Should a Declining Discount Rate Be Used in Project Analysis?

by

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September 12, 2013

Abstract

In project analysis, the rate at which future benefits and costs are discounted often determines whether a project passes the benefit-cost test. This is especially true of projects with long horizons, such as projects to reduce greenhouse gas (GHG) emissions. The benefits of reduced GHG emissions last for centuries, but mitigation costs are borne today. Whether such projects pass the benefit-cost test is especially sensitive to the rate at which future benefits are discounted. In evaluating public projects, France and the United Kingdom use discount rate schedules in which the discount rate applied today to benefits and costs occurring in future years declines with maturity: the rate used today to discount benefits from year 200 to year 100 is lower than the rate used to discount benefits in year 100 to the present In the United States, the Office of Management and Budget (OMB) recommends that project costs and benefits be discounted at a constant exponential rate, although this rate may be lower for projects that affect future generations. This raises a familiar, but difficult, question: how should governments discount the costs and benefits of public projects, especially projects that affect future generations? In this paper we ask what principles should be used to determine the rates at which to discount the costs and benefits of regulatory programs. Specifically, we ask whether these principles suggest that a declining discount rate (DDR) schedule should be used in project evaluation—similar to the approach followed in the UK and France—or whether benefits and costs should be discounted at a constant exponential rate. We conclude that the arguments in favor of a DDR are compelling, and merit serious consideration by regulatory agencies in the United States.

*At a workshop held at Resources for the Future in September of 2011, twelve of the authors were asked by the U.S. Environmental Protection Agency (EPA) to give advice on principles to be used in discounting the benefits and costs of projects that affect future generations. Maureen L. Cropper chaired the workshop.

Should a Declining Discount Rate Schedule Be Used in Project Analysis?

Introduction

In project analysis, the rate at which future benefits and costs are discounted often determines whether a project passes the benefit-cost test. This is especially true of projects with long horizons, such as projects to reduce greenhouse gas (GHG) emissions. The benefits of reduced GHG emissions last for centuries, but mitigation costs are borne today. Whether such projects pass the benefit-cost test is especially sensitive to the rate at which future benefits are discounted. In evaluating public projects, France and the United Kingdom use discount rate schedules in which the discount rate applied today to benefits and costs occurring in future years declines with maturity: the rate used today to discount benefits from year 200 to year 100 is lower than the rate used to discount benefits in year 100 to the present (see Figure 1).[1] In the United States, the Office of Management and Budget (OMB) recommends that project costs and benefits be discounted at a constant exponential rate, although this rate may be lower for projects that affect future generations.[2] This raises a familiar, but difficult, question: how should governments discount the costs and benefits of public projects, especially projects that affect future generations?

In this paper we ask what principles should be used to determine the rates at which to discount the costs and benefits of regulatory programs. Specifically, we ask whether these principles suggest that a declining discount rate (DDR) schedule should be used in project

[1] The discount rates pictured in Figure 1 are forward discount rates; that is, rates used today to compare benefits occurring at dates t and $t+1$.
[2] For intra-generational projects OMB (2003) recommends that benefit-cost analyses be performed using a discount rate of 7%, representing the pre-tax real return on private investments and, alternately, a discount rate of 3%, representing the "social rate of time preference."

evaluation—similar to the approach followed in the UK and France—or whether benefits and costs should be discounted at a constant exponential rate. We conclude that the arguments in favor of a DDR are compelling, and merit serious consideration by regulatory agencies in the United States.

The basic argument for a DDR is a simple one: if shocks to the interest rates (and therefore on the consumption growth rates) are persistent, this will result in a declining schedule of efficient discount rates. Two branches of the literature about declining discount rates have emerged over the last decade. The first branch is based on the consumption approach to discounting; i.e., on the extended Ramsey rule. The second is expected net present value approach. We briefly summarize each approach.

The theory of benefit-cost analysis dictates that the sure benefits and costs of a project should be converted to consumption units and discounted to the present at the consumption rate of interest—the rate at which society would trade consumption in year t for consumption in the present.[3] Ignoring uncertainty, this approach leads to the Ramsey discounting formula, in which the discount rate applied to net benefits at time t, ρ_t, equals the sum of the utility rate of discount (δ) and the rate of growth in consumption between t and the present (g_t), weighted by (minus) the elasticity of marginal utility of consumption (η): $\rho_t = \delta + \eta \cdot g_t$.

As is well known, the standard Ramsey formula for the consumption rate of discount can be extended to handle uncertainty about the rate of growth in consumption by subtracting a third "precautionary" term from the formula (Mankiw 1981; Gollier 2002). If growth is subject to

[3] Throughout the paper we ignore uncertainty in the stream of benefits and costs associated with a project, effectively assuming that these have been converted to certainty-equivalents. This allows us to focus on the term structure of risk free discount rates.

independently and identically distributed shocks, this is unlikely to significantly alter the consumption rate of discount. If, however, shocks to growth are positively correlated over time, the precautionary term in the Ramsey formula may become sizeable in absolute value for long horizons, leading to a declining term structure of discount rates (see Gollier 2012 for an extended survey). Uncertainty about the mean and variance of the rate of growth in consumption can also lead to a declining risk free discount rate (Gollier 2008; Weitzman 2004, 2007).

The other branch of this literature, the Expected Net Present Value (ENPV) approach, was initially developed by Weitzman (1998, 2001, 2007). He showed that the uncertainty about future discount rates justifies using a decreasing term structure of discount rates today. Computing ENPVs with an uncertain but constant discount rate is equivalent to computing NPVs with a certain but decreasing "certainty-equivalent" discount rate. More specifically, a probability distribution over the discount rate under constant exponential discounting in the future should induce us to use a declining term structure of discount rates today. Other literature has used a reduced-form approach to estimating certainty-equivalent discount rates based on historical time series of interest rates (for example, Newell and Pizer 2003; Groom et al. 2007; Hepburn et al. 2009). Gollier and Weitzman (2010) have attempted to reconcile these two branches of the literature (the ENPV approach and the extended Ramsey rule).

In the remainder of this paper we discuss these two approaches in detail. The next section discusses the use of the Ramsey formula as a basis for discounting and spells out conditions under which uncertainty in the rate of growth in per capita consumption will lead to a DDR. This is followed by a discussion of the expected net present value approach. In each case we discuss difficulties that would be encountered in using the approach to generate an empirical schedule of discount rates for regulatory impact analysis. We also contrast the use of a DDR schedule with current practice, as dictated by the US Office of Management and Budget (OMB) Guidelines (OMB 2003). The paper concludes by discussing circumstances under which a DDR will or will not lead to problems of time consistency.

The Ramsey Formula as a Basis for Discounting

In benefit-cost analysis, the consumption rate of discount is usually approached from the perspective of a social planner who wishes to maximize the social welfare of society (Dasgupta 2008; Goulder and Williams 2012). The utility of persons alive at t, u_t, is given by an increasing, strictly concave function of consumption (which can be broadly defined to include both market and non-market goods and services), c_t, i.e., $u_t = u(c_t)$, and it is assumed that the planner maximizes the discounted sum of the utilities of current and future generations. In evaluating investment projects, a social planner would be indifferent between \$1 received at time t and \$ε today if the marginal utility of \$ε today equaled the marginal utility of \$1 at time t.[4]

$$u'(c_0)\varepsilon = e^{-\delta t}u'(c_t) \tag{1}$$

Equation (1) assumes that the planner's utility function is additively separable, that the utility received from a given level consumption is constant over time, and that future utility is discounted at the rate δ. Solving equation (1) for ε, the present value of \$1 in year t,

$$\varepsilon = \frac{e^{-\delta t}u'(c_t)}{u'(c_0)} = e^{-\rho_t t} \tag{2}$$

[4] In this paper, c_t represents the average consumption of people alive at time t. In an intergenerational context, t is often interpreted as indexing different generations; however, it need not be. It can simply represent average consumption in different time periods, some of whom may be the same people. A discussion of models that distinguish individuals within and across generations is beyond the scope of our paper.

where ρ_t denotes the annual consumption rate of discount between periods 0 and *t*. If we assume that $u(c)$ exhibits constant relative risk aversion (CRRA) [$u(c) = c^{(1-\eta)}/(1-\eta)$], then ρ_t can be written using the familiar Ramsey formula

$$\rho_t = \delta + \eta \cdot g_t \tag{3}$$

where η is both the coefficient of relative risk aversion and (minus) the elasticity of marginal utility with respect to consumption, and g_t is the annualized growth rate of consumption between time 0 and time *t*.[5]

In equation (3), δ is the rate at which society (i.e., the social planner) discounts the utility of future generations. A value of $\delta = 0$ says that we judge the utility of future generations to contribute as much to social welfare as our utility. For any generation, η describes how fast the marginal utility of consumption declines as consumption increases. Higher values of η imply that the marginal utility of consumption declines more rapidly as consumption increases. The standard interpretation of (3) is that, when g_t is positive, the social planner will discount the utility of consumption of future generations because future generations are wealthier. To illustrate, if $g_t = 1.3\%$ annually, per capita consumption in 200 years will be over 13 times higher than today. So it makes sense to discount the utility of an extra dollar of consumption received 200 years from now. And the planner will discount it at a higher rate the faster the marginal utility of consumption decreases as consumption rises.

[5] In the Ramsey formula, η captures the intertemporal elasticity of substitution between consumption today and consumption in the future, the coefficient of relative risk aversion and inequality aversion. More sophisticated treatments (Epstein and Zin 1991; Gollier 2002) separate attitudes toward time and risk, as discussed more fully below.

The Ramsey Formula When the Growth Rate of Consumption is Uncertain

The rate of growth in consumption is likely to be uncertain, especially over long horizons. Allowing for uncertainty in the rate of growth in per capita consumption alters the Ramsey formula. We begin with the case in which shocks to consumption are independently and identically distributed, which yields the extended Ramsey formula. If the growth rate of consumption is a random walk that is normally distributed with mean μ_g and variance σ_g^2, this uncertainty adds a third term to the Ramsey formula (Mankiw 1981, Gollier 2002):[6]

$$\rho_t = \delta + \eta\mu_g - 0.5\,\eta^2\sigma_g^2 \qquad (4)$$

The last term in (4) is a precautionary effect: uncertainty about the rate of growth in consumption reduces the discount rate, causing the social planner to invest more for the future.[7] The magnitude of the precautionary effect is, however, likely to be small, at least for the United States. Suppose that $\delta = 0$, and $\eta=2$, as suggested by Gollier (2008) and Dasgupta (2008). Using annual data from 1889 to 1978 for the United States, Kocherlakota (1996) estimated μ_g to be 1.8 percent and σ_g to equal 3.6 percent. This implies that the precautionary effect is 0.26 percent and that ρ = 3.34 percent (rather than 3.6 percent, as implied by equation (3)).[8]

Shocks to consumption may have a larger impact on the discount rate if they represent catastrophic risks. Pindyck and Wang (2012) examine the discounting implications of the risk of global events that could cause a substantial decline in the capital stock or in the productivity of the capital stock. Examples of catastrophes include an economic collapse on the order of the

[6] To be more precise, suppose that $ln(c_i/c_{i-1})$, is independently and identically normally distributed with mean μ_g and variance σ_g^2.

[7] A necessary condition for this to hold is that the planner be prudent (i.e., that the third derivative of $u(c)$ be positive), which is satisfied by the CRRA utility function.

[8] Gollier (2011) finds that the size of the precautionary effects is much larger for other countries, especially developing countries.

Great Depression, nuclear or bioterrorism, a highly contagious "mega-virus" that kills large numbers of people, or an environmental catastrophe, such as the melting of the West Antarctic Ice Sheet.

Suppose that catastrophic risk is modeled as a Poisson process with mean arrival rate λ, and that, if a catastrophe occurs, consumption falls by a random percentage ξ.[9] This subtracts $\eta\lambda E(\xi)$ from the right-hand side of (4), thus reducing the discount rate. How important is this last term? Recent estimates of λ and $E(\xi)$ based on panel data by Barro (2006, 2009) and others put $\lambda \approx .02$ and $E(\xi) \approx 0.3$ to 0.4. Thus if $\eta = 2$, the adjustment would be about -1.2 percent to -1.6 percent.

Uncertain Consumption and the DDR

As equation (4) illustrates, independently and identically normally distributed shocks to consumption growth with known mean and variance result in a constant consumption rate of discount. As discussed below, the consumption rate of discount may decline with maturity if shocks to consumption are positively correlated over time, or if the rate of change in consumption is independently and identically distributed with unknown mean or variance.

Gollier (2012, Chapter 8) proves that if shocks to consumption are positively correlated and $u(c)$ exhibits CRRA, ρ_t will decline.[10] The intuition behind this is that positive shocks to consumption make future consumption riskier, increasing the strength of the precautionary effect in equation (4) for distant time horizons. To illustrate, a possible form that shocks to

[9] Equation (4) assumes that, in continuous time, the log of consumption evolves according to arithmetic Brownian motion: $d\, lnc_t = \mu_g dt + \sigma_g dz$. Catastrophic risk could be modeled by adding a term $-\lambda dq$ to this expression, where dq represents a Poisson (jump) process with mean arrival rate λ.

[10] Formally Gollier shows that if $ln(c_i/c_{i-1})$ exhibits "positive quadrant dependence" and $u'''(c) > 0$, ρ_t will decline as t increases.

consumption could take is for $ln(c_t/c_{t-1}) \equiv x_t$, the percentage growth in consumption at t, to follow an AR(1) process

$$x_t = \varphi x_{t-1} + (1-\varphi)\mu + u_t \quad (5)$$

where u_t is independently and identically normally distributed with constant variance. Mathematically, equation (5) will generate a declining discount rate on average, provided $0 < \varphi < 1$.[11] To be precise, the precautionary effect is multiplied by the factor $(1-\varphi)^{-2}$ as t goes to infinity (Gollier 2008).

Various models of per capita consumption growth have been estimated for the United States (e.g., Cochrane 1988; Cecchetti, Mark, and Lam 2000), and these could be used to empirically estimate a DDR using the extended Ramsey formula. The persistence in the rate of change in per capita consumption in the United States, based on historic data, is likely to result in a discount rate that declines slowly over time. In the case of equation (5), Gollier (2008) reports an estimate of $\varphi = 0.3$, based on the literature, which implies a very gradual decline in the discount rate. The same is true of the certainty-equivalent discount rate based on the regime-switching model of Cecchetti, Mark, and Lam (2000). The efficient discount rate in the positive growth regime declines from 4.3 percent today to 3.4 percent after 100 years.

The approach to parameterizing the extended Ramsey formula described in the previous paragraphs is based on the assumption that the nature of the stochastic consumption-growth process can be adequately characterized by econometric models estimated using historical data.

[11] The term structure will evolves over time over the business cycle. In recessions, the short term discount rate can be low enough (because of the low growth rate in the short run) that the term structure will be temporarily increasing.

The consumption-based asset pricing literature suggests that this is not the case.[12] To quote Weitzman (2007), "People are acting in the aggregate like there is much more . . . subjective variability about future growth rates than past observations seem to support." This argues for treating μ_g and σ_g as uncertain. Subjective uncertainty about the trend and volatility in consumption growth, as modeled in Weitzman (2007, 2004) and Gollier (2008), will lead to a declining discount rate.

The form of the planner's subjective uncertainty about the mean rate of growth in consumption clearly influences the path of the efficient discount rate. The assumptions in Weitzman (2004) cause the efficient discount rate to decline linearly, eventually becoming negative. Gollier (2008) presents examples that yield non-negative paths for the efficient discount rate.

Gollier (2008) proves that, when the rate of growth in log consumption follows a random walk and the mean rate of growth depends on an uncertain parameter θ [$\mu_g = \mu_g(\theta)$], the certainty-equivalent discount rate, R_t, will decline over time. Figure 2 demonstrates the path of R_t for case of $\delta = 0$, $\eta = 2$ and $\sigma_g = 3.6$ percent. The mean rate of growth in consumption is assumed to equal 1 percent and 3 percent with equal probability. This yields a certainty-equivalent discount rate that declines from 3.8 percent today to 2 percent after 300 years—a path that closely resembles the French discounting schedule in Figure 1. The choice of other distributions for θ will, of course, lead to other DDR paths.

[12] The extended Ramsey formula does a poor job of explaining the equity premium puzzle: the large gap between the mean return on equities and risk-free assets.

How to Parameterize the Ramsey Formula?

To empirically implement a DDR using the extended Ramsey formula requires estimates of δ and η as well as information about the process governing the growth of per capita consumption. Below we discuss both prescriptive and descriptive approaches to quantifying δ and η.

δ and η as Policy Parameters

Many economists regard the Ramsey approach to discounting, which underlies the theory of cost-benefit analysis, as a normative approach. This implies that its parameters should reflect how society values consumption by individuals at different points in time; i.e., that δ and η should reflect social values. The question is how these values should be measured.

Many—but not all—of us agree with Frank Ramsey that it is ethically indefensible to discount the utility of future generations, except possibly to take account of the fact that these generations may not exist. This implies that $\delta = 0$, or a number that reflects the probability that future generations will not be alive. Stern (2006), for example, assumes that the hazard rate of extinction of the human race is 0.1 percent per year.

The parameter η plays three roles in the Ramsey formula: it represents the intertemporal elasticity of substitution between consumption today and consumption in the future, the coefficient of relative risk aversion, and it reflects intergenerational inequality aversion. This complicates estimation of η: a researcher will obtain difference values for η depending on which approach is taken (Atkinson et al. 2009; Groom and Maddison 2013). One could also argue (Traeger 2009) that discounting should be based on a richer characterization of preferences than those underlying the Ramsey formula; for example, Epstein-Zin preferences (Epstein and Zin

1991) separate risk aversion from the elasticity of intertemporal substitution of consumption. While Epstein-Zin preferences have been useful in explaining individual household behavior in financial markets, it is not clear that they provide an appropriate foundation for social preferences.

We therefore adhere to the Ramsey formula and argue that, from a normative perspective, η should be interpreted as reflecting the maximum sacrifice one generation should make to transfer income to another generation (Dasgupta 2008; Gollier et al. 2008). To make this more concrete, Appendix Table A1 describes the maximum sacrifice that society believes a higher income group (A) should make to transfer 1 dollar to the poorer income group (B), as a function of η. When group A is twice as rich as group B and $\eta = 1$ the maximum sacrifice is \$2; when $\eta = 2$, the maximum sacrifice is \$4.

Should δ and η Reflect Observed Behavior in Public Policy?

How, empirically, should η be determined? One approach is to examine the value of η implied by decisions that society makes to redistribute income, such as through progressive income taxes. In the United Kingdom, socially revealed inequality aversion, based on income tax schedules, has fluctuated considerably since the Second World War, with a mean of 1.6 (Groom and Maddison 2013). Applying this value to climate policy would assume (1) that the U.K. government has made the "right" choice in income redistribution and (2) that income redistribution within a country and period is the same as income redistribution between countries and over time. Tol (2010) estimates the consumption rate of international inequity aversion, as revealed by decisions on the level and allocation of development aid, at 0.7.

It is also possible to elicit values of η and δ using stated preference methods. The issue here is whose preferences are to be examined and how. As Dasgupta (2008) has pointed out, it is important to examine the implications of the choice of η and δ for the fraction of output that a social planner would choose to save. Ceteris paribus, a lower value of η implies that society would choose to save a larger proportion of its output to increase the welfare of future generations. The implications of the choice of δ and η would need to be made clear to the subjects queried.[13]

Some of us are, however, skeptical of the validity of using stated preference methods, especially as applied to lay people who may not appreciate theoretical constructs such as "pure time preference," "risk-free investment," and "benevolent social planner." We suggest that a check on the reasonableness of results obtained from direct questioning methods would be to compare the resulting values of ρ_t with the return on risk-free investment. This may not represent the consumption rate of discount; however, as noted above, it is currently viewed as a surrogate for the consumption rate of discount by OMB (2003), and is more readily observable.

Should δ and η Reflect Observed Behavior in Financial Markets?

In the simple Ramsey formula, the parameter η also represents the coefficient of relative risk aversion, suggesting that η could be estimated from observed behavior in financial markets.[14] Although some of us favor this approach, others object to the use of these estimates on two grounds: they reflect the preferences of people proportionate to their activity in financial

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[13] There is also the issue of how to aggregate preferences. One approach to this problem is to characterize equilibrium discount rates in an economy in which agents differ in their rate of time preference (Gollier and Zeckhauser 2005) and possibly in their assumptions about future growth in consumption (Jouini, Marin and Napp 2010).

[14] The macroeconomic literature on the coefficient of relative risk aversion is summarized by Meyer and Meyer (2005).

markets, and they do not reflect intergenerational consumption tradeoffs, making them inappropriate as estimates of η in a social welfare function.

The use of financial market data to estimate η raises the broader issue of whether the consumption rate of discount should reflect observed behavior and/or the opportunity cost of capital. The descriptive approach to social discounting (Arrow et al., 1996), epitomized by Nordhaus (1994, 2007), suggests that δ and η should be chosen so that ρ_t approximates market interest rates. In base runs of the Nordhaus Dynamic Integrated Climate-Economy (DICE) 2007 model, $\delta = 1.5$ and $\eta = 2$. DICE is an optimal growth model in which g_t and ρ_t are determined endogenously. In DICE 2007, ρ_t ranges from 6.5 percent in 2015 to 4.5 percent in 2095 as consumption growth slows over time (Nordhaus 2007).

This raises the question: should we expect the consumption rate of discount in equation (3) to equal the rate of return to capital in financial markets, and, if not, what should we do about this? In an optimal growth model (e.g., the Ramsey model), the consumption rate of discount in (3) will equal the marginal product of capital along an optimal consumption path. If, for example, the social planner chooses the path of society's consumption in a one-sector growth model, ρ_t will equal the marginal product of capital along an optimal path. What if society is not on an optimal consumption path? Then theory tells us that we need to calculate the social opportunity cost of capital—we need to evaluate the present discounted value of consumption that a unit of investment displaces—and use it to value the capital used in a project when we conduct a cost-benefit analysis (Dasgupta, Marglin and Sen 1972). But, once this is done—once all quantities have been converted to consumption equivalents—the appropriate discount rate to judge whether a project increases social welfare is the consumption rate of discount (ρ_t).

A potential problem is that converting all costs and benefits to consumption units can, in practice, be difficult. This argues for using the rate of return to capital to measure the discount rate, when a project displaces private investment. This is, in effect, what OMB recommends when it suggests using a 7 percent real discount rate. A discount rate of 7 percent is “an estimate of the average pretax rate of return on private capital in the U.S. economy” (OMB 2003) and is meant to capture the opportunity cost of capital when “the main effect of the regulation is to displace or alter the use of capital in the private sector.”

The Expected Net Present Value Approach

The Ramsey formula provides a theoretical basis for intergenerational discounting and also suggests that the discount rate schedule is likely to decline over time due to uncertainty about the rate of growth in per capita consumption.

An alternate approach to modeling discount rate uncertainty is the expected net present value (ENPV) approach. Suppose that an analyst discounts net benefits at time t, $Z(t)$, to the present at a constant exponential rate r, so that the present value of net benefits at time t equals $Z(t)exp(-rt)$.[15] If the discount rate r is fixed over time but uncertain, then the expected value of net benefits is given by

$$A(t)Z(t) = E(exp(-rt))Z(t) \tag{6}$$

where expectation is taken with respect to r. $A(t)$ is the expected value of the discount factor and $R_t \equiv -(dA_t/dt)/A_t$ is the instantaneous certainty-equivalent discount rate. If the probability

[15] We assume that $Z(t)$ represents certain benefits. If benefits are uncertain we assume that they are uncorrelated with r and that $Z(t)$ represents certainty-equivalent benefits. A referee notes that this approach is at variance with the approach in corporate finance, in which discount rates are adjusted to reflect the riskiness of a project.

distribution over r is stationary, then, because the discount factor is a convex function of r, the certainty-equivalent discount rate, R_t, will decline over time (Weitzman 1998, 2001).[16]

This is illustrated in Table 1, which contrasts the present value of $1,000 received at various dates using a constant discount rate of 4 percent versus a constant discount rate that equals 1 percent and 7 percent with equal probability. The convexity of the discounting function guarantees that the present value computed using the mean discount rate of 4 percent is always smaller than the expected value of the discount factor.[17] The relationship between the expected NPV in any two adjacent years can be expressed in terms of a certainty-equivalent discount rate. For example, the expected NPV of $1000 received in year 101 is $182.53, and in year 100 is $184.40, which is $182.53*(1.0102). This 1.02% change is the certainty-equivalent discount rate used to discount benefits from year 101 to 100. As Table 1 illustrates, the certainty-equivalent discount rate is less than the mean discount rate and declines over time, as the present values at 1% dominate the expected NPV.

This was first pointed out in the context of intergenerational discounting by Weitzman (1998, 2001). In "Gamma Discounting," Weitzman showed that, if uncertainty about r is described by a gamma distribution with mean μ and variance σ^2, the certainty-equivalent discount rate is given by

$$R_t = \mu/[1 + t\sigma^2/\mu] \quad (7)$$

The gamma distribution provides a good fit to the responses Weitzman obtained when he asked over 2,000 Ph.D. economists what rate should be used to discount the costs and benefits

[16] Gollier and Weitzman (2010) discuss the theoretical underpinnings for the expected net present value approach. The approach is consistent with utility maximization in the case of a logarithmic utility function.
[17] Formally, $E(exp(-rt)) > exp(-E(r)t))$ by Jensen's inequality.

associated with programs to mitigate climate change. The associated mean (4 percent) and standard deviation (3 percent) of responses lead to the schedule of certainty-equivalent discount rates in Table 2.

It is, however, important to consider the underlying source of uncertainty that generates a declining discount rate schedule. On the one hand there are differences in opinion concerning how the future will turn out with regard to the returns to investment, growth, and, hence, the discount rate. Over such time horizons there is genuine uncertainty about these quantities, which will be resolved in the future. Weitzman (2001), however, argued that this source of disagreement among experts represents the "tip of the iceberg" compared to differences in normative opinions on the issue of intergenerational justice. Rather than reflecting uncertainty about the future interest rate, which falls naturally into the positive/descriptive school, variation in normative opinions reflects irreducible differences on matters of ethics. Here variation reflects heterogeneity rather than uncertainty. In our view, disagreement among experts that reflects differing preferences, rather than underlying uncertainty about the economy, should not form the basis for establishing a declining discount rate schedule.

In contrast, if expert responses represent forecasts, Freeman and Groom (2012) argue that they should be combined to reduce forecasting error, as is typical in the literature on combining forecasts (e.g., Bates and Granger 1969). Where experts are unbiased and independent, the appropriate measure of variation is the standard error, which is smaller by a factor of the inverse of the sample size. Freeman and Groom (2012) show that, in this case, the most appropriate methods of combining forecasts lead to a much slower decline in the discount rate than the original Gamma Discounting approach. Generally their methods suggest that the decline will be more rapid the greater the dependence between expert forecasts.

The declining certainty-equivalent discount rate in Gamma Discounting follows directly from Jensen's inequality and the fact that the distribution over the discount rate is constant over time. The more general case in which the discount rate varies over time gives us

$$A(t) = E[exp(-\sum_{\tau=1 \dots t} r_\tau)] \tag{8}$$

In this case, the shape of the R_t path depends on the distribution of the per-period discount rate $\{r_\tau\}$. If $\{r_\tau\}$ are independently and identically distributed, the certainty-equivalent discount rate is constant. In equation (8), there must be persistence in uncertainty about the discount rate for the certainty-equivalent rate to decline. If, for example, shocks to the discount rate are correlated over time, as in equation (9),

$$r_t = \pi + e_t \quad \text{and} \quad e_t = ae_{t-1} + u_t, \quad |a| \leq 1 \tag{9}$$

the certainty-equivalent discount rate will decline over time (Newell and Pizer 2003).[18]

Empirical Estimates of the DDR Schedule for the United States

The approach followed in the empirical DDR literature is to view r_t as representing the return on government bonds (e.g., the return on Treasury bonds) and to develop models to forecast r_t. The empirical DDR literature includes models of interest rate determination for the United States (Newell and Pizer 2003; Groom et al. 2007); Australia, Canada, Germany, and United Kingdom (Hepburn et al. 2009; Gollier et al. 2008); and France, India, Japan, and South Africa (Gollier et al. 2008). We focus on the empirical DDR literature as applied to the United States.

[18] In equation (9), the interest rate follows an AR(1) process. In estimating (9) it is typically assumed that $\pi \sim N(\mu_\pi, \sigma_\pi^2)$, and $\{u_t\} \sim$ i.i.d. $N(0, \sigma_u^2)$.

Using two centuries of data on long-term, high quality, government bonds (primarily U.S. Treasury bonds), Newell and Pizer (2003) estimate reduced-form models of U.S. bond yields, which they use in turn to estimate certainty-equivalent discount rates over the next 400 years. The authors assume that interest rates follow an autoregressive process. This is given by equation (9) in the case of AR(1).[19] Equation (9) implies that the mean interest rate is uncertain, and that deviations from the mean interest rate will be more persistent the higher is a.[20] When $a = 1$, interest rates follow a random walk.

To illustrate the implications of persistence, if $a = 1$, $\mu_\pi = 4$ percent, $\sigma_\pi^2 = 0.52$ percent and $\sigma_u^2 = 0.23$ percent, the certainty-equivalent discount rate declines from 4 percent today to 1 percent 100 years from now. In contrast, a value of $a < 1$ (a mean-reverting model) implies that interest rates will revert to μ_π in the long run. When $a = 0.96$, $\mu_\pi = 4$ percent, $\sigma_\pi^2 = 0.52$ percent and $\sigma_u^2 = 0.23$ percent, the certainty-equivalent discount rate is 4.0 percent today and 3.6 percent 100 years from now (Newell and Pizer 2003).

Newell and Pizer use results from their preferred specifications of random walk and mean-reverting models to simulate the path of certainty-equivalent discount rates.[21] In the random walk model the certainty-equivalent discount rate falls from 4 percent today to 2 percent in 100 years; in the mean-reverting model, a certainty-equivalent discount rate of 2 percent is

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[19] The authors estimate autoregressive models in the logarithms of the variables ($lnr_t = ln\pi + e_t$) to avoid negative interest rates. Their preferred models are AR(3) models in which $e_t = a_1e_{t-1} + a_2e_{t-2} + a_3e_{t-3} + u_t$.

[20] The authors demonstrate that the instantaneous certainty-equivalent interest rate corresponding to (9) is given by $R_t = \mu_\pi - t\sigma_\pi^2 - \sigma_u^2 f(a,t)$ where $f(a,t)$ is increasing in a and t. How fast the certainty-equivalent interest rate declines depends on the variance in the mean interest rate as well as on how persistent are shocks to the mean interest rate (i.e., on a).

[21] The preferred models are AR(3) models, estimated using the logarithms of the variables (see footnote 18). The random walk model imposes the restriction that $a_1 + a_2 + a_3 = 1$.

reached only in 300 years. The authors cannot reject the random walk hypothesis but investigate the implications of both models for calculating the marginal social cost of carbon.[22]

The subsequent literature, following the literature in finance, has estimated more flexible reduced-form models of interest rate determination. Groom et al. (2007) estimate five models for the United States using the same data as Newell and Pizer (2003). The first two are random walk and mean-reverting models identical to those in Newell and Pizer (2003); the third is an autoregressive integrated generalized autoregressive conditional heteroskedasticity (AR-IGARCH) model that allows the conditional variance of the interest rate (held fixed in equation (9)) to vary over time; the fourth is a regime-switching model that allows the interest rate to shift randomly between two regimes that differ in their mean and variance. The final model, which outperforms the others in within- and out-of-sample predictions, is a state space model. This is an autoregressive model that allows both the degree of mean reversion and the variance of the process to change over time.[23] Freeman et al. (2013) further extend the literature by adjusting the data series used by Newell and Pizer (2003) and Groom et al. (2007) for inflation.[24] They find that a declining DDR is robust to a more rigorous treatment of inflation.

Figure 3 contrasts the path of certainty-equivalent rates from the random walk model of Newell and Pizer (2003), the state-space model of Groom et al. (2007) and Freeman et al.

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[22] The point estimate of $a_1 + a_2 + a_3 = 0.976$ with a standard error of 0.11. The authors also note that the mean-reverting model, when estimated using data from 1798 through 1899, over-predicts interest rates in the first half of the 20th century.

[23] In the state-space model $r_t = \pi + a_t r_{t-1} + e_t$, where $a_t = \sum \lambda_i a_{t-1} + u_t$. e_t and u_t are serially independent, zero-mean, normally distributed random variables, whose distributions are uncorrelated. The authors compare the models using the root mean squared error of within- and out-of-sample predictions.

[24] Newell and Pizer (2003) and Groom et al. (2007) use annual market yields on long-term government bonds for the period 1798-1999. Starting in 1950 nominal interest rates are converted to real ones using the expected rate of inflation forecast by the Livingston Survey of professional economists.

(2013)'s preferred model.[25] The certainty-equivalent rates from the state space model decline more rapidly than rates produced by the random walk model for the first 100 years, leveling off at about 2 percent. The random walk model yields a certainty-equivalent discount rate of 2 percent at 100 years and 1 percent in year 200, declining to about 0.5 percent when $t = 400$. The DDR path corresponding to Freeman et al. (2013) is initially higher than Groom et al. (2007) but declines more rapidly at long horizons than Newell and Pizer (2003).

The DDR schedules pictured in Figure 3 make a considerable difference to estimates of the social cost of carbon, compared to using a constant exponential discount rate of 4 percent. Using damage estimates from Nordhaus (1994), all 3 sets of authors compute the marginal social cost of carbon as the present discounted value of global damages from emitting a ton of carbon in 2000, discounted at a constant exponential rate of 4 percent and using certainty-equivalent rates from their models. Using a constant exponential rate of 4 percent, the social cost of carbon is $10.70 (2013 US$). It is $19.50 per ton of carbon using the random walk model in Newell and Pizer (2003), $27.00 using the state space model in Groom et al. (2007), and $26.10 per ton using the preferred model in Freeman et al. (2013) (2013 US$).

The DDR and Time Consistency

An issue that frequently arises in the context of the DDR is whether the use of a declining discount rate will lead to time-inconsistent decisions. It is well known (Gollier et al. 2008) that a

[25] The preferred model in Freeman et al. (2013) is an augmented autoregressive distributed lag model. Simulations have been run for all 3 models assuming a mean interest rate of percent per annum.

policy chosen by a decisionmaker who maximizes a time-separable expected utility function will be time consistent if expected utility is discounted at a constant exponential rate.[26]

The problem of time consistency can, however, arise in an ENPV framework if the DDR schedule does not change over time. If an analyst were to evaluate future net benefits using the discounting schedule in "Gamma Discounting" (Weitzman 2001) in 2012 and the schedule did not change over time, a program that passed the benefit cost test in 2012 would not necessarily pass it in a later year, depending on the time pattern of net benefits.

Of course, if new information becomes available that alters the DDR schedule, the analyst will want to re-evaluate the ENPV of the program. Because new information is available, a reversal of the outcome of the benefit-cost analysis would not constitute time inconsistency. Newell and Pizer (2003) argue that an analyst, when using historical data to estimate a DDR, will naturally update estimates of the DDR as more information becomes available. This obviates the problem of time inconsistency. In a regulatory setting, however, such updating may occur only infrequently.[27] The question of updating is an issue to which more thought should be given.

Concluding Remarks

The use of a DDR in project evaluation would have important implications for how regulations are evaluated in the United States. Currently, OMB requires that benefits and costs be discounted at a constant exponential rate, although a lower discount rate than the required 3 percent and 7 percent may be used as a sensitivity analysis when a project yields benefits to future generations. In contrast, France and the United Kingdom use declining discount rate

[26] Constant exponential discount is a sufficient condition for time consistency but not a necessary one. See Heal (2005) for other conditions that will yield time-consistent decisions. It is a necessary condition for an optimal policy to be both time consistent and stationary.
[27] The U.K. discount rate schedule in Figure 1 has been in place since 2003 (HM Treasury 2003).

schedules in which all costs and benefits occurring in the same year are discounted at a rate that declines over time. Who is correct?

Theory provides compelling arguments for a declining certainty-equivalent discount rate. In the Ramsey formula, uncertainty about the future rate of growth in per capita consumption can lead to a declining consumption rate of discount, assuming that shocks to consumption are positively correlated. This uncertainty in future consumption growth rates may be estimated econometrically based on historic observations, or it can be derived from subjective uncertainty about the mean rate of growth in mean consumption or its volatility.

The path from theory to a numerical schedule of discount rates requires estimates of δ, η and the process generating g_t. The UK discounting schedule pictured in Figure 1 assumes that $\delta = 1.5$ and $\eta = 1$ (HM Treasury, Annex 6 2003). The initial value of $g = 2$ percent, implying $\rho = 3.5$ percent. The DDR path is a step function that approximates the rate of decline in the discount rate in Newell and Pizer's random walk model (OXERA 2002).

The ENPV approach is less theoretically elegant and does not measure the consumption rate of discount as given by the Ramsey formula. It is, however, empirically tractable and corresponds to the approach currently recommended by OMB for discounting net benefits, when expressed in consumption units (OMB 2003). The empirical ENPV literature has focused on models of the rate of return on long-term, high quality, government debt. And, in the United States, the literature suggests that the certainty-equivalent rate is declining over time. Results from the empirical DDR literature are, however, sensitive to the model estimated, the data series used to estimate the model, and how the data are smoothed and corrected for inflation.

Clearly, judgment is required in estimating a DDR schedule, whichever approach is used. And, as emphasized above, the DDR schedule should be updated as time passes and more data become available. Setting forth a procedure for estimating a DDR would, however, be an improvement over OMB's current practice of recommending fixed discount rates that are rarely updated.

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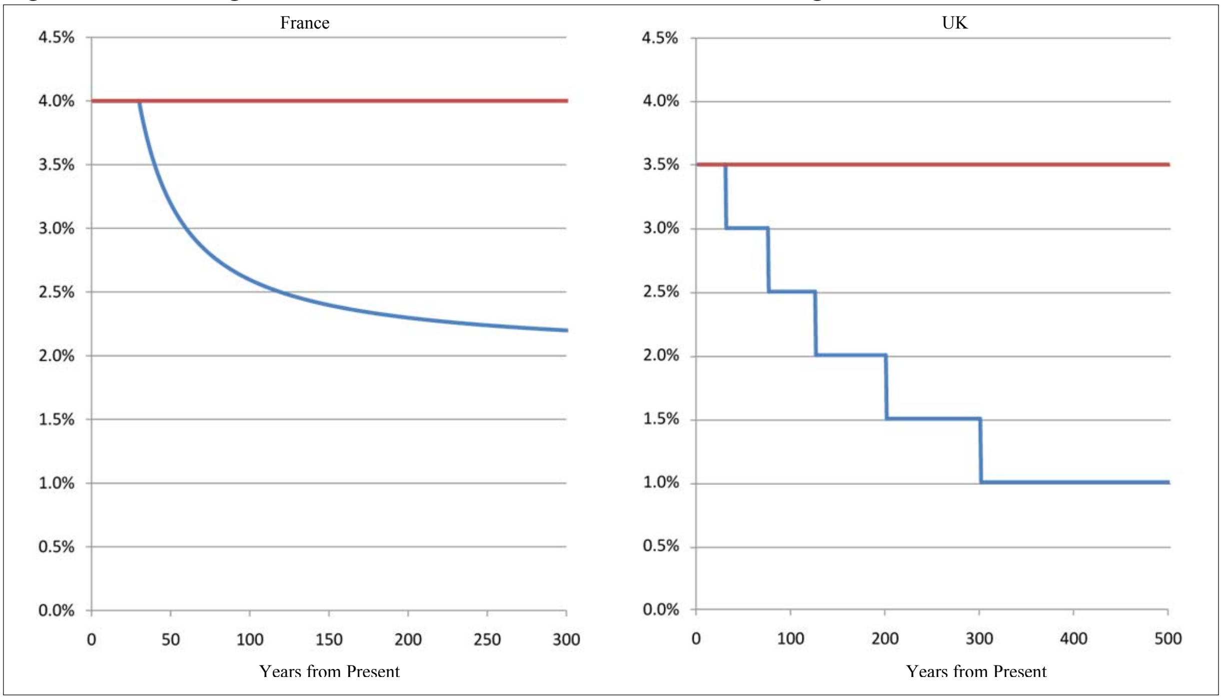
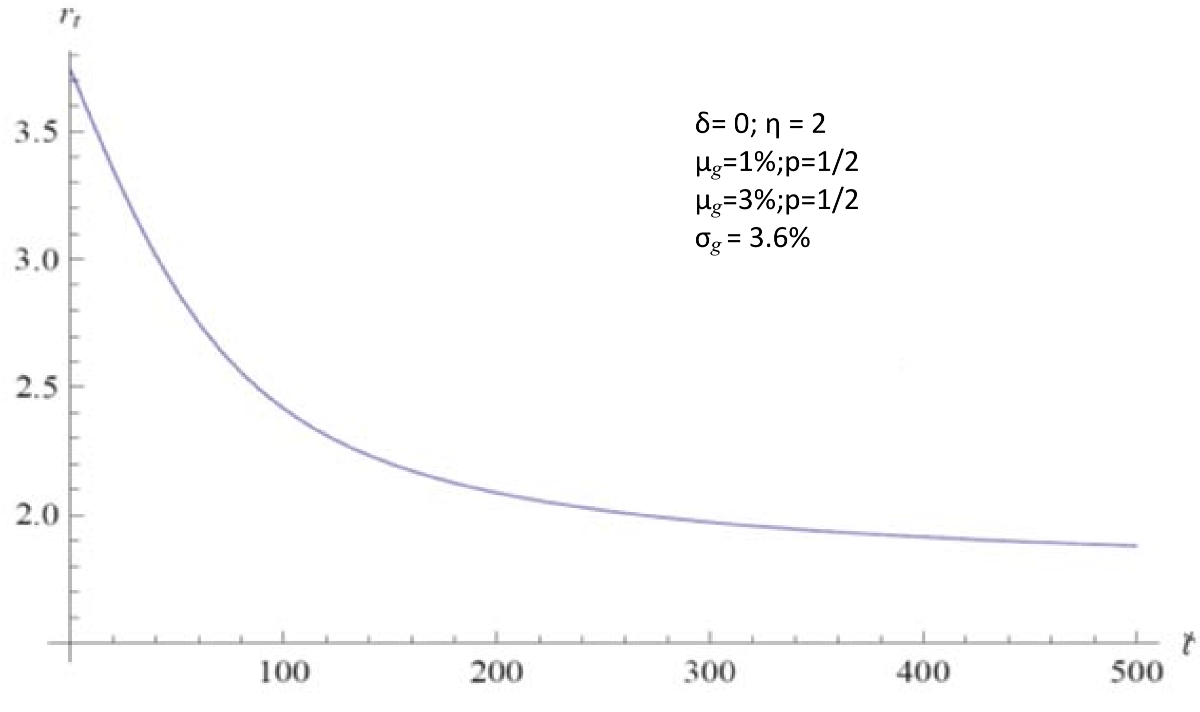
Figure 1. Declining Discount Rates in France and the United Kingdom



Source: Sterner, Damon, and Mohlin (2012)

Figure 2. Certainty-Equivalent Discount Rate Assuming Per Capita Consumption Follows a Random Walk with Uncertain Mean ($\mu = \mu(\theta)$)



Source: Gollier (2008)

Please insert Figure 1 from Science as Figure 3.

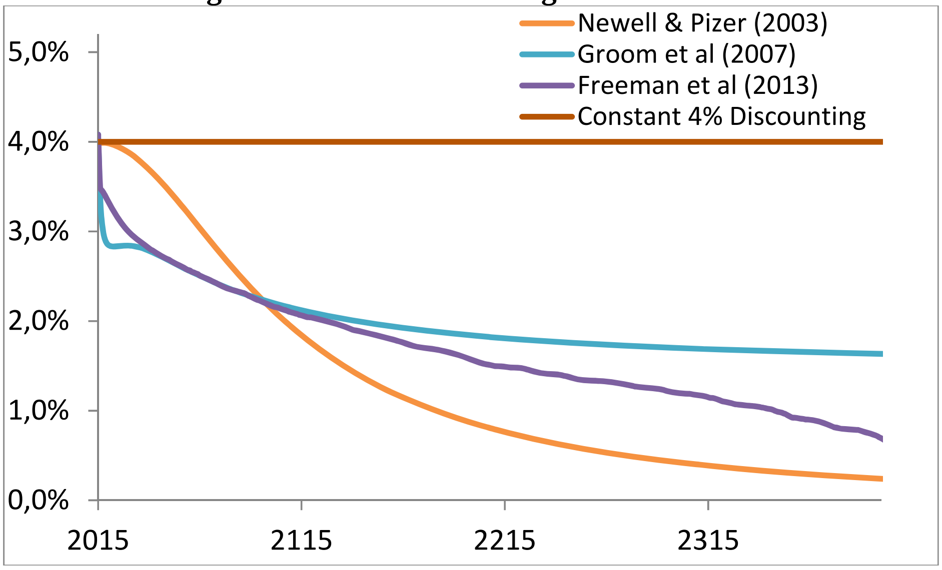


Table 1. Present Value of a Cash Flow of $1000 Received After t Years.

t	Discount Rates				
	1%	4%	7%	equally likely 1% or 7% Expected Value	Certainty Equivalent
1	990.05	960.79	932.39	961.22	3.94%
10	904.84	670.32	496.59	700.71	3.13%
50	606.53	135.34	30.2	318.36	1.28%
100	367.88	18.32	0.91	184.4	1.02%
150	223.13	2.48	0.03	111.58	1.01%
200	135.34	0.34	0	67.67	1.01%
300	49.79	0.01	0	24.89	1.01%
400	18.32	0	0	9.16	1.01%

Table 2. Discount Rate Schedule from Weitzman (2001)

Time period	Name	Marginal discount rate (Percent)
Within years 1 to 5 hence	*Immediate* Future	4
Within years 6 to 25 hence	*Near* Future	3
Within years 26 to 75 hence	*Medium* Future	2
Within years 76 to 300 hence	*Distant* Future	1
Within years more than 300 hence	*Far-Distant* Future	0

Source: Weitzman (2001)

Appendix Table A1. Maximum Acceptable Sacrifice from Group A to Increase Income of Group B by $1

η	Group A Income = 2*Group B Income	Group A Income = 10*Group B Income
0	1.00	1.00
0.5	1.41	3.16
1	2.00	10.00
1.5	2.83	31.62
2	4.00	100.00
4	16.00	10000.00

Gollier et al. (2008)