Competing in Network Industries: Divide and Conquer*

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Abstract

The paper examines a competition game between a dominant network and a challenging network with price-discrimination and differentiated networks. Domination is captured through an adequate resolution of the coordination game played by consumers, that favours one firm over the other, and is interpreted as a reputation effect. Price-discrimination in this context has a strong impact because cross-subsidization allows a firm to coordinate the choices of consumers, reducing the impact of reputation effects. Competitive strategies subsidize the participation of some consumers in order to create a bandwagon effect on others. This intensifies competition and reduces average equilibrium prices. Under perfect price-discrimination, both networks prefer to be compatible, because bandwagon effects are due to the incompatibility between networks. Price-discrimination promote efficiency by reducing the extent of excess inertia, but new features of excess momentum and market unstability appear. A network may also have incentives to unilateraly degrade the quality for some targeted group of consumers in order to weaken competition.

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1 Introduction

While there is already a substantive literature on competition with network effects (see Katz and Shapiro (1994) and Economides (1996)), one feature of modern networks that has not received considerable attention is that network effects are often not isotropic: members may join for different reasons and value both the service and the participation of others in very different ways. In conjunction with that, and partly because of that, most suppliers of network goods practice some form of price-discrimination. For instance, telecommunication operators or software suppliers discriminate between residential and professional users, as well as geographic areas. This paper is concerned with situations involving differentiated users of the network and price-discrimination. This general set-up covers a wide range of activities, such as intermediation activities, telecommunication services, postal services, multi-media services, advertising, Internet services, operating systems and software applications, research centers,....

In the presence of network externalities, consumers face a coordination problem in their purchasing decision that may generate multiple equilibria (Katz and Shapiro (1985)). A key determinant of the success of a network is the consumers' confidence on the ability of a network to grow.[1] When consumers tend to perceive an established firm as focal and to coordinate on its network, this effect, that can be interpreted as a reputation effect, creates barriers to entry. This is indeed the main motive for fast entry and aggressive strategies in building a customer base in infant stages of network industries.

In this context, price-discrimination creates a very particular environment as it allows for aggressive strategies that subsidize some consumers to join and

[1] See Arthur (1989), David (1985), Farell and Saloner (1986), Katz and Shapiro (1986,1992).

exploit the network externalities to recover the subsidy on other consumers (a divide-and-conquer strategy)[2]. Such a strategy allows a network to overcome the coordination problem by transferring part of the surplus to targeted customers and creating a bandwagon effect. This is the static version of the dynamic mechanism that leads a network to subsidize earlier customers in an attempt to build the customer base.[3]

The paper studies the interaction between the reputation effects at work with network externalities and the new complex marketing strategies that emerge when a differential price treatment of consumers is possible. It examines a competition game between an established network and a challenging network, where the former benefits from a strong reputational advantage. The population is decomposed into groups and third degree price-discrimination is possible. Network effects may occur within groups (intra-group) and between groups (inter-group). Different groups may have different valuations for the goods, as well as for the participations of other groups to the network. This allows for a wide range of differentiation (both horizontal and vertical) and of network effects.

The reputation of the established network is modeled by selecting a particular equilibrium of the subgame played by consumers when they choose where to buy, which can be interpreted as a particular choice of consumer's equilibrium beliefs. With positive network externalities, there is a well defined selection criterion that captures the idea of a reputation advantage for the established network, which amounts to impose that a consumer will buy from the established network whenever this is the outcome of at least one equilibrium of the allocation game. This is referred to as *Domination*

[2] See Innes and Sexton (1993) for similar ideas exploiting increasing returns to scale.

[3] See Bensaid and Lesne (1996) and Cabral, Salant and Woroch (1999) applications to dynamic monopoly pricing.

in Beliefs and it maximizes the dominant firm market share for any price configuration.

To overcome its reputation disadvantage, a newcomer relies on divide-and-conquer strategies as described above. This means that the nature of competition is changed in a non-trivial manner. Firms will target some consumers by offering advantageous conditions, and the outcome of the market game may exhibit a substantive amount of cross-subsidization.

This forces the established firm to set on average prices at a much lower level than it would do with uniform prices. It turns out that it is impossible for a network to capture in equilibrium the surplus generated by the inter-group network externalities.

The intuition behind the result is the following. Consider two firms I and C (with zero cost) competing for two consumers A and B. Suppose that A and B have identical valuations for the two network goods, and that they receive an extra value v if they both join the same firm. With uniform prices, there is an equilibrium where one firm (say I) sells to both consumers at price v. The reason is that if a consumer is convinced that the other consumer joins I, it will be reluctant to join C unless it undercut $I's$ price by an amount v. Now suppose that price-discrimination is allowed and that I charges a uniform price p^I. The same logic applies but C can act as follows: it charges a price below $p^I - v$ to consumer A (divide) and a price $p^I + v$ to consumer B (conquer). With such a price structure, A joins firm C irrespective of what B does because in relative terms, C offers him a value larger than the value attached to the participation of B (it is a dominant strategy for A to join C). Knowing that, B must then choose between buying alone from I and joining A in the network of C. He is thus willing to pay the premium v to join firm C. Notice that C achieves to serve the two consumers irrespective of the nature of the coordination process that determinates the allocation of consumers, and that it obtains the profit $2p^I$. The result is that equilibrium profits vanish in equilibrium.

This reasoning will imply in particular that with an homogeneous population and perfect price discrimination, any equilibrium involves an efficient allocation of consumers, which stands at odds with inefficiency results that emerge with uniform pricing.

With network goods, each individual is not only a consumer but also an input, reflecting the fact that his participation to the network creates value for other consumers. As an input, the consumer is a scarce resource. Firms then compete to sell to consumers but at the same time compete to buy the inputs, i.e. the participation of individuals. Price-discrimination exacerbates this dual nature of competition by allowing the firm to decide which individuals are treated like an input rather than a buyer.

The effect is even stronger when inter-group network effects are asymmetric. Consumers who value less the participation of others then become the object of an intense competition as they are more value-enhancing relatively to others. To see that suppose that A doesn't care for the participation of B. Then the firm that succeeds to sell to A gains a competitive advantage on B, equal to v, $B's$ willingness to pay to join A. The competition for A then dissipates the profit, and may even prevent the existence of a pure strategy equilibrium.

One consequence of this intense competitive process is that, when intra-group network effects are small and in particular when perfect price discrimination is feasible, both firms are better off by selling compatible goods, even one that benefits from a strong reputational advantage. With (almost) perfect price-discrimination, incompatible networks are extremely aggressive in building market shares, while being enable to capture the extra surplus generated by network effects. Compatibility, by suppressing the interaction between the relative size of each network and the willingness to pay of consumers, eliminates the motive for cross-subsidization. Provided that the network goods are differentiated, even slightly, removing the incentives to enlarge market share restores the benefits of differentiation, with a weakening of the competitive pressure and market segmentation. Unless one firm can exploit large intra-group network effects, a peaceful exploitation of differentiation by compatible networks is then more profitable than a head-to-head confrontation between incompatible networks.

The second consequence is that price-discrimination reduces the degree of inertia that may occur in network industries, as well as barriers to entry. It may even be the source of excess momentum, with consumers switching to the challenger's network, while this is inefficient. This is at odds with standard results because the model is solved under assumptions that generate excess inertia when prices are uniform. Even when the incumbent offers a uniformly higher quality, it may fail to cover the market. The reason is that the challenger can capture a positive share of the value of network effects with a divide-and-conquer strategies. If the quality differential in favor of the incumbent is smaller than this share, it will fail to cover the market.

A final point that emerges is that networks' quality choices may not be efficient. A network may in particular degrade the quality it offers to some targeted groups (and not for others), as a mean to increase the degree of horizontal differentiation and to induce market sharing. This can be viewed as a precommitment device for the network, which doing so, commits not to compete for this group.

The paper is organized as follows. Section 2 presents the general model along with the notion of reputation. Section 3 presents the analysis of competitive strategies and cross-subsidization. Section 4 discusses the effect of compatibility. Section 5 analyses the situation where the population is homogeneous, while Section 6 presents a full-fledged analysis of the case with two groups. Section 7 provides further discussions of the results. Section 8 concludes.

2 The model

2.1 A monopoly network

Consider an incumbent sole supplier of a network good with a production cost normalized to 0, denoted I. The good is consumed by J different types of users, each represented by a group composed of a mass m_j of identical consumers. The set of groups is denoted $\mathcal{J}$. The perceived quality of the good varies across types of users, denoted v_j^I for type j users (intrinsic value). Consumers of a given group may value differently the participation of every other group to their network. Denoting by n_j^I the mass of consumers of type j buying from I, the valuation of a consumer j for the participation of n_l^I members of group l is $\alpha_{jl} n_l^I$, where the coefficients α_{jl} are nonnegative. I shall refer to network effects between two different groups as inter-group network effects, and to the externality within groups as intra-group network effects. The case $\alpha_{jj} = 0$ and $m_j = 1$ can be interpreted as a situation where j is a single individual, and thus as one of perfect price discrimination. Overall the gross utility that a type j consumer derives from consumption of the network good is $v_j^I + \sum_{l=1}^{J} \alpha_{jl} n_l^I$.

Firm I is able to charge a different price for each group, and thus choose a vector of J prices $P^I = \{p_1^I, ..., p_J^I\}$. Given these prices, each consumer decides whether to buy or not. Consumers coordinate on a rational expectation equilibrium of this allocation game.[4] Due to network effects, there may be a multiplicity of such equilibria.

The maximal prices that a monopoly firm can set are $v_j^I + \sum_{l=1}^{J} \alpha_{jl} m_l$. At these prices, each consumer is willing to buy provided that it anticipates that all others do. I shall from now on impose that prices are below this maximal monopoly price. Whether the monopolist is able to sell at these prices or not depend on the coordination process of consumers. The reason is that there is also another equilibrium allocation of consumers where none of them buy.

More generally, for any vector of prices such that $p_j^I \geq v_j^I$ for all j, there is the possibility that consumers fail to coordinate and that the firm doesn't sell at all. Thus when the consumers coordination process leads to the least favorable allocation, the monopolist must set at least one price p_j^I below v_j^I. Suppose that $p_1^I < v_1^I$, then a member group 1 buys irrespective of what the others are doing. Given that, I can set a price above v_2^I for group 2: any price $p_2^I < v_2^I + \alpha_{21} m_1$ ensures that group 2 buys as well, since given that group 1 buys, a member of group 2 buys irrespective of what the others are doing. Using this reasoning recursively, we see that I can conquer the market by setting prices $p_j^I = v_j^I + \sum_{l<j} \alpha_{jl} m_l$, even faced with the least favorable market conditions. Obviously the reasoning doesn't depend on the order of the group so that, if $\sigma(.)$ denotes an order (a permutation) on the set of the groups, prices $p_j^I = v_j^I + \sum_{\sigma(l)<\sigma(j)} \alpha_{jl} m_l$ (or slightly below) allow I to conquer the market as well. Thus, even in the worst case, the monopoly can extract a substantial part of the value of network externalities.

The general conclusion from this section is that price-discrimination may

[4] Each individual's consumption decision maximizes his utility given the prices and the equilibrium allocation of other consumers, inculding others members of its group.

serve two purposes for a monopoly network. First it may serve the standard purpose of increasing the rent extracted from high value users, while selling to low value users. Second, it may help the network to overcome problems due to coordination failure.

2.2 Two competing networks

Suppose now that the incumbent I competes with a challenger C for the provision of network goods. The goods are differentiated and are allowed to be partially compatible. The intrinsic value of good k for a type j consumer is $v_j^k > 0$. Let θ measure the degree of compatibility as follows: if n_l^I and n_l^C members of group l buy from I and C respectively, a type j consumer buying from firm k benefits of network effects $\alpha_{jl}\left(n_j^k + \theta n_j^{-k}\right)$, where $-k$ denotes the other firm. The case $\theta = 0$ correspond to full incompatibility, while $\theta = 1$ corresponds to full compatibility. In what follows, the market will covered by the two firms in any case. In particular, the issue of the impact of compatibility on the size of the market is avoided.

Given that $n_l^I + n_l^C = m_l$, the utility writes as $U_j^k = v_j^k + \theta \sum_{l=1}^J \alpha_{jl} m_l + (1-\theta) \sum_{l=1}^J \alpha_{jl} n_l^k - p_j^k$. The agent receives a direct utility v_j^k plus industry level network effects $\theta\alpha_{jl}m_l$. She enjoys an extra benefits $(1-\theta)\alpha_{jl}m_l$ from network effects when group l joins the same network. Define:

Definition 1 $u_j^k = v_j^k + \theta \sum_{l=1}^J \alpha_{jl} m_l$, $\beta_{jk} = (1-\theta)\alpha_{jl}$.

The valuation of good k by a type j consumer is thus

$$U_j^k = u_j^k + \sum_{l=1}^J \beta_{jl} n_l^k - p_j^k.$$

With such a formulation, the model is compatible with both vertical differentiation and horizontal differentiation.

This paper focuses on the case where the network externalities at the firm level are not too large compared to the intrinsic value of the service. More specifically it is assumed that:

Assumption 1: *For all* j, $u_j^C \geq \sum_{l \neq j} \beta_{jl} m_l$.

Stated in terms of compatibility level, assumption 1 reduces to

$$\theta \geq \bar{\theta} = \frac{\sum_{l \neq j} \alpha_{jl} m_l - v_j^C}{2 \sum_{l \neq j} \alpha_{jl} m_l + \alpha_{jj} m_j}$$

Notice that $\bar{\theta}$ is less than $\frac{1}{2}$ and it is negative if $v_j^C > \sum_{l \neq j} \alpha_{jl} m_l$. The assumption can thus be interpreted alternatively as resulting from the fact that network effects are small compared to the value of direct services, or from the fact that there is enough compatibility. To give an example, ISPs services (connectivity, email, webpage hosting) can be seen as involving no network effects at the firm level, the introduction of instant messenging creates a network externality but it remains limited compared to the value of the base services. Backbones provide another example: β_{jl} captures the relative benefits from being in the same network when the quality of interconnection is not perfect. The benefits of using the same text editor are confined within small communities, and presumably not of the same level than the value of using a text editor.

However this assumption is not just a technical one. The case where the intrinsic value is small compared to the network effects leads to a different analysis of pricing strategies. It is analyzed in Caillaud-Jullien (2000b), and discussed in Section 7.

The competitive game is composed of two stages:

Stage 1: Firms I and E simultaneously set prices P^I and P^E

Stage 2: Consumers simultaneously decide which good to buy.[5]

Faced to prices $P^I = \{p_1^I, .., p_J^I\}$ and $P^C = \{p_1^C, ..., p_J^C\}$, consumers coordinate on a rational expectation equilibrium allocation (REA). Denote by $\mathcal{A}(P^I, P^C)$ an allocation rule that assigns to every price vectors a REA. Denote by $\Pi^k(P^I, P^C, \mathcal{A})$ the profit of firm k when the prices are P^I and P^C, and consumers are allocated according to $\mathcal{A}$. An equilibrium consists in an allocation rule $\mathcal{A}$, and equilibrium prices for the pricing game with profit functions $\Pi^k(P^I, P^C, \mathcal{A})$. The choice of the allocation rule is then key to the determination of equilibrium prices. We can interpret this choice as a reflecting consumers' beliefs (their conjectures about the composition of the two networks), and thus as a reputation effect.

The general idea of the paper is to capture the fact that I is dominant through the fact that consumers' beliefs are biased in favor of firm I. In a somewhat tautological way, the incumbent is dominant when all consumers believe it is. This generates self-fulfilling beliefs that resolve the coordination problem of consumers in favor of I. While the notion of favorable beliefs may be ambiguous in a general context,[6] the following result shows that it takes the form of a simple selection criteria when there are positive network externalities.

Lemma 1 *Fix the prices P^I and P^C, and consider all the REA of consumers for these prices. Let $\mathcal{K}^I$ be the set of groups buying from I in **at least one** REA and $\mathcal{K}^C$ be the set of groups buying from C in **all** REA. Then there*

[5] In this model, consumption is exclusive. See Caillaud and Jullien (2001) for an analysis of non-exclusive consumption.

[6] With negative externalities, consumers will tend to separate from each other. Then beliefs favourable to I would correspond to beliefs that others join C. Given that expectations must be rational in equilibrium, they may not be a simple and non ambiguous way to define beliefs that favor I over C.

exists a REA, denoted $\mathcal{D}^I(P^I, P^C)$, *such that: all groups within* $\mathcal{K}^I$ *buy from* I, *and only groups within* $\mathcal{K}^C$ *buy from* C.

Proof. See appendix. ■

The key feature is that the value of a network uniformly increases when new consumers are added to its customer base. According to a bandwagon effect, displacing consumers from C to I raises the incentives to join I for all individuals and reduces the incentives to join C. Thus there exists a unique REA that at the same time maximizes the market share of the incumbent and minimizes the market share of the challenger. This follows from the fact that there are strategic complementarities in the consumers allocation process (see Topkis (1979), Vives(1990), Milgrom-Roberts (1990)).

It is assumed that firm I dominates the coordination process in the sense that consumers coordinate on $\mathcal{D}^I(P^I, P^C)$, referred to as domination is beliefs:

Assumption 2 (Domination in Beliefs):

For almost all (P^I, P^C), $\mathcal{A}(P^I, P^C) = \mathcal{D}^I(P^I, P^C)$;

For all (P^I, P^C), $\Pi^C(P^I, P^C, \mathcal{A}) \leq \lim_{\varepsilon \longrightarrow 0} \Pi^C(P^I, P^C, \mathcal{D}^I(P^I, P^C - \varepsilon))$.

The second part of the assumption is intended to avoid inexistence problems that may be created by a discontinuity at an indifference point of consumers. Notice that the domination in beliefs implies that consumers will not always coordinate according to the Pareto criterion.

To illustrate the implications, suppose that $\beta_{jl} = 0$ for all j, l. Then domination in beliefs implies that group j buys from I whenever $u_j^I + \beta_{jj} m_j - p_j^I > u_j^C - p_j^C$. Let us define the "stand-alone" value differential as:

Definition 2 $\delta_j = u_j^I - u_j^C + \beta_{jj} m_j$.

With no inter-group network effects, I sells to group j in equilibrium whenever $\delta_j \geq 0$. The model is then similar to a Bertrand competition game with a difference δ_j between the quality of $I's$ good and the quality of $C's$ good.

Lemma 2 *When $\beta_{jl} = 0$ for $l \neq j$, firm I sells to group j if and only if $\delta_j \geq 0$. The maximal equilibrium profit $\sum_{j=1}^{J} \max\{\delta_j, 0\} m_j$ for I and $\sum_{j=1}^{J} \max\{-\delta_j, 0\} m_j$ for C.*

Proof. Immediate from what precedes. ■

In particular, under assumption 2, the value of the intra-groups network externalities can be attributed to I. This property extends to the general case. Indeed, consider now the case with inter-group network effects. Let us fix the equilibrium behavior of all groups except group j, under the assumption that consumers coordinate on $\mathcal{D}^I(P^I, P^C)$. Denote $\mathcal{K}^I$ the set of groups buying from I and B_j^k the value of inter-group network effects that a member of group j derives from firm k. Then, if $u_j^I + \beta_{jj} m_j + B_j^I - p_j^I > u_j^C + B_j^C - p_j^C$, group j must buy from I. To see that, let us impose that group j buys from I and apply lemma 1 on the other groups. This generates a new allocation of customers. Since assigning group j to I has raised or leaved unchanged the value of $I's$ good, groups within $\mathcal{K}^I$ still buy from I. The value of inter-group network effects for group j at I is thus at least B_j^I in this new allocation, making it rational to buy from I. We have thus generated an allocation where groups within $\mathcal{K}^I$ and group j buys from I. The fact that the equilibrium allocation is $\mathcal{D}^I(P^I, P^C)$ implies that this construction can't increase the size of $\mathcal{K}^I$ and thus that group j belongs to $\mathcal{K}^I$. Thus group j buys from I if $\delta_j + B_j^I - p_j^I > B_j^C - p_j^C$.

In all the paper, only δ_j matters and they will be no difference in treatment between the intra-group network effects and the intrinsic value of good I. The main problem will rather be to determine how the presence of inter-group network effects affects the competition game.

3 Price-discrimination and network effects

This first part of the paper focuses on the analysis of the strategic effects at work with inter-group network effects. Under the assumption 2 (Domination in Beliefs), intuition would suggest that the incumbent should benefit from the presence of any network effects, and in particular inter-group network effects. This is not the case because the incumbent must account for the possibility that firm C uses price-discrimination to overcome its disadvantage in terms of consumers' beliefs. To do so, C has to favor one group over the other. C can charge a low or even negative price for one group if selling to this group allows to sell to another group at a high price. To follow insights from Innes and Sexton (1993), one may view the situation as one in which C faces consumers who have the possibility to form a coalition (to join I). To prevent the formation of the coalition, C needs to "bribe" some groups. However it needs not bribe all groups but only enough of them to ensure that the value of the sub-coalition composed of the remaining groups is reduced to a point where it becomes unattractive. The challenger's optimal strategy is then to subsidize some groups and to charge high prices on other groups: a "divide-and-conquer" strategy. This strategy is so efficient in solving the coordination problem that it allows C to overcome its presumed disadvantage in terms of consumers' beliefs and to capture part of the value of network externalities.

To illustrate this effect, suppose that there are only two groups and that I sells to both groups in equilibrium. Assuming that $\beta_{12} \leq \beta_{21}$, let us build the best strategy that would enable C to attract both groups.

Let j denote an arbitrary group of consumers and l the other group. For any pair of prices such that $p_j^C \geq p_j^I - \delta_j - \beta_{jl} m_l$, assumption 2 implies that group j buys from I if the other group does. To build a profitable strategy that attracts all consumers, C needs to set one price below $p_j^I - \left(\delta_j + \beta_{jl} m_l\right)$. Such a strategy implies that C charges a negative price for at least one group.[7] Suppose that C sets the price p_1^C just below $p_1^I - \delta_1 - \beta_{12} m_2$. Then group 1 buys from the challenger irrespective of what the other group does. This is known by group 2 so that to attract group 2, the challenger needs only to set a price p_2^C such that $u_2^C + \beta_{21} m_1 - p_2^C \geq u_2^I + \beta_{22} m_2 - p_2^I$. Indeed the choice for a member of group 2 is now between joining group 1 or staying with I. Under the above condition, the former dominates unambiguously. The maximal price that C can set on group 2 is thus $p_2^I - \delta_2 + \beta_{21} m_1$. By attracting group 1, the challenger has reduced the attractiveness of the incumbent for group 2 by an amount $\beta_{21} m_1$, and increased its own attractiveness by the same amount. The overall profit from this strategy is $\left(p_1^I - \delta_1 - \beta_{12} m_2\right) m_1 + \left(p_2^I - \delta_2 + \beta_{21} m_1\right) m_2$. If the incumbent sells to both groups in equilibrium, this must be non-positive:

$$p_1^I m_1 + p_2^I m_2 \leq \sum_{j=1}^{2} \delta_j m_j - (\beta_{21} - \beta_{12}) m_1 m_2 \tag{1}$$

We see that the maximal profit is bounded above by $\sum_{j=1}^{2} \delta_j m_j$. The next part of this section is devoted to extend this analysis.

[7] Given that cost are normalised to zero, a negative price should be interpreted as a price below marginal cost. If the effect is large, the price can be not only below marginal cost but indeed negative. The best interpretation in this case is that the customer receives free access to the network, and is subsidized through additional free services (gifts, lotteries, ...). This can also be interpreted as the result of costly advertising campaigns.

3.1 The incumbent's maximal profit

Suppose that I covers the market at prices P^I. Since I dominates in beliefs, there exist a REA where it covers the market whenever for all groups, the maximal utility at I, $u_j^I + \sum_{l=1}^{J} \beta_{jl} m_l - p_j^I$ is larger than the minimal utility at C, $u_j^C - p_j^C$. This condition writes as $p_j^I - p_j^C \leq \delta_j + \sum_{l \neq j} \beta_{jl} m_l$ *for all j*. Firm I must necessarily set prices such that

$$p_j^I \leq \delta_j + \sum_{l \neq j} \beta_{jl} m_l, \tag{2}$$

which ensures that the maximal utility that a type j consumer can derive from I is larger than the minimal utility obtained when buying from C at a zero price. If it were not the case, the challenger could attract a type j with a positive price.

Notice that assumption 1 implies that $\delta_j + \sum_{l \neq j} \beta_{jl} m_l \leq u_j^I + \beta_{jj} m_j$. Thus condition (2) implies that $p_j^I \leq u_j^I + \beta_{jj} m_j$, which ensures that group j buys from I if it doesn't buy from C, even if C sells to all other groups.

Let us now assume that prices p_j^I verify (2) and suppose that C tries to sell to a subset $\mathcal{K}$ of groups. For conciseness take $\mathcal{K}$ $=\{1, 2, ..., K\}$. The challenger has to set at least one price small enough so that $u_j^I + \sum_{l=1}^{J} \beta_{jl} m_l - p_j^I \leq u_j^C - p_j^C$, which ensures the willingness of a member of group j to buy alone. Say that the price p_1^C is slightly below $p_1^I - \delta_1 - \sum_{l>1} \beta_{1l} m_l$. The resulting price is such that it is a dominant strategy for a member of group 1 to buy from C. Given that it is commonly known that group 1 joins C, any price p_2^C such that $u_2^I + \sum_{l \geq 2} \beta_{2l} m_l - p_2^I \leq u_2^C + \beta_{21} m_1 - p_2^C$ induces a member of group 2 to buy from C. C can thus convince group 2 to join with a price p_2^C slightly below $p_2^I - \delta_2 - \sum_{l>2} \beta_{2l} m_l + \beta_{21} m_1$. The process can then continue: given p_1^C and p_2^C, it is commonly known that members of groups 1 and 2 join C, so that a member of group 3 buys at a price that makes it more attractive to join groups 1 and 2, as opposed to staying with groups 3 and above. More generally, group j is charged by C the largest price that ensures that its members buy given that groups $l < j$ join. The pricing strategy is thus build in such a way that after j rounds of elimination of dominated strategies, there are j groups for which the only remaining strategy is to buy from C. The resulting prices are:[8]

$$\begin{aligned} p_1^C &= p_1^I - \delta_1 - \sum_{l>1} \beta_{1l} m_l \\ p_j^C &= p_j^I - \delta_j + \sum_{l<j} \beta_{jl} m_l - \sum_{l>j} \beta_{jl} m_l \end{aligned}$$

Summing up over the groups we obtain the challengers' profit:

$$\sum_{j=1}^{K} p_j^C m_j = \sum_{j=1}^{K} p_j^I m_j - \sum_{j=1}^{K} \delta_j m_j + \sum_{j=1}^{K} \sum_{l=j+1}^{K} \left(\beta_{lj} - \beta_{jl}\right) m_l m_j - \sum_{j=1}^{K} \sum_{l=K+1}^{J} \beta_{jl} m_l m_j$$

More generally, the challenger has the choice of the set of groups targeted, and of the order in which groups are subsidized. Indeed C would benefit from subsidizing the groups valuing less the network effects, since this allows the subsidy to be smaller than the value of the network externalities that C can extract from the other groups. To compute equilibrium conditions, we must thus account for any possible order between groups. Denote $\sigma(.)$ a permutation on the set of groups, where $\sigma(l) > \sigma(j)$ means that j is ranked before l. The interpretation is that C sets a price $p_j^C = p_j^I - \delta_j + \sum_{\sigma(l)<\sigma(j)} \beta_{jl} m_l - \sum_{\sigma(l)>\sigma(j)} \beta_{jl} m_l$ for group j, such that a member of this group is willing to join provided that it is sure that all groups ranked below join as well. The maximal profit that C can get on a subset $\mathcal{K}$ of groups is now obtained by summing the prices over the groups and optimizing on the

[8] Here equality generates the maximal profit but one should think of C as setting prices slightly below equality.

order. Define

$$\Omega_{\mathcal{K}} = \max_{\sigma(.)}\left\{\sum_{j\in\mathcal{K}}\sum_{\substack{l\in\mathcal{K}\\ \sigma(l)>\sigma(j)}}(\beta_{lj}-\beta_{jl})m_l m_j\right\} \quad (3)$$

$$B_{\mathcal{K}} = \sum_{j\in\mathcal{K}}\sum_{l\notin\mathcal{K}}\beta_{jl}m_l m_j. \quad (4)$$

$\Omega_{\mathcal{K}}$ extends the term $\sum_{j=1}^{K}\sum_{l=j+1}^{K}\left(\beta_{lj}-\beta_{jl}\right)m_l m_j$ to any ranking of the groups, the maximization coming from the optimal choice by C of the order of targeting. This term has a very simple interpretation: when group j is attracted "before" group l, C must give a subsidy $\beta_{jl}m_l$ to the members of group j equal to their opportunity cost of leaving group l, but C can charge an extra $\beta_{lj}m_j$ to the members of group l corresponding to the value of joining group j. The net effect is then $(\beta_{lj}-\beta_{jl})m_l m_j$.

$B_{\mathcal{K}}$ is the total value of inter-group network effects between groups within $\mathcal{K}$ and groups outside $\mathcal{K}$.

By selling to the subset $\mathcal{K}$, the challenger can obtain at most $\sum_{j\in\mathcal{K}}p_j^I m_j - \sum_{j\in\mathcal{K}}\delta_j m_j + \Omega_{\mathcal{K}} - B_{\mathcal{K}}$. Whenever the incumbent I covers the market in equilibrium, there must be no subset $\mathcal{K}$ of groups that C could attract with positive profits. This amounts to the fact that:

$$\text{for all } \mathcal{K},\ \sum_{j\in\mathcal{K}}p_j^I m_j \leq \sum_{j\in\mathcal{K}}\delta_j m_j - \Omega_{\mathcal{K}} + B_{\mathcal{K}}. \quad (5)$$

This provides us with bounds on the total profit derived by I from any particular subset of groups. This bound includes three components. The first component includes the absolute advantage of I on the groups targeted (δ_j), captured by the " stand alone" value differential (quality differential augmented by the value intra-group network effects). The second term $\Omega_{\mathcal{K}}$ captures the impact on profits of $C's$ discrimination between groups. The last term captures the fact that when C chooses not to sell to some groups, it must compensate its clients for the value of networks effects they would receive if they stayed with I.

Clearly an upper bound on the profit that I can obtain by selling to all groups is the bound obtained for the set of all groups $\mathcal{J}$.

Proposition 1 *If I sells to all groups in equilibrium, its profit is less or equal $\bar{\Pi}^I = \sum_{j=1}^{J} \delta_j m_j - \Omega_{\mathcal{J}}$. Moreover there exist prices P^I such that the best-response of C is not to sell at all and I obtains this maximal profit.*

Proof. See appendix. ■

The results show that in a Stackelberg pricing game with I as a leader and C as a follower, I could obtain this maximal profit by setting adequate prices. Given this result, it is fairly straightforward to derive a bound on $I's$ profit when it doesn't cover the market.

Corollary 1 *If I sells to groups within $\mathcal{K}$ in equilibrium, and C to the other groups, I's profit is smaller than $\sum_{j \in \mathcal{K}} \delta_j m_j - \Omega_{\mathcal{K}} - B_{\mathcal{K}}$.*

The bound is the same as before but accounts for the fact that since groups outside $\mathcal{K}$ buy from C, the value of $C's$ good is augmented by the corresponding value $B_{\mathcal{K}}$ of inter-group effects. While the same logic applies, we will see that in the case where I doesn't cover the market this bound may not always be attainable. In other words, a Stackelberg leader firm I may not be able to obtain this level of profit under some circumstances. The reason is that the bound is obtained by fixing the prices set by C for the groups it serves in equilibrium and varying the other prices. It thus ignore the possibility that the challenger uses more complex strategies involving all the prices.

The main implication of the results is that I can't benefit from the presence of inter-group network effects, despite assumption 2. When network effects are symmetric ($\beta_{jl} = \beta_{lj}$), the effects of inter-group network externalities in the divide and conquer strategy cancel out and $\Omega_{\mathcal{K}}$ vanishes. Inter-group network effects between its clients are neutral for I in this case. Then introducing some asymmetry can only worsen the case for the incumbent.

Lemma 3 *For all $\mathcal{K}$, $\Omega_{\mathcal{K}} \geq 0$, with equality if and only if for all j, l within $\mathcal{K}$, $\beta_{jl} = \beta_{lj}$.*

Proof. See appendix. ■

In particular, when I sells to all groups in $\mathcal{K}$ it can't obtain more than $\sum_{j \in \mathcal{K}} \delta_j m_j$ which is the maximal profit without inter-group network effects.

3.2 The challenger's maximal profit

Let us now consider the maximal profit of C when it sells to all the groups within $\mathcal{L}$ and I sells to the groups within the complement subset $\mathcal{K}$. The key difference with before is that I can capture all the surplus from network effects by exploiting its domination and needs not rely on cross-subsidization. Indeed I can capture the whole clientele of C, thus $\mathcal{L}$, by setting prices $p_j^I \leq p_j^C + u_j^I - u_j^C + \sum_{l=1}^{J} \beta_{jl} m_l$ for these groups (assuming here that $p_j^C < u_j^C$). Thus I could capture the "stand alone" value differential as well as the full value of the network effects for the groups buying $C's$ good. Notice that, by doing so, I raises also its value for its initial clients (groups within $\mathcal{K}$) by a total amount $B_{\mathcal{L}}$. It can thus also raise its prices on its initial clients. This generates a total potential value differential in favor of I equal to the sum of the quality differential and the total increase in the value of network effects associated with I when it attracts the whole population instead of groups within $\mathcal{K}$ only.

Proposition 2 *The profit that* C *obtains when it sells to groups within* $\mathcal{L}$ *and* I *sells to groups within* $\mathcal{K} = \mathcal{J} \backslash \mathcal{L}$ *is less or equal to* $\sum_{j \in \mathcal{L}} (u_j^C - u_j^I) m_j - \sum_{j \in \mathcal{L}} \sum_{l=1}^{J} \beta_{jl} m_l m_j - B_{\mathcal{K}}$.

Proof. See Appendix. ■

The proof shows that C could achieve this profit when acting as a Stackelberg leader. Notice that when C covers the market, this bound reduces to:

$$\bar{\Pi}^C = \sum_{j \in \mathcal{J}} (u_j^C - u_j^I) m_j - \sum_{j,l \in \mathcal{J}} \beta_{jl} m_l m_j.$$

The profit is the same as if buying from C generates no network externality. The challenger's profit is the difference between the total intrinsic utility it offers to its customers and the total surplus that the incumbent would generate by attracting all of them, which includes not only the network effects for $C's$ customers but also the increase in the utility of $I's$ customers.

4 Being compatible or not?

The first natural question that bounds on profits allow to address is whether firms would prefer to sell compatible goods. To analyze the issue let us compare the profit under full compatibility ($\theta = 1$) and partial compatibility ($\theta < 1$).

When the network goods are fully compatible, a consumer buying the good from firm k benefits from network externalities with all consumers. The utility derived by a type-j consumer buying from firm k is then $U_j^k = v_j^k + \sum_{l=1}^{J} \alpha_{jl} m_l - p_j^k$. For any individual, the comparison between the two goods reduces to the comparison between $v_j^I - p_j^I$ and $v_j^C - p_j^C$. Network effects becomes irrelevant for the analysis of the competition game which reduces to a standard Bertrand type game, where each group constitutes a specific market and only intrinsic values matters. Firm I sells to group j if $v_j^I \geq v_j^C$, while firm C sells to group j if $v_j^C > v_j^I$. The maximal profit that firm $k = I, C$ can expect in equilibrium when network goods are compatible is $\sum_{j=1}^{J} \max\{v_j^k - v_j^{-k}, 0\}$, that is taken to be the equilibrium profit under compatibility.

It is fairly straightforward to see that firm C prefers to be compatible, because network effects work at its disadvantage. The key question is whether I prefers to be compatible or not. Looking at this issue from $I's$ perspective, there are several conflicting effects. Clearly the presence of intra-group network externalities gives an advantage to I over C and tends to generate a preference for incompatibility. On the other hand, inter-groups networks effects hurt the incumbent. Given a set of clients $\mathcal{K}$, they force I to reduce its prices to prevent divide and conquer strategies by C. Moreover, the incumbent may have to sell to some groups of consumers despite the fact that $\delta_j < 0$, because there is an opportunity cost of letting them buying from the competitor. In other words the traditional incentive to reach a large population when there are network externalities is preserved, although firms are not able to appropriate the efficiency gains associated with these externalities. Choosing to be compatible is then one way to escape from this logic and to keep only profitable customers.

Proposition 3 *Assume that $\beta_{jj} = 0$ for all j (perfect price discrimination) and $1 > \theta > \bar{\theta}$. Then the profit that a firm obtains in any pure strategy equilibrium is smaller than its equilibrium profit under full compatibility ($\theta = 1$).*

The inequality becomes strict as soon as either $\alpha_{jl} > 0$ for all j, l and

$\alpha_{jl} \neq \alpha_{lj}$ for some j, l, or $\alpha_{jl} \neq \alpha_{lj}$ for all j, l.

Under perfect price-discrimination, both firms prefer to be compatible. Notice that given that the bounds on the profits of I are derived under the most favorable conditions for I, this conclusion holds for any choice of the market allocation of consumers and doesn't depend on assumption 2. When networks are incompatible, the ability to exploit inter-group network effects, with divide-and-conquer strategies, strengthens competition and is extremely harmful for profits. The incumbent has to bear the cost of this exacerbation of competition, and prefers to be compatible.

With intra-group network effects, results become ambiguous. Denote by $\bar{\Omega}_{\mathcal{K}}$ and $\bar{B}_{\mathcal{K}}$ the respective values of $\Omega_{\mathcal{K}}$ and $B_{\mathcal{K}}$ evaluated for $\theta = 0$ (thus for $\beta_{jl} = \alpha_{jl}$). If I sells to a subset $\mathcal{K}$ of groups in equilibrium, its maximal profit is smaller than $\sum_{j \in \mathcal{K}} (v_j^I - v_j^C) m_j + (1-\theta) \left(\sum_{j \in \mathcal{K}} \beta_{jj} m_j^2 - \bar{\Omega}_{\mathcal{K}} - \bar{B}_{\mathcal{K}} \right)$. The term $\bar{\Omega}_{\mathcal{K}}$ is a measure the asymmetry while $\bar{B}_{\mathcal{K}}$ is a measure of the level of inter-group network effects. Thus I prefers to be compatible if there are strong and asymmetric inter-group network effects, and relatively small inter-group network effects.

Installed bases and lock in of some consumers may also change the conclusion. Suppose for instance that each network as an installed base of consumers I^I and I^C that cann't move. Then the utility writes $U_j^k = v_j^k + \theta(I^I + I^C + \sum_{l=1}^{J} \alpha_{jl} m_l) + (1-\theta) I^k + (1-\theta) \sum_{l=1}^{J} \alpha_{jl} n_l^k - p_j^k$. The bound on I's profit under prefect price discrimination is thus $\sum_{j \in \mathcal{K}} (v_j^I - v_j^C) m_j + (1-\theta) \left(I^I - I^C - \bar{\Omega}_{\mathcal{K}} - \bar{B}_{\mathcal{K}} \right)$ and it decreases with the level of compatibility if the diffrence between installed bases $I^I - I^C$ is large enough.

5 Homogeneous population

Consider a population of identical individuals of total size M, divided into J subgroups of size m_j. Suppose also that individuals care only about the mass of consumers buying the same good as them. For this situation, parameters are

H: $\beta_{jj} = \beta_{jl} = \beta,\ u_j^I = u^I,\ u_j^C = u^C.$

When prices are uniform and assumption 2 holds, I covers the market with a maximal price $p^I = u^I - u^C + \beta M\mathrm{I}$ if $u^C - u^I \leq \beta M$. Otherwise C covers the market. Consider now price-discrimination.

We have $\delta_j = u^I - u^C + \beta m_j$ and $\Omega_{\mathcal{K}} = 0$ for all $\mathcal{K}$. It is rather intuitive (and proved in appendix) that in equilibrium one of the two firms covers the market. Given that, the maximal profit that can be obtained by firm C is $(u^C - u^I)M - \beta M^2$, where βM^2 is the total value of network externalities. On the other hand the total value of the intra-group network externalities is $\sum_{j=1}^{J} \beta m_j^2$ with a maximal profit for I equal to $(u^I - u^C)M + \beta \sum_j m_j^2$.

Proposition 4 *Assume H, and define* $h = \sum_j \left(\frac{m_j}{M}\right)^2$, *then:*
If $\frac{u^C - u^I}{\beta M} \leq h$, *I covers the market with profit* $\Pi^I \leq (u^I - u^C + \beta h M)M$;
If $h < \frac{u^C - u^I}{\beta M} < 1$, *there is no pure strategy equilibrium;*
If $1 \leq \frac{u^C - u^I}{\beta M}$, *firm C covers the market with profit* $\Pi^C \leq (u^C - u^I - \beta M)M$.

Proof. See appendix. ■

h is the ratio of the total value of intra-group network effects over the total value of network effects, can be interpreted as a measure of the concentration of the groups.When the challenger has a higher quality than I but not too high, there is no equilibrium. The reason is that I can exploit its reputation advantage while C can exploit the potential of discrimination. The two effects act in opposite directions and prevent any stability of the competitive process. This points to the fact that price discrimination may be the source of instability in network industries.

As the number of groups increases, or more generally as the concentration decreases, it becomes less and less likely that I captures the market when $u^C > u^I$. For a given total population, h is maximal when the subpopulations are of equal size. On the other hand, the ability to separate a small group from the rest of the population has a low impact on $I's$ profit. The reason is that it is extremely costly for the challenger to persuade this small group to leave the large group. And the gain from doing so is small as the propensity of members of the large group to join the small group is proportional to the size of this group.

At the limit when the number of groups increases with a fixed population, the model converges to the situation of perfect price-discrimination.[9]

Corollary 2 *Assume H. Under perfect price-discrimination, firm I captures the market with probability* 1 *if and only if it offers a higher quality than firm C $(u^I > u^C)$.*

With perfect price discrimination, the weakest firm is able to overcome the coordination problem and to pass the full value of the surplus to its customers, which limits potential inefficiencies. One conjecture is that if assumption 2 (domination in beliefs) is not imposed, any pure strategy equilibrium is efficient so that the highest quality network emerges.

[9] Alternatively, perfect price-discrimination is obtained by setting $\beta_{jj} = 0$ instead of β and $m_j = 1$.

6 Heterogeneous population

Let us now turn to the equilibrium analysis in the case of an heterogeneous population. For this assume that the population is composed of two groups. For the sake of presentation, the groups are ranked:

Assumption 3: $J = 2,\ \beta_{12} < \beta_{21}$.

Thus group 1 is the group that values less the externality. Assumption 3 is maintained throughout this section. We first examine the equilibria where one firm covers the market.

If firm k covers the market, the maximal profit $\bar{\Pi}^k$it can obtain is:

$$\begin{aligned}\bar{\Pi}^I &= \delta_1 m_2 + \delta_2 m_2 - (\beta_{21} - \beta_{12}) m_1 m_2;\\ \bar{\Pi}^C &= -\delta_1 m_2 - \delta_2 m_2 - (\beta_{21} + \beta_{12}) m_1 m_2.\end{aligned}$$

Consider an equilibrium where I covers the market. From the analysis of $C's$ best response it must verify:

$$p_j^I \leq \delta_j + \beta_{jl} m_l, \tag{6}$$

$$0 \leq p_1^I m_1 + p_2^I m_2 \leq \bar{\Pi}^I; \tag{7}$$

along with the condition for the market allocation:

$$p_j^C \geq p_j^I - \delta_j - \beta_{jl} m_l. \tag{8}$$

The question is now whether one can set prices for C in such a way that I doesn't deviate. This requires to bind condition 8 for both groups. When the two prices of I are nonnegative, this is sufficient to obtain an equilibrium. However, with heterogeneity, p_j^I is negative if $\delta_j + \beta_{jl} m_l < 0$. I will then be tempted to serve only the other group l. One way to look at this issue is to analyze the opportunity cost of selling to group j. This costs (at least) the subsidy $-\delta_j - \beta_{jl} m_l$ given to group j. But since group j buys from I instead of C, the valuation of $I's$ good by the other group increases by an amount equal to the value of the inter-group network effect $\beta_{lj} m_j$, while the valuation of $C's$ good decreases by the same amount. This allows I to increase the price on group l by twice this amount. The net opportunity cost for I to serves group j is thus $-\left(\delta_j + \beta_{jl} m_l\right) m_j - 2\beta_{lj} m_j m_l$. Per member of group j we obtain an opportunity cost:

$$\Delta_j^I = -\delta_j - \beta_{jl} m_l - 2\beta_{lj} m_l.$$

I prefers to cover the market rather than to sell to l only when this opportunity cost is negative.

Let us now turn to the case where C covers the market. It must be the case that

$$p_j^C \leq -\delta_j + \beta_{jl} m_l \tag{9}$$

$$0 \leq p_1^C m_1 + p_2^C m_2 \leq \bar{\Pi}^C \tag{10}$$

Things are a bit different because now prices must be such that buying from C is a dominant strategy for the members of at least one group. It is shown in the proof that it must be group 1 because it is the easiest to attract. Given that, selling or not to group 2 doesn't affect the decision of group 1. The opportunity cost for C of selling to group 2 is simply (per member)

$$\Delta_2^C = \delta_2 - \beta_{21} m_1.$$

On the other hand, prices are such that group 2 buys only if group 1 does. The analysis of the opportunity cost of selling to group 1 is thus the same as the analysis for I when it covers the market. It includes the direct opportunity cost $(\delta_1 - \beta_{12} m_2)\, m_1$ plus the indirect cost $-2\beta_{21} m_1 m_2$, hence an opportunity cost per member:

$$\Delta_1^C = \delta_1 - \beta_{12}m_2 - 2\beta_{21}m_2.$$

The next proposition shows that an equilibrium with one firm covering the market exists provided that the profit is nonnegative and that the opportunity cost of selling to any of the two groups is negative:

Proposition 5 *There exists an equilibrium where firm k covers the market (with profit $\Pi^k \in [0, \bar{\Pi}^k]$) if and only if $\bar{\Pi}^k \geq 0$, $\Delta_1^k \leq 0$ and $\Delta_2^k \leq 0$.*

The analysis of market sharing situations follows the same lines. Suppose that firm I serves group j and firm C serves group l. Then it must be profitable for I not to sell to group l. The relevant criteria is again the opportunity cost for I of selling to group l when it sells to group j. Indeed I could raise its profit by $p_l^C m_l - \Delta_l^I m_l$ covering the market. This means that the maximal profit of C is bounded above by $\Delta_l^I m_l$.

For firm C there are differences because the strategy analyzed before to derive the opportunity cost for C of selling to group j may not be the exact opposite of the strategy that C would use to conquer the market in a deviation from a market sharing equilibrium. When C is selling to group 1 only, this doesn't really make a difference, because the choice always involves choosing between conquering group 2 with a price $p_2^I - \Delta_2^I$, or not.

But when C sells to group 2, we have to take this fact into account. Here, C would conquer the market by setting a very low price $p_1^I - \delta_1 - \beta_{12}m_1$ for group 1 and raising the price for group 2 by $2\beta_{21}m_1$. Because $\beta_{21} > \beta_{12}$, this is more profitable than just proposing a price $p_1^C = p_1^I - \delta_1 + \beta_{12}m_1$. Hence a bound on the price of I is: $p_1^I \leq \delta_1 + \beta_{12}m_1 - 2\beta_{21}m_1$.

We then obtain:

Proposition 6 *There exists an equilibrium where firm I sells to group j with price p_j^I and firm C sells to group l with price p_l^C if and only:*
$\Delta_1^C + 2\beta_{12}m_2 \geq p_1^I \geq 0$ *and* $\Delta_2^I \geq p_2^C \geq 0,\ if\ j = 1$ *and* $l = 2$;
$\Delta_1^I \geq p_1^C \geq 0$ *and* $\Delta_2^C \geq p_2^I \geq 0,\ if\ j = 2$ *and* $l = 1$.

Proof. See appendix. ■

The global equilibrium configuration is always the same and is depicted on Figure 1.

INSERT FIGURE 1

In the middle range, a pure strategy equilibrium fails to exist, and in particular no firm can cover the market with probability one. This occurs when the firms offer goods of similar characteristics and intra-group network effects are small. The situation is then highly unstable and it is not possible to predict which firm will sell.

We can also see that market sharing equilibria and market covering equilibria hardly coexist. Indeed the equilibrium is unique except in one case where there are two equilibria: one in which C covers the market, and one in which I sells only to the low externality group 1.

6.1 Welfare: excess inertia or excess momentum?

Let us compare the equilibrium configuration with the efficient allocation. Given that there is enough flexibility in designing prices to transfer the surplus from one group to another, the welfare criterion used here is total surplus. Total welfare is maximal with market sharing whenever $(\beta_{12} + \beta_{21})m_1m_2$ is smaller $\min\{\delta_1, -\delta_2\}$ or $\min\{-\delta_1, \delta_2\}$, which corresponds to a strong pattern of horizontal differentiation. Otherwise firm I should cover the market if $\delta_1 + \delta_2 > 0$, while firm 2 should cover the market if $\delta_1 + \delta_2 < 0$. Dotted lines in Figures 1 show the delimitations of the various range of quality differentials (for $\beta_{jj} = 0$).

In what follows, *excess inertia* refers to the cases where it would be socially optimal to reduce the market share of I. Our domination assumption favoring the incumbent, it tends to generate some inertia. In our model, the extent of excess inertia increases with the size of intra-group network effects. This is a standard result, so let us now focus on the case where there is no intra-group network effects: $\beta_{11} = \beta_{22} = 0$. Figure 1 clearly shows that there can be excess inertia which can take the form of inefficient market covering by I or inefficient market sharing, but also occurs when a pure strategy equilibrium fails to exists implying that C can't cover the market with probability one while it should.

But there is also the possibility of *excess momentum*, which occurs when it would be optimal to increase the market share of firm I and corresponds to the grey areas in Figure 1. It can take two forms. First, I may fail to cover the market with probability 1 while it should. Second, C may cover the market in equilibrium while it would be more efficient to share the market.

In a context of a sequential pricing game where I is a Stackelberg leader, excess momentum emerges for a much wider range of parameter values (see Jullien 2000). In particular when I is a leader and there is enough horizontal differentiation, it may choose to accommodate by inducing an allocation with market sharing, in situations where total welfare maximization would require that both groups buy from I.

7 Discussion and extensions

7.1 Strategic degradation of quality for targeted customers

One of the general principle that emerges is that head-to-head competition to conquer the whole market is extremely costly when price-discrimination is possible. One way to escape from such situation of intense competition is to achieve enough horizontal differentiation. In the present model, this means shifting from a market covering equilibrium to a more peaceful market sharing situation. When a firm controls the quality of the good at the individual level, it can reach this objective by degrading quality for some customers.[10]

To illustrate this phenomenon, suppose that I covers the market in equilibrium with maximal profit $\bar{\Pi}^I = (\delta_1 + \beta_{12}m_2)m_1 + (\delta_2 - \beta_{21}m_1)\, m_2 > 0$. Suppose in addition that $\delta_1 + \beta_{12}m_2 < 0$ (which is compatible with equilibrium conditions). Consider what happens if I succeeds in degrading quality for group 1 (holding u_2^I constant) up to a point where the new quality differential $\hat{\delta}_1 < -(\beta_{12} + 2\beta_{21})m_2$. Then the new equilibrium involves market sharing: I sells to group 2 only with profit $(\delta_2 - \beta_{21}m_1)\, m_2 > \bar{\Pi}^I$. It is thus profitable to do so. The point here is that I would like to commit not to compete on group 1, because the lack of commitment creates situations where I is 'forced' to include group 1 in its network despite a competitive hedge in favor of C on this group. A targeted degradation of quality is one way to achieve such a commitment. The same phenomenon may hold for C as well.

More generally when a network can choose the technology and affect perceived qualities, and when it can't gain a large quality advantage on both

[10] Both the motive for quality reduction and the way it is achieved differ substantially from models of damaged goods and screening as Denekere and McAfee (199) and Hahn (2000).

groups, it will have incentives to shift its technological choices toward the preferred technology of one group and the least preferred technology for the other group. This may result in inefficiencies in technological choices and even in the choice of a dominated technology.

7.2 Large network effects.

In the paper, network effects are assumed to be small compared to intrinsic values, or at least networks are assumed to be partially compatible so that the network effects have a relatively small impact (Assumption 1). For pure network goods that are totally incompatible, the only source of value is the presence of other users. Then assumption 1 will not hold. The situation were $u_j^C < \sum_{l \neq j} \beta_{jl} m_l$ is dealt with in Caillaud-Jullien (2000a and 2000b), along with other issues that emerge in the context of intermediation. Nothing is changed when the market is shared, but the analysis of market covering by I differs. The key point is that I may set a price $p_j^I > u_j^I + \beta_{jj} m_j$ and still cover the market, which never occurs under assumption 1. By setting $p_j^I > u_j^I + \beta_{jj} m_j$, I can generate an extra profit $p_j^I - u_j^I - \beta_{jj} m_j$ on group j that the challenger will be unable to compete away with a divide and conquer strategy. The reason is that if C convinces group l to join with a low price, group j stops to buy from I. The largest price that C can set for group j is then $u_j^C + \beta_{jl} m_l$, independent of p^I and smaller than $p_j^I - \delta_j + \beta_{jl} m_l$. The challenger's ability to compete is thus reduced. This effect allows I to extract part of the value of the inter-group externality without increasing the profit that C can generate by stealing $I's$ clients (the value vanishes as soon as group l joins C). The incumbent's profit may then exceed $\sum_j \delta_j m_j$. Moreover the equilibria exhibit a much stronger pattern of cross-subsidization.

7.3 Equilibrium and domination

One of the problem encountered is that an equilibrium may fail to exist. The question is then whether this is due to the assumption of domination in beliefs (assumption 2) that gives too much weight to firm I. In the case of an homogenous population for instance, a pure strategy equilibrium always exists when assumption 2 is dropped. But although this helps to extend the range of existence (at the cost of multiplicity), the inexistence problem remains if firms are not differentiated and there is some asymmetry in network externalities. In particular, with two groups, a pure strategy equilibrium doesn't exist if $\sum_j \left(\left| u_j^I - u_j^C \right| m_j + \beta_{jj} m_j^2 \right) < \min\{\beta_{12}, \beta_{21} - \beta_{12}\} m_1 m_2$. The high instability of situations of competition between similar networks thus appears to be a robust conclusion.

8 Conclusion

It is commonly admitted that network externalities are sources of market failures that favor incumbents and generate barriers to entry. This paper points to the fact that when competitive strategies exploit the potential of price-discrimination, the nature of competition is more complex. Networks are vulnerable to strategies that build a customer base with a subsidy and exploit the bandwagon effect. While this issue has been addressed in a dynamic context (see for instance Katz and Shapiro (1986,1992), or Fudenberg and Tirole (2000)), it also occurs with price-discrimination in a static framework. In particular, when the scope for price discrimination is high, the competitive pressure is stronger in the presence of network externalities than without. This can be seen as a source of instability for network industries. One striking feature is that some conclusions obtained under uniform prices are reversed under price-discrimination, such as compatibility choices or inertia.

Obviously the model of Bertrand competition is extreme and its conclusions need to tempered for practical purposes. The model suggests that it may be easier than expected for a superior technology to enter, provided that the quality improvement is large enough. On the other hand, for small quality improvements, the fact that competition is intense may act as a stronger barrier to entry than reputation effects. While a firm may expect to succeed in building its reputation and conquering the market in a reasonable delay under uniform pricing, thus recovering its sunk costs, the intensity of ex-post competition with price-discrimination may reduce the profitability of entry.

One caveat is that the equilibrium concept used in the paper may be too demanding on the part of consumers. They are assumed to perfectly anticipate the choices of other consumers when choosing where to buy. This requires that they observe all the prices and make correct inference upon them, both assumptions being strong. While a static model provides key insights, a more dynamic perspective for the joint evolution of prices and demand is certainly called for.

The analysis doesn't address risk and financial issues. Viewed in a dynamic context, divide-and-conquer strategies require high financial possibilities as they imply negative cash-flows for some period of time. Firms may have difficulties in finding the required external funding for such a strategy because the analysis also suggests that the prospects of winning the market are uncertain. Even in our static context, the strategies may be very risky if demand characteristics are imperfectly known, because a firm faces the risk of attracting only unprofitable users.

While the paper focuses on third-degree discrimination, it would be worth investigating the implications of second-degree price discrimination for competition. This includes also the choices of bundles of services and of technologies in multi-attribute networks. Indeed most networks offer various services that may be bundled in complex ways. The present model can be interpreted as one with multiple services provided that each individual buys only one service. It would be worth extending the analysis to situations where consumers can choose various combinations of services.

It is also worth investigating further the implications of competition for technological choices and compatibility decisions. Basically, networks face two possibilities. They can try to differentiate their services under compatible technologies, or they can try to conquer the market by providing high quality with incompatible technology. The paper shows that this issue is complex because the intensity of competition increases when network are incompatible.

As mentioned in the paper, the analysis of pure network goods (no intrinsic value) that are totally incompatible doesn't fit into the paper's framework. The analysis in Caillaud and Jullien (2000a and b), focusing on matching services, shows that the competitive pressure is weaker in this case. It also shows that multi-homing possibilities (the possibility to join several networks at a time) modify the nature of the pricing strategies, and that allowing for complex pricing schemes that succeed in linking the price to the size of the network increases the competitiveness of the industry.

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A Appendix to Section 2

Proof of lemma 1. Consider two continuation equilibria $i = 1, 2$, and the respective $\mathcal{K}_i^I$ and $\mathcal{K}_i^C$. Then

$$\begin{aligned} j &\in \mathcal{K}_i^I \Rightarrow u_j^I + \sum_{l\in\mathcal{K}_i^I} \beta_{jl} m_l - p_j^I \geq \max\{u_j^C + \sum_{l\in\mathcal{K}_i^C} \beta_{jl} m_l - p_j^C, 0\} \\ j &\in \mathcal{K}_i^C \Rightarrow u_j^C + \sum_{l\in\mathcal{K}_i^C} \beta_{jl} m_l - p_j^C \geq \max\{u_j^I + \sum_{l\in\mathcal{K}_i^I} \beta_{jl} m_l - p_j^I, 0\} \\ j &\in \mathcal{J}\backslash\mathcal{K}_i^C \cup \mathcal{K}_i^I \Rightarrow 0 \geq \max\{u_j^I + \sum_{l\in\mathcal{K}_i^I} \beta_{jl} m_l - p_j^I, u_j^C + \sum_{l\in\mathcal{K}_i^C} \beta_{jl} m_l - p_j^C\} \end{aligned}$$

Now suppose that all groups in $\mathcal{J}\backslash\mathcal{K}_1^C \cap \mathcal{K}_2^C$ don't buy from C, and that all groups in $\mathcal{K}_1^I \cup \mathcal{K}_2^I$ buy from I. Since for the firms in $\mathcal{J}\backslash\mathcal{K}_1^C \cap \mathcal{K}_2^C$

$$\max\{u_j^I + \sum_{l\in\mathcal{K}_1^I\cup\mathcal{K}_2^I} \beta_{jl}m_l - p_j^I, 0\} \geq u_j^C + \sum_{l\in\mathcal{K}_1^C\cap\mathcal{K}_2^C} \beta_{jl}m_l - p_j^C,$$

the minimal benefit that a member of a group in $\mathcal{J}\backslash\mathcal{K}_1^C\cap\mathcal{K}_2^C$ can obtain when not buying from C is larger than the maximal benefit it can gain when buying from C. Therefore the optimal strategy is either to buy from I or not to buy at all. Moreover for a consumer in $\mathcal{K}_1^I\cup\mathcal{K}_2^I$, $u_j^I+\sum_{l\in\mathcal{K}_1^I\cup\mathcal{K}_2^I}\beta_{jl}m_l-p_j^I \geq 0$. Thus the optimal strategy is indeed to buy from I.There is thus an equilibrium with $\mathcal{K}_1^I\cup\mathcal{K}_2^I \subset \mathcal{K}^I$ *and* $\mathcal{K}^C \subset \mathcal{K}_1^C\cap\mathcal{K}_2^C$. Taking a maximal element completes the proof. ■

B Appendix to Section 3

Lemma 4 *Given prices P^I such that $p_j^I \leq u_j^I + \beta_{jj}m_j$, the maximal profit that C can obtain is smaller or equal to Π if f:*

$$\forall\mathcal{K} \subset \mathcal{J},\ \sum_{j\in\mathcal{K}} p_j^I m_j \leq \sum_{j\in\mathcal{K}} \delta_j m_j - \Omega_{\mathcal{K}} + B_{\mathcal{K}} + \Pi. \tag{11}$$

Proof. It is true if $J = 1$. Suppose the proposition holds up to $\#\mathcal{J} = J - 1$ and consider $\#\mathcal{J} = J$. Suppose first that C decides to sell to only $J - 1$ groups, leaving outside one group (say $j = J$) by setting p_{2J} high. Given that group J stay with I, the problem of attracting $K < J$ groups within J groups is the same as with only $J - 1$ groups but with a utility for group j equal to $u_j^I + \beta_{jJ}m_J + \sum_{l=1}^{J-1}\beta_{jl}n_l^I - p_j^I$ when buying from I. Condition 11 for $K < J$ then ensures that attracting less than J group can't yield more than Π.

Consider now attracting all the groups. For at least one of the group the price p_j^C must be smaller than $p_j^I - (\delta_1 + \sum_{l\neq j}\beta_{jl}m_l)$. Let us say it is group 1. We then set $p_1^C = p_1^I - \left(\delta_1 + \sum_{l>1}\beta_{1l}m_l\right)$ and group 1 buys from C for sure. Here the problem is reduced to the $J - 1$ groups $j \geq 2$, but now with utility $u_j^I + \sum_{l=2}^J \beta_{jl}n_l^I - p_j^I$ at firm I, and utility $u_j^C + \sum_{l=2}^J \beta_{jl}n_l^C + \beta_{j1}m_1 - p_j^C$ at C. The total profit on the $J-1$ groups is less than $\Pi - (p_1^I - (\delta_1 + \sum\beta_{1l}m_l))m_1$, which is precisely condition 11 for $\sigma(1) = 1$. Therefore C can't get more than Π in the worst case faced to the pricing strategy of firm 1. ■

Proof of proposition 1. From lemma 4, the incumbent serves all the market if and only if

$$for\ all\ \mathcal{K} : \sum_{j\in\mathcal{K}} p_j^I m_j \leq \sum_{j\in\mathcal{K}} \delta_j m_j - \Omega_{\mathcal{K}} + B_{\mathcal{K}}. \tag{12}$$

W.l.o.g. let us assume that $\Omega_{\mathcal{J}}$ is obtained for the order $1, 2, ...J$. Suppose that I sets prices

$$p_j^I = \delta_j - \sum_{l<j} \beta_{jl} m_l + \sum_{l>j} \beta_{jl} m_{l.} \tag{13}$$

Then $\sum_{j\in\mathcal{J}} p_j^I m_j = \sum_{j\in\mathcal{J}} \delta_j m_j - \Omega_{\mathcal{J}}$.

Fix a subset $\mathcal{K}$ and let σ be such that $\Omega_{\mathcal{K}} = \sum_{j\in\mathcal{K}} \sum_{\substack{l\in\mathcal{K} \\ \sigma(l)>\sigma(j)}} (\beta_{lj} - \beta_{jl}) m_l m_j$. Suppose that $\sum_{j\in\mathcal{K}} p_j^I m_j > \sum_{j\in\mathcal{K}} \delta_j m_j - \Omega_{\mathcal{K}} + B_{\mathcal{K}}$ and let us show that it leads to contradiction. The condition writes

$$\sum_{j\in\mathcal{K}} \left(\sum_{l<j} \beta_{jl} m_l - \sum_{l>j} \beta_{jl} m_l \right) m_j < \sum_{j\in\mathcal{K}} \sum_{\substack{l\in\mathcal{K} \\ \sigma(l)>\sigma(j)}} (\beta_{lj} - \beta_{jl}) m_l m_j - \sum_{j\in\mathcal{K}} \sum_{l\notin\mathcal{K}} \beta_{jl} m_l m_j$$

Define $\hat{\sigma}$ as the order $\mathcal{K}$:

$$\begin{aligned} \hat{\sigma}(j) &< \hat{\sigma}(l)\ if\ j \in \mathcal{K}\ and\ l \notin \mathcal{K}, \\ \hat{\sigma}(j) &< \hat{\sigma}(l) \Longleftrightarrow \sigma(j) < \sigma(l)\ if\ j \in \mathcal{K}\ and\ l \in \mathcal{K}, \\ \hat{\sigma}(j) &< \hat{\sigma}(l) \Longleftrightarrow j < l\ if\ j \notin \mathcal{K}\ and\ l \notin \mathcal{K}. \end{aligned}$$

Then tedious computation shows that

$$\sum_{j\in\mathcal{J}} \sum_{l>j} (\beta_{lj} - \beta_{jl}) m_l m_j < \sum_{j\in\mathcal{J}} \sum_{\hat{\sigma}(l)>\hat{\sigma}(j)} \left(\beta_{lj} - \beta_{jl}\right) m_l m_j - 2 \sum_{j\in\mathcal{K}} \sum_{\substack{l\notin\mathcal{K} \\ l<j}} \beta_{lj} m_j m_l$$

But this implies that $\Omega_{\mathcal{J}} < \sum_{j\in\mathcal{J}} \sum_{\hat{\sigma}(l)>\hat{\sigma}(j)} \left(\beta_{lj} - \beta_{jl}\right) m_l m_j$,a contradiction. Therefore, it must be the case that (12) holds. ■

Proof of corollary 1. Let us fix the prices of I at their equilibrium values. Suppose that C set prices for groups $j \notin \mathcal{K}$ at their equilibrium values minus an arbitrarily small amount, and attempt to design prices for groups $j \in \mathcal{K}$ so as to attract some of them. In doing so C can't loose its equilibrium customers (groups $j \notin \mathcal{K}$) because our criterion for the allocation of consumers ensures that bringing new customers to C (which reduces the attractiveness of I) can't induce C' customers to change their behavior and buy from I (otherwise they would have done so in the equilibrium configuration). We can then analyze $C's$ strategy as above but taking into account the fact that groups outside $\mathcal{K}$ buy from C. The resulting reduced game is thus the same game as above played on the groups $j \in \mathcal{K}$ only, but where the intrinsic value of the good proposed by the challenger u_j^C is replaced by $u_j^C + \sum_{l \notin \mathcal{K}} \beta_{jl} m_l$. An upper bound on the profit of that I can derive by selling to groups within $\mathcal{K}$ is thus the maximal profit of I when its covers a market consisting of only groups in $\mathcal{K}$, with the new "stand-alone" value differential defined as $\delta_j - \sum_{l \notin \mathcal{K}} \beta_{jl} m_l$ instead of δ_j. The result then follows. ∎

Proof of lemma 3. Denote $\Omega_{\mathcal{J}}^{\sigma} = \sum_{j \in \mathcal{J}} \left(\sum_{\sigma(l) > \sigma(j)} (\beta_{lj} - \beta_{jl}) m_l m_j \right)$. For any order σ on $\mathcal{J}$ there is an exact reverse ordering $\hat{\sigma}$ and it verifies $\Omega_{\mathcal{J}}^{\sigma} = -\Omega_{\mathcal{J}}^{\hat{\sigma}}$. Hence the maximum over all permutations is non-negative. The maximum is zero iff $\Omega_{\mathcal{J}}^{\sigma} = 0$ for all σ. Suppose this is the case. Fix j, l. Consider a permutation σ such that $\sigma(j) = J - 1$ and $\sigma(l) = J$, and $\bar{\sigma}$ that coincides with σ except that $\bar{\sigma}(j) = J$ and $\bar{\sigma}(l) = J - 1$. Then $0 = \Omega_{\mathcal{J}}^{\sigma} = \Omega_{\mathcal{J}}^{\bar{\sigma}} + 2(\beta_{lj} - \beta_{jl}) m_j m_l$ which implies that $\beta_{jl} = \beta_{lj}$. ∎

Proof of proposition 2. In equilibrium I sets prices for $j \in \mathcal{K}$ at

$$p_j^I = u_j^I - \max\{u_j^C + \sum_{l \in \mathcal{L}} \beta_{jl} m_l - p_j^C, 0\} + \sum_{l \in \mathcal{K}} \beta_{jl} m_l$$

which is the maximal price at which a member of group j is willing to buy from I when groups $l \in \mathcal{K}$ do. If I decides to attract groups in $\mathcal{H} \subset \mathcal{L}$, it can do so by setting prices for all groups in $\mathcal{K} \cup \mathcal{H}$, including those groups it already serves:

$$\hat{p}_j^I = u_j^I - \max\{u_j^C + \sum_{l \in \mathcal{L} \setminus \mathcal{H}} \beta_{jl} m_l - p_j^C, 0\} + \sum_{l \in \mathcal{K} \cup \mathcal{H}} \beta_{jl} m_l$$

The gain in profit is then $\sum_{j \in \mathcal{H}} \hat{p}_j^I m_j + \sum_{j \in \mathcal{K}} (\hat{p}_j^I - p_j^I) m_j$. Notice first that for $j \in \mathcal{K}$, the price differential $\hat{p}_j^I - p_j^I$ is larger than $\sum_{l \in \mathcal{H}} \beta_{jl} m_l m_j$, with equality when $p_j^C \geq u_j^C + \sum_{l \in \mathcal{L}} \beta_{jl} m_l$.. The gain in $I's$ profit is thus minimal when C sets very high prices for these groups, with a lower bound $\sum_{j \in \mathcal{H}} \hat{p}_j^I m_j + \sum_{j \in \mathcal{K}} \sum_{l \in \mathcal{H}} \beta_{jl} m_l m_j$.

If C sells to all groups in $\mathcal{L}$, it sets prices such that for all subset $\mathcal{H}$:

$$\sum_{j\in\mathcal{H}} p_j^C m_j \leq \sum_{j\in\mathcal{H}} (u_j^C - u_j^I) m_j + \sum_{j\in\mathcal{H}} \sum_{l\in\mathcal{L}\setminus\mathcal{H}} \beta_{jl} m_l m_j - \sum_{j\in\mathcal{H}} \sum_{l\in\mathcal{K}\cup\mathcal{H}} \beta_{jl} m_l m_j - \sum_{j\in\mathcal{K}} \sum_{l\in\mathcal{H}} \beta_{jl} m_l m_j$$

An upper bound on the profit is then obtained for $\mathcal{H} = \mathcal{L}$:

$$\sum_{j\in\mathcal{L}} p_j^C m_j \leq \sum_{j\in\mathcal{L}} (u_j^C - u_j^I) m_j - \sum_{j\in\mathcal{L}} \sum_{l=1}^{J} \beta_{jl} m_l m_j - B_{\mathcal{K}}.$$

If it is a Stackelberg leader, C can obtain this profit by setting a price $p_j^C = u_j^C - u_j^I - \left(\sum_{l=1}^J \beta_{jl} m_l - \sum_{l\in\mathcal{K}} \beta_{lj} m_l\right)$ for all groups within $\mathcal{L}$. ∎

C Appendix to Section 4

Proof of proposition 3. Denote $\Pi_{\mathcal{K}}^k$ the bound derived in section 3 on firm k profit when I sells to $\mathcal{K}$. Section 3 shows that if there is an equilibrium where I sells to $\mathcal{K}$ with profit Π^k, then there exists a strategy for its opponents that allows it to cover the market with an extra profit larger then $\Pi^k - \Pi_{\mathcal{K}}^k$. Thus $\Pi^k \leq \max_{\mathcal{K}} \left\{\Pi_{\mathcal{K}}^k\right\}$. But this implies that $\Pi^I < \max_{\mathcal{K}} \left\{\sum_{j\in\mathcal{K}} (v_j^I - v_j^C) m_j\right\} \leq \sum_{j\in\mathcal{J}} \max(v_j^I - v_j^C, 0) m_j$, while $\Pi^C < \sum_{j\in\mathcal{J}} \max(v_j^C - v_j^I, 0) m_j$. ∎

D Appendix to Section 5

Proof of proposition 4. First we show that one firm cover the market in equilibrium. Suppose that this is not the case so that I sells to groups in $\mathcal{K}$ only. Then it must be the case that

$$\begin{aligned} 0 &\leq \Pi^I \leq (u^I - u^C) \sum_{j\in\mathcal{K}} m_j + \beta \sum_{j\in\mathcal{K}} m^2 - \beta \left(\sum_{j\in\mathcal{K}} m_j\right) \left(\sum_{j\notin\mathcal{K}} m_j\right), \\ 0 &\leq \Pi^C \leq -(u^I - u^C) \sum_{j\notin\mathcal{K}} m_j - \beta \sum_{j\notin\mathcal{K}} m^2 - \beta \left(\sum_{j\in\mathcal{K}} m_j\right) \left(\sum_{j\notin\mathcal{K}} m_j\right), \end{aligned}$$

The second inequality implies $u^I - u^C \leq -\beta \sum_{j\in\mathcal{K}} m_j$. Given that we obtain from the first inequality $0 < -\beta \left(\sum_{j\in\mathcal{K}} m_j\right)^2 + \beta \sum_{j\in\mathcal{K}} m_j^2$ which is not possible. It follows that either I or C covers the market.

Suppose that $u^C - u^I \leq \beta hM$. Consider equilibrium prices $0 \leq p_j^I \leq u^I - u^C + \beta m_j$, $p_j^C = p_j^I - \beta M$. C doesn't sell and we know that it is playing a best response. The maximal deviation profit that I can obtain is

$$\sum_{j \in \mathcal{K}} (p_j^I + \beta \sum_{j \in \mathcal{K}} m_j - \beta M) m_j \leq \sum_{j \in \mathcal{K}} p_j^I m_j.$$

So this is an equilibrium as long as I obtains a positive profit.

Suppose that $u^C - u^I \geq \beta M$. Let $\hat{p}_j^I$ be the prices exhibited in the proof of proposition 1 that allows I to obtain $\bar{\Pi}^I$: $\hat{p}_j^I = u^I - u^C + \beta \sum_{l \leq j} m_l - \beta \sum_{l > j} m_l$. Let $p_j^I = \hat{p}_j^I + \frac{\Pi}{M}$, $p_j^C = \frac{\Pi}{M}$, $0 \leq \Pi \leq \bar{\Pi}^C$. At these prices there is a REA where C covers the market (and another one where I covers the market). C can't obtain more than Π because its maximal profit is $\sum p_j^I m_j - \bar{\Pi}^I = \Pi$. I can't obtain a positive profit because its profits is bounded by $\sum p_j^C m_j - \bar{\Pi}^C \leq 0$. Therefore this an equilibrium. It is also clear that these are the unique one as I can't sell at a nonnegative price (C would attract such a group for sure with a zero price).

When $\beta M > u^C - u^I > \beta hM$, no firm can cover the market with positive profit so that no pure strategy equilibrium exists. ■

E Appendix to Section 6

Proof of proposition 5 . First a firm can't cover the market in equilibrium unless $\bar{\Pi}^k \geq 0$.

Suppose that $\bar{\Pi}^I \geq 0$. Set the prices p_j^I verifying conditions 6 and 7. Then $C's$ best response is not to sell at all. The smallest possible prices for C are $p_j^C = p_j^I - \delta_j - \beta_{jl} m_l$: at these prices, each consumer is indifferent between C and I. Moreover, it is impossible for I to sell at a group above p_j^I. Thus this is an equilibrium if I prefers to cover the market than to sell to one group only. To sell to group j alone, I must set a price $\hat{p}_j^I$ such that $u_j^I + \beta_{jj} m_j - \hat{p}_j^I \geq u_j^C + \beta_{jl} m_l - p_j^C$, which amounts to $\hat{p}_j^I = p_j^I - 2\beta_{jl} m_l$. This is not profitable if $-2\beta_{jl} m_1 m_2 \leq p_l^I m_l$. Thus an equilibrium must verify $p_j^I m_j + p_l^I m_l \geq p_j^I m_j - 2\beta_{jl} m_l m_j$ or : $p_l^I \geq -2\beta_{jl} m_j$. Equilibrium prices then exist if $\delta_j + \beta_{jl} m_l \geq -2\beta_{lj} m_l$ for both groups or $\Delta_j^I \leq 0$.

Suppose now that $\bar{\Pi}^C \geq 0$. Choose prices p_j^C verifying conditions 9 and 10. I can't obtain a positive profit because its profits is the maximum between $\sum p_j^C m_j - \bar{\Pi}^C$, $\left(p_1^C + \delta_1 - \beta_{12} m_2\right) m_1$ and $\left(p_2^C + \delta_2 - \beta_{21} m_1\right) m_2$, and they are all nonpositive. C can't obtain more than Π^C by covering the market if

$\sum p_j^I m_j - \bar{\Pi}^I \leq \Pi^C$. The unique prices that are compatible with the fact that C covers the market and obtains at most Π^C are $p_1^I = p_1^C + \delta_1 + \beta_{12}m_2$, $p_2^I = p_2^C + \delta_2 - \beta_{21}m_1$. Notice that at these prices, group 1 buys alone from C. So C would cover the market only if $p_2^C \geq 0$. On the other hand, C could reduce its price by $2\beta_{21}m_1m_2$ and sell only to group 2. So C would cover the market only if $p_1^C m_1 \geq -2\beta_{21}m_1m_2$. We can find such prices whenever $-2\beta_{21}m_2 \leq -\delta_1 + \beta_{12}m_2$ and $0 \leq -\delta_2 + \beta_{21}m_1$, which reduces to $\Delta_1^C \leq 0$ and $\Delta_2^C \leq 0$. ∎

Proof of proposition 6. Assume that I sells to group 1. Each consumers should prefer to stay with its group rather than to join the other group. This yields

$$\begin{aligned} u_1^I + \beta_{11}m_1 - \max\left\{u_1^C + \beta_{12}m_2 - p_1^C, 0\right\} &\geq p_1^I \\ u_2^C - \max\left\{u_2^I + \beta_{22}m_2 + \beta_{21}m_1 - p_2^I, 0\right\} &\geq p_2^C \end{aligned}$$

Clearly to sustain the equilibrium, the best is to put prices p_1^C and p_2^I at their minimal value given profits, which yields:

$$p_1^I = p_1^C + \delta_1 - \beta_{12}m_2 \textit{ and } p_2^C = p_2^I - \delta_2 - \beta_{21}m_1$$

Equilibrium conditions then write for C:

$$\begin{aligned} p_2^C m_2 &\geq p_1^I m_1 - \delta_1 m_1 - \beta_{12}m_1m_2 \\ &\quad + u_2^C m_2 - \max\left\{u_2^I m_2 + \beta_{22}m_2^2 - p_2^I m_2, 0\right\} + \beta_{21}m_1m_2 \\ p_2^C m_2 &\geq p_1^I m_1 + p_2^I m_2 - \delta_1 m_1 - \delta_2 m_2 + \beta_{12}m_1m_2 - \beta_{21}m_1m_2 \end{aligned}$$

For I we then get

$$p_1^I m_1 \geq p_2^C m_2 + p_1^C m_1 + \delta_1 m_1 + \delta_2 m_2 + (\beta_{12} + \beta_{21})\, m_1m_2.$$

Overall this yields equilibrium conditions

$$\begin{aligned} \delta_1 + (\beta_{12} - 2\beta_{21})m_2 &\geq p_1^I \\ \delta_1 - \beta_{12}m_2 &\geq p_1^I \\ -\delta_2 - (2\beta_{12} + \beta_{21})\, m_1 &\geq p_2^C \end{aligned}$$

Given that $\beta_{12} \leq \beta_{21}$, the second constraint is implied by the first so that we obtain:

$$\Delta_1^C + 2\beta_{12}m_2 \geq p_1^C \textit{ and } \Delta_2^I \geq p_2^C.$$

If I sells to group 2 the three conditions are the same, reverting the role of the two groups. Given that $\beta_{12} \leq \beta_{21}$, the relevant constraints are now $\Delta_2^C = \delta_2 - \beta_{21}m_1 \geq p_2^I$ and $\Delta_1^I \geq p_1^C$. ∎

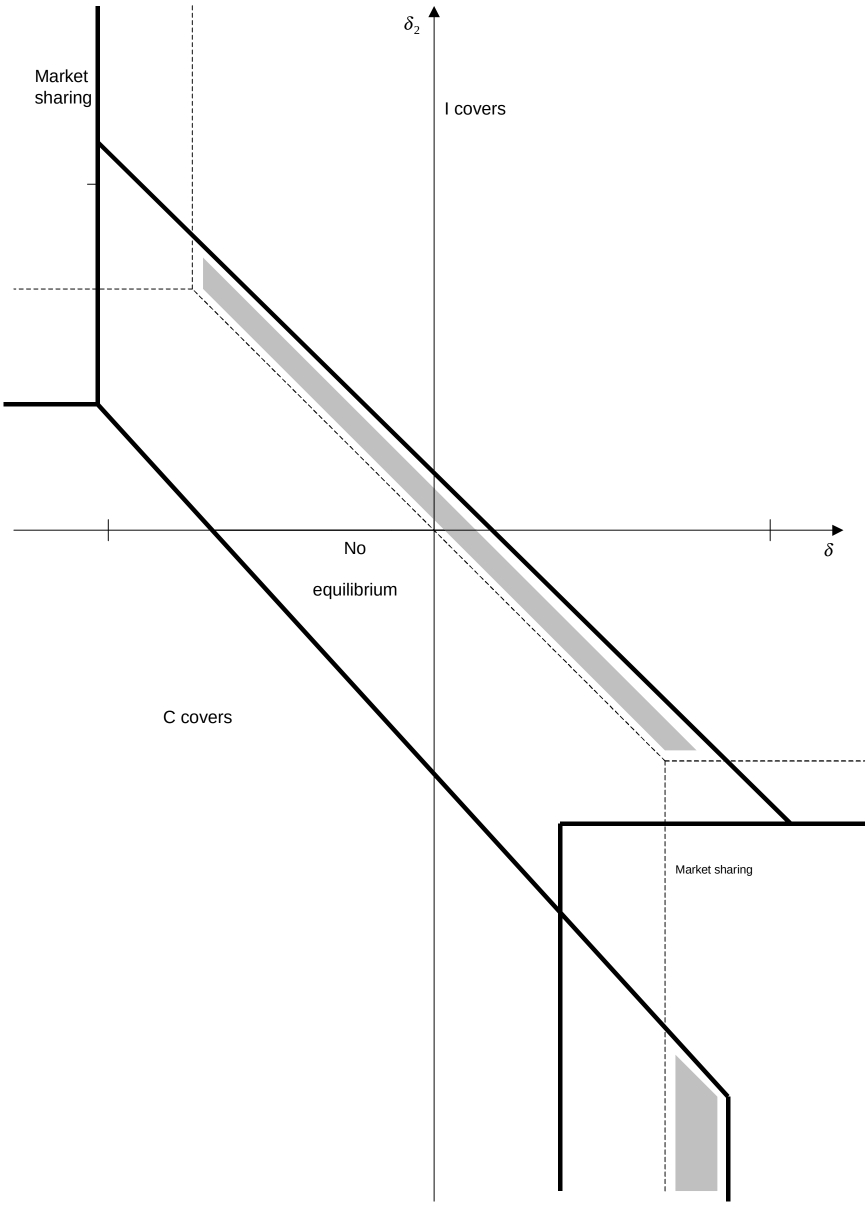

Figure 1: Equilibrium configuration