The Social Cost of Air Traffic Delays

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This version December 2014

Abstract:
The so-called buffer time or buffer delay allows airlines to control for excessive delays by introducing extra time in their schedule in addition to what is technically required. We study the differences between unregulated markets - where airlines are free to fix their buffer times strategically - and a situation where a social planner would control for time schedules, and in particular the buffer time. To do so, we use a calibrated model of a network of three cities - one of them being a hub - served by a single airline. Welfare losses that follow from delays are relatively small as compared to the potential benefits that would follow from a decrease in ticket prices. The model thus advocates that, at least for the connections that are considered, fares rather than delays should be the focus of institutions aiming at enhancing passengers’ welfare.

JEL Classification: R41, L50, L93
Keywords: Airlines, Delays, Social Optimum, Calibration

Acknowledgements: The authors are grateful for their comments and suggestions to our referee Martin Dresner and participants at GARS06 (Amsterdam), our discussants Peter Forsyth at the TPUG07 conference (Chicago) and Cletus Coughlin at the ITEA 2010 Kuhmo Nectar Conference and also participants at the seminar "Economie du secteur aérien" organized by the “Ministère de l’Ecologie, de l’Energie, du Développement Durable et de la Mer” 2010. Detailed comments from Jan Brueckner, Benoit Durand, Philippe Gagnepain, Milena Petrova and Patrick Rey have been very helpful. Yet, all remaining errors are ours. We are thankful to the Direction Generale de l'Aviation Civile (DGAC) of the French Ministry of Transport for the provision of data.
1. Introduction

The permanent delays that plague air traffic question the economist about the possible reasons for this apparent market failure. Leaving aside the traditional explanation – namely congestion – this paper is aimed at explaining how the transportation system as a whole copes with exogenous events that result in delays on single flights. As we show below, neither the minimum travel time, nor the difference between the realized and scheduled arrival times are actually of any help to assess the social cost of delays. This cost is to be measured in reference to a socially optimal schedule, which does not reduce the travel time to its minimum, nor necessarily wash out any delay. While the socially optimal schedule should in general differ from the one corresponding to profit maximization, we evidence that, at least on the network we consider, the resulting cost is small as compared to the benefits that would follow a decrease in fares.

During the last decades, airports have regularly experienced delays. According to the Central Office for Delays Analysis about 40 percent of European flights have been delayed by at least 5 minutes between 2003 and 2010. During that same period, according to the Bureau of Transportation Statistics, more than 20 percent of flights landed with at least 15 minutes delays in the US. These figures decreased over the last years as a result of the 2008-2009 economic crisis, during which the total number of flights went down by 6.6 percent, after years with a growth rate of about 3.5 percent. However, the observed delays remain high and, since January 2010, passenger traffic is again growing in all regions of the globe. This growth is expected to continue: Airbus forecasts a yearly traffic growth of 4.7 percent for the period 2013-2033; Boeing’s forecast, although slightly smaller still amounts to 4.1 percent, meaning that traffic is expected to double in less than 17 years. Delays are thus unlikely to disappear any soon.

There are essentially two channels that should be acted upon in order to reduce delays. First, an increase in airport and air traffic management capacity could be needed, at least to cope with the traffic growth. Second, congestion charges should be introduced for an efficient use of existing capacities. For both action channels, an estimation of the costs of delays is of primary importance.

Although the estimates may differ substantially, the impact of delays over the economy is unanimously considered as significant. In the U.S., the Joint Economic Committee (2008) evaluated the cost of delays (for both airlines and passengers) at $ 31 billion for 2007, while the Air Transport Association (2013) estimated it to amount to $ 7 billion for 2012. In 1996, as a

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1Sources: Airbus, 2014 and Boeing, 2014.
consequence, the Federal Aviation Administration created the National Center of Excellence for Aviation Operators (NEXTOR), a university-industry-government partnership aimed at developing capabilities for modeling, designing and evaluating the performance and safety of partially decentralized, more flexible, advanced technology concepts for air traffic management and airports. In Europe, delays are estimated to cost airlines between €1.3 and €1.9 billion a year.\(^2\) To deal with this issue, the Commission is promoting the so-called Single European Sky and the Single European Sky ATM Research Program (SESAR), which is the European version of NEXTOR.\(^3\) SESAR also sustains the development of alternatives to air transportation, such as the High-Speed Rail System.\(^4\) Delays are considered to be such a burden that the Commission proposed specific measures to protect consumers. (See European Commission, 2001.) Since 2005, monetary compensations are to be provided to passengers in case of denied boarding to all flights from the E.U. and/or with destination in the E.U. (See European Commission, 2005.) It is also the case for “long delays.” Moreover, the E.U. Court of Justice ruled on 19th November 2009 that when the delay is longer than three hours, compensations may be payable as if the flight was cancelled.\(^5\)

Delays are usually considered to be the direct product of congestion, which in turn is considered to be, at least in part, a consequence of externalities attached to air traffic. As a matter of fact, an airline has no incentive to account for the burden its flights’ schedule may impose on others operations. Congestion charges may help inducing firms to “internalize” the above mentioned externalities and they are broadly accepted as key to achieve efficiency in congested airports and reduce overall delays. See, among others, Levine (1969), Carlin and Park (1970), Park (1971) or Morrison et al. (1989). Obviously, the higher its market share, the larger the proportion of externalities internalized by an airline, thus the smaller the optimal congestion charges. Daniel (1995) was the first to study the internalization of congestion at hub airports. Brueckner (2002; 2005) shows that congestion is fully internalized at airports dominated by a monopolist. Similar results are obtained by Pels and Verhoef (2004), Janic and Stough (2006), Zhang and Zhang (2006) or Ater (2012). However, empirical evidence is weak as suggested by Morrison and Winston (2007) or Rupp (2009). Moreover, congestion pricing involves several issues. First, as suggested by Schank (2005), it is hard to implement it effectively. Second, given the lack of competitors (at least on many European routes) and the barrier for new airlines to enter at least in most of European hubs, linking congestion fees and market power in the

\(^3\) See [http://www.sesarju.eu/](http://www.sesarju.eu/)
\(^5\) Joined cases of Sturgeon v Condor Flugdienst GmbH and Bock and others v Air France SA C-402 and 423/07
design of optimal taxation (i.e. charging more small operators) has undesirable (anti-competitive) effects that may well outweigh expected benefits.

A closer look at the actual management of airlines – more precisely flight scheduling and operation management – may however provide us with interesting insights on the issue of delays. Airlines can indeed leverage delays through different channels, but mainly through the so-called flight-time buffer, which is defined as the extra-time introduced in the schedule in addition to the minimum required travel time. Flight-time buffer is a very widespread practice, in particular among airlines whose network is organized around a hub. It provides indeed a simple precautionary method to ensure that all passengers are able to get their connections in time. Flight-time buffer can yet turn out to be useful also for carriers that do not offer any connections, i.e., low cost airlines. In fact, by “padding” the schedule, airlines are able to recover from delays more easily; they improve the predictability of rotations and their punctuality performance with respect to published schedules.

Thus, this practice does not only reduce the costs to passengers, which usually consider less painful to have a longer journey than to endure unexpected delays; it also drives down airlines’ operational costs. However, most airlines do not choose a flight buffer time that leads to an average delay equal to zero. As pointed out in a report produced by Cook et al. (2004), an airline finds it worth adding minutes of flight-time buffer to the schedule only “up to the point at which the cost of doing this equals the expected cost of the tactical delays they are designed to absorb.” Clearly, the very fact that some type of “delays” may be profitable to firms does not say anything about its social desirability. Here we nevertheless evidence that “delays” can also benefit society, and in particular, positive flight-time buffer may also benefit travelers.

Indeed, although delays are often considered as a totally undesirable phenomenon, they may however also entail some profits both for airlines and consumers. Mayer and Sinai (2003) already point out that delays cannot be explained by congestion externalities only. When delays are measured as the difference between actual travel time and minimum travel time, they observe that delays are not reduced to zero at the optimum. In their model, transport services are organized according to a hub-and-spoke system in which a single round-trip flight from (and to) the hub connected with N airports generates 2N different journeys. There are thus network benefits attached to clustering flights at the hub as it may allow more passengers to connect.

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6 It is a common practice of airlines operating at hubs in the U.S. to schedule longer flight times at the end of the day on a same market operated with the same aircraft. Some airlines also introduce more buffer time in the morning to avoid concatenating delays all over the day. In France, this practice is not allowed and airlines must choose a single scheduled time for all their flights on a given market operated with the same aircraft.

7 See Cook et al. (2004).
Then, the authors provide evidence that longer “delays” (waiting time) at the hubs are the efficient equilibrium outcome for an airline equating marginal (congestion) costs of an additional flight with its marginal (network) benefits.

The very idea of social gains coming from congestion is also present in Betancor and Nombela (2002) who show that although it may generate delays, an increase in frequency of services can raise the welfare of travelers. Nombela et al. (2004) suggest that socially optimum flight-time buffer is likely to be strictly positive. This is to say that congestion or delays are not to be regarded as evidence of system distress.

More generally, the assessment of any transportation system cannot spare an explicit reference to social optimum. Some delays might be desirable, even from the travelers’ point of view. Therefore, when attempting to estimate “social costs of delays,” it is economically restrictive to consider the sole observed delays and give them a monetary value by coining a value of time. This is the approach taken until now in the literature, which is not profuse while there is a profuse literature of value of time (See for instance Shires and Jong, 2009.). Indeed, few studies have been devoted to the estimation of the costs of delays in air transportation. The research undertaken by Nombela et al. (2002) and the reports by the Institut du Transport Aérien (ITA 2000), Ball et al. (2010) and by Cook et al. (2004) are the main references. Their results are summarized in Table 1 in the Appendix.8 These three reports consider observed delays and flight-time buffers for the estimation of cost. In all cases, however, the estimated values of airline and passenger costs depend heavily upon the value of time estimates that are taken from previous works and are relatively heterogeneous.

It is manifest that, when estimating the inefficiencies streaming from delays, one should not consider the difference between realized travel time and minimum travel time. For society, the real costs of delays stream from the difference between actual travel time and optimum travel time, namely the expected travel time when flight-time buffers are set to their (socially) optimum level. The social cost of “delays” (observed and buffer) is therefore the loss in welfare that follows from a socially non optimal scheduling decided by the firm. And the sole “delay” for which the firm can be unequivocally rebuked is the difference between the scheduled flight-time buffer and the optimal one. This is at least the approach adopted in our model and, more generally, the view we advocate.

Characterizing the optimal policy and providing a guideline for its implementation would be a very challenging exercise in a real context. Given its complexity (and the potential costs attached to it), we rather choose to assess the potential benefits of such a policy through a

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8 All tables and figures are gathered in an Appendix at the end of the text.
The results of our calibration exercise suggest that, in the considered case, (i) Optimal flight-time buffers are smaller than actual flight-time buffers (i.e., travelers would prefer to have shorter journeys, even at a cost of more delays) and (ii) the welfare losses that follow from sub-optimal scheduling are relatively small as compared to the potential benefits that would follow from other regulatory efforts such as a decrease in ticket prices. However, these results are tied to the features of the particular network we study here.

To sum up, this paper aims at making precise the real issues at stake when facing the problem of delays in air-transportation. It also provides a methodology in order to estimate the social cost of delays. The latter is illustrated with a simple calibrated model. The paper is organized as follows. In section 2, we introduce the notations of the model. Section 3 presents the firm’s and passengers’ problems and outlines how demand parameters can be recovered. The maximization of social welfare is discussed in section 3.3. In section 4, we describe the data for the market under scrutiny, namely the city pairs formed by Toulouse, Paris and Nice. Then using the calibrated demand parameters and these data, we apply the proposed methodology in section 5. The last section presents our conclusions.

2. Notations and definitions for travel time

Consider a direct flight between two cities $i$ and $j$. The flow of passengers between these cities is denoted $X_{ij}$. The travel time between cities $i$ and $j$ is affected by a stochastic delay $\varepsilon_{ij}$ distributed according to some density function $f_{ij}$.

Airlines control these delays announcing larger scheduled travel time than the minimum (and in most of the cases, average) observed travel time. The minimum technical time required to fly between the two cities is denoted, $T_{ij}$, and we denote as $\tau_{ij}$ the flight-time buffer, i.e. the extra-time introduced in the schedule in addition to the minimum required travel time. The scheduled travel time is therefore the sum of the minimum required travel time, $T_{ij}$, and the flight-time buffer $\tau_{ij}$. Under this setting passengers suffer an apparent delay only if the stochastic delay is larger than the flight-time buffer, $\varepsilon_{ij} > \tau_{ij}$, as shown in Figure 1.

Passengers often use a connecting flight in order to reach their final destination. The flow of passengers between cities $i$ and $f$ flying by a hub is denoted $X_{ihj}$ with the index $h$ referencing the

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9 Sensible in the sense that the simulated situation is based upon a reduced network but observed data in France.
hub. Under optimal conditions, passengers between $i$ and $j$ connecting at the hub $h$ require some minutes to get their connection. Let $\Delta_{ihj}$ represent the minimum time technically required to reach the departure gate after landing. Airlines could sell tickets with a connecting time equal to $\Delta_{ihj}$. However, missing a connection represents a high cost for passengers. In order to reduce this risk and similarly to the flight-time buffer introduced on direct flights, carriers schedule the connecting flights at a later time by introducing a connection buffer $\delta_{ihj} > 0$. The connecting time is thus $\Delta_{ihj} + \delta_{ihj}$. Passengers are expected to arrive to the hub at time $T_a + \tau_a$ and depart at time $T_a + \tau_a + \Delta_{ihj} + \delta_{ihj}$. If no connection buffer is introduced, any apparent delay on the first flight implies a missed connection. As airlines introduce connection buffer, the apparent delay must be higher than the buffer to miss the connecting flight. By doing so, carriers increase the expected travel time of connecting passengers. There is thus a first trade-off between the costs and benefits of the connection buffer, $\delta_{ihj}$. Observe that this trade-off regards connecting passengers only. Ater (2012) suggests that hub-carriers spread out their waves of arriving and departing flights to control for delays when their market share increases, and consequently impose longer connecting time for their passengers.

Hence, as $\delta_{ihj}$ is strictly positive, the expected travel time is larger than the minimum possible time, which says that missing the plane is costly. We also know that even if buffer time is included, delays remain possible. The plane will arrive too late with a positive probability since increasing expected travel time is costly. Thus, when the realization of the stochastic delay exceeds the flight-time buffer and the connection buffer, $\epsilon_a > T_a + \delta_{ihj}$, airlines may consider delaying departure of the second flight by $\gamma_{hj}$ minutes to wait for connecting passengers. This is a real delay as opposed to $\delta_{ihj}$, i.e., scheduling later the departure. It benefits connecting passengers but creates a cost to all passengers of the flight departing from the hub. We define $\gamma_{hj}$ as the “held-flight” buffer. Thus passengers with an arrival delay, $\epsilon_a$, smaller than $\tau_a + \delta_{ihj} + \gamma_{hj}$ will not miss their connecting flight. We expect that $\gamma_{hj} \leq \delta_{ihj}$ as marginal benefits from both buffers, passengers gaining their connection in case of small delays, are equal while the marginal costs of adding one minute of held-flight buffer $\gamma_{hj}$ is higher than the marginal cost of the connection buffer $\delta_{ihj}$. Likewise we can expect $\gamma_{hj} > 0$, as missing the connection has a cost for both passengers and airlines. In other words, some diffusion of delays is to be observed. Note, however, that the later phenomenon is conditional on the delay being “sufficiently large”,

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i.e., larger than the buffers, $\epsilon_{ih} > \tau_{ih} + \delta_{ih}$, and yet, not “too large” as the costs of delaying the flight must not overcome the gains for the connecting passengers.

## 3. Model Specification

### 3.1. The Passenger’s problem

We assume that passenger’s demand is affected linearly by the price $P$ and the expected travel time for any city pair. According to our definitions for flight time, connection and held-flight buffers, three different expected travel times are possible according to the city of origin and destination. Hence there exist three types of demands corresponding to the three types of markets, each associated to one city pair:

- Markets arriving to the hub $h$, with demand $X_{ih}$;
- Markets departing from the hub with a demand $X_{hi}$;
- Markets with a connection at the hub, with a demand $X_{ihj}$.

**Markets arriving to the hub**

In the simplest case, i.e., flights arriving to the hub, the demand is expressed as follows:

$$X_{ih} = \alpha_{ih} + \beta_{ih} \left( P_{ih} + \nu \left( T_{ih} + \tau_{ih} + Ed_{ih} \right) \right),$$

where $\alpha_{ih} \geq 0$ and $\beta_{ih} \leq 0$ are constant parameters specific to the routes. The parameter $\nu \geq 0$ is common to all routes and denotes the passengers’ value of expected travel time. The expected travel time is equal to the scheduled time, $T_{ih} + \tau_{ih}$, plus the average or expected delay, $Ed_{ih}$. Koster et al. (2013) show that the mean delay is a good proxy for the total expected passenger costs.

Delays, as random events, pose higher costs for passengers than costs linked to scheduled travel time, $\nu$. Small (1993), Khattak (1995) or De Palma and Rochat (1996) estimate these cost of delays between 1.03 and 2.69 times the cost of scheduled travel time. We decompose the cost of delays as the product between the value of time $\nu$ and a parameter $\theta$ larger than 1. The demand is then rewritten as
\[ X_{ih} = \alpha_{ih} + \beta_{ih} \left( P_{ih} + \nu \left( T_{ih} + \tau_{ih} + \Theta \int_{\tau_{ih} + \epsilon}^{\infty} (\epsilon - \tau_{ih}) f_{ih} (\epsilon) d\epsilon \right) \right), \]  

\hspace{1cm} \text{(2)}

Under this specification, delays represent a linear cost even though passengers penalize more severely long than short delays. There are several possibilities to introduce a nonlinear approach in the valuation of time.\(^{10}\) We follow Mahmassani and Chang (1987), who propose a “band of indifference” so that passengers do not punish carriers for small delays. Travelers suffer a cost, \( \nu \theta \), only if delays are superior to a significant threshold, \( \epsilon \), such that:

\[ X_{ih} = \alpha_{ih} + \beta_{ih} \left( P_{ih} + \nu \left( T_{ih} + \tau_{ih} + \Theta \int_{\tau_{ih} + \epsilon}^{\infty} (\epsilon - \tau_{ih} - \epsilon) f_{ih} (\epsilon) d\epsilon \right) \right), \]  

\hspace{1cm} \text{(3)}

\[ \text{Markets departing from the hub} \]

Passengers flying from the hub, \( h \), to a spoke, \( j \), also benefit from a direct flight. However, their expected travel time is subject to a higher uncertainty. If the realized delay attached to flights from \( i \) to \( h \) exceeds the flight-time and connection buffers, that is, if \( \epsilon_{ih} > \left( \tau_{ih} + \delta_{ih} \right) \), the carrier may introduce a held-flight buffer, \( \gamma_{hj} \), allowing connecting passengers to get their connection. This decision is based on the aggregated distribution of delays from flights arriving at the hub with connecting passengers.

The expected travel time for passengers flying from the hub is increased by the expected held-flight buffer introduced by the airline. Their demand is specified as:

\[ X_{hj} = \alpha_{hj} + \beta_{hj} \left( P_{hj} + \nu \left( T_{hj} + \tau_{hj} + \Theta \text{Ed}_{hj} \right) \right), \]  

\hspace{1cm} \text{(4)}

where \( \text{Ed}_{hj} \) represents the expected delay for the flight between \( h \) and \( j \) and is given by

\[ \text{Ed}_{hj} = \int_{\tau_{hj} + \delta_{hj} + \gamma_{hj}}^{\tau_{hj} + \delta_{hj} + \epsilon_{hj}} (\epsilon_{ih} - \tau_{ih} - \delta_{ih} + \epsilon_{hj} - \tau_{hj} - \epsilon) f_{hj} (\epsilon_{hj}) f_{ih} (\epsilon_{ih}) d\epsilon_{hj} d\epsilon_{ih} + \]

\[ \int_{\tau_{hj} + \delta_{hj} + \gamma_{hj}}^{\epsilon_{hj}} f_{hj} (\epsilon_{hj}) d\epsilon_{hj} + \int_{\tau_{hj} + \delta_{hj} + \gamma_{hj}}^{\epsilon_{hj}} f_{ih} (\epsilon_{ih}) d\epsilon_{ih} \int_{\tau_{hj} + \epsilon}^{\infty} (\epsilon_{hj} - \tau_{hj} - \epsilon) f_{hj} (\epsilon_{hj}) d\epsilon_{hj} \]  

\hspace{1cm} \text{(5)}

\[ \text{Markets with a connection at the hub} \]

\hspace{1cm} \text{---} 

\(^{10}\) See, for instance, Carrion and Levinson (2012) for a review on value of time reliability.
Finally, connecting passengers flying from $i$ to $j$ through the hub face the longest and most complex journey. They risk suffering a delay at the end of their trip, $\varepsilon_{ij}$. However, their total travel time is mainly determined by the delays observed at their first flight, $\varepsilon_{ih}$. If the latter are small, their journey equals the minimum travel time required for the trip. If instead they are between $\tau_{ih} + \delta_{ih} + \gamma_{ij}$, the airline introduces a held-flight buffer, $\gamma_{ij} = \varepsilon_{ih} - \tau_{ih} - \delta_{ih}$, to ensure the connection. If delays $\varepsilon_{ih}$ are bigger than $\tau_{ih} + \delta_{ih} + \gamma_{ij}$ passengers lose their connecting flight and wait for the next scheduled flight. This implies an extra waiting time that we denote $\omega_{ij}$. The airline gives a compensation $C_{ij}$ to each passenger losing his/her connection. Thus, the demand is obtained as:

$$X_{ij} = \alpha_{ij} + \beta_{ij} \left( P_{ij} - C_{ij} \left( 1 - F_{ih} \left( \tau_{ih} + \delta_{ih} + \gamma_{ij} \right) \right) + \nu \left( T_{ih} + \tau_{ih} + \Delta_{ih} + \delta_{ij} + T_{ij} + \tau_{ij} + \theta Ed_{ij} \right) \right)$$

where

$$Ed_{ij} = \left\{ \begin{array}{c}
\int_{\tau_{ih} + \delta_{ih}}^{\tau_{ih} + \delta_{ih} + \gamma_{ij}} \left( \varepsilon_{ih} - \tau_{ih} - \delta_{ih} + \varepsilon_{ij} - \tau_{ij} - \omega_{ij} \right) f_{ih} \left( \varepsilon_{ih} \right) d\varepsilon_{ih} \\
\int_{\tau_{ih} + \delta_{ih}}^{\tau_{ih} + \delta_{ih} + \gamma_{ij}} f_{ih} \left( \varepsilon_{ih} \right) d\varepsilon_{ih}
\end{array} \right\}$$

3.2. The Airline’s Problem

We assume that an airline in a monopolistic position is serving all markets, which is a common case within regional markets. In particular, despite the liberalization process in air transportation, there is still a low level of competition on European markets as suggested by Billette de Villemeur (2004) or Neven et al. (2006). The airline offers direct flights between any city $i$ and the hub $h$ and offers indirect services from $i$ to $j$ through $h$. Therefore the three types of demand presented in the previous section correspond to three types of markets, each associated with one city pair:

The cost to convey $X_{ih}$ passengers from city $i$ to the hub $h$ is assumed to obey the following functional form:
\[ C_{ih} (X_{ih}) = a + (c + bX_{ih})T_{ih} + \tau_{ih}C_{\tau} + C_{\varepsilon} \max \left( \varepsilon_{ih} - \tau_{ih}, 0 \right), \]  \hspace{1cm} (7) 

where \( a, c \) and \( b \) are strictly positive parameters, \( T_{a} \) represents the minimum travel time between the cities, \( \tau_{a} \) is the flight-time buffer and \( C_{\tau} \) is the cost linked to adding one minute of flight-time buffer, which is expected to be smaller than the cost for a minute of observed delay \( C_{\varepsilon} \).

Observe that airlines face a tradeoff since increasing buffer time (and its cost \( C_{\tau} \)) reduces the observed delays (and their cost \( C_{\varepsilon} \max (\varepsilon_{a} - \tau, 0) \)). These delays are distributed according to a cumulative distribution \( F_{\varepsilon}(\varepsilon) \). The delay distribution is considered to be (exogenously) given. In particular, like the optimal network they do not depend upon (the pattern of) flows. In this sense, the model does not consider congestion issues or network optimality.

The firm maximizes the expected total profit by choosing the prices, the flight-time buffer, the connection buffer and the held-flight buffer. The profit is a function of the length of delays. If \( \varepsilon_{a} < (\tau_{a} + \delta_{a}) \), connecting passengers catch their flights. The firm does not need to introduce held-flight buffer and profits are at their maximum level,

\[
\Pi \left( P_{ih}, P_{hj}, P_{hij}, \delta_{ij}, \gamma_{ij}, \tau_{ij}, \tau_{hj} \mid \varepsilon_{ih} < (\tau_{ih} + \delta_{ij}) \right) = \sum_{i} \sum_{j} \left( P_{ih} \cdot X_{ih} - \delta_{ij} C_{\delta} - (\tau_{ih} + \tau_{hj}) C_{\tau} \right) + \sum_{i} \left( P_{ih} \cdot X_{ih} - a - (c + b(X_{ih} + \sum_{j} X_{hij}))T_{ih} - C_{\varepsilon} \int_{\tau_{ih}}^{\tau_{hj}} (\varepsilon_{ih} - \tau_{ih}) f_{ih}(\varepsilon_{ih}) d\varepsilon_{ih} \right) \]

\[
+ \sum_{j} \left( P_{hj} \cdot X_{hj} - a - (c + b(X_{hj} + \sum_{i} X_{hij}))T_{hj} - C_{\varepsilon} \int_{\tau_{hj}}^{\tau_{ih}} (\varepsilon_{hj} - \tau_{hj}) f_{hj}(\varepsilon_{hj}) d\varepsilon_{hj} \right) \]  \hspace{1cm} (8) 

\( C_{\varepsilon} \) represents the cost per minute for the airline of suffering a delay. It is assumed to be the same for all flights, but our results remain unchanged if we assume different values for each route/plane. Let \( C_{\delta} \) represent the opportunity cost for airlines, i.e., the cost airlines support by choosing how long planes stay at airports and how much time they fly. When an airline raises the connection buffer at the hub, it also drives down the number of plane rotations; therefore the plane spends more time on the ground than in the air, and consequently costs per day decrease. If the airline reduces the connection buffer, the plane spends less time at the hub and increases its rotations in a day so that total costs rise.
Notice that under this setting, considering prices for round trip tickets instead of one way tickets would not affect Equation (8). The number of passengers in each market is on average the same for any direction, i.e.; the flow of passengers between Madrid and Paris is equal to the flow between Paris and Madrid, say. However, prices may be different for passengers buying a roundtrip from Paris to Madrid rather than from Madrid to Paris. We would therefore still require different prices for different markets.

If \( \varepsilon_{ih} > (\tau_{ih} + \delta_{ij}) \) the airline can introduce a held-flight buffer equal to \( \varepsilon_{ih} - \tau_{ih} - \delta_{ij} \) at a cost per minute of \( C_r \), or passengers flying from \( i \) to \( j \) lose their connecting flight and the carrier pays a penalty \( C_y \) per passenger due to the new regulation on delays. \( (\varepsilon_{ih} - \tau_{ih} - \delta_{ij})C_y \) is subtracted from (8) in the first case and \( C_y X_{hj} \) in the second one.

Hence total expected profit results from the addition of the three possible cases, i.e., small delays, delays requiring to wait for connecting passengers and large delays with passengers missing their connections, and we obtain:

\[
\Pi \left( P_{ih}, P_{hj}, \delta_{ij}, \gamma_{ij}, \tau_{ih}, \tau_{hj} \right) = \\
\sum_i \left( P_{ih} X_{ih} - a - (c + b(X_{ih} + \sum_j X_{ij}))T_{ih} - C_\varepsilon \int_{\tau_{ih}}^{\varepsilon_{ih}} \left( \varepsilon_{ih} - \tau_{ih} \right) f_{ihj}(\varepsilon_{ih})d\varepsilon_{ih} \right) + \\
\sum_j \left( P_{hj} X_{hj} - a - (c + b(X_{hj} + \sum_i X_{ij}))T_{hj} - C_\varepsilon \int_{\tau_{hj}}^{\varepsilon_{hj}} \left( \varepsilon_{hj} - \tau_{hj} \right) f_{hij}(\varepsilon_{hj})d\varepsilon_{hj} \right) + \\
\sum_{ij} \left( P_{ihj} X_{ij} - \delta_{ij} C_\delta - (\tau_{ih} + \tau_{hj})C_\tau - C_y X_{hj} \left( 1 - F_{ihj}(\tau_{ih} + \delta_{ij} + \gamma_{ij}) \right) \right) - \\
C_y \sum_{ij} \left( \int_{\tau_{ihj} + \delta_{ij}}^{\tau_{ihj} + \delta_{ij} + \gamma_{ij}} \left( \varepsilon_{ih} - \tau_{ih} - \delta_{ij} + \varepsilon_{hj} - \tau_{hj} \right) f_{hij}(\varepsilon_{hj}) f_{ihj}(\varepsilon_{ih}) d\varepsilon_{ih} d\varepsilon_{hj} \right) 
\]

(9)

The firm maximizes profit choosing \( P_{ih}, P_{hj}, \tau_{ih}, \tau_{hj}, \delta_{ij} \), and \( \gamma_{ij} \). At equilibrium, the profit maximizing conditions together with the demand equations must be satisfied. From this system of equations we can recover all unknown parameters \( (\alpha_{ih}, \beta_{ih}, \alpha_{hj}, \beta_{hj}, \nu, \theta, C_\alpha, C_\gamma) \).

### 3.3. Welfare

Our aim is to evaluate the difference between optimal social welfare and welfare at equilibrium. Welfare results from the addition of passengers’ surplus and airline’s profits. Passengers’ surplus can be recovered once the parameters of the demand have been calibrated.
Indeed the representative passenger is assumed to maximize his net utility given by
\[ U_j = P_j X_j - \nu E_t X_j, \]
where \( U_j \) is the gross utility that consumers obtain from travelling and \( E_t \) is the expected travel time. To maximize its net utility, the condition
\[ U'(X_j) = P_j + \nu E_t = \tilde{P}_j \]
must be satisfied, where \( \tilde{P}_j \) is the generalized price. Once we calibrate the parameters of demand, the gross utility obtained by the passengers can be recovered by integrating the generalized price, \( P_j \) over \( X_j \), namely:

\[
X_j = \alpha_j + \beta_j \left( P_j + \nu E_t \right) = \alpha_j + \beta_j \tilde{P}_j \quad \Rightarrow \quad \tilde{P}_j = \frac{X_j - \alpha_j}{\beta_j}
\]

\[
U_j = \int_0^{X_j} U'(x)dx = \int_0^{X_j} \tilde{P}(x)dx = \frac{(X_j)^2}{2\beta_j} - \frac{\alpha_j X_j}{\beta_j}, \tag{10}
\]

where each demand \( X_j \) responds to Equations (3), (4) and (6).

We compute the connection, flight-time and held-flight buffers (\( \delta_{ih}, \tau_{ih}, \gamma_{ih} \)) maximizing social welfare:

\[
\max_{\tau_{ih}, \gamma_{ih}, \delta_{ih}} \left[ U_{ih} + U_{hj} + U_{ihj} - \text{Costs for passengers} + \text{Profits of the Firm} \right] \tag{11}
\]

**Definition**: The difference between welfare evaluated at \( \delta_{ih}^*, \tau_{ih}^*, \tau_{hj}^* \) and \( \gamma_{ih}^* \) and welfare at equilibrium represents the social cost of delays.

Of course, the exact value of social cost of delays depends on the underlying market conditions. Market conditions will determine the chosen prices according to the imposed \( \delta_{ih}, \tau_{ih}, \tau_{hj} \) and \( \gamma_{ih} \).

4. **Data**

Our model is validated in a network composed of the city-pairs Toulouse-Paris \((i,h)\), Paris-Nice \((h,j)\) and Toulouse-Nice \((i,h,j)\), where Paris operates as a hub. Data are available for all flights in the network during May and June of 2004. At the time Toulouse –Paris was the 5th
busiest city pair in Europe and Paris – Nice the 9th. Data for frequencies of flights, number of passengers, capacity and scheduled travel time are summarized in Table 2 for each direct route of the main airline. Although one competitor was present on the routes Toulouse-Paris and Paris-Nice, the degree of competition was low since it only offered 4 daily flights on each route (compared with the 23 and 20 flights offered by the incumbent) transporting 14.8 percent and 16.3 percent of daily passengers respectively.

On average 5 percent of passengers arriving to Paris from Toulouse and Nice took another plane to get to their final destination. We assume that connecting passengers on Paris-Nice represent 5 percent of the total passengers on this route since the airline’s decision about the connection and held-flight buffers is made as a function of the total number of connecting passengers in the studied market.

Table 3 presents the unknown parameters and the values used in our calibration some of which are detailed in this section. We had information on more than 100 listed prices for different kinds of consumers under different conditions, yet we do not observe the final choice of consumers and the percentages of business and leisure travelers remain unknown. It is therefore impossible to compute the average price paid by a representative consumer. However our robustness analysis, in Section 5.1, shows that prices have minor effects over the calibration of demand, and do not affect the optimal buffer and extra time so that the gain in welfare remains unaffected.

Arrival delays on each route, \( E_{ij} \), are assumed to be distributed according to a gamma distribution. Its scale and shape parameters are estimated by maximum likelihood. Although the airline’s decisions are determined according to the aggregated distribution of delays from all flights arriving at the hub, we assume that this aggregated distribution of arrivals is equal to the distribution of the single flight arriving at the hub that we are considering. Delays are stochastic. Nevertheless, as stated previously, they can be controlled introducing a flight-time buffer for the schedule and a held-flight buffer at the tactical level. Table 4 presents the average delays for our markets. Although most delays occur when the load of planes is higher than the average (peak traffic), long delays occur at flights with lower load than the average delayed flight. This implies that airlines have more control over delays than just flight-time buffer as we assume in our model.

The cost of adding a held-flight buffer minute, \( C_j \), is available from different studies as shows Table 1. Estimations by Nombela et al. (2002) and Cook et al. (2004; 2011) include costs due to lost market share and lost corporate image. These costs are captured in our model by the very dependence of the passengers’ demand on the expected travel time. Therefore we
considered a lower range of values proposed by ITA (2000), between 35.5 and 50.9 €/min.\textsuperscript{11} The same range of values is considered for $C_\gamma$. As the airline mostly operates the same airplane model, we assume that $b$ and $C_\gamma$ are the same for all routes. We consider low values for $b$, between 0.005€ and 0.03€, since $b$ measures the variable cost per passenger and minute on a given flight, i.e., the costs per kilometer derived from the increase in gasoline consumption from an additional passenger.

Finally, $C_f$ is the hard cost for the airline of a passenger losing his/her connection, that is to say compensations and rebooking. We are assuming that at the studied period these costs were zero. In section 5.1 we study the effect on social welfare of the inclusion of compensations as proposed by the E.U. Commission.

5. Results

The proposed values for the calibration exercise as well as the calibrated parameters are presented in Table 5. The absolute values of the price elasticities are larger than 1 as expected, given the monopoly assumption: 1.02 for Toulouse-Paris, 1.02 for Paris-Nice and 1.03 for Toulouse-Paris-Nice.

The cost of delays resulting from the product of the ratio $\theta$ and the value of buffer time $\nu$ remains pretty stable with values comprised between 0.85 and 0.95 Euros per minute (51-57 Euros per hour) even if we apply significant changes to any parameter in the calibration exercise. This can be considered as high when compared to the values proposed in the studies compiled in Table 1. However, the rapport prepared by the Commissariat Général à la Stratégie et à la Prospective (Quinet, 2013) presents values between 52 and 72 Euros for air transport according to the travel purposes for short delays (less than 10 minutes) on an unspecified distance within France.

It is interesting to notice the role of non-linearities in the cost of delays on the calibration. The ratio $\theta$ increases with the threshold fixed for significant delays (obtaining always values superior to 1 from a threshold larger than or equal 5 minutes of delay). This sustains the hypothesis that the cost of delays is not linear and increases with the size of the delay.

\textsuperscript{11} Nonetheless, given the thorough information provided by Cook \textit{et al.} (2004) we can also compute the cost for the studied route considering only at-gate and taxi delay costs of the airplanes used in the routes. We obtain an average cost of 15.7 Euros per minute. This value implies a lower estimated value of time at the calibration of demand, but it does not imply any change on the optimal choice of buffer and extra delays.
With respect to buffer time, $\nu$, our value of 0.69 euros per minute (41.4 euros per hour) is as expected lower than the cost of delays. The study by Cook et al. (2004) proposes a value between 0 and 16.3 euros per flight-time buffer minute for airlines. However the study states that “these are fairly rudimentary estimates.” Besides, it takes into account only the airline’s costs while we consider the overall effects over airlines and passengers. The study by ITA (2000) assumes that the passengers cost of flight-time buffer is equal to the cost of delays which seems to be far from reality. It also assumes that airlines’ cost for flight-time buffer is even slightly bigger than the cost of delays, which makes unreasonable the existence of the flight-time buffer. We could also compare this value with the proposed values of time in the literature, for instance Bickel et al. (2006) proposed values between 16 and 38€ for France suggestions: , “and Quinet 2013 values between 52 and 72”.

Given the calibrated demands, we can calculate the flight-time buffer, connection buffer and held-flight buffer that maximize social welfare: $\delta_{ij}^* = 12.91$, $\gamma'_{ij} = 0$, $\tau_{ih}^* = 11.75$ and $\tau_{ij}^* = 9.67$ minutes. Whatever the changes applied to the parameters used in the study, at the optimal solution all buffers decrease. At the social optimum, passengers enjoy a smaller average travel time and face a higher probability of suffering delays and thus more chance of losing their connection. The held-flight buffer disappears at the optimum due to the high cost of introducing held-flight buffer for the carrier and due also to some factors characterizing our markets such as the low probability of losing a connection, the low extra waiting time or the low number of connecting passengers that would profit from this buffer compared to the number of passengers who would suffer its consequences. If some of these factors are attenuated we find optimal solutions where the held-flight buffer is again larger than zero while it remains always lower than the value at equilibrium.

If these values are imposed on the airline, the gain in welfare for society represents only a 1.43 percent increase with respect to the welfare observed at equilibrium. Under this optimal solution, demand increases in the three considered routes even if prices increase. For Paris-Toulouse-Nice the demand increases by 7 percent while the price increases by 6.8 percent. Toulouse-Paris’ passengers increase by 4.5 percent coupled with a price increase of 4.4 percent. Finally we observe an increase of 3 percent in the demand for Paris-nice and 2.9 percent increase in the price.

5.1. Sensitivity analysis
The effects on demand calibration and optimal welfare of a measurement error or a variation are negligible for most of the parameters. For instance, changes in the minimum time required for passengers to connect, $\Delta_{ihj}$, or changes in the variable cost per passenger, $b$, have insignificant effects over the calibration of demand and the optimal social choice. Other parameters such as $C_{ij}$, have more relevant effects. From the conditions that must be satisfied at equilibrium we know that changes in the cost of the held-flight buffer affect proportionally the introduction of the held-flight buffer by airlines. This affects also the gain in welfare when we compel the optimal levels of $\delta_{ihj}$ and $\gamma_{ij}$ which, however, do not suffer any significant variation. Other variables require a more detailed analysis.

**Changes in Prices:** The selection of prices has a small effect over the calibration of demand, and does not affect the optimal flight-time buffer, connection buffer and held-flight buffer so that the gain in welfare remains unaffected. In particular, if price were 25 percent lower only parameters $\alpha$ and $\beta$ are modified on each demand, but no change is observed on the cost of delays and the optimal schedule choices. Still, this price reduction has important effects over the absolute level of welfare since it affects the consumers’ surplus. With the actual setting, welfare increases almost by 13 percent. Instead, decreasing flight-time buffer by more than a half increases welfare only by 1.4 percent (the same effect can be obtained with a 2.5 percent price reduction). Therefore, effects of changes in prices over welfare are of first order magnitude while changes of flight-time buffer produce a second order effect over welfare.

**Introduction of delay compensations $C_{ij}$:** At equilibrium delay compensations were set equal to zero. We analyze the effects of introducing compensations for passengers losing their connections given the low probability of suffering long delays on the studied markets. Any compensation leads to higher prices for connecting passengers and a decrease on welfare. Only very high compensations affect the buffer choices, increasing especially the minutes of buffer at the connecting airport. Still, the effects remain minor for small compensations. Although the airline could also decide to increase the flight-time buffer for the second segment of the flight (Paris-Nice), it would have to compensate all the direct passengers with a price reduction. If we calibrate the highest compensation for delays proposed by the European Commission for a market with less than 1500 km, which attains 250€, welfare diminishes by 3.8 percent, more than double the possible gain from imposing optimal delays.

**Changes over the number of connecting passengers:** Ceteris paribus, for a higher number of connecting passengers, we expect to find a smaller cost of delays and therefore a smaller gain in welfare. In fact, for a similar connection buffer and a higher number of connecting passengers (which implies that direct flight passengers have decreased), the probability of passengers
loosing connections rests unchanged while their weight over the market increases. Therefore, the cost of delays is less important than in the case where connecting passengers are fewer. Vice versa, if we believe that the number of connecting passengers was smaller than what we assumed, we would expect a higher cost of delays.

Cost of delays estimations are sensible to changes in the number of connecting passengers, especially when we decrease it. By contrast, the optimal flight-time buffer and held-flight buffer remain almost unaltered. If we decrease the number of connecting passengers by 20 percent, we observe that the cost of delays increases to 1.09€/ minute (an increase of 27 percent). Conversely when we increase this value by the same proportion, the cost of delays decreases to 0.26€/minute (-19 percent) and the gain in welfare to 6.15€ (-10 percent). If we keep increasing the number of connecting passengers the cost of delays decreases. Still, almost no effect is observed over optimal buffer times and gains in welfare. Also, a reduction (or an increase) in the number of connecting passengers is accompanied by a reduction (or increase) in the minutes of held-flight buffer introduced to wait for connecting passengers, which implies an opposite effect over the calibration of buffer time.

**Changes in the distribution of exogenous delays:** Changes on the distribution of delays have minor effects both over the calibration and the optimal choices. Exogenous changes due to shocks or changes in security laws have also small effects over welfare with respect to the reduction or increase of delays for this particular network. The effect could instead be significant if it implied the entry of new competitors on congested airports.

### 6. Conclusions

Travel time delays constitute a widespread phenomenon in air-transportation. This paper is a first-attempt to make precise the issues at stake in order to draw a consistent policy. It also provides a methodology to estimate the social costs of delays. The latter is illustrated by the means of a simple calibration.

We consider a single, profit-maximizing operator. Complex pricing schemes do not come as an issue since we adopt a representative agent approach. All passengers have the same value of time and, for each city-pair, demand is derived from quasi-linear preferences as represented by quadratic utilities.

With this simple yet (in our view) realistic model, we obtain very clear-cut results from a calibration exercise performed with exhaustive data over a two-month period. Airlines should
decrease their flight-time buffer. That is to say, a socially optimal schedule would result in shorter journeys but more apparent delays.

The effects over welfare of these changes, however appear to be small. There are several reasons for this. First, the low number of connecting passengers over the sample and the high frequency of services. Second and more importantly, scheduling is only one dimension of the analysis. As long as pricing is not subject to any constraint, firms are able to extract a fair amount of consumer (gross) surplus. Thus, because an increase in consumer surplus ultimately leads to an increase in their profits, airlines account for traveler benefits while taking their scheduling decisions. This is to say: the only difference between profit-maximizing and socially optimal scheduling stem from pricing imperfections.

While free scheduling yields very little inefficiency from a social welfare point of view, clearly, the other side of the coin is that profit-maximizing pricing is likely to be very harmful, absent competition. If public intervention is to be considered, this is the place.

Overall and in any case, the proposed E.U. policy on compensation for long delays appears either to be ineffective or to result in reduced social welfare.

This paper has several limitations. First, as a consequence of the representative agent approach, travelers have an identical value of time. It follows that market segmentation is exogenous. Would consumers have had heterogeneous characteristics, optimal choice theory would have indeed provided a natural endogenous split of travelers across available services. Second, passengers are risk neutral. This is obviously a point to take into account and we plan to look at the consequences of risk aversion in the near future. Observe, however, that the latter can only change numerical estimates. All conclusions drawn here are robust to the introduction of risk aversion as they do not hinge upon the particular values attached to time losses. They directly follow from the economic mechanisms at hand.

Second, the model assumes that the distribution of delays is exogenous and therefore it proposes only two tools to control the level of congestion:

- the extra delays that are included on the second flight due to the delay on the first flight
- the proposed values for the cost of adding a held-flight buffer minute, \( C_f \), include congestion costs

Wiping off this assumption would probably enhance the results and provide lesser impact of the optimal policy: with congestion the increase of delays implied by the optimum will be reached with a lower increase of traffic, and the difference between the optimal solution and the monopolist solution will be smaller than without congestion.
Finally, some may point to the monopoly assumption as being quite restrictive. Yet, according to Tournut (2004), 60 percent of the routes in the world are operated through a monopolistic position. And, according to Billette de Villemeur (2004), the figure raises to 85 percent for the routes over the French territory. Obviously, optimal delays (hence costs) depend upon the market situation. Thus, whenever competition occurs (within the air-transportation mode or across transportation modes), it has to be taken into account in order to derive consistent empirical estimates. That said, we are rather confident that our main conclusions would persist with such an enrichment of our model.
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market power ». Journal of Urban Economics 60 (2): 229-47. 
Appendix: Figures and Tables

Figures

Figure 1: Flight-time buffer and distribution of delays

Figure 2: Distribution of delays affecting connecting passengers
### Tables

#### Table 1: Summary on Studies of Costs of Air Traffic Delays for Europe

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kind of delays</strong></td>
<td>Schedule and Buffer</td>
<td>Schedule</td>
<td>Schedule and Buffer</td>
<td>Schedule and Buffer*</td>
</tr>
<tr>
<td><strong>Airlines</strong></td>
<td></td>
<td>39.4 - 48.6 €/min</td>
<td>83.3 €/min</td>
<td>$4.6 billion</td>
</tr>
<tr>
<td><strong>Estimated costs</strong></td>
<td></td>
<td>0.57 – 0.73 €/min</td>
<td>0.26 €/min</td>
<td>$0.63/min</td>
</tr>
<tr>
<td><strong>Passengers</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Flight-time buffer are estimated in a theoretical way but not included on the final estimation of costs.

#### Table 2: Average data May –June 2004

<table>
<thead>
<tr>
<th>Direct Flights</th>
<th>Toulouse-Paris</th>
<th>Paris-Nice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total passengers</td>
<td>177414</td>
<td>166831</td>
</tr>
<tr>
<td>Total number of flights</td>
<td>1432</td>
<td>1228</td>
</tr>
<tr>
<td>Average Passengers per flight</td>
<td>123.9</td>
<td>135.9</td>
</tr>
<tr>
<td>Travel time (minutes)</td>
<td>80</td>
<td>85</td>
</tr>
<tr>
<td>Frequencies a</td>
<td>23.5</td>
<td>20.1</td>
</tr>
<tr>
<td>Airplane b</td>
<td>A320</td>
<td>A320</td>
</tr>
<tr>
<td>Capacity c</td>
<td>161.9</td>
<td>168.1</td>
</tr>
<tr>
<td>Average occupation</td>
<td>76.5%</td>
<td>80.8%</td>
</tr>
</tbody>
</table>

a Average frequency of flights per day; b Most frequent plane; c Average capacity of the planes operated on the route.
### Table 3: Results from the calibrated demands

<table>
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<tr>
<th>Values proposed for the calibration</th>
<th>Unknown parameters</th>
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<tr>
<td>$X_{ih}$ 118.47</td>
<td>$\gamma_{bh}$ 0.81</td>
</tr>
<tr>
<td>$X_{bj}$ 130.52</td>
<td>$C_{\gamma}$ 40</td>
</tr>
<tr>
<td>$X_{abj}$ 5.34</td>
<td>$C_{\varepsilon}$ 40</td>
</tr>
<tr>
<td>$P_{ih}$ 80</td>
<td>$b$ 0.02</td>
</tr>
<tr>
<td>$P_{bj}$ 95</td>
<td>$\Delta_{abj}$ 20</td>
</tr>
<tr>
<td>$P_{abj}$ 120</td>
<td>$\delta_{abj}$ 25.04</td>
</tr>
<tr>
<td>$T_{ih}$ 55</td>
<td>$\omega_{abj}$ 40</td>
</tr>
<tr>
<td>$\tau_{ih}$ 25</td>
<td>$\omega_{abj}$ 40</td>
</tr>
<tr>
<td>$T_{bj}$ 64</td>
<td>$C_{ij}$ 0</td>
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<tr>
<td>$\tau_{bj}$ 21</td>
<td>$\theta$ 1.25</td>
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<tr>
<td>$\varepsilon$ 10</td>
<td></td>
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### Table 4: Average Delays for flights and passengers

<table>
<thead>
<tr>
<th>At Departure</th>
<th>At Arrival</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flights</td>
<td>29.3%</td>
</tr>
<tr>
<td></td>
<td>4.4 min</td>
</tr>
<tr>
<td>Flights with delay &gt; 15 min</td>
<td>7.7%</td>
</tr>
<tr>
<td></td>
<td>35.3 min</td>
</tr>
<tr>
<td>Passengers</td>
<td>31.9%</td>
</tr>
<tr>
<td></td>
<td>5.1 min</td>
</tr>
<tr>
<td>Passengers with delay &gt; 15 min</td>
<td>8.4%</td>
</tr>
<tr>
<td></td>
<td>36.2 min</td>
</tr>
<tr>
<td>Values proposed for the calibration</td>
<td>Results</td>
</tr>
<tr>
<td>------------------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>$X_{ih}$ 118.47 $\gamma_{hj}$ 0.81</td>
<td>$\alpha_{ih}$ 323.35</td>
</tr>
<tr>
<td>$X_{hj}$ 130.52 $C_{\gamma}$ 40</td>
<td>$\beta_{ih}$ -1.50</td>
</tr>
<tr>
<td>$X_{ihj}$ 5.34 $C_{\epsilon}$ 40</td>
<td>$\alpha_{hj}$ 346.60</td>
</tr>
<tr>
<td>$P_{ih}$ 80 $b$ 0.02</td>
<td>$\beta_{hj}$ -1.39</td>
</tr>
<tr>
<td>$P_{hj}$ 95 $\Delta_{ihj}$ 20</td>
<td>$\alpha_{ihj}$ 17.46</td>
</tr>
<tr>
<td>$P_{ihj}$ 120 $\delta_{ihj}$ 25.04</td>
<td>$\beta_{ihj}$ -0.05</td>
</tr>
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<td>$T_{ih}$ 55 $\delta_{ihj}$ 20</td>
<td>$\nu$ 0.69</td>
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<tr>
<td>$\tau_{ih}$ 25 $\omega_{ihj}$ 40</td>
<td>$C_{\gamma}$ -3.19</td>
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<tr>
<td>$T_{hj}$ 64 $C_{lf}$ 0</td>
<td>$C_{\tau}$ -65.13</td>
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<td>$\tau_{hj}$ 21 $\theta$ 1.25</td>
<td>Cost of delays $\theta \nu = 0.8625$</td>
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<td>$\gamma$ 10</td>
<td>Welfare 36154€</td>
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Welfare 36154€