Working Paper Series n° 706

March, 2012

"Identifying New Shocks from SVAR"

Patrick FEVE and Ahmat JIDOUD

Institut d'Economie Industrielle (IDEI) – Manufacture des Tabacs
Aile Jean-Jacques Laffont – 21, allée de Brienne – 31000 TOULOUSE – FRANCE
Tél. + 33(0)5 61 12 85 89 – Fax + 33(0)5 61 12 86 37 – **www.idei.fr** – contact@cict.fr

INSTITUT D'ECONOMIE INDUSTRIELLE

Identifying News Shocks from SVARs

Patrick Feve*
Toulouse School of Economics (GREMAQ, IDEI, IUF and Banque de France)

Ahmat Jidoud
Toulouse School of Economics (GREMAQ)

March, 2012

Abstract

This paper investigates the reliability of SVARs to identify the dynamic effects of news shocks. We show analytically that the dynamics implied by SVARs, using both long–run and short–run restrictions, are biased. However, the bias vanishes as long as news shocks account for most of the variability of the endogenous variable and the economy exhibits strong forward–looking behavior. Our simulation experiments confirm these findings and further suggest that the number of lags is a key ingredient for the success of the VAR setup. Furthermore, a simple correlation diagnostic test shows that news shocks identified using both restrictions are found to exhibit a correlation close to unity, provided that news shocks drive an overwhelming part of aggregate fluctuations.

Keywords: News shocks, SVARs, Identification, Diagnostic Test, Non–fundamentalness

JEL Class.: C32, C52, E32

Introduction

This paper contributes to the expanding literature towards the empirical relevance of anticipated shocks, labeled as news shocks. Using a structural vector error correction model for total factor productivity (TFP) and stock prices, Beaudry and Portier (2005, 2006) suggest an identification procedure allowing to uncover anticipated shocks. They find out that innovations in the growth rate of TFP are largely anticipated. Furthermore, these news shocks on TFP account for more than half of the forecast error variance of consumption, output and hours. In a similar framework, Beaudry and

*Address: Université de Toulouse 1, Aile Jean-Jacques Laffont, 21 Allée de Brienne, 31000, Toulouse, France. email: patrick.feve@univ-tlse1.fr. We thank the Co–Editor, D. McMillin, and an anonymous referee for helpful comments and suggestions. We also thank M. Dupaigne, R. Farmer, J–P. Florens, G. Lorenzoni, R. Pancrazi, P. Pintus, F. Portier, G. Saint–Paul and A. Seymen for for fruitful discussions. We also thank participants at the Macro Workshop (Toulouse, march 2010), "New Challenge for Macroeconomic Policy" workshop (Marseille, May 2010), T2M conference 2011 (Montreal), EEA-ESEM 2011 (Oslo) and 2nd SEEK Conference 2012 (Mannheim). The views expressed herein are those of the authors and should not be interpreted as reflecting those of the Banque de France.

Lucke (2009) provide a complete assessment of the leading forces of aggregate fluctuations and show that news shocks unveil to be the main drivers of business cycle.[1]

The objective of this paper is to answer to the following question: under which conditions are Structural Vector AutoRegressions (SVARs) successful at identifying news shocks and their dynamic effects? Two arguments motivate this question. The first is that SVARs are widely used as useful tools for the validation and estimation of DSGE models. Accordingly, failures of SVARs to provide reliable results transmit into wrong model selection and erroneous policy prescriptions (see Christiano et al., 2006). Second and more importantly, identifying news shocks in the SVAR setup happens to be a tedious task.[2] Indeed, the presence of news shocks in the economy may induce a non–fundamental time series representation of the data (see Fève, Matheron and Sahuc, 2009 and Leeper, Walker and Yang, 2009). Such non fundamentalness corrupts the identification of structural shocks from past and current data, an assumption taken as given in the VAR analysis.[3]

To identify and understand the implications of news shocks, we use a SVAR setup with two observable variables under the solution of a simple linear forward-looking model with rational expectations. The first observable variable can be interpreted in several ways: growth rate of TFP (as in Beaudry and Portier, 2006), fiscal policy (see Leeper, Walker and Yang, 2009), dividend growth rate or any stochastic forcing variable that can be subjected to news shocks. The second observable variable represents an endogenous decision variable that heavily depends in a forward looking fashion on the anticipated and unexpected shocks. In spite of its abusive simplicity, the structural model considered here as the Data Generating Process (DGP) allows us to understand the implications of news shocks in a SVAR framework given its analytical tractability. Furthermore, we adopt the approach proposed by Beaudry and Portier (2005, 2006). Their empirical analysis consists in two steps. First, they apply sequentially long–run and short–run restrictions on the VAR model to identify the news shocks. Second, they compute the correlation between the two news shocks recovered from the two identification strategies. Such an indicator is intended to show the power of the two identification approaches to uncover the relevance of the identified news shocks.[4] In Beaudry and Portier (2005, 2006), this procedure is actually shown to identify correctly the structural shocks in that the empirical correlation is found to be positive and close to one, suggesting that positive news shocks in productivity are preceded by stock market booms.

[1]A large part of the business cycle literature tackles also the issue of news shocks using Dynamic Stochastic General Equilibrium (DSGE) models among which Davis (2007), Fujiwara and Shintani (2008), Schmitt–Grohé and Uribe (2008) and Khan and Tsoukalas (2009). For example, Schmitt–Grohé and Uribe (2008) show that standard RBC models augmented with real rigidities (habits formation in consumption and leisure, investment adjustment costs and variable capacity utilization) generate news driven business cycles and anticipated shocks explain more than two thirds of the predicted aggregate fluctuations.

[2]See Beaudry and Portier (2005) and (2006) and Beaudry and Lucke (2009) for the estimation of news TFP shocks from SVARs. See also, e.g., Ramey (2009) and Mertens and Ravn (2009a) for quantitative investigations about the usefulness of SVARs for the identification tax policy shocks.

[3]See the references in Leeper, Walker and Yang (2009) about non–fundamentalness issues in rational expectation econometrics.

[4]Other empirical strategies have been implemented in a VAR setup. Barsky and Sims (2010) identify news shocks as those explaining the overwhelming fluctuations in TFP. Mertens and Ravn (2009b) propose an augmented fiscal SVAR estimator which is robust to the presence of anticipation effects.

A number of key results arise from our paper. First, we find that the estimated impulse responses function (IRF) using either a long–run or a short–run restriction are biased. This is particularly true when the news contains longer anticipation horizon. Second, a short–run restriction performs better than a long–run restriction. In particular, a SVAR model with a long–run restriction yields biased estimated responses even in the case where the VAR model does not display non–fundamentalness. Third, the estimated bias is strongly reduced when the fraction of fluctuations in the economy driven by news shocks is substantial and the VAR model is estimated using a sufficient number of lags. Indeed, estimating a VAR model with a number of lags smaller than the length of news implies a lag truncation–bias. Finally, performing the simple correlation diagnostic test of Beaudry and Portier, we obtain that the correlation between the innovations identified using long run restrictions and those obtained with short–run restrictions is almost equal to one when anticipated shocks mostly drive the fluctuations in the model and the economy is subject to strong forward–looking dynamic. These findings are both obtained from analytical results and simulation experiments.

The paper is organized as follows. In a first section, we expound our reference setup and we discuss non fundamentalness issues. The second section reports the identified dynamic responses using both long–run and short–run restrictions. The third section assesses the reliability of SVARs from different simulation experiments. The last section concludes.

1 The Setup

We use a simple model as the DGP and investigate under which conditions VAR models admit a non–fundamental representation.

1.1 The Model

The model economy takes the following form[5]

$$\begin{aligned} y_t &= aE_t y_{t+1} + bE_t \Delta x_{t+1}, \quad (1)\\ \Delta x_t &= \sigma_\varepsilon \varepsilon_{t-q} + \sigma_u u_t \quad \sigma_\varepsilon, \sigma_u > 0 \quad , \quad (2) \end{aligned}$$

where y_t denotes a single endogenous variable and x_t is a single exogenous variable, specified in first–difference. E_t denotes the expectation operator conditional on the information set in period t, *i.e.* when agents must take their decisions about y_t. The parameters a and b in the behavioral equation (1) are assumed to be non-zero. For simplicity, hereafter, we normalize b to unity.[6] Equation (1) naturally emerges from any optimization problem in stochastic equilibrium models. Typically, equations (1)–(2) define the log–linear equilibrium conditions for an asset–pricing model where y_t denotes the log of the

[5]We have also investigated another model economy of the form $y_t = aE_t y_{t+1} + b\Delta x_t$. Our quantitative findings are almost identical. The results are available from the authors upon request.

[6]Our main findings are unaffected by this normalization. The results are also available from the authors upon request.

price–dividend ratio and Δx_t the growth rate of exogenous dividends. Equation (2) is the backbone of our analysis. It states that, at any point in time t, private agents observe the two components of Δx_t: an anticipated component observed $q \geq 1$ periods in advance, ε_{t-q} and an unanticipated component, u_t. Furthermore, these shocks are assumed to be serially uncorrelated with zero mean and unit variance and are mutually uncorrelated at all leads and lags.

Excluding sunspots (*i.e.* we impose $|a| < 1$) and bubbles (*i.e.* we restrict the solution to satisfy $\lim_{T\to\infty} E_t a^T y_{t+T} = 0$) and using the process (2), we obtain the solution (or reduced form) for y_t

$$y_t = \sigma_\varepsilon \sum_{i=0}^{q-1} a^{q-1-i} \varepsilon_{t-i} \tag{3}$$

Equation (3) together with equation (2) represent the DGP. We assume that the variables Δx_t and y_t are observed by the econometrician but she cannot distinguish between the two shocks driving Δx_t. This observability problem is made more pernicious as the econometrician is faced with two permanent shocks.[7] In a more compact way our DGP writes as

$$Z_t = \mathcal{H}(L) v_t, \tag{4}$$

where $Z_t = (\Delta x_t, y_t)'$, $v_t = (\varepsilon_t, u_t)'$ and the matrix $\mathcal{H}(L)$ is given by

$$\mathcal{H}(L) = \begin{pmatrix} \sigma_\varepsilon L^q & \sigma_u \\ \sigma_\varepsilon \sum_{i=0}^{q-1} a^{q-1-i} L^i & 0 \end{pmatrix}.$$

1.2 Non–fundamentalness issues

The issue of this paper is to investigate under which conditions, SVARs can properly uncover the true dynamic responses of a DGP that does not admit a fundamental times series representation. A formal definition of fundamentalness is the following (see Alessi, Barigozzi, and Capasso, 2008)

Definition 1. *Consider a covariance stationary process Z_t. Then the representation $Z_t = \mathcal{H}(L)v_t$ is fundamental if: i) v_t is a white noise vector; ii) $\mathcal{H}(L)$ has no poles of modulus less or equal than unity, i.e. it has no poles inside the unit disc; iii)* $\det \mathcal{H}(z)$ *has no roots of modulus less than unity, i.e. it has no poles outside the unit disc* $\det \mathcal{H}(z) \neq 0, \quad \forall z \in \mathbb{C} \quad s.t. \quad |z| < 1$.

Given $\mathcal{H}(.)$ in (4), we deduce

$$\mathcal{H}(z) = \begin{pmatrix} \sigma_\varepsilon z^q & \sigma_u \\ \sigma_\varepsilon \sum_{i=0}^{q-1} a^{q-1-i} z^i & 0 \end{pmatrix}.$$

It follows that $\det \mathcal{H}(z) = -\sigma_u \sigma_\varepsilon \sum_{i=0}^{q-1} a^{q-1-i} z^i$. When $q = 1$, the determinant is given by $\sigma_u \sigma_\varepsilon \neq 0$ for non-zero values of σ_u and σ_ε. The system is accordingly fundamental. However for any anticipation

[7]Notice that we follow the empirical strategy adopted by Beaudry and Portier (2005), (2006) by assuming that the variable subject to news shocks is observed, together with y_t. In their paper, the observed forcing variable x_t is the log of TFP and y_t is defined as the excess return on stock prices.

horizon beyond two periods ($q \geq 2$), the roots μ_i of $\det \mathcal{H}(z)$ are such that $|\prod_{i=0}^{q-1} \mu_i| = |a^{q-1}| < 1$. This implies that at least one root lies within the unit circle. Consequently the system (4) is non fundamental as long as $q \geq 2$. For example, when $q = 2$, we obtain $|z| = 1/|a| > 1$.

2 Estimation and Identification

The above results show that when $q \geq 2$, the DGP does not admit a fundamental bi–variate representation. Then, we restrict our analytical calculations to $q = 1, 2$. We consider an econometrician whose objective is to identify news shocks using SVARs. For such a purpose, the econometrician first estimates an appropriate VAR(p) model from the observed variables Δx_t and y_t, where p denotes the number of lags. Then she applies structural restrictions on the estimated VAR model to identify the relevant shocks.

We consider a VAR(1) model as a simple way to statistically represent the solution of our structural model.[8] The estimated VAR model is given by

$$\begin{pmatrix} \Delta x_t \\ y_t \end{pmatrix} = A \begin{pmatrix} \Delta x_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} \upsilon_{1t} \\ \upsilon_{2t} \end{pmatrix}, \tag{5}$$

where $\upsilon_t = (\upsilon_{1,t}, \upsilon_{2,t})'$ is the vector of canonical errors and

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}.$$

The elements of the matrix A are estimated using linear projections of Δx_t and y_t on their own lagged values.

2.1 The Fundamental Case

When $q = 1$, the DGP admits a VAR(1) representation

$$\begin{pmatrix} \Delta x_t \\ y_t \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta x_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} \sigma_u u_t \\ \sigma_\varepsilon \varepsilon_t \end{pmatrix} \tag{6}$$

This VAR(1) representation suggests that the econometrician will find that current values of the endogenous variable y_t may convey information useful to forecast the future values of the exogenous shock Δx_{t+1}. The estimated VAR model under our DGP thus implies that the endogenous variable y_t *Granger causes* the exogenous variable Δx_t. This represents an additional illustration of pitfalls in the use of causality test (see Hamilton, 1994, for another example).

Before proceeding with SVARs, we compute two matrices that are useful for the identification of news shocks. The VAR(1) model admits a VMA(∞) representation, $Z_t = B(L)\upsilon_t$, with $Z_t = (\Delta x_t, y_t)'$ and

[8] Analytical and tractable solutions are available only for $p = 1$. We provide simulation results for a higher number of lags and show that they are in line with the theoretical ones.

$B(L) = (I - AL)^{-1}$. The covariance matrix of the canonical innovations is thus given by

$$\Omega_\upsilon = \sigma_\varepsilon^2 \begin{pmatrix} \theta & 0 \\ 0 & 1 \end{pmatrix},$$

where $\theta = \sigma_u^2/\sigma_\varepsilon^2$. This parameter accounts for the relative variance of standard unexpected shocks to news shocks. As we will show latter, this parameter will play a central role in the ability of the econometrician to consistently identify the news shock. We define the long–run covariance matrix (the spectral density of the vector Z_t at zero frequency) as $\Sigma^{LR} = B(1)\Omega_\upsilon B(1)'$, where Ω_υ is the covariance matrix of the canonical innovations and $B(1) = (I - A)^{-1}$. We deduce the long–run covariance matrix

$$\Sigma^{LR} = \sigma_\varepsilon^2 \begin{pmatrix} 1+\theta & 1 \\ 0 & 1 \end{pmatrix}$$

Long–Run Restriction Let $\eta_t = (\eta_{1,t}, \eta_{2,t})'$ be the vector of structural shocks. We have the structural representation $Z_t = B(L)\upsilon_t \equiv C(L)\eta_t$, where $C(L) = B(L)S$ and S is a non-singular matrix constructing the innovations υ_t as linear combinations of structural disturbances η_t. As usual, we impose an orthogonality assumption on the structural shocks, which combined with a scale normalization implies $Var(\eta_t) = I_2$. However, this is not enough to identify S and following Blanchard and Quah (1989), we impose a long–run restriction. The news shock is then identified as the only shock with a long–run effect of the level of x_t (see Beaudry and Portier, 2006). Given the ordering of Z_t, $C(1)$ must be lower triangular. This amounts to imposing that $C(1)$ be the Cholesky decomposition of $\Sigma^{LR} = C(1)C(1)'$. Given this identity, we can easily recover $C(1)$ and accordingly $S = B(1)^{-1}C(1) \equiv (I - A)C(1)$.

$$C(1) = \sigma_\varepsilon \begin{pmatrix} \sqrt{1+\theta} & 0 \\ \frac{1}{\sqrt{1+\theta}} & \sqrt{\frac{\theta}{1+\theta}} \end{pmatrix} \quad , \quad S = \frac{\sigma_\varepsilon}{\sqrt{1+\theta}} \begin{pmatrix} \theta & -\sqrt{\theta} \\ 1 & \sqrt{\theta} \end{pmatrix}$$

We assess this SVAR at identifying news shocks through the dynamic responses of Δx_t and y_t. These IRFs are given by (in parentheses, we report the true responses)

$$\begin{aligned} \frac{\partial \Delta x_t}{\partial \eta_{1t}} &= \frac{\theta}{\sqrt{1+\theta}}\sigma_\varepsilon \ \ (\geq 0) \ , & \frac{\partial y_t}{\partial \eta_{1t}} &= \frac{1}{\sqrt{1+\theta}}\sigma_\varepsilon \ \ (\leq \sigma_\varepsilon) \\ \frac{\partial \Delta x_{t+1}}{\partial \eta_{1t}} &= \frac{1}{\sqrt{1+\theta}}\sigma_\varepsilon \ \ (\leq \sigma_\varepsilon) \ , & \frac{\partial y_{t+1}}{\partial \eta_{1t}} &= 0 \ \ (= 0). \end{aligned}$$

These dynamic responses are driven by θ (the relative size of standard surprise shocks with respect to news shocks) and are thus biased, if we except the response of y_t at one lag. Although private agents receive news about future value of x in the DGP, the econometrician mistakenly rejects the presence of news as the variable Δx_t contemporaneously responds to η_{1t}. Interestingly, the smaller is θ, the smaller the bias. Hence, as the fraction of fluctuations driven by news shocks gets larger ($\theta \to 0$), the SVAR consistently identifies news shocks.

To provide an intuition for such a result, we recover the identified shock using long–run restrictions as a function of the structural shocks, ε_t and u_t:

$$\begin{aligned}\eta_{1,t} &= \frac{1}{\sigma_\varepsilon\sqrt{1+\theta}}\left\{\sigma_u u_t + \sigma_\varepsilon \varepsilon_t\right\} \\ &= \sqrt{\frac{\theta}{1+\theta}}u_t + \frac{1}{\sqrt{1+\theta}}\varepsilon_t\end{aligned}$$

The econometrician does not identify the true news shocks but rather a weighted average of the anticipated and the unanticipated shocks. Only when the relative volatility of news shocks is substantial ($\sigma_\varepsilon >> \sigma_u$ or $\theta \to 0$) in the economy, will the identified shock be the true one.

Short–Run restrictions In this setup, the econometrician uses some prior information to restrict the impact response of x_t. The restriction is imposed now on the matrix $\tilde{C}(0) = B(0)\tilde{S} \equiv \tilde{S}$, the matrix of contemporaneous responses (see, Sims, 1980). Notice that we use again an orthogonality assumption on the structural shocks and a scale normalization. News shocks are assumed to have a zero impact on Δx_t. This corresponds to $s_{11} = 0$ or to the following restriction on the canonical disturbance $\upsilon_{1,t} = s_{12}\tilde{\eta}_{2,t}$, where the vector of innovations is now $\tilde{\eta}_t = (\tilde{\eta}_{1,t}, \tilde{\eta}_{2,t})'$. The previous system rewrites as $Z_t = B(L)\upsilon_t \equiv \tilde{C}(L)\tilde{\eta}_t$ and

$$\tilde{S} = \begin{pmatrix} 0 & s_{12} \\ s_{21} & s_{22} \end{pmatrix}.$$

In this SVAR model, the short–run variance covariance matrix of the system is given $\tilde{C}(0)\tilde{C}(0)' = B(0)\Omega_\upsilon B(0)'$. This implies that $\tilde{S}\tilde{S}' = \Omega_\upsilon$, *i.e.* $\tilde{S}$ is a Cholesky decomposition of the variance covariance matrix Ω_υ of the canonical residuals. Using this decomposition, we obtain

$$\tilde{S} = \sigma_\varepsilon \begin{pmatrix} 0 & \sqrt{\theta} \\ 1 & 0 \end{pmatrix}.$$

The IRFs at zero and one lag are given by (in parentheses, we report the true responses)

$$\begin{aligned}\frac{\partial \Delta x_t}{\partial \tilde{\eta}_{1t}} &= 0 \ \ (=0) \ , & \frac{\partial y_t}{\partial \tilde{\eta}_{1t}} &= \sigma_\varepsilon \ \ (=\sigma_\varepsilon) \ , \\ \frac{\partial \Delta x_{t+1}}{\partial \tilde{\eta}_{1t}} &= \sigma_\varepsilon \ \ (=\sigma_\varepsilon) \ , & \frac{\partial y_{t+1}}{\partial \tilde{\eta}_{1t}} &= 0 \ \ (=0) \ .\end{aligned}$$

These responses are exactly those implied by the DGP of the underlying economy. Thus, the estimated dynamics implied by a short–run restriction are independent from θ. No matter, how these forces are allocated, the econometrician does perfectly identify the news shock. This is because the news shock is perfectly uncovered under this identification scheme ($\tilde{\eta}_{1t} = \varepsilon_t$).

The Correlation Diagnostic Test Beaudry and Portier (2006) have performed a test allowing to assess the empirical plausibility of the news shock hypothesis. Formally speaking, this simple diagnostic test consists in computing the correlation between the identified news shocks recovered from long–run and short–run restrictions ($\eta_{1,t}$ and $\tilde{\eta}_{1,t}$ in our previous notations) and see how this correlation evolves. Beaudry and Portier (2006) obtain a correlation close to one and conclude that this result strongly supports their empirical findings about the relevance of news shocks.

Using the identified news shocks from the long–run and short–run restrictions, we get

$$corr(\eta_{1t}, \tilde{\eta}_{1t}) = \frac{1}{\sqrt{1+\theta}}$$

This result shows when news shocks are the dominant source of fluctuations ($\theta \to 0$), the long–run identification strategy provides accurate estimates and the correlation then tends to unity.

2.2 The Non–Fundamental Case

When $q = 2$, the DGP is now defined by

$$\Delta x_t = \sigma_u u_t + \sigma_\varepsilon \varepsilon_{t-2} \quad (7)$$

$$y_t = a\sigma_\varepsilon \varepsilon_t + \sigma_\varepsilon \varepsilon_{t-1} \quad (8)$$

Using these two equations as the DGP, we assume that the econometrician seeks to estimate a VAR(1) model.[9] The resulting matrix A in (5) is given by

$$A = \begin{pmatrix} 0 & \phi \\ 0 & a\phi \end{pmatrix}.$$

where $\phi = 1/(1+a^2)$. Notice that the previous remarks about the *Granger* causality still apply. Given the DGP and the matrix A of the VAR model, the canonical innovations $\upsilon_{1,t}$ and $\upsilon_{2,t}$ can be expressed in terms of the structural shocks

$$\begin{pmatrix} \upsilon_{1,t} \\ \upsilon_{2,t} \end{pmatrix} = \begin{pmatrix} \sigma_u u_t - a\phi\sigma_\varepsilon \varepsilon_{t-1} + (1-\phi)\sigma_\varepsilon \varepsilon_{t-2} \\ a\sigma_\varepsilon \varepsilon_t + (1-a^2\phi)\varepsilon_{t-1} - a\phi\varepsilon_{t-2} \end{pmatrix}. \quad (9)$$

We then deduce $var(\upsilon_{1,t}) = (\theta + 1 - \phi)\sigma_\varepsilon^2$, $var(\upsilon_{2,t}) = (\phi + a^2)\sigma_\varepsilon^2$ and $cov(\upsilon_{1,t}, \upsilon_{2,t}) = -a\phi\sigma_\varepsilon^2$. The covariance matrix of the canonical innovations is thus given by:

$$\Omega_\upsilon = \sigma_\varepsilon^2 \begin{pmatrix} \theta + 1 - \phi & -a\phi \\ -a\phi & \phi + a^2 \end{pmatrix}$$

From Ω_υ, $B(1) = (I - A)^{-1}$ and $\Sigma^{LR} = B(1)\Omega_\upsilon B(1)'$, we obtain the long–run covariance matrix

$$\Sigma^{LR} = B(1)\Omega_\upsilon B(1)' = \frac{\sigma_\varepsilon^2}{1-a\phi} \begin{pmatrix} (1+\theta)(1-a\phi) & 1 \\ 1 & \frac{\phi+a^2}{1-a\phi} \end{pmatrix}.$$

[9] Obviously, fitting a VAR(1) model to these data will lack information about the relevant dynamics. See Section 3 for various illustrations using simulated data.

Long–Run Restriction Applying a long–run restriction on the estimated VAR(1) model yields

$$\begin{aligned} C(1) &= \frac{\sigma_\varepsilon}{1-a\phi}\kappa \begin{pmatrix} (1-a\phi)\sqrt{1+\theta} & 0 \\ \frac{1}{\sqrt{1+\theta}} & \sqrt{(\phi+a^2)(1+\theta)-1} \end{pmatrix} , \\ S &= \frac{\sigma_\varepsilon}{1-a\phi} \begin{pmatrix} (1-a\phi)\sqrt{1+\theta} - \frac{\phi}{\sqrt{1+\theta}} & -\phi\sqrt{(\phi+a^2)(1+\theta)-1} \\ \frac{1-a\phi}{\sqrt{1+\theta}} & (1-a\phi)\sqrt{(\phi+a^2)(1+\theta)-1} \end{pmatrix}. \end{aligned}$$

Furthermore we have $C(L) = B(L)S = B(L)B(1)^{-1}C(1)$. The structural model is given by

$$\begin{aligned} \begin{pmatrix} \Delta x_t \\ y_t \end{pmatrix} &= \begin{pmatrix} 0 & \phi \\ 0 & a\phi \end{pmatrix} \begin{pmatrix} \Delta x_{t-1} \\ y_{t-1} \end{pmatrix} \\ &+ \frac{\sigma_\varepsilon}{1-a\phi} \begin{pmatrix} (1-a\phi)\sqrt{1+\theta} - \frac{\phi}{\sqrt{1+\theta}} & -\phi\sqrt{(\phi+a^2)(1+\theta)-1} \\ \frac{1-a\phi}{\sqrt{1+\theta}} & (1-a\phi)\sqrt{(\phi+a^2)(1+\theta)-1} \end{pmatrix} \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \end{pmatrix}. \end{aligned}$$

The moving–average representation is obtained by computing $B(L) = \sum_{i=0}^{\infty} A^i L^i$. The form of A implies that $A^i = (a\phi)^{i-1}A$ for $i = 1, 2, \ldots$. The IRF at horizon $i \geq 1$ is therefore given by the product $A^i S = (a\phi)^{i-1}AS$. We concentrate our analysis on the dynamics implied by news 0, 1 and 2 periods after the shock.

On impact, we have (in parentheses, we report the true responses)

$$\frac{\partial \Delta x_t}{\partial \eta_{1,t}} = \frac{\sigma_\varepsilon}{1-a\phi}\left[(1-a\phi)\sqrt{1+\theta} - \frac{\phi}{\sqrt{1+\theta}}\right] \quad (\lesseqgtr 0) \quad , \quad \frac{\partial y_t}{\partial \eta_{1,t}} = \frac{\sigma_\varepsilon}{\sqrt{1+\theta}} \quad (\lesseqgtr a\sigma_\varepsilon).$$

The estimated responses are biased. However the size of this bias is essentially determined by how much news shocks are relevant in the economy (the value of θ) and the strength of the forward–looking behavior of the economy (the value of a). To see this, assume that $\theta \to 0$ and $a \to 1$. Then the estimated responses become

$$\lim_{\theta\to 0, a\to 1} \frac{\partial \Delta x_t}{\partial \eta_{1,t}} = 0 \quad , \quad \lim_{\theta\to 0, a\to 1} \frac{\partial y_t}{\partial \eta_{1,t}} = \sigma_\varepsilon.$$

Under these two restrictions, the estimated IRFs are unbiased. The intuition of this result is the following. First, the parameter a controls the severity of the non–fundamentalness problem as shown in section 1.2. The closer is a to unity, the less severe is the non–invertibility problem raised by news shocks (see Sims, 2009, for a similar statement). Second, news shocks must represent most of fluctuations in the underlying economy, as in the fundamental case. Similar results follow when we consider IRF at one and two lags.

Let us now consider the responses of x and y one period after the shock

$$\frac{\partial \Delta x_{t+1}}{\partial \eta_{1,t}} = \frac{\phi\sigma_\varepsilon}{\sqrt{1+\theta}} \quad (> 0) \quad , \quad \frac{\partial y_{t+1}}{\partial \eta_{1,t}} = \frac{a\phi\sigma_\varepsilon}{\sqrt{1+\theta}} \quad (< \sigma_\varepsilon).$$

Notice that the estimated responses are biased, even if we impose $\theta \to 0$ and $a \to 1$. A possible explanation of this result is that the VAR model does not include enough lags. Indeed, inspecting the time series properties of $\upsilon_{1,t}$ and $\upsilon_{2,t}$ in (9) shows that these two errors terms display a sizeable degree of serial correlation. This will be shown in our simulation experiments in section 3. Finally, two periods after the shock, we obtain

$$\frac{\partial \Delta x_{t+2}}{\partial \eta_{1,t}} = \frac{\phi^2 \sigma_\varepsilon}{\sqrt{1+\theta}} \quad (< \sigma_\varepsilon) \quad , \quad \frac{\partial y_{t+2}}{\partial \eta_{1,t}} = \frac{(a\phi)^2 \sigma_\varepsilon}{\sqrt{1+\theta}} \quad (> 0).$$

When $\theta \to 0$ and $a \to 1$, the SVAR model yields biased estimates. Our previous statement about lag truncation prevails as we will show in the next section.

Short–Run restrictions Applying now a short–run restriction on the estimated VAR(1) model yields

$$\tilde{S} = \frac{\sigma_\varepsilon}{\sqrt{1+\theta-\phi}} \begin{pmatrix} 0 & 1+\theta-\phi \\ \sqrt{(\phi+a^2)\theta + a^2(1-\phi)} & -a\phi \end{pmatrix}.$$

The IRFs are given by the elements of the matrix $A^i\tilde{S} = (a\phi)^{i-1}A\tilde{S}$ for $i = 1, 2, \ldots$. The identification strategy imposes a zero response of x on impact, by construction equal to the true one. The response of y on impact is (in parentheses, we report the true responses)

$$\frac{\partial y_t}{\partial \tilde{\eta}_{1,t}} = \sigma_\varepsilon \psi \quad (> a\sigma_\varepsilon) \qquad \text{and} \qquad \lim_{\theta \to 0} \frac{\partial y_t}{\partial \tilde{\eta}_{1,t}} = |a|\sigma_\varepsilon \quad .$$

where $\psi = \sqrt{((\phi+a^2)\theta + a^2(1-\phi))/(1+\theta-\phi)}$. When $\theta \to 0$ and $a > 0$, a short–run restriction allows to perfectly uncover the true impact responses of x and y to a news shock. This is in contrast with the long–run identification scheme.

One period after the shock, we obtain the following IRFs

$$\frac{\partial \Delta x_{t+1}}{\partial \tilde{\eta}_{1,t}} = \phi \sigma_\varepsilon \psi \quad (> 0) \quad , \quad \frac{\partial y_{t+1}}{\partial \tilde{\eta}_{1,t}} = a\phi\sigma_\varepsilon\psi \quad (\lesseqgtr \sigma_\varepsilon).$$

As in the long–run restriction case, the estimated response with a short–run restriction is biased and polluted by the two key parameters θ and a. Finally, two periods after the shock, Δx and y respond as follow

$$\frac{\partial \Delta x_{t+2}}{\partial \tilde{\eta}_{1,t}} = a\phi^2 \sigma_\varepsilon \psi \quad (\lesseqgtr \sigma_\varepsilon) \quad , \quad \frac{\partial y_{t+2}}{\partial \tilde{\eta}_{1,t}} = a^2\phi^2\sigma_\varepsilon\psi \quad (> 0).$$

When $\theta \to 0$ and $a \to 1$, the estimated responses are biased, as in the case with a long–run restriction.

The Correlation Diagnostic Test Using the two previous SVARs, we can determine the relation between the two identified shocks $\eta_{1,t}$ and $\tilde{\eta}_{1,t}$ and the structural shocks of the DGP:

$$\begin{aligned} \eta_{1,t} &= \frac{1}{\sigma_\varepsilon(1+\theta)} \left\{ u_t + \frac{a\phi}{1-a\phi}\varepsilon_{t-1} + \frac{1-\phi}{1-a\phi}\varepsilon_{t-2} \right\} \\ \tilde{\eta}_{1,t} &= \frac{a\phi}{\sigma_\varepsilon} \sqrt{\frac{1+\theta-\phi}{(\phi+a^2)(1+\theta)-1}} \left\{ \frac{1}{1+\theta-\phi}u_t - \varepsilon_{t-1} + (1-\phi)\varepsilon_{t-2} \right\} \end{aligned}$$

Direct calculations yield

$$corr(\eta_{1t}, \tilde{\eta}_{1t}) = \frac{\gamma}{\lambda}$$

where

$$\gamma = [a\phi + (1-\phi)^2]\theta + (1-\phi)^2$$

$$\lambda = (1+\theta)(1-a\phi)\sqrt{(\theta+1-\phi)[(\phi+a^2)(1+\theta)-1]}$$

Taking limits at relevant values shows that

$$\lim_{\theta\to 0, a\to 1} \gamma = \frac{1}{4} \quad , \quad \lim_{\theta\to 0, a\to 1} \lambda = \frac{1}{4} \quad ,$$

which leads to

$$\lim_{\theta\to 0, a\to 1} corr(\eta_{1t}, \tilde{\eta}_{1t}) = 1.$$

The condition $\theta \to 0$ ($\sigma_\varepsilon \gg \sigma_u$) is necessary to get a correlation diagnostic test close to unity. Interestingly, this corresponds to a situation where the two identification schemes yield dynamic responses close to the true ones. However, the ability of these two restrictions to properly uncover the true responses hinges also on the strength of the forward–looking behavior of the endogenous variable y. Higher forward–looking dimension ($a \to 1$) allows to dilute the negative effects of non–fundamentalness on the estimated dynamics.

3 Simulation Experiments

We now use the model (4) to simulate artificial data, over which we estimate SVARs with long–run and short–run restrictions. To compute artificial time–series of the variables of interest, we draw $N = 1000$ independent random realizations of the innovations u_t and ε_{t-q}. For given values of a, σ_u and σ_ε, we compute $N = 1000$ equilibrium paths for Δx_t and y_t. These simulations are conducted for different values of q, *i.e* $q = [1, 2, 4]$. In all experiments, the sample size is equal to 250 time periods, as it is usually the case with actual data. In order to reduce the influence of initial conditions, the simulated sample includes 1000 initial points which are subsequently discarded before the estimation of VAR models. For each draw, the number of lags in VAR models is set to $p = [1, 2, 4, 8]$, a range of values typically used in empirical studies. In order to evaluate the relative performance of the different approaches, we compute the equally weighted (over horizons) cumulative Mean Absolute Error (MAE) and Root Mean Square Error (MSE).[10] A summary of simulation results is reported in Table 1. This table contains the MAE and the RMSE for different values of q, $\sigma_u/\sigma_\varepsilon \equiv \sqrt{\theta}$, a and p. It also includes the correlation between the two news shocks, identified using either a long–run restriction or a short–run restriction. Figures 1–10 illustrates the results included in Table 1.

We investigate several issues previously raised in the analytical part of the analysis: long–run versus short–run restrictions, the relative size of shocks, the number in lags in VARs, the correlation diagnostic test and the forward–looking dimension.

[10]The equally weighted cumulative MAE and RMSE at horizon h is defined as $(1/h)\sum_{i=0}^{h} mae_i$ and $(1/h)\sum_{i=0}^{h} mes_i$, respectively, where $mae_i = (1/N)\sum_{j=1}^{N} |irf_i(model) - irf_i(svar)^j|$ and $mse_i = ((1/N)\sum_{j=1}^{N}(irf_i(model) - irf_i(svar)^j)^2)^2$ represent the MAE and the MSE at horizon i, respectively. $irf_i(model)$ denotes the model's impulse response and $irf_i(svar) = (1/N)\sum_{j=1}^{N} irf_i(svar)^j$ the mean of impulse responses over the N simulation experiments obtained from SVARs.

Long–run versus Short–run restrictions The SVAR model with a short–run restriction performs better than the one with a long–run restriction. The MAE and the RMSE for both x and y are always smaller. This is especially verified when the standard–error of the expected and unexpected shock are equal ($\sigma_u/\sigma_\varepsilon = 1$). In this case, the identification assumption with a long–run restriction is strongly violated since two shocks have a long run effect of the same size on the variable x. This result holds, even if the DGP does not raise any non–fondamentalness issue. To see this, consider the case where $q = 1$. In this situation, the DGP is invertible and admits an exact VAR(1) representation, independent from the value of a. The first part of Table 1 includes the simulation results with $q = 1$ for different values of p and $\sigma_u/\sigma_\varepsilon$ and Figures 1 and 2 report the dynamic responses of x and y. When $\sigma_u = \sigma_\varepsilon$, a SVAR model with long–run restriction over–estimates the contribution of the news shock to x since this identification scheme cannot separate the two permanent shocks while it under–estimates the response of y (see the first panel of Figure 2). At the same time, the short–run restriction almost perfectly uncovers the news shock and its dynamic effect of x and y. These results are consistent with our theoretical findings in the fundamental case. Indeed, using long–run restriction the econometrician identifies a weighted average of the two permanent shocks

$$\eta_{1,t} = \frac{\sigma_u}{\sqrt{\sigma_u^2 + \sigma_\varepsilon^2}} u_t + \frac{\sigma_\varepsilon}{\sqrt{\sigma_u^2 + \sigma_\varepsilon^2}} \varepsilon_t \quad .$$

When a short–run restriction is applied to SVAR, the news shock is correctly identified since $\tilde{\eta}_{1,t} = \varepsilon_t$. This can be easily seen from Figures 1 and 2: the IRFs obtained using a short–run restriction mimic very well those of the DGP while IRFs obtained from a long–run restriction entail a bias. Consequently, the MAE et RMSE are smaller with a short–run restriction. When $\sigma_u << \sigma_\varepsilon$, the discrepancy between the two approaches is strongly reduced because the weight allocated to the news shocks is higher and then a SVAR with long–run restriction is able to properly identify the news shock.

The relative size of shocks When σ_u decreases (relative to σ_ε), the MAE and RMSE of the two SVARs decrease. This is especially true when we consider non–fundamental cases ($q > 1$) and the SVAR model with a long–run restriction. However, this is the case only when the VAR model includes a sufficient number of lags (see Table 1 and more specifically figures 8 and 9). Indeed, when $q = 1$, the error (MAE and RMSE) increase with $\sigma_u/\sigma_\varepsilon$ for any selected lags $p \in [1, 8]$. Conversely, when q takes larger values (say, 2 and 4), a small number of lags ($p < q$) does not imply a decrease in the estimation error. For example, when $q = 4$, selected $p = 1$ or $p = 2$, small ratio $\sigma_u/\sigma_\varepsilon$ is not associated with small estimation errors. However as the number of lags in the VAR model is such that $p \geq q$ the error monotonically decreases with the ratio $\sigma_u/\sigma_\varepsilon$. In such a case, the correlation between the two identified news shocks increases and tends to unity for small $\sigma_u/\sigma_\varepsilon$. These results confirm our previous analytical findings that a small value of $\sigma_u/\sigma_\varepsilon$ (or equivalently $\theta \to 0$) is not sufficient to obtain consistent estimated of the true dynamic responses.

The number of lags in VARs Our previous analytical results are obtained under the strong assumption that a VAR(1) model accurately represents the dynamics of x and y as implied by the

DGP. So, we remain silent about the lag–truncation bias in VAR models. Simulation experiments bring insightful information on this issue. Increasing the number of lags in the VAR model allows to reduce the error (MAE and RMSE). This is especially true when q takes larger value. The first panel of Figures 3 and 4 reports the dynamic responses of x and y when $q = 2$ for $\sigma_u/\sigma_\varepsilon = 1$. When a VAR model with $p = 1$ is estimated under this DGP, SVARs (either with long–run and short–run restrictions) face some troubles in reproducing the true IRFs. The problem is more pronounced when these dynamic responses are identified from a SVAR model with a long run restriction, which implies a biased response of x on impact. Estimating a VAR with a number of lags $p < q$ induces a lag–truncation bias (see Ravenna, 2007). Given that the VAR(p) model does not well approximate the true DGP, the econometrician will face a sizeable bias when omitting lags. Increasing the number of lags helps at reducing the bias essentially when a short–run restriction is used, whereas the accuracy of the SVAR with a long–run restriction slightly improves. This finding is reinforced when we consider $q = 4$. The estimated dynamic responses of x and y are reported in the first panel of Figures 5 and 6. Again, the two SVARs poorly mimic the true IRFs when $p < q$. For $p = 1$, the two SVAR models behaves similarly, except on impact. As p increases, the SVAR model with a short–run restriction more accurately reproduces the true IRFs than the SVAR model with a long–run restriction. The latter still implies an immediate response of x. However, for q small, *i.e.* $q = 1$, increasing p does not improve the accuracy of SVARs, since the DGP admits a VAR(1) representation. In this situation only, including a larger number of lags leads to an over–parametrization of the VAR model and affects the precision (RMSE) of the estimated responses. Furthermore, gains from increasing the number of lags in a VAR model are more substantial for small ratios of $\sigma_u/\sigma_\varepsilon$ as witnessed by the second panels of Figures 3–6.

The correlation diagnostic test An interesting result emerging from these simulations is that the correlation diagnostic test can lead to spurious conclusions. Such misleading conclusions are drawn when the number of lags p is too small regarding the length of expected shock q. To see this, let us consider again the case where $q = 2$ and $\sigma_u = \sigma_\varepsilon$. When $p = 1$, the correlation is 0.7079, whereas it is equal to 0.6009 when $p = 2$ (see Table 1). This finding is illustrated in Figure 7. This figure reports the value of the correlation between the two identified news shock, when $\sigma_u/\sigma_\varepsilon$ varies on the $[0.01, 1]$ interval. When $q = 1$, the correlation unambiguously decreases with $\sigma_u/\sigma_\varepsilon$. However, when $q = 2$ or $q = 4$, this correlation is not monotonic with $\sigma_u/\sigma_\varepsilon$ for p too small regarding the selected value of q. Let us consider the case where $q = 4$. The last panel of Figure 7 shows that the correlation displays an hump–shaped pattern when $p = 1$ and $p = 2$. This finding is explained by Figures 8 and 9, that report the RMSE of x for different values of $\sigma_u/\sigma_\varepsilon$ and the two identification schemes. When $p = q = 1$, increasing $\sigma_u/\sigma_\varepsilon$ unambiguously implies larger errors in the estimated responses (see the first panel of this figure). However, when $q = 2$ or $q = 4$, this is not the case when $p < q$. The RMSE is only monotonic with $\sigma_u/\sigma_\varepsilon$ when the VAR model includes a sufficient number of lags. These results mean that the correlation diagnostic test is meaningful only when the VAR(p) model is properly specified with respect to anticipation horizon of news shock.

The forward–looking dimension The effect of a on the reliability of SVARs is rather small when the number of lags is appropriately chosen. Conversely, when $p < q$, the value of a can deeply affect the correlation between the innovations (see the Table 1). This finding is illustrated by Figure 10. When $p = 1$ and $q = 4$, increasing a leads to a larger correlation between the two innovations. Conversely, when $p = q = 4$, the effect of a on this correlation is very small. Similar findings also hold for the errors (MAE, RMSE) of the estimated responses. Notice that these findings with $q = 4$ echo those obtained from our analytical results with $q = 1$. Indeed, we have shown that when $q = 1$, all the results are independent from the values of a. Moreover, our simulation experiments confirm these asymptotic results in finite sample[11]

4 Conclusion

This paper inspects under which conditions, SVARs can be used to properly identify news shocks. Indeed, the presence of news shocks in the economy may induce a non–fundamental time series representation of the data. Such non fundamentalness corrupts the identification of shocks using SVARs. Then the estimated IRFs obtained from long–run and short–run restrictions entail a sizeable bias. Both analytical and simulation based results shows that the anticipation horizon, the forward–looking dimension, the number of lags in VARs and the relative size of news shocks in the economy matter for the reliability of SVARs in identifying news shocks and their dynamic effects.

[11]This is why we do not report simulation results for different values of a when $q = 1$ in Table 1.

References

Alessi, L., Barigozzi, M. and M. Capasso, (2008) “A review of nonfundamentalness and identification in Structural VAR models”, ECB *Working Paper* n 922, European Central Bank.

Barsky, R. and E. Sims (2010) “News Shocks and Business Cycles”, *manuscript*, University of Notre Dame.

Beaudry, P. and F. Portier, (2005) “The news view of ecconomic fluctuations: evidence from aggregate Japanese data and sectoral US data”, *Journal of the Japanese and International Economics*, 191, 635-652.

Beaudry, P. and F. Portier, (2006) “Stock prices, news and economic fluctuations”, *American Economic Review*, 96, 96(4): pp. 1293–1307.

Beaudry, P. and B. Lucke, (2009) “Letting different views about business fluctuations compete”, NBER *Working Paper* 14950.

Blanchard, O.J. and D. Quah (1989) “The Dynamic Effects of Aggregate Demand and Supply Disturbances”, *American Economic Review*, 79(4), pp. 655–673.

Christiano, L., Eichenbaum, M. and R. Vigfusson, (2006) “Assessing Structural VAR models”, NBER Macroeconomics Annual, Vol. 21, pp. 1–105

Davis, J., (2007) “News and the Term Structure in General Equilibrium”, manuscript

Fève, P., J. Matheron, and J-G. Sahuc, (2009) “On the dynamic implications of news shocks”, *Economic Letters*, 102, pp. 96–98

Fujiwara, I. and M. Shintani, (2008) “Can News be a Major Source of Aggregate Fluctuations? A Bayesian DSGE approach”, IMES *Working Paper* 2008-E-16.

Hamilton, J. (1994) *Time Series Analysis.* Princeton University Press.

Khan, H. and J. Tsoukalas, (2009) “The Quantitative Importance of News Shocks in Estimated DSGE Model”, Carleton Economic Papers 09-07, Carleton University, Department of Economics.

Leeper, E., T. Walker, and S. Yang, (2009) “Fiscal Foresight and Information Flows”, NBER *Working Paper* 14630.

Mertens, K. and M. Ravn, (2009a) ”Understanding the Aggregate Effects of Anticipated and Unanticipated U.S. Tax Policy Shocks”, forthcoming *Review of Economic Dynamics.*

Mertens, K. and M. Ravn, (2009b) ”Measuring the Impact of Fiscal Policy in the Face of Anticipation: a Structural VAR Approach”, *manuscript.*

Ramey, V. (2009) “Identifying Government Spending Shocks: It’s All in the Timing”, NBER *Working Paper* 15464, forthcoming *Quarterly Journal of Economics.*

Ravenna, F. (2007) "Vector Autoregressions and Reduced Form Representations of DSGE Models", *Journal of Monetary Economics*, vol. 57(7), pp. 2048–2064.

Schmitt–Grohé, S. and M. Uribe, (2008) "What's news in business cycles", NBER *Working Paper* 14215.

Sims, C. (1980) "Macroeconomics and Reality", *Econometrica*, 48, pp. 1–48.

Sims, E. (2009) "Non invertibilities and Structural VARS", *manuscript*, University of Notre Dame

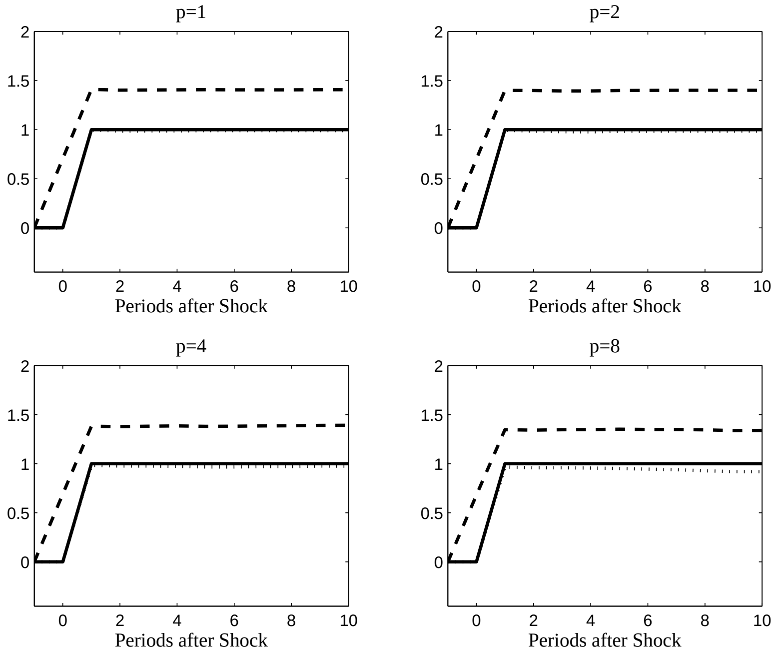
Table 1: Simulation Results

				MAE				RMSE				
				Long–Run		Short–Run		Long–Run		Short–Run		
q	$\sigma_u/\sigma_\varepsilon$	a	p	x	y	x	y	x	y	x	y	Corr
1	1	0.90	1	0.6552	0.2743	0.0941	0.0677	0.4489	0.1033	0.0360	0.0146	0.4978
			2	0.6500	0.3050	0.1234	0.0980	0.4498	0.1276	0.0431	0.0212	0.4990
			4	0.6398	0.3305	0.1766	0.1519	0.4461	0.1511	0.0575	0.0352	0.5007
			8	0.6154	0.3593	0.2389	0.2104	0.4257	0.1822	0.0817	0.0562	0.4913
1	0.1	0.90	1	0.2530	0.2442	0.0748	0.0747	0.0835	0.0827	0.0167	0.0166	0.9859
			2	0.2722	0.2643	0.1051	0.1050	0.0968	0.0967	0.0233	0.0233	0.9824
			4	0.2928	0.2846	0.1584	0.1579	0.1132	0.1136	0.0375	0.0372	0.9750
			8	0.3154	0.3062	0.2155	0.2134	0.1319	0.1329	0.0591	0.0577	0.9569
2	1	0.90	1	0.6561	0.4045	0.2401	0.2549	0.4666	0.2238	0.0579	0.0556	0.7079
			2	0.6674	0.3665	0.2484	0.2354	0.4801	0.1830	0.1017	0.0844	0.6009
			4	0.6569	0.3542	0.2646	0.2357	0.4724	0.1707	0.1047	0.0751	0.5685
			8	0.6353	0.3554	0.3089	0.2617	0.4525	0.1757	0.1283	0.0860	0.5433
		0.99	1	0.6630	0.3889	0.2494	0.2543	0.4766	0.2081	0.1242	0.1088	0.6611
			2	0.6648	0.3610	0.2501	0.2289	0.4765	0.1768	0.1027	0.0799	0.5951
			4	0.6554	0.3441	0.2690	0.2306	0.4703	0.1620	0.1078	0.0724	0.5523
			8	0.6347	0.3481	0.3159	0.2617	0.4516	0.1713	0.1337	0.0863	0.5193
	0.1	0.90	1	0.3612	0.3354	0.2412	0.2264	0.1605	0.1534	0.1049	0.0985	0.9797
			2	0.2655	0.2571	0.1635	0.1632	0.6648	0.3610	0.2501	0.2289	0.9914
			4	0.2766	0.2656	0.2082	0.2073	0.1055	0.1049	0.0595	0.0591	0.9819
			8	0.2912	0.2774	0.2559	0.2532	0.1162	0.1155	0.0823	0.0804	0.9631
		0.99	1	0.3527	0.3353	0.2375	0.2320	0.1572	0.1537	0.1028	0.0993	0.9927
			2	0.2646	0.2559	0.1669	0.1667	0.0959	0.0952	0.0454	0.0453	0.9909
			4	0.2764	0.2651	0.2129	0.2121	0.1054	0.1048	0.0622	0.0618	0.9810
			8	0.2911	0.2772	0.2622	0.2594	0.1162	0.1157	0.0863	0.0844	0.9621
4	1	0.90	1	0.6545	0.4588	0.4173	0.4274	0.4880	0.2852	0.2278	0.2163	0.8221
			2	0.6698	0.4545	0.3677	0.3752	0.5057	0.2823	0.1972	0.1876	0.7840
			4	0.7045	0.4743	0.4250	0.4005	0.5467	0.2904	0.2144	0.1889	0.6720
			8	0.6686	0.4072	0.3920	0.3331	0.5010	0.2193	0.1949	0.1367	0.6257
		0.99	1	0.6900	0.4414	0.4602	0.4473	0.5379	0.2614	0.2735	0.2273	0.7061
			2	0.6832	0.4255	0.4066	0.3794	0.5259	0.2485	0.2323	0.1864	0.6972
			4	0.6932	0.4522	0.4414	0.3990	0.5302	0.2633	0.2280	0.1877	0.6534
			8	0.6613	0.3620	0.4134	0.3236	0.4906	0.1786	0.2143	0.1312	0.5701
	0.1	0.90	1	0.4657	0.4587	0.4282	0.3991	0.2449	0.2531	0.2200	0.2083	0.8937
			2	0.4309	0.3960	0.3780	0.3452	0.2161	0.2055	0.1875	0.1701	0.9417
			4	0.2412	0.2320	0.2703	0.2697	0.0828	0.0820	0.0930	0.0928	0.9941
			8	0.2465	0.2289	0.2801	0.2773	0.0870	0.0842	0.0993	0.0977	0.9746
		0.99	1	0.4630	0.4401	0.4428	0.4293	0.2476	0.2408	0.2365	0.2226	0.9843
			2	0.4129	0.3885	0.3764	0.3642	0.2078	0.2004	0.1886	0.1793	0.9878
			4	0.2354	0.2257	0.2888	0.2884	0.0810	0.0800	0.1058	0.1056	0.9932
			8	0.2447	0.2257	0.3015	0.2991	0.0864	0.0836	0.1145	0.1129	0.9722

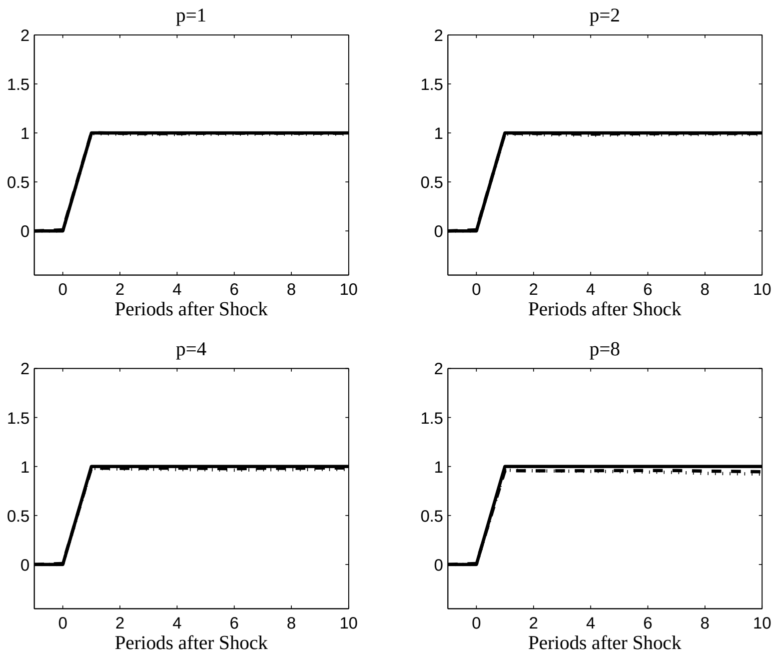
Note: MAE: equally weighted (over horizons) cumulative Mean Absolute Error; RMSE: equally weighted (over horizons) cumulative Root Mean Square Error. Long–Run: SVAR with a long–run restriction; Short–Run: SVAR with a short–run restriction; the sample size is equal to 250 time periods; The simulated sample includes 1000 initial points which are subsequently discarded before the estimation of VAR models; The selected horizon for IRFs is 11. 1000 Monte-Carlo experiments.

Figure 1: IRFs of the variable x_t ($q = 1$)

(a) $\sigma_u = 1$



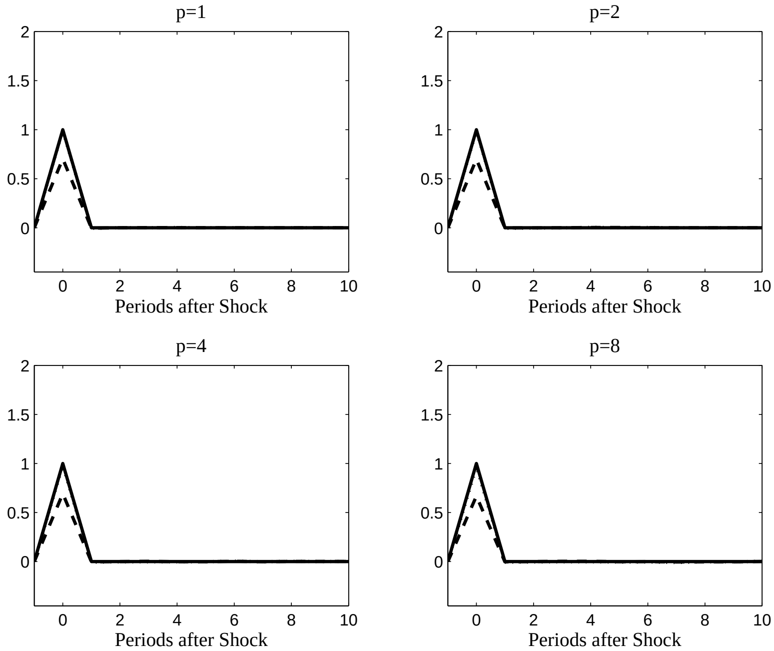
(b) $\sigma_u = 0.1$



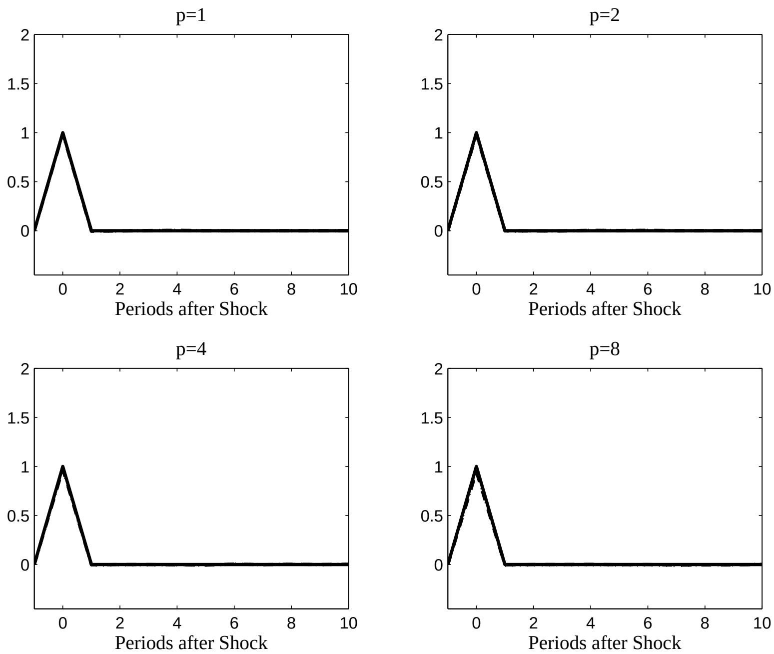
Note: The solid line corresponds to the true IRF. The dashed line corresponds to the IRF with a long–run restriction. The dotted line corresponds to the IRF with a short–run restriction. The results are obtained from our benchmark calibration. The selected horizon for IRFs is 11. The size of the sample is equal to 250. The average values for IRFs are obtained from 1000 Monte-Carlo experiments.

Figure 2: IRFs of the variable y_t $(q = 1)$

(a) $\sigma_u = 1$



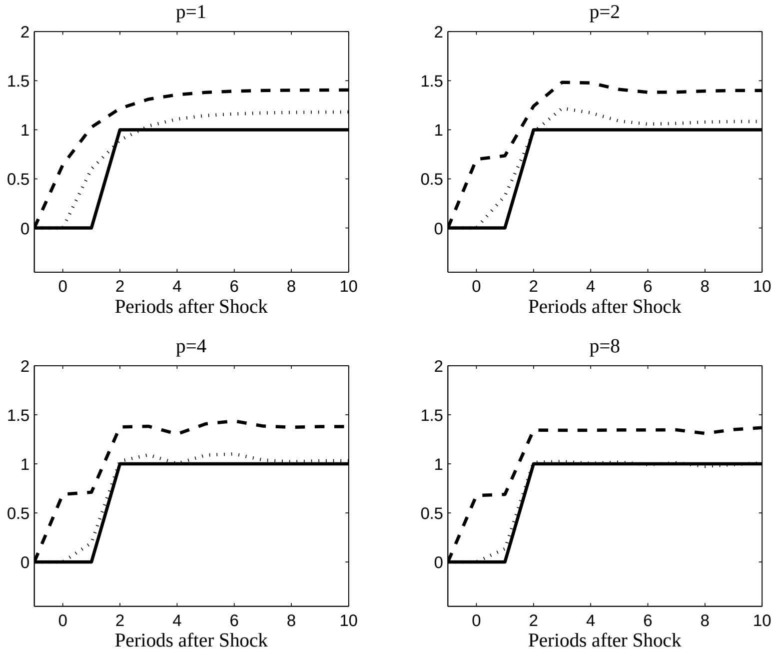
(b) $\sigma_u = 0.1$



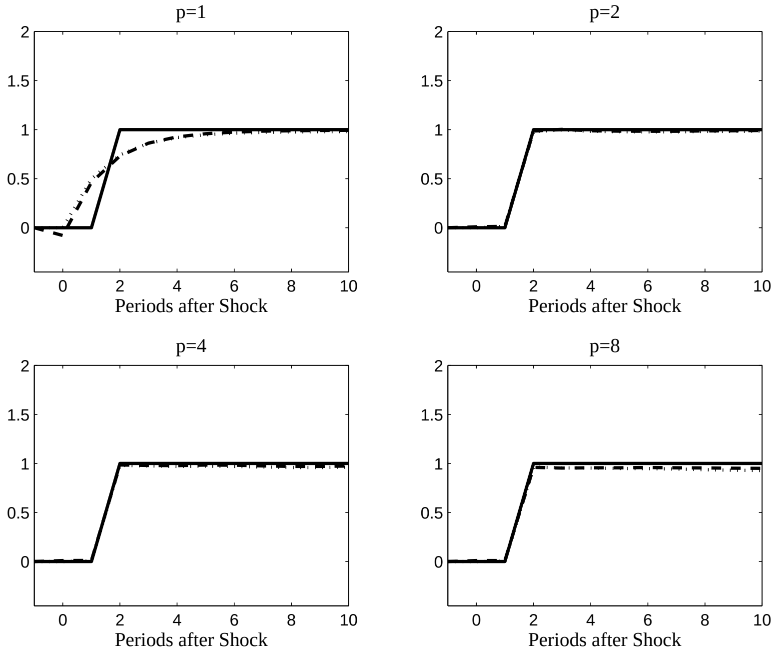
Note: The solid line corresponds to the true IRF. The dashed line corresponds to the IRF with a long–run restriction. The dotted line corresponds to the IRF with a short–run restriction. The results are obtained from our benchmark calibration. The selected horizon for IRFs is 11. The size of the sample is equal to 250. The average values for IRFs are obtained from 1000 Monte-Carlo experiments.

Figure 3: IRFs of the variable x_t $(q = 2)$

(a) $\sigma_u = 1$



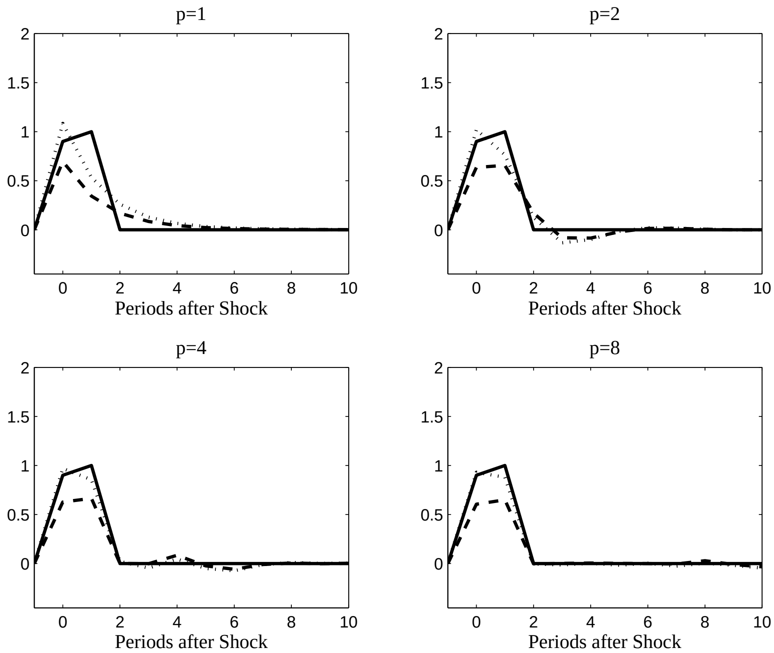
(b) $\sigma_u = 0.1$



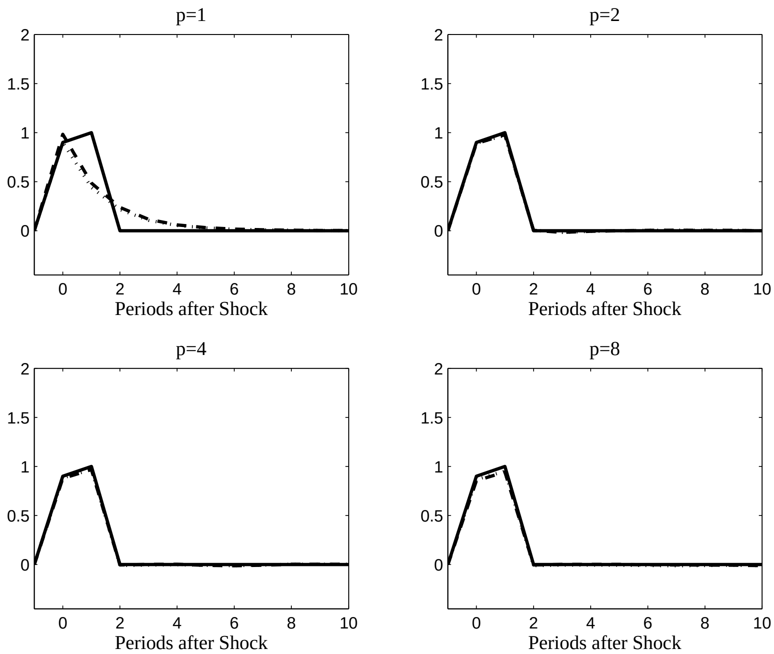
Note: The solid line corresponds to the true IRF. The dashed line corresponds to the IRF with a long–run restriction. The dotted line corresponds to the IRF with a short–run restriction. The results are obtained from our benchmark calibration. The selected horizon for IRFs is 11. The size of the sample is equal to 250. The average values for IRFs are obtained from 1000 Monte-Carlo experiments.

Figure 4: IRFs of the variable y_t ($q = 2$)

(a) $\sigma_u = 1$



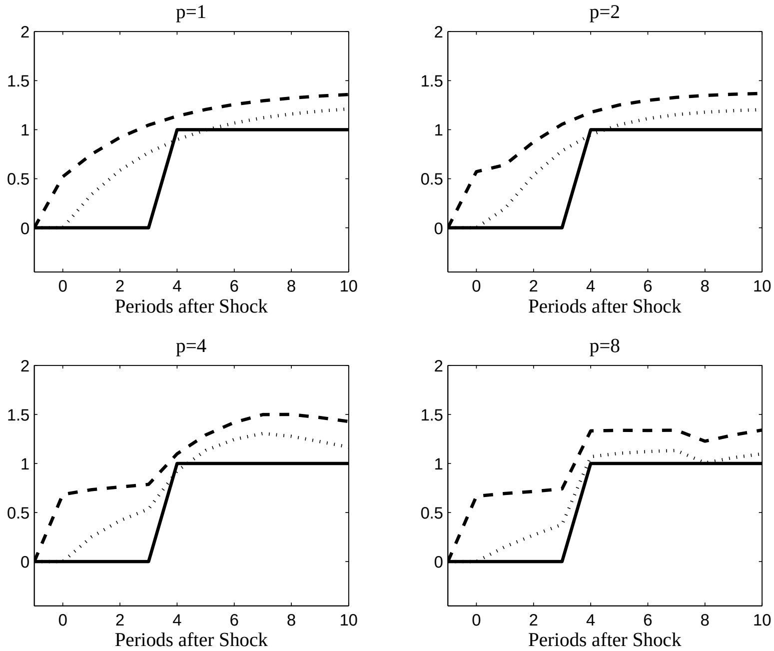
(b) $\sigma_u = 0.1$



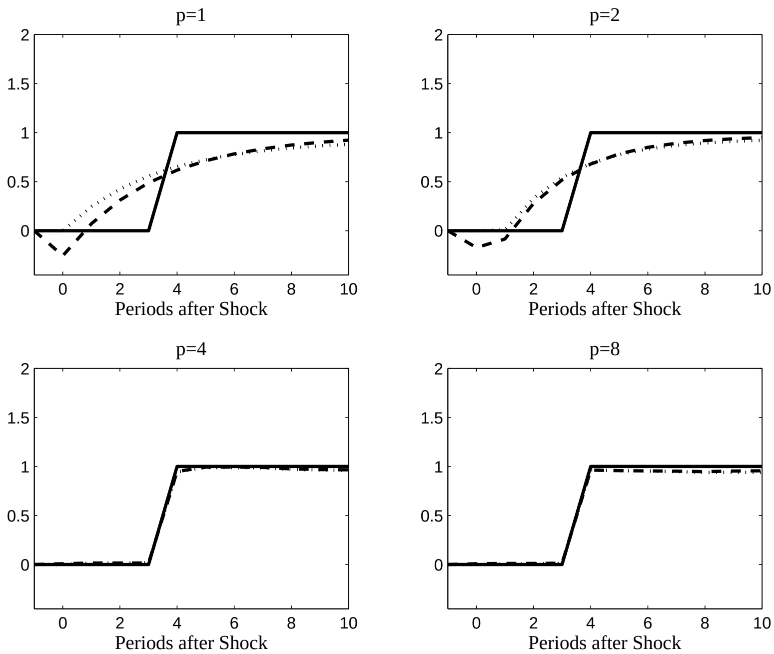
Note: The solid line corresponds to the true IRF. The dashed line corresponds to the IRF with a long–run restriction. The dotted line corresponds to the IRF with a short–run restriction. The results are obtained from our benchmark calibration. The selected horizon for IRFs is 11. The size of the sample is equal to 250. The average values for IRFs are obtained from 1000 Monte-Carlo experiments.

Figure 5: IRFs of the variable x_t $(q = 4)$

(a) $\sigma_u = 1$



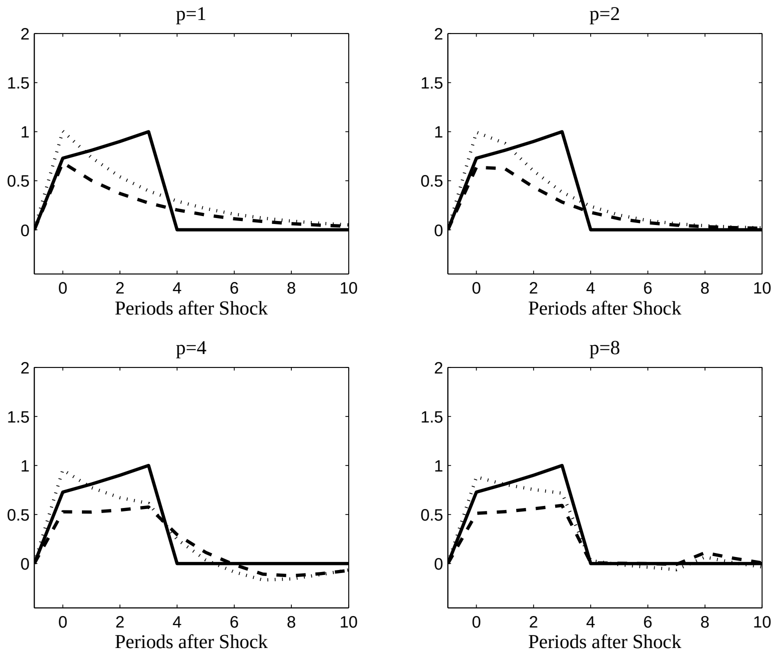
(b) $\sigma_u = 0.1$



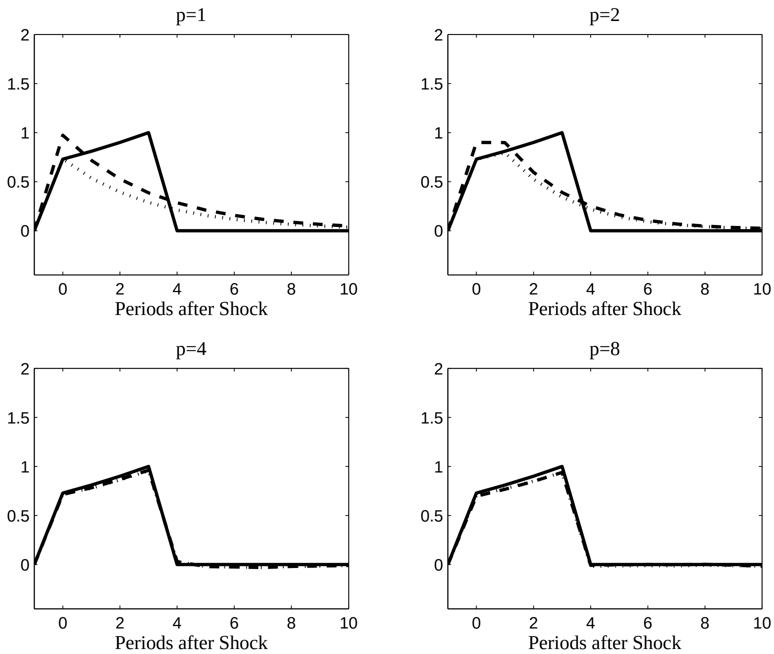
Note: The solid line corresponds to the true IRF. The dashed line corresponds to the IRF with a long–run restriction. The dotted line corresponds to the IRF with a short–run restriction. The results are obtained from our benchmark calibration. The selected horizon for IRFs is 11. The size of the sample is equal to 250. The average values for IRFs are obtained from 1000 Monte-Carlo experiments.

Figure 6: IRFs of the variable y_t ($q = 4$)

(a) $\sigma_u = 1$



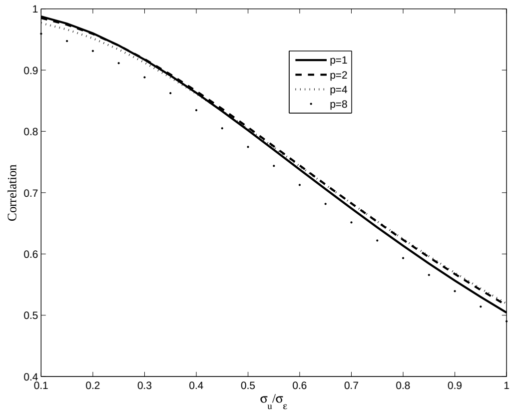
(b) $\sigma_u = 0.1$



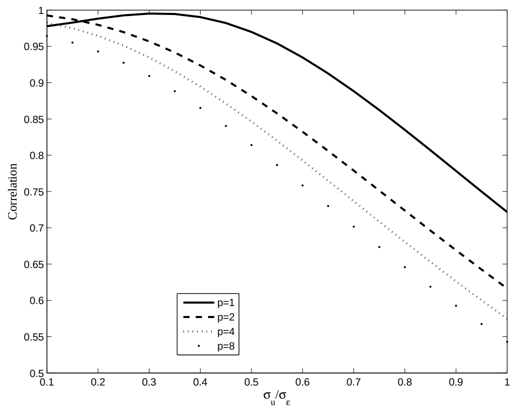
Note: The solid line corresponds to the true IRF. The dashed line corresponds to the IRF with a long–run restriction. The dotted line corresponds to the IRF with a short–run restriction. The results are obtained from our benchmark calibration. The selected horizon for IRFs is 11. The size of the sample is equal to 250. The average values for IRFs are obtained from 1000 Monte-Carlo experiments.

Figure 7: Sensitivity of the Correlation to $\sigma_u/\sigma_\varepsilon$

(a) $q = 1$



(b) $q = 2$



(c) $q = 4$

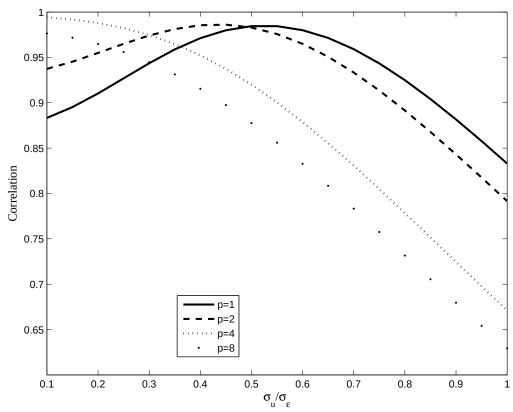
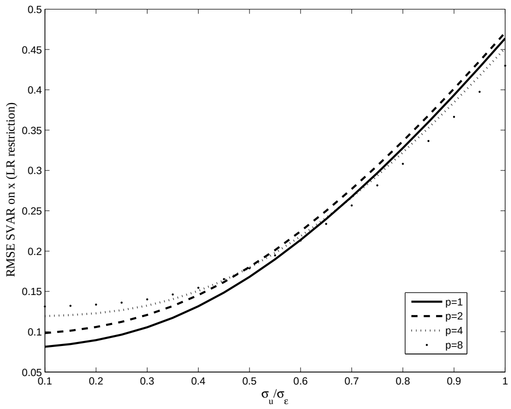
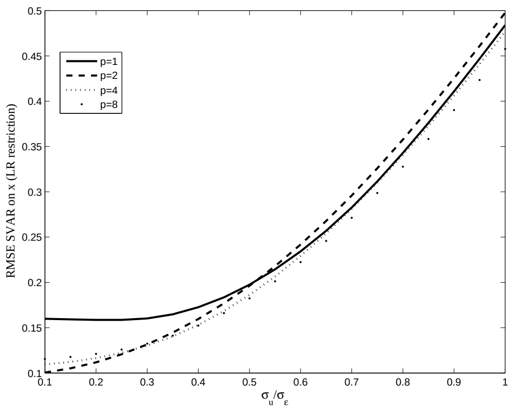


Figure 8: Sensitivity of the RMSE on x (Long–Run Restriction) to $\sigma_u/\sigma_\varepsilon$

(a) $q = 1$



(b) $q = 2$



(c) $q = 4$

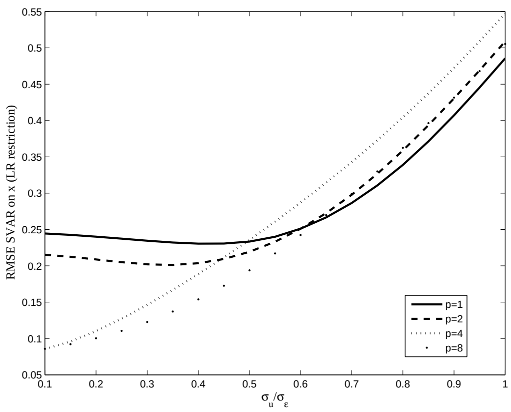
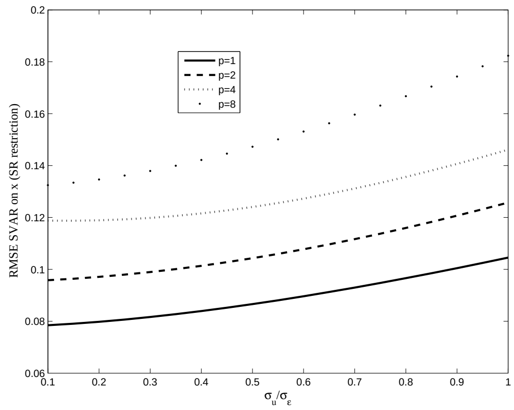
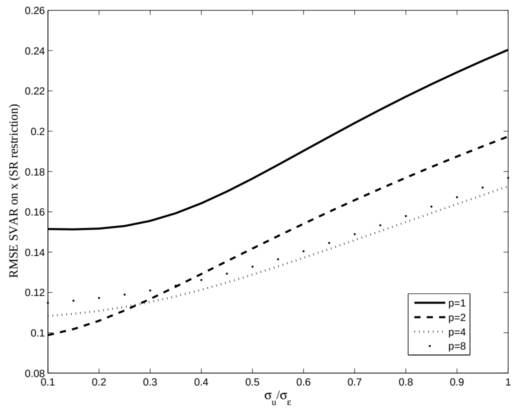


Figure 9: Sensitivity of the RMSE on x (Short–Run Restriction) to $\sigma_u/\sigma_\varepsilon$

(a) $q = 1$



(b) $q = 2$



(c) $q = 4$

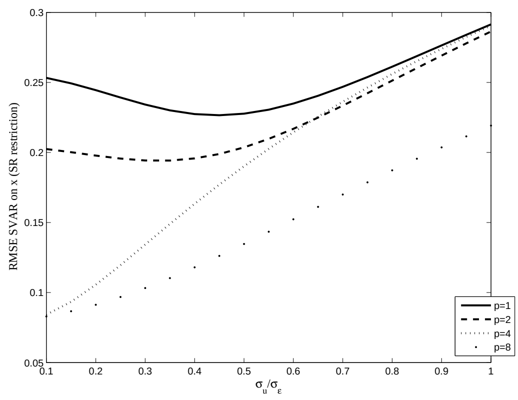
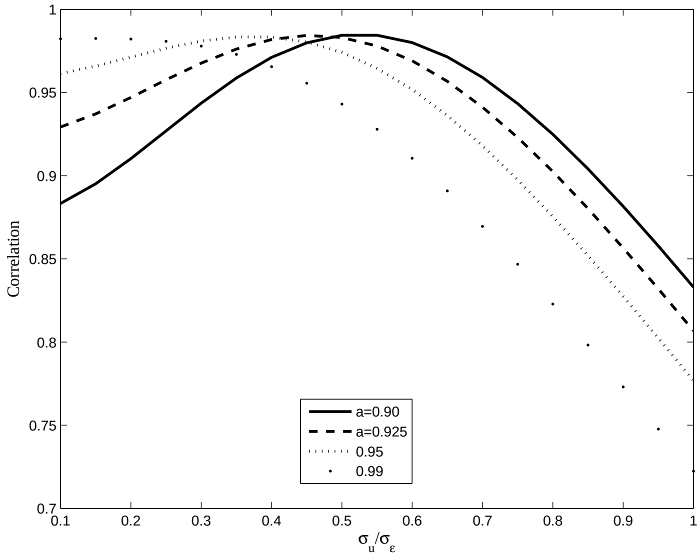


Figure 10: Sensitivity of the Correlation to a

(a) $p = 1, q = 4$



(b) $p = 4, q = 4$