Reputational Cheap Talk with Misunderstanding∗

Alexander Guembel
Saïd Business School
University of Oxford

Silvia Rossetto
Warwick Business School

and

Toulouse School of Economics

February 27, 2009

∗We would like to thank Itay Goldstein, Gary Gorton, Thomas Mariotti, Volker Nocke, Marco Ottaviani, Oren Sussman, the audience at the Finance Research workshop at the Saïd Business School, and in particular the advisory editor and an anonymous referee for very helpful comments. We also gratefully acknowledge Daniel Rexhausen’s contribution to this paper in its early stages. Guembel would like to thank the Wharton School, where part of this research was carried out, for their kind hospitality. All remaining errors are ours.

Correspondence address: Alexander Guembel, Toulouse School of Economics, 21, Allee de Brienne, 31000 Toulouse, France, Tel.: ++33 561 12 85 74, e-mail: alexander.guembel@sbs.ox.ac.uk
Reputational Cheap Talk with Misunderstanding

Abstract

We consider a cheap talk game with a sender who has a reputational concern for an ability to predict a state of the world correctly, and where receivers may misunderstand the message sent. When communication between the sender and each receiver is private, we identify an equilibrium in which the sender only discloses the least noisy information. Hence, what determines the amount of information revealed is not the absolute noise level of communication, but the extent to which the noise level may vary. The resulting threshold in transmission noise for which information is revealed may differ across receivers, but is unrelated to the quality of the information channel. When information transmission has to be public, a race to the bottom results: the cut-off level for noise of transmitted information now drops to the lowest cut-off level for any receiver in the audience.

JEL Classification: D82, G14, G20.

Keywords: Communication, noise, cheap talk, reputational concerns.

1. Introduction

This paper investigates communication between a sender who has a reputational concern for making correct predictions, and a receiver who may misunderstand the sender. The sender has an unknown ability to learn correctly about the realization of a state of the world and he can choose whether to communicate a prediction about this state to receivers. The probability of a misunderstanding is affected by two aspects of communication: Firstly, the transmission noise inherent in the communication problem, which we assume to be privately observed by the sender. Secondly, the quality of the information channel, which is publicly observable but may differ across receivers. When communication between the sender and each receiver is private, we identify an equilibrium in which the sender only discloses the least noisy information. Hence, what determines the amount of information revealed is not the absolute noise level of communication, but the extent to which the noise level may vary.
There is no reason why this is directly related to the quality of the information channel. As a result, information may be disclosed to some receivers and not to others, unrelated to the quality of the receiver’s information channel.

When the sender is constrained to sending one message to all receivers (public communication), information is only revealed when the noise level can fall no further for any receiver. This is because the reputational loss with some receivers from revealing noisier information is not compensated for by any reputational gain with other receivers. We thus predict a race to the bottom of transmission noise when the audience widens.

The question how much information to make available when misunderstanding is possible, is important in many real world situations of private and public communication. A firm, for example, has to decide which information to disclose to analysts and shareholders. Such disclosures have become subject to ‘Regulation Fair Disclosure’, which prohibits firms from disclosing information selectively. It has been argued that this regulatory change has led to a reduction in information disclosure, particularly of more complex, and qualitative information. We discuss this example in more detail in section 4. Similarly, a central bank faces the question whether or not it should make transcripts of policy committee meetings public (see Woodford, 2005) when the potentially complex arguments behind policy decisions may be misinterpreted by some market participants. Winkler (2002) and Carpenter (2004) argue that the potential for misunderstanding by the market greatly affects the effectiveness of a central bank’s policies and central banks are therefore naturally concerned about the risks involved in disclosing information.

Clearly, the examples provided here differ by the precise objectives of the sender and one model will not be able to capture the richness that each present in reality. In all the examples, however, a reputational concern for competence by the sender is an important (although maybe not the only) motive for determining communication strategies. As such we believe our paper identifies an important mechanism underlying communication decisions.

In order to highlight the robustness and limitations of the mechanism, we discuss the following extensions: that (i) the sender receives a monetary reward from transmitting information, (ii) he is privately informed about his ability, and (iii) the difficulty of the prediction is privately known to the sender.

Standard ‘cheap-talk’ games, based on the work by Crawford and Sobel (1982) assume that the sender has a directional bias over the message, because his utility is directly affected by the receiver’s message-contingent action. We depart from this framework by assuming that the sender’s objective is his reputational concern. This type of problem has been considered in specific models by Scharfstein and Stein (1990) and Trueman (1994) who
look at herding incentives when two agents with reputational concerns choose an action sequentially. In a series of papers, Ottaviani and Sorensen (2006a, b, c) characterize more comprehensively the implications of reputational concerns in cheap-talk games. They show that generically a fully informative equilibrium cannot exist, because in such an equilibrium an expert’s expected reputation differs across his signals, providing an incentive to deviate. In equilibrium, an expert can at best transmit information about the direction of his signal, but not its intensity. We consider a special case where this problem does not occur, namely when signals are binary (as for example also in Scharfstein and Stein, 1990). This allows us to identify more clearly the role played by the possibility of misunderstanding. Moreover, unlike the above papers, we allow the sender to refrain from sending any message. This enables us to investigate the conditions under which information will or will not be transmitted.

Blume, Board and Kawamura (2007) introduce the possibility of communication error in the set-up of Crawford and Sobel (1982). They show that such noise increases the amount of information transmitted in equilibrium and thereby improves welfare. This contrasts with our finding whereby less information is transmitted when communication errors are possible.1

Finally, Farrell and Gibbons (1989) extend the Crawford and Sobel (1982) model to explore how public compared to private disclosure affects the credibility of the sender’s message. Our mechanism differs from theirs, because it is not based on directional biases that would lead to considerations about message credibility.

The remainder of this paper proceeds as follows. Section 2 describes the model when communication between the sender and each receiver is private. Its solution is presented in Section 3, while Section 4 considers public communication. Section 5 discusses extensions of the basic model. Section 6 concludes. All proofs are in the Appendix.

2. The model

There are six dates 1, ..., 6 and Figure 1 summarizes the time line of the model. There is one agent called the sender and N agents called receivers. At date 1 nature chooses agents’ types, a state of the world and the transmission noise of the communication problem. The sender’s ability type is \( a \in \{L, H\} \). With probability \( q \) the sender has high ability \( (a = H) \), and with \( 1 - q \) he has low ability \( (a = L) \). The sender’s ability is not observable by any of the agents, including the sender himself.2 Receivers \( i \in \{1, ..., N\} \) can be of types \( t_i \in [\underline{t}, \overline{t}] \)

1Dewatripont and Tirole (2005) also explicitly address the problem of misunderstanding, although in a different framework. They focus on the moral hazard problem that arises when improving the understanding in communication requires costly effort.

2This is in line with a large literature on implicit incentives through career concerns, starting with Holmstrom (1982). The idea behind the assumption is that if the agent begins his career with no informational
and $t_i$ is common knowledge.

The state of the world is $\omega \in \{0, 1\}$ with equally likely realizations ex ante. It only becomes publicly observable at date 6. The transmission noise $c \in [0, 1]$ is drawn from the probability density $h(c) > 0 \ \forall \ c \in [0, 1]$ and it determines the probability with which receivers understand the sender correctly. Details of how this works will be described below. Moreover, $c$, $\omega$ and $a$ are independent of one another. The realization of $c$ is privately observed by the sender at date 2 and is never observed by receivers.

At date 3 the sender can ‘enter’ or ‘quit’. If he enters he will receive private information about $\omega$ at date 4 and a non-trivial communication game between sender and receiver ensues (described below). If he quits, the sender moves directly to the final date of the game. One can think of ‘enter’ and ‘quit’ as an expert’s decision whether or not to cover a specific issue, based on his knowledge of the likelihood with which a receiver will later misunderstand the sender.

We first consider the game where the sender communicates privately with each receiver and assume (i) that the sender can take an enter or quit decision with respect to each receiver individually, and (ii) the sender’s decision to enter or quit is observable only to the receiver concerned. In Section 4 we consider the case where the sender is constrained to take the same, publicly observable action with respect to all receivers.

We now describe in more detail the subform that follows if the sender chooses ‘enter’ at date 3. In that case he receives at date 4 a private signal $s \in \{0, 1\}$ about the state $\omega$. The probability with which the signal is correct depends on the sender’s ability and is defined as follows

$$\mu_a \equiv \text{prob}(s = 0|\omega = 0) = \text{prob}(s = 1|\omega = 1).$$

Assume that both types of senders receive informative signals, but the high ability sender’s signal is of higher quality so that $\mu_H > \mu_L > \frac{1}{2}$. Moreover, denote the sender’s uncontingent probability of receiving a correct signal by $\mu \equiv q\mu_H + (1 - q)\mu_L$.

At date 5 the sender sends a message $m_i \in \{0, 1\}$ to receiver $i$ who observes $\hat{m}_i \in \{0, 1\}$, which is a noisy version of $m_i$. In particular, with a probability $f(c, t_i) \geq \frac{1}{2}$ the receiver observes $\hat{m}_i = m_i$ and with complementary probability he observes $\hat{m}_i \neq m_i$. Assume that $f(c, t_i)$ is decreasing in $c$ and increasing in $t_i$. Thus messages for which transmission noise advantage and performance is publicly observable, then all players have the same information regarding the agent’s type. In Section 5 we discuss the case where the sender is privately informed about his own type at date 1.
Nature chooses sender type $a \in \{L, H\}$, state of the world $\omega \in \{0,1\}$, noise level $c \in [0,1]$, receiver types $t_i \in [l, r]$. Sender privately observes $c$, sender enters / quits, sender privately observes $s$, sender sends $m$ and receiver observes $\hat{m}$, all agents observe $\omega$.

Figure 1: Time Line

is high are more likely to be observed incorrectly by the receiver. Conversely, receivers with a higher type $t_i$ are more likely to observe a message correctly. We can thus think of $t_i$ as the quality of receiver $i$’s information channel. Finally, denote by $c_\omega$ the highest element of $\arg \max_c f(c, t_i)$. That is to say, $c_\omega$ denotes the lowest level of noise such that any further reduction would not improve the probability of a correct understanding of receiver $i$. Obviously, if $f(c, t_i)$ is strictly decreasing in $c$, then $c_\omega = 0$. However, if $f$ is weakly decreasing, a receiver’s maximum ability of understanding may be reached at a value $c_\omega > 0$.

Finally at date 6 all agents observe $\omega$. Each receiver then forms a belief about the sender’s type by Bayesian updating, taking into account his own type $t_i$ and conditioning on the observed message $\hat{m}_i$ and the true state $\omega$. We define the sender’s reputation in the eyes of receiver $i$ by the probability with which the sender has high ability conditional on the receiver’s observation $\hat{m}_i$ and the true state $\omega$. We thus use the notation

$$q(\hat{m}_i, \omega) \equiv \Pr(a = H|\hat{m}_i, \omega).$$  \hspace{1cm} (1)

Note that the parameter $c$ and the true message $m_i$ are never observed by the receiver and can thus not be used to form posterior beliefs about the sender’s ability.

As mentioned above, if the sender chooses ‘quit’ at date 1, he moves directly to the final date, i.e., he does not receive any signal and sends no message. We denote the resulting reputation by $q^{\text{quit}}$.

The sender’s objective is to maximize his expected final reputation with each receiver. We
define a communication strategy as (i) a decision to enter or quit depending on the receiver’s type $t_i$ and the transmission noise $c$, (ii) if the sender chooses to enter, a type $t_i$ dependent message $m_i$ about $s$, where $m_i$ can be conditioned on $c, t_i$, and $s$. We also allow for mixed strategies and denote by $\sigma_s(m_i)$ the probability that message $m_i$ is sent when signal $s$ is observed.

If the sender chooses to enter, the receiver will form beliefs about $c$ from his observation of the entry decision. We can denote by $\hat{h}(c)$ the probability density of $c$ conditional on having observed entry. Denote by

$$F_{t_i} = \int_{c=0}^{1} f(c, t_i) \hat{h}(c) dc$$

(2)

the receiver’s expected probability that his observed message corresponds to the one sent, i.e., $\hat{m}_i = m_i$. Similarly, we can denote by $g_{\hat{m}_i}(\omega)$ the probability of state $\omega$ in the eyes of receiver $i$ conditional on having observed $\hat{m}_i$.

3. Equilibrium in private communication

We now consider the sender’s communication strategy when he communicates privately with each receiver. Since there is no link between the sender’s communication strategy with respect to one or another receiver, we can consider the communication strategies for each receiver in isolation. For notational simplicity we drop the index $i$ throughout this section.

An equilibrium is then defined as follows. The sender chooses his action at each date so as to maximize his expected reputation at date 6. The receiver updates his belief about the sender’s ability using Bayes’ rule and given his belief about the sender’s communication strategy. In equilibrium the receiver’s belief about the sender’s communication strategy must be correct.

At nodes of the game where Bayes’ rule cannot be used, we make the following assumption throughout the paper. If the sender’s decision to ‘quit’ is off the equilibrium path, we assume that observing ‘quit’ induces the belief that the sender has high ability with probability $q$, i.e., a sender’s decision to quit is independent of his ability. This assumption is important, because other off-the-path beliefs could support an equilibrium in which a sender always enters: If ‘quit’ is off the equilibrium path and induces the belief that the sender has low ability, then no sender would ever wish to quit.

The assumption on the off-the-path belief can be justified under a plausible perturbation of the game. Suppose a sender can only predict some of the time and does not enter when
he cannot predict (maybe because he is simply unaware of the existence of a prediction opportunity). If, in addition, the availability of a prediction opportunity is uncorrelated with the sender’s ability, observing no entry is always on the equilibrium path and leads to a reputation update of $q$. It should be noted, however, that there are other plausible perturbations of the game that would not generate an update of $q$, namely if the ability to predict was itself correlated with the sender’s type.

If it is on the equilibrium path for a sender to quit for some value of $c$, then his reputation from choosing to quit simply remains $q$. This follows from the fact that the sender cannot condition on his ability and that ability and transmission noise are independent. Together with our assumption on the off-the-path belief we can thus write $q^{\text{quit}} = q$.

Note that non-informative ‘babbling’ equilibria typically exist in models of cheap talk. In our case babbling equilibria involve the following. The sender enters at any arbitrary set of realizations of $c$ and then randomizes between $m = 1$ and $0$ with probabilities that are independent of the signal realization $s$. In that case $\hat{m}$ is uninformatice and the sender’s reputation update is equal to $q$ regardless of $\hat{m}$ and $\omega$. The more interesting question is whether there might be equilibria that actually convey information about $\omega$, at least for some values of $c$. In order to tackle this question we first define what we mean by ‘informative’ and then investigate properties of equilibrium.

**Definition 1** An equilibrium is informative about $\omega$ if $g_{\hat{m}}(\omega) \neq \frac{1}{2}$ for some $\hat{m}$ along the equilibrium path.

Since the prior probability of $\omega$ is $\frac{1}{2}$ for both realizations, the above definition implies that the receiver changes his belief about $\omega$ in response to observing a message $\hat{m}$, at least following some realizations of $c$. Note that if the receiver changes his belief about $\omega$ upon receiving, say $\hat{m} = 1$ then he will also change his belief following $\hat{m} = 0$. This can be shown as follows.

Suppose, for example, that $g_1(1) > \frac{1}{2}$. Using the fact that

$$g_1(1) = \frac{1}{2} \frac{\Pr (\hat{m} = 1|\omega = 1)}{\Pr (\hat{m} = 1|\omega = 1) + \frac{1}{2} \Pr (\hat{m} = 1|\omega = 0)},$$

it follows that $g_1(1) > \frac{1}{2}$ is equivalent to $\Pr (\hat{m} = 1|\omega = 1) > \Pr (\hat{m} = 1|\omega = 0)$. Moreover, denoting by $p^1 \equiv \Pr (\hat{m} = 1|s = 1)$ and $p^0 \equiv \Pr (\hat{m} = 1|s = 0)$, where

$$p^1 = \sigma_1(1) F_t + \sigma_1(0) (1 - F_t),$$

$$p^0 = \sigma_0(1) F_t + \sigma_0(0) (1 - F_t),$$

7
we can write

\[
\begin{align*}
\text{Pr} (\hat{m} = 1 | \omega = 1) &= \mu p^1 + (1 - \mu) p^0, \text{ and} \\
\text{Pr} (\hat{m} = 1 | \omega = 0) &= (1 - \mu) p^1 + \mu p^0.
\end{align*}
\]

Using the above, we can thus conclude that

\[
g_1 (1) > \frac{1}{2} \iff p^1 > p^0.
\]

From analogous calculations for \( \hat{m} = 0 \) it follows that \( p^1 > p^0 \iff g_0 (1) < \frac{1}{2} \), which proves the claim made above.

We will now use definition 1 so as to describe properties of equilibria.

**Lemma 1** If an equilibrium is informative about \( \omega \), then, following entry, the sender always uses a pure strategy in \( m \).

The intuition for this result is as follows. If an equilibrium is informative about \( \omega \), this implies that the sender does not randomize between \( m = 1 \) and \( 0 \) unconditionally on \( s \). The receiver can therefore infer from his observation of \( \hat{m} \), that the sender was more likely to have received one signal rather than another. This in turn allows the receiver to make an inference at date 6 about the likelihood with which the sender observed a correct signal \( s = \omega \). The receiver therefore updates his belief about the sender’s ability so that the sender’s posterior reputation will not remain at \( q \). But since different messages \( m \) have different probabilities of yielding ‘high’ or ‘low’ reputation updates the sender is not indifferent between the messages he sends. He will therefore choose a pure strategy.

With this preliminary result we can now investigate properties of equilibrium, in particular whether there are equilibria which are informative about \( \omega \). The next Proposition speaks to this question.

**Proposition 1** There exists an equilibrium which is informative about \( \omega \), and in which the sender enters for \( c \leq \underline{c} \) and quits otherwise. There does not exist any equilibrium which is informative about \( \omega \) in which the sender enters for \( c > \underline{c} \).

According to Proposition 1 the sender will avoid transmitting information to a receiver for any level of noise \( c > \underline{c} \). The intuition for this result is the following. The reputation of a sender who does not enter remains unchanged at \( q \). The sender who enters at the highest level of \( c \) for which entry occurs in a candidate equilibrium gets a reputation that is (weakly) below \( q \), because the receiver assigns a weakly lower than the true probability to a
transmission error. Moreover, this reputation is strictly below $q$, unless $f(c, t)$ is independent of $c$ whenever entry occurs.

Note that the value $c$ depends on the function $f(c, t)$ and the quality $t$ of the information channel. In the case where $f(c, t)$ is strictly decreasing in $c$, we get $c = 0$ and therefore no information at all will be revealed. If $f(c, t)$ is only weakly increasing, at least for some types of receivers, we might expect to see that a sender treats receivers differently depending on the quality of their information channel.

An interesting question this discussion raises is how the cut-off level for $c$ is related to the quality of the communication channel. Consider the following example. Suppose there are three receiver types $t_1, t_2$ and $t_3$ ordered by an increase in the quality of their information channel. Let $f(c, t_1) = K_1$ and $f(c, t_3) = K_3$ with $K_3 > K_1$. Moreover, suppose $f(c, t_2) \in (K_1, K_3)$ is strictly decreasing in $c$. The cut-off levels are then given by $c_1 = c_3 = 1$, $c_2 = 0$ and thus not monotonic in the quality of the communication channel. The example illustrates that the sender’s communication decision is determined by the extent to which the quality of communication varies with transmission noise. Information revelation is therefore not directly linked to the absolute quality of the communication channel.

4. Equilibrium in public communication

We now consider the case where the sender is not allowed to communicate privately with each receiver. Instead he is constrained to communicate publicly, if he enters, by sending one message to all receivers. Although only one message is sent we allow each receiver $i$ to have his own realization of $\hat{m}_i$, i.e., the communication channel (and its quality) may still differ across receivers. Like before, we assume that receivers do not communicate with each other, i.e., they do not learn how other receivers understood the sender’s message, or what conclusions they reach regarding the sender’s reputation.

The sender’s objective is now to maximize the weighted average expected reputation across all receivers. We make no restrictions on the weights, except that they are strictly positive for all receivers. Denote by $C_{\text{min}} = \inf \{c_1, ..., c_i, ..., c_N\}$. Note that the result from Lemma 1 also holds in the context of public communication. This is because it holds for each individual receiver and therefore also if the sender needs to choose the same communication

---

3 Note that one could also consider the case where a sender can send a different message to each receiver, but that each such message can also be observed by all other receivers.

4 If the information yields an advantage over which receiver’s compete, it is plausible to assume that receivers would not disclose $\hat{m}_i$ to others. For example financial investors may wish to trade on the information they receive from an analyst and therefore keep their signal secret.
strategy for all receivers. We can then show the following.

**Proposition 2** There exists an equilibrium which is informative about $\omega$, and in which the sender enters for $c \leq C_{\min}$ and quits otherwise. There does not exist any equilibrium which is informative about $\omega$ in which the sender enters for $c > C_{\min}$.

The level at which the sender is thus willing to communicate publicly is determined by the lowest cut-off level $c_j$ of any of the receivers. This results in a race to the bottom in the level of transmission noise of messages passed on by the sender in equilibrium.

An example that illustrates the relevance of the paper’s mechanism is the regulatory change introduced in October 2000 by the U.S. Securities and Exchange Commission. It adopted ‘Regulation Fair Disclosure’ (Reg FD), according to which firms are no longer allowed to disclose material information to investors and financial analysts selectively. One of the main reasons put forward in favor of Reg FD was that the commonly practiced selective disclosure led to an unfair distribution of information across financial analysts and investors. The SEC argued that firms granted privileged access to information to some analysts whom they then expected to make favorable recommendations to investors.\(^5\)

An important question, however, that remained unaddressed by the SEC was the impact on the overall amount and type of information that firms would be willing to disclose under Reg FD. Opponents of Reg FD argued vehemently that its adoption would lead to a reduction in information disclosure by firms in the form of “information brownout.” Bailey et.al. (2003) summarize the argument by saying that “Communication will be reduced to “sound bites,” “boilerplate” disclosures, or large amounts of nonmaterial and raw information of little value to analysts and the public at large” (p.2488). One specific reason for reduced information disclosure put forward by practitioners was that “Reg FD will result in firms disclosing less high-quality information for fear that [...] individual investors will misinterpret the information provided” (Bushee, Matsumoto and Miller, p.618 (2004)).

There is evidence suggesting that this fear was well founded. Bushee, Matsumoto and Miller (2004) find that firms which used closed conference calls for information disclosure prior to the adoption of Reg FD were significantly more reluctant to do so afterwards. In surveys of analysts conducted by the Association of Investment Management and Research, and the Security Industry Association, 57% and 72% of respondents respectively felt that less substantive information was disclosed by firms in the months following the adoption of Reg FD. Gomes, Gorton and Madureira (2007) find a post Reg FD increase in the cost of capital.

\(^5\)Note also that the analysis of Section 3 suggests that discrimination in private disclosure may indeed be an equilibrium. This is consistent with the SEC’s concern about unequal distribution of information.
for smaller firms and firms with a stronger need to communicate complex information. This again supports the argument that the public disclosure requirement reduced the amount of information available to investors, particularly information that is, arguably, more liable to lead to misunderstanding.

5. Discussion of results

Since our model is very simple this Section discusses its limitations and possible extensions.

5.1 Monetary reward for entering

One obvious limitation of the model is that the sender’s incentive to enter is weak by construction. It would be strengthened if there were additional benefits from entering, such as a direct monetary gain $M$. It is straightforward to see that in such a case entry could be sustained for some levels of $c > c$. At the same time there would still be a cut-off point beyond which no entry would occur, if the monetary payment is not too high relative to the reputational concern. To see why this is the case consider the condition for entry, which is now $E(q) \geq q - M$. We can then find that belief $f'$ which solves $E(q) = q - M$. If $M$ is sufficiently low then $f' > \frac{1}{2}$. Moreover, we know from the proof of Proposition 1 that the sender’s expected reputation $E(q)$ is a strictly increasing function in $f(c,t_i)$ and therefore decreasing in $c$. When $f' > \frac{1}{2}$ it follows directly that there may exist levels of $c$ such that $f(c,t_i) < f'$ and thus no entry occurs for those values of $c$. Hence, adding a monetary payoff for making a disclosure yields an equilibrium in which the incentive to avoid disclosure at high levels of $c$ is traded off against the monetary loss from not entering.

5.2 Sender is privately informed about his ability

A related issue arises if one allows the sender to be privately informed about his own ability when he chooses whether or not to enter. Since the entry decision can then be contingent on $a \in \{L,H\}$, not entering may in some cases reflect negatively on the sender’s type. This provides more incentives to enter and reveal information in the sense that typically there will be an equilibrium in which entry occurs, even if $f$ is a strictly monotone function in $c$.

5.3 The signal quality is variable and the sender has private information about it

We have assumed throughout that the probability with which a sender observes the state $\omega$ correctly is only a function of the sender’s (unknown) ability. In addition, one may wish
to allow for a situation in which the sender’s signal quality depends on a privately known attribute of the communication problem. This would capture the idea that a sender (of either ability) finds some things easier to predict than others. Whether the sender faces an ‘easy’ or a ‘difficult’ prediction problem, may, however, not be known to receivers.

Formally, one could thus make $\mu_H$ and $\mu_L$ decreasing functions of a parameter that captures the difficulty of the prediction problem and about which the sender is privately informed. A higher difficulty of the prediction problem thus increases signal noise. For the sake of brevity we omit a formal elaboration of this extension. It is, however, fairly straightforward to see that the communication strategy would be very similar to the case studied in Section 3. That is to say, the sender would wish not to enter when the signal quality is below a cut-off level and he only enters if the signal quality is at its maximum.

6. Conclusions

This paper provides a simple model of cheap talk with reputational concerns by the sender, when there is a chance that the receivers of information may misunderstand the sender. We showed that the possibility of misunderstanding provides the sender with a strong incentive not to transmit any information. The basic mechanism at work is the following. Since the receiver does not observe the likelihood of a misunderstanding, he bases his belief about the sender’s ability on the average probability of a mistake in equilibrium. But under this rule, the sender loses in terms of expected reputation whenever the true probability of a transmission error is above the average of the receiver’s belief about it. He therefore prefers not to transmit information in these instances, which leads to unravelling. The sender is only willing to transmit information at the lowest noise levels. This leads to omission of information in equilibrium and, in the case of public communication, to a race to the bottom in the transmission noise for which communication occurs.

The model we provide can serve as a basis for future research in several ways. Firstly, it would be interesting to consider some extensions of a mainly technical nature. Foremost, in the basic model, the sender does not know his own ability and a full treatment of this case is left for future research. Secondly, we view our model as identifying one driving force among others in real world communication situations. A task for future research is to investigate models that describe more fully a specific communication problem, integrating the mechanism of the paper with other relevant driving forces. This would provide a richer set of insights, tailored to specific instances of communication encountered in the real world.
7. Appendix

Proof of Lemma 1. If the equilibrium communication strategy is informative about \( \omega \) it must be the case that \( g_{\hat{m}}(\omega) \neq \frac{1}{2} \) for some realization \( c' \) of \( c \). Suppose, without loss of generality that, following \( c' \), \( g_1(1) > \frac{1}{2} \) and hence \( p^1 > p^0 \). Moreover, from (2) we know that \( F \geq \frac{1}{2} \). However, if \( p^1 > p^0 \) this implies the stronger condition \( F > \frac{1}{2} \) and therefore it must be that \( \sigma_1(1) > \sigma_0(1) \).

Consider now the reputation updates in this candidate equilibrium. These are given from Bayesian updating as follows

\[
q(1,1) = \frac{\mu_H p^1 + (1 - \mu_H) p^0}{\mu p^1 + (1 - \mu) p^0},
\]
\[
q(0,1) = \frac{\mu_H (1 - p^1) + (1 - \mu_H) (1 - p^0)}{\mu (1 - p^1) + (1 - \mu) (1 - p^0)},
\]
\[
q(1,0) = \frac{\mu_H p^0 + (1 - \mu_H) p^1}{\mu p^0 + (1 - \mu) p^1},
\]
\[
q(0,0) = \frac{\mu_H (1 - p^0) + (1 - \mu_H) (1 - p^1)}{\mu (1 - p^0) + (1 - \mu) (1 - p^1)}.
\]

After some straightforward calculations we can verify that if \( \mu_H > \mu \) (which holds by assumption) then

\[
p^1 > p^0 \iff q(1,1) > q(0,1) \iff q(0,0) > q(1,0).
\]

In the next step consider the sender’s optimal choice of \( \sigma_s(m) \), given the receiver’s beliefs. Again without loss of generality, suppose the sender observes \( s = 1 \). Note that the sender can deviate from the equilibrium mixing strategy. Denote by \( \tilde{p} \) the probability that \( \hat{m} = 1 \) given \( s = 1 \) if the sender deviates to \( \tilde{\sigma}_1(1) \). The sender’s expected final reputation is then given by

\[
E(q) = \mu \tilde{p}_1 q(1,1) + \mu (1 - \tilde{p}_1) q(0,1) + (1 - \mu) \tilde{p}_1 q(1,0) + (1 - \mu) (1 - \tilde{p}_1) q(0,0).
\]

This can be re-written as

\[
E(q) = \tilde{p}_1 (\mu q(1,1) - q(0,1)) - (1 - \mu) (q(0,0) - q(1,0)) + \mu q(0,1) + (1 - \mu) q(0,0).
\]

Moreover, after some calculations we can show that \( p^1 > p^0 \) implies that

\[
\mu (q(1,1) - q(0,1)) - (1 - \mu) (q(0,0) - q(1,0)) > 0.
\]
It follows that the sender’s expected reputation is strictly increasing in $\tilde{p}^1$. Since $F_t > \frac{1}{2}$ the sender maximizes $\tilde{p}^1$ by setting $\tilde{\sigma}_1(1) = 1$. \hfill ■

**Proof of Proposition 1.** The proof proceeds by first showing that entry for $c > \underline{c}$ cannot be an equilibrium. It then shows existence of an equilibrium with entry for $c \leq \underline{c}$.

The first part of the proof proceeds by contradiction. Suppose it was an equilibrium for the sender to enter for some $c > \underline{c}$ and denote by $c_{\text{max}}$ the highest level of $c$ for which the sender enters with strictly positive probability in equilibrium. The receiver’s inference about the probability of correct understanding is drawn from his observation of the sender’s decision to enter. This is given by $F_t$ from (2).

Using the fact that the message following entry is in pure strategies, we can set, without loss of generality, $\sigma_1(1) = 1$ and $\sigma_0(0) = 1$. From (3) and (4) we thus get $p^1 = F_t$ and $p^0 = 1 - F_t$. It follows that $q(1,1) = q(0,0)$ and $q(0,1) = q(1,0)$. Using $q^+ \equiv q(1,1)$ and $q^- \equiv q(0,1)$ we can write

$$q^+ = \frac{\mu_H F_t + (1 - \mu_H)(1 - F_t)}{\mu F_t + (1 - \mu)(1 - F_t)},$$

$$q^- = \frac{\mu_H (1 - F_t) + (1 - \mu_H)F_t}{\mu (1 - F_t) + (1 - \mu)F_t}. \tag{5}$$

The sender’s expected reputation if he enters at $c$ is then

$$E(q) = (\mu f(c,t) + (1 - \mu)(1 - f(c,t))) q^+$$

$$+ (\mu (1 - f(c,t)) + (1 - \mu) f(c,t)) q^-.$$

If the sender chooses to quit instead he can guarantee himself a reputation of $q$ at any level of $c$. If entering yields an expected reputation below $q$, the sender would therefore quit. We can then show after some algebra that

$$E(q) < q \Leftrightarrow F_t > f(c,t). \tag{7}$$

Note that $E(q)$ is strictly increasing in $f(c,t)$, and thus decreasing in $c$. If entering at $c_{\text{max}}$ is an equilibrium action it must be that entering at some other levels $c < c_{\text{max}}$ is strictly preferred over quitting. This is because $f(c,t)$ must be strictly decreasing at some point between $c_{\text{max}}$ and $\underline{c}$ and therefore $E(q)$ must be strictly higher at some point $c < c_{\text{max}}$ than at $c_{\text{max}}$. The equilibrium set of points for $c$ at which entry occurs is thus strictly larger than $\{c_{\text{max}}\}$. It follows that $f(c_{\text{max}},t) < F_t < f(\underline{c},t)$. But from (7) it then follows that the sender would not enter at $c_{\text{max}}$. Since this argument applies to any putative $c_{\text{max}}$ it cannot be an equilibrium for the sender to enter for any $c > \underline{c}$. 

14
In order to show existence consider the following. If the sender chooses to enter only if \( c \leq c \) then \( F_t = f(c, t) \) and \( E(q) = q \) for all \( c \) at which the sender enters. Moreover, there is no incentive to deviate by entering for \( c > c \) as in that case \( F_t > f(c, t) \) and from (7) entering would not be optimal.

**Proof of Proposition 2.** We first prove the non existence result by contradiction. Suppose the sender chooses to enter for some \( c > C_{\text{min}} \) and denote by \( c_{\text{max}} > C_{\text{min}} \) the highest level of \( c \) for which the sender chooses to enter in this candidate equilibrium. We can then define a sub-set of the population of receivers, labelled \( \Gamma \), to which those receivers with \( c_i < c_{\text{max}} \) belong. Note that in this candidate equilibrium \( \Gamma \) must be a non-empty set.

Consider first the sender’s expected reputation with those receivers who do not belong to \( \Gamma \). Each receiver \( i \notin \Gamma \) knows that any noise level for which the sender chooses to enter has the same probability \( f(c, t_i) \) of misunderstanding. Their belief about \( f(c, t_i) \) is therefore \( F_{t_i} = f(c, t_i) \). From (7) it thus follows that for all \( c \) that result in the sender choosing to enter in the candidate equilibrium, we have \( E(q_i) = q \).

Consider now the sender’s expected reputation with receivers \( i \in \Gamma \). From the proof of Proposition 1 we know that their belief about \( f(c, t_i) \) satisfies \( F_{t_i} > f(c_{\text{max}}, t_i) \). This follows from the fact that the sender has a strict incentive to enter for some levels of \( c < c_{\text{max}} \) and thus their belief about the expected probability of understanding correctly is strictly larger than at the highest level of noise for which the sender enters in equilibrium. As a result \( E(q_i) < q \) for \( i \in \Gamma \).

If, instead of entering, the sender deviates to quit when \( c = c_{\text{max}} \) his expected reputation increases from below \( q \) to \( q \) for those receivers who are in \( \Gamma \), while it remains unchanged at \( q \) with those receivers who are not in \( \Gamma \). He thus has a strict incentive not to enter. The same argument holds whenever \( \Gamma \) is a non-empty set.

When \( \Gamma \) is the empty set, and receivers believe that the sender enters for all \( c \leq C_{\text{min}} \) it is straightforward to see that the sender strictly prefers not to enter for any \( c > C_{\text{min}} \), while he is indifferent between entering and not entering for \( c \leq C_{\text{min}} \). This proves existence.

**References**


