Discounting, inequalities and economic convergence

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Abstract

The aim of this paper is to examine the impact of inequalities and economic convergence on the efficient discount rate, in the absence of any risk-sharing scheme. We consider an economy in which the initial consumption level and the distribution of consumption growth are heterogeneous. The benchmark case is when inequalities are permanent and relative risk aversion is constant. The discount rate is not affected by inequalities in that case. We first relax the assumption on risk aversion, and we derive conditions under which permanent inequalities reduce the discount rate. If relative prudence is larger than unity, an increase in economic convergence always raises the efficient discount rate. In a realistic calibration exercise, we show that the effect of economic convergence is to triple the discount rate, from less 2% to more than 6%.

Keywords: Prudence, temperance, concordance, discount rate.

1 Introduction

In this paper, we address the question of the impact of current and future wealth inequalities on the choice of the discount rate. Most models aimed at determining the rate at which one should discount future cash flows assume that there is a representative agent in the economy.[1] As is well-known, this assumption allows for the existence of wealth inequalities, as long as risks can be shared efficiently and credit markets are efficient. However, these assumptions are rather unrealistic, in particular when we consider long time horizons. Commitment problems, transaction costs, adverse selection and moral hazard limits the ability to reallocate consumption across states of nature and through time in our society. Even within the European Community, individual countries are strongly reluctant to share risk with other countries of the EC, as illustrated by the 2010 Greek episode. Our aim is to examine the impact of inequalities on the efficient discount rate without assuming that consumption is efficiently allocated across states and through time.

Ramsey (1928) provides the benchmark model to determine the efficient discount rate, i.e., the minimum rate of return of safe investment projects that makes them socially desirable to implement. Putting aside the standard preference for the present, the main ingredient of this model is the aversion to consumption fluctuations over time, which is modelled by the concavity of the utility function $u(c)$. If agents expect that their income will increase over time, they will accept to save some of their income today only if the return on their saving is large enough to compensate for the increased intertemporal consumption inequality that it will generate. The so-called Ramsey rule tells us that this discount rate net of the rate of impatience equals the product of the growth rate of consumption and the index of relative risk aversion, which is measured by $-cu''(c)/u'(c)$. Adding an uncertain growth rate into the picture has been done by Hansen and Singleton (1983), Gollier (2002), or Weitzman (2007) for example. Prudent agents want to save more when their future become more uncertain. At the collective level, this implies a reduction of the discount rate. In the small, this precautionary effect is proportional to the product of the variance of the growth rate of consumption by the relative aversion to downside risk, which is measured by $c^2u'''(c)/u'(c)$ (Keenan and

[1] A noticeable exception is Azar and Sterner (1996) and Emmerling (2010) in the context of climate change.

Snow (2005)).

Let us introduce inequalities into this model. They can take the form of heterogeneities in the individual levels of initial consumption, or in the rate at which this consumption will increase over time. As soon as risk-sharing or credit markets are inefficient, the notion of discount rate becomes problematic, since the intertemporal marginal rate of substitution becomes individual-specific. One euro transferred from today to the future has a social value that depends on the characteristics of the beneficiary of this transfer. In this paper, we consider an investment project whose all costs and benefits are equally shared in the population.

Suppose first that there is no economic convergence across dynasties or countries. This means that inequalities are stable through time. Under the veil of ignorance, this type of inequalities is equivalent to adding the same proportional risk $\widetilde{z}$ to final consumption at all dates. In other words, it is equivalent to determining the impact of a change of marginal utility function from u' to v', with $v'(c) = Eu'(c\widetilde{z})$. What is the effect of this additional permanent risk on the discount rate? Signing the impact on the wealth effect requires comparing the relative risk aversion of u and v. Following a methodology developed in Gollier and Pratt (1996) and Gollier and Kimball (1996), we show that adding this permanent risk raises the concavity of the indirect utility function v – and therefore raises the discount rate – only if some restrictive conditions related to the fourth derivative of u. Similarly, signing the impact of this permanent level of inequalities on the precautionary effect requires comparing the relative aversion to downside risk of u and v. We show that doing this necessitates conditions on the fifth derivative of the utility function. In the special case of a power utility function, neither the wealth effect nor the precautionary effect is affected by permanent inequalities, so that they have no effect on the discount rate.

But the degree of inequalities is not stable through time. Several authors have tested the plausibility of economic convergence, i.e., poor regions tend to grow faster then rich ones in per capita terms. For example, Barro and Sala-i-Martin (1992) exploited data on personal incomes in 48 U.S. states since 1840, and obtained clear evidence of convergence. Convergence reduces inequalities over time. Under the veil of ignorance, it reduces the uncertainty for future generations. It is thus intuitive that economic convergence raises the discount rate, since it tends to switch off the precautionary motive for a small discount rate. We prove that this intuition is correct by using a simple definition of comparative convergence from Tchen (1980) and Epstein and Tanny (1980).

2 The discount rate for a uniform allocation of cash-flows

We consider a model with two arbitrary dates 0 and t. Agents differ on their initial wealth c_0 and on their expectations about their consumption at date t. The distributions of initial and final consumption are characterized by random variable $\widetilde{c}_0$ and $\widetilde{c}_t$, respectively. Let $\widetilde{y}$ denote the gross growth rate of consumption between 0 and t, which implies that $\widetilde{c}_t = \widetilde{c}_0 \widetilde{y}$.If $\widetilde{y}$ is independent of $\widetilde{c}_0$, we say that consumption inequalities are permanent. In the second part of this paper, we will allow $\widetilde{y}$ and $\widetilde{c}_0$ to be statistically related, thereby allowing the possibility of economic convergence. We treat $\widetilde{c}_0$ and $\widetilde{y}$ as exogeneous random variables. This intertemporal allocation of consumption may or may not be efficient.

We suppose that all agents have the same von Neumann-Morgenstern utility function u, and the same rate of pure preference for the present δ. We assume that u is differentiable up to the fifth order. The social welfare function is the sum of individual discounted expected utility:

$$SWF = \sum_{t=0} e^{-\delta t} E\left[u(\widetilde{c}_t)\right].$$

Notice that the expectation operator in this equation plays two roles. First, it computes the expected utility $E\left[u(\widetilde{c}_t) \mid c_0\right]$ of an agent with an initial consumption c_0. Second, it takes the mean the individual expected utility levels. In this model, two agents i and j with the same c_0 but different expectations $\widetilde{y}_i$ and $\widetilde{y}_j$ will be treated as if they would both have the same expectations described by $(\widetilde{y}_1, 1/2; \widetilde{y}_2, 1/2)$.

We are interested in characterizing the impact of inequalities and economic convergence on the socially efficient discount rate. In order to define it, we consider a sure investment project that reduces all agents' current consumption by ε and that increases all agents' consumption at date t by $\varepsilon \exp(rt)$. Because marginal rates of substitution are generally not equalized in this model, the way in which cash-flows are allocated in the economy will matter for the determination of the efficient discount rate. It is crucial to keep in mind that we consider a uniform allocation of costs and benefits in this paper. The socially efficient discount rate r_t is the internal rate of return r of the project such that implementing the project has no effect at the margin on SWF. It yields

$$r_t = \delta - \frac{1}{t} \ln \frac{Eu'(\widetilde{c}_0 \widetilde{y})}{Eu'(\widetilde{c}_0)}. \tag{1}$$

The benchmark case is obtained without any inequality, i.e., when $\widetilde{c}_0$ has a Dirac distribution δ_{c_0}. If the support of $\widetilde{y}$ is in a small neighborhood of c_0, which is the case when t is small, we can estimate $u'(c_0 y)$ by using a second-order Taylor expansion around c_0. As shown for example in Gollier (2010), it yields the following approximation, which is usually referred to as the extended Ramsey rule (Hansen and Singleton (1983)):

$$r_t \simeq \delta + R_1(c_0) \left[g_t - 0.5 R_2(c_0) \sigma_t^2 \right], \tag{2}$$

where $g_t = t^{-1} E(y-1)$ is the expected growth rate of consumption between 0 and t, and $\sigma_t^2 = t^{-1} Var(y)$ is the annualized variance of the growth rate of consumption over the period. We also define

$$R_i(c) = -\frac{c u^{[i+1]}(c)}{u^{[i]}(c)} \tag{3}$$

as the relative concavity of the ith derivative of u, which is itself denoted $u^{[i]}$. For example, R_1, R_2 and R_3 denote respectively relative risk aversion, relative prudence and relative temperance. Equation (2) is referred to as the "extended Ramsey rule". The right-hand side of equation (2) exhibits the three determinants of the efficient discount rate: impatience, the wealth effect and the precautionary effect. The wealth effect measured by $R_1 g_t$ is positive if the expected growth rate g_t is positive. In that case, investing for the future raises intertemporal inequalities, which is bad for intertemporal welfare. We are thus willing to sacrifice more of current wealth only if it is compensated by a positive return of the investment. Technically, this wealth effect comes from the fact that the marginal utility of consumption is smaller in the future if one believes that one will be wealthier in the future. The intensity of the wealth effect is proportional to relative risk aversion R_1, which measures the speed at which marginal utility decreases when wealth increases.

The precautionary effect is measured by $-0.5R_1R_2\sigma_t^2$. Under positive prudence ($u''' \geq 0$, or $R_2 \geq 0$), this precautionary effect tends to reduce the discount rate. Intuitively, the uncertainty on future growth makes prudent people more willing to transfer consumption to the future. This corresponds to the well-known precautionary saving motive (Leland (1968), Drèze and Modigliani (1972)). At the collective level, this increased willingness to transfer consumption to the future takes the form of a reduction of the discount rate. Technically, this effect comes from the fact that under positive prudence, the convexity of marginal utility implies that the uncertainty on future consumption raises the expected marginal utility of future consumption. As shown by Kimball (1990), the uncertainty of future consumption has an effect on the willingness to save that is equivalent to a sure reduction of the growth rate of consumption equaling the precautionary premium. This precautionary premium is approximately equal to $0.5R_2\sigma_t^2$, where the index of relative prudence R_2 measures the degree of convexity of marginal utility. Because that equivalent reduction in the growth rate has an effect on the efficient discount rate that is proportional to R_1, we see that the precautionary effect is proportional to the product of R_1 and R_2.

3 The effect of inequalities without convergence

In this section, we assume that the economy exhibits no tendency of economic convergence. This means that $\widetilde{c}_0$ and $\widetilde{y}_t$ are independent random variables. All individual consumption levels fluctuate proportionally to each others: when Mr Smith's consumption level doubles, so does Mr Jones' consumption level.

Let us first consider a simple benchmark case in which $u'(c) = c^{-\gamma}$, where $\gamma = R_1$ is the constant degree of relative risk aversion. Notice that for such power functions, the index of aversion R_i is constant and equal to $R_1 + i - 1$. In that case, we obtain the following sequence of equalities:

$$r_t = \delta - \frac{1}{t}\ln\frac{E\widetilde{c_0}^{-\gamma}\widetilde{y}^{-\gamma}}{E\widetilde{c_0}^{-\gamma}} = \delta - \frac{1}{t}\ln\frac{E\widetilde{c_0}^{-\gamma}E\widetilde{y}^{-\gamma}}{E\widetilde{c_0}^{-\gamma}} = \delta - \frac{1}{t}\ln E\widetilde{y}^{-\gamma}. \tag{4}$$

This means that in this benchmark case, the existence of an initial inequality in consumption has no effect on the socially efficient discount rates, and on its term structure. If we assume that $\log\widetilde{y}$ is normally distributed with mean μ and variance σ_y^2, this equation can be rewritten as

$$r_t = \delta - \frac{1}{t}\ln E\exp(-\gamma\log\widetilde{y}) = \delta + \gamma\mu - 0.5\gamma^2\sigma_y^2, \tag{5}$$

because the Arrow-Pratt approximation is exact in that case. Notice that it implies that equation (2) is exact under this specification.

We hereafter determine conditions on u under which the existence of no-convergent inequalities raises the socially efficient discount rate. This is the case if

$$\frac{Eu'(\widetilde{c}_0\widetilde{y}_t)}{Eu'(\widetilde{c}_0)} \leq \frac{Eu'(\overline{c}_0\widetilde{y})}{u'(\overline{c}_0)}, \tag{6}$$

where $\overline{c}_0 = E\widetilde{c}_0$ is the average consumption level at date $t = 0$. Let $\widetilde{c}_0$ equal $\overline{c}_0\widetilde{z}$, with $E\widetilde{z} = 1$. Using the fact that $\widetilde{z}$ and $\widetilde{y}$, we define the indirect utility function v with $v'(c) = Eu'(c\widetilde{z})$. The above condition can then be rewritten as follows:

$$\frac{Ev'(\overline{c}_0\widetilde{y})}{v'(\overline{c}_0)} \leq \frac{Eu'(\overline{c}_0\widetilde{y})}{u'(\overline{c}_0)}. \tag{7}$$

If this inequality is reversed, no-convergent economic inequalities reduce the efficient discount rate. We hereafter consider two different context of economic growth. In the first context, all individual consumption levels increase by a sure positive rate $y - 1 > 0$. In the second context, all individual consumption levels are multiplied by an uncertain factor with a zero expected growth.

3.1 The growth rate is a sure positive constant

Suppose that there exists some scalar $k > 1$ such that $\widetilde{y} = k$ almost surely. In such a context, equation (7) can be rewritten as follows:

$$\frac{v'(\overline{c}_0 k)}{u'(\overline{c}_0 k)} \leq \frac{v'(\overline{c}_0)}{u'(\overline{c}_0)}. \tag{8}$$

Since no restriction limits the choice of $\overline{c}_0$, a necessary and sufficient condition is that the left-hand side of the inequality be non-increasing in $k \in \mathbb{R}^+$. This is the case if and only if $-\overline{c}v''(\overline{c})/v'(\overline{c})$ is larger than $-\overline{c}u''(\overline{c})/u'(\overline{c})$ for all $\overline{c}$, where $\overline{c}$ plays the role of $\overline{c}_0k$. This means that adding inequalities $\widetilde{z}$ into the picture reduces the relative aversion to intertemporal fluctuations of consumption, i.e.,

$$E\widetilde{z} = 1 \quad \Rightarrow \quad -\frac{E\overline{c}\widetilde{z}u''(\overline{c}\widetilde{z})}{Eu'(\overline{c}\widetilde{z})} \geq R_1(\overline{c}). \tag{9}$$

This condition is very intuitive. Because we assume that growth is certain and positive, the efficient discount rate is larger than the rate of pure preference for the present because of the wealth effect. Because marginal collective utility is decreasing, transferring consumption from the future to the present raises current felicity more than it reduces future felicity. This argument to raise the discount rate depends the speed at which marginal collective utility decreases with consumption, which is measured by the aversion to intertemporal fluctuations of consumption $-E\widetilde{c}u''(\widetilde{c})/Eu'(\widetilde{c})$. Inequalities $\widetilde{z}$ raises the discount rate in this context if it raises this aversion. This is reminiscent of a problem raised by Gollier and Pratt (1996): Under which condition does a zero-mean risk raise the absolute aversion to other independent additive risks? This condition, that they called "risk vulnerability", depends upon the sign of the fourth derivative of the utility function. Problem (9) is similar, but it differs on the basis that we consider here multiplicative risks and relative risk aversion.

Suppose first that the intensity of inequalities is small, so that $\widetilde{z} = 1+\lambda\widetilde{\varepsilon}$, with $E\widetilde{\varepsilon} = 0$ and λ is small. Let us define function $\widehat{R}_1$ as follows:

$$\widehat{R}_1(\lambda) = -\frac{E\overline{c}(1+\lambda\widetilde{\varepsilon})u''(\overline{c}(1+\lambda\widetilde{\varepsilon}))}{Eu'(\overline{c}(1+\lambda\widetilde{\varepsilon}))}.$$

It is easy to check that

$$\widehat{R}_1(0) = R_1(\overline{c}), \quad \widehat{R}_1'(0) = 0, \text{ and } \widehat{R}_1''(0) = \sigma_{\widetilde{\varepsilon}}^2 R_1(\overline{c})R_2(\overline{c})\left(R_3(\overline{c}) - 2 - R_1(\overline{c})\right),$$

so that

$$\widehat{R}_1(\lambda) = R_1(\overline{c}) + 0.5\lambda^2\sigma_{\widetilde{\varepsilon}}^2 R_1(\overline{c})R_2(\overline{c})\left(R_3(\overline{c}) - 2 - R_1(\overline{c})\right) + o(\lambda^3).$$

Using the above equality and assuming that the intensity λ of inequalities is small, condition (9) can be rewritten as

$$R_2(\overline{c})\left[R_3(\overline{c}) - 2 - R_1(\overline{c})\right] \geq 0. \tag{10}$$

This condition is necessary and sufficient for a small degree of inequalities to raise the efficient discount rate when the growth of consumption is a sure positive constant. We hereafter assume that agents are prudent ($R_2 > 0$). Thus, this condition means that the degree of relative temperance is larger than 2 plus the degree of relative risk aversion. Eeckhoudt and Schlesinger (2006) have defined temperance in a very intuitive way. Consider two independent zero-mean risk $\widetilde{\varepsilon}_1$ and $\widetilde{\varepsilon}_2$. If one prefers a 50-50 chance lottery to have either $\widetilde{\varepsilon}_1$ or $\widetilde{\varepsilon}_2$ to another lottery with a 50-50 chance to get $\widetilde{\varepsilon}_1 + \widetilde{\varepsilon}_2$ or nothing, one is said to be temperant. Relative temperance R_3 measures this aversion to the aggregation of zero-mean risks. As shown by Eeckhoudt and Schlesinger (2006), one is temperant if and only if $u^{[4]}$ is negative, i.e., if u'' is concave. Notice that risk aversion corresponds to the parallel concept where $\widetilde{\varepsilon}_1$ and $\widetilde{\varepsilon}_2$ are two sure losses, and R_1 measures the aversion to this aggregation of losses. Condition (10) tells us that the intensity of temperance must not be too small compared to risk aversion, compared to the case of a power utility function, for which condition (10) holds as an equality. Thus this condition means that the difference between the aversions to the aggregation of zero-mean risks and to the aggregation of sure losses must be larger than in the case of power utility functions, where it equals two.

Let us now relax the assumption that the intensity of inequalities is small. We can rewrite left condition in (9) as follows:

$$ER_1(\overline{c}\widetilde{z})u'(\overline{c}\widetilde{z}) \geq R_1(\overline{c})Eu'(\overline{c}\widetilde{z}).$$

We hereafter show that this condition holds when R_1 is decreasing and convex. Indeed, this implies that

$$ER_1(\overline{c}\widetilde{z})u'(\overline{c}\widetilde{z}) \geq ER_1(\overline{c}\widetilde{z})Eu'(\overline{c}\widetilde{z}) \geq R_1(\overline{c})Eu'(\overline{c}\widetilde{z}).$$

The first equality comes from the fact that R_1 and u' are comonotone, and the second inequality comes from the convexity of R_1. Thus, when relative risk aversion is decreasing and convex in consumption, consumption inequalities always raise the efficient discount rate when the growth rate of consumption is a positive constant.

This sufficient condition is quite restrictive. Let us look for the necessary and sufficient condition. Condition (9) can be rewritten as follows:

$$Ef(\widetilde{c}) = 0 \quad \Longrightarrow \quad Eg(\widetilde{c}) \leq 0, \tag{11}$$

with

$$f(c) = \widetilde{c} - \overline{c} \quad \text{and} \quad g(c) = cu''(c) + R_1(\overline{c})u'(c)$$

We can apply the diffidence theorem[2] (Gollier and Kimball (1996), Gollier (2001)), which states that a necessary and sufficient condition for (11) is $g(c) \geq g'(\overline{c})f(c)/f'(\overline{c})$. It yields the following proposition.

Proposition 1 *Suppose that all individual consumption levels grow at the same sure positive rate. Inequalities raise the efficient discount rate if and only if for all c and $\overline{c}$ in the domain of consumption, we have that*

$$\overline{c}u'(c)\left(R_1(c) - R_1(\overline{c})\right) \geq u'(\overline{c})R_1(\overline{c})(c - \overline{c})\left(R_1(\overline{c}) + 1 - R_2(\overline{c})\right). \tag{12}$$

A sufficient condition is that R_1 be decreasing and convex. Condition (10) is necessary.

Of course, the necessary and sufficient condition for inequalities to reduce the efficient discount rate is the symmetric condition where the inequality in (12) is reversed. A sufficient condition is that R_1 be increasing and concave. These results are essentially negative in the sense that the sign of impact of inequalities on the efficient discount rate depends on sophisticated conditions (12) that relies on the fourth derivative of the utility function.

However, there is a realistic case in which the above proposition is useful. Suppose that

$$u'(c) = (c - c_{\min})^{-\gamma},$$

where $c_{\min} > 0$ is some minimum level of subsistence. In that case, we obtain that $R_1(c) = \gamma c/(c - c_{\min})$, which is decreasing and convex in the relevant domain of consumption $c > c_{\min}$. Thus, with such preferences, inequalities always raise the efficient discount rate when economic growth is certain.

[2]The diffidence theorem can be expressed as follows. Suppose that f and g are twice differentiable in their joint domain D and that there exists $\overline{c} \in D$ such that $f(\overline{c}) = g(\overline{c}) = 0$ and $f'(\overline{c}) \neq 0$. Condition (11) holds for all random variables $\widetilde{c}$ whose support is in D if and anly if $g(c) \geq g'(\overline{c})f(c)/f'(\overline{c})$ for all $c \in D$. A necessary condition is that $g''(\overline{c}) \geq g'(\overline{c})f(\overline{c})/f'(\overline{c})$.

3.2 The growth rate entails a small zero-mean risk

In this section, we still assume that all agents face the same growth of their consumption, but we now assume that this growth is uncertain . To isolate the precautionary effect, let us hereafter assume that $\widetilde{y} = 1+\widetilde{k}$, with $E\widetilde{k} = 0$. In other words, the expected growth of consumption is zero. Equation (6) that guarantees that inequalities raise the discount rate can be rewritten as follows:

$$\frac{Ev'(\overline{c}_0(1+\widetilde{k}))}{v'(\overline{c}_0)} \leq \frac{Eu'(\overline{c}_0(1+\widetilde{k}))}{u'(\overline{c}_0)}. \tag{13}$$

Let $S(c) = c^2 u'''(c)/u'(c) = R_1(c)R_2(c)$ denote the product of relative risk aversion and relative prudence. This index is often referred to as the aversion to downwards risk (Keenan and Snow (2005), Modica and Scarcini (2005), Crainich and Eeckhoudt (2007), Emmerling (2010)). Obviously, if $\widetilde{k}$ is small, condition (13) holds if and only if $v'''(\overline{c}_0)/v'(\overline{c}_0) \leq u'''(\overline{c}_0)/u'(\overline{c}_0)$, i.e., if and only if

$$\frac{Ec^2\widetilde{z}^2 u'''(\overline{c}_0\widetilde{z})}{Eu'(\overline{c}_0\widetilde{z})} = \frac{ES(\overline{c}_0\widetilde{z})u'(\overline{c}_0\widetilde{z})}{Eu'(\overline{c}_0\widetilde{z})} \leq S(\overline{c}_0). \tag{14}$$

By applying the diffidence theorem, we obtain the following results.

Proposition 2 *Suppose that all individuals face the same small zero-mean growth risk on their consumption. Inequalities raise the efficient discount rate if and only if for all c and $\overline{c}$ in the domain of consumption, we have that*

$$u'(\overline{c})\left(R_1(c)R_2(c) - R_1(\overline{c})R_2(\overline{c})\right) \leq u''(\overline{c})(c-\overline{c})R_2(\overline{c})\left(R_3(\overline{c}) - 2 - R_1(\overline{c})\right). \tag{15}$$

A sufficient condition is that $S = R_1R_2$ be increasing and concave. A necessary condition is given by

$$R_2(\overline{c})\left[2 - R_1(\overline{c})R_2(\overline{c}) + R_3(\overline{c})(R_4(\overline{c}) - 4)\right] \leq 0 \tag{16}$$

for all $\overline{c}$ in the consumption domain.

Proof: Conditions (15) and (16) are obtained by applying the diffidence theorem. The sufficiency condition is proved as follows. Suppose that S is increasing. It implies that $S(\overline{c}_0 z)$ and $u'(\overline{c}_0 z)$ are anti-comonotone. By the covariance rule, it implies that

$$ES(\overline{c}_0\widetilde{z})u'(\overline{c}_0\widetilde{z}) \leq ES(\overline{c}_0\widetilde{z})Eu'(\overline{c}_0\widetilde{z}).$$

Assuming that S is concave implies that $ES(\overline{c}_0\widetilde{z}) \leq S(\overline{c}_0)$ by Jensen inequality. Combining this observation with the above inequality immediately implies condition (14). ■

Reciprocally, economic inequalities reduce the discount rate if and only if inequality (15) is reversed. A sufficient condition is that S be decreasing and convex. The reversed inequality in (16) is a necessary condition. The power utility function is the limiting case in which conditions (15) and (16) hold as equalities. Observe that our necessary condition (16) is quite sophisticated, since it relies on R_4, i.e., on the fifth derivative of the utility function.

As in the previous section, let us consider the case $u'(c) = (c - c_{\min})^{-\gamma}$, with $c_{\min} > 0$. Because $S(c) = \gamma(\gamma+1)(c/(c-c_{\min}))^2$, S is decreasing and convex in the relevant consumption domain $c > c_{\min}$. It implies that when consumers face the same small zero-mean growth risk, inequalities reduce the efficient discount rate. From the previous section, we know that the same effect prevails when the the growth is a sure negative constant. Combining these two results, we conclude that inequalities reduce the efficient discount rate when the risk on economic growth is small and has a negative expectation. The effect of inequalities on the discount rate is intrinsically ambiguous when the expected consumption growth is positive, in the sense that it is negative when the expected growth is small, and it is positive when the expected growth is large.

This phenomenon is illustrated in Figure 1. Following for example Ogaki and Zhang (2001), suppose that $u'(c) = (c-c_{\min})^{-\gamma}$, where $c_{\min}$ is normalized to unity and $\gamma = 2$. In the spirit of the "Twin Peaks" cross-country distribution of incomes documented for example by Quah (1997), suppose that the distribution of initial consumption levels is $\widetilde{c}_0 \sim (2, 1/2; 10, 1/2)$. Suppose also that the growth of individual consumption is $\widetilde{y} \sim (1+g-4\%, 1/2; 1+g+4\%, 1/2)$, where g is the expected growth rate. In Figure 1, we have drawn the efficient discount rate as a function of the expected growth rate g in the unequal economy. The dashed curve corresponds to the efficient discount rate when all agents have the same initial consumption $\overline{c}_0 = 6$. When $g = 0$, only the precautionary effect is at play as examined in this section, and the

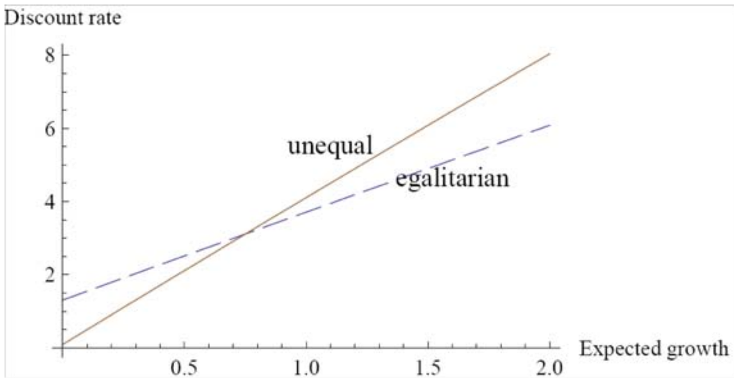

Figure 1: The effect of inequalities on the discount rate (in %) as a function of the expected growth rate g (in %). We assume that $u'(c) = (c-1)^{-2}$, $\delta = 2\%$, $\widetilde{c}_0 \sim (2, 2/3; 10, 1/3)$, no convergence, and $\widetilde{y} \sim (1+g-4\%, 1/2; 1+g+4\%, 1/2)$,

impact of inequalities on the discount rate is negative. For larger expected growth rates, the effect of inequalities is reversed.

The reader should also be made aware of the fact that condition (14) is necessary and sufficient for inequalities to raise the discount rate only when the growth risk is small. When the growth risk is not restricted to be small, this condition is not sufficient, as shown in Gollier and Kimball (1996). They provide sufficient, necessary, and necessary and sufficient conditions on u and v in the general case. One should use them in combination with the definition of v, with $v'(c) = Eu'(c\widetilde{z})$, to relax the assumption that the growth risk is small in the above proposition. Given the already complex analysis in the small, we decided to leave this for future research.

4 The effect of economic convergence

In the previous section, we have assumed that all agents face the same uncertainty about the growth rate of their future consumption. We examined the impact of this permanent level of inequalities on the efficient discount rate. In this section, we examine another problem. We take the initial inequalities expressed by $\widetilde{c}_0$ as given, and we examine the role of economic convergence on the choice of the discount rate. This convergence takes the form of a negative statistical dependence between $\widetilde{c}_0$ and the growth rate $\widetilde{y}$ of individual consumption.

The notion of a reduction in statistical dependence that is useful in this context was first developed by Tchen (1980) and Epstein and Tanny (1980). We identify the notion of more economic convergence to Tchen's notion of less concordance between the initial condition and the future expectations. This means that a greater initial consumption z_i goes with less optimistic expectations about future growth. It is easiest to define their concept of "less concordance" by assuming that both $\widetilde{c}_0 = \overline{c}_0 \widetilde{z}$ and $\widetilde{y}$ have a discrete support. Suppose that $\widetilde{z}$ and $\widetilde{y}$ can take respectively values $z_1 < z_2 < ... < z_n$ and $y_1 < y_2 < ... < y_p$. Let $p_{ij} = \Pr[\widetilde{z} = z_i, \widetilde{y} = y_j]$ be the joint probability that $\widetilde{z} = z_i$ and $\widetilde{y} = y_j$. We have that $0 \leq p_{ij} \leq 1$ and $\Sigma_i \Sigma_j p_{ij} = 1$. We compare two economic contexts represented respectively by probability matrices $P = [p_{ij}]$ and $\widehat{P} = [\widehat{p}_{ij}]$. We define a "marginal-preserving reduction in concordance" as any transformation in the probability distribution from P to $\widehat{P}$ that satisfies the following property:

$$\exists (i, i') \in \{1, ..., n\}^2, \ \exists (j, j') \in \{1, ..., p\}^2, \ i < i', \ j < j', \ \exists \varepsilon > 0 :$$

$$\widehat{p}_{ij} = p_{ij} - \varepsilon; \ \widehat{p}_{i'j'} = p_{i'j'} - \varepsilon; \ \widehat{p}_{ij'} = p_{ij'} + \varepsilon; \ \widehat{p}_{i'j} = p_{i'j} +, ;$$

whereas all other probabilities are unchanged. In words, $\widehat{P}$ is obtained from P by substracting probability mass ε from the "concordant states" (z_i, y_j) and $(z_{i'}, y_{j'})$, and by adding probability mass ε to the "discordant states" $(z_i, y_{j'})$ and $(z_{i'}, y_j)$. This is illustrated in Figure 2. Observe that marginal-preserving reductions in concordance do not affect the marginal distributions of $\widetilde{z}$ and $\widetilde{y}$. Following Tchen (1980) and Epstein and Tanny (1980), we say that $(\widetilde{z}, \widetilde{y})$ undergoes a reduction in concordance if the new joint distribution of this pair can be obtained from the original one by a sequence of marginal-preserving reductions in concordance. They showed that this is the case if and only if for all (z, y) in the support of $(\widetilde{z}, \widetilde{y})$,

$$\widehat{F}(z, y) \leq F(z, y), \tag{17}$$

where F and $\widehat{F}$ are the initial and final cumulative distribution functions of $(\widetilde{z}, \widetilde{y})$. A less concordant cdf concentrates less probability mass in any South-East quadrangle of $\mathbb{R}^{\nvDash}$.

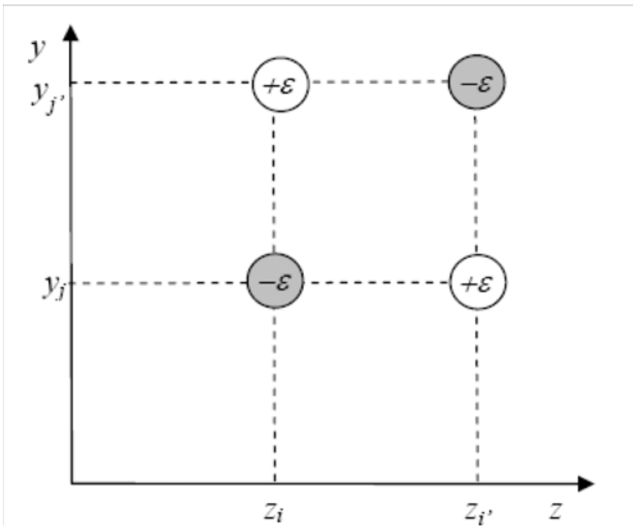

Figure 2: Transfer of probability masses that yields a marginal-preserving reduction in concordance.

Definition 1 *Consider two economies that are characterized by the cumulative distributions F and $\widehat{F}$ of $(\widetilde{z}, \widetilde{y})$. We say that economy $\widehat{F}$ exhibits more convergence than economy F if and only if condition (17) holds for all (z, y) in the support of $(\widetilde{z}, \widetilde{y})$.*

This yields immediately the following property of economic convergence, which relies on the notion of First-order Stochastic Dominance (FSD): For any z in the support of $\widetilde{z}$, the increase in economic convergence yields a FSD-improvement in the distribution of $\widetilde{y} \mid \widetilde{z} \leq z$. Adding the condition that this change in distribution does not affect the marginal distributions of $\widetilde{z}$ and $\widetilde{y}$ yields an alternative definition of economic convergence.

We now examine the impact of an increase in economic convergence on the efficient discount rate r_t defined by equation (1). Because the marginal distribution of $\widetilde{c}_0$ is unaffected by it, we see that the increase in economic convergence raises r_t if and only if it reduces $Eh(\widetilde{z}, \widetilde{y})$, with $h(z, y) = u'(\overline{c}_0 zy))$. Let us consider more generally any function $h : \mathbb{R}^{\nvDash} \rightarrow \mathbb{R}$. From the definition of a marginal-preserving reduction in concordance of $(\widetilde{z}, \widetilde{y})$, it is clear that it reduces $Eh(\widetilde{z}, \widetilde{y})$ if and only if

$$-\varepsilon\left[h(z,y)+h(z',y')\right]+\varepsilon\left[h(z,y')+h(z',y)\right]\leq 0$$

for all $\varepsilon > 0$,This is true if and only if h is supermodular. In fact, Tchen (1980) and Epstein and Tanny (1980) proved that any reduction in concordance reduces Eh if and only if h is supermodular. Interestingly enough, the reduction in concordance of $(\widetilde{z}, \widetilde{y})$ implies that $\widetilde{z}+\widetilde{y}$ becomes less risky in the sense of Rothschild-Stiglitz.[3]

The following proposition is a direct application of the above result, with $h(z,y) = Eu'(\overline{c}_0 zy)$. The supermodularity of this h function is equivalent to the condition that relative prudence R_2 is uniformly larger than unity.

Proposition 3 *Any increase in economic convergence raises the efficient discount rate if and only if relative prudence $R_2(c)$ is larger than unity, for all c in the domain of consumption.*

The intuition of this result is quite simple. Under the veil of ignorance, the increased economic convergence reduces the uncertainty about the log consumption $\ln \widetilde{c}_t = \ln \overline{c}_0 \widetilde{z} + \ln \widetilde{y}$. Under prudence, this tends to raise the efficient discount rate. However, it also reduces the expected future consumption, since $h(\widetilde{z}, \widetilde{y}) = \widetilde{z}\widetilde{y}$ is supermodular. This wealth effect tends to reduce the efficient discount rate. Thus, the precautionary effect generated by the increased convergence needs to be large enough to guarantee the result. The presence of the counterbalancing wealth effect explains why the condition is that R_2 be larger than unity rather than 0. Notice that in the power case with $u'(c) = c^{-\gamma}$, $R_2 = \gamma + 1$, so that this condition is always satisfied. Emmerling (2010) calibrates this model by using the SRES A2 baseline scenario proposed in the last IPCC Report (2007) with 9 regions exhibiting economic convergence.

Let us reexamine the simple numerical exercise of the previous section with $u'(c) = (c - c_{\min})^{-2}$, which implies that $R_2(c) = (\gamma+1)c/(c - c_{\min})$, which is larger than unity in the consumption domain $c >_{\min}$. We assume that the marginal distributions of $\widetilde{c}_0$ and $\widetilde{y}$ are as in the previous section, with $g = 2\%$:

[3]To prove this, observe that the reduction in concordance raises $E\phi(\widetilde{z} + \widetilde{y})$ for all ϕ concave. Moreover, the reduction in concordance does not affect $E(\widetilde{z} + \widetilde{y})$, since the marginal distributions are unaffected.

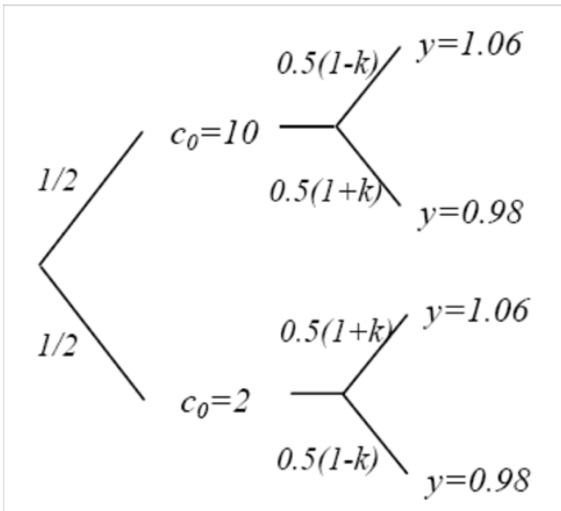

Figure 3: Parameter k is an index of economic convergence.

$\widetilde{c}_0 \sim (2, 1/2; 10, 1/2)$ and $\widetilde{y} \sim (0.98, 1/2; 1.06, 1/2)$. However, conditional to being initially poor ($c_0 = 2$), the probability of the high consumption growth is $0.5(1+k)$, with $k > 0$. Similarly, conditional to being initially wealthy ($c_0 = 10$), the probability of the high consumption growth is a smaller $0.5(1-k)$. This economic context is represented in Figure 3. Parameter $k \in [0, 1]$ is an index of economic convergence, since an increase in k yields a marginal-preserving reduction in concordance. We represented in Figure 4 the relation between the index of economic convergence and the efficient discount rate, for the original calibration with $c_{\min} = 1$, and for the CRRA case $c_{\min} = 0$. As predicted by the above proposition, an increase in economic convergence raises the efficient discount rate. This effect is in fact quite dramatic for $c_{\min} = 1$, since the discount rate goes from 8.04% in the absence of convergence ($k = 0$) up to 24.29% in the case of maximum convergence ($k = 1$).

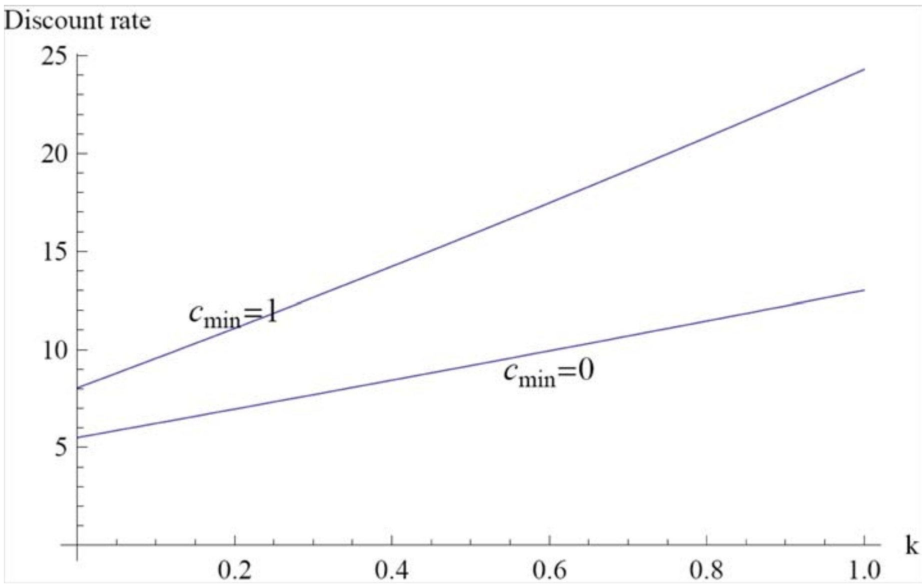

Figure 4: The discount rate as a function of the index of economic convergence k. We assume that $u'(c) = (c - c_{\min})^{-2}$, $\delta = 2\%$, and the economic context is described in Figure 3.

4.1 Calibration

In this section, we calibrate a simple – but realistic – specification of the model in order to estimate the effect of economic convergence on the discount rate. Our calibration is based on the ERS International Macroeconomic data set that gives us estimation of the GDP/cap for 190 countries over the period 1969-2009. Because of the extremely large heterogeneity of the 190 country sizes, we defined a set of 13 regions that are relatively homogenous in size and in socio-economic structure.[4] In Table 1 and Figure 6, we exhibit the regional pairs $(\log c_0, \log y)$, where $\log c_0$ is the logarithm of the regional GDP/cap in 1969, and $\log y$ is the increase in the logarithm of the regional GDP/cap between 1979 and 2009.

Suppose also that $u'(c) = c^{-\gamma}$. Under the hypothesis that there is no convergence, i.e., inequalities are permanent, we know that inequalities have no effect on the discount rate. The regional data set described above yields $\mu = E \log \widetilde{y} = 0.9047$ and $\sigma_y^2 = Var \log \widetilde{y} = 0.5128$. Let us assume that $\log \widetilde{y}$ is normally distributed, which implies that the discount rate r_t is described by equation (5). Assuming $\delta = 0$ and $\gamma = 2$, we get $r_t = 78.38\%$. Expressed on an annual basis, it yields $r = 1.96\%$.

Let us now alternatively recognize that regional inequalities are not permanent. To test this, let us regress $\log y$ with respect to $\log c_0$:

$$\log y = 2.89 - 0.26 \log c_0 + \widetilde{\varepsilon}. \tag{18}$$

The t-statistic of the slope coefficient β equals -2.41, so that it is significantly different from 0. The R2 of the regression is 0.35. This estimation provides a strong basis to accept the hypothesis of economic convergence. We also get that $Var(\widetilde{\varepsilon}) = 0.31$. Let p_i denote the population size of region i in 1969. Equation (1) can then be rewritten as follows:

$$r = \delta - \frac{1}{40} \ln \frac{\left(Ee^{-\gamma(2.89+\widetilde{\varepsilon})}\right) \sum_{i=1}^{13} p_i c_{0i}^{-\gamma(1+\beta)}}{\sum_{i=1}^{13} p_i c_{0i}^{-\gamma}}.$$

If we assume that $\widetilde{\varepsilon}$ is normally distributed, we get that

$$Ee^{-\gamma(2.89+\widetilde{\varepsilon})} = e^{-\gamma(2.89-0.5\gamma Var(\widetilde{\varepsilon}))}.$$

[4] In this process, we ignored 2.53% of the world population because of the difficulty to allocate some countries to an homogenous region. This is the case for example for Switzerland, Taiwan, and East European countries not in the EU27.

Using $\beta = -0.26$ and $Var(\widehat{\varepsilon}) = 0.31$ together with the actual distribution of $\widetilde{c}_0$ in 1969, we obtain $r = 6.45\%$. The existence of economic convergence raises the efficient discount rate from 1.96% to 6.45%. This effect is surprisingly large. It is in part explained by the phenomenal growth rate of the Chinese economy during the period.[5] Under the veil of ignorance, the plausibility for poor countries to experience a growth rate similar to China over the last 40 years yields a strong decrease in risk for future generations. This basically reverses the precautionary argument:[6] Because of economic convergence, the future looks less risky than the present!

5 Conclusion

The recent debate on the intensity of the fight against climate change has raised the crucial question of the choice of the discount rate. The traditional determinants of this rate are the wealth effect and the precautionary effect. These effects are simple to estimate when there is a representative agent in the economy, i.e., when there is no inequalities and no asymmetric shocks to income flows, or when risks are shared efficiently in the economy. These assumptions are unrealistic. The aim of this paper was to explore the impact of inequalities on the discount rate. When there is no economic convergence, that is when inequalities are permanent, the sign of this impact relies on sophisticated conditions involving the fourth and fifth derivatives of the utility function. Assuming a power utility function with a minimum level of subsistence, a permanent level of inequalities raises the discount rate through the wealth effect, and reduces the discount rate through the precautionary effect.

A simpler conclusion of this paper is that the effect of inequalities on the discount rate is mostly driven by how the degree of inequalities evolves over time. If we believe that regional economies tend to convergence as suggested by empirical evidence, then the discount rate should be positively impacted

[5] If China is removed from the data set, the R2 of the regression goes down to 0.09.The β coefficient goes up to -0.08, and is not anymore statistically significant. The efficient discount rate equals 2.97% and 4.26%, respectively without and with convergence.

[6] Notice indeed that the Ramsey rule without uncertainty and inequality would yields $r = \gamma\mu$, which corresponds here to 181%, or 4.52% per annum. Because the efficient discount rate of 6.45% is larger, it means that the precautionary effect coming from risk and inequalities *raises* the discount rate.

by this phenomenon. Indeed, under prudence, the existence of relatively large inequalities today compared to the future raises the marginal utility cost of investing for the future.

Several extensions to these results should be explored. First, we assumed in this paper that the costs and benefits of the investment are equally distributed among all consumers in the economy. One should also examine other distributions of cash flows, for example when individual benefits or costs are proportional to GDP/cap. Because risks are not efficiently shared and credit markets are inefficient, marginal rates of substitution are not equalized, which implies that the allocation of cash-flows matters for the economic evaluation. Second, more sophisticated stochastic processes for the n-country economic dynamics should be considered in the calibration and in the estimation of the model. Third, we did not really address in this paper the problem of the term structure of discount rates, which may depend upon complex relations between growth and inequalities.

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REGION	POPULATION 1969	GDP/cap	
		1969	2009
North America	224 125 073	20 745	41 213
Latin America	278 029 524	2 841	5 242
EU15	339 500 557	15 834	33 410
EU27- EU15	94 697 749	3 452	9 053
Former Soviet Union	233 466 117	2 773	4 302
China	798 640 508	128	2 494
Japan	103 171 831	13 466	32 818
Southeast Asia	284 800 658	454	1 829
South Asia	707 314 884	247	814
Oceania	19 546 597	14 075	24 662
Middle East	100 739 686	3 319	5 415
North Africa	68 930 972	1 013	2 359
Sub-Saharan Africa	287 946 358	1 030	997

Figure 5:

Table 1: Real GDP per capita for baseline regions 1969-2009.
Source: ERS International Macroeconomic Data Set

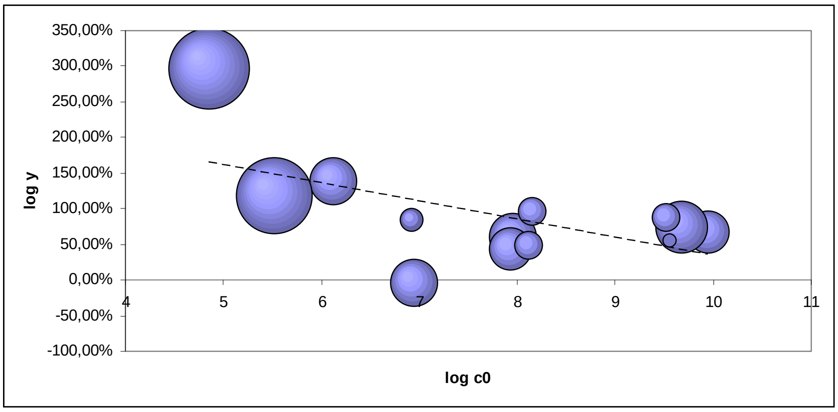

Figure 6: Change in log consumption as a function of log consumption in 1969. The size of the circles is proportional to regional population in 1969.