THEORETICAL AND EMPIRICAL ANALYSIS OF THE "JUSTE RETOUR" RULE

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1. An introduction to auction theory and its applications

An auction is a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants.

1.1. Practical and theoretical considerations

Auctions are of considerable interest on both theoretical and practical grounds.

On a practical ground, the value of goods exchanged each year by auctions or procurements is huge. In the private sector a large number of commodities are sold by auctions, such as antiques, arts, books, or machinery. This trend has recently been considerably accelerated by the popularity of internet auction sites. However, the most prominent auctioneer in the United States is the federal government that uses auctions to sell treasury bills, mobile phone licenses (spectrum auctions), oil leases, corporations, and it uses procurements to buy goods and services from the private sector. In fact, the government purchases from private firms via procurement account for about ten percent of the gross domestic product.[1]

On a theoretical ground, auction is a perfect paradigm for games with incomplete information. Indeed, it provides an ideal way of approaching, both theoretically and empirically, the question of price formation when each buyer and the seller have different information regarding the value of the object for sale. The theory of auction has considerably expanded in the economics literature since Vickrey's (1961) seminal work. This trend was particularly pronounced over the last two decades, due to the development of appropriate game theoretic tools.

1.2. Different auction mechanisms

Four basic auction designs are typically used in real life :

The English auction is the most commonly used to sell goods. In this design, the price is successively raised until one bidder remains. The essential feature of the English auction is that at any point in time each bidder knows the level of the current highest bid.

The Dutch auction is the converse of the English auction. The auctioneer calls an initial high price and then lowers the price until one bidder accepts the current

[1] A procurement may be seen as an inverse auction where the auctioneer wishes to buy a good or a service The bidders submit tenders specifying the price (and sometimes other characteristics such as the quality) at which they can provide this good or service. In the remainder we present the theory of auction, but it obviously also applies to procurements.

price.

In the First Price sealed bid auction, the potential buyers submit sealed bids, and the highest bidder is awarded the item for the price he bided. The crucial difference between English and First Price sealed bid auction is that in the former bidders are able to observe their rivals bids and they can revise their strategy accordingly, while in the latter, each bidder can submit only one bid.

Under the Second Price sealed bid auction, bidders submit sealed bids having been told that the highest bidder wins the item, but pays a price equal to the second highest bid. While this auction has useful theoretical properties, it is seldom used in practice.

1.3. Common versus Private Values Models

Much of the existing literature on auction theory analyses the Independent Private Values paradigm. In this model, a single indivisible object is to be sold to several symmetric and risk neutral bidders. Each bidder knows his own valuation, but not those of the other participants. The private values are assumed to be drawn independently from the same distribution which is common knowledge. This model applies, for instance, to antique auctions, where bidders have heterogenous tastes and buy for they own use and not for resale.

At the other extreme, consider the sale of mineral rights. This item has a single objective value to all participants, which is the amount of oil laying beneath the ground. Therefore, the item for sale is assumed to have a true value, which is unknown at the time of the auction, but each participants formed an unbiased estimate of this true value. This setting corresponds to the concept of Common Value auction described by Milgrom and Weber (1982). Note that in this type of auctions, a bidder may win an auction but realizes ex post that he has overpaid given the actual value of the item. This situation, known as the "winner's curse" as been often observed in offshore oil drilling rights procurements.

1.4. Basic auction Model

The general auction model can be summarized as follow : there are N bidders each of which is endowed with a privately known 'type' or 'signal' $\xi_i \in \Re^k$. ξ_i may represent the value to bidder i of the object for sale in an auction, or, alternatively, the cost to bidder i of producing the good demanded in a procurement. The types $\xi = (\xi_1, ..., \xi_N)$ are drawn from a joint distribution with cumulative distribution function (hereafter c.d.f.) $F(\xi)$ and density $f(\xi)$. Let $F_i(\xi_i)$ denotes the marginal c.d.f. of ξ_i and $f_i(\xi_i)$ the corresponding density.

Unobserved types are transformed into actual bids by means of transformations $x_i = \varphi_i(\xi_i)$. The bid x_i provides player i a given utility $U_i(x_1, ..., x_N; \xi_1, ..., \xi_N)$. Finally, one can apply a given game theoretic solution concept (typically the Nash Equilibrium concept) to this game in order to determine $\varphi_i^*(\xi_i)$ the optimal bid function.

Note that the except under fairly restrictive assumptions, whose empirical validity are often questionable, the equilibrium strategy $\varphi_i^*(\xi_i)$ cannot be derived analytically. As an alternative Armantier, Florens and Richard (2000) developed the concept of Constrained Strategic Equilibrium (CSE). This concept can serve as an approximation to analytically or numerically untractable Nash Equilibrium strategies, or it can be thought of as a simple decision rule (a 'Rule of Thumb') based upon intuition or past experience, and used by boundedly rational agents in place of complicated and computationally demanding Nash equilibrium strategies. The efficiency and the versatility of the CSE have been demonstrated in the analysis of a procurement in the French aerospace industry where bidders submit bi-dimensional tenders in price and quality and the winner is determined by the highest price/quality ratio. In addition, several theoretical complications, such as bidders asymmetry or collusion, are taken into considerations. Note that this model could not be analyzed with traditional approaches. In other words, the CSE concept provides a powerful tool to extend the reach of empirical and theoretical analysis to complex auction games for which there does not exist an analytical solution.

1.5. Empirical Analysis of Auction Models

There are two main approaches to the empirical analysis of auction models.

The first one is based upon reduced form econometric models. The private signals being unobserved, bids are then explained by exogenous variables such as the number of bidders, the reservation price, and some characteristics relative to the participants, or the object to be sold.

The second approach, based upon structural models, has been mainly adopted within the last decade. These models, rely upon the assumption that observed bids have been generated by an equilibrium strategy. Since this equilibrium bid function relates bids to private information, which is unobserved and modeled as random, the bids are also random with a distribution that is uniquely determined by the distribution of private signals. The structural model then defines, a non linear mapping from the distribution of the exogenous variables to the distribution of the observables (the bids). This mapping is implicitly defined by the game theoretic solution adopted. Provided identification is achieved, the estimation of the distribution of observed bids determines the structural elements. Such methods have been originally proposed by Laffont, Vuong and Ossard (1991) and Donald and Paarsch (1993). More recently, Florens, Protopopescu, and Richard (2000) have developed a very flexible estimation method applicable to a wide range of game theoretic models.

Note that the reduced form estimation does not rely on a particular theoretical model. Therefore, the scope of this approach is limited to testing few theoretic predictions. Structural models on the other hand are based upon the theoretical model. This provides a crucial advantage, as the estimates may be used to make important policy analysis.

1.6. Theoretical and Empirical Contributions

Auction theory has both normative and positive aspects. Indeed, it has provided crucial improvements to existing institutions by addressing the following questions: How can the auctioneer extract the most profit from the bidders (i.e. receive the highest possible price in an auction, or, equivalently, pay the smallest possible price in a procurement)? Which auction mechanism and/or reserve price policy should the auctioneer select? How can social efficiency be attained (i.e. the good is awarded to the bidder with the highest valuation in an auction, or, equivalently, the contract is awarded to the firm with the lowest cost in a procurement)? Should the auctioneer release any information he may have regarding the value of the object for sale? What can the auctioneer do to counter collusion among bidders? How should a bidder select optimally his bid? How may a bidder avoid the "winner's curse"? The answers to these questions however typically depend upon the characteristics of the model. It is therefore often necessary to estimate the parameters of the model to make realistic theoretical predictions or policy recommendations.

Empirical research in auction over the past two decades had two main objectives: The first is to test some implications of the theory using reduced form econometric models. Examples include the test of the revenue equivalence theorem (Mead 1967, Hansen 1985), the effect of competition on bids and the auctioneer profit (Gaver and Zimmerman 1977, or Brannman, Klein, and Weiss 1984), and the existence and consequences of the "winner's curse" (Capen, Clapp and Campbell 1971, Hendricks, Porter and Gertler 1986). The second objective is to estimate structurally some characteristics of either the auction, the auctioneer or the bidders in order to make sound policy recommendations.

2. Procurement with the "Juste Retour" Rule

In this section we propose a simple model accounting for the "Juste Retour" rule used in practice by the European Spatial Agency to built satellite on behalf of the European Community. Note, however, that the environment of the auction model developed here is drastically simplified, but this preliminary analysis illustrates the type of quantitative results one may obtained in a more realistic framework. Note also that, due the complex allocation mechanism, this model cannot be solved analytically. The subsequent analysis will therefore rely upon the numerical concept of Constrained Strategic Equilibrium.

2.1. The Simple Model

An international agency has to built a complex piece of equipment (e.g. a satellite) on behalf of M countries. The original project is divided into M subprojects consisting of smaller equipments of comparable size. Firm i_{js} ($i_{js} = 1, ..., N_{js}$) in country j ($j = 1, ..., M$) can realize only one subproject s ($s = 1, ..., M$) at a cost $c_{i_{js}}$. For the sake of illustration we assume that the costs $c_{i_{js}}$ are i.i.d. across firms, countries and subprojects with cumulative distribution F and common support $\Xi = [0, 1]$.

The subprojects are sold during M simultaneous sealed bid procurements. Firms submit prices $P_{i_{js}}$ for the object they are able to built. The "Juste Retour" rule consists in allocating the goods in order to minimize the agency total expenditure given that each country is allocated exactly one subproject. In other words, the agency selects an allocation vector $\delta = (\delta_{1_{11}}, ..., \delta_{N_{MM}})$, where $\delta_{i_{js}} = 1$ when subproject s is allocated to firm i_{js} in country j and $\delta_{i_{js}} = 0$ otherwise, such that

$$\underset{\delta}{Min} \sum_{\substack{i_{js}=1,...N_{js} \\ j=1,...,M \\ s=1,...,M}} \delta_{i_{js}} P_{i_{js}} \quad s.t. \quad \left(\begin{array}{ll} \sum\limits_{\substack{i_{js}=1,...N_{js} \\ j=1,...,M}} \delta_{i_{js}} = 1 & \forall s = 1, ..., M \\ \sum\limits_{\substack{i_{js}=1,...N_{js} \\ s=1,...,M}} \delta_{i_{js}} = 1 & \forall j = 1, ..., M \end{array} \right) . \quad (2.1)$$

This allocation rule is a simplified version of an actual procurement mechanism used in practice by the European Spatial Agency to built satellite on behalf of the European Community. Comparable Rules are used in the attribution of federal projects to different US states.

Firms are assumed to be risk neutral so that the utility can be represented by the payoff function. The utility of firm i_{js} in country j is then given by

$$U\left(c_{i_{js}}, P\right) = \left(P_{i_{js}} - c_{i_{js}}\right)\delta_{i_{js}} \quad ,$$

where $P = (P_{1_{11}}, ..., P_{N_{MM}})$. The expected utility conditional on a cost type $c_{i_{js}}$ can be written

$$\widehat{U}\left(c_{i_{js}}, P\right) = \left(P_{i_{js}} - c_{i_{js}}\right) G\left(P_{i_{js}}\right) \quad ,$$

where $G(.)$ is a survival function representing the probability of winning a subproject conditional on a bid $P_{i_{js}}$. Note that the determination of $G(.)$ requires to solve a complex combinatorial problem.

The Nash equilibrium of the game can be determined analytically when $M = 2$. As M increases however, the probability that a firm wins a subproject becomes extremely complex to derive analytically. This implies not only that the Nash equilibrium may not be obtained in closed form, but also that approximation techniques based upon the first order conditions of the problem are unapplicable. The CSE therefore appear to be currently the only amenable solution concept to analyze theoretically auctions with the "Juste Retour" rule. In addition, the CSE would allow us to include features in the model that matches more realistically actual auctions in the European spatial industry. Such features include an allocation of the goods proportional to the countries' contribution, some heterogeneity across goods, firms and/or countries, a dependence across firms' costs, and allowing firms to subcontract a portion of their subproject.

2.2. Economic Questions

One of the benefits of the CSE approach is that it allows to analyze problems that could not be treated before. For instance, the "Juste Retour" mechanism is clearly inefficient, in the sense that firms with the lowest costs are not necessarily awarded a subproject. This issue has recently raised growing concerns within the European community and a commission was hired to analyze the merits of the "Juste Retour" mechanism. Conclusions were reached based upon principles, but it was not possible to quantify the exact extent of the inefficiencies. The CSE approach now permits to conduct a quantitative comparison between the "Juste Retour" and the traditional procurement allocation rule.

In particular we can now answer the following questions: what are the quantitative consequences of the "Juste Retour" rule on: i) the total price paid by the agency? ii) the firms profits? iii) overall competition among firms and/or countries? iv) the social efficiency of the allocation?

In addition, the CSE allows one to run some policy simulations in order to establish the effects of slightly modifying the rules of the auction. In particular, one might be interested in knowing how the answers to questions i) to iv) would be affected if the projects were divided in a larger number of subprojects, or if the number of firms per project and/or the number of firms per country was increased.

Finally, we could also analyze the Meta-Game in which countries must first decide how much too contribute to the projects, and then subprojects are allocated to firms proportionally to their country's contribution. The analysis of this sequential game is even more challenging since countries must consider the probability of their own firms to win the subsequent auction, when deciding how much to contribute to the project.

2.3. Simulation Results

For the sake of illustration we assume that only one firm per country can realize a given subproject (i.e. $N_{js} = 1 \ \forall j, s$). The number of countries and subprojects is set to two (i.e. $M = 2$) in the first application and four (i.e. $M = 4$) in the second. The costs $c_{i_{js}}$ are assumed to follow a beta distribution with parameters vector $\theta = (4, 4)$. The object of this simulation is to compare the traditional auction mechanism (where the subprojects are awarded to the firms submitting the lowest price) to the "Juste Retour" mechanism. The equilibrium bid functions under each allocation mechanism are presented in Figures 1 and 2. Quantitative results are summarized in Tables 1 and 2.

The comparison between allocation mechanisms indicates, as expected, that firms are more aggressive in the traditional procurement. Indeed, in graphs 1 and 2 a firm with the same cost will submit a smaller bid in the traditional auction compare to an auction with the "Juste Retour Rule". Note that the difference between bids is more important for low cost firms, which are also the most likely to win an auction. In other words, the difference between bid functions will have important consequence for the revenue of the auctioneer. This result is confirmed in Table 2 where the prices paid by the agency are more than 17%

higher under the "Juste Retour" rule. This, in turn, leads firms' profit to increase by approximately 26%. Finally, the cost of the winning firm would be almost 14% lower if subprojects were to be sold in a traditional procurement.

These results should be interpreted with caution as they are simulated for a pre-set distribution of costs. The magnitude of the results however seems to be preserved with different distributions.[2] To be fully conclusive, we would need to estimate the cost distribution of the competing firms from the observation of actual auction outcome.

2.4. Object of the Proposed Analysis

The analysis we propose to conduct may be divided in four successive steps:

- Step 1: Data analysis.

Using a sample of past auctions using the "Juste Retour" rule, we propose to estimate (or calibrate) the model presented previously, in order to approximate the distribution of the costs of the firms participating in the auctions. This step, although of limited economic interest, is essential as it is the foundation of our subsequent economic analysis.

- Step 2: Analysis of the current environment.

Once the distribution of costs has been estimated, we are now in position to analyze the current environment, both from the agency and the firms perspectives. In particular, we can quantify for the auctions in our sample each firm's cost and profit, and we can evaluate the difference between the price paid by the agency, and the actual cost of the product delivered.

- Step 3: Comparison with traditional auction mechanism.

Once the cost of each firm is determined, we can evaluate what would have been their optimal behavior in a traditional auction. We can then compare what would have been the outcomes in a traditional auction, with those observed in the auction with the "Juste Retour" rule. As previously mentioned, we could

[2] We have tried Normal, Lognormal, Gamma and Weibull distributions, truncated on $[0, 1]$, and with various parameters value. The price paid by the agency is invariably between 12% and 28% higher under the "Juste Retour" mechanism.

in particular estimate how much the agency overpaid, and how much the firms benefited from the "Juste Retour" rule. We can also evaluate how much the "Juste Retour" rule has contributed to the distribution of the contracts across the participating countries. To do this we just need to verify whether, given the estimated cost of each firm, the attribution of the contracts under a traditional auction format would have been concentrated among a limited number of countries and/or firms.

This quantitative analysis would therefore shed some light on the actual social efficiency of the "Juste Retour" allocation rule.

- Step 4: Counter-speculative analysis:

Finally, the objective of the last step is to conduct a counter-speculative analysis in order to explore whether changing some aspects of the "Juste Retour" auction mechanism could improve some of its properties. For instance, we could examine whether dividing the project in a larger number of subprojects, or providing incentives to increase the number of participating firms, would improve competition, and reduce the financial inefficiencies compared to a traditional auction

It has to be noted that the analysis we just oulined is modest in terms of the sophistication of the theoretic model applied, but ambitious with respect to the questions addressed. However, if this initial analysis yields interesting conclusions, then a larger study, based on a more elaborated and more realistic theoretic model, would be in order to confirm these results.