“THE LAFFER CURVE IN AN INCOMPLETE-MARKETS ECONOMY”

Patrick FEVE, Julien MATHERON and Jean-Guillaume SAHUC
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PATRICK FÈVE, JULIEN MATHERON, AND JEAN-GUILLAUME SAHUC

ABSTRACT. This paper investigates the characteristics of the Laffer curve in a neoclassical growth model of the US economy with incomplete markets and heterogeneous agents. The shape of the Laffer curve changes depending on which of transfers or government debt are varied to balance the government budget constraint. While the Laffer curve has the traditional shape when transfers vary, it looks like a horizontal S when debt varies. In this case, fiscal revenues can be associated with up to three different levels of taxation. This finding occurs because the tax rates change non-monotonically with public debt when markets are incomplete.


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P. Fève, Toulouse School of Economics, patrick.feve@univ-tlse1.fr.
J. Matheron, Banque de France, julien.matheron@banque-france.fr.
J.-G. Sahuc, Banque de France, jean-guillaume.sahuc@banque-france.fr.

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The Laffer curve, i.e. the inverted-U-shaped relation between fiscal revenues and tax rates, has often been a key motivation for fiscal reforms. The best known example was the Economic Recovery Tax Act of 1981. Driven by the belief that the US were lying on the prohibitive part of the Laffer curve, the Reagan administration engineered a major tax cut, expected to be self-financed. While the actual location of the US along the Laffer curve subsequently raised controversies\(^1\), the Laffer curve concept has figured prominently in policy debates ever since. Against the current backdrop of fiscal consolidation, it has become even more relevant.

In this paper, we study issues related to the shape of the Laffer curve in the context of a neoclassical growth model with incomplete markets and heterogeneous, liquidity-constrained agents (IM for short in the remainder). There are at least two reasons why the shape of this curve might change in such an environment. First, because of market incompleteness and liquidity constraints, agents self-insure by accumulating more assets (Aiyagari, 1994) and supplying more labor (Pijoan-Mas, 2006) than in a complete-market (CM) setup. As a consequence, the fiscal bases prove to be less tax-elastic than in a CM context, thus pushing potentially the top of the Laffer curve farther to the right. Second, and more importantly, the after-tax interest rate’s response to tax changes will not a priori be invariant to which of public debt or transfers are varied to balance the government budget constraint as taxes are varied, as opposed to a CM setup in which the after-tax interest rate would be invariant to debt and transfers. This is so because public debt and transfers offer different partial insurance to households, thus altering in different manners their incentive to self-insure. In such a setup, we argue, there is no sense in which one can define a Laffer curve irrespective of which variable is chosen to balance the government budget constraint.

To address this issue, we introduce the concept of conditional Laffer curves. Holding public debt constant, we let transfers vary over a predetermined range and adjust one of the different tax rates considered in the analysis to balance the government budget constraint. This yields a relation linking fiscal revenues to the tax rate conditional on transfers. Holding transfers constant and varying debt, we can define similarly a Laffer curve conditional on public debt. As we show, in a CM setup, the two conditional Laffer curves coincide exactly, which is the mere reflection of the irrelevance of public debt and transfers for the equilibrium allocation and price system.\(^2\) In an IM setup, however, the picture changes radically.

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\(^1\)See e.g. Feldstein (1986) for a skeptical view summarizing much of the debate around the Laffer curve at that time. See also Blinder (1981).

\(^2\)In other words, given a change in distortionary taxes, the resulting allocation does not depend on transfers and/or public debt, which is just Ricardian equivalence at play.
While the Laffer curve conditional on transfers has the traditional inverted-U shape, its counterpart conditional on debt looks like a horizontal S. In this case, there can be one, two, or three tax rates compatible with a given level of fiscal revenues. The regular part of this curve (the one which indeed looks like an inverted U) is associated with positive government debt while the odd part (the one which makes the curve look like a horizontal S) is associated with negative debt levels.

To understand this odd shape, consider a simplified setup with only labor income taxes and constant shares of government consumption and transfers in output. The derivative of fiscal revenues as a share of GDP with respect to the debt-output ratio is the sum of two terms: (i) the interest rate and (ii) a multiplicative term combining the debt-output ratio and the sensitivity of the real interest rate to this ratio. In a CM model, this multiplicative term is zero because the interest rate is invariant to the debt-output ratio. In an IM setup however, as shown in Aiyagari and McGrattan (1998), public debt crowds out physical capital in households portfolio, so that the interest rate increases with the debt-output ratio. As a consequence, the sign of the derivative of fiscal revenues (as a share of GDP) with respect to the debt-output ratio depends on the sign of public debt, as does the sign of the derivative of the labor income tax with respect to the debt-output ratio. For positive debt levels, these derivatives are positive. However, for sufficiently negative debt levels, they are negative. At the same time, since debt crowds out physical capital, output will decline with the debt-output ratio. The level of fiscal revenues being the product of output and the share of fiscal revenues in output, we obtain that fiscal revenues first decline with debt, then increase, and then decline again. Combined with the non-monotonic response of the labor income tax, this translates into a horizontal-S shape of the Laffer curve.

To explore quantitatively these issues, we consider a neoclassical model along the lines of Aiyagari and McGrattan (1998) and Flodén (2001). In this economy, households are subject to persistent, uninsurable, idiosyncratic productivity shocks and face a borrowing constraint. The model includes distortionary taxes on labor, capital, and consumption. These taxes are used to finance a constant share of government consumption in output, lump-sum transfers, and interest repayments on accumulated debt. The model is calibrated to the US economy to mimic great ratios as well as moments related to the wealth distribution, using the method devised by Domeij and Heathcote (2004) and Heathcote (2005). As in Trabandt and Uhlig (2011), we then study the steady-state conditional Laffer curves associated with each of the three tax considered.

A nice feature of our setup is that it nests the standard neoclassical model. Setting the variance of idiosyncratic labor productivity shocks to zero and eliminating the borrowing constraint, our model boils down to the standard neoclassical model with distortionary taxation. This makes easier a quantitative comparison of the two versions.
Our main quantitative findings are the following. First, when transfers are varied, the Laffer curves in the IM economy look broadly like their CM counterparts. In our benchmark calibration, the revenue-maximizing labor income tax rate is 50 percent, compared to a CM counterpart equal to 49 percent. For the capital income tax, the revenue-maximizing tax is equal to 58 percent, as opposed to 53 percent in the CM economy. Second, when debt is varied instead of transfers, the local maximum of fiscal revenues in the regular part is associated with a labor income tax rate equal to 56 percent. For the capital income tax, the local revenue-maximizing tax rate in the regular part of the Laffer curve is about 51 percent. Both for capital and labor income taxes, though, higher revenues could be reached at lower taxes in the odd part of the curve. We also find no Laffer effect for consumption taxes. However, even for consumption taxes, when debt is varied, we obtain an odd portion of the Laffer curve. Thus, for all three taxes considered in the paper, there exist two levels of fiscal revenues associated with a given tax rate in the odd part of the curve: A high (resp. low) level related to negative (resp. positive) public debt.

Our results also have the corollary implication that the Laffer curves (conditional on transfers) are not invariant to the level of public indebtedness. This is potentially very important in the current context of high public debt-output ratios in the US and other advanced economies. It turns out that the Laffer curves are only mildly affected by the debt-output ratio, provided the latter is positive. However, for negative levels of public debt, e.g. as in Norway and Sweden (with net debt to GDP amounting to $-165.5$ percent and $-17.6$ percent, respectively, according to International Monetary Fund, 2013), we find that the Laffer curves can be significantly higher than their benchmark counterparts, especially when it comes to capital income taxes.

Finally, we ask whether the odd portions of the Laffer curves conditional on debt is relevant from a policy point of view. To answer this, we compute the welfare maximizing taxes and debt-output ratios. As in Röhrs and Winter (2010), we obtain that the optimal debt-output ratio is negative, translating into optimal tax rates which precisely lie on the odd portions of the Laffer curves associated with each of the three tax rates considered. Thus, the odd part of the Laffer curves is not just a theoretical curiosity. At the same time, as one might argue, the exact optimal quantity of debt crucially depends on the stochastic properties of idiosyncratic shocks. However, we show in our robustness analysis that the odd portions of the Laffer curves are robust to alternative specifications of idiosyncratic shocks.

This paper is related to previous studies investigating taxation and/or public debt in an IM setup. A first strand, exemplified by Aiyagari and McGrattan (1998) and Flodén (2001), established that a proportional income tax rate changes non-monotonically with debt. However, this literature did not explore how this feature could impact on the shape
of the Laffer curve. Röhrs and Winter (2010) extended recently this analysis to a carefully calibrated multi-tax environment. However, they too ignored the implications for the Laffer curve. Our paper complements this literature by focussing on how the conditional Laffer curve changes as debt or transfers are varied. A second strand has explored the Laffer effect in the context of IM models. For example, Flodén and Lindé (2001) found that the Laffer curve peaks when labor income tax is approximatively 50 percent or more. However, their analysis abstracts from public debt. More recently, Ljungqvist and Sargent (2008) and Alonso-Ortiz and Rogerson (2010) revisited in an IM setup the effects of labor taxation studied by Prescott (2004). Ljungqvist and Sargent (2008) and Alonso-Ortiz and Rogerson (2010) compared the Laffer curves under IM and CM. Focusing on labor income taxes, they obtained that the prohibitive part of the Laffer curve in the IM case differs only mildly from what obtains in the CM version of their model. However, they too abstract from government debt. Our paper complements these works by insisting more on the role of public debt and by considering various forms of distortionary taxes.

The rest of the paper is organized as follows. In section 1, we expound the IM model and define the steady-state equilibrium under study. We introduce formally the concept of conditional Laffer curves. Section 2 is devoted to the quantitative results. We first discuss our calibration strategy and then explore the extent to which the Laffer curves change when computed in an IM model. Section 3 explores the robustness of our results to alternative assumptions on government consumption and on the stochastic process for idiosyncratic shocks. The last section briefly concludes.

1. THE MODEL

In this section, we describe the model economy and define the associated steady-state equilibrium. We then discuss how to construct Laffer curves in this setup. In particular, we insist on the dependence of the Laffer curve on which of public debt or transfers is varied when taxes are varied.

1.1. Environment. We consider a discrete-time economy without aggregate risk. Time is indexed by \( t \in \{1, 2, \ldots\} \). The final good \( Y_t \), which we take as the numeraire, is produced by competitive firms, according to the Cobb-Douglas technology

\[
Y_t = K_t^\theta (Z_t N_t)^{1-\theta},
\]

where \( \theta \in (0, 1) \) denotes the elasticity of production with respect to capital, \( K_t \) and \( N_t \) are the inputs of physical capital and efficient labor, respectively, and \( Z_t \) is an exogenous technical progress index, evolving according to \( Z_{t+1} = (1 + \gamma) Z_t \) with \( Z_0 = 1, \gamma > 0 \). Firms rent capital and efficient labor on competitive markets, at rates \( r_t + \delta \) and \( w_t \),
respectively, where $\delta \in [0, 1]$ is the depreciation rate of physical capital, $w_t$ is the wage rate, and $r_t$ is the interest rate.

The economy is inhabited by a continuum of agents, of measure one. Each agent’s time endowment is normalized to 1 and can be allocated to market work $h_t$ or to leisure $1 - h_t$. Agents have preferences over consumption $c_t$ and leisure defined by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\eta \log(c_t) + (1 - \eta) \log(1 - h_t)] \right\}$$

with $c_t \geq 0$ and $0 \leq h_t \leq 1$. Here $\beta \in (0, 1)$ is the subjective discount factor, $E_0 \{ \cdot \}$ is the mathematical expectation conditioned on the individual state at date $t = 0$, and $\eta \in (0, 1)$ is the relative weight of consumption in utility.

In each period, households receive an uninsurable shock $s_t > 0$ to their labor productivity. These shocks are assumed to be i.i.d. across agents and evolve over time according to a Markov process, with bounded support $S$ and stationary transition function $Q(s, s')$. These idiosyncratic productivity shocks are normalized so that the unconditional mean of their logarithm is equal to zero, i.e. $E\{ \log(s) \} = 0$. An individual agent’s efficient labor is thus $s_th_t$, with corresponding labor earnings given by $(1 - \tau_N)w_t s_th_t$, where $\tau_N$ denotes the labor income tax. In addition, agents self-insure by accumulating $a_t$ units of assets which pay the after-tax rate of return $(1 - \tau_A)r_t$, where $\tau_A$ denotes the capital income tax. These assets can consist of units of physical capital and/or government bonds. Once arbitrage opportunities have been ruled out, each asset has the same rate of return. Also, agents must pay a consumption tax $\tau_C$. Finally, they receive lump-sum transfers $T_t$. Thus, an agent’s budget constraint is

$$(1 + \tau_C)c_t + a_{t+1} \leq (1 - \tau_N)w_t s_th_t + (1 - \tau_A)r_ta_t + T_t.$$

As in Aiyagari and McGrattan (1998), borrowing is exogenously restricted by the following constraint

$$a_{t+1} \geq 0.$$

There is finally a government in the economy. The government issues debt $B_{t+1}$, collects tax revenues, rebates transfers, and consumes $G_t$ units of final good. The associated budget constraint is given by

$$B_{t+1} = (1 + r_t)B_t + T_t + G_t - (\tau_A r_tA_t + \tau_N w_t N_t + \tau_C C_t)$$

where $C_t$ and $A_t$ denote aggregate (per capita) consumption and assets held by the agents, respectively.

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4The transition $Q$ has the following interpretation: for all $s \in S$ and for all $S_0 \in \mathcal{S}$, where $\mathcal{S}$ denotes the Borel subsets of $S$, $Q(s, S_0)$ is the probability that next period’s individual productivity lies in $S_0$ when current productivity is $s$. 
1.2. Equilibrium Defined. In the remainder of this paper, we focus exclusively on the steady state of an appropriately de-trended version of the above economy. Growing variables are de-trended by dividing them by \( Y_t \). De-trended variables are referred to with a hat. In the benchmark specification, the ratio of government consumption to output \( \hat{c} \) is constant. In the robustness section, we also consider an alternative case in which the level of government consumption (in deviation from \( Z_t \)) is constant.

We let \( A \) denote the set of possible values for assets \( \hat{a} \) and let the joint distribution of \( \hat{a} \) and individual productivity \( s \) across agents be denoted \( x(\hat{a}, s) \). Thus, for all \( A_0 \times S_0 \in \mathcal{A} \times \mathcal{S} \), \( x(A_0, S_0) \) is the mass of agents with assets in \( A_0 \) and idiosyncratic productivity in \( S_0 \), where \( \mathcal{A} \times \mathcal{S} \) denotes the Borel subsets of \( A \times S \).

We can now write an agent’s problem in recursive form

\[
\begin{aligned}
\nu(\hat{a}, s) &= \max_{\hat{c}, \hat{h}, \hat{a}'} \left\{ \eta \log(\hat{c}) + (1 - \eta) \log(1 - \hat{h}) + \beta \int_{S} \nu(\hat{a}', s') \mathcal{Q}(s, ds') \right\} \\
\text{s.t.} \quad & (1 + \tau_C)\hat{c} + (1 + \gamma)\hat{a}' \leq (1 - \tau_N)\hat{\omega}h + (1 + (1 - \tau_A)\hat{a} + \hat{T}, \\
& \hat{a}' \geq 0, \quad \hat{c} \geq 0, \quad 0 \leq h \leq 1.
\end{aligned}
\]

We can thus define a stationary, recursive equilibrium in the following way.

**Definition 1**—A steady-state, recursive competitive equilibrium is a constant system of prices \( \{r, \hat{\omega}\} \), a vector of constant policy variables \( (\tau_C, \tau_A, \tau_N, \hat{T}, \hat{\omega}, \hat{B}) \), a value function \( \nu(\hat{a}, s) \), time-invariant decision rules for an individual’s assets holdings, consumption, and labor supply \( \{g_a(\hat{a}, s), g_c(\hat{a}, s), g_h(\hat{a}, s)\} \), a measure \( x(\hat{a}, s) \) of agents over the state space \( A \times S \), and aggregate quantities \( \hat{A} \equiv \int \hat{a}dx, \hat{C} \equiv \int g_c(\hat{a}, s)dx, N \equiv \int sg_h(\hat{a}, s)dx \) and \( \hat{K} \) such that:

(i) The value function \( \nu(\hat{a}, s) \) solves the agent’s problem stated in equation (1), with associated decision rules \( g_a(\hat{a}, s) \), \( g_c(\hat{a}, s) \) and \( g_h(\hat{a}, s) \);

(ii) Firms maximize profits and factor markets clear, so that

\[
\hat{\omega} = \frac{1 - \theta}{N},
\]

\[
r + \delta = \frac{\theta}{\hat{K}}.
\]

(iii) Tax revenues equal government expenses

\[
\tau_N\hat{\omega}N + \tau_Ar\hat{A} + \tau_C\hat{C} = \hat{T} + \hat{G} + (r - \gamma)\hat{B};
\]

(iv) Aggregate savings equal firm’s demand for capital plus government’s debt

\[
\hat{A} = \hat{K} + \hat{B};
\]

(v) The distribution \( x \) is invariant

\[
x(A_0, S_0) = \int_{A_0 \times S_0} \left\{ \int_{A \times S} 1_{\{\hat{a}' = g_a(\hat{a}, s, x)\}} \mathcal{Q}(s, ds')dx \right\} da'ds'.
\]
for all \( A_0 \times S_0 \in \mathcal{A} \times \mathcal{S} \), where \( \mathbf{1}_{\{\cdot\}} \) is an indicator function taking value one if the statement is true and zero otherwise.

With a slight abuse of notation, we define the stationary level of output \( \hat{Y} = Y_t / Z_t \). It is linked to \( \hat{K} \) and \( N \) through \( \hat{Y} = \hat{K}^{\theta/(1-\theta)} \Omega N \).

For comparison purposes, we also consider a version of the model in which (i) we impose idiosyncratic labor income shocks \( s_t \) set to their average value and (ii) we relax the borrowing constraint. Notice that in this CM environment, the distinction between effective labor \( H = \int g_h(\hat{a}, s) dx \) and efficient labor \( N \) is no longer useful since both quantities coincide (up to a multiplicative constant). We thus incorporate a productivity scale factor \( \Omega \) in front of \( N_t \) in the production function to compensate the CM economy for the average labor productivity effect present in the IM economy (i.e. the relative difference between \( N \) and \( H \)). Doing so, we make sure that in the benchmark calibration described below, all economies share the same interest rate, the same effective labor \( H \), and the same stationary production level \( \hat{Y} \). This is done by defining \( \hat{Y} = \hat{K}^{\theta/(1-\theta)} \Omega N \).

1.3. The Laffer Curves. From the government budget constraint, fiscal revenues (as a share of GDP) \( \hat{R} \) are given by

\[
\hat{R} = \tau_N \hat{w}N + \tau_A r\hat{K} + \tau_C \hat{C}.
\]

\( \hat{R} \) is then converted to level according to \( R = \hat{R} \times \hat{Y} \). Notice that the level of fiscal revenues \( R \) is defined net of fiscal receipts from taxing public bonds return.

Traditionally, the steady-state Laffer curve associated with \( \tau_i, i \in \{N, A, C\} \) is defined as follows. Let \( \tau_i \) vary over an admissible range, holding the other two taxes constant. The Laffer curve is then the locus \((\tau_i, R) \) which relates the level of fiscal revenues \( R \) to the tax rates \( \tau_i \). This definition of the Laffer curve correctly takes into account the general equilibrium effects induced by a tax change, as argued by Trabandt and Uhlig (2011). For example, a given change in \( \tau_N \) will modify \( x, g_a, g_h, \) and \( g_c \), so that it will also impact on all the fiscal bases.

However, notice that in this definition, no reference is made as to how the government balances its budget constraint when \( \tau_i \) varies. Indeed, in equilibrium, we must always have

\[
\hat{R} = \hat{G} + \hat{T} + [(1 - \tau_A)r - \gamma] \hat{B},
\]

so that a given change in one of the three tax rates is associated with a corresponding adjustment in either \( \hat{T} \) or \( \hat{B} \).\(^5\)

\(^5\)Recall that, unless otherwise stated, \( \hat{G} \) is constant in all our experiments.
In a CM setup, one can abstract from these adjustments, as shown in the following proposition.

**Proposition 1**—In a CM setup, the steady-state Laffer curve associated with \( \tau_i, i \in \{N, A, C\} \) is invariant to which of \( \hat{T} \) or \( \hat{B} \) are adjusted to balance the government budget constraint.

*Proof.* See Appendix A. \( \square \)

This proposition establishes that in a CM setup, given a change in one of the three distorting taxes, adjusting lump-sum transfers or public debt is of no consequence for the equilibrium allocation and price system thus implying the same Laffer curve. This is just Ricardian equivalence at play, which, in the present context, manifests itself notably through the invariance of the after-tax interest rate to changes in \( \hat{T} \) or \( \hat{B} \).

In an IM setup, however, the invariance of the after-tax interest rate does not hold. Indeed, the after-tax interest rate is affected by the fact that capital and government bonds provide partial insurance to households. The cost of this insurance is reflected in the lower rate of return on those assets. When the government issues more debt, it effectively decreases the price of capital, thus lowering the insurance cost associated with holding capital. This translates into an increasing interest rate. By the same line of reasoning, since an increase in transfers also provides partial insurance to households, it also translates into an increasing interest rate. Hence, it is a priori unclear how balancing the government budget constraint via either \( \hat{T} \) or \( \hat{B} \) can feedback on the Laffer curve. As a consequence, in an IM setup, there is no sense in which one can define a Laffer curve independently from the way in which the government budget constraint is balanced.

In order to organize our discussion, it is thus convenient at this stage to define the concept of a steady-state conditional Laffer curve as follows.

**Definition 2**—Let \( \hat{B} \) be fixed and let \( \hat{T} \) vary over an admissible range. Let \( \tau_i(\hat{T}), i \in \{N, A, C\} \), denote the tax rate that balances the government budget constraint, holding the other two taxes constant, and let \( R(\hat{T}) \) denote the associated level of government revenues. The corresponding steady-state conditional Laffer curve is the locus \((\tau_i(\hat{T}), R(\hat{T}))\) relating tax rates to fiscal revenues. One can alternatively define a locus \((\tau_i(\hat{B}), R(\hat{B}))\) by varying \( \hat{B} \) over an admissible range, holding \( \hat{T} \) constant.

In the following discussion, we will refer to the locus \((\tau_i(\hat{T}), R(\hat{T}))\) as the Laffer curve conditional on transfers, and to the locus \((\tau_i(\hat{B}), R(\hat{B}))\) as the Laffer curve conditional on debt. Definition 2 leads us to the following proposition.

**Proposition 2**—In a CM setup, the steady-state conditional Laffer curves \((\tau_i(\hat{T}), R(\hat{T}))\) and \((\tau_i(\hat{B}), R(\hat{B}))\) coincide, for all \( i \in \{N, A, C\} \).
This proposition establishes that in a CM setup, the notion of conditional Laffer curves serves no special purpose since both curves coincide. In the rest of this paper, we focus on analyzing to what extent they differ in an IM setup.

2. Quantitative Results

In this section, we calibrate the IM and CM models in order to analyze quantitatively its predictions relative to the conditional Laffer curves discussed above.

2.1. Calibration and solution method. The model is calibrated to the US economy. A period is taken to be a year. Preferences are described by two parameters, \( \eta \) and \( \beta \). We pin down \( \eta \) so that aggregate hours worked \( H \equiv \int g_h(a,s)dx \) equal 0.25. The subjective discount factor \( \beta \) is set so that the after tax interest rate is equal to 4 percent, as in Trabandt and Uhlig (2011).

The fiscal parameters \( \hat{\beta} \) and \( \hat{G} \) are set to match the debt-output ratio and the government consumption-output ratio reported by Trabandt and Uhlig (2011), i.e. \( \hat{\beta} = 0.63 \) and \( \hat{G} = 0.18 \). The tax rates are calibrated to match estimates of effective tax rates computed using the methodology developed by Mendoza, Razin, and Tesar (1994). This yields \( \tau_N = 0.28 \), \( \tau_A = 0.38 \), and \( \tau_C = 0.05 \). Using these parameters, the benchmark value of the transfer-output ratio \( \hat{T} \) is endogenously computed so as to balance the government budget constraint, yielding \( \hat{T} = 7.4 \) percent.

To calibrate the stochastic process \( \{s_t\} \), we follow Domeij and Heathcote (2004) and Heathcote (2005). We assume that \( \{s_t\} \), evolves over time according to a three-state Markov chain, with support \( S = \{s_1, s_2, s_3\} \) and transition matrix \( Q \). The typical element \( Q_{ij} \) denotes the probability of reaching state \( j \) from state \( i \). We impose the following structure on \( Q \)

\[
Q = \begin{pmatrix}
Q_{11} & 1 - Q_{11} & 0 \\
(1 - Q_{22})/2 & Q_{22} & (1 - Q_{22})/2 \\
0 & 1 - Q_{11} & Q_{11}
\end{pmatrix}.
\]

As in Heathcote (2005) and Domeij and Heathcote (2004), this transition matrix implies that households cannot move between the high and low productivity levels directly, that the fractions of high and low productivity households are equal, and that the probabilities of moving from the medium productivity state into either of the others are the same. Finally, as discussed in the previous section, we further impose the restriction \( E\{\log(s_t)\} = 0 \).
Given the above restrictions, this leaves four free parameters to be calibrated: $\bar{s}_1$, $\bar{s}_2$, $Q_{11}$, and $Q_{22}$. We pin down their values by matching four calibration targets: the Gini coefficient of the wealth distribution, the share of wealth held by the 40 percent poorest, $\rho(\log(s_t))$ the autocorrelation of the logarithm of labor earnings, and $\sigma(\log(s_t))^2$ the variance of the logarithm of labor earnings. The first two calibration targets are taken from Díaz-Gimenez, Glover, and Ríos-Rull (2011). In particular, they report that the Gini index is equal to 0.816 and the share of aggregate wealth held by the 40 percent poorest amounts to 0.9 percent. The last two correspond to the values reported by Heathcote (2005) and Domeij and Heathcote (2004). In particular, we seek to match $\rho(\log(s_t)) = 0.9$ and $\sigma(\log(s_t))^2 = 0.05/(1 - 0.9^2)$.

The calibration is summarized in table I. The procedure described above implies $\beta = 0.97$ in the IM economy, versus $\beta = 0.98$ in the CM economy. This illustrates that, given

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**Table I. Calibration Summary**

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Incomplete Markets</th>
<th>Complete Markets</th>
<th>Calibration Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>0.3398</td>
<td>0.3058</td>
<td>$H = 0.25$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9680</td>
<td>0.9808</td>
<td>$(1 - \tau_A)r = 0.04$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Shocks</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{s}_1$</td>
<td>0.2131</td>
<td>Wealth held by 40 percent poorest, 0.9 percent</td>
</tr>
<tr>
<td>$\bar{s}_2$</td>
<td>1.0160</td>
<td>Gini wealth, 0.816</td>
</tr>
<tr>
<td>$Q_{11}$</td>
<td>0.9001</td>
<td>$\rho(\log(s_t))$, 0.9</td>
</tr>
<tr>
<td>$Q_{22}$</td>
<td>0.9855</td>
<td>$\sigma(\log(s_t))^2$, 0.05/(1 − 0.9^2)</td>
</tr>
</tbody>
</table>

| Technology | |
| $\theta$ | 0.3800 |
| $\delta$ | 0.0700 |
| $\gamma$ | 0.0200 |

| Fiscal Block | Trabandt and Uhlig (2011) |
| $\tau_N$ | 0.2800 |
| $\tau_A$ | 0.3600 |
| $\tau_C$ | 0.0500 |
| $\hat{B}$ | 0.6300 |
| $\hat{G}$ | 0.1800 |

**Note:** $\rho(\log(s_t))$ stands for the first-order serial correlation $\log(s_t)$ and $\sigma(\log(s_t))$ stands for the standard error of $\log(s_t)$.
the self-insurance motive in an IM setup, it takes a smaller discount factor than in a CM economy to reach the same target for the after-tax interest rate. Similarly, we obtain \( \eta = 0.34 \) in the IM setup, versus \( \eta = 0.31 \) in the CM setup. When it comes to the parameter governing the idiosyncratic earning shocks, we obtain \( \bar{s}_1 = 0.21, \bar{s}_2 = 1.02, \) and \( \bar{s}_3 = 3.77 \). This means that when an \( \bar{s}_2 \) agent turns \( \bar{s}_1 \), her labor earnings are cut by almost 80 percent, while if she turns \( \bar{s}_3 \), her labor earnings increase by 271 percent. Corresponding to these labor earning shocks are transition probabilities which imply that conditional on being \( \bar{s}_1 \) (resp. \( \bar{s}_3 \)), an agent has a 90 percent probability of not changing status. Similarly, conditional on being \( \bar{s}_2 \), an agent has a 98.6 percent probability of not changing status.

The solution method is now briefly described. Given the calibration targets for the debt-output ratio and the tax rates, we postulate candidate values for the interest rate \( r \) and aggregate efficient labor \( N \). We then solve the government budget constraint for the transfer-output ratio. To do so, we use the representative firm’s first-order conditions, which give us values for \( \hat{K} \) and \( \hat{w} \), and the aggregate resource constraint, from which we back out \( \hat{C} \). Given these, we solve the agents problem using the endogenous grid method proposed by Carroll (2006), adapted to deal with endogenous labor supply, in the spirit of Barillas and Fernandez-Villaverde (2007). Using the implied decision rules, we then solve for the stationary distribution as in Ríos-Rull (1999) and use it to compute aggregate quantities. We then iterate on \( r \) and \( N \) and start the whole process all over again until the markets for capital and labor clear. For a given \( N \), the interest rate is updated via a hybrid bisection-secant method. The bisection part of the algorithm is activated whenever the secant would update \( r \) to a value higher than the CM interest rate (which would result in diverging private savings, as shown in Aiyagari, 1994). Once the market-clearing \( r \) is found, \( N \) is updated with a standard secant method.

To compute the conditional Laffer curves, we adapt the previous algorithm as follows. We first vary either the transfer-output ratio or the debt-output ratio over pre-specified ranges. At each grid point, given the postulated pair \((r, N)\), the government steady-state budget constraint is balanced by adjusting one of the three tax rates considered, holding the other two constant. Given values for the debt-output ratio or the transfer output ratio, we then solve for the agents decision rules and for the stationary distribution. We then iterate on \( r \) and \( N \) as described above.

2.2. Labor Income Taxes. Figure 1 describes how the conditional Laffer curve associated with labor income taxes \( \tau_N \) is constructed when the transfer-output ratio \( \hat{T} \) is varied.

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6 Further details are reported in the technical appendix.
7 In doing so, we exploit the special structure of the first order condition on \( h \) induced by the specific functional form adopted for the utility function.
Figure 1. Construction of the Laffer Curve Conditional on Transfers - Labor Income Taxes

Note: The benchmark fiscal setup is identified with a circle in the curves above. The plain gray line corresponds to the incomplete-market economy and the dashed gray curve is associated with the complete-market economy. The welfare-maximizing tax in the CM setup is identified with a triangle; the welfare-maximizing tax in the IM setup is identified with a square.

Panel A (top left graph) shows the relation between the level of fiscal revenues $R(\hat{T})$ and $\hat{T}$. Panel B (top right graph) shows the corresponding relation between $\tau_N(\hat{T})$ and $\hat{T}$. Finally, panel C (bottom graph) is a combination of the previous two relations. The plain gray line corresponds to the IM setup and the dashed gray line is associated with the CM economy.

Both in the IM and CM economies, the Laffer curve conditional on $\hat{T}$ has the classic inverted-$U$ shape, as displayed in panel C. To understand this shape, consider a simplified setup in which $\tau_A = \tau_C = 0$. In this configuration, the government budget constraint simplifies to

$$\hat{R}(\hat{T}) \equiv (1 - \theta)\tau_N(\hat{T}) = \hat{G} + \hat{T} + (r - \gamma)\hat{B}.$$  

Since $\hat{G}$ and $\hat{B}$ are held constant, assuming differentiability, we obtain from the above equation

$$\frac{\partial \hat{R}}{\partial \hat{T}} = (1 - \theta)\frac{\partial \tau_N}{\partial \hat{T}} = 1 + \frac{\partial r}{\partial \hat{T}}\hat{B}.$$  

In the CM economy, since $\partial r/\partial \hat{T} = 0$ (see the proof of proposition 1), fiscal revenues as a share of GDP $\hat{R}(\hat{T})$ unambiguously increase when $\hat{T}$ increases. As the above equation shows, this also implies that labor income taxes increase with $\hat{T}$. In turn, output declines
when taxes rise. The level of fiscal revenues $R$ is thus the product of a term which declines with $\hat{T}$ and another one which is an increasing function of $\hat{T}$. This yields the inverted-$U$ shape obtained for $R(\hat{T})$ in the CM setup.

In the IM setup, changes in transfers impact on the steady-state interest rate. This is so because higher transfers reduce the self-insurance motive and thus reduce capital accumulation by private agents. We thus expect $\partial r/\partial \hat{T}$ to be positive. Since $\hat{B}$ is positive in our benchmark calibration, we obtain that $\hat{R}$ increases with $\hat{T}$. For the same insurance motive, higher transfers also reduce the aggregate labor supply and the capital stock. This is reinforced by the fact that higher transfers come hands in hands with higher labor taxation. Since both $\hat{N}$ and $\hat{K}$ decline, aggregate output $\hat{Y}$ also declines. In turn, the level of fiscal revenues $R$ is the product of $\hat{R}$ and $\hat{Y}$, yielding the inverted-$U$ shape.

Since, in both setups, $\tau_N$ is an increasing function of $\hat{T}$ (see panel B), the locus $(\tau_N(\hat{T}), R(\hat{T}))$ inherits the inverted-$U$ shape obtained for $(\hat{T}, R(\hat{T}))$, thus yielding a classic Laffer curve. In the general case, when $\tau_C$ and $\tau_A$ are non-zero, the above reasoning still holds but must also take into account the responses of $\hat{K}$ and $\hat{C}$ to changes in $\hat{T}$. These endogenous responses combine together to define the curves reported in figure 1.

Notice that the conditional Laffer curve in the IM setup clearly resembles its CM counterpart. The key difference appears in the high $\tau_N$ region of panel C. Here, a given tax rate generates relatively more fiscal revenues than in the CM setup. This is clearly due to the relative inelasticity of labor supply in the IM economy (the aggregate labor elasticity to taxation is lower in the IM economy than in the CM economy). This translates into a tax rate maximizing revenues equal to 50.01 percent in the IM economy and 48.64 percent in the CM economy. This allows the government to raise 15.56 percent more revenues than in the benchmark calibration in the IM economy and only 13.77 percent in the CM economy. To sum up, when transfers are adjusted, resorting to a CM model or to an IM model to characterize the shape and peak of the labor income tax Laffer curve has only mild consequences.

Panel C in figure 1 also reports the optimal labor income tax, i.e. the values of $\tau_N$ maximizing social welfare, given values for $\hat{B}$, $\hat{G}$, $\tau_A$, and $\tau_C$. In the CM model, the optimal $\tau_N$ is equal to 4.8 percent (see the triangle in panel C). In the IM model, when transfers are varied, the optimal labor income tax is equal to 26 percent instead.

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8One can show that this happens whenever consumption and leisure are normal goods.
9See the technical appendix for figures reporting the response of aggregate variables to $\hat{T}$.
10Given the utility function adopted in the paper, social welfare is equal to $\int v(\hat{a}, s)dx + \eta \log(\hat{Y})$. This corresponds to a measure of welfare in deviation from $Z_t$. Notice also that since the ratio $\hat{G}$ is held constant throughout, we are comparing economies with different levels of government consumption. As in Aiyagari and McGrattan (1998), we interpret this as an approximation to a setup where $G_t$ would be chosen optimally.
We turn now to the Laffer curve for labor income taxes $\tau_N$ conditional on the debt-output ratio $\hat{B}$. Figure 2 describes how this curve is constructed. Panel A (top left graph) shows the relation between $R(\hat{B})$ and $\hat{B}$. Panel B (top right graph) shows the corresponding relation between $\tau_N(\hat{B})$ and $\hat{B}$. Finally, panel C (bottom graph) is a combination of the previous two relations. The black line corresponds to the IM setup and the dashed gray line is associated with the CM economy.

When debt is varied, the conditional Laffer curve now looks like an $S$ oriented horizontally. In the left part of the graph, for relatively low tax levels, the Laffer curve has an increasing branch which continuously reaches the usual pattern as labor income taxes decrease. This junction takes place in what appears to be a minimum tax level which is close to 25 percent. Interestingly, the minimum labor income tax obtains for a debt-output ratio close to $-96$ percent. Above this level, there can be one, two, or three tax rates associated
with a given level of fiscal revenues. Put differently, there can be two levels of fiscal revenues associated with the same tax rate in the odd part of the Laffer curve conditional on debt: A high (resp. low) level associated with negative (resp. positive) debt. Also, in the regular part of this Laffer curve (i.e. the part that is indeed inverted-U-shaped), the revenue-maximizing labor income tax is 56 percent, allowing the government to raise 17.83 percent more revenues than in the benchmark situation.

What explains the odd shape of the Laffer curve in the left part of figure 2 when the debt-output ratio is varied? To gain an insight, imagine once again a simplified setting in which $\tau_C = \tau_A = 0$. Assuming differentiability of fiscal revenues with respect to $\hat{B}$, one gets

$$\frac{\partial \hat{R}}{\partial \hat{B}} = (1 - \theta) \frac{\partial \tau_N}{\partial \hat{B}} = (r - \gamma) + \hat{B} \frac{\partial r}{\partial \hat{B}}.$$ 

Now, since public debt crowds out capital in the households portfolio, we expect $\partial r / \partial \hat{B} > 0$. Indeed, as shown by Aiyagari and McGrattan (1998), when $\hat{B}$ is large, $\hat{K}$ gets smaller, which makes the equilibrium interest rate $r$ increase. Conversely, when $\hat{B}$ is negative and large in absolute value, private wealth $\hat{A}$ shrinks and the aggregate level of capital $\hat{K}$ increases, which makes the equilibrium interest rate decrease.

Thus the term $\hat{B} \partial r / \partial \hat{B}$ changes sign when $\hat{B}$ changes sign. For a sufficiently negative debt-output ratio, we can thus observe a change in the sign of $\partial \hat{R} / \partial \hat{B}$ and, since $\hat{R}(\hat{B}) = (1 - \theta) \tau_N(\hat{B})$, a corresponding change in the sign of $\partial \tau_N / \partial \hat{B}$.

At the same time, $\hat{K}$ and $N$ decrease with $\hat{B}$, so that $\hat{Y}$ is also decreasing with $\hat{B}$. Thus the level of fiscal revenues $R(\hat{B})$ obtains as the product of a relation that changes sign and another one which is strictly decreasing, thus yielding a horizontal-S shape (see panel A). Now, given the non-monotonic response of $\tau_N(\hat{B})$ (see panel B), the Laffer curve conditional on debt, which is a combination of panels A and B, exhibits a horizontal-S shape too (panel C).

Starting from a negative debt-output ratio $\hat{B}$, output, $\tau_N$, and $R$ are large. As the government sells more and more assets, i.e. as $\hat{B}$ increases, output and $\tau_N$ decline, so that $R$ declines too. This corresponds to the odd part of the Laffer curve. In this region, there are two forces at play. First, as $\hat{B}$ increases, the capital stock decreases, thus implying declining real wages, resulting in declining aggregate labor $N$. Second, since $\tau_N$ also decreases, agents are willing to supply more labor. It turns out that the first force dominates. Once the minimal tax is reached, $\tau_N$ and $\hat{R}$ start to increase again while $\hat{Y}$ is still declining.

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11The results are reported in the technical appendix.
12It is important at this stage to emphasize that the odd shape of the Laffer curve conditional on debt has nothing to do with a pathological behavior of the labor supply. In particular, the technical appendix reports that efficient hours are a decreasing function of $\hat{B}$ that closely mimic the behavior of hours worked in a CM setup.
This corresponds to the regular part of the Laffer curve, i.e. that which actually looks like an inverted U. In this region, increases in $\tau_N$ first dominate the disincentive effects of taxation, up to the maximal tax rate from which disincentive effects start to dominate.

In the general case, when $\tau_C$ and $\tau_A$ are non-zero, the above reasoning still holds but must also take into account the responses of $\hat{K}$ and $\hat{C}$. These endogenous responses combine together to define the particular point at which fiscal revenues exhibit the odd shape identified above. This also defines the minimal labor income tax.

The question then naturally arises as to the policy relevance of this odd section of the Laffer curve conditional on debt. To investigate this, we compute as before the tax rate maximizing social welfare. It turns out that in the IM model with debt varied, the optimal labor income tax is equal to 26.80 percent (see the square in figure 2). The associated debt-output ratio is close to $-205$ percent.\textsuperscript{13} This means that the optimal tax rate precisely lies on the odd portion of the Laffer curve. This suggests that this part of the curve is not just a theoretical curiosity but is actually relevant from a policy perspective.

2.3. Capital Income Taxes. Figure 3 reports three Laffer curves associated with variations in $\tau_A$. The dashed gray curve corresponds to the CM economy. The plain gray line is the Laffer curve conditional on transfers $\hat{T}$ in the IM setup. Finally, the black line is the Laffer curve conditional on debt $\hat{B}$ in the IM economy. To save space, we dispense with a complete description of how the conditional Laffer curves are constructed, given that they parallel closely the steps explained before.

In the case when transfers are varied, the conditional Laffer curve associated with $\tau_A$ has the standard inverted-U shape. It has the overall same shape as the curve that would obtain in the CM economy, as shown in figure 3. Once again, the key difference appears in the high $\tau_A$ region of the graph. Here, a given tax rate generates relatively more fiscal revenues than in the CM setup. This is clearly due to the relative inelasticity of agents saving behavior in the IM economy. As in Aiyagari (1994), in this kind of economy where contingent assets are ruled out, agents self-insure by accumulating relatively more assets (physical capital or public debt) than in a CM setup. This translates into a tax rate maximizing revenues equal to 58.0 percent in the IM economy versus 53.0 percent in the CM economy. The government may then raise 3.14 percent more revenues than in the benchmark calibration in the IM economy and only 2.0 percent in the CM economy. Notice that the optimal capital income tax (holding $\hat{G}$, $\hat{B}$, $\tau_N$, and $\tau_C$ constant) is close to 14 percent, as opposed to 0 percent in the CM version of the model.

\textsuperscript{13}The optimal negative debt-output ratio confirms results previously obtained by Röhrs and Winter (2010).
As was the case for labor income taxes, when the debt-output ratio $\hat{B}$ is varied, we reach very different conclusions (see the black curve in figure 3). Under this assumption too, the Laffer curve looks like an S oriented horizontally. In the left part of the graph, for relatively low tax levels, the Laffer curve has an increasing branch which reaches the regular pattern as capital income taxes decrease. Once again, this junction takes place in what appears to be a minimum tax level which is close to 25 percent. Interestingly, the minimum capital income tax obtains for a debt-output ratio close to $-129$ percent. Above this level, there can be one, two, or three tax rates associated with a given level of fiscal revenues. Also, in the regular part of this Laffer curve (i.e. the part that is indeed inverted-U-shaped), the revenue-maximizing capital income tax is 51.5 percent, allowing the government to raise 1.4 percent more revenues than in the benchmark situation. We find that the optimal tax on capital (holding $\hat{G}$, $\hat{T}$, $\tau_N$, and $\tau_C$ constant) is about 36 percent. Once again, the associated tax-revenue mix lies on the odd part of the Laffer curve and is associated with a debt-output ratio of $-180.6$ percent.

2.4. Consumption Taxes. Figure 4 reports three Laffer curves associated with variations in $\tau_C$, defined in the exact same way as before. The dashed gray one corresponds to the CM economy, as above. The plain gray line is the Laffer curve associated with the IM economy, conditional on transfers $\hat{T}$. Finally, the black line is the Laffer curve in the IM economy, conditional on the debt-output ratio $\hat{B}$.
As in Trabandt and Uhlig (2011), the Laffer curve associated with $\tau_C$ does not exhibit a peak, either in the CM setup or in the IM setup with adjusted transfers. In the latter, fiscal revenues are slightly higher than in the former. Fundamentally, in both settings, taxing consumption is like taxing labor (both taxes show up similarly in the first-order condition governing labor supply). A difference, though, is that in an IM economy such as ours, agents with a low labor productivity choose not to work whenever they hold enough assets. Clearly, those agents would not suffer from labor income taxation but do suffer from consumption taxes. Combined with the relative inelasticity of labor supply in the IM setup, this explains why the government can raise more revenues in this framework than in the CM setup.

As in the previous sections, when the debt-output ratio $\hat{B}$ is varied, we reach different conclusions (see the black curve in figure 4). Under this assumption too, in the left part of the graph, for relatively low tax levels, the Laffer curve has an increasing branch which reaches the regular pattern as consumption taxes decrease. Once again, this junction takes place in what appears to be a minimum tax level which is close to 1.42 percent, associated with a debt-output ratio close to $-110.8$ percent. Above this level, there can be two tax rates associated with a given level of fiscal revenues.

Interestingly, when transfers are varied, the optimal $\tau_C$ (holding the other fiscal parameters constant) is 12.8 percent (as opposed to 0 percent in the CM setup). In contrast,
when debt is varied, the optimal $\tau_C$ is about 3.4 percent (very close to actual data) and is located once more on the odd part of the curve.

2.5. **Corollary.** Propositions 1 establishes that the Laffer curves associated with $\tau_i$, $i \in \{N, A, C\}$ do not depend on the debt-output ratio in a CM setup. Put another way, the Laffer curves in an economy with a debt-output ratio of 63 percent are the same as those in an economy with a debt-output ratio of 200 percent or $-100$ percent. However, the previous analyses suggest that we should not expect this property to hold in an IM framework, due to the general equilibrium feedback effect of public debt on the after-tax interest rate. To investigate this, this section studies how variations on the steady-state debt-output ratio impact on the Laffer curve conditional on transfers.

The results are reported in figure 5. Panels A, B, and C report the Laffer curves associated with labor income taxes, capital income taxes, and consumption taxes, respectively. For each tax considered in the analysis, three Laffer curves (conditional on transfers) are drawn, each associated with a different debt-output ratio. The dashed lines correspond to the benchmark calibration, in which $\hat{B} = 0.63$. The plain and the dotted lines correspond to alternative economies with $\hat{B} = -1$ and $\hat{B} = 2$, respectively, holding all the other parameters to their benchmark value. As before, the circles indicate the benchmark calibration.
Several interesting results emerge. First, panels A and C show that for positive debt-output ratios, the Laffer curves associated with labor income taxes $\tau_N$ or consumption taxes $\tau_C$ hardly differ. This suggests that a high degree of public indebtedness will not impact much on the shape of the Laffer curve (conditional on transfers). In particular, for a steady-state debt-output ratio equal to 200 percent, the maximal labor income tax rate is 52 percent, yielding 15.19 percent more fiscal revenues than in the benchmark calibration, as opposed to a maximal tax of 50 percent and 15.56 percent more fiscal revenues in the case when the steady-state debt-output ratio is calibrated to 63 percent. To some extent, this is reassuring given the current fiscal context in the US. However, panels A and C also show that for negative debt levels, the Laffer curves on $\tau_N$ and $\tau_C$ are somewhat higher than their benchmark counterparts. In particular, for a steady-state debt-output ratio equal to $-100$ percent, the maximal labor income tax rate is 50 percent, yielding 19.25 percent more fiscal revenues than in the benchmark calibration.

More striking differences emerge from panel B which shows the Laffer curves associated with capital income taxation. First, the dotted line, corresponding to a debt-output ratio of 200 percent, is lower than its benchmark counterpart. This means that a higher ratio of public indebtedness reduces the extent to which the government can tax capital income. In particular, for a steady-state debt-output ratio equal to 200 percent, the maximal capital income tax rate is 54 percent, yielding 1.67 percent more fiscal revenues than in the benchmark calibration, as opposed to a maximal tax of 58 percent and 3.14 percent more fiscal revenues in the case when the steady-state debt-output ratio is calibrated to 63 percent. Second, the plain line, corresponding to a debt-output ratio of $-100$ percent, is much higher than its benchmark counterpart. In particular, for a steady-state debt-output ratio equal to $-100$ percent, the maximal capital income tax rate is 74 percent, yielding 10.77 percent more fiscal revenues than in the benchmark calibration, considerably more than what obtains when the debt-output ratio is calibrated to 63 percent.

3. Robustness

In this section, we explore the robustness of our results to alternative assumptions regarding government consumption and the stochastic process governing idiosyncratic productivity shocks.

3.1. Alternative Assumption on Government Consumption. As in Aiyagari and McGrattan (1998), Flodén (2001), and Röhrs and Winter (2010), we have assumed a constant share of government consumption in output, i.e. a constant $\hat{G}$. In general, this is not an innocuous assumption. For example, in the CM version of our model, this resulted in an optimal tax rate on labor income different from 0. More broadly, this assumption has strong implications on the tax bases response to changes in taxes. This is so because
it imposes a certain pattern on the way government consumption crowds out private consumption.\textsuperscript{14}

In this section, we relax our previous assumption and impose instead that $G_t/Z_t$ be constant. Thus as $\hat{Y}$ decreases, the share of government consumption in output increases. We redo our Laffer curve calculations under this alternative assumption. In general, this does not raise special problems, except when we consider the Laffer curve associated conditional on $\hat{B}$. In this case, we must add an extra loop on the government budget constraint to make sure that the after tax interest rate is never updated to values higher than $(1 + \gamma)/\beta - 1$. If it were the case, our algorithm would never converge, for reasons discussed in Aiyagari (1994).

The results are reported on figure 6. The figure contains three subplots, each associated with one of the three tax rates considered in the analysis. In each case, we report three Laffer curves: the one deriving from the CM version of the model, the one in the IM model when transfers are adjusted, and the one in the IM model when debt is varied instead. As before, we also report the welfare-maximizing taxes. In those exercises, the resulting government consumption-output ratio ranges from 14 percent to 30 percent as opposed to 18 percent in the benchmark calibration.

\textsuperscript{14} Notice also that welfare comparisons are not well defined when the level of government spending is not fixed.
Figure 7. Laffer Curves - Alternative Assumption on \( s_t \)

Note: Level of fiscal revenues as a function of tax rates when \( s_t \) is calibrated as in Flodén (2001). Fiscal revenues are normalized by revenues in the benchmark fiscal setup, identified with a point in the curves above. IM stands for incomplete markets and CM for complete markets.

We obtain qualitatively the same results as in the previous section. When transfers are adjusted, the IM and CM Laffer curves have a similar pattern. However, when debt is adjusted instead, the Laffer curve exhibits once again the horizontal S shape. Several differences with respect to the constant \( \hat{G} \) case are worth mentioning. First, consistent with standard results, the optimal taxes are all equal to zero in the CM model (see the triangle in figure 6). In the IM model with transfers adjusted, we also find much lower optimal tax rates than before. The optimal capital income and consumption taxes should be set to zero while the optimal labor income tax is 4 percent. These values are not even remotely close to the actual values for taxes. In contrast, when debt is adjusted, we obtain optimal values for \( \tau_N \), \( \tau_A \), and \( \tau_C \) equal to 27.56 percent, 11.00 percent, and 5.21 percent, respectively. Except for capital income taxes, these values are very close to the actual values for taxes. In each case, the optimal debt-output ratios are lower than −300 percent. Second, the Laffer curve associated with \( \tau_A \) is now somewhat steeper than when \( \hat{G} \) was held constant. Also, when debt is adjusted, the capital income tax covers a much wider range of values than under a constant government consumption-output ratio.

3.2. Alternative Assumption on Idiosyncratic Shocks. Up to now, we focused on a specification for idiosyncratic productivity shocks that allowed the benchmark IM to reproduce key characteristics of the wealth distribution. As suggested by Röhrs and Winter (2010), such an endeavor is crucial if one is interested in assessing the optimal quantity
of debt. As a matter of fact, we found that the odd part of the Laffer curve contains the tax-revenue mix corresponding to this optimal level, which was found to be negative, as convincingly argued by Röhrs and Winter (2010).

This may suggest that the odd part of the Laffer curve could prove sensitive to the calibration of the idiosyncratic labor earning shocks. To investigate this, we recompute the Laffer curves using an alternative specification for idiosyncratic shocks \( \{s_t\} \). We follow Flodén (2001) and assume that \( \log(s_t) \) follows an AR(1) process \( \log(s_t) = \rho \log(s_{t-1}) + \sigma_c \epsilon_t \) with \( \epsilon_t \sim N(0, 1) \), \( \rho = 0.9 \) and \( \sigma_c = 0.21 \). This process is then approximated with a discrete Markov chain, using the method developed by Tauchen (1986). To do so, we allow for 7 states in the discrete Markov chain. Holding all the other parameters to their previous value, the model now delivers a Gini coefficient for the wealth distribution equal to 0.64, implying a much lower degree of wealth inequality than in the benchmark calibration. Similarly, under this alternative calibration, the wealth held by the 40 percent poorest amounts to 2.6 percent of total wealth. The bottom line is that under this alternative calibration, we obtain a very different wealth distribution.\(^{15}\)

Figure 7 reports the Laffer curves obtained under this alternative calibration. As before, we also report the curves associated with the CM version of the model. What conclusion can we draw from this exercise? First, we obtain slightly different levels of optimal taxes, though this is not particularly striking. However, when debt is adjusted, we no longer obtain negative levels for the optimal debt-output ratio. This is just a confirmation of results obtained by Röhrs and Winter (2010). Second, and more interestingly, the odd part of the Laffer curve which arises when debt is adjusted still appears under this alternative calibration. Thus, the mere existence of this phenomenon does not seem to depend particularly on our benchmark calibration for idiosyncratic shocks.

4. Conclusion

In this paper, we have inspected how allowing for liquidity-constrained agents and incomplete financial markets impacts on the shape of the Laffer curve. To address this question, we formulated a neoclassical growth model along the lines of Aiyagari and McGrattan (1998). The model was calibrated to the US economy to mimic great ratios as well as moments related to the wealth distribution. We paid particular attention to which of debt or transfers is adjusted to balance the government budget constraint as taxes are varied. While in a complete-market (CM) framework, this does not matter, opting to adjust debt rather than transfers can potentially make a big difference in an incomplete-market (IM) setup.

\(^{15}\)We also considered a version in which \( \eta \) and \( \beta \) are re-calibrated to match the same targets as before. This does not change our qualitative conclusions.
Our main quantitative findings are the following. When it comes to labor and capital income taxes, the CM and IM models deliver similar Laffer curves when transfers are adjusted. The slippery slope is a little bit farther to the right in the IM model. This results from households using labor supply and savings to self-insure, allowing for a greater level of taxation. However, when public debt is adjusted, the shape of the Laffer curve is dramatically affected: The conditional Laffer curve now looks like a horizontal S. First, for a positive debt, the slippery curve moves to the right as in the case of lump-sum transfers adjustments. Second, for a negative public debt, government revenues increase with debt. This implies that three tax rates compatible with the same level of fiscal revenues can exist. Finally, when consumption tax is considered, the CM and IM models exhibit broadly similar shapes when transfers are adjusted, each of them displaying no peak. Once again, when public debt is adjusted instead, there exist two consumption taxes delivering the same level of fiscal revenues. The horizontal S shape of Laffer curves in the IM setup comes from the non-monotonic response of tax rates to changes in public debt.

REFERENCES


The complete-market economy is the representative-agent version of the IM framework, where we have eliminated idiosyncratic shocks and the borrowing constraint. Once growing variables have been de-trended by \( Y_t \), the associated steady state is then solution to the system
\[
\hat{C} + (\gamma + \delta) \hat{K} + \hat{G} = 1, \tag{2}
\]
\[
(1 + \gamma) = (1 + (1 - \tau_A)r)\beta, \tag{3}
\]
\[
\hat{C} = \frac{\eta}{1 - \eta} \frac{1 - \tau_N}{1 + \tau_C} \hat{w}(1 - H), \tag{4}
\]
\[
\hat{w} = (1 - \theta) / H, \tag{5}
\]
\[
r + \delta = \theta / \hat{K}. \tag{6}
\]
Equation (2) is the resource constraint. Equation (3) is the steady-state Euler equation. Equation (4) is the marginal rate of substitution between consumption and labor. Equations (5) and (6) are the representative firm’s first order conditions.

We solve for the steady-state values in the standard way. First, notice that by combining the Euler equation (3) with the condition on optimal use of capital by firms (6), one gets
\[
\hat{K} = \frac{\beta \theta (1 - \tau_A)}{1 + \gamma - \beta [1 - (1 - \tau_A) \delta]}.
\]
Having solved for \( \hat{K} \), one can use the resource constraint (2) to solve for \( \hat{C} \). Then, combining the first-order condition on labor supply (4) with the labor demand by firms (5), one can solve for \( H \).

It thus turns out that the steady-state allocation \((\hat{C}, \hat{K}, H, \hat{Y})\) and the steady-state price system \((r, \hat{w})\) do not depend on either \( \hat{T} \) or \( \hat{B} \). In turn, tax revenues \( R \) depend only on the tax system \((\tau_N, \tau_A, \tau_C)\), the steady-state allocation, and on the steady-state price system. Thus, for each \( i \in \{N, A, C\} \), the Laffer curve associated with \( \tau_i \) is independent from either \( \hat{T} \) or \( \hat{B} \).

**Appendix B. Proof of Proposition 2**

Given proposition 1, it is sufficient to establish the existence of a bijective relationship between \( \hat{T} \) and \( \tau_i \) and between \( \hat{B} \) and \( \tau_i, i \in \{N, A, C\} \), to prove that the two conditional Laffer curves \((\tau_i(\hat{T}), R(\hat{T}))\) and \((\tau_i(\hat{B}), R(\hat{B}))\) coincide in a CM setup.

To this end, let \( \bar{r} \equiv (1 - \tau_A)r \) denote the after tax interest rate. We thus have
\[
\bar{r} = \frac{1 + \gamma}{\beta} - 1.
\]
Notice that the CM steady-state system implies that $\tilde{r}$ does not depend on any of the three tax rates considered. Fiscal revenues (as a share of GDP) $\hat{R}$ are then

$$\hat{R} = (1 - \theta) \tau_N + \tau_C \hat{C} + \frac{\tau_A}{1 - \tau_A} \tilde{r} \hat{K}.$$ 

B.1. **Labor income tax.** Let us first consider the labor income tax $\tau_N$. $\hat{K}$ does not depend on this tax rate and, by (2), neither $\hat{C}$ does. It follows that

$$\frac{\partial \hat{R}}{\partial \tau_N} = 1 - \theta.$$

At the same time, it must be the case that

$$\hat{R} = \hat{G} + \hat{T} + (\tilde{r} - \gamma) \hat{B}.$$

Recall that $\tilde{r}$ does not depend on any of the three tax rates. Assuming that $\hat{T}$ is adjusted and $\hat{B}$ is held fixed, we thus obtain

$$\frac{\partial \hat{T}}{\partial \tau_N} = \frac{\partial \hat{R}}{\partial \tau_N} = 1 - \theta > 0.$$

Now, assume that $\hat{B}$ is adjusted and $\hat{T}$ is held fixed, we obtain

$$\frac{\partial \hat{B}}{\partial \tau_N} = \frac{1 - \theta}{\tilde{r} - \gamma} > 0.$$

The inequality follows from the fact that $\beta \in (0, 1)$ since

$$\tilde{r} - \gamma = (1 + \gamma) \left( \frac{1}{\beta} - 1 \right).$$

It follows that the relation between $\tau_N$ and $\hat{T}$ and the relation between $\tau_N$ and $\hat{B}$ are both strictly increasing and thus bijective. It is thus equivalent to vary $\tau_N$ and adjust $\hat{T}$ (resp. $\hat{B}$) or vary $\hat{T}$ (resp. $\hat{B}$) and adjust $\tau_N$.

B.2. **Consumption tax.** Now, consider the consumption $\tau_C$. Reasoning as above, we obtain

$$\frac{\partial \hat{T}}{\partial \tau_C} = \hat{C} > 0.$$

$$\frac{\partial \hat{B}}{\partial \tau_C} = \frac{\hat{C}}{\tilde{r} - \gamma} > 0.$$

It follows that the relation between $\tau_C$ and $\hat{T}$ (resp. $\hat{B}$) is strictly increasing and thus bijective. It is thus equivalent to vary $\tau_C$ and adjust $\hat{T}$ (resp. $\hat{B}$) or vary $\hat{T}$ (resp. $\hat{B}$) and adjust $\tau_C$. 
B.3. Capital income tax. Finally, consider the capital income tax $\tau_A$. Differentiating fiscal revenues (as a share of GDP) with respect to $\tau_A$ yields

$$\frac{\partial \hat{R}}{\partial \tau_A} = C \frac{\partial \hat{C}}{\partial \tau_A} + \frac{\tilde{r}}{1 - \tau_A} \left( \frac{1}{1 - \tau_A} \hat{K} + \tau_A \frac{\partial \hat{K}}{\partial \tau_A} \right).$$

In turn, the partial derivative of $\hat{K}$ with respect to $\tau_A$ is

$$\frac{\partial \hat{K}}{\partial \tau_A} = -\frac{\beta \theta (1 + \gamma - \beta)}{[1 + \gamma - \beta (1 - (1 - \tau_A) \delta)]^2} < 0,$$

and the partial derivative of $\hat{C}$ with respect to $\tau_A$ is

$$\frac{\partial \hat{C}}{\partial \tau_A} = \frac{(\gamma + \delta) \beta \theta (1 + \gamma - \beta)}{[1 + \gamma - \beta (1 - (1 - \tau_A) \delta)]^2} > 0.$$

We thus obtain

$$\frac{\partial \hat{R}}{\partial \tau_A} = \theta (1 + \gamma - \beta) [\beta C (1 + \gamma + \delta) (1 + \gamma - \beta (1 - \delta))] [1 + \gamma - \beta (1 - (1 - \tau_A) \delta)]^2 > 0.$$

At the same time, it must be the case that

$$\frac{\partial \hat{R}}{\partial \tau_A} = (\tilde{r} - \gamma) \frac{\partial \hat{B}}{\partial \tau_A} = \frac{\partial \hat{T}}{\partial \tau_A}.$$

Thus, once again, it follows that the relation between $\tau_A$ and $\hat{T}$ (resp. $\hat{B}$) is strictly increasing and thus bijective. It is thus equivalent to vary $\tau_A$ and adjust $\hat{T}$ (resp. $\hat{B}$) or vary $\hat{T}$ (resp. $\hat{B}$) and adjust $\tau_A$.

B.4. Summing up. For each $i \in \{N, A, C\}$, we found that there exists a bijective relationship between $\hat{T}$ and $\tau_i$ and between $\hat{B}$ and $\tau_i$. Thus, the Laffer curve obtained by varying $\tau_i$ and letting $\hat{T}$ (resp. $\hat{B}$) adjust coincide with the conditional Laffer curve obtained by varying $\hat{T}$ (resp. $\hat{B}$) and letting $\tau_i$ adjust. By proposition 1, we thus obtain that in a CM setup, the steady-state conditional Laffer curves coincide.
THE LAFFER CURVE IN AN INCOMPLETE-MARKETS ECONOMY

PATRICK FÈVE, JULIEN MATHERON, AND JEAN-GUILLAUME SAHUC

ABSTRACT. This paper is a quantitative investigation into the characteristics of the Laffer curve in a neoclassical growth model with incomplete markets and heterogeneous, liquidity-constrained agents. We show that the shape of the Laffer curves related to taxes on labor, capital and consumption dramatically changes depending on which of transfers or government debt are adjusted to make the government budget constraint hold. When transfers are adjusted, the Laffer curve has the traditional shape. However, when debt is adjusted, the Laffer curve looks like a horizontal \( S \), in which case fiscal revenues can be associated with up to three different levels of taxation. This finding occurs because the tax rates change non monotonically with public debt when markets are incomplete.


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Patrick Fève: Toulouse School of Economics, Banque de France, patrick.feve@univ-tlse1.fr.

Julien Matheron: Banque de France, julien.matheron@banque-france.fr.

Jean-Guillaume Sahuc: Banque de France, jean-guillaume.sahuc@banque-france.fr.

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1. Introduction

The Laffer curve, i.e. the inverted-$U$-shaped relation between fiscal revenues and tax rates, has often been a key motivation for fiscal reforms. The best known example was the Economic Recovery Tax Act of 1981. Driven by the belief that the US were lying on the prohibitive part of the Laffer curve, the Reagan administration engineered a major tax cut, expected to be self-financed. While the actual location of the US along the Laffer curve subsequently raised controversies, the mere Laffer curve concept has figured prominently in policy debates ever since.\(^1\)

From an analytical point of view, the Laffer curve is a general equilibrium phenomenon. Indeed, a change in a given tax will generally affect all fiscal bases, through its impact on all individual decision rules. In the literature, these general-equilibrium effects have been mainly studied in the context of complete-market (CM) frameworks (see, for example, Fullerton, 1982, Trabandt and Uhlig, 2011). In this paper, we investigate how allowing for incomplete markets and heterogeneous, liquidity-constrained agents (IM for short in the remainder) affects the characteristics of the Laffer curve.

There are at least two reasons why this curve might change in such an environment. First, because of market incompleteness and liquidity constraints, agents self-insure by accumulating more assets (Aiyagari, 1994) and supplying more labor (Pijoan-Mas, 2006) than in a CM setup. As a consequence, the fiscal bases prove to be less tax-elastic than in a CM context, thus pushing potentially the top of the Laffer curve farther to the right. Second, Ricardian equivalence fails in an IM setup with liquidity-constrained agents. Consequently, the interest rate’s response to tax changes will not a priori be invariant to which of public debt or transfers is adjusted to make the government budget constraint hold as taxes are varied. This opens the possibility for general equilibrium effects on tax bases that differ according to which variable is adjusted, thus potentially affecting the shape of the Laffer curve. This is the main focus of the present paper.

We explore quantitatively these questions in the context of a neoclassical model along the lines of Aiyagari and McGrattan (1998), Flodén (2001), and Röhrs and Winter (2010). In our economy, households are subject to persistent idiosyncratic productivity shocks and face a borrowing constraint. The model includes distortionary taxes on labor, capital, and consumption. These taxes are used to finance a constant share of government consumption in output, lump-sum transfers, and interest repayments on accumulated debt.\(^2\) The model is calibrated to the US economy to mimic great

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\(^1\)See e.g. Feldstein (1986) for a skeptical view summarizing much of the debate around the Laffer curve at that time. See also Blinder (1981).

\(^2\)A nice feature of our setup is that it nests the standard neoclassical model. Setting the variance of idiosyncratic labor productivity shocks to zero and eliminating the borrowing constraint, our model boils down to the standard neoclassical model with distortionary taxation. This makes easier a quantitative comparison of the two versions.
ratios as well as moments related to the wealth distribution, using the method devised by Domeij and Heathcote (2004) and Heathcote (2005). As in Trabandt and Uhlig (2011), we then study the steady-state Laffer curves associated with each of the three tax considered. To investigate how the shape of these curves depends on which of debt or transfers are adjusted, we proceed by varying either the transfer-output ratio or the debt-output ratio over prespecified ranges. In each case, the government steady-state budget constraint is balanced by adjusting one of the three tax rates considered, holding the other two taxes constant. Doing so, we are able to compute parametric Laffer curves, i.e. loci linking fiscal revenues to the tax rate used to balance the government budget constraint. We end up with six curves, depending on which tax is adjusted and on which of debt or transfers are varied.

Our main findings are the following. Consider first the Laffer curve associated with labor income taxes. When transfers are varied, the Laffer curve in the IM economy looks like its CM counterpart. It has the standard inverted-U shape, though its top is a little bit farther to the right, reflecting a smaller tax-elasticity of fiscal bases in the IM environment. In our benchmark calibration, the revenue-maximizing tax rate is 51%, compared to a CM counterpart equal to 49%. When debt is varied instead of transfers, the picture changes radically. The Laffer curve now looks like a horizontal S in the IM economy. Under this configuration, there can be one, two, or three tax rates compatible with a given level of fiscal revenues. The regular part of the Laffer curve (the one which indeed looks like an inverted U) is associated with positive government debt while the awkward part (the one which makes the curve look like a horizontal S) is associated with negative debt levels. Under this configuration, the local maximum of fiscal revenues in the regular part is associated with a tax rate equal to 56%, as opposed to 49% for the CM economy, though higher fiscal revenues can be reached at lower tax rates when considering the awkward portion of the curve. We obtain broadly similar conclusions when it comes to capital income taxes. When transfers are varied, the associated Laffer curve looks like its CM counterpart. Once again, the top of the Laffer curve is a little bit farther to the right, reflecting less tax-elastic fiscal bases. The revenue-maximizing tax is equal to 58%, as opposed to 53% in the CM economy. When debt is varied instead, we obtain an awkward Laffer curve that looks like a horizontal S once again. The local revenue-maximizing tax rate in the regular part of the Laffer curve is about 54%, though higher revenues could be reached at lower taxes in the awkward part of the curve. Finally, we find no Laffer effect for consumption taxes though, when debt is varied, we still obtain an awkward portion associated with negative debt levels.

To understand these awkward shapes, imagine a simplified setup in which only labor income taxes are non zero. Thus, fiscal revenues as a share of GDP, are proportional to the labor income tax. Suppose in addition that the debt-output ratio is varied as taxes are adjusted, while government expenditures and transfers (as a share of GDP) remain constant. It follows that the derivative of
fiscal revenues (as a share of GDP) with respect to the debt-output ratio is the sum of two terms: (i) the interest rate and (ii) a multiplicative term combining the debt-output ratio and the sensitivity of the real interest rate to this ratio. In the CM version, this multiplicative term is zero, due to Ricardian equivalence. In our IM setup however, Ricardian equivalence fails. As shown in Aiyagari and McGrattan (1998), public debt then crowds out physical capital in households portfolio, so that the interest rate is monotonically increasing with the debt-output ratio. As a consequence, the sign of the derivative of fiscal revenues (as a share of GDP) with respect to the debt-output ratio depends on the sign of public debt. For positive debt levels, this derivative is positive. Conversely, for sufficiently negative debt levels, this derivative is negative. At the same time, since debt crowds out physical capital, output will decline with the debt-output ratio. Thus, starting from a sufficiently negative debt level, output is high and so too are fiscal revenues and the labor income tax. As the government sells more and more assets, revenues, the tax rate, and output decline. This corresponds to the awkward part of the Laffer curve. For a sufficiently positive debt level, the share of revenues in output starts to increase again while output, at the same time, continues to decrease due to the disincentive effect of increasing taxes. This corresponds to the usual shape of the Laffer curve.

Are these awkward portions of the Laffer curves relevant from a policy point of view? To answer this question, we compute the welfare maximizing taxes and debt-output ratios. As in Röhrs and Winter (2010), we obtain that the optimal debt-output ratio is negative, translating into optimal tax rates which precisely lie on the awkward portions of the Laffer curves associated with each of the three tax rates considered. Thus, the awkward part of the Laffer curves are not just theoretical curiosities. As one might argue, the exact optimal quantity of debt crucially depends on the stochastic properties of idiosyncratic shocks. However, we show in our robustness analysis that the awkward portions of the Laffer curves are robust to alternative specifications of idiosyncratic shocks.

This paper is related to previous studies investigating taxation and/or public debt in an IM setup. A first strand, exemplified by Aiyagari and McGrattan (1998) and Flodén (2001), established that a proportional income tax rate changes non-monotonically with debt. However, this literature did not explore how this feature could impact on the shape of the Laffer curve. Röhrs and Winter (2010) extended recently this analysis to a carefully calibrated multi-tax environment. However, they too ignored the implications for the Laffer curve. Our paper complements this literature by focussing on how the Laffer curve changes as debt or transfers are varied. A second strand has explored the Laffer effect in the context of IM models. For example, Flodén and Lindé (2001) found that the the Laffer curve peaks when labor income tax is approximatively 50% or more. However, their analysis abstracts from public debt. More recently, Ljungqvist and Sargent (2008) and Alonso-Ortiz and Rogerson (2010) revisited in an IM setup the incentive issues of labor taxes raised by Prescott (2004). Ljungqvist and Sargent (2008) and Alonso-Ortiz and Rogerson (2010) compared the Laffer
curves under IM and CM. Focusing on labor income taxes, they obtained that the prohibitive part of the Laffer curve in the IM case is weakly affected. However, they too abstract from government debt. Our paper complements these works by insisting more on the role of public debt and by considering various forms of distortionary taxes.

The rest of the paper is organized as follows. In section 2, we expound the IM model and define the steady-state equilibrium under study. Section 3 is devoted to the quantitative results. We first discuss our calibration strategy and then explore the extent to which the Laffer curves change when computed in an IM model. Section 4 explores the robustness of our results to alternative assumptions on government consumption and on the stochastic process for idiosyncratic shocks. The last section briefly concludes.

2. The Model

In this section, we describe the model economy used in our quantitative experiment.

2.1. Environment. We consider a discrete-time economy without aggregate risk. Time is indexed by \( t \in \mathbb{N} \). The final good \( Y_t \), which we take as the numeraire, is produced by competitive firms, according to the Cobb-Douglas technology

\[
Y_t = K_t^\theta (Z_t N_t)^{1-\theta},
\]

where \( \theta \in (0, 1) \) denotes the elasticity of production with respect to capital, \( K_t \) and \( N_t \) are the inputs of physical capital and efficient labor, respectively, and \( Z_t \) is an exogenous technical progress index, evolving according to \( Z_{t+1} = (1 + \gamma)Z_t \) with \( Z_0 = 1, \gamma > 0 \). Firms rent capital and efficient labor on competitive markets, at rates \( r_t + \delta \) and \( w_t \), respectively, where \( \delta \in [0, 1] \) is the depreciation rate of physical capital, \( w_t \) is the wage rate, and \( r_t \) is the interest rate.

The economy is inhabited by a continuum of agents, of measure one. Each agent’s time endowment is normalized to 1 and can be allocated to market work \( h_t \) or to leisure \( 1 - h_t \). Agents have preferences over consumption \( c_t \) and leisure defined by

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t) \right\}
\]

with \( c_t \geq 0 \) and \( 0 \leq h_t \leq 1 \). Here \( \beta \in (0, 1) \) is the subjective discount factor, \( E_0 \{ \cdot \} \) is the mathematical expectation conditioned on the individual state at date \( t = 0 \), and \( u(c, 1 - h) \) is a well-behaved utility function, assumed to be homogeneous of degree \( 1 - \sigma_c \) in \( c \), with \( \sigma_c > 0 \).

In each period, households receive an uninsurable shock \( s_t > 0 \) to their labor productivity. These shocks are assumed to be i.i.d. across agents and evolve over time according to a Markov process,
with bounded support $S$ and stationary transition function $Q(s, s')$. These idiosyncratic productivity shocks are normalized so that the unconditional mean of their logarithm is equal to zero, i.e. $E\{\log(s)\} = 0$. An individual agent’s efficient labor is thus $s_t h_t$, with corresponding labor earnings given by $(1 - \tau_N) w_h s_t h_t$, where $\tau_N$ denotes the labor income tax. In addition, agents self-insure by accumulating $a_t$ units of assets which pay the after-tax rate of return $(1 - \tau_A) r_t$, where $\tau_A$ denotes the capital income tax. These assets can consist of units of physical capital and/or government bonds. Once arbitrage opportunities have been ruled out, each asset has the same rate of return. Also, agents must pay a sales tax $\tau_C$. Finally, they perceive transfers $T_t$. Thus, an agent’s budget constraint is

$$(1 + \tau_C) c_t + a_{t+1} \leq (1 - \tau_N) w_h s_t h_t + (1 - \tau_A) r_t a_t + T_t.$$

Borrowing is exogenously restricted by an “ad hoc” constraint

$$a_{t+1} \geq 0.$$ 

There is finally a government in the economy. The government issues debt $B_{t+1}$, collects tax revenues, rebates transfers, and consumes $G_t$ units of final good. The associated budget constraint is given by

$$B_{t+1} = (1 + r_t) B_t + T_t + G_t - (\tau_A r_t A_t + \tau_N w t N_t + \tau_C C_t)$$

where $C_t$ and $A_t$ denote aggregate (per capita) consumption and assets held by the agents, respectively.

2.2. Equilibrium Defined. In the remainder of this paper, we focus exclusively on the steady state of an appropriately detrended version of the above economy. Growing variables are detrended by dividing them by $Y_t$. Detrended variables are referred to with a hat. In the benchmark specification, the ratio of government expenditures to output $\hat{G}$ is constant. In the robustness section, we also consider an alternative case in which the level of government expenditures (in deviation from $Z_t$) is constant.

We let the joint distribution of assets $\hat{a}$ and individual productivities $s$ across agents be denoted $x(\hat{a}, s)$. Thus, for all $A_0 \times S_0 \in \mathcal{A} \times \mathcal{S}$, $x(A_0, S_0)$ is the mass of agents with assets in $A_0$ and idiosyncratic productivity in $S_0$, where $\mathcal{A} \times \mathcal{S}$ denotes the Borel subsets of $A \times S$.

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3The transition $Q$ has the following interpretation: for all $s \in S$ and for all $S_0 \in \mathcal{S}$, where $\mathcal{S}$ denotes the Borel subsets of $S$, $Q(s, S_0)$ is the probability that next period’s individual productivity lies in $S_0$ when current productivity is $s$. 
We can now write an agent’s problem in recursive form

$$v(\hat{a}, s) = \max_{\hat{c}, h, \hat{a}'} \left\{ u(\hat{c}, 1 - h) + \beta \int_S v(\hat{a}', s') Q(s, ds') \right\}$$

s.t. \( (1 + \tau_C)\hat{c} + (1 + \gamma)\hat{a}' \leq (1 - \tau_N)\hat{w}sh + (1 + (1 - \tau_A)r)\hat{a} + \hat{T}, \) \( \hat{a}' \geq 0, \hat{c} \geq 0, 0 \leq h \leq 1, \) \( (1) \)

where \( \beta \equiv \tilde{\beta}(1 + \gamma)^{1 - \sigma_c} \) denotes the growth-adjusted discount factor.

For convenience, we restrict \( \hat{a} \) to belong to the compact set \( A = [0, \hat{a}_M] \), where \( \hat{a}_M \) is a large number.\(^4\) We can thus define a stationary, recursive equilibrium in the following way.

**Definition 1**—Given a vector of constant policy parameters \((\tau_C, \tau_A, \tau_N, \hat{T}, \hat{G}, \hat{B})\), a steady-state, recursive competitive equilibrium is a constant system of prices \(\{r, \hat{w}\}\), a value function \(v(\hat{a}, s)\), time-invariant decision rules for an individual’s assets holdings, consumption, and labor supply \(\{g_a(\hat{a}, s), g_c(\hat{a}, s), g_h(\hat{a}, s)\}\), a measure \(x(\hat{a}, s)\) of agents over the state space \(A \times S\), and aggregate quantities \(\hat{A} = \int \hat{a} dx, \hat{C} = \int g_c(\hat{a}, s) dx, N = \int s g_h(\hat{a}, s) dx, \) and \(\hat{K}\) such that:

(i) The value function \(v(\hat{a}, s)\) solves the agent’s problem stated in eq. (1), with associated decision rules \(g_a(\hat{a}, s), g_c(\hat{a}, s)\) and \(g_h(\hat{a}, s)\);

(ii) Firms maximize profits and factor markets clear, so that

\[
\hat{w} = \frac{1 - \theta}{N},
\]
\[
r + \delta = \frac{\theta}{\hat{K}},
\]

(iii) Tax revenues equal government expenses

\[
\tau_N\hat{w}N + \tau_Ar\hat{A} + \tau_C\hat{C} = \hat{T} + \hat{G} + (r - \gamma)\hat{B};
\]

(iv) Aggregate savings equal firm’s demand for capital plus Government’s debt

\[
\hat{A} = \hat{K} + \hat{B};
\]

(v) The distribution \(x\) is invariant

\[
x(A_0, S_0) = \int_{A_0 \times S_0} \left\{ \int_{A \times S} 1_{\{\hat{a}' = g_a(\hat{a}, s, x)\}} Q(s, s') dx \right\} d\hat{a}' ds',
\]

for all \(A_0 \times S_0 \in \mathcal{A} \times \mathcal{S}\), where \(1_{\{\}}\) is an indicator function taking value one if the statement is true and zero otherwise.

With a slight abuse of notation, we define the stationary level of output \(\hat{Y} = Y_t / Z_t\). It is linked to \(\hat{K}\) and \(N\) through \(\hat{Y} = \hat{K}^{\theta/(1 - \theta)} N\).

\(^4\)\(\hat{a}_M\) is selected so that the decision rule on assets for an individual with the highest individual productivity crosses the 45-degree line below \(\hat{a}_M\).
2.3. The Laffer Curves. From the government budget constraint, fiscal revenues (as a share of GDP) $\hat{R}$ are given by

$$\hat{R} = \tau_N \hat{w}N + \tau_A r \hat{K} + \tau_C \hat{C}.$$ 

$\hat{R}$ is then converted to level according to $R = \hat{R} \times \hat{Y}$. Notice that the level of fiscal revenues $R$ is defined net of fiscal receipts from taxing public bonds return.

In the remainder, we consider three Laffer curves, each relating fiscal revenues $R$ to one of the three tax rates ($\tau_N, \tau_A, \tau_C$) considered here, holding the other two taxes constant. As argued by Trabandt and Uhlig (2011), this is the appropriate definition of the Laffer curve since it correctly takes into account the general equilibrium effects induced by a tax change. For example, a given change in $\tau_N$ will modify $x, g_a, g_h, \text{and } g_c$, so that it will also impact on all the fiscal bases.

In equilibrium, we must always have

$$\hat{R} = \hat{G} + \hat{T} + [(1 - \tau_A)r - \gamma] \hat{B},$$

so that a given change in one of the three tax rates is associated with a corresponding adjustment in either $\hat{T}$ or $\hat{B}$. Depending on which variable is adjusted, we consider two sub-cases for each possible Laffer curve. In each case, we hold the ratio of government expenditures to output $\hat{G}$ constant. In the robustness section, we explore the consequence of holding the level of public expenditures constant (in deviation from $Z_t$).

3. Quantitative Results

In this section, we calibrate the IM model in order to analyze quantitatively its predictions relative to the Laffer curves discussed above.

3.1. Calibration and solution method. A period is taken to be a year. The momentary utility function is

$$u(c, 1 - h) = \eta \log(c) + (1 - \eta) \log(1 - h),$$

as is standard in the literature. This amounts to imposing $\sigma_c = 1$. Preferences are then described by two parameters, $\eta$ and $\beta$. We pin down $\eta$ so that aggregate hours worked $H \equiv \int g_h(a, s)dx$ equal 0.25. The subjective discount factor $\beta$ is set so that the after tax interest rate is equal to 4%, as in Trabandt and Uhlig (2011).

The fiscal parameters $\hat{B}$ and $\hat{G}$ are set to match the debt-output ratio and the public expenditure-output ratio reported by Trabandt and Uhlig (2011), i.e. $\hat{B} = 0.63$ and $\hat{G} = 0.18$. The tax rates are calibrated to match estimates of effective tax rates computed using the methodology developed by Mendoza, Razin, and Tesar (1994). This yields $\tau_N = 0.28$, $\tau_A = 0.38$, and $\tau_C = 0.05$. Using these
parameters, the benchmark value of the transfer-output ratio $\hat{T}$ is endogenously computed so as to balance the government budget constraint.

To calibrate the stochastic process \{s_t\}, we follow Domeij and Heathcote (2004) and Heathcote (2005). We assume that \{s_t\}, evolves over time according to a three-state Markov chain, with support $S = \{\bar{s}_1, \bar{s}_2, \bar{s}_3\}$ and transition matrix $Q$. The typical element $Q_{ij}$ denotes the probability of reaching state $j$ from state $i$. We impose the following structure on $Q$

$$Q = \begin{pmatrix} Q_{11} & 1 - Q_{11} & 0 \\ (1 - Q_{22})/2 & Q_{22} & (1 - Q_{22})/2 \\ 0 & 1 - Q_{11} & Q_{11} \end{pmatrix}.$$  

As in Heathcote (2005) and Domeij and Heathcote (2004), this transition matrix implies that households cannot move between the high and low productivity levels directly, that the fractions of high and low productivity households are equal, and that the probabilities of moving from the medium productivity state into either of the others are the same. Finally, as discussed in the previous section, we further impose the restriction $E\{\log(s_t)\} = 0$.

Given the above restrictions, this leaves four free parameters to be calibrated: $\bar{s}_1$, $\bar{s}_2$, $Q_{11}$, and $Q_{22}$. We pin down their values by matching four calibration targets: the Gini coefficient of wealth distribution, the share of wealth held by the 40% poorest, $\rho(\log(s_t))$ the autocorrelation of $\log(s_t)$, and $\sigma(\log(s_t))^2$ the variance of $\log(s_t)$. The first two calibration targets are taken from Díaz-Giménez, Glover, and Ríos-Rull (2011). In particular, they report that the Gini index is equal to 0.816 and the share of aggregate wealth held by the 40% poorest amounts to 1.1%. The last two correspond to the values reported by Heathcote (2005) and Domeij and Heathcote (2004). In particular, we seek to match $\rho(\log(s_t)) = 0.9$ and $\sigma(\log(s_t))^2 = 0.05/(1 - 0.9^2)$. The calibration is summarized in table I.

For comparison purposes, we also consider a version of the previous model in which (i) we impose idiosyncratic labor income shocks $s_t$ set to their average value and (ii) we relax the borrowing constraint. The model is recalibrated along the lines of Trabandt and Uhlig (2011), as described above.\(^5\) Importantly, the parameters $\eta$ and $\beta$ must be recalibrated to match the same calibration targets as those imposed in the IM economy. Notice that in this CM environment, the distinction between effective $H$ and efficient $N$ labor is no longer useful since both quantities coincide. We thus incorporate a productivity scale factor $\Omega$ in front of $N_t$ in the production function to compensate the CM economy for the average labor productivity effect present in the IM economy (i.e. the relative difference between $N$ and $H$). Doing so, we make sure that in the benchmark calibration described

\(^5\)See appendix A for details on the solution.
Table I. Calibration Summary

<table>
<thead>
<tr>
<th></th>
<th>Incomplete Markets</th>
<th>Complete Markets</th>
<th>Calibration Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.3391</td>
<td>0.3057</td>
<td>$H = 0.25$</td>
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<tr>
<td>$\beta$</td>
<td>0.9683</td>
<td>0.9808</td>
<td>$(1 - \tau_A)r = 0.04$</td>
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<tr>
<td><strong>Technology</strong></td>
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<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.3800</td>
<td></td>
<td>Trabandt and Uhlig (2011)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0700</td>
<td></td>
<td>Trabandt and Uhlig (2011)</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{s}_1$</td>
<td>0.2023</td>
<td>0</td>
<td>Wealth held by 40% poorest</td>
</tr>
<tr>
<td>$\bar{s}_2$</td>
<td>1.0184</td>
<td>0</td>
<td>Gini wealth</td>
</tr>
<tr>
<td>$Q_{11}$</td>
<td>0.9001</td>
<td>0</td>
<td>$\rho(\log(s))^a$</td>
</tr>
<tr>
<td>$Q_{22}$</td>
<td>0.9862</td>
<td>0</td>
<td>$\sigma(\log(s))^b$</td>
</tr>
<tr>
<td><strong>Fiscal Block</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_N$</td>
<td>0.2800</td>
<td></td>
<td>Trabandt and Uhlig (2011)</td>
</tr>
<tr>
<td>$\tau_A$</td>
<td>0.3600</td>
<td></td>
<td>Trabandt and Uhlig (2011)</td>
</tr>
<tr>
<td>$\tau_C$</td>
<td>0.0500</td>
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<tr>
<td>$\hat{B}$</td>
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<tr>
<td>$\hat{G}$</td>
<td>0.1800</td>
<td></td>
<td>Trabandt and Uhlig (2011)</td>
</tr>
</tbody>
</table>

$^a\rho(\log(s))$ stands for the first-order serial correlation $\log(s_t)$

$^b\sigma(\log(s))$ stands for the standard error of $\log(s_t)$.

above, all economies share the same interest rate, the same effective labor $H$, and the same stationary production level $\hat{Y}$. Clearly, we have $\hat{Y} = \hat{K}^{\theta/(1-\theta)}\Omega N$.

The solution method is now briefly described. Given values for the debt-output ratio and the tax rates, we postulate candidate values for the interest rate $r$ and aggregate efficient labor $N$. We then solve the government budget constraint for the transfer-output ratio. To do so, we use the representative firm’s first-order conditions, which give us values for $\hat{K}$ and $\hat{w}$, and the aggregate resource constraint, from which we back out $\hat{C}$. Given these, we solve the agents problem using the endogenous grid method proposed by Carroll (2006), adapted to deal with endogenous labor supply, in the spirit of Barillas and Fernandez-Villaverde (2007). Using the implied decision rules, we then solve for the stationary distribution as in Ríos-Rull (1999) and use it to compute aggregate average wages.

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6See appendix B for further details.

7In doing so, we exploit the special structure of the first order condition on $h$ induced by the specific functional form adopted for $u$. 

---
quantities. We then iterate on $r$ and $N$ and start the whole process all over again until the markets for capital and labor clear. For a given $N$, the interest rate is updated via a hybrid bisection-secant method. The bisection part of the algorithm is activated whenever the secant would update $r$ to a value higher than the CM interest rate (which would result in diverging private savings, as shown in Aiyagari, 1994). Once the market-clearing $r$ is found, $N$ is updated with a standard secant method.

To compute the Laffer curves, we adapt the previous algorithm as follows. We first vary either the transfer-output ratio or the debt-output ratio over prespecified ranges. For each grid point, the government steady-state budget constraint is balanced by adjusting one of the three tax rates considered, holding the other two taxes constant. Given values for the debt-output ratio and the transfer output ratio, we then solve for the agents decision rules and for the stationary distribution. We then iterate on $r$ and $N$ as described above.

3.2. Laffer Curves on Labor Income Taxes. Figure 1 reports three Laffer curves associated with variations in $\tau_N$, computed as indicated above. The gray one corresponds to the CM economy. The dark, plain line is the Laffer curve associated with the IM economy, when transfers $\hat{T}$ are varied. Finally, the dark, dashed line is the Laffer curve in the IM economy, when the debt-output ratio $\hat{B}$ is varied instead. In each case, fiscal revenues have been normalized by their benchmark value, indicated by a disk on the figure.
In the case when transfers are adjusted, the Laffer curve associated with $\tau_N$ has the standard inverted-$U$ shape. It clearly resembles the curve that would obtain in the CM economy, as shown in figure 1. The key difference appears in the high $\tau_N$ region of the graph. Here, a given tax rate generates relatively more fiscal revenues than in the CM setup. This is clearly due to the relative inelasticity of labor supply in the IM economy. As argued by Pijoan-Mas (2006), in this kind of economy, agents tend to supply more labor as part of their desire to self-insure. Put another way, the aggregate labor elasticity to taxation is lower in the IM economy than in the CM economy. This translates into a tax rate maximizing revenues equal to 51.35% in the IM economy and 48.64% in the CM economy. This allows the government to raise 15.58% more revenues than in the benchmark calibration in the IM economy and only 13.78% in the CM economy. To sum up, when transfers are adjusted to make the steady-state government budget constraint hold, resorting to a CM model or to an IM model to characterize the shape and peak of the labor income tax Laffer curve can have important consequences. The maximum tax rate is higher by 2.7% in the IM economy. In turn, this implies that the government can permanently raise revenues higher by 1.8%.

This is further illustrated in figure 2, which reports changes in steady-state output ($\hat{Y}$), (after tax) interest rate (($1 - \tau_A)r$), effective hours ($H$), physical capital ($\hat{K}\hat{Y}$), the debt-output ratio ($\hat{B}$), and the transfer-output ratio ($\hat{T}$) when $\tau_N$ is varied (see the dark and gray, plain lines). As is clear from this picture, except for $r$, the variables considered look very similar in the IM and CM cases. This is particularly striking when it comes to the transfer-output ratio. Indeed, when transfers are adjusted, there is no discernible difference between the curves obtained in the IM and CM setups. This is reminiscent of the quasi-aggregation result obtained in Krusell and Smith (1998). More recently, the debate between Prescott (2004) and Ljungqvist and Sargent (2008) came to a similar conclusion: using an IM or a CM model does not change much the aggregate conclusion one draws from labor income tax experiments. Nevertheless, for high tax rates, physical capital in the IM setup is slightly below the CM level. The reason why is that transfers help people to self-insure, thus reducing the need to accumulate assets. As a consequence, the after tax interest rate $(1 - \tau_A)r$ rises with $\tau_N$. Due to a relatively lower physical capital, effective labor turns out to be also lower in the IM economy than in the CM economy.

Figure 1 also reports optimal labor income taxes, i.e. the values of $\tau_N$ maximizing social welfare, given values for $\hat{B}$, $\hat{G}$, $\tau_A$, and $\tau_C$.\(^8\) In the CM model, the optimal $\tau_N$ is equal to 4.88% (see the triangle in figure 1). In the IM model, when transfers are varied, the optimal labor income tax is

\(^8\)Given the particular utility function adopted at the calibration step, social welfare is equal to $\int v(\hat{a}, s)dx + \eta \log(\hat{Y})$. This corresponds to a measure of welfare in deviation from $Z_t$. Notice also that since the ratio $G$ is held constant throughout, we are comparing economies with different levels of government consumption. As in Aiyagari and McGrattan (1998), we interpret this as an approximation to a setup where $G_t$ would be chosen optimally.
equal to 26.03% instead (see the star in figure 1), close to its actual value (see the disk in figure 1). Thus, even though the quasi-aggregation result obtained in Krusell and Smith (1998) seems to hold in our setup, we obtain, as Ljungqvist and Sargent (2008), that there can be significantly different policy implications in an IM setup compared to its CM counterpart.

Finally, we report on figure 3 the decomposition by fiscal bases of the Laffer curve associated with $\tau_N$. More precisely, the figure reports separately fiscal revenues from labor income taxes, from capital income taxes, and from consumption taxes, each as a function of $\tau_N$. The figure makes clear that the similitude in shape of the curves in the IM and CM economies does not result from a composition effect. In both economies, the Laffer curve shape is dominated by the response of the labor income tax basis, as in Trabandt and Uhlig (2011).

In contrast, when the debt-output ratio $\hat{B}$ is adjusted, we reach very different conclusions (see the dark, dashed curve in figure 1). Under this assumption, the Laffer curve looks like an S oriented horizontally. In the left part of the graph, for relatively low tax levels, the Laffer curve has an increasing branch which continuously reaches the usual pattern as labor tax income taxes decrease. This junction takes place in what appears to be a minimum tax level which is close to 25%. Interestingly, the minimum labor income tax obtains for a debt-output ratio close to $-110\%$. Above this level, there can be one, two, or three tax rates associated with a given level of fiscal revenues. Also, in
the regular part of this Laffer curve (i.e. the part that is indeed inverted-U-shaped), the revenue maximizing labor income tax is 55.57%, allowing the government to raise 17.92% more revenues than in the benchmark situation.

What explains the awkward shape of the Laffer curve in the left part of figure 1 when the debt-output ratio is adjusted? To gain an insight, imagine a simplified setting in which $\tau_C = \tau_A = 0$, so that, expressed as a share of GDP, fiscal revenues are $\hat{R} = \tau_N (1 - \theta)$. The steady-state government budget constraint now writes

$$\hat{R} = \hat{G} + \hat{T} + (r - \gamma) \hat{B}.$$  

Assuming differentiability with respect to $\hat{B}$, one gets

$$\frac{\partial \hat{R}}{\partial \hat{B}} = (r - \gamma) + \hat{B} \frac{\partial r}{\partial \hat{B}}.$$  

Now, since in this non-Ricardian economy, public debt crowds out capital in the households portfolio, we expect $\partial r / \partial \hat{B} > 0$. Indeed, as shown by Aiyagari and McGrattan (1998), when $\hat{B}$ is large, $\hat{K}$ gets smaller, which makes the equilibrium interest rate $r$ increase. Conversely, when $\hat{B}$ is negative and large in absolute value, private wealth $\hat{A}$ shrinks and the aggregate level of capital $\hat{K}$ increases, which makes the equilibrium interest rate decrease.

Thus the term $\hat{B} \partial r / \partial \hat{B}$ changes sign when $\hat{B}$ changes sign. For a sufficiently negative debt-output ratio, we can thus observe a change in the sign of $\partial \hat{R} / \partial \hat{B}$ and, since $\hat{R} = \tau_N (1 - \theta)$, correspondingly, a change in the sign of $\partial \tau_N / \partial \hat{B}$. Clearly, this would be impossible in a CM economy since, there,
\[ \frac{\partial r}{\partial B} = 0. \] By construction, this cannot happen either in the IM economy in which \( B \) is constant and \( T \) is adjusted.

At the same time, \( \hat{K} \) and \( N \) decrease with \( \hat{B} \), so that \( \hat{Y} \) is also decreasing with \( \hat{B} \). Thus, starting from a negative debt-output ratio \( \hat{B} \), output, \( \tau_N \), and \( R \) are large. As the government sells more and more assets, i.e. as \( \hat{B} \) increases, output and \( \tau_N \) decline, so that \( R \) declines too. This corresponds to the awkward part of the Laffer curve. In this region, there are two forces at play. First, as \( \hat{B} \) increases, the capital stock decreases, thus implying declining real wages, resulting in declining aggregate labor \( N \). Second, since \( \tau_N \) also decreases, agents are willing to supply more labor. It turns out that the first force dominates. Once the minimal tax is reached, \( \tau_N \) and \( \hat{R} \) start to increase again while \( \hat{Y} \) is still declining. This corresponds to the regular part of the Laffer curve, i.e. that which actually looks like an inverted \( U \). In this region, increases in \( \tau_N \) first dominate the disincentive effects of taxation, up to the maximal tax rate from which disincentive effects start to dominate.

In the general case, when \( \tau_C \) and \( \tau_A \) are non-zero, the above reasoning still holds but must also take into account the responses of \( K \) and \( \dot{A} \). These endogenous responses combine together to define the particular point at which fiscal revenues exhibit the awkward shape identified above. This also defines the minimal labor income tax. The question then naturally arises as to the policy relevance of this awkward section of the Laffer curve. To investigate this, we compute as before the tax rate maximizing social welfare. It turns out that in the IM model with debt varied, the optimal labor income tax is equal to 26.72% (see the square in figure 1). The associated debt-output ratio is close to -200%.\(^9\) This means that the optimal tax rate precisely lies on the awkward portion of the Laffer curve. This suggests that this part of the curve is not just a theoretical curiosity but is actually relevant from a policy perspective.

The steady-state responses of aggregate quantities are reported in figure 2. As \( \hat{B} \) gets smaller and smaller, the labor income tax \( \tau_N \) gets higher and higher. This corresponds to the awkward part of the Laffer curve. In this part of the graph, physical capital \( \hat{K} \) increases sharply (and concomitantly, private assets \( \hat{A} \) decline). This leads to a marked decline in the real interest rate \( r \). Since \( \hat{K} \) increases, the level of output also increases, though at a slower pace. This is due to the fact that aggregate hours rise at a slower pace too and even decline for high enough a labor income tax. In the regular part of the Laffer curve, as \( \hat{B} \) gets higher and higher, so too does the labor income tax. The increase in debt leads to a rise in private assets but to a decline in physical capital. Since the labor income tax increases, aggregate labor declines. In the end, output also declines.

Is there a limit to what the government can do by accumulating bigger and bigger assets? To answer this, imagine a limit situation in which \( \hat{B} \) is so negative that \( \hat{A} \) has been driven to zero. In

\(^9\)The optimal negative debt-output ratio confirms results previously obtained by Röhrs and Winter (2010).
this case, private agents no longer hold any assets and simply face static consumption and labor choices. Their income derives from wages and transfers only. From such a static program, one can easily establish that an individual labor supply $h$ is a decreasing function of $\tau_N$. Also, there is a tax rate above which agents no longer supply any labor, in which case, there is no production and the economy ceases to exist. Now, from the government budget constraint, one can see that $\tau_N$ decreases with $\hat{B}$ in this particular situation. Thus, as $\hat{B}$ gets smaller and smaller (i.e. as the government accumulates more and more physical capital), the labor income tax gets higher and higher. The limit to what the government can do, of course, is the particular $\hat{B}$ which leads to a $\tau_N$ so high that agents are no longer willing to supply any labor.

3.3. Laffer Curves on Capital Income Taxes. Figure 4 reports three Laffer curves associated with variations in $\tau_A$. As before, the gray one corresponds to the CM economy, as above. The dark, plain line is the Laffer curve associated with the IM economy, when transfers $\hat{T}$ are adjusted to make the government budget constraint hold. Finally, the dark, dashed line is the Laffer curve in the IM economy, when the debt-output ratio $\hat{B}$ is adjusted instead.

In the case when transfers are adjusted, the Laffer curve associated with $\tau_A$ has the standard inverted-U shape. It has the overall same shape as the curve that would obtain in the CM economy, as shown in figure 4. Once again, the key difference appears in the high $\tau_A$ region of the graph. Here, a given tax rate generates relatively more fiscal revenues than in the CM setup. This is clearly due to the relative inelasticity of agents saving behavior in the IM economy. As in Aiyagari (1994), in this kind of economy where contingent assets are ruled out, agents self-insure by accumulating relatively more assets (physical capital or public debt) than in a CM setup. This translates into a tax rate maximizing revenues equal to 57.73% in the IM economy and 53.01% in the CM economy. This allows the government to raise 3.11% more revenues than in the benchmark calibration in the IM economy and only 2.0% in the CM economy. Notice that the difference is less marked than with $\tau_N$. Notice that the optimal capital income tax (holding $\hat{G}$, $\hat{B}$, $\tau_N$, and $\tau_C$ constant) is close to 15%, as opposed to 0% in the CM version of the model.

This is further illustrated in figure 5, which reports changes in steady-state output ($\hat{Y}$), (after tax) interest rate ($(1 - \tau_A)r$), effective hours ($H$), physical capital ($\hat{K}\hat{Y}$), the debt-output ratio ($\hat{B}$), and the transfer-output ratio ($\hat{T}$) when $\tau_A$ is varied (see the dark and gray, plain lines). As is clear from this picture, except for $r$, the variables considered look similar in the IM and CM cases. For high tax rates, effective labor and physical capital turn out to be higher in the IM economy than in the CM economy. Notice that in this experiment, the after tax interest rate declines, in spite of an increase in transfers. This is because the rise in $\tau_A$ more than compensate the rise in $r$ consecutive to a decline in $K$. 
As in the previous section, when the debt-output ratio $\hat{B}$ is adjusted, we reach very different conclusions (see the dark, dashed curve in figure 4). Under this assumption too, the Laffer curve looks like an $S$ oriented horizontally. In the left part of the graph, for relatively low tax levels, the Laffer curve has an increasing branch which reaches the regular pattern as capital income taxes decrease. Once again, this junction takes place in what appears to be a minimum tax level which is close to 25%. Interestingly, the minimum capital income tax obtains for a debt-output ratio close to -31.86%. Above this level, there can be one, two, or three tax rates associated with a given level of fiscal revenues. Also, in the regular part of this Laffer curve (i.e. the part that is indeed inverted-U-shaped), the revenue maximizing capital income tax is 53.75%, allowing the government to raise 1.40% more revenues than in the benchmark situation. As with labor income taxes, we find that the optimal tax on capital (holding $\hat{G}$, $\hat{T}$, $\tau_N$, and $\tau_C$ constant) is close to its benchmark value. However, once again, the associated tax-revenue mix lies on the awkward part of the Laffer curve and is once again associated with a negative level for the optimal debt-output ratio.

Figure 5 helps understand what is happening in this case. When the debt-output ratio is increased (high $\tau_A$ region), physical capital is crowded-out by public debt, just as in the previous section. This implies an increase in the real interest rate, despite the increase in $\tau_A$. Thus, fiscal receipts from capital income taxation increase. At the same time, capital income taxation discourages individual
labor supply through two channels. First, since the stock of capital declines, so too does the real wage. Second, the increase in public debt helps soften the liquidity constraint and thus mitigates the need to self-insure through saving and working longer hours. Yet, because transfers (as a share of GDP) are held constant, agents work relatively harder than in the CM economy. As a consequence, fiscal receipts from labor income taxation decline. For sufficiently high tax rates on capital income, this decline more than compensates the rise in fiscal receipts from capital taxation.

In contrast, when the debt-output ratio is negative, the economy experiences a large inflow of physical capital. The real interest rate declines as capital increases (i.e. as the government accumulates more and more assets). Private wealth also shrinks, which forces private agents to work more. As a consequence, the capital income basis decreases while labor income basis increases. The combination of these effects implies that fiscal revenues increase when public debt is more and more negative. In addition, for moderately negative debt-output ratios, the government rents physical capital to firms and rental revenues are sufficiently high that $\tau_A$ can decrease. When the government holds too many assets, rental revenues are no longer sufficient to cover transfers and final good expenditures. At this stage, the government increases $\tau_A$ again.

3.4. Laffer Curves on Consumption Taxes. Figure 6 reports three Laffer curves associated with variations in $\tau_C$, defined in the exact same way as before. As usual, the gray one corresponds to the
Figure 6. Laffer Curves – Consumption Tax

**Note:** Level of fiscal revenues as a function of labor income tax $\tau_A$. Fiscal revenues are normalized by revenues in the benchmark fiscal setup, identified with a point in the curves above. IM stands for incomplete markets and CM for complete markets.

CM economy, as above. The dark, plain line is the Laffer curve associated with the IM economy, when transfers $\hat{T}$ are adjusted to make the government budget constraint hold. Finally, the dark, dashed line is the Laffer curve in the IM economy, when the debt-output ratio $\hat{B}$ is adjusted instead.

As in Trabandt and Uhlig (2011), the Laffer curve associated with $\tau_C$ does not exhibit a peak, either in the CM setup or in the IM setup with adjusted transfers. In the latter, fiscal revenues are slightly higher than in the former. Figure 7 reports changes in steady-state output ($\hat{Y}$), (after tax) interest rate ($(1 - \tau_A)\hat{r}$), effective hours ($\hat{H}$), physical capital ($\hat{K}\hat{Y}$), the debt-output ratio ($\hat{B}$), and the transfer-output ratio ($\hat{T}$) when $\tau_C$ is varied (see the dark and gray, plain lines). As is clear, the difference between the CM and IM economies is rather mild, even when it comes to the real interest rate. In both economies, as expected, output, hours, and capital decline when $\tau_C$ rises. However, for all $\tau_C$ considered, hours, capital, and output are slightly higher in the IM case than in the CM economy. Fundamentally, in both settings, taxing consumption is like taxing labor (both taxes show up similarly in the first order condition governing labor supply). A difference, though, is that in an IM economy such as ours, agents with a low labor productivity choose not to work whenever they hold enough assets. Clearly, those agents would not suffer from labor income taxation but do suffer from consumption taxes. Combined with the relative inelasticity of labor supply in the IM setup, this explains why the government can raise more revenues in this framework than in the CM setup.
As in the previous sections, when the debt-output ratio $\hat{B}$ is adjusted, we reach different conclusions (see the dark, dashed curve in figure 6). Under this assumption too, in the left part of the graph, for relatively low tax levels, the Laffer curve has an increasing branch which reaches the regular pattern as consumption taxes decrease. Once again, this junction takes place in what appears to be a minimum tax level which is close to 0.02, associated with a debt-output ratio close to $-63.09\%$. Above this level, there can be two tax rates associated with a given level of fiscal revenues.

Interestingly, when transfers are varied, the optimal $\tau_C$ is zero (holding the other fiscal parameters constant). In contrast, when debt is varied, the optimal $\tau_C$ is about 4% and is located once more on the awkward part of the curve.

4. Robustness

In this section, we explore the robustness of our results to alternative assumptions regarding government consumption and the stochastic process governing idiosyncratic productivity shocks.

4.1. Alternative Assumption on Government Expenditures. As in Aiyagari and McGrattan (1998), Flodén (2001), and Röhrs and Winter (2010), we have assumed a constant share of government consumption in output, i.e. a constant $\hat{G}$. In general, this is not an innocuous assumption. For
example, in the CM version of our model, this resulted in an optimal tax rate on labor income different from 0. More broadly, this assumption has strong implications on the tax bases response to changes in taxes. This is so because it imposes a certain pattern on the way government consumption crowds out private consumption.

Thus, in this section, we relax our previous assumption and impose instead that $G_t/Z_t$ be constant. Thus as $\hat{Y}$ decreases, the share of government consumption in output increases. We redo our Laffer curve calculations under this alternative assumption. In general, this does not raise special problems, except when we consider the Laffer curve associated with $\tau_A$ when $\hat{B}$ is varied. In this case, we must add an extra loop on the government budget constraint to make sure that the real after tax interest rate is never updated to values higher than $1/\beta − 1$. If it were the case, our algorithm would never converge, for reason discussed in Aiyagari (1994).

The results are reported on figure 8. The figure contains three subplots, each associated with one the three tax rates considered in the analysis. In each case, we report three Laffer curves: the one deriving from the CM version of the model, the one in the IM model when transfers are adjusted, and the one in the IM model when debt is adjusted instead. As before, we also report the welfare-maximizing taxes.

**Figure 8. Laffer Curves - Alternative Assumption on $G_t$**

*Note:* Level of fiscal revenues as a function of tax rates when $G_t/Z_t$ is held constant. Fiscal revenues are normalized by revenues in the benchmark fiscal setup, identified with a point in the curves above. IM stands for incomplete markets and CM for complete markets. The level of public expenditures is assumed to be constant.
We obtain qualitatively the same results as in the previous section. When transfers are adjusted, the IM and CM Laffer curves have a similar pattern. However, when debt is adjusted instead, the Laffer curve exhibits once again the horizontal S shape. Several differences with respect to the constant $\hat{G}$ case are worth mentioning. First, consistent with standard results, the optimal taxes are all equal to zero in the CM model (see the triangle in figure 8). In the IM model with transfers adjusted, we also find much lower optimal tax rates than before. The optimal capital income and consumption taxes should be set to zero while the optimal labor income tax is 4%. These values are not even remotely close to the actual values for taxes. In contrast, when debt is adjusted, we obtain optimal values for $\tau_N$, $\tau_A$, and $\tau_C$ equal to 27.48%, 15.25%, and 5.11%, respectively. Except for capital income taxes, these values are very close to the actual values for taxes. In each case, the optimal debt-output ratios are lower than $-300\%$. Second, the Laffer curve associated with $\tau_A$ is now somewhat steeper than when $\hat{G}$ was held constant. Also, when debt is adjusted, the capital income tax covers a much wider range of values than under a constant government consumption - output ratio.

4.2. Alternative Assumption on Idiosyncratic Shocks.

**Figure 9. Laffer Curves - Alternative Assumption on $s_l$**

Note: Level of fiscal revenues as a function of tax rates when $s_l$ is calibrated as in Flodén (2001). Fiscal revenues are normalized by revenues in the benchmark fiscal setup, identified with a point in the curves above. IM stands for incomplete markets and CM for complete markets. The level of public expenditures is assumed to be constant.
Up to now, we focused on a specification for idiosyncratic productivity shocks that allowed the benchmark IM to reproduce key characteristics of the wealth distribution. As suggested by Röhrs and Winter (2010), such an endeavor is crucial if one is interested in assessing the optimal quantity of debt. As a matter of fact, we found that the awkward part of the Laffer curve contains the tax-revenue mix corresponding to this optimal level, which was found to be negative, as convincingly argued by Röhrs and Winter (2010). Does this mean that the awkward part of the Laffer curve is specific to our particular calibration of idiosyncratic productivity shocks?

To answer this question, we recomputed our Laffer curves using an alternative specification for idiosyncratic shocks \( \{s_t\} \). To do so, we follow Flodén (2001) and assume that \( \log(s_t) \) follows an AR(1) process \( \log(s_t) = \rho \log(s_{t-1}) + \sigma \epsilon_t \) with \( \rho = 0.9 \) and \( \sigma = 0.21 \). This process is then approximated with a discrete Markov chain, using the method developed by Tauchen (1986). To do so, we allow for 7 states in the discrete Markov chain. Holding all the other parameters to their previous value, the model now delivers a Gini coefficient for the wealth distribution equal to 0.64. The bottom line is that under this alternative calibration, we obtain a very different wealth distribution.

Figure 9 reports the Laffer curves obtained under this alternative calibration. As before, we also report the curves associated with the CM version of the model. What conclusion can we draw from this exercise? First, we obtain slightly different levels of optimal taxes, though this is not particularly striking. However, when debt is adjusted, we no longer obtain negative levels for the optimal debt-output ratio. This is just a confirmation of results obtained by Röhrs and Winter (2010). Second, and more interestingly, the awkward part of the Laffer curve which arises when debt is adjusted still appears under this alternative calibration. Thus, the mere existence of this phenomenon does not seem to depend particularly on our benchmark calibration for idiosyncratic shocks.

5. Conclusion

In this paper, we have inspected how allowing for liquidity-constrained agents and incomplete financial markets impacts on the shape of the Laffer curve. In doing so, we paid particular attention which of debt or transfers is adjusted to make the government budget constraint hold as taxes are varied. While in a Ricardian framework, this does not matter, opting to adjust debt rather than transfers can potentially make a big difference in a non-Ricardian setup.

To address this question, we then formulated a neoclassical growth model with liquidity-constrained agents and incomplete financial markets along the lines of Aiyagari and McGrattan (1998). The model was calibrated to the US economy to mimic great ratios as well as moments related to the wealth distribution. We then investigated how the Laffer curve changes shape.
Our main findings are the followings. When it comes to labor and capital income taxes, the benchmark and IM models deliver similar Laffer curves when transfers are adjusted. The slippery slope is a little bit farther to the right in the IM model. This results from households using labor supply and savings to self-insure, allowing for a greater levels of taxation. However, when debt is adjusted, the shape of the Laffer curve is dramatically affected: the Laffer curve now looks like a horizontal S. First, for a positive debt, the slippery curve moves to the right as in the case of lump-sum transfers adjustments. Second, for a negative public debt, government revenues increase with debt. This implies that there can exist three tax rates compatible with the same level of fiscal revenues. Finally, when consumption tax are considered, the CM and IM models exhibit broadly similar shapes when transfers are adjusted, each of them displaying no peak. Once again, when debt is adjusted instead, there exist two consumption taxes delivering the same level of fiscal revenues. This result comes again from the non-monotonic response of the consumption tax rate to changes in public debt in the IM setup.
The complete-markets economy is the representative-agent version of the IM framework, where we have eliminated idiosyncratic shocks. Once growing variables have been detrended by $Y_t$, the associated steady state is then solution to the system

$$\dot{C} + (\gamma + \delta)\dot{K} + \dot{G} = 1$$

$$\dot{C} = \frac{\eta}{1 - \eta \frac{1 - \tau_N}{1 + \tau_C}} \dot{w}(1 - H)$$

$$\dot{w} = (1 - \theta)/H,$$

$$r + \delta = \theta/\dot{K}.$$  

The first equation is the resource constraint. The second equation is the steady-state Euler equation on capital. The third equation is the first-order condition on labor. The last two equations are the representative firm’s first order conditions.

We recursively solve for the steady-state values in the standard way. First, notice that by combining the Euler equation on capital with the condition on optimal use of capital by firms, one gets

$$\dot{K} = \frac{\beta \theta (1 - \tau_A)}{1 + \gamma - \beta [1 - (1 - \tau_A)\delta]}.$$  

Using this, we can solve for the consumption-output ratio according to

$$\dot{C} = 1 - [(\gamma + \delta)\dot{K} + \dot{G}].$$

Now, using the first order condition on labor supply together with the condition on optimal use of labor, one gets

$$H = \frac{1}{1 + \frac{1 - \eta}{(1 - \theta) \eta} \frac{1 + \tau_C}{1 - \tau_N} \dot{C}}.$$  

Finally, we modify the production function to make sure that the level of stationary output $\dot{Y}$ in the CM economy coincides with that in the IM economy. To do so, define $\Omega = \dot{N}_{IM}/H_{IM}$, where the subscript $IM$ refers to the IM setup variables. Then

$$Y_t = K_t^{\Omega}(\Omega Z_t H_t)^{1 - \theta}.$$  

$\Omega$ can be interpreted as a mean-preserving spread correcting for the average labor productivity effect present in the IM economy.
Appendix B. Solving for the Decision Rules with the Endogenous Grid Method

In this appendix, we describe how we implement the endogenous grid method to solve for the agents’ decision rules. At this stage, we assume that $r$ and $\hat{w}$ are known and take them parametrically, together with tax rates appearing in an individual’s budget constraint.

We set a grid of values for $\hat{a}'$, denoted by $G_a$. In practice, we select an exponential grid, with 2000 points. The algorithm is initialized by postulating an approximate decision rule for $\hat{a}''$, which we denote by $\hat{g}_a^{(0)}$. Also, we define a numerical tolerance parameter $\epsilon$; in practice $\epsilon = 1 e - 8$. We then implement the following steps:

1. Given $\hat{g}_a^{(i)}$, for each $(\hat{a}', s') \in G_a \times S$, compute
   - next period’s labor supply
     $$\hat{g}_h^{(i)}(\hat{a}', s') = \max \left\{ 0, 1 - (1 - \eta) \left[ 1 + \frac{[1 + (1 - \tau_A)r]\hat{a}' + \hat{T} - (1 + \gamma)\max\{0, \hat{g}_a^{(i)}(\hat{a}', s')\}}{(1 - \tau_N)\hat{w}s'} \right] \right\}.$$  
   - next period’s cash on hand
     $$m^{(i)}(\hat{a}', s') = (1 - \tau_N)\hat{w}s'\hat{g}_h^{(i)}(\hat{a}', s') + [1 + (1 - \tau_A)r]\hat{a}' + \hat{T},$$  
   - next period’s Lagrange multiplier
     $$\lambda^{(i)}(\hat{a}', s') = \frac{\eta}{m^{(i)}(\hat{a}', s') - (1 + \gamma)\max\{0, \hat{g}_a^{(i)}(\hat{a}', s')\}}.$$
2. For each $(\hat{a}', s') \in G_a \times S$, compute
   - the current period Lagrange multiplier:
     $$\bar{\lambda} = [1 + (1 - \tau_A)r]\frac{\beta}{1 + \gamma}\mathbb{E}\{\lambda^{(i)}(\hat{a}', s')|s\},$$
   - the current period consumption
     $$\bar{c} = \frac{\eta}{(1 + \tau_C)\bar{\lambda}},$$
   - the current period cash on hand
     $$\bar{m} = (1 + \tau_C)\bar{c} + (1 + \gamma)\hat{a}'.$$
3. Using $\bar{m}$, $m^{(i)}$ and $G_a$, update $\hat{g}_a^{(i)}$ via an interpolation procedure and thus compute $\hat{g}_a^{(i+1)}$.
4. If $\|\hat{g}_a^{(i+1)} - \hat{g}_a^{(i)}\| < \epsilon$, stop, else go back to step 1.
References


