymbol{U}\left(x^j, y^j, \frac{I^j}{wn^j}; \theta^j\right) - \phi \sum_{j=1}^{4} \pi^j y^j - \phi D \right]^{-\eta} \\
&+ \mu \left[\boldsymbol{O}_D(L, K, D) - 1 \right] = 0, && \text{(A24)} \\
\frac{\partial \pounds}{\partial K} &= \mu \left[\boldsymbol{O}_K(L, K, D) - r \right] = 0, && \text{(A25)} \\
\frac{\partial \pounds}{\partial w} &= -\sum_{j=1}^{4} (n^j L^j) \frac{\partial \pounds}{\partial I^j} = 0. && \text{(A26)}
\end{aligned}$$

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