Chicken & Egg: Competing Matchmakers*

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Abstract

We examine a Bertrand competition game between two intermediaries offering matching services between two sides of a market. Indirect network externalities arise as the probability of finding one's match with a given intermediary increase with the number of agents of the other side who use the services of this intermediary. We formalize some specificities of intermediation on the Internet by allowing registration and transaction prices, and multiple registration. When only registration fees are used and agents register to at most one cybermediary, there exists an equilibrium where one firm corners the market with positive profits, as well as zero profit equilibria where the firms share the market. Introducing either fees that are contingent on successful matching or the possibility of registration with two intermediaries drastically reduces the profits of a dominant firm. Moreover, with multiple registration, new types of positive-profit equilibria emerge where both matchmakers are active and one side of the market registers with both cybermediaries.

1 Introduction

Intermediation activities play a prominent role in intermediate or final markets where there is a lot of differentiation and dispersion of buyers and

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sellers, e.g. in real estate markets, financial markets and banking activities, distribution, ... Intermediaries provide services whose purpose is to help agents with specific needs find other agents with matching characteristics and conclude transactions: these include search, certification, screening, advertising, legal support, loan and credit facilities, etc. For example, real estate agents post ads for properties on sale, look for buyers with matching demand, screen them e.g. with respect to their credit history, write sale contracts and propose financial packages. In the brick and mortar economy, intermediaries often buy and resell goods, e.g. intermediaries in the distribution sector or import / export business. On the other hand, the development of new technologies of information and communication has triggered a drastic reduction in the costs of gathering, processing and using large batches of data, and e-commerce and Internet-related activities have brought informational intermediation to the forefront of the "new economy". This paper analyzes a model of imperfect competition among providers of informational intermediation services, hence with a particular relevance for intermediation on the Internet.

Overall, the revenue generated by intermediation activities on the Internet amounts to \$100 billions for the US in 1999, i.e. about 17% of the total revenue generated by all economic activities that are related to the Internet. Although the figures are still limited, the trend shows that intermediation will be a critical component of the new economy.[1] Despite this increasing weight, issues related to the content of services proposed on the World Wide Web have not been much studied compared to issues about infrastructure

[1] See the study by the University of Austin, at www.internetindicators.com, for a decomposition and evaluation of different types of activities related to Internet, the survey of The Economist on e-commerce for a general presentation, and Kaplan and Sawhney (2000) for a discussion of auctions sites and the various types of aggregation.

and access.[2] Admittedly, intermediation and other services involved in the electronic commerce have existed for a long time in other, more traditional businesses and have been studied in these contexts.[3] But the development of new technologies puts the emphasis on new dimensions such as the possibility of dealing with huge data bases, the possibility of simultaneous using several intermediaries' services and the possibility of precisely monitoring transactions. Consequently, our analysis of intermediation activities, with a particular focus on Internet-based intermediation, takes into account the role of economies of scale in data collection and the new competitive conditions and business strategies that prevail under these specific conditions.

Matchmaking in general is a typical example of informational intermediation. As an example, consider the case of individuals who, when they feel lonely, turn to dating agencies, which are increasingly web-based matchmakers, such as www.match.com. Men who register with such intermediary enter their specific profile into the intermediary's data base. If a woman's profile coincides, both users are informed of a possible match. Although some intermediaries are specific to well-identified religious or social communities, one individual can register with several intermediaries at the same time. Matchmakers usually charge users a flat rate on a monthly basis as long as they are registered in the data base.

Other examples of matchmaking are provided by portals such as yahoo, msn or netscape-center, that help websurfers find the URL-addresses of specific websites they are interested in. The choice of which portal to visit first depends to some extent on how large the data bases are and how often

[2] See Shapiro-Varian (1998) for a general presentation of the economics of the Internet.

[3] See e.g. Diamond (1984), Rubinstein-Wolinsky (1987), Yanelle (1989). A key feature of these models is that intermediaries buy and resale while in ours they only match buyers and sellers. The issue of network effect that emerges in our context translates into the potential rationing of demand in these models.

they are updated, since access and use are basically free. Specific websites and other announcers, on the other hand, often have to pay portals for each visit they receive that originates from the general purpose portal with which they are registered. More sophisticated pricing, based on priority orders, e.g. access to top-screen banners, is also possible.

Many B2B and trade-oriented B2C websites, such as demand or supply aggregators or auction sites, usually provide a bundle of services, among which intermediation is critical. The website esteel.com , for example, records types and characteristics of orders, and connects buyers and sellers who want to trade some given quality of steel with some well-specified properties. Auction-sites provides trading mechanisms, but at the same time, they provide information and matching services by making it known that a given good is on sale, by identifying the tastes of users and signalling when something of interest comes up, by providing means for the buyer to assess the quality or the aspect of the good, the reputation of the sellers, and by providing guarantees to trade safely.

The main feature we want to emphasize in these situations is the presence of network externalities. Direct network externalities are straightforward to identify. As an example, demand aggregators such as mobshop.com can propose better trade conditions to participating buyers, the larger the number of orders they aggregate. Similarly, increased participation of sellers at the same market place intensifies competition and reduce prices.[4] The specificity of informational intermediation, however, lies more in *indirect network externalities*, where users of one side of the market have larger expected gains from intermediation, the larger the number of users on the other side of the market. Women visiting a dating-services provider benefit from a

[4] See Baye-Morgan (2000) for an analysis of this point in the context of monopoly intermediation.

larger data base of men's profiles, and conversely. Buyers of an intermediate good or service enjoy facing a large number of sellers as they have access to more diversity, while sellers benefit from more buyers as they increase their chances of selling their product for a better price. Indirect network externalities give rise to a "chicken-and-egg" problem: in order to attract buyers, an intermediary should have a large base of registered sellers, but these will indeed be willing to register only if they expect many buyers to show up at the intermediary.

The second feature is about intermediation pricing and possibilities of price discrimination. Users who ask for intermediation services are not homogenous and consequently, they can be charged different prices for intermediation. In auction sites, for example, buyers and sellers have to pay a transaction fee for each sale, but sellers usually have to pay registration fees too, to open a new auction for a given good. Websites and websurfers are treated differently by multi-purpose portals as well.

Combining these two features, we propose a model of competition by price-discriminating intermediaries that are subject to indirect network externalities. The combination of externalities across two categories of users and of price discrimination across these different categories opens the possibility of rich business strategies. For instance, an intermediary may subsidize the participation of some users in order to attract other participants. Such strategies drastically affect the outcome of competition compared to the more standard situations with competition and network externalities (see e.g. Katz-Shapiro(1985, 1994) and Farell-Saloner (1985)).[5]

We characterize the different equilibrium outcomes of imperfect compe-

[5] In a sense the problem can be seen as a problem of network competition with price discrimination (Jullien (2001)). Our work is thus related to the work of Innes-Sexton (1997) which shows how a monopoly may price-discriminate between similar consumers to prevent the formation of consumers' coalitions.

tition situations, depending on the pricing instruments that matchmakers use and on the possibility of users to use several intermediaries' services simultaneously. More precisely, we investigate a Bertrand duopoly competition game between two matchmakers who propose to match two sides of a market. The demand addressed by users of one side of the market to a given matchmaker depends on matchmakers' prices and on the number of users of the other side who demand intermediation from this matchmaker. In our model, matchmakers can rely on two pricing instruments, registration fees paid ex-ante, and a transaction fee paid ex-post when a transaction takes place between two matched parties.

We analyze the existence and sustainability of concentrated market structures, where one firm monopolizes the intermediation market, as well as of more balanced market structures. We also provide an analysis in terms of maximal profits that can be sustained in equilibrium and how details on pricing instruments and users behavior affect these sustainable profits and possibly limit the possibility of capturing users' surplus.[6]

We start the investigation by assuming that users can only demand intermediation from one matchmaker. Market monopolization is always possible. When matchmakers compete in registration fees only, it yields positive profits for the monopolizing matchmaker. Introducing transaction fees reduces market power and sustainable profits vanish: monopolization is still possible but all profit opportunities are exhausted by the need to protect the monopoly position. It is shown that one's best strategy to protect market share, as well as to conquer the market, requires to set the maximal feasible transaction fee along with low registrations fees. Other more balanced equilibria, that all imply zero profits, exist as well but only when there are

[6] Despite this, the paper is not policy-oriented and the reader should not expect competition policy recommendations for intermediation markets.

strong limitations on the size of the transaction fees that a matchmaker can enforce.

Allowing for the possibility that users register with two matchmakers at the same time changes the nature of competitive strategies because convincing some users to register becomes easier. The issue shifts partly from attracting users to attracting transactions. While the introduction of transaction fees still reduces market power, there always exist equilibria with positive profits. Because matching technologies are imperfect, the efficient allocation may involve global multi-homing, where all users register with both matchmakers, or single-homing. Monopolization is still possible, although inefficient ex-post monopolization can only occur if there is a limitation on transaction fees. On the other hand, an equilibrium with global multi-homing exists when this is efficient, and it involves positive profits for both matchmakers. In all these equilibria, the firm that obtains the highest profits sets a zero transaction fee. The other firm may either be inactive, or act as a "second-source" by proposing a positive transaction fee. Users then rely primarily on the services of the matchmaker that is "cheap on transactions", and when it doesn't perform the match, they turn to the second source or not depending on the equilibrium.

With multiple registration by users, however, we show that inefficient equilibria can exist where users on one side of the market ask for intermediation from both matchmakers, but not users on the other side. Moreover these equilibria may generate the highest level of profits for both intermediaries.

The paper is organized as follows. Section 2 spells out the details of the model of intermediation we use. Section 3 analyzes the case where users can only rely on the services of one matchmaker or the other. Section 4 formalizes users' behavior more precisely and takes into account the possibility of using multiple matchmakers' services at the same time. We conclude in section 5.

2 A model of intermediation

2.1 The framework

• *Matching.* Consider a simple matching model with two populations, labelled $i = 1$ and 2, each consisting of a continuum of mass 1 of agents. Agents can be viewed as buyers and sellers. Each buyer has specific needs and demands a specific version of a given good or service, and each seller offers a specific version of the good. For each agent, there exists a unique matching agent on the other side of the market. The gain from trade between a buyer and a seller equals 1 if they perfectly match, 0 otherwise;[7] that is, we assume no substitutability among the different versions of the good. Perfectly matched agents follow an efficient bargaining process which yields a linear sharing of the trade surplus, with share u_i for the type-i agent such that $u_1 + u_2 = 1$, and better bargaining position for 2-agents, that is: $u_2 \geq \frac{1}{2} \geq u_1$.

A pair of agents on each side of the market realizes that they match once they meet, but ex ante there is no chance that a particular i-agent finds its perfect match by just picking randomly within the j-population. There exists a matching technology that allows to process, select and use information on agents. A firm k endowed with this matching technology acts as an intermediary or matchmaker between both sides of the market. The technology works as follows. The matchmaker builds a data base with the characteristics of the i-agents who register with him. Given the charac-

[7] These are gross of transaction fees, see below.

teristics of a given j-agent, for $j \neq i$, the technology analyzes this data base and finds this specific j-agent's match with probability λ, if it belongs to the data base; the search fails otherwise. Hence, if $n_i \leq 1$ agents of type i, drawn randomly within population i, register with a matchmaker, a j-agent finds its match through this intermediary with probability $\lambda n_i \in [0, 1]$. Let c_i denote the cost of one i-agent using the technology, which includes the agent's personal cost, the matchmaker's costs of registration and information processing.[8] We let $c = c_1 + c_2$ denote the total cost and we assume that intermediation is efficient enough, so that a matchmaker could serve the market with positive profit, namely: $\lambda > c$.

• *Pricing instruments.* Two matchmakers, $k \in \mathbb{K} = \{I, E\}$, compete on the intermediation market using the same technology. Matchmakers have two types of pricing instruments. First, matchmaker k can charge each user of type i an up-front connection or registration fee p_i^k. The use of flat rates, on a monthly or annual basis, is quite common in e-commerce. It is important to notice that we do not restrict registration prices to be non-negative. A negative price can be interpreted as the consequence of gifts given to joining members, or as the result of the addition of free services to the basic matching service.[9]

As a second pricing instrument, matchmaker k can charge a transaction fee t^k on the gains from trade that takes place through k's intermediation services, so that the net surplus, on which matching agents bargain, is reduced

[8] In some cases, intermediaries finance themselves through advertising. In these cases, c_i should include the advertising revenue that a customer-i generates. This means that the cost c_i can be negative.

[9] Suppose that there exist services that substitute perfectly to money: a quantity q of services costs q per user for a utility q. If the firm charges a price $\bar{p}_j$ with services q_j, we can define $p_j = \bar{p}_j - q_j$: the utility is $n_i u_j - p_j$ and the profit p_j. A positive price can correspond to $\bar{p}_j > 0$ and $q_j = 0$, a negative price to $\bar{p}_j = 0$ and $q_j > 0$.

to $(1-t^k)$.[10] When transactions do not give rise to physical or monetary exchanges, such as for pure informational intermediation or pure matching, transaction fees are difficult to implement. When transactions involve trade and monetary exchange, they may also be difficult or costly to monitor, or else it may be costly to prove that they actually took place through the matchmaker's intermediation services. Negative transaction fees would induce arbitrary pairs of agents to pretend they match simply to collect the fee. Moreover, if transaction fees exceeded the gains from trade, no transaction would take place. The analysis is developed assuming that $t^k \in [0, \bar{t}]$ with $0 \leq \bar{t} \leq 1$, and comparative statics will be performed on this upper bound $\bar{t}$. So, the case where transaction fees are not feasible corresponds to $\bar{t} = 0$, while the case without restriction corresponds to $\bar{t} = 1$. Notice that there is no trade distortion associated with the use of transaction fees.

Matchmaker k cannot charge prices such that agents of type i receive negative expected surplus from trade. We thus restrict ourselves to prices such that

$$\lambda u_i(1-t^k) - p_i^k \geq 0. \tag{1}$$

For a given matchmaker k, we will focus on pricing instruments in $\mathbb{P}^k$, the set of available prices, with:

$$\mathbb{P}^k = \left\{ P^k = \left(p_1^k, p_2^k, t^k\right), 0 \leq t^k \leq \bar{t} \text{ and (1) for } i = 1, 2 \right\}.$$

Let r_i^k denote the maximum revenue extracted by k on i-user's transactions given prices P^I, that is $r_i^k \equiv p_i^k + \lambda u_i t^k$. Then, (1) can be written as: $r_i^k \leq \lambda u_i$, and we will describe P^k using (r_1^k, r_2^k, t^k) when convenient.

[10] The use of fees, proportional to the transaction price, is standard in auction websites. Piecewise-linear transaction fees have also been introduced. The fees on final value at aucland.com are 3%, while at eBay.com they amount for 5%, 2.5% or 1.25% of the transaction price depending on the level of this price.

• *Multihoming.* We shall consider the possibility that agents register with both matchmakers, which we call a multi-homing strategy. This modelling option corresponds to a situation where a firm that sells goods or services, actually posts an advertisement with several intermediaries, or posts a web-page at several intermediaries' websites. Doing so increases the probability of reaching consumers by making the firm known by the population visiting one of these intermediaries' websites.[11]

Let $\mathbb{S}_i$ denote the choices available to i-agents. We will thus examine situations where $\mathbb{S}_i=\{\{I\},\{E\}\}$, referred to as a situation with exclusive intermediation services, and $\mathbb{S}_i=\{\{I\},\{E\},\{I,E\}\}$, referred to as a situation with no-exclusivity. Let $\mathcal{N}=\left\{n_i^I, n_i^E, n_i^{\mathbb{K}}\right\}_{i=1,2}$ denote the distribution of agents across matchmakers, with n_i^k the number (proportion) of agents of type i who register with matchmaker k only, and $n_i^{\mathbb{K}}$ the number of agents of type i registering with both matchmakers. Finally, for any subset of matchmakers $S \subset \mathbb{S}_i$ and prices $P=(P^I, P^E)$, let $U_i(P, S, \mathcal{N})$ denote the net (indirect) expected utility of a i-agent registering with all intermediaries within S and only with these.[12] Similarly, let $\pi^k(P, \mathcal{N})$ denote matchmaker k's profits from charging prices P^k and facing prices P^{-k} and a number of users given by the distribution $\mathcal{N}$. The precise forms of $U_i(.)$ and $\pi^k(.)$ will be made explicit in each case of interest.

2.2 Timing and equilibrium

The situations we consider in the following all have the same structure. In a first stage, both matchmakers set prices P^k simultaneously and non-

[11] Other modelling options could be more appropriate under some circumstances, in particular options that follow more closely the consumers' search process: a consumer starts looking through his/her preferred intermediary and then, if still not matched, visits his/her second preferred intermediary provided the prospects of continuing the search and the probability assessment of finding a match are high enough.

[12] By definition, $U_i(P, \emptyset, \mathcal{N})=0$.

cooperatively. The resulting price system P is publicly observable. Then, in a second stage, users simultaneously choose which matchmakers (if any) to register with, yielding a distribution of users across intermediaries $\mathcal{N}$. Given the assumption of a continuum of users on each side of the market, the setting does not exactly correspond to a game. But the definitions below are easily adapted from the standard concept of subgame-perfect equilibrium.

Definition 1 *: A distribution of users $\mathcal{N}$ is an equilibrium distribution for a price system P if for all $S \in \mathbb{S}_i$:*

$$n_i^S > 0 \Longrightarrow \forall S' \in \mathbb{S}_i,\ U_i(P, S, \mathcal{N}) \geq U_i(P, S', \mathcal{N}).$$

A market allocation is a mapping $\mathcal{N}(.)$ that associates to each price P an equilibrium distribution of users $\mathcal{N}(P)$.

In words, if some i-user registers only with k, then he must be as well off as if he registered instead with the other matchmaker or with both matchmakers (when alllowed). And if some i-user registers with both intermediaries, then registering with just one of them must not be a better option. Note that, as a function of prices P, $n_j^k(P) + n_j^{\mathbb{K}}(P)$ determines the demand for matchmaker k's services by users of type j.

There can be multiple market allocations. Although most of our results do not rely on point predictions about the equilibrium outcome, we will use a mild refinement to focus on reasonable market allocations. This refinement amounts to ruling out increasing demand functions, as it basically requires that when the prices $P^k = \left(p_1^k, p_2^k, t^k\right)$ charged by a matchmaker increase, the number of users that register with this matchmaker, that is $n_j^k + n_j^{\mathbb{K}}$, cannot increase.

Definition 2 *: A market allocation $\mathcal{N}(.)$ is monotone if $\forall k$, $n_j^k(P^k, P^{-k}) + n_j^{\mathbb{K}}(P^k, P^{-k})$ is non-increasing in P^k.*

So, we rule out the cases where users become pessimistic about how many users register with matchmaker k, when k decides to charge lower prices. Higher prices per se cannot be interpreted as good news about the matchmaker's market share.

Note that monotonicity is not very restrictive. In particular, it has no bite when, say, p_1^k increases while p_2^k decreases; in such cases, we impose no restriction on market allocations. The restriction is implied by a selection criterion, that would impose that users coordinate on a Pareto undominated allocation (for users only). It is also a consequence of the selection criterion used in Jullien (2001), where users are assumed to coordinate so as to maximize the market share of firm I, thereby reflecting a reputation effect.

The only caveat is that prices may be viewed as a signal of quality. In our model, the "quality" of the intermediation services depends on the mass of users registering and so, a low price could be perceived as a bad signal triggering a reduction in demand. But this effect is conceivable only if intermediaries have a better information on demand than consumers when they set prices, which is not the case in our model. We conjecture that a more detailed dynamic process would deliver the monotonicity restriction as a more natural property of equilibrium.

A market allocation $\mathcal{N}(.)$ generates a reduced-form price-setting game among matchmakers, where payoff functions are given by $\pi^k(P, \mathcal{N}(P))$. An equilibrium is then defined straightforwardly:

Definition 3 *: An equilibrium is a pair* $(P^*, \mathcal{N}(.))$, *where (i)* $\mathcal{N}(.)$ *is a monotone market allocation and (ii)* P^* *is a Nash equilibrium of the reduced form pricing game induced by the market allocation (or "demand functions")* $\mathcal{N}(.)$.

Intuitively, an equilibrium consists of a set of prices charged by matchmakers and of a description of how users choose among them. The allocation of users corresponds to a system of demand functions addressed to each matchmaker. Given how (almost) all users allocate, the demand of a particular user only concerns matchmaking services that are preferred because they provide the best mix between being relatively cheap and generating relatively high network externalities for the user. Given that i-users are identical ex-ante, aggregate demand functions are easily obtained. Once demand is characterized, the first stage amounts to a classical price setting game.

It is convenient to interpret this equilibrium concept as a rational expectation equilibrium where, following the choice of a price system P, each infinitesimal user has expectations about how all users will allocate among the different matchmakers, and in equilibrium expectations are common and fulfilled. We shall use this interpretation repeatedly to explain why, for a given price system, there may exist several market allocations: it will be argued that common pessimistic beliefs about the number of users registering with one intermediary may indeed prevent any user from registering with this intermediary, thereby justifying these pessimistic beliefs.[13]

3 Competition for exclusive services

In this section, we make the critical assumption that *intermediation services are exclusive*, which means that agents can only register with matchmaker I or E, but not with both. This may be because the data concerning an agent in a data base are proprietary. Or, a seller cannot have several parallel procedures running at the same time at various auction sites to sell a given

[13] That network externalities are a source of multiplicity of equilibria is a well-known phenomenon, see e.g. Farell-Saloner (1985), Katz-Shapiro (1985, 1994).

good, or at least he would suffer a cost of doing so, in terms of reputation loss or future exclusion from the website where he defaults.

The consequence of these assumptions is simply that only singleton sets of matchmakers need be considered in the definition of U_j and π^k:

$$U_j(P, \{k\}, \mathcal{N}) = n_i^k \lambda u_j (1 - t^k) - p_j^k$$

$$\pi^k(P^k, \mathcal{N}) = \sum_{i=1,2} n_i^k (p_i^k - c_i) + \lambda n_1^k n_2^k t^k.$$

Although our model is symmetric, the presence of network externalities opens the possibility for endogenously asymmetric market structures. We will study first *dominant firm equilibria*, where only one matchmaker, I for incumbent in this case, is active on the intermediation market although there are no fixed cost of entry for the other matchmaker: such equilibria are characterized by $n_i^E = 0$ and $n_i^I = 1$ for $i = 1, 2$. Then, we will study *market segmentation equilibria*, where both matchmakers have positive market shares.[14]

3.1 Dominant firm equilibria

A dominant firm equilibrium arises as long as there exists no pricing strategy that enables E to capture a positive market share on both populations at non-negative profits. From definition (3), an equilibrium price system must be sustained by a specific market allocation. If it exists, a dominant firm equilibrium price system (P^I, P^E) can be sustained by a "bad expectation" (or pessimistic) market allocation against E, that is, by a market allocation such that, after any deviation in prices P'^E by E, users coordinate on an

[14] In all the paper, we are interested in equilibria where all users are served so that the market is fully covered.

equilibrium distribution with zero market share for E, $n_i^E(P^I, P'^E) = 0$, whenever possible. The basic intuition for our results can then be grasped by analyzing E's best response to a set of prices P^I charged by I, when the resulting market allocation relies on pessimistic beliefs about E.

For a price system $P = (p_1^I, p_2^I, t^I, p_1^E, p_2^E, t^E)$, there exists a "bad expectation" market allocation against E, where $n_i^E(P) = 0$ and $n_i^I(P) = 1$, as long as:

$$\lambda u_i(1 - t^I) - p_i^I \geq -p_i^E. \tag{2}$$

There is no way for E to gain positive market share by charging registration fees such that (2) holds for both $i = 1, 2$, since every user expects the others to register with I and then has no incentives to register with E.

To get a positive market share despite pessimistic beliefs, E must adopt a *divide-&-conquer* strategy (hereafter DC-strategy), namely subsidize one group of users and extract the ensuing externality benefit on the other group of users. Let us explain why. First, E must target a population i and charge:

$$p_i^E < r_i^I - \lambda u_i.$$

So, E must subsidize group i: $p_i^E < 0$. Then, an equilibrium distribution of users necessarily satisfies $n_i^E = 1$, and E can escape the curse of pessimistic users' beliefs. This strategy is costly since i-users are subsidized. But E can recoup some of these losses for it has now market power over the j-population, $j \neq i$. Indeed, as j-users now rationally expect all i-users to register with E, they have to compare $\lambda u_j(1 - t^E) - p_j^E$ with $(-p_j^I)$ and 0 and they will decide to register with E provided:

$$r_j^E = \lambda u_j t^E + p_j^E < \lambda u_j + \inf\left\{p_j^I, 0\right\}.$$

Therefore, E can choose the registration fee for j-users and the transaction fee in $\mathbb{P}^E$ under this constraint so as to maximize profits. Both instruments are perfect substitutes with respect to collecting profits on j-users, but for a fixed value of r_j^E, charging high transaction fees $t^E = \bar{t}$ allows to obtain an extra revenue $\lambda u_i \bar{t}$, because j-users only pay a share u_j of this fee. It thus reduces the cost for E of attracting population i without jeopardizing the shift from I to E by i-users. So, maximal profits for E (with pessimistic beliefs) is obtained at the maximal transaction fee $\bar{t}$, and they are larger when transaction fees are feasible (when $\bar{t} > 0$).

To deny E an active participation in the market, I's pricing strategy must thus be designed so that no such DC-strategy for E is profitable. When transaction fees are feasible, it therefore becomes more costly for the dominant firm to deter E from getting a market share as a larger share of profits has to be foregone to protect the monopoly position. The next proposition summarizes these results.[15]

Proposition 1 : *(i) With exclusive intermediation services, there exist dominant-firm equilibria, where I captures all users, and E is inactive.*
(ii) The highest profits for the dominant firm I are given by:

$$\bar{\pi}^I = \inf\{\lambda u_1(1-\bar{t}); \sup\{\lambda(u_2-u_1)(1-\bar{t}); c-\lambda\bar{t}\}\}.$$

In this high-profits equilibrium, firm I's charges a maximal transaction fee $t^I = \bar{t}$ *and, when* $c < \bar{c} \equiv \lambda\bar{t} + \lambda(u_2-u_1)(1-\bar{t})$*, it subsidizes the low-externality group (population* 1*) participation with* $p_1^I < 0$*.*

In equilibrium, the dominant firm charges prices such that there exists no profitable DC-strategies for E (there is some flexibility for E's equilibrium prices). Following a deviation from equilibrium prices by one intermediary,

[15] Proofs are relegated in the Appendix.

users coordinate on a pessimistic allocation w.r.t. this deviating intermediary.[16]

Note first that, given the surplus left to users, setting low registration fees is always more profitable for a firm than setting low transaction fees. The reason is that it is in the best interest of a matchmaker, either the dominant firm trying to keep its customers or the challenging firm trying to "poach" them, to design prices that are the most attractive for users even when they hold pessimistic beliefs against this matchmaker. For E, it is the only way to convince some group to join, while for I it helps convince a group to stay if the other leaves. So, firms opt for the maximal transaction fee $\bar{t}$.

The cost for E of the DC-strategy where 1-users are subsidized by E is at least $\lambda u_1(1-\bar{t})-p_1^I+c$ and the gain can at best be equal to $\lambda u_2+\lambda u_1\bar{t}=\lambda\bar{t}+\lambda u_2(1-\bar{t})$ (with maximal transaction fees and if p_2^I is non-negative). If $c>\bar{c}$, such a strategy is unprofitable even for $p_1^I>0$, and I can afford to capture some of group 1's surplus. If $c<\bar{c}$, however, the dominant firm must increase the cost of this DC-strategy by subsidizing 1-users ($p_1^I<0$), thereby making it worthless for E to follow this DC-strategy. In this case, I subsidizes 1-users in equilibrium to prevent E from doing so. Of course, a similar cost-benefit analysis applies for the other DC-strategy that aims at subsidizing 2-users, but since $u_2>u_1$, this strategy is less profitable for E, hence easier to deter with $p_2^I>0$.

As already suggested by the notation, our result can be interpreted in a model with sequential entry, where an incumbent I first chooses (and commits to) its prices, and a potential entrant follows. The prices P^I and profit π^I then correspond to the highest-profit, entry-deterrence equilibrium.

[16] Note that, given that E has no market share in equilibrium, the monotonicity requirement has no bite when E deviates from equilibrium prices.

Despite the possibility of entry and the absence of any fixed costs of entry, the incumbent is able to monopolize the market and to deter the entrant from entering. Users' optimistic beliefs constitute the key factor that determines entry barriers. However the effectiveness of these barriers to deter entry relies on limitations of the pricing instruments of the entrant.

Corollary 1 *: When full transaction fees are feasible, i.e. when* $\bar{t} = 1$*, all dominant-firm equilibria generate zero profits for the dominant matchmaker.*

When $\bar{t} = 1$, transaction fees are sufficient to capture all the surplus from transactions. Thus firm E can use the registrations fees to subsidize both groups for access by an amount equal to the total surplus $\lambda - c$. As a result the dominant firm has to leave the full surplus to the users.

3.2 Market segmentation equilibria

More balanced market structures can also arise, where both matchmakers are active, have a positive market share, and all users are served.

Consider a candidate equilibrium with price system $P = (P^I, P^E)$ and market allocation such that on the equilibrium path, $0 < n_i^I = 1 - n_i^E < 1$ for $i = 1, 2$. Both types of users must then be indifferent between both matchmakers, that is:

$$n_i^I \lambda u_j (1 - t^I) - p_j^I = n_i^E \lambda u_j (1 - t^E) - p_j^E \geq 0. \tag{3}$$

Starting from P, a deviation by matchmaker k to prices $P'^k = (p_1'^k, p_2'^k, t'^k) < P^k$ must give rise to a market allocation such that, by the monotonicity condition, $n_i'^k \geq n_i^k$, where we let $n_i'^k$ denote $n_i^k(P'^k, P^{-k})$. Using (3), it follows that:

$$\begin{aligned} n_i'^k \lambda u_j (1 - t'^k) - p_j'^k &> n_i^k \lambda u_j (1 - t^k) - p_j^k \\ &= n_i^{-k} \lambda u_j (1 - t^{-k}) - p_j^{-k} \geq n_i'^{-k} \lambda u_j (1 - t^{-k}) - p_j^{-k}, \end{aligned}$$

and the only possible market allocation is given by: $n_i'^k = 1$. In words, a small cut in prices enables an intermediary to capture both sides of the market entirely. The Bertrand logic prevails. It is therefore not surprising that all equilibria with two active matchmakers are characterized by zero profits.

Proposition 2 *: Suppose intermediation services are exclusive. When* $u_2 > \frac{2}{3}$*, there exists no equilibria where both matchmakers are active. When* $u_2 \leq \frac{2}{3}$*, there exists* $\tau(u_2) < 1$ *such that there exists equilibria with both matchmakers active if and only if* $\bar{t} \leq \tau(u_2)$*: in these equilibria, both intermediaries charge identical prices, they serve half the users of each population and they receive zero profit.*

An equilibrium with two active intermediaries is inefficient, as each agent is matched with only probability $\frac{1}{2}$. It.exists only if users have no strongly asymmetric positions when deciding on how to share the matching surplus and if the possibilities of imposing transaction fees are limited. Such market segmentation equilibria are symmetric. They are The reason is that a DC-strategy with only registration fees does not enable a matchmaker to capture the whole increase in social surplus it creates by attracting all users, but only the differential between the values created for each group: $\lambda(u_2 - u_1)$. When this differential is large, this is sufficient to eliminate the inefficient outcomes. But when it is small (less than half the total surplus), matchmakers do not gain enough from improving the allocation.

Under market sharing, competition between both matchmakers is of a Bertrand type and pushes profits down to zero. While natural, this result is in marked contrast with the one obtained in Proposition 1. Our symmetric setting is one of fierce competition where profits vanish if users hold beliefs that are not biased against one matchmaker. Nevertheless, positive profits and monopoly power emerge if users are more optimistic with respect to one matchmaker than the other. The source of bias in the users' beliefs is not formalized, as it is not payoff-relevant: it may be related to sunspots, or any other kind of self-fulfilling prophecy, or to elements of history that are not accounted for. Its consequences are however dramatic: one intermediary serves all the intermediation market and makes positive profits, while the other is denied any market share.

Overall, the possibility of using sophisticated pricing instruments (here, transaction fees) has strong consequences on the outcome of imperfect competition in the intermediation market. It clearly reduces profits. In dominant firm equilibria, the maximal profit decreases with $\bar{t}$, and with full transaction fees, the dominant firm is unable to draw any positive profits from its market dominance. It also limits potential inefficiencies in the equilibrium allocation, as it raises the ability of an intermediary to generate efficiency gains and capture some of these gains through its many pricing instruments, hence the elimination of more balanced equilibria. The prediction in terms of equilibrium is then pretty sharp, at least if upward sloping demand functions are ruled out: the only reasonable equilibria that can emerge involve market dominance of one matchmaker, without market power, since profits are null.

4 Multi-homing

Exclusivity of intermediation services is quite a restrictive assumption in many circumstances, in particular when focusing on intermediation on the Internet. As an example, a websurfer looking for some specific good or service will usually visit several intermediation service providers and register with many matchmakers to increase his chances of finding a good match. Similarly, firms offering various services register with different intermediaries in order to benefit from their different users bases. It may be the case that some intermediaries impose an exclusivity clause when registering users so as to avoid this type of behavior and ensure that their efforts in processing the users' specific demands will end up with a transaction. This occurs with real estate agents trying to find buyers for a given property. It may also occur when registering involves a process where the user's profile is defined and precisely documented, so that the matchmaker could argue that such information is proprietary and cannot be used with another matchmaker. In order to assess the strategic and efficiency reasons that motivate such exclusivity clauses, one must nevertheless investigate the consequences of competition with non-exclusive services. This section is therefore devoted to the analysis of imperfect competition between two matchmakers who offer non-exclusive intermediation services, so that users may engage in "multi-homing", that is, they may use both matchmakers' services simultaneously.

When users of both types engage in multi-homing, they register with both matchmakers and it is possible that both intermediaries perform the match at the same time. Users will then choose to conclude the transaction via the intermediary that imposes the lowest transaction cost. When both transaction fees are equal, we assume that users randomize evenly between the matchmakers. The utility of user i is now given by:

$$U_i(P, \{k\}, \mathcal{N}) = \lambda(n_{-i}^k + n_{-i}^{\mathbb{K}})u_i(1 - t^k) - p_i^k \tag{4}$$

$$U_i(P, \mathbb{K}, \mathcal{N}) = \sum_{k=I,E} U_i(P, \{k\}, \mathcal{N}) - \lambda^2 n_{-i}^{\mathbb{K}} u_i(1 - \max\{t^1, t^2\}) \tag{5}$$

The last term captures the possibility that both intermediaries find the match.

Non-exclusivity of services opens the possibility of equilibria where some or all users adopt a multi-homing strategy in equilibrium. The exhaustive analysis of all the possible types of equilibria is beyond the scope of this paper. We rather choose to illustrate the main features of this framework by assuming that the market allocation is such that i-users either all use a multi-homing strategy or all use a single-homing strategy.[17] The basic intuition for our results can again be grasped from a careful analysis of E's best response to a set of available prices $P^I \in \mathbb{P}^I$ charged by I, when the resulting market allocation relies on pessimistic beliefs about E. Then, we successively study the existence and properties of global multi-homing equilibria, where all users engage in multi-homing, of dominant firm equilibria, and of market sharing equilibria where only one population of users engage in multi-homing.

4.1 Best-response analysis

For a given price P^I, we analyze the most profitable choice of prices P^E by E if "bad expectations" prevail against E. This will enable us to evaluate best deviation profits by E in various equilibrium configurations.

A bad-expectation distribution of users sets the number of users registering with E to zero, $n_i^E = n_i^{\mathbb{K}} = 0$, for $i = 1, 2$, whenever:

$$\begin{aligned} \lambda u_i - r_i^I &\geq -p_i^E, \\ \lambda u_i - r_i^I &\geq \lambda u_i - r_i^I - p_i^E, \end{aligned}$$

which reduces to

$$p_i^E \geq 0, \; i = 1, 2.$$

[17] We conjecture that there is no equilibrium where both choices coexist for the same users.

A profitable strategy for E must necessarily be a DC-strategy, where a group of users is subsidized with negative registration fees. The difference with the case of exclusivity is that any negative price ensures that the users register with E, since they need not forgive on registering with I to do so. In the following, we let i denote the group of users that is subsidized and j denote the other group. So, $p_i^E < 0$. Even if $n_j^I = 1$, i-users register with *E as a second home,* to cash in the subsidy *while still maintaining their registration with I* to benefit from externality benefits. So, *attracting one side of the market is less costly for E than with exclusive services.*

This strategy only makes sense if it generates a bandwagon effect and helps attract j-users, $j \neq i$. With non-exclusive services, however, a DC-strategy that subsidizes i-users may lead to $n_i^E = 0$ and $n_i^{\mathbb{K}} = 1$. Despite they register with E, i-users may still register with I ($n_i^{\mathbb{K}} = 1$), and so, j-users may continue registering only with I ($n_j^I = 1$). Using (4)-(5), this occurs if $p_i^E < 0$ and

$$\begin{aligned} r_j^E &\geq r_j^I \\ r_j^E &\geq \lambda u_j \left[1 - \lambda + \lambda \max\{t^I, t^E\}\right], \end{aligned}$$

in which case E gets no profits as users coordinate on a "bad-expectation" market allocation with $n_i^{\mathbb{K}} = n_j^I = 1$.[18] The first condition ensures that j-users prefer I to E when they register only with one intermediary. The second condition ensures that they don't use a multi-homing strategy and can be interpreted as follows. Consider an hypothetical scenario where users have to pay the transaction fee at all intermediaries performing a match (a matching fee). In this scenario, a j-user registered with I decides to register also with E by comparing the cost r_j^E to the increase in the expected value

[18] We extend our previous concept of bad-expectation market allocation by choosing, more generally, the users' distribution that yields the lowest profits for E.

of transactions $\lambda(1-\lambda)u_j$, which accounts for the fact that there is no additional value if I performs the match. Compared to that scenario, the user saves also with probability λ^2 a payment $\max\{t^I, t^E\}$, because in case of a double match, it will pay only the smallest transaction fee.

A best-response for E, under bad expectations, must then satisfy:

$$r_j^E < \max\left\{r_j^I; \lambda u_j\left[1-\lambda+\lambda\max\{t^I,t^E\}\right]\right\}. \tag{6}$$

Hence, *attracting the other, profitable side of the market generates smaller profits than with exclusivity, as the externality surplus cannot be entirely captured.*

Under $p_i^E < 0$ and (6), all users register with E. Whether or not they also register with I determines the profitability of E's pricing strategy. For the viewpoint of E, this really matters if $t^I < t^E$, since otherwise all the matches performed by E generate a revenue t^E. Assuming that $t^I < t^E$, all users use multi-homing if

$$\begin{aligned} r_1^I &\leq \lambda u_1\left[1-\lambda+\lambda t^E\right] \\ r_2^I &\leq \lambda u_1\left[1-\lambda+\lambda t^E\right] \end{aligned}$$

Let θ_i be defined as follows:

$$r_i^I = \lambda u_i\left[1-\lambda+\lambda\theta_i\right].$$

We see that users engage in multi-homing if $t^E \geq \max\{\theta_1,\theta_2\}$. Several cases need then to be considered for the optimal DC-strategy:

- E may choose to serve users as a *second-source:* E charges $t^E \geq t^I$ and all users engage in multi-homing ($n_i^{\mathbb{K}} = n_j^{\mathbb{K}} = 1$) so that in case of a double match, users choose to conclude the transaction via I. E only processes the transaction when the match has failed at I. Formally, E

sets $p_i^E < 0$, and (6) implies:

$$\begin{aligned} r_j^E &< \lambda u_j \left[1 - \lambda + \lambda t^E\right] \\ \theta_j &\leq t^E \leq \bar{t}. \end{aligned}$$

- E may instead choose prices so as to process all transactions after a match; a first type of such strategy consists in fixing $t^E < t^I$, which we call a *first source strategy*. Then, irrespective of whether users find a match at I and E or whether they only get matched at E, E will get all transactions per successful match. Formally, there exists i with $p_i^E < 0$, and (6) becomes:

$$r_j^E < \lambda u_j \left[1 - \lambda + \lambda \max\{\theta_j; t^I\}\right].$$

- The other type of strategy that enables E to process all transactions after a match, consists in choosing $t^E \geq t^I$ and inducing one group of users (at least) to register only with E. As long as

$$r_h^k \leq \lambda u_h \left[1 - \lambda + \lambda t^E\right], \text{ for } h = 1, 2 \text{ and } k = I, E,$$

  there exists a distribution of users with global multi-homing. To get one group to register with E only, E can try:

  - to prevent i-users to register with I; since

  $$r_i^E < \lambda u_i t^E \leq \lambda u_i \left[1 - \lambda + \lambda t^E\right],$$

    this requires that: $t^I \leq t^E < \theta_i$;

  - or to prevent j-users to register with I, which imposes that:

  $$\lambda u_j \left[1 - \lambda + \lambda t^E\right] < r_j^I \quad \text{and} \quad r_j^E < r_j^I,$$

    and therefore: $t^I \leq t^E < \theta_j$.

The following proposition provides a complete characterization of E's best response, in particular w.r.t. the type of business strategy that E can use to fight its way to the intermediation market when users hold pessimistic beliefs against it.

Proposition 3 *: Let $\bar{t}^I = \max\{\theta_1, \theta_2, t^I\}$ and $h_0 = \arg\max_h \theta_h$; E's best response to prices P^I (if E sells) when users hold pessimistic beliefs w.r.t. E, is characterized as follows:*

- *If $\bar{t}^I > \bar{t}$,*

$$\pi^E = \max_{i,j \neq i} \{\lambda \bar{t} u_i + \lambda u_j [1 - \lambda + \lambda \max\{\theta_j; \bar{t}\}]\} - c;$$

  the strategy is such that $t^E = \bar{t}$ and E prevents h_0-users to register with I.

- *If $\frac{(1-\lambda)u_1}{u_1+\lambda u_2}\bar{t} \leq \bar{t}^I \leq \bar{t}$,*

$$\pi^E = \lambda \bar{t}^I u_1 + \lambda u_2 \left[1 - \lambda + \lambda \bar{t}^I\right] - c;$$

  the strategy is such that $t^E = \bar{t}^I$, and if $t^I = \bar{t}^I$, E adopts a first source strategy while if $t^I < \bar{t}^I$, E prevents h_0-users to register with I.

- *If $\bar{t}^I \leq \frac{(1-\lambda)u_1}{u_1+\lambda u_2}\bar{t}$, E adopts a second-source strategy which yields:*

$$\pi^E \equiv \lambda(1-\lambda)[u_2 + u_1 \bar{t}] - c.$$

4.2 Global multi-homing equilibria

Armed with the characterization in Proposition 3 we can now perform the equilibrium analysis. Doing so, we shall rely also on the monotonicity requirement which imposes that, starting from the candidate equilibrium prices, no matchmaker can loose market share by decreasing (one of) its prices.

We start with equilibria such that users of both types register with both matchmakers, a situation we call global multi-homing. Given prices that satisfy $\lambda u_i \geq p_i^k + \lambda u_i t^k = r_i^k$, a global multi-homing allocation is such that $n_1^{\mathbb{K}} = n_2^{\mathbb{K}} = 1$ and requires:

$$\lambda(1-\lambda)u_i + \lambda^2 u_i \max\{t^I, t^E\} \geq r_i^k \quad \text{for all } i \text{ and } k. \tag{7}$$

The condition states that, given that a i-user registers with one intermediary, he is willing to join also the second matchmaker if all j-users do so. They may be two motives for that. First because matching technologies are not perfect, this increases the likelihood to be matched by an amount $\lambda(1-\lambda)$. Second, when the transaction fee is smaller at the second intermediary, this reduces transaction cost in case of a double match.

We obtain:

Proposition 4 *: A global multi-homing equilibrium exists iff $\lambda(1-\lambda) \geq c$. When it holds, there exists an equilibrium that jointly maximizes the profits of the two matchmakers, with $\lambda - c > \pi^I \geq \pi^E = \lambda(1-\lambda) - c$. When $\bar{t} > 0$, it implies $t^I < t^E$ and $\pi^I > \pi^E$. When transaction fees are not feasible ($\bar{t} = 0$), both matchmakers earn the same profits, $\lambda(1-\lambda) - c$ and play a symmetric role.*

The condition that $\lambda(1-\lambda) \geq c$ is equivalent to the fact that it is efficient to have two matchmakers, for the purpose of insuring against the possibility of a failed match. The proposition shows that global multi-homing is indeed an equilibrium whenever this is efficient. A necessary condition is of course that the matching technology is not very effective (λ not too close to 1).

A surprising result is that, when the condition holds, there exists a multi-homing equilibrium where profits are unambiguously maximum and this equilibrium is not symmetric: both matchmakers play different roles.

Firm I sets a low transaction fee and acts as a first source of intermediation, that is as the provider through which transactions are implemented whenever possible, while E sets a high transaction fee and acts as a second source, concluding transactions between trading partners who have not been matched elsewhere. Overall E is cheaper in terms of registration fees for both categories of users, but once registered with E, all users are still willing to register with I because this allows them to save on transaction fees if they are matched. The equilibrium configuration exhibits endogenous differentiation between the matchmakers.

4.3 Dominant firm equilibria

We now turn to the existence of dominant firm equilibria, and as in the previous section, we let I denote the dominant firm and E the potential entrant. Given prices that satisfy $\lambda u_i \geq p_i^k + \lambda u_i t^k = r_i^k$, a dominant firm allocation is such: $n_i^I = 1$ and $n_i^{\mathbb{K}} = n_i^E = 0$. A necessary condition is that:

$$0 \leq p_i^E \quad \text{for all } i, \tag{8}$$

since for prices $p_i^E < 0$, i-users are willing to register with E at least to cash in the subsidy.

The next proposition determines the level of profits for matchmaker I that can be sustained in a dominant firm equilibrium.

Proposition 5 *: A dominant firm equilibrium exists if and only if $c \geq \lambda(1-\lambda)[u_2 + u_1\bar{t}]$. The equilibrium with highest-profit for the dominant firm yields profits equal to $\frac{(1-\lambda)u_1}{u_1+\lambda u_2}(\lambda - c)$ when*

$$c \leq \lambda u_2 \left[1 - \lambda + \lambda\bar{t}\right] + \lambda\bar{t}u_1, \tag{9}$$

and $c - \lambda\bar{t}$ otherwise. They are always bounded from above by c.

Proposition 5 proves that there exist dominant firm equilibrium when multi-homing equilibria do not exist as well as for a range of parameters where multi-homing equilibria exist (and are efficient). In a dominant firm equilibrium, both firms may charge identical prices but users coordinate on registering with one dominant firm. The condition for existence guarantees that a second source strategy would not be profitable for E.

It is interesting to explain how I deters any other possible deviations by E in such an equilibrium. First, any profit that can be attained in a dominant firm equilibrium can be supported by strategies with a zero transaction fee, $t^I = 0$: matchmaker I does not have to use transaction fees to extract its profits, registration fees enable it to attain these profits. This contrasts with the results obtained in the case of exclusivity. Indeed under exclusivity the dominant firm protects its market share by making the "Divide" part of a DC-strategy costly for E. With multi-homing, dividing is easy, so that I must raise the cost the Conquer part for E. This is best achieved by setting a low transaction fee which ensures that E services are used only as a second source in case of multi-homing.

When transactions fees are not feasible, I is not constrained while E has less pricing instruments to get a market share. The sole possibility for E is to subsidize a population of users and to undercut the registration fee for the other population. With both registration fees equal to c, no profitable DC-strategy is profitable for E, but of course, I's profits are limited by c.

Corollary 2 *: When transaction fees are not feasible ($\bar{t} = 0$), a dominant firm equilibrium exists if and only if $c \geq \lambda(1-\lambda)u_2$ and the highest attainable profit for I in such an equilibrium is equal to c.*

If available, however, transaction fees may constitute a weapon that the entrant E can profitably use. Consider first that case where $\bar{t}$ is small. The dominant firm sets prices such that $t^I = 0$ and $r_i^I = p_i^I = c - \lambda \bar{t} u_j$. For these prices, the best entry strategy for E, given by Proposition 3, is a DC-strategy where $t^E = \bar{t}$ and 1-users are diverted from I (register only with E).[19] This can be implemented by subsidizing either 1-users or 2-users, say i-users, and setting registration fees so that r_j^E is almost equal to r_j^I. The profit from entry is at most $r_j^I + \lambda \bar{t} u_i - c = 0$. Entry is not profitable because the transaction fee has limited power and I can easily fight back by slightly reducing its registration fees. Note that, for subsequent use, the best DC-strategy induces both populations of users to register with E only.

For $\bar{t}$ small, the profitability of the DC-strategy for E is exogeneously limited by the fact that E cann't rely on high transaction fees to recoup part of the subsidy to group 1. I can thus set relatively high prices, although decreasing with $\bar{t}$.

When $\bar{t}$ increases, however, and reaches the point where (9) holds, this exogeneous limitation on transaction fees becomes ineffective. For $\bar{t}$ large and with prices $p_i^I = c - \lambda \bar{t} u_j$, feasible transaction fees are such that E can capture the market with positive profits. I must then find an alternative way to limit the maximal transaction fee that E can set if it wishes to be the sole source of intermediation. This is achieved by a reduction of $I's$ registration fees, making multi-homing more attractive for any type of users. To deter entry, I must thus adopt a different pricing strategy. One key point is that it is not sufficient for I to ensure that 1-users prefer multi-homing to E only for t^E in the range profitable for E. I must ensure that both types of users prefer multi-homing, since it is sufficient that one group diverts from I and the other uses multi-homing for E to be the first-source of transcations. This requires to reduce the two registration fees, even the highest p_2^I. I then

[19] As long as $c > \lambda \bar{t}$, $\bar{t} < \theta_2 < \theta_1$.

deters entry by setting prices $\frac{p_1^I}{\lambda u_1} = \frac{p_2^I}{\lambda u_2} = \frac{c+(1-\lambda)u_1}{u_1+\lambda u_2}$ so that E's profits are null. The change in I's entry-deterring strategy implies a discontinuity in its behavior and in its profits, reflected in Proposition 5.

For higher values of $\bar{t}$, the strategy described in the previous paragraph continues to deter entry and so, I's behavior and profits are not affected by $\bar{t}$ once (9) holds. The only difficulty is that, when $\bar{t}$ becomes very large, the second source strategy yields larger revenues for E and eventually becomes profitable.

Corollary 3 *: When transaction fees are fully feasible ($\bar{t} = 1$), a dominant firm equilibrium exists if and only if $c \geq \lambda(1-\lambda)$, hence exactly when it is efficient. The highest attainable profit is $\frac{(1-\lambda)u_1}{u_1+\lambda u_2}(\lambda - c)$.*

One key implication here is that under multi-homing and no restriction on transaction fees, an efficient equilibrium always exists, and it may involve positive profits for active firms.

4.4 Market-sharing equilibria

As for the case of exclusive services, other market configurations can emerge where users of at least one type are indifferent between registering with one matchmaker or the other, but do not adopt a multi-homing strategy on the equilibrium path. Market segmentation refers to a situation where both matchmakers are active and all users register with one and only one matchmaker so that there is no overlap between the matchmakers' market shares. So, $n_i^k > 0 = n_i^{\mathbb{K}}$ for all i and k, and $0 < n_i^I = 1 - n_i^E < 1$. In a market segmentation equilibrium, users are indifferent between registering with I or with E, but they prefer single-homing to multi-homing, so that the equilibrium market allocation satisfies:

$$\lambda n_{-i}^{I} u_i (1 - t^I) - p_i^I = \lambda n_{-i}^{E} u_i (1 - t^E) - p_{-i}^{E} = 0 \quad \text{for all } i \tag{10}$$

The discussion of this type of equilibrium follows the discussion of market segmentation when intermediation services are exclusive. Matchmakers profits are necessarily null under market segmentation. The new condition is that users must receive a zero surplus because they have the option of registering to both intermediaries which provides them with the sum of the utility levels obtained with each intermediary.

Proposition 6 *When intermediation services are not exclusive, there does not exist market segmentation equilibria provided that $\lambda \neq 2c$, that is generically.*

Proof. From the proof of Proposition 2, a market segmentation equilibrium has to be symmetric, with symmetric market allocation, and implies zero profit for both matchmakers. Moreover, from (10), prices must verify $p_i + \frac{\lambda}{2} u_i t = \frac{\lambda}{2} u_i$. Summing the equalities for over i yields: $p_1 + p_2 + \frac{\lambda}{2} t = \frac{\lambda}{2}$. The zero-profit condition then implies that $\frac{\lambda}{2} = c$, a condition on the parameters that generically does not hold. ■

More interestingly, though, non-exclusivity of intermediation services generates another type of market configuration where one group of users registers with one and only one matchmaker, while the other group adopts a multi-homing strategy, and both matchmakers serve both sides of the market. We call such an equilibrium a "market-sharing" equilibrium as it involves some sort of mild competition, as seen below, with respect to users who use multi-homing and for whom both matchmakers look more like complements than like substitutes in terms of intermediation service provision.

In the remaining of this subsection, we shall denote i, the group using multi-homing ($n_i^{\mathbb{K}} = 1$), and j the other, single-homing group ($n_j^{\mathbb{K}} = 0,\ 0 < n_j^I = 1 - n_j^E < 1$). The market allocation necessarily satisfies:

$$\begin{aligned} \lambda n_j^k u_i &\geq p_i^k + \lambda n_j^k u_i t^k \quad \text{for all } k = I, E & (11) \\ \lambda u_j &\geq r_j^I = r_j^E \geq \lambda(1-\lambda)u_j + \lambda^2 u_j \max\{t^I, t^E\}, & (12) \end{aligned}$$

using $r_j^k = p_j^k + \lambda u_j t^k$ as before, and profits are nonnegative if

$$\pi^k = p_i^k - c_i + n_j^k(p_j^k + \lambda t^k - c_j) \geq 0. \tag{13}$$

The analysis of deviations relies on Proposition 3, except for deviations by undercutting, where monotonicity imposes some restrictions. These restrictions can easily be obtained as follows. Suppose first that, starting from the equilibrium prices, matchmaker k reduces registration fees for the multi-homing users, that is p_i^k; keeping the distribution of users unchanged satisfies monotonicity and so, this type of deviation cannot be profitable for k. Suppose then that matchmaker k reduces registration fees for the single-homing users, that is p_j^k; then, by monotonicity, it cannot loose the population of type i. It follows that all j-users strictly prefer to register with k and, by (12), only with k. Matchmaker k thus captures the entire market with profit equal to:

$$p_i^k + p_j^k + \lambda t^k - c.$$

To guarantee that undercutting on type-j users is not profitable, it must be that profit generated by j-users' registering with k is nonpositive, that is:

$$p_j^k + \lambda t^k \leq c_j. \tag{14}$$

In a market-sharing equilibrium, matchmakers necessarily make losses on the single-homing side of the market, even taking into account transaction fees. Note that (14) also guarantees that undercutting on transaction fees is not profitable either.

Taking into account other possible deviations, the following proposition presents a partial analysis of market-segmentation equilibria, for the case where matching is perfect, so that the efficient equilibrium involves only one active firm..

Proposition 7 : *Suppose intermediation services are non-exclusive and matching is perfect ($\lambda = 1$). A sufficient condition for the existence of a market-sharing equilibrium is that $c_h \leq \frac{u_h}{2}$ for $h = 1, 2$. Under these sufficient conditions, the maximal aggregate profit is attained in a symmetric market-sharing equilibrium such that:*

(i) When $\bar{t} \leq \frac{c_i}{u_i} \leq \frac{c_j}{u_j}$, the individual profit is equal to $\max_h\{\frac{u_h}{2} - c_h\}$;

(ii) When $\frac{c_i}{u_i} < \bar{t} \leq \frac{c_j}{u_j}$, the individual profit is equal to $\frac{u_i}{2} - c_i$;

(iii) When $\frac{c_i}{u_i} \leq \frac{c_j}{u_j} < \bar{t}$, the individual profit is equal to $\frac{c_j}{u_j} u_i - c_i$;

(iv) One type of users register with both intermediaries (i-users for (ii) and (iii)). Users of the other side register with only one intermediary (half with each).

According to Proposition 7, there exist positive-profits, market-sharing equilibria where both intermediaries are active, one group of users registers with both intermediaries and the other registers with one and only one matchmaker. In particular, when for each population of users, the personal and matchmakers' costs of registration and the costs of information-processing are small compared to the surplus of intermediation, market-sharing equilibria exist, irrespective of whether matchmakers have access to a limited set or a full set of pricing instruments.

The feasibility of transaction fees is however critical to determine the profitability associated with market-sharing equilibria. Consider the case of intermediation on the Internet as an example where costs are possibly small. Market-sharing equilibria have then the following feature. Users of one type, say firms, register with both matchmakers and, given that each user of the other type, say each consumer, visits one intermediation website or the other, firms are sure to get matched. Conversely, consumers are sure to find their match at any intermediary's website since firms post ads with both sites, so they only register with one intermediary, the least costly one. This forces matchmakers to charge consumers identical prices, equal to the costs of serving them, hence $p_j = c_j$ close to zero.

As for the registration fee on firms, both matchmakers would be willing to jointly extract the network externality benefits generated, that is to charge an entry fee equal to half these benefits: $p_i = \frac{u_i}{2}$. This would yield intermediation for each matchmaker equal to $(\frac{u_i}{2} - c_i)$. When transaction fees are not feasible, the best deviation for a matchmaker corresponds to a DC-strategy where j-users are slightly subsidized and i-users are captured through slightly reduced registration fees; this corresponds to deviation profits equal $\frac{u_i}{2} - c$, hence nonprofitable. So, when feasible transaction fees are small, joint extraction of multihomers' surplus constitutes an equilibrium. With respect to intermediation on the Internet, we get a striking and quite realistic result that consumers get basically free access to intermediation websites, and they find their match easily since all firms register with all intermediation websites. Matchmakers' profits come from charging entry or registration fees upon firms who wish to enter their profile and post an ad, so as to benefit from visiting consumers.

When transaction fees are fully feasible, note first that the strategy of joint extraction of multihomers' surplus does not rely on the availability of transaction fees. The profitability of DC-strategies does, however ! In particular, a deviating matchmaker can subsidize both population of users with slightly negative registration fees, while setting transaction fees equal to $\max_h \left\{\frac{p_h}{u_h}\right\} = \frac{1}{2}$, for small costs. Then, i-users are induced to register with, and only with, the deviating matchmaker, and the deviation could be profitable if:

$$\frac{1}{2} - c > \frac{u_i}{2} - c_i,$$

which holds if costs are small (precisely, if $c_j < \frac{u_j}{2}$). The joint extraction of multihomers' surplus is then limited as prices must be set so as to deter this type of DC-strategy, that is:

$$\frac{p_i}{u_i} - c = p_i - c_i \Longleftrightarrow p_i = \frac{u_i c_j}{u_j} < \frac{u_i}{2}.$$

Consequently, the feasibility of transaction fees affects the nature of equilibrium prices, as well as the maximum profits that can be achieved in market-sharing equilibria. When registration and information-processing costs tend to be negligeable, strictly positive profits of the order or $\frac{u_2}{2}$ can be achieved if transaction fees are impossible, while equilibrium profits necessarily vanish when transaction fees are unconstrained. With rich possibilities in terms of pricing instruments, matchmakers have many opportunities to generate efficiency gains and to capture the ensuing profits; so, inefficient market configurations where a group of users splits between both matchmakers can be easily upset and only limited market power is left in the hands of matchmakers.

Notice that for $\lambda = 1$, the maximal profit in a dominant firm equilibrium is $\max\{0, c - \bar{t}\}$ so that when the value of the intermediation service is large or transaction fees are large, even a firm will prefer to share the market to being dominant.

4.5 Summarizing the results without exclusivity

The previous subsections in section 4 have proposed the analysis of several types of equilibrium configurations that can emerge when matchmakers compete in nonexclusive intermediation services. We wish here to give a short summary of these results.

First, for small enough costs of registration and of information-processing, the efficient market configuration is a global multi-homing market structure. This efficient configuration can be sustained as an equilibrium with strict market power by the duopolist matchmakers, and hence strictly positive profits. Moreover, endogenous differentiation arises then, with one matchmaker specializing as a second source with high transaction fees, provided these are feasible. So, in order to benefit from sources of matching, users register with several exchange websites and simply choose a priori to transact through the exchange where transaction is less heavily taxed. Note that maximal equilibrium profits weakly increase when the constraint on the availability of transaction fees is relaxed.

Even when global multi-homing is efficient, other types of equilibria also exist. Nonexclusivity, however, drastically modifies the nature of equilibria. Recall that under exclusivity, equilibria can be either dominant firm equilibria or market-segmentation equilibria. Here, market-segmentation equilibria disappear, and dominant firm equilibria, with only one active exchange website, do not exist if costs are negligeable.

Second, for large costs of registration and information-processing, global multi-homing are inefficient market allocations and the sole type of efficient equilibrium involves a dominant firm with some market power.

Nonetheless, market-sharing equilibria, where some users adopt a multi-homing strategy, may exist. Exchange websites then charge marginal cost for one group of users and jointly extract part of the matching surplus from the other group. As for global multi-homing, this market configuration exhibits some sort of smooth competition. However, the allocation of users is not efficient, and if the set of pricing instruments is rich enough, in particular if transaction fees are available, equilibrium profits are limited (and vanish as costs become negligeable). It may however be preferred by both firms to a dominant firm equilibrium.

The impact of transaction fees remains contrasted, however: the possibility of transaction fees improves profits resulting from global multi-homing, while it constitutes a major threat to equilibrium and reduces sustainable profits in dominant firm or market-sharing equilibria.

5 Conclusion

This paper has proposed a framework to analyze imperfect competition between matchmakers, with a particular emphasis on model specifications that may reflect relevant features of the intermediation activity on the Internet. A key aspect of intermediation is its "chicken & egg" nature: users of type 1 are interested in the services of a given exchange only if they expect users of type 2, with whom they want to be matched, to rely mostly on the services of the same exchange; and users of type 2 will indeed rely on this exchange if they expect enough users of type 1 to use the services of this exchange. Which class of users should then be attracted first, and how ?

We have provided answers in terms of characterizing the business strategies that make sense to get a bite on such a market. These are divide-and-conquer strategies, where one side of the market is subsidized and profits are made on the other side of the market, which can be more easily captured. The possibility of such business strategies have strong consequences in terms of market equilibrium and market structures that are likely to emerge. Multiplicity of outcomes is the rule, as should be expected. But not all equilibria have the same properties, in particular in terms of market monopolization and of sustainable profits.

When intermediation services require exclusivity and fees cannot be based on the realization of transactions, reasonable market equilibria involve either one dominant matchmaker, who monopolizes the market and makes strictly positive profits, or more balanced situations where fierce competition à la Bertrand erases all possibilities of profits from intermediation. This suggests that concentrated market structures and monopolization are likely to be observed in practice on such markets, along with a low level of profits. But intermediation providers on the Internet may implement sophisticated pricing strategies. Taking these into account opens the possibility of richer strategic considerations. Too many ways of stealing the competitors' business appear. Unsurprisingly, the strategic situation is very unstable and the only equilibrium situation that is tenable is for a firm to exert dominance on the intermediation market, i.e. to be the sole supplier of intermediation services, without enjoying any market power as potential entrants create a strong disciplinary device for the dominant firm. In some sense, this market is extremely contestable.

Intermediation services usually are not exclusive and users often heavily rely on the services of several intermediation providers. Users visit many intermediation websites, firms advertise on many market places,... When this possibility is introduced and the costs of intermediation are small enough, dominant firm equilibria or any other type of market-segmentation equilibrium where matchmakers' market shares do not overlap are deeply modified; either they do not exist, or they imply limited or zero profits. There exist equilibria where matchmakers share the market more peacefully and appear more as complements than as substitutes for intermediation. Market structures with overlapping market shares are then likely to emerge, where one or two groups of users rely on several matchmakers to satisfy their needs.

In these situation, the availability of a pricing instrument such as transaction fees has contrasted consequences. It mainly constitutes an additional weapon to be used for upsetting an equilibrium configuration. This threat limits market power. It is compensated by the possibility of endogenous differentiation in pricing, when global multi-homing is considered, and it does not reduce equilibrium profits then. But there is no such counter-benefit for other types of equilibria, and profits are unambiguously reduced in these other cases.

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A Exclusivity

Proof of Proposition 1. Consider the candidate equilibrium prices P^I and market allocation $\mathcal{N}(.)$. Assume that $n_i^E(P^I, P^E) = 0$ for all P^E such that $p_i^E > r_i^I - \lambda u_i$, $i = 1, 2$.

Following the text, consider a DC-strategy where E subsidizes 1-users with $p_1^E = r_1^I - \lambda u_1$ (slightly below) and attract 2-users with $r_2^E = \lambda u_2 + \inf \{p_2^I, 0\}$ (slightly below). The resulting profits are:

$$r_1^E + r_2^E - c = r_1^I + \lambda u_1 t^E - \lambda u_1 + \lambda u_2 + \inf \{p_2^I, 0\} - c,$$

and symmetrically for the other DC-strategy where 2-users are subsidized. $E's$ profit is maximal for $t^E = \bar{t}$, and for $P^I = (p_1, p_2, t)$ to be supported as a dominant firm equilibrium, it must necessarily be that:

$$p_1 + \lambda u_1 t + \inf \{p_2, 0\} + \lambda u_1 \bar{t} \leq c - \lambda(u_2 - u_1) \quad (15)$$

$$p_2 + \lambda u_2 t + \inf \{p_1, 0\} + \lambda u_2 \bar{t} \leq c + \lambda(u_2 - u_1), \quad (16)$$

and that I's profits are non-negative: $p_1 + p_2 + \lambda t \geq c$.

Conversely, any vector of prices $P^I = (p_1, p_2, t)$ satisfying (15), (16), (1) and the non-negative profit condition can be sustained as a dominant firm equilibrium with $P^E = P^I$ and the following market allocation:

- For the equilibrium prices P, users are pessimistic w.r.t. E: $n_i^I(P) = 1$ and $n_i^E(P) = 0$;
- For any deviation by E to prices $P'^E = (p_1^E, p_2^E, t^E)$ such that for both $i = 1, 2$, $p_i^E \geq p_i - \lambda u_i(1 - t)$, users hold pessimistic beliefs w.r.t. E so that $n_1^E(P^I, P'^E) = n_2^E(P^I, P'^E) = 0$;
- For any deviation by I to prices $P'^I = (p_1^I, p_2^I, t^I) \leq P^I$, $n_1^I(P'^I, P^E) = n_2^I(P'^I, P^E) = 1$ by monotonicity, while for any other deviation by I, $n_1^I(P'^I, P^E) = n_2^I(P'^I, P^E) = 0$.
- For other deviations, choose any market allocation compatible with monotonicity.

With this market allocation, I looses all its market if it raises any price, while E cannot obtain a positive profit. So this is an equilibrium.

To prove existence, let us maximize $I's$ profit $\pi^I = p_1 + p_2 + \lambda t - c$ subject to (15) , (16) and (1). First note that given $p_i + \lambda u_i t$, the conditions are the less stringent when $t = \bar{t}$. Then, given that $t = \bar{t}$, if $p_2 \leq 0$, (15) implies that $\pi^I \leq \lambda(u_2 - u_1)\bar{t} - \lambda(u_2 - u_1) \leq 0$. Thus we can restrict attention to cases where $t^I = \bar{t}$ and $p_2 \geq 0$. Condition (15) implies $p_1 + \lambda u_1 \bar{t} < c - \lambda(u_2 - u_1) - \lambda u_1 \bar{t} < \lambda u_1$. Using $u_1 + u_2 = 1$, the constraints then reduce to:

$$\begin{aligned} p_1 &\leq c - 2\lambda u_1 \bar{t} - \lambda(u_2 - u_1) = c - \lambda\bar{t} - \lambda(u_2 - u_1)(1 - \bar{t}) \\ p_2 + \inf\{p_1, 0\} &\leq c - 2\lambda u_2 \bar{t} + \lambda(u_2 - u_1) = c - \lambda\bar{t} + \lambda(u_2 - u_1)(1 - \bar{t}) \\ 0 &\leq p_2 \leq \lambda u_2(1 - \bar{t}) \end{aligned}$$

- If $c - \lambda\bar{t} < \lambda(u_2 - u_1)(1 - \bar{t})$, the solution is obtained at

  $$\begin{aligned} p_1 &= c - \lambda\bar{t} - \lambda(u_2 - u_1)(1 - \bar{t}) < 0 \\ p_2 &= \inf\{\lambda u_2(1 - \bar{t}); 2\lambda(u_2 - u_1)(1 - \bar{t})\} \end{aligned}$$

  which yields profits given by:

  $$\bar{\pi}^I = \inf\{\lambda u_1(1 - \bar{t}); \lambda(u_2 - u_1)(1 - \bar{t})\}.$$

- If $c - \lambda\bar{t} > \lambda(u_2 - u_1)(1 - \bar{t})$, then p_1 can be positive and the solution is

  $$\begin{aligned} p_1 &= c - \lambda\bar{t} - \lambda(u_2 - u_1)(1 - \bar{t}) > 0 \\ p_2 &= \inf\{\lambda u_2(1 - \bar{t}); c - \lambda\bar{t} + \lambda(u_2 - u_1)(1 - \bar{t})\} \end{aligned}$$

  which yields profits given by:

  $$\bar{\pi}^I = \inf\{\lambda u_1(1 - \bar{t}); c - \lambda\bar{t}\}.$$

■

Proof of Proposition 2. Denote $\rho_i^k = p_i^k + \lambda n_j^k u_i t^k$. A possible deviation for k is to slightly undercut prices P^k and capture all the market for a deviation profit (almost) equal to:

$$(\rho_1^k - c_1) + (\rho_2^k - c_2) + \lambda\left[(1 - n_1^k)u_2 + (1 - n_2^k)u_1\right]t^k. \tag{17}$$

A price system P and market allocation n_i^k can emerge in an equilibrium only if these deviations are not profitable: $\forall k$,

$$\begin{aligned} (\rho_1^k - c_1)n_1^k + (\rho_2^k - c_2)n_2^k \geq\ & (\rho_1^k - c_1) + (\rho_2^k - c_2) \\ & +\lambda\left[(1 - n_1^k)u_2 + (1 - n_2^k)u_1\right]t^k \end{aligned}$$

or equivalently:

$$(\rho_1^k - c_1)(1 - n_1^k) + (\rho_2^k - c_2)(1 - n_2^k) \leq -\lambda \left[(1 - n_1^k)u_2 + (1 - n_2^k)u_1\right] t^k. \tag{18}$$

Moreover, equilibrium profits have to be non-negative:

$$(\rho_1^k - c_1)n_1^k + (\rho_2^k - c_2)n_2^k \geq 0. \tag{19}$$

Using the fact that $n_i^{-k} = 1 - n_i^k$, (3) yields:

$$\rho_j^E = (1 - 2n_i^I)\lambda u_j + \rho_j^I.$$

Substituting for ρ_j^E and for n_j^E in (18) and (19) yields the following system of inequalities:

$$(\rho_1^I - c_1)n_1^I + (\rho_2^I - c_2)n_2^I \geq 0 \tag{20}$$

$$(\rho_1^I - c_1)(1 - n_1^I) + (\rho_2^I - c_2)(1 - n_2^I) + (1 - 2n_2^I)(1 - n_1^I)\lambda u_1 + (1 - 2n_1^I)(1 - n_2^I)\lambda u_2 \geq 0 \tag{21}$$

$$(\rho_1^I - c_1)(1 - n_1^I) + (\rho_2^I - c_2)(1 - n_2^I) \leq -\lambda \left[(1 - n_1^I)u_2 + (1 - n_2^I)u_1\right] t^I \tag{22}$$

$$(\rho_1^I - c_1)n_1^I + (\rho_2^I - c_2)n_2^I + (1 - 2n_2^I)n_1^I\lambda u_1 + (1 - 2n_1^I)n_2^I\lambda u_2 \leq -\lambda \left[n_1^I u_2 + n_2^I u_1\right] t^E \tag{23}$$

(20) and (23) imply:

$$(1 - 2n_2^I)n_1^I u_1 + (1 - 2n_1^I)n_2^I u_2 \leq -\lambda \left[n_1^I u_2 + n_2^I u_1\right] t^E \leq 0, \tag{24}$$

while (21) and (22) imply:

$$(1 - 2n_2^I)(1 - n_1^I)u_1 + (1 - 2n_1^I)(1 - n_2^I)u_2 \geq \lambda \left[(1 - n_1^I)u_2 + (1 - n_2^I)u_1\right] t^I \geq 0. \tag{25}$$

Considering only the inequalities of the LHS with respect to 0, it must necessarily be the case that either $(1 - 2n_2^I)$ and $(1 - 2n_1^I)$ have opposite signs or are both equal to 0. If they have opposite signs and, say, $(1-2n_2^I) < 0 < (1 - 2n_1^I)$, then $n_2^I > \frac{1}{2} > n_1^I$. So, (24) implies that:

$$\frac{(1 - 2n_1^I)u_2}{\mid 1 - 2n_2^I \mid u_1} \leq \frac{n_1^I}{n_2^I} < 1,$$

while (25) implies:

$$\frac{(1-2n_1^I)u_2}{\mid 1-2n_2^I \mid u_1} \geq \frac{1-n_1^I}{1-n_2^I} > 1.$$

Hence a contradiction.

So, it is necessary that $n_i^k = \frac{1}{2}$ for all i and k. Moreover, reintroducing the middle terms in inequalities (24) and (25), this in turn implies that $t^I = t^E = 0$. From (3), the symmetry of the allocation implies that $\rho_i^I = \rho_i^E$, hence $p_i^I = p_i^E = p_i$. Only symmetric equilibria, with symmetric market allocation can exist. Moreover, inequalities (20)-(23) imply that equilibrium profits have to be equal to zero:

$$p_1 + p_2 = c$$

To summarize, candidate equilibrium must necessarily be symmetric, with common prices such that $P^I = P^E = (p_1, p_2, t = 0)$, $p_i \leq \lambda \frac{u_i}{2}$ and $p_1 + p_2 = c$.

From the preceding proof, (p_1, p_2) is indeed an equilibrium if no firm can improve its profit with a DC-strategy, so that P must satisfy (15) and (16) in addition. There exists an equilibrium if the following system has a solution (p_1, p_2):

$$\begin{aligned}
p_1 + \inf\{p_2, 0\} + \lambda u_1 \bar{t} &\leq c - \lambda(u_2 - u_1), \\
p_2 + \inf\{p_1, 0\} + \lambda u_2 \bar{t} &\leq c + \lambda(u_2 - u_1), \\
p_2 &\leq \lambda \frac{u_2}{2}, \\
p_1 &= c - p_2 \leq \lambda \frac{u_1}{2}.
\end{aligned}$$

Note that if this system has a solution for $\bar{t}$, then the same prices form a solution of the system with $\bar{t}' < \bar{t}$. Note moreover that for $\bar{t} = 1$, the first two inequalities would imply negative prices; so, for $\bar{t} = 1$, there exists no solution.

Finally, for $\bar{t} = 0$, this system of inequalities reduces to the existence of p_2 such that:

$$\begin{aligned}
\lambda(u_2 - u_1) &\leq p_2 \\
\inf\{0, p_2 - c\} &\leq \lambda(u_2 - u_1) \\
c - p_2 &\leq \lambda \frac{u_1}{2} \\
p_2 &\leq \lambda \frac{u_2}{2}.
\end{aligned}$$

Given that $c < \lambda u_1 \leq \frac{\lambda}{2}$, such a p_2 exists if and only if $u_2 \leq \frac{2}{3}$. ■

B Multi-homing

Proof of Proposition 3. We follow the steps of analysis provided in the text.

First, E may first choose to act as *second-source.* There exists i with $p_i^E < 0$ and for $j \neq i$,

$$\begin{aligned} r_j^E &< \lambda u_j \left[1 - \lambda + \lambda t^E\right] \\ \theta_j &\leq t^E \leq \bar{t}. \end{aligned}$$

E's profits are then given by

$$\begin{aligned} p_i^E + p_j^E + \lambda(1-\lambda)t^E - c &< \lambda u_j \left[1 - \lambda + \lambda t^E\right] - \lambda u_j t^E + \lambda(1-\lambda)t^E - c \\ &= \lambda(1-\lambda)\left[u_j + t^E u_i\right] - c. \end{aligned}$$

Choosing $t^E \lesssim \bar{t}$, $p_i^E \lesssim 0$ and $r_j^E \lesssim \lambda u_j \left[1 - \lambda + \lambda t^E\right]$, and then optimizing w.r.t. i, E's profits can be made almost equal to:

$$\begin{aligned} \pi^{SS} &\equiv \lambda(1-\lambda)[u_2 + u_1 \bar{t}] - c, \quad \text{if } \theta_2 \leq \bar{t} \\ &= \lambda(1-\lambda)\left[u_1 + u_2 \bar{t}\right] - c, \quad \text{if } \theta_1 \leq \bar{t} < \theta_2. \end{aligned}$$

Secondly, E may set prices such that $t^E < t^I$. In this case, there exists i with $p_i^E < 0$ and

$$r_j^E < \lambda u_j \left[1 - \lambda + \lambda \max\{\theta_j; t^I\}\right].$$

E's profits are given by

$$r_i^E + r_j^E - c < \lambda t^E u_i + \lambda u_j \left[1 - \lambda + \lambda \max\{\theta_j; t^I\}\right] - c.$$

Setting optimally t^E as close as possible to t^I, with p_i^E and r_j^E as large as possible, yields profits almost equal to:

$$\pi^{Fi} = \lambda u_i t^I + \lambda u_j \left[1 - \lambda + \lambda \max\{\theta_j; t^I\}\right] - c.$$

Thirdly, E may choose $t^E \geq t^I$, subsidize i-users with $p_i^E < 0$ (and guarantee (6)) and prevent the same i-users to register with I with $t^E < \theta_i$. E's profits are given by:

$$r_i^E + r_j^E - c < \lambda t^E u_i + \lambda u_j \left[1 - \lambda + \lambda \max\{\theta_j; t^E\}\right] - c.$$

Setting optimally $t^E \lesssim \bar{\theta}_i = \inf\{\theta_i, \bar{t}\}$, with p_i^E and r_j^E as large as possible, yields profits almost equal to:

$$\pi^{ii} = \lambda u_i \bar{\theta}_i + \lambda u_j \left[1 - \lambda + \lambda \max\{\theta_j; \bar{\theta}_i\}\right] - c.$$

Finally, E may choose $t^E \geq t^I$, subsidize i-users with $p_i^E < 0$ (and guarantee (6)) and prevent the other group, that is j-users, to register with I with by setting $t^E < \theta_j$ and $r_j^E < r_j^I$. E's profits are given by:

$$r_i^E + r_j^E - c < \lambda t^E u_i + \lambda u_j \left[1 - \lambda + \lambda \theta_j\right] - c.$$

Setting optimally $t^E \lesssim \bar{\theta}_j = \inf\{\theta_j, \bar{t}\}$, with p_i^E and r_j^E as large as possible, yields profits almost equal to:

$$\pi^{ij} = \lambda u_i \bar{\theta}_j + \lambda u_j \left[1 - \lambda + \lambda \theta_j\right] - c.$$

To compare the various profit levels obtained for different pricing policies, note first that if there exists h, such that $t^I < \theta_h$, then

$$\begin{aligned}
\pi^{Fi} &= \lambda u_i t^I + \lambda u_j \left[1 - \lambda + \lambda \max\{\theta_j; t^I\}\right] - c \\
&\leq \lambda u_i \bar{\theta}_h + \lambda u_j \left[1 - \lambda + \lambda \max\{\theta_j; \bar{\theta}_h\}\right] - c \\
&\leq \begin{cases} \lambda u_i \bar{\theta}_i + \lambda u_j \left[1 - \lambda + \lambda \max\{\theta_j; \bar{\theta}_i\}\right] - c & \text{if } h = i \\ \lambda u_i \bar{\theta}_j + \lambda u_j \left[1 - \lambda + \lambda \theta_j\right] - c & \text{if } h = j \end{cases}
\end{aligned} \tag{26}$$

so that either $\pi^{Fi} \leq \pi^{ii}$ or $\pi^{Fi} \leq \pi^{ij}$. So, acting as a first source with global multi-homing is dominated by a strategy that induce some type of users not to register with I.

Consider first the easiest case where $\theta_{h_0} = \bar{t}^I = \max\{\theta_1, \theta_2, t^I\} > \bar{t}$. The second source strategy is feasible only if $\theta_{-h_0} \leq \bar{t}$, and yields, after rearranging: $\lambda(1-\lambda)\bar{t} + \lambda(1-\lambda)(1-\bar{t})u_{-h_0} - c$. The first source strategy is dominated. And among the four possibilities yielding π^{ii} or π^{ij}, with $i = h_0$ or $j = h_0$, the maximum can easily be written as:

$$\max_{i, j \neq i} \left\{\lambda \bar{t} u_i + \lambda u_j \left[1 - \lambda + \lambda \max\{\theta_j; \bar{t}\}\right]\right\} - c, \tag{27}$$

which is clearly larger than the profits under the second-sourcing strategy, when feasible. (27) therefore gives the best deviation profits for E.

Consider now the case where $\bar{t}^I = \max\{\theta_1, \theta_2, t^I\} \leq \bar{t}$. Let us first prove that among the strategies that allow E to process all transactions after a match, the highest profits are given by

$$\lambda \bar{t}^I u_1 + \lambda u_2 \left[1 - \lambda + \lambda \bar{t}^I\right] - c. \tag{28}$$

Note that if $t^I = \bar{t}^I$, the result is trivial since having one group not register with I is not a feasible strategy; hence the profits are π^{Fi}, maximized for $i = 2$. When there exists h_0 such that $t^I < \theta_{h_0} = \bar{t}^I \leq \bar{t}$, (26) proves that the first source strategy is dominated. Among the other strategies,

$$\begin{aligned}\pi^{(-h_0)(-h_0)} &= \lambda u_{-h_0}\theta_{-h_0} + \lambda u_{h_0}\left[1-\lambda+\lambda\theta_{h_0}\right] - c \\ &\leq \lambda u_{-h_0}\theta_{h_0} + \lambda u_{h_0}\left[1-\lambda+\lambda\theta_{h_0}\right] - c = \pi^{(-h_0)h_0}\end{aligned}$$

and similarly

$$\begin{aligned}\pi^{h_0(-h_0)} &= \lambda u_{h_0}\theta_{-h_0} + \lambda u_{-h_0}\left[1-\lambda+\lambda\theta_{-h_0}\right] - c \\ &\leq \lambda u_{h_0}\theta_{h_0} + \lambda u_{h_0}\left[1-\lambda+\lambda\theta_{h_0}\right] - c = \pi^{h_0 h_0}.\end{aligned}$$

Moreover, $\pi^{(-h_0)h_0}$ and $\pi^{h_0 h_0}$ are indeed feasible if $t^I < \theta_{h_0} = \bar{t}^I \leq \bar{t}$, even if $\theta_{-h_0} < t^I$. Now, the highest of both is precisely characterized by (28). So, when $\bar{t}^I \leq \bar{t}$, the best-response profit correspond either to (28) or to $\pi^{SS} = \lambda(1-\lambda)[u_2 + u_1\bar{t}] - c$. It is given by (28) if

$$\lambda\bar{t}^I u_1 + \lambda u_2\left[1-\lambda+\lambda\bar{t}^I\right] \geq \lambda(1-\lambda)[u_2 + u_1\bar{t}]$$

that is if:

$$\frac{(1-\lambda)u_1}{u_1+\lambda u_2}\bar{t} \leq \bar{t}^I \leq \bar{t},$$

and by π^{SS} otherwise. Q.E.D. ■

Proof of Proposition 4. Let us first look for symmetric multi-homing equilibria with transaction fees equal to t. If $t > 0$, a slight undercutting of $t'^E < t$ cannot lead to a decrease in E's market share. E could then make sure that it attracts all users and gets the transaction fee for all matches it performs. As one of the two firms obtains in equilibrium only a share of its matches, it would raise its profits. Thus, $t = 0$ necessarily.

For $t = 0$, summing conditions (7), we get $\lambda(1-\lambda) \geq p_1^k + p_2^k$. Equilibrium profit is nonnegative only if $\lambda(1-\lambda) \geq c$. If $\lambda(1-\lambda) \geq c$, a symmetric multi-homing equilibrium implies zero transaction fees. Condition (7) is compatible with profits equal to $\lambda(1-\lambda) - c$ only at prices $p_i^k = \lambda(1-\lambda)u_i$, which implies $\bar{t}^I = 0$. Clearly, undercutting prices (which cannot trigger bad-expectation market allocation by monotonicity) cannot be profitable. For any other deviation by E, there is a bad-expectation market allocation against E; choose $\mathcal{N}(P)$ such that this is the case. The best deviation with these pessimistic beliefs yields from Proposition 3:

$$\lambda(1-\lambda)[u_2 + u_1\bar{t}] - c < \lambda(1-\lambda) - c.$$

Therefore, a deviating firm cannot get more then the equilibrium profit $\lambda(1-\lambda) - c$. These profits correspond to the maximum profits attained within the set of symmetric multi-homing equilibria.

Consider now an asymmetric multi-homing equilibrium with $0 \leq t^I < t^E \leq \bar{t}$. First, it cannot profitable for I to undercut prices. E could however undercut I with a lower transaction fee (still preserving multi-homing market allocation, by monotonicity), thereby becoming first source instead of second source. Such a strategy is not profitable if t^I is small enough, that is:

$$t^I \leq (1-\lambda)t^E \tag{29}$$

For the other deviations, we can apply Proposition 3. From (7), it comes $\bar{t}^I \leq t^E = \bar{t}^E \leq \bar{t}$, with a straightforward extension of the notation where $\bar{t}^E = \max\{\theta_1^E, \theta_2^E, t^E\}$ and θ_i^E defined by $r_i^E = \lambda u_i \left[1 - \lambda + \lambda\theta_i^E\right]$. From (7), E's profits are bounded from above by $\lambda(1-\lambda) - c$, which must then be nonnegative. So, the other conditions for nonexistence of a profitable deviation are easily obtained:

$$\begin{aligned}
\lambda\bar{t}^I u_1 + \lambda u_2\left[1 - \lambda + \lambda\bar{t}^I\right] &\leq r_1^E + r_2^E - \lambda^2 t^E \leq \lambda(1-\lambda) &(30)\\
\lambda(1-\lambda)[u_2 + u_1\bar{t}] &\leq r_1^E + r_2^E - \lambda^2 t^E \leq \lambda(1-\lambda) &(31)\\
\lambda t^E u_1 + \lambda u_2\left[1 - \lambda + \lambda t^E\right] &\leq r_1^I + r_2^I &(32)\\
\lambda(1-\lambda)[u_2 + u_1\bar{t}] &\leq r_1^I + r_2^I. &(33)
\end{aligned}$$

It is clear that if an asymmetric multi-homing equilibrium exists with r_i^E, there also exists one with $\hat{r}_i^E = \lambda(1-\lambda)u_i + \lambda^2 u_i t^E$, and the same t^E, which yields a profit for E exactly equal to: $\lambda(1-\lambda) - c$. This allows existence more easily and improves I's profit. For such equilibria, (31) trivially holds and (30) becomes:

$$\lambda\bar{t}^I u_1 + \lambda u_2\left[1 - \lambda + \lambda\bar{t}^I\right] \leq \lambda(1-\lambda)$$

which can be written as:

$$\begin{aligned}
t^I &\leq \frac{(1-\lambda)u_1}{u_1 + \lambda u_2} &(34)\\
r_i^I &= p_i^I + \lambda u_i t^I \leq \left[\frac{\lambda + u_1}{\lambda u_2 + u_1}\right]\lambda(1-\lambda)u_i. &(35)
\end{aligned}$$

It is therefore possible to find an asymmetric multi-homing equilibrium with prices p_i^I and $\hat{r}_i^E$ if there exists (t^I, t^E) such that:

$$\begin{aligned}
\inf\{1-\lambda+\lambda t^E, (1-\lambda)(\frac{\lambda+u_1}{\lambda u_2+u_1})\} &\geq \max\{(1-\lambda)\left[u_2+u_1\bar{t}\right], u_2(1-\lambda+\lambda t^E)+u_1 t^E\} \\
(1-\lambda)\inf\{t^E, \frac{u_1}{\lambda u_2+u_1}\} &\geq t^I.
\end{aligned}$$

It is easy to check that for any t^E with $0 \leq t^E \leq \bar{t} \leq 1$, the first set of inequalities holds and so, such an equilibrium always exists. I's profit is larger than or equal to $\lambda(1-\lambda)-c$, hence not smaller than E's profit. Moreover, in an asymmetric multi-homing equilibrium, I's profit is maximal for t^E as large as possible, hence given by

$$\lambda \inf\{1-\lambda+\lambda\bar{t}, (1-\lambda)(\frac{\lambda+u_1}{\lambda u_2+u_1})\} - c,$$

which is smaller than $\lambda - c$ and strictly larger than E's maximal profit. ■

Proof of Proposition 5. Recall first the necessary conditions provided in the text are: $r_i^I \leq \lambda u_i$ and $p_i^E \geq 0$. Then, note that it is not necessary to look at undercutting strategies with the monotonicity restriction since I could not possibly gain from undercutting while the market allocation following E's undercutting is not restricted at all since E has no market share in a dominant firm equilibrium. Therefore, we only need to guarantee that E's best response profits in Proposition 3 are non-positive.

The conditions can be split in two parts, depending on whether $\bar{t}^I \leq \bar{t}$ or $\bar{t}^I > \bar{t}$. If P^I is such that $\bar{t}^I \leq \bar{t}$, conditions are: for all $i = 1, 2$,

$$\begin{aligned}
\lambda(1-\lambda)\left[u_2+u_1\bar{t}\right] &\leq c && (36)\\
\lambda t^I u_1 + \lambda u_2\left[1-\lambda+\lambda t^I\right] &\leq c && (37)\\
\lambda\theta_i u_1 + \lambda u_2\left[1-\lambda+\lambda\theta_i\right] &\leq c. && (38)
\end{aligned}$$

The resulting profits for I are given by

$$r_1^I + r_2^I - c = \lambda(1-\lambda) + \lambda^2\left[u_1\theta_1 + u_2\theta_2\right] - c.$$

They are unaffected by t^I as long as r_i^I (or θ_i) are maintained constant. Moreover, reducing t^I relaxes the condition for existence. So, we investigate the condition for existence with $t^I = 0$, and (37) can then be omitted as it is implied by (36). So, if (36) holds, existence conditions in this range coincide with the conditions for existence of θ_1, θ_2, smaller or equal to $\bar{t}$, satisfying (38) and such I's profits are non-negative. These reduces to:

$$\lambda(1-\lambda)\left[u_2 + u_1\bar{t}\right] \leq c \leq \lambda\left[1-\lambda+\lambda\inf\left\{\frac{c-\lambda(1-\lambda)u_2}{\lambda(u_1+\lambda u_2)};\bar{t}\right\}\right].$$

Straightforward algebra proves that one condition is implied by $c \leq \lambda$. So, we are left with necessary conditions for existence with $\bar{t}^I \leq \bar{t}$:

$$\lambda(1-\lambda)\left[u_2 + u_1\bar{t}\right] \leq c \leq \lambda\left[1-\lambda+\lambda\bar{t}\right].$$

Under these circumstances, the highest profit for I is given by:

$$\inf\left\{\lambda\left[1-\lambda+\lambda\bar{t}\right]-c;\frac{(\lambda-c)}{u_1+\lambda u_2}(1-\lambda)u_1\right\}.$$

If, instead, P^I is such that $\bar{t}^I \geq \bar{t}$, Proposition 3 implies that conditions are:

$$\begin{aligned}
\lambda\bar{t}u_1 + \lambda u_2\left[1-\lambda+\lambda\max\{\theta_2,\bar{t}\}\right] &\leq c \\
\lambda\bar{t}u_2 + \lambda u_1\left[1-\lambda+\lambda\max\{\theta_1,\bar{t}\}\right] &\leq c.
\end{aligned}$$

These can be written as:

$$\max\left\{r_i^I;\lambda u_i\left[1-\lambda+\lambda\bar{t}\right]\right\} \leq c-\lambda\bar{t}u_j,$$

for $i = 1, 2$ and $j \neq i$, and consequently,

$$\lambda u_2\left[1-\lambda+\lambda\bar{t}\right]+\lambda\bar{t}u_1 \leq c. \tag{39}$$

If this condition is not satisfied, there can be no dominant firm equilibrium with $\bar{t}^I \geq \bar{t}$. If it holds, I's profits in a dominant equilibrium would be bounded from above by $c - \lambda\bar{t}$, which under (39) is positive.

Conversely, suppose that

$$\lambda(1-\lambda)[u_2+u_1\bar{t}] \leq c < \lambda u_2\left[1-\lambda+\lambda\bar{t}\right]+\lambda\bar{t}u_1. \tag{40}$$

Then, prices $P^I = P^E$ such that $t^I = 0$ and

$$r_i^I = \lambda u_i\frac{c+(1-\lambda)u_1}{u_1+\lambda u_2},$$

can be supported in a dominant firm equilibrium with bad expectation equilibria against any deviating matchmaker, except undercutting deviation by I. Maximal profits for I are then equal to $\frac{(\lambda-c)}{u_1+\lambda u_2}(1-\lambda)u_1$. Similarly, if

$$\lambda u_2\left[1-\lambda+\lambda\bar{t}\right]+\lambda\bar{t}u_1 \leq c < \lambda\left[1-\lambda+\lambda\bar{t}\right],$$

then prices $P^I = P^E$ such that $t^I = 0$ and $r_i^I = \lambda u_i [1 - \lambda + \lambda \bar{t}]$ can be supported in a dominant firm equilibrium, and $r_i^I = c - \lambda \bar{t} u_j$ can also be. The maximal profit for I is given by:

$$\max \{c - \lambda \bar{t}; \lambda [1 - \lambda + \lambda \bar{t}] - c\}.$$

This maximum is attained for $\lambda [1 - \lambda + \lambda \bar{t}] - c$ if

$$c \leq \lambda(1 - \lambda)\frac{(1 - \bar{t})}{2},$$

which is not compatible with the lower bound on c for existence. So, in this case, I's maximal profits are: $c - \lambda \bar{t}$. Finally, if

$$\lambda [1 - \lambda + \lambda \bar{t}] \leq c \leq \lambda u_1,$$

prices $P^I = P^E$ such that $t^I = 0$ and $r_i^I = c - \lambda \bar{t} u_j$ can be supported in equilibrium with maximal profits for I equal to $c - \lambda \bar{t}$.

As a final point, the existence condition implies that $c \geq \lambda(1 - \lambda)u_2$ and therefore:

$$\frac{(\lambda - c)}{u_1 + \lambda u_2}(1 - \lambda)u_1 \leq \lambda(1 - \lambda)u_1 < c.$$

And so, I's profits are always smaller or equal to c. ■

Proof of Proposition 7. For $(p_i^k, p_j^k, t^k, n_j^k)$ to be supported as a market-sharing equilibrium, (11), (12), (13) and (14) must necessarily hold. Other deviations, as given in Proposition 3, must also be nonprofitable. So, if $\bar{t}^k > \bar{t}$,

$$\begin{aligned} \pi^{-k} &\geq \lambda \bar{t} u_i + \lambda(1 - \lambda)u_j + \lambda^2 \max \{r_j; \bar{t} u_j\} - c && (41) \\ \pi^{-k} &\geq \lambda \bar{t} u_j + \lambda(1 - \lambda)u_i + \lambda^2 \max \left\{p_i^k + n_j^{-k} u_i t^k; \bar{t} u_i\right\} - c, && (42) \end{aligned}$$

while if $\bar{t}^k \leq \bar{t}$,

$$\pi^{-k} \geq \lambda u_1 \bar{t}^k + \lambda(1 - \lambda)u_2 + \lambda^2 u_2 \bar{t}^k - c. \quad (43)$$

The set of all these conditions is also sufficient to characterize an equilibrium with a bad expectation market allocation after any deviation that does not simply consists in undercutting, along the lines developed in the proof of Proposition 3.

If t^k is reduced, while holding ρ_i^k and $r_j^k = r_j$ constant, then $\bar{t}^k$ (weakly) decreases and π^k is unchanged since,

$$\pi^k = p_i^k - c_i + n_j^k(r_j - c_j).$$

Hence, this change only relaxes the set of constraints. So, without loss of generality, we can look for existence conditions of a market-sharing equilibrium with $t^k = 0$, for $k = I, E$, hence with $p_j^k = p_j$ and $\bar{t}^k = \max\{\frac{p_j}{u_j}, \frac{p_i^k}{u_i}\}$.

Let us now assume that $\lambda = 1$ and $u_h \geq 2c_h$, $h =, 1, 2$. Our objective is to maximize total profit $\pi^1 + \pi^2$. For this we will distinguish various cases depending on the value of $\bar{t}^k$ at the maximal profit allocation.
Case 1: $\bar{t}^k \geq \bar{t},\ k = 1, 2$.

Notice that this is only possible if $\bar{t} < \frac{1}{2}$.

Deviations are taken into account through (41) and (42). Moreover, if there exists a market-sharing equilibrium with (p_i^k, p_j, n_j^k), then there also exists a market-sharing which is symmetric and such that $p_i = \frac{p_i^I + p_i^E}{2}$, $n_j^k = \frac{1}{2}$, p_j is unchanged and total profits of intermediation are preserved. Hence, we can narrow our analysis to the search for the highest-profit, symmetric equilibrium with:

$$\begin{aligned}
p_i &\leq \frac{u_i}{2} \\
p_j &\leq c_j \\
\bar{t}u_i + p_j - c &\leq p_i - c_i + \frac{1}{2}(p_j - c_j) \\
\bar{t}u_j + p_i - c &\leq p_i - c_i + \frac{1}{2}(p_j - c_j) \\
\max\{\frac{p_i}{u_i}, \frac{p_j}{u_j}\} &\geq \bar{t}
\end{aligned}$$

The last condition reduces to:

$$2\bar{t}u_j - c_j \leq p_j \leq 2p_i + c_j - 2\bar{t}u_i. \tag{44}$$

Increasing p_i relaxes the constraints and increases profits. So, $p_i = \frac{u_i}{2}$ maximizes profits, as well as:

$$p_j = c_j.$$

Condition (44) requires that

$$\bar{t} \leq \frac{c_j}{u_j}.$$

When this holds, profits are $\frac{u_i}{2} - c_i$ which is the maximal feasible profit given that $p_j \leq c_j$. We thus conclude that when $\bar{t} \leq \frac{c_j}{u_j}$, the maximal equilibrium profit is indeed $\frac{u_i}{2} - c_i$, when i-users register with both intermediaries.

Case 2: $\bar{t} > \frac{c_j}{u_j}$.

Suppose that $\bar{t}^1 \leq \bar{t} \leq \bar{t}^2$. As $\frac{p_j}{u_j} \leq \frac{c_j}{u_j} \leq \frac{1}{2} < \bar{t}$, this case requires $\bar{t}^2 = \frac{p_i^2}{u_i} \geq \bar{t} > \frac{p_i^1}{u_i}$. Equilibrium conditions imply

$$p_i^1 - c_i + n_j^1(p_j - c_j) \geq \bar{t}u_j + p_i^2 - c$$

But this implies that $p_i^1 > p_i^2$, which contradicts $\bar{t} < \frac{p_i^2}{u_i}$.

Hence the equilibrium verifies $\bar{t}^k < \bar{t}$, $k = 1, 2$.

Deviations are now taken into account through (43). As in the previous case, one can narrow the study to the case of the highest-profit, symmetric equilibrium, hence with conditions:

$$\begin{aligned} p_i &\leq \frac{u_i}{2} \\ 0 \leq p_j &\leq c_j \\ \max\left\{\frac{p_j}{u_j}; \frac{p_i}{u_i}\right\} - c &\leq p_i - c_i + \frac{1}{2}(p_j - c_j). \end{aligned}$$

This last condition can be written as:

$$u_i\left[\frac{p_j + c_j}{2}\right] + \frac{p_j - c_j}{2} \leq u_j p_i \leq u_i\left[\frac{p_j + c_j}{2}\right]$$

Then, maximal profits are obtained for $p_i = \frac{u_i c_j}{u_j}$ and $p_j = c_j$, and profits are nonnegative if

$$\pi = \frac{u_i}{u_j}c_j - c_i \geq 0,$$

that is, if $\frac{c_j}{u_j} \geq \frac{c_i}{u_i}$.

Overall we find that when $\frac{c_j}{u_j} \geq \frac{c_i}{u_i} \geq \bar{t}$, there can be two types of equilibria, and the maximal profit is obtained when users $h_0 \in \arg\max_h \left\{\frac{u_h}{2} - c_h\right\}$ register with both intermedairies. When $\frac{c_j}{u_j} \geq \bar{t} > \frac{c_i}{u_i}$, then there is only one type of equilibria and i-users register with both intermediaries with price $\frac{u_i}{2}$. When $\bar{t} > \frac{c_j}{u_j} \geq \frac{c_i}{u_i}$, then there is only one type of equilibria and i-users register with both intermediaries, with price $\frac{u_i}{u_j}c_j$. ■