Venture Capital and Knowledge Transfer*

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Abstract

This paper explores a new role for venture capitalists, as knowledge intermediaries. A venture capital investor can communicate valuable knowledge to an entrepreneur, facilitating innovation. The venture capitalist can also communicate the entrepreneur's innovative knowledge to other portfolio companies. We study the costs and benefits of these two forms of knowledge transfer, and their implications for investment, innovation, and product market competition. The model also sheds light on the choice between venture capital and other forms of finance, and the determinants of the decision to seek patent protection for innovations.

Keywords: venture capital, knowledge intermediaries, contracts, innovation, competition, patents.

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1 Introduction

Innovative start-up firms often produce valuable new knowledge. Investors who are closely involved with the start-ups they finance, such as venture capitalists[1], typically have direct access to this innovative knowledge, while outsiders do not. These investors are therefore in a very favorable position to act as knowledge intermediaries, transferring knowledge between the different companies they are involved with[2]. This paper investigates the role of venture capitalists in knowledge transfer. Much of the theoretical literature has explored instead their role as monitors and/or providers of advice and support. We abstract from these to focus on knowledge transfer.

Evidence on knowledge transfer by venture capitalists is difficult to obtain, but several empirical studies suggest it plays an important role. Some direct evidence comes from Gonzalez-Uribe (2013), who finds that venture capitalists diffuse knowledge about their existing patented innovations among their portfolio companies. There is also indirect evidence, highlighting the importance of knowledge transfer in other, similar settings. Helmers et al. (2013) find that information transmission through interlocking boards of directors has a significant positive effect on innovation. Asker and Ljungqvist (2010) show that firms are disinclined to share investment banks with other firms in the same industry, but only when the firms engage in product-market competition (suggesting concern over the possibility of knowledge transfer to competitors)[3].

We develop a theoretical model to study the costs and benefits of knowledge transfer, as well as the implications for investment, firm performance and innovation. The model has an *ex-ante* innovation stage, followed by an *ex-post* commercialization stage. An entrepreneur with an innovative project may develop a valuable innovation at the end of the first stage; the innovation then has to be commercialized in the second stage in order to yield financial returns at the end. We begin by studying the case where the valuable innovation cannot be protected through a patent. We analyze two forms of knowledge transfer by the venture capitalist (VC) who funds the project: *ex ante*, the VC may, by incurring a private cost C, communicate useful knowledge obtained from other firms to the entrepreneur. This *inward* knowledge transfer helps the entrepreneur to develop a valuable innovation. *Ex post*, once the entrepreneur has innovated

[1]Gorman and Sahlman (1989) find that lead venture investors visit each portfolio company an average of 19 times per year and spend 100 hours in direct contact (onsite or by phone) with the company. Sahlman (1990) highlights venture capitalists' involvement with their portfolio companies in a variety of ways, including the recruitment and compensation of key individuals, strategic decisions, and links with suppliers and customers. Bottazzi et al. (2008) provide further evidence of active involvement by venture capitalists and frequent interaction with their portfolio firms. Kaplan and Strömberg (2003) show that venture capitalists often hold seats on the board, as well as substantial voting and control rights.

[2]Many of these will be innovative start-ups, although it is worth noting that venture capitalists also often serve on boards of mature public firms (see Celikyurt et al. (2012)).

[3]Atanasov et al. (2008) find that 47% of a sample of VC-related lawsuits involve allegations of "tunneling" (wrongful transfers of assets, expropriation of profitable opportunities, etc.), suggesting that concern over reputation is not always sufficient to deter such behavior. Knowledge transfer is typically much harder to demonstrate, and hence easier to undertake.

successfully, the VC may communicate this innovative knowledge to other firms. We assume this *outward* knowledge transfer has a beneficial effect on the other companies, yielding a gain, G, for the VC. However, it also reduces the entrepreneur's expected profitability through a competition effect, parameterized by k. In general, the parameters C, G and k can vary across the firms in a VC portfolio, depending on the characteristics of the project and the resulting innovation. For example, some innovations may generate greater positive spillovers than others, affecting G, while the extent to which knowledge sharing leads to erosion of profits through competition may vary with industry and product characteristics, affecting k and C.

We study optimal contracts between the entrepreneur and the VC: depending on parameter values, the two forms of knowledge transfer can emerge as substitutes or complements, with quite different implications for innovation and profitability. Outward knowledge transfer has a direct negative impact on profitability through the competition effect, but also an indirect positive impact because it relaxes the venture capitalist's participation constraint; the first of these channels tends to reduce entrepreneurial effort, while the second tends to increase it. We then explore the entrepreneur's choice between VC and non-VC (no knowledge transfer) finance. The main drawback of VC finance is due to the cost of inducing the VC *not* to transfer knowledge outwards when G is below a critical threshold. When the cost C is relatively high, the trade-off between the two forms of finance is non-monotonic in the benefit G: for low values of G, the entrepreneur is indifferent; for intermediate values, he strictly prefers non-VC finance, while for higher values, VC finance dominates.

In section 4, we go on to study the case of patentable innovations. We allow for some uncertainty over the outcome of patent applications, and for the fact that the patent application process can disclose information to competitors. Our objective in this section is to investigate the determinants of the decision to apply for patent protection. We find that these differ depending on how the firm is financed, with VC-funded firms exhibiting a greater propensity to apply for a patent (holding constant the quality of the innovation). This is not due to fear of expropriation by the VC, but rather to the fact that the VC's role as knowledge intermediary offers protection against the loss associated with expropriation by competitors following information disclosure.

Our results provide a rationale for the empirical evidence showing that VC-funded firms have substantially higher patent counts[4]. In our model, this is due to two effects: first, inward knowledge transfer by the VC increases the probability of a valuable innovation; second, VC-funded firms are more likely to apply for patent protection. Currently available empirical evidence has indeed established a causal impact of venture capital on innovation measured by patent counts[5], but cannot distinguish between these two possibilities. This will require data analogous to that used by Helmers et al. (2013): they are able to exploit the occurrence of an exogenous

[4]See, for example, Graham et al. (2009).

[5]Kortum and Lerner (2000) , Mollica and Zingales (2007), Ueda and Hirukawa (2008).

corporate governance reform and an exogenous change to the patent system in India to identify a positive effect of board interlocks on R&D spending, as well as a separate positive effect on patenting propensity.

While we often focus attention on innovation, our analysis also highlights other important implications of knowledge transfer by venture capitalists. We note one of them here, namely the *pro-competitive* effect on product markets. This needs to be weighed against the (very different) anti-competitive effect of venture capital studied by Cestone and White (2003), suggesting a complex link between venture capital and product market competition - which merits empirical investigation.

The remainder of the paper is organized as follows. We complete this section by discussing the related theoretical literature. Section 2 presents the baseline model. We study the case of innovations that cannot be patented in section 3, and patentable innovations in section 4. Section 5 concludes.

1.1 Relationship to theoretical literature

There is a large theoretical literature on the role of venture capitalists, which focuses primarily on monitoring[6] and advice/support[7]. We add a new role, as knowledge intermediaries. In this respect, the closest papers to ours are Bhattacharya and Chiesa (1995), Ueda (2004) and Yosha (1995). Bhattacharya and Chiesa consider an economy with many industries: in each industry, two rival firms engage in an R&D race. There are two banks in the economy. Bhattacharya and Chiesa compare bilateral financing, in which each bank finances only one of the rivals in each industry, with multilateral financing, in which each bank provides half of the funding of each rival in each industry. Being one of the financiers gives access to any knowledge produced by the firm at the interim stage. At this stage, financiers decide whether to disclose the knowledge produced by one firm to its rival: this is the link with our paper. The setting is completely different though, and the main focus of Bhattacharya and Chiesa is the effect of a commitment to knowledge sharing on firms' *ex-ante* incentives to invest in R&D. Yosha (1995) also studies the choice between bilateral and multilateral financing, under the assumption that the latter entails a lower cost but greater leakage of information to competitors[8,9].

Ueda (2004) explores the trade-off between bank and VC finance under the assumption that venture capitalists, unlike banks, may steal an entrepreneur's idea at the *ex-ante* financing stage

[6] See, for example, Dessí (2005) and Holmstrom and Tirole (1997).

[7] See, among others, Bottazzi et al. (2009),Casamatta (2003), Cestone (2014) , Cumming et al. (2005) , De Bettignies and Brander (2007), Dessí (2010), Hellmann (1998), Jeng and Wells (2000), Lerner and Schoar (2005), Repullo and Suarez (2000, 2004), Riyanto and Schwienbacher (2006), and Schmidt (2003).

[8] Thus higher quality firms, who have more to lose from information leakage, prefer bilateral financing.

[9] See also Bhattacharya and Ritter (1983), who examine the trade-off between information disclosure to competitors and raising finance on better terms on capital markets.

(before the project is undertaken); on the other hand, venture capitalists have greater ability to evaluate projects.[10] We focus instead on knowledge transfer after the project has been funded and undertaken.

A few other papers have studied the choice between venture capital and bank finance, focusing on quite different trade-offs from those examined in our paper. Winton and Yerramilli (2008) assume that venture capitalists have a greater ability to evaluate possible continuation strategies for the firm. A trade-off arises because VCs are also assumed to have a higher cost of capital. Landier (2003) views the choice between VC and bank finance as determined by a hold-up problem: when investors need protection against hold-up by the entrepreneur, venture capital with staged financing is preferred; when the entrepreneur needs protection against hold-up by investors, long-term bank finance is preferred.

2 The Baseline Model

The model has two stages, with three corresponding dates, $t = 0, 1, 2$. All agents (entrepreneur and investors) are assumed to be risk neutral and protected by limited liability.

2.1 Project

Consider an entrepreneur (start-up firm) endowed with an innovative investment project. The project starts with an innovative idea and requires a contractible initial investment I (money) at the beginning of the first stage (date 0). During the first stage, the idea may be developed into a valuable innovation. For example, we can think of the entrepreneur as having an idea for a new product to begin with; he then undertakes some initial production and carries out the tests/trials required to establish that it works well and satisfies appropriate quality standards. If the first stage is successful, the innovation then needs to be commercialized: here the entrepreneur's effort is crucial, key strategic decisions have to be made, new personnel may need to be recruited, and so on[11]. We assume that if the innovation has been developed successfully (at date 1), and in the absence of knowledge transfer (see below), the project will finally succeed at date 2 with probability e, where e captures the entrepreneur's effort during the second stage. Irrespective of the entrepreneur's effort, success is never certain, thus $e < 1$. If the initial innovative idea fails to be developed into a valuable innovation[12], the project's success probability is reduced;

[10] See also Biais and Perotti (2008), who study an entrepreneur's decision to hire experts when different forms of expertise are valuable but experts may steal a good idea, and Hellmann and Perotti (2011), who examine the costs and benefits of circulating initially incomplete ideas (completion versus appropriation).

[11] We focus here on entrepreneurs, who will manage the business and try to make it succeed, rather than pure inventors, who may prefer to exit as soon as they have developed a valuable innovation.

[12] For expositional convenience, we will refer to this as the "no innovation" outcome.

for simplicity, we assume it is equal to zero. If the project succeeds at date 2, it yields verifiable returns R; if it fails, it yields nothing ($R > 0$).

2.2 Entrepreneur

The entrepreneur has no initial monetary wealth, and needs to raise finance from outside investors. If he is able to secure outside funding and undertake the project (and absent knowledge transfer, see below), he develops a successful innovation with probability π. He then chooses his effort level e, where $0 \leq e < 1$, and the cost of effort is given by $c(e) \equiv \frac{1}{2}e^2$. To make the analysis interesting, we assume that $R > \frac{I}{\pi}$, otherwise the project would not be worth financing (absent knowledge transfer). Given our assumptions about effort, we normalize both R and I to be less than one[13].

2.3 Investors

Investors provide the initial funding I for the project. We assume they are competitive, earning zero expected profits in equilibrium.

In our model the main difference between venture capitalists and other investors lies in the venture capitalists' close connections[14] with their portfolio firms, implying that venture capitalists (henceforth VCs) can transfer knowledge relatively easily between the firms they fund. In particular, we assume that VCs would find it easier to transfer knowledge than any outsiders, including other, arm's length investors, since they interact closely and repeatedly with the entrepreneur, and have privileged access to information throughout the time in which the innovation is being developed. For simplicity, we capture this difference by assuming that VCs, unlike other investors, can transfer knowledge. As we shall see, this brings about both benefits and costs. To focus on the trade-off between these costs and benefits, we abstract from other roles played by venture capitalists, such as monitoring or screening, which have been studied extensively in the theoretical literature on venture capital.

2.4 Knowledge transfer

We consider two forms of knowledge transfer. The VC may communicate valuable knowledge to the entrepreneur (e.g. information acquired through his involvement with other portfolio firms) during the first stage, while the innovation is being developed. We model this as increasing the probability of a valuable innovation, from π to $\pi + \tau$ ($\tau > 0$). The VC incurs a private cost C

[13] In the simplest case and absent knowledge transfer considerations, the socially optimal effort is given by $e^* = \arg\max_e eR - \frac{1}{2}e^2$. The first order condition tells us $e^* = R$. Since we have assumed $e < 1$, we must also have $R < 1$. Given that $R > \frac{I}{\pi}$, this further implies $I < 1$.

[14] See footnote 1.

in doing this (e.g. opportunity cost of time, effort, or lower expected returns on his investment in other portfolio firms). We refer to this as inward knowledge transfer, or *ex ante* knowledge transfer because it occurs in the first stage of our model. The second form of knowledge transfer is outward, or *ex post*, knowledge transfer, whereby the VC transfers knowledge to another firm once the entrepreneur has successfully developed an innovation, in a way that is beneficial to the other firm (and to the VC), but has an adverse effect on the entrepreneur's profitability. We model this as bringing a private benefit of value $G > 0$ to the VC, while decreasing the success probability of the entrepreneur's project from e to ke, with $0 < k < 1$. We shall also refer to this form of knowledge transfer as expropriation.

We assume that the entrepreneur does not observe whether the VC transfers knowledge outward, and that both forms of knowledge transfer cannot be contracted on explicitly. The VC will therefore engage in one, or both, if, and only if, this is in his interest. Finally, we allow for the possibility that, when the VC does not expropriate the entrepreneur's innovative knowledge, some of his competitors may later succeed in doing so (e.g. reverse engineering), or may independently develop an equivalent innovation, which also reduces the success probability of the entrepreneur's project from e to ke. We shall treat these two possibilities together, assuming they occur with probability $\mu > 0$. For expositional convenience we will refer to them simply as expropriation (by competitors).

2.5 Contract design

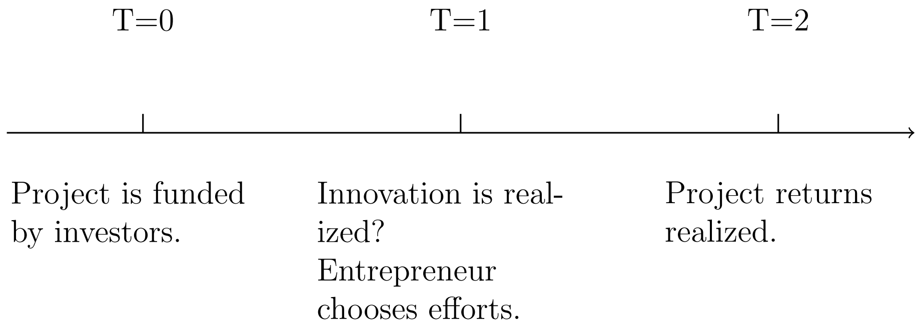
Contracts specify the investor's (venture capitalist's) financial contribution at the beginning (I), and a sharing rule for final returns, R.

2.6 Patent protection

Section 3 focuses on innovative knowledge that cannot, by its very nature, be protected from expropriation by a patent. In section 4 we go on to examine patentable innovations. We assume that, once he has successfully developed an innovation, the entrepreneur can apply for a patent. The application is approved with probability $\beta < 1$.[15] If the application is approved, expropriation is no longer feasible, and knowledge transfer to other firms can only occur through licensing. If the application is rejected, the innovation remains vulnerable to expropriation. Moreover, we allow for a higher probability of expropriation by competitors in this case, $\alpha > \mu$, reflecting leakage of information through the patent application.

[15] We treat β as a parameter of the model, capturing the efficiency of the patent system, and/or the characteristics of the product or process.

Figure 1: Time Line



2.7 Time line

Figure 1 shows the timeline for the baseline model.

3 Non-patentable innovations

We begin by considering innovative knowledge that cannot, by its very nature, obtain patent protection. Section 4 will study patentable innovations. We examine first the case where the entrepreneur raises the required external funding from a non-VC investor, then go on to analyze the case of VC funding.

3.1 Non-VC investor

At date 0, the entrepreneur secures external funding for his project from a non-VC investor. The contract signed with the investor maximizes the entrepreneur's expected payoff, subject to guaranteeing zero expected profits to the investor (since we are assuming that investors are competitive). The contract specifies the investor's capital contribution, I, and the share of final returns going to each party: R_e^N for the entrepreneur, $R - R_e^N$ for the investor. To study the optimal contracting problem, we apply backward induction and start with the effort decision of the entrepreneur at the second stage. The optimal effort level exerted by the entrepreneur is given by[16] $e^N = argmax_e \;\; e(1-\mu+\mu k)R_e^N - \frac{1}{2}e^2$. The first order condition gives us $e^N = \omega R_e^N$, where $\omega = (1-\mu+\mu k)$.

[16] Recall that expropriation is not observed by the entrepreneur: he therefore chooses his effort knowing that other firms will expropriate with probability μ, and that when this happens his probability of success will be reduced to ke.

Thus, the optimal contract solves:

$$
\begin{aligned}
\max_{R_e^N} \quad & \pi[e^N(1-\mu+\mu k)R_e^N - \frac{1}{2}(e^N)^2] \\
s.t. \;\; e^N &= \omega R_e^N \quad (IC_e) \\
\pi e^N \omega (R - R_e^N) &\geq I \quad (PC_i) \\
&\Longleftrightarrow \\
\max \quad & R_e^N \\
s.t. \;\; y &\geq \frac{I}{\pi\omega^2} \\
where \;\; y &= R_e^N(R - R_e^N), \omega = 1-\mu+\mu k
\end{aligned}
$$

When condition ($I \leq \frac{\pi\omega^2 R^2}{4}$) [17] is satisfied[18], the optimal contract is given by $R_e^N \geq \frac{R}{2}$, where R_e^N is the largest root of $\pi\omega^2 R_e^N(R - R_e^N) = I$.

3.2 VC investor

We now study how the contracting problem differs when the entrepreneur obtains external finance from a venture capitalist. As discussed earlier, we focus on one, so far under-explored difference between venture capitalists and other investors: by virtue of their close involvement with portfolio firms, VCs can more easily transfer knowledge between them. From the perspective of the entrepreneur in our model, knowledge transfer can take two forms. The first is inward (ex-ante) knowledge transfer, whereby the VC communicates valuable knowledge to him during the innovation stage. The second is outward (ex-post) knowledge transfer, whereby the VC transfers the entrepreneur's knowledge to other firms once he has developed a valuable innovation, in a way that reduces the entrepreneur's profitability (expropriation). Recall from section 2 that outward knowledge transfer reduces the entrepreneur's success probability from e to ke ($k < 1$), while yielding a private benefit of value G to the venture capitalist.

We model inward knowledge transfer as increasing the probability of a valuable innovation from π to $\pi + \tau$, where $\tau > 0$. The VC incurs a private cost $C > 0$ (e.g. opportunity cost of time, effort, or lower expected returns on his investments in other portfolio firms). Formally, our modeling of inward knowledge transfer is analogous to models of "advice and support" in the theoretical literature on venture capital. We differ from these models in considering also the role of outward knowledge transfer, and the interaction between the two.

When the entrepreneur turns to a VC for external finance, he can choose between four different contracting possibilities. He can design the contract to induce the VC to engage in both types

[17] Note that from PC_i, we have $\pi\omega^2 R_e^N(R - R_e^N) = I$ at the optimum,and the maximum value of $R_e^N(R - R_e^N)$ is $\frac{R^2}{4}$ when R_e^N equals $\frac{R}{2}$. Thus it is never optimal for the entrepreneur to set $R_e^N < \frac{R}{2}$.

[18] If this condition is not satisfied, the entrepreneur cannot raise the funding needed to undertake his project.

of knowledge transfer, only one type, or no knowledge transfer. In what follows, we characterize the optimal contract for each of these possible choices. We then study the entrepreneur's optimal choice.

3.2.1 Outward (ex-post) knowledge transfer, or expropriation

We begin by considering the case where the VC only transfers knowledge outward. This reduces the entrepreneur's probability of success from e to ke, while yielding a private benefit $G > 0$ for the VC. The optimal contract solves the following problem (P1):

$$\begin{aligned} & \max_{R_e^{VN}} \ \pi[ke^{VN}R_e^{VN} - \frac{1}{2}(e^{VN})^2] \\ s.t.\ e^{VN} &= kR_e^{VN} \quad (IC_e) \\ \pi[ke^{VN}(R - R_e^{VN}) + G] &\geq I \quad (PC_{VC}) \\ \tau[ke^{VN}(R - R_e^{VN}) + G] &\leq C \quad (IC_{VC}\ ex\ ante) \\ G + ke^{VN}(R - R_e^{VN}) &\geq \omega e^{VN}(R - R_e^{VN}) \quad (IC_{VC}\ ex\ post) \end{aligned}$$

Comparing this with the equivalent problem for the non-VC investor case, we see that the entrepreneur's incentive constraint, (IC_e), is modified to allow for the fact that the VC always expropriates ex post, reducing the probability of success. On the other hand, the private benefit G relaxes the venture capitalist's participation constraint, (PC_{VC}), making it possible to offer more high-powered monetary incentives to the entrepreneur (higher R_e^{VN}). In addition, we have two new constraints. Since we are considering the case without inward knowledge transfer, it must be the case that the VC has no incentive to transfer knowledge to the entrepreneur; i.e. the private cost C is greater than the expected financial return to the VC (IC_{VC} *ex ante*). Finally, it must be the case that the VC expects a net gain from transferring the entrepreneur's knowledge to competitors (IC_{VC} *ex post*); i.e. the private benefit G is greater than the reduction in the VC's expected return on his investment in the entrepreneur's project.

The solution to P1 is described in the following lemma.

Lemma 1 *Inducing the VC investor to transfer knowledge outwards (to other firms) but not inwards (to the entrepreneur) requires that* $\frac{I}{\pi} \leq \frac{C}{\tau}$ *and* $G \geq \frac{\omega - k}{\omega}\frac{I}{\pi}$. *If* $G \geq \frac{I}{\pi}$, *the optimal contract sets* $R_e^{VN} = R$. *When the inequality holds strictly, the VC will make an additional payment* F *ex ante, beyond* I, *so that the participation constraint holds as an equality; i.e.* $\pi G = I + F$. *If* $G < \frac{I}{\pi}$, *the optimal contract,* R_e^{VN}, *is determined by the largest root of the following equation:*

$$\pi[k^2 R_e^{VN}(R - R_e^{VN}) + G] = I;$$

The problem has a solution only when condition $\pi[\frac{k^2R^2}{4}+G] \geq I$ *is satisfied.*

Proof. See Appendix. ■

The intuition for Lemma 1 is as follows: If the cost C is too low ($\frac{C}{\tau} < \frac{I}{\pi}$), it is not possible to induce the VC to participate (which requires that his expected gain from innovative success be sufficiently large) without transferring knowledge inwards (which increases the probability of innovative success). Similarly, it is not possible to induce the VC to participate and to expropriate ex post if the private benefit from expropriation is too low. The final condition simply requires the investment cost, I, not to be too high relative to the expected benefits from the project, which include its financial returns as well as the venture capitalist's private benfit from expropriation. When the private benefit G and the cost C are not too low, the optimal contract is determined by the participation constraint of the VC.

In order to build intuition for the results to follow, it is helpful to compare Lemma 1 under the assumption that the cost C is very high with the corresponding result for the non-VC investor case. By assumption, we can ignore the possibility of inward knowledge transfer by the VC. The choice between VC and non-VC funding then depends on the magnitude of G, in a non-monotonic way. For $G < \frac{\omega-k}{\omega}\frac{I}{\pi}$, it is not possible to induce the venture capitalist to participate *and* expropriate: in this case, the optimal contract will be the same for the two types of investor, entailing no knowledge transfer, implying that the entrepreneur will be indifferent between VC and non-VC finance. There is then a threshold value of the private benefit, call it $\hat{G}$, ($\hat{G} > \frac{\omega-k}{\omega}\frac{I}{\pi}$) [19], such that for $\frac{\omega-k}{\omega}\frac{I}{\pi} < G < \hat{G}$, the entrepreneur will strictly prefer non-VC finance, while for $G > \hat{G}$, he will strictly prefer VC finance. The intuition for this result is straightforward. For intermediate values of G, the gain from expropriation is not sufficient to compensate for its negative effect on the probability of success. The optimal contract then is the one with the non-VC investor studied earlier. Note that offering the same contract to the VC investor would induce him to expropriate: thus non-VC finance is strictly preferred. For larger values of G, the beneficial effect of the venture capitalist's expected gain from expropriation on his participation constraint dominates, making VC finance optimal for the entrepreneur.

We now relax the assumption that the cost C is very high, and go on to examine the optimal contract designed to induce the venture capitalist to transfer knowledge inward as well as outward.

[19] Note that $\hat{G}$ must be larger than the value of G, call it $\bar{G}$, such that the optimal VC contract with (only) outward knowledge transfer entails the same returns R_e for the entrepreneur as the optimal contract with non-VC finance. This follows from the fact that the entrepreneur's expected utility with non-VC financing is $\frac{1}{2}\pi\omega^2R_e^2$, while for VC financing with outward knowledge transfer it is $\frac{1}{2}\pi k^2 R_e^2$. Since $R_e^N(R-R_e^N) = \frac{I}{\pi\omega^2}$, and $R_e^{VN}(R-R_e^{VN}) = \frac{\frac{I}{\pi}-G}{k^2}$, $\bar{G}$ is the value such that $\frac{I}{\pi\omega^2} = \frac{\frac{I}{\pi}-G}{k^2}$, i.e. $\bar{G} = \frac{\omega+k}{\omega}\frac{\omega-k}{\omega}\frac{I}{\pi}$, which is larger than $\frac{\omega-k}{\omega}\frac{I}{\pi}$. Therefore, $\hat{G} > \frac{\omega-k}{\omega}\frac{I}{\pi}$.

3.2.2 Inward (ex ante) and outward (ex post) knowledge transfer

When the VC transfers knowledge both inwards ("advice") and outwards ("expropriation"), we know that the entrepreneur's effort level e^{VN} is determined by $argmax_e \; keR_e^{VN} - \frac{1}{2}e^2 = kR_e^{VN} \quad (IC_e)$, since the probability of success is reduced to ke by expropriation. The venture capitalist's participation constraint is given by:

$$(\pi + \tau)[ke^{VN}(R - R_e^{VN}) + G] \geq I + C \quad (PC_{VC})$$

reflecting the higher probability of innovation success $(\pi + \tau)$ due to advice, as well as the private benefit G due to expropriation. There are two incentive constraints for the VC. First, he has to be induced to advise ex ante:

$$\tau[ke^{VN}(R - R_e^{VN}) + G] \geq C \quad (IC_{VC} \; ex \; ante)$$

Second, he has to be induced to expropriate ex post:

$$G + ke^{VN}(R - R_e^{VN}) \geq \omega e^{VN}(R - R_e^{VN}) \quad (IC_{VC} \; ex \; post)$$

The optimal contract that induces the venture capitalist to advise ex ante and expropriate ex post is determined by the following optimization problem (P2):

$$\begin{aligned}
&\max_{R_e^{VN}} \; (\pi + \tau)[ke^{VN}R_e^{VN} - \frac{1}{2}(e^{VN})^2] \\
&s.t. \; (IC_e) \\
&(PC_{VC}) \\
&(IC_{VC} \; ex \; ante) \\
&(IC_{VC} \; ex \; post)
\end{aligned}$$

The solution to P2 is provided in Lemma 2.

Lemma 2 *When $G < \frac{\omega - k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, it is not possible to induce the VC to transfer knowledge ex post. When $G \geq max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, the optimal contract is $R_e^{VN} = R$; when $max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\} > G \geq \frac{\omega - k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, the optimal contract is the largest root of the following equation:*

$$k^2 R_e^{VN}(R - R_e^{VN}) + G = max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\};$$

The optimal contract will also entail an ex ante fee when the VC participation constraint is *slack, to ensure the VC earns zero expected rents. The solution holds only when condition* $(\pi+\tau)[\frac{1}{4}k^2R^2+G]\geq I+C$ *is satisfied, otherwise, the problem has no solution.*

Proof. See Appendix. ■

Comparing this with the result for the optimal contract that induces only outward knowledge transfer reveals that when both contracts are feasible (requiring the condition $\frac{I}{\pi}\leq\frac{C}{\tau}$ to hold), the optimal contract which induces both inward and outward knowledge transfer in general offers a lower stake in the project's financial returns to the entrepreneur (lower R_e^{VN}). Specifically, this will be the case when the inequality holds strictly ($\frac{I}{\pi}<\frac{C}{\tau}$). Thus the project's probability of final success, once a valuable innovation has been developed, is lower in this case. On the other hand, the probability of a successful innovation is higher. Essentially, when the cost of advice is relatively high, the entrepreneur has to relinquish a higher share of final returns to the VC to induce him to transfer knowledge inwards: this increases the likelihood of innovating successfully ex ante, but reduces entrepreneurial effort ex post.

When the cost of inward knowledge transfer is relatively low ($\frac{I}{\pi}>\frac{C}{\tau}$), on the other hand, the only feasible contract is the one that induces both types of knowledge transfer.

3.2.3 Inward (ex ante) knowledge transfer, or advice

We now study the optimal contract when the entrepreneur chooses to induce only inward knowledge transfer by the VC. Following a successful innovation, the project's success probability is given by e if there is no expropriation by others (with probability $1-\mu$), and ke otherwise. The entrepreneur's expected probability of success when he chooses his effort level is therefore equal to ωe where $\omega=1-\mu+\mu k$, implying that effort is given by $e^{VN}=argmax_e\ \omega eR_e^{VN}-\frac{1}{2}e^2=\omega R_e^{VN}$ (IC_e). The optimal contract with ex-ante knowledge transfer but no expropriation ex post is determined by the following program (P3):

$$\begin{aligned}
&& \max_{R_e^{VN}}\ (\pi+\tau)[\omega e^{VN}R_e^{VN}-\frac{1}{2}(e^{VN})^2] \\
s.t.\ e^{VN} &=& \omega R_e^{VN}\ \ (IC_e) \\
(\pi+\tau)[\omega e^{VN}(R-R_e^{VN})] &\geq& I+C\ \ (PC_{VC}) \\
\tau[\omega e^{VN}(R-R_e^{VN})] &\geq& C\ \ (IC_{VC}\ ex\ ante) \\
G+ke^{VN}(R-R_e^{VN} &\leq& \omega e^{VN}(R-R_e^{VN})\ \ (IC_{VC}\ ex\ post)
\end{aligned}$$

Comparing this program with those studied earlier, we see that the private benefit G no longer appears in the venture capitalist's participation constraint or in his ex-ante incentive constraint. His ex-post incentive constraint now induces him *not* to transfer knowledge ex post. The solution to P3 is described by Lemma 3.

Lemma 3 *If the entrepreneur seeks to induce the VC to transfer knowledge ex ante but not ex post:*

- *when* $G < \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$*, the optimal contract is the largest root of the following equation:*

$$\omega^2 R_e^{VN}(R - R_e^{VN}) = max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\};$$

- *when* $G \geq \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$*, the optimal contract is the largest root of the following equation:*

$$(\omega - k)\omega R_e^{VN}(R - R_e^{VN}) = G$$

The optimal contract will entail a fee ex ante if the VC participation constraint is slack. The problem has a solution only when the following conditions are satisfied: $\frac{1}{4}\omega^2 R^2 \geq max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ *and* $G \leq \frac{1}{4}(\omega - k)\omega R^2$*.*

Proof. See Appendix. ■

Lemma 3 shows that inducing only inward knowledge transfer requires the venture capitalist's private benefit from outward knowledge transfer to be below a critical threshold value. When the advice cost is relatively low ($\frac{C}{\tau} < \frac{I+C}{\pi+\tau}$), there are two possibilities: either the VC participation constraint binds, or his ex-post incentive constraint (requiring him to refrain from expropriation) binds. Conversely, when the advice cost is relatively high ($\frac{C}{\tau} > \frac{I+C}{\pi+\tau}$), either his ex-ante incentive constraint (requiring him to transfer knowledge inwards) binds, or his ex-post incentive constraint binds.

3.2.4 No Knowledge Transfer

Finally, we study under what conditions the venture capitalist chooses not to engage in any form of knowledge transfer. In this case, the VC acts in the same way as the non-VC investor: the difference lies in the constraints that must be satisfied for the VC to refrain from transferring knowledge, yielding a different optimization problem for the entrepreneur and a different resulting contract. The optimal contract solves the following program (P4):

$$\begin{aligned}
& \max_{R_e^{NN}} \;\; \pi[\omega e^{NN} R_e^{NN} - \frac{1}{2}(e^{NN})^2] \\
s.t.\ e^{NN} &= \omega R_e^{NN} \;\; (IC_e) \\
\pi\omega e^{NN}(R - R_e^{NN}) &\geq I \;\; (PC_{VC}) \\
\tau\omega e^{NN}(R - R_e^{NN}) &< C \;\; (IC_{VC}\ ex\ ante) \\
G + k e^{NN}(R - R_e^{NN}) &\leq \omega e^{NN}(R - R_e^{NN}) \;\; (IC_{VC}\ ex\ post)
\end{aligned}$$

The solution to P4 is described in the following lemma.

Lemma 4 *The VC chooses not to transfer knowledge ex ante or ex post in the following two cases:*

- *when $\frac{I}{\pi} \leq \frac{C}{\tau}$ and $G < \frac{\omega-k}{\omega}\frac{I}{\pi}$. The optimal contract is determined by the largest root of the following equation:*

$$\pi\omega^2 R_e^{NN}(R - R_e^{NN}) = I;$$

- *when $\frac{I}{\pi} \leq \frac{C}{\tau}$ and $G \geq \frac{\omega-k}{\omega}\frac{I}{\pi}$. The optimal contract is the largest value such that IC_{VC} ex post is binding:*

$$(\omega - k)\omega R_e^{NN}(R - R_e^{NN}) = G$$

The optimal contract will entail a fee ex ante if the VC participation constraint is slack. The problem has a solution only when conditions $\frac{\pi\omega^2 R^2}{4} \geq I$ and $G \leq \frac{1}{4}(\omega - k)\omega R^2$ are satisfied.

Proof. See Appendix. ■

Lemma 4 shows that there are two cases of interest. Both require the cost of advice C to be relatively high, to deter inward knowledge transfer by the VC. In the first case, the venture capitalist's private benefit from expropriation G is sufficiently low not to tempt him, given his stake in the financial returns of the entrepreneur's project (required to satisfy his participation constraint). In the second case, the private benefit from expropriation is larger, and the VC has to be offered a higher share of financial returns to ensure he does not expropriate. Thus in the first case, the optimal contract with the VC is the same as with the non-VC investor, and the entrepreneur is indifferent between raising external finance from a VC or a non-VC investor. In the second case, the optimal contract with the VC differs from the one with the non-VC investor because of the binding ex-post incentive constraint for the VC: in this case, the entrepreneur will prefer to raise funding from a non-VC investor.

3.2.5 Choice of contract under VC finance

Using the results summarized by Lemmas 1 to 4, we can study the entrepreneur's optimal choice of contract when he raises external finance from a venture capitalist. We will then be able to examine the tradeoffs involved in obtaining funding from a VC relative to a non-VC investor. The following proposition states under which conditions each type of VC contract is preferred by the entrepreneur.

Proposition 1 *The entrepreneur's choice of VC contract is determined by the relative size of $G, C,$ and I:*

- *when $\frac{C}{\tau} < \frac{I}{\pi}$, the optimal contract will always induce the VC to transfer knowledge ex ante. There exists a cutoff value $G^* > \frac{\omega - k}{\omega}\frac{I+C}{\pi+\tau}$, such that*
  - *when $G > G^*$, the optimal contract will be the one that induces knowledge transfer ex ante and ex post;*
  - *when $G < G^*$, the optimal contract will be the one yielding only knowledge transfer ex ante.*
- *when $\frac{C}{\tau} > \frac{I}{\pi}$, there exist two cutoff values, G^{**}, where $G^{**} > \frac{\omega-k}{\omega}\frac{C}{\tau}$, and G^{***}, where $\frac{\omega-k}{\omega}\frac{I}{\pi} < G^{***} < \frac{\omega-k}{\omega}\frac{C}{\tau}$, such that*
  - *when $G > G^{**}$, the optimal contract will always induce the VC to transfer knowledge ex post. For $\frac{C}{\tau}$ below a cutoff value, the contract will also induce the VC to transfer knowledge ex ante.*
  - *when $G^{**} \geq G \geqslant G^{***}$, the optimal contract will either induce knowledge transfer ex ante or it will induce knowledge transfer ex post (depending on the magnitude of $\frac{C}{\tau}$, G, k and ω).*
  - *when $G < G^{***}$, the optimal contract will never induce knowledge transfer ex post. It will induce knowledge transfer ex ante for $\frac{C}{\tau}$ below a threshold value.*

Proof. See Appendix. ■

The intuition for the first part of Proposition 1 is very simple: If the advice cost is low ($\frac{C}{\tau} < \frac{I}{\pi}$) , the optimal contract will induce advice with expropriation when the expropriation benefit is high ($G > G^*$), and advice without expropriation when the expropriation benefit is low ($G < G^*$).

If the advice cost is relatively high, $\frac{C}{\tau} > \frac{I}{\pi}$, there are several possible outcomes. When the expropriation benefit is sufficiently high, the optimal contract will always induce expropriation;

it may also induce advice provided the advice cost is not too high. Conversely, when the expropriation benefit is sufficiently low, the optimal contract will never induce expropriation; it may again induce advice provided the advice cost is not too high. For intermediate values of the expropriation benefit G, two possibilities emerge. The optimal contract may entail advice without expropriation: inducing the VC to transfer knowledge ex ante means he has to be given a relatively high share of financial returns, which deters expropriation ex post, given that the private benefit from expropriation is not so large. Alternatively, the optimal contract may entail expropriation without advice: this implies that the VC is given a relatively low share of financial returns, which leads him to transfer knowledge ex post, but does not induce him to advise ex ante. Thus for intermediate values of G, the two forms of knowledge transfer are substitutes. They become complements for higher values of G (provided the cost of ex-ante knowledge transfer is not too high).

3.2.6 Choosing between VC and non-VC finance

We can now study the trade-offs faced by the entrepreneur in choosing between VC and non-VC finance. These are described by the following result.

Proposition 2 *The entrepreneur' s choice between VC and non-VC finance is determined below:*

- *when $\frac{C}{\tau} < \frac{I}{\pi}$, there exist two cutoff values, $G^{\#}$, where $\frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau} < G^{\#} \leqslant G^{*}$, and $G^{\#\#}$, with $G^{\#\#} \geqslant G^{*}$, such that:*
  - *when $G \leqslant G^{\#}$, VC finance is preferred. The optimal VC contract induces (only) knowledge transfer ex ante.*
  - *when $G^{*} \geqslant G > G^{\#}$, the entrepreneur chooses between VC finance with (only) knowledge transfer ex ante and non-VC finance, with higher τ and lower $C/G/\mu$ favoring VC finance.*
  - *when $G^{\#\#} \geqslant G > G^{*}$, the entrepreneur chooses between non-VC finance and VC finance with knowledge transfer ex ante and ex post, with higher $\tau/G/\mu$ and lower C favoring VC finance.*
  - *when $G > G^{\#\#}$, VC finance is preferred. The optimal VC contract induces knowledge transfer ex ante and ex post.*
- *When $\frac{C}{\tau} > \frac{I}{\pi}$, there is a cutoff value μ^{*} such that:*
  -
  - *If $\mu < \mu^{*}$, then there exists a cutoff value $G^{\#\#\#}$, where $G^{\#\#\#} > G^{**}$, such that*

  * *when $G < G^{**}$, the entrepreneur chooses between VC finance with (only) knowledge transfer ex ante and non-VC finance;*
  * *when $G^{\#\#\#} > G > G^{**}$, the entrepreneur chooses between non-VC finance and VC finance with knowledge transfer ex ante and ex post;*
  * *when $G > G^{\#\#\#}$, VC finance with ex-post knowledge transfer is preferred. The optimal contract will also induce knowledge transfer ex ante if $\frac{C}{\tau}$ is sufficiently low.*

– *If $\mu \geq \mu^*$, then there exists a cutoff value $G^{\#\#\#\#}$, where $G^{**} > G^{\#\#\#\#} > G^{***}$, such that*
  * *when $G < G^{\#\#\#\#}$, the entrepreneur chooses between VC finance with (only) knowledge transfer ex ante and non-VC finance;*
  * *when $G^{**} > G > G^{\#\#\#\#}$, VC finance is preferred. The entrepreneur chooses between knowledge transfer ex ante and ex post;*
  * *when $G > G^{**}$, VC finance with ex-post knowledge transfer is preferred. The optimal contract will also induce knowledge transfer ex ante if $\frac{C}{\tau}$ is sufficiently low.*

Proof. See Appendix. ■

The intuition for the first part of this result is as follows. We know from our earlier results that when the cost of *ex-ante* knowledge transfer is low, in the sense that $\frac{C}{\tau} < \frac{I}{\pi}$, the optimal VC contract always induces this form of transfer; in addition, it induces *ex-post* knowledge transfer if, and only if, $G > G^*$. Thus, for $G \leqslant G^*$, the entrepreneur will choose between non-VC finance and VC finance with (only) *ex-ante* knowledge transfer. In particular, when $G \leqslant G^{\#}$, VC finance is preferred, since it increases the probability of a successful innovation at a low cost. There may then be an interval, with $G^* \geqslant G > G^{\#}$, where, depending on parameter values, non-VC finance is preferred: for this to be the case it must be that the VC *ex-post* incentive constraint (requiring him not to expropriate) is binding in the optimal VC contract, implying that the VC has to be allocated a relatively high share of financial returns, which tends to reduce entrepreneurial effort.

For $G > G^*$, the entrepreneur chooses instead between non-VC finance and VC finance with both forms of knowledge transfer. There may be a range with $G^{\#\#} \geqslant G > G^*$, where non-VC finance is preferred, depending on parameter values. Essentially this is because the optimal VC contract induces *ex-post* knowledge transfer, but this may be due to the high cost of giving the VC incentives not to expropriate, whereas preventing expropriation by non-VC investors is costless. On the other hand, when the benefit from expropriation is sufficiently large ($G > G^{\#\#}$), VC finance with both forms of knowledge transfer is always preferred.

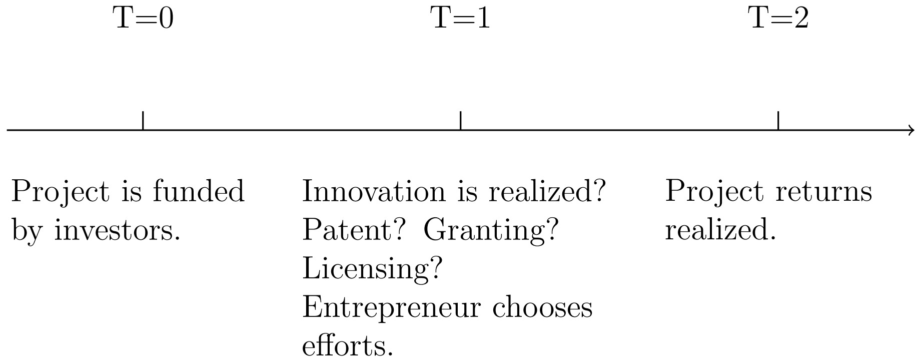
Turning to the second part of the result, we know from our previous analysis that when the cost of *ex-ante* knowledge transfer is above a critical threshold, $\frac{C}{\tau} > \frac{I}{\pi}$, the optimal choice of

VC contract is less simple. In particular, we saw that for an intermediate range of values of the private benefit from expropriation, $G^{**} \geq G \geqslant G^{***}$, the optimal VC contract could entail either *ex-ante* or *ex-post* knowledge transfer, depending on parameter values. Intuitively, the contract with *ex-post* knowledge transfer becomes relatively more attractive as μ increases, since higher values of μ imply that competitors are more likely to expropriate later on, unless pre-empted by the VC. We can therefore distinguish between choices when μ is above or below a cutoff value μ^* (to which corresponds a value ω^*, since by definition $\omega = (1 - \mu + \mu k)$). In both cases, for G sufficiently low the entrepreneur chooses between non-VC finance and VC finance with *ex-ante* knowledge transfer, the latter being preferred when $\frac{C}{\tau}$ is below a critical threshold. Conversely, for G sufficiently high the entrepreneur will always prefer VC finance, with *ex-post* knowledge transfer (and *ex-ante* knowledge transfer if $\frac{C}{\tau}$ is below a critical threshold). Where the two cases differ, as expected, is for intermediate values of G: when $\mu \geq \mu^*$(i.e. when the probability of expropriation by competitors is high), VC finance is preferred over a wider range of values for G. Specifically, we find that for $G^{**} > G > G^{\#\#\#\#}$, VC finance is preferred and the two forms of knowledge transfer by the VC are substitutes when $\mu \geq \mu^*$ (the entrepreneur chooses between VC finance with *ex-ante* knowledge transfer and VC finance with *ex-post* knowledge transfer), while for $\mu < \mu^*$, the entrepreneur chooses between VC with *ex-ante* knowledge transfer and non-VC finance. For $G^{\#\#\#} > G > G^{**}$, moreover, VC finance is again preferred when $\mu \geq \mu^*$, while the entrepreneur chooses between non-VC and VC finance when $\mu < \mu^*$.

4 Patentable innovations and the decision to seek patent protection

In this section, we extend the analysis to patentable innovations. We incorporate a crucial feature of the way patent systems work in practice: typically there is some uncertainty as to whether a patent application will be successful, even for commercially valuable innovations. Moreover, the patent application itself often reveals information that may be beneficial to competitors. We model this by assuming that, following the development of a valuable innovation, the entrepreneur can apply for a patent: this application will be approved with probability $\beta < 1$. The parameter $\beta > 0$ captures the efficiency of the patent system, industry characteristics, and the characteristics of the innovation. We also assume that, if the patent application is rejected, the leakage of information from the patenting application increases the probability of subsequent expropriation by competitors from μ to α, with $1 > \alpha > \mu$. This assumption is motivated by empirical evidence from the 2008 Berkeley Patent Survey: Graham, Merges, Samuelson and Sichelman (2010) analyze the responses from 1332 early stage companies founded since 1998 and find that 35% cite "Did not want to disclose information" as a reason for not seeking patent protection

Figure 2: Time Line



for their innovations[20]. If a patent is granted, there are two possibilities. Either the patent is used to exclude competitors: in this case the entrepreneur's project succeeds with probability e. Alternatively, the intellectual property can be licensed: this yields revenue $L \geqslant G$ for the firm, while the project succeeds with reduced probability ke. This captures the idea that private knowledge transfer by the VC may yield a lower benefit than licensing, as it cannot be done through an explicit legal contract.

Our main interest here is to explore the decision to seek patent protection, and how it differs depending on whether the entrepreneur raises external finance from a VC or a non-VC investor. For simplicity, we abstract from ex-ante (inward) knowledge transfer, and focus on ex-post (outward) knowledge transfer by the VC, which is the one directly affected (ruled out) when the innovation is protected by a patent. The timing of the model is illustrated in Figure 2.

4.1 Non-VC investor

We begin by studying optimal contracts between the entrepreneur and a non-VC investor.

4.1.1 Non-VC investor: patent used to exclude competitors

When the firm chooses to use the patent to exclude competitors, the effort level exerted by the entrepreneur following patent approval is given by $e^P = argmax_e \;\; eR_e^P - \frac{1}{2}e^2$, and in case of patent rejection it is $e^R = argmax_e \;\; ezR_e^R - \frac{1}{2}e^2$, where $z \equiv 1 - \alpha + \alpha k$. The first order conditions give us: $e^P = R_e^P, e^R = zR_e^R$. The optimal contract solves the following maximization problem

[20] The survey highlights substantial differences across industries, with the proportion of respondents citing information disclosure as a reason not to seek patent protection varying from 59% in biotechnology to 25% in software.

(P5):

$$\max_{R_e^P, R_e^R} \quad \pi\{\beta[e^P R_e^P - \frac{1}{2}(e^P)^2] + (1-\beta)[e^R z R_e^R - \frac{1}{2}(e^R)^2]\}$$
$$s.t. \;\; \pi\{\beta e^P(R - R_e^P) + (1-\beta)e^R z(R - R_e^R)\} \geq I \quad (PC_i)$$
$$e^P = R_e^P, e^R = zR_e^R \quad (IC_e)$$

The solution to P5 is described by Lemma 5.

Lemma 5 *The optimal contract satisfies $R_e^P = R_e^R = \hat{R}$, and $\hat{R}$ is the largest root of $\pi[\beta + (1-\beta)z^2]\hat{R}(R - \hat{R}) = I$ provided that $I \leq \frac{R^2}{4}\pi[\beta + (1-\beta)z^2]$.*

Proof. See Appendix. ■

4.1.2 Non-VC investor: licensing

When the firm licenses its intellectual property, the probability of project success decreases from e to ke. Therefore, the effort level of the entrepreneur following patent approval is altered: $e^L = argmax_e keR_e^L - \frac{1}{2}e^2 = kR_e^L$. The effort level in case of patent rejection is unchanged, i.e., $e^R = zR_e^R$.

We can see that in general it is optimal to allocate all the license revenue L to the investor, since this relaxes his participation constraint, making it possible to maximize the share of the final project return given to the entrepreneur, which induces higher entrepreneurial effort. The two channels through which the licensing decision affects the entrepreneur's payoff are: on the one hand, licensing reduces the probability of project success, which decreases the expected payoff of the entrepreneur; on the other hand, licensing relaxes the investor's participation constraint, giving a higher share of the final returns to the entrepreneur, which increases his expected return.

The optimization problem of the entrepreneur (P6) is:

$$\max_{R_e^L, R_e^R} \quad \pi\{\beta[ke^L R_e^L - \frac{1}{2}(e^L)^2] + (1-\beta)[e^R z R_e^R - \frac{1}{2}(e^R)^2]\}$$
$$= \frac{\pi}{2}[\beta(kR_e^L)^2 + (1-\beta)(zR_e^R)^2]$$
$$s.t. \;\; \pi\{\beta[k^2 R_e^L(R - R_e^L) + L] + (1-\beta)z^2 R_e^R(R - R_e^R)\} \geq I$$

The solution to P6 is given by Lemma 6.

Lemma 6 *When $L = \frac{I}{\pi\beta}$, the optimal contract is $R_e^L = R_e^R = R$, and the VC earns the license fee L. when $L < \frac{I}{\pi\beta}$, the problem has the following interior solution: 1. $R_e^L = R_e^R \equiv \hat{\hat{R}}$;*

2. $\hat{\hat{R}}$ is the largest root of $\pi(\beta k^2 + (1-\beta)z^2)\hat{\hat{R}}(R - \hat{\hat{R}}) = I - \pi\beta L$; 3. The condition $I \leq \frac{\pi R^2}{4}(\beta k^2 + (1-\beta)z^2) + \pi\beta L$ must be satisfied. When $L > \frac{I}{\pi\beta}$, the investor is willing to provide more initial capital than the required amount I, i.e., $I + Z = L\pi\beta$, where Z denotes the difference between the initial investment I and the investor's initial capital contribution.

Proof. See Appendix. ■

4.1.3 The patenting decision with non-VC finance

Comparing Lemma 6 with Lemma 5, we see that if the license fee L were reduced to zero, using the patent to exclude competitors would clearly be preferred, since the benefit from licensing disappears, while the project's probability of success is reduced by licensing. As L increases, the entrepreneur's expected utility from the licensing contract increases monotonically, while the expected utility from the patent to exclude competitors contract is unchanged. Thus for L above some threshold value, the entrepreneur's preference switches in favor of the licensing contract. Comparing Lemma 5 with our earlier results for non-VC finance without patents, we also see that there is a clear trade-off between applying for a patent with which to exclude competitors, and not applying for a patent at all. Specifically, it is optimal to apply for a patent to exclude competitors only if the expected benefit from applying for the patent, due to the ability to protect the innovation if the patent is approved, outweighs the expected cost, due to information disclosure (i.e., $\beta + (1-\beta)z^2 > \omega^2$).

The following result describes the entrepreneur's optimal choice between the three possible options with non-VC finance: apply for a patent and, if approved, use it to exclude competitors; apply for a patent and, if approved, license the innovation; do not apply for patent protection.

Proposition 3 *There exist two cutoff values L^N and L^P,[21] such that $L^P > L^N > L^*$, where $L^* = \frac{(1-k^2)I}{\pi[\beta+(1-\beta)z^2]}$ is the licensing value such that $\hat{\hat{R}} = \hat{R}$, and:*

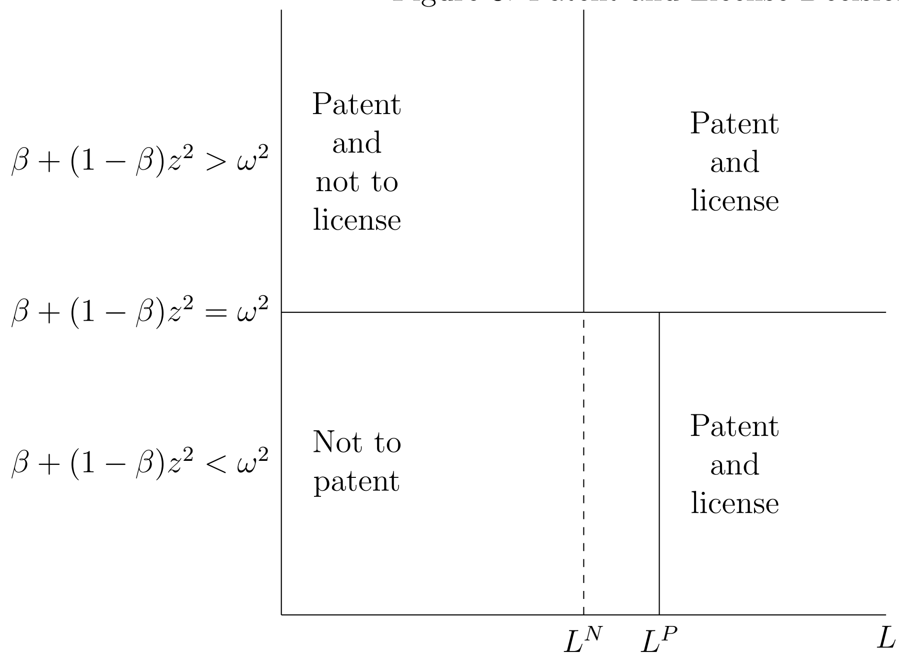
1. *When $\beta + (1-\beta)z^2 > \omega^2$, it is optimal to apply for a patent. When $L \leq L^N$, it is also optimal to use the patent to exclude competitors, while when $L \geq L^N$, it is optimal to license.*
2. *When $\beta + (1-\beta)z^2 < \omega^2$, applying for a patent and licensing is preferred if $L > L^P$. Otherwise, if $L < L^P$, it is optimal not to apply for a patent.*

Proof. See Appendix. ■

The tradeoffs described by the Proposition are illustrated in Figure 3.

[21] The values of L^N and L^P are given in the appendix.

Figure 3: Patent and License Decision



4.2 VC investor

The entrepreneur's choice is somewhat more complicated when he raises external finance from a venture capitalist, and is studied below. There are in principle six possible options: (1) apply for a patent, use it to exclude competitors if the patent is approved; otherwise induce the VC to transfer knowledge; (2) apply for a patent, use it to exclude competitors if the patent is approved; otherwise induce the VC not to transfer knowledge; (3) apply for a patent, license if the patent is approved; otherwise induce the VC to transfer knowledge; (4) apply for a patent, license if the patent is approved; otherwise induce the VC not to transfer knowledge; (5) do not apply for a patent; induce the VC to transfer knowledge; (6) do not apply for a patent; induce the VC not to transfer knowledge. However, the options where the VC does not transfer knowledge yield the same outcome in terms of knowledge transfer as non-VC finance, and a lower expected utility for the entrepreneur if the VC incentive constraint (ensuring that he does not transfer knowledge) is binding. Thus non-VC finance is preferred. Without loss of generality, we can therefore focus on the three options that entail knowledge transfer by the VC.

For expositional convenience we assume that $G < \frac{I}{\pi}$, i.e., the expected gain from expropriation would never be sufficient, on its own, to induce the VC to fund the entrepreneur, and similarly $L < \frac{I}{\pi}$,implying that the licensing fee is not enough to recover all the investment cost of the project.[22].

[22]These assumptions reduce the number of cases to be considered, without affecting the main insights from our results.

4.2.1 VC investor: Patent used to exclude competitors

When the patent is used to exclude competitors, the entrepreneur's effort level will be $e^P = argmax_e \;\; eR_e^{VP} - \frac{1}{2}e^2 = R_e^{VP}$ if the patent is granted, and $e^R = argmax_e \;\; ekR_e^{VR} - \frac{1}{2}e^2 = kR_e^{VR}$ if the patent is rejected, since in the latter case the VC will expropriate.

The optimal contract is defined by the following problem (P7):

$$\max_{R_e^{VP}, R_e^{VR}} \quad \pi\{\beta[e^P R_e^{VP} - \frac{1}{2}(e^P)^2] + (1-\beta)[e^R k R_e^{VR} - \frac{1}{2}(e^R)^2]\}$$
$$s.t. \;\; e^P = R_e^{VP}, e^R = kR_e^{VR} \quad (IC_e)$$
$$\pi\{\beta e^P(R - R_e^{VP}) + (1-\beta)[e^R k(R - R_e^{VR}) + G]\} \geq I \quad (PC_{VC})$$
$$G + ke^R(R - R_e^{VR}) \geq ze^R(R - R_e^{VR}) \quad (IC_{VC})$$

The solution to problem (P7) is given by Lemma 7.

Lemma 7 *Define the threshold values* $C_2 = \frac{k(z-k)}{\beta+(1-\beta)kz}\frac{I}{\pi}$, $C_3 = \frac{z-k}{(1-\beta)z}[\frac{I}{\pi} - \frac{1}{4}\beta R^2]$. *Then:*

- *when* $G \geq C_2$, *the optimal contract specifies* $R_e^{VP} = R_e^{VR} = \hat{\hat{R}}$, *where* $\hat{\hat{R}}$ *is the largest value such that* PC_{VC} *is binding;*

- *when* $C_2 > G \geq C_3$, *the optimal contract specifies* $\hat{R}_e^{VR} > \hat{R}_e^{VP}$, *where* $\hat{R}_e^{VR}$ *is the largest value such that* IC_{VC} *is binding, and given* $\hat{R}_e^{VR}$, $\hat{R}_e^{VP}$ *is the largest value such that* PC_{VC} *is binding;*

- *when* $G < C_3$, *it is not possible to induce the VC to participate and transfer knowledge.*

Proof. See Appendix. ■

Lemma 7 tells us that if the expropriation benefit G is large enough, then it is optimal to give the entrepreneur the same share of final returns if the patent is granted and if the patent is rejected; this share is determined by the binding participation constraint for the VC. As G decreases, the incentive constraint of the VC can no longer be satisfied. Therefore, the share of returns going to the VC when the patent is rejected needs to be reduced, while his share of returns when the patent is approved increases to satisfy the participation constraint as an equality. Finally if G is too low, it is not possible to induce the venture capitalist to participate and transfer knwoledge.

4.2.2 VC investor: licensing

The entrepreneur's effort when a patent is granted and then licensed is given by $e^L = argmax_e \, ekR_e^{VL} - \frac{1}{2}e^2 = kR_e^{VL}$. The optimal contract in this case solves the following problem (P8):

$$\max_{R_e^{VL}, R_e^{VR}} \quad \pi\{\beta[ke^L R_e^{VL} - \frac{1}{2}(e^L)^2] + (1-\beta)[ke^R R_e^{VR} - \frac{1}{2}(e^R)^2]\}$$

$$s.t. \;\; e^L = kR_e^{VL}, \;\; e^R = kR_e^{VR} \;\; (IC_e)$$

$$\pi\{\beta[L + ke^L(R - R_e^{VL})] + (1-\beta)[G + ke^R(R - R_e^{VR})]\} \geq I \;\; (PC_{VC})$$

$$G \geq (z-k)kR_e^{VR}(R - R_e^{VR}) \;\; (IC_{VC})$$

The solution to (P8) is summarized in Lemma 8.

Lemma 8 *Define the threshold values* $H_2 \equiv \frac{z-k}{\beta k + (1-\beta)z}(\frac{I}{\pi} - \beta L)$*, and* $H_3 \equiv \frac{z-k}{(1-\beta)z}[\frac{I}{\pi} - \beta L - \frac{\beta k^2 R^2}{4}]$*. then:*

when $G \geq H_2$*, it is optimal to specify the same share of returns for the entrepreneur when the patent is granted or rejected, determined as the largest share that satisfies the binding VC participation constraint;*

when $H_2 > G \geq H_3$*, it is optimal to set* $R_e^{VR} > R_e^{VL}$*. Here* R_e^{VR} *is the largest value such that* IC_{VC} *is binding; while* R_e^{VL} *is the value such that* PC_{VC} *is binding given* R_e^{VR}*;*

when $G < H_3$*, it is not possible to induce the VC to participate and expropriate.*

Proof. See Appendix. ■

4.2.3 The patenting decision with VC finance

We first investigate the decision to apply for patent protection under VC finance:

Lemma 9 *Under VC finance, it is always optimal to apply for patent protection.*

Proof. See Appendix. ■

We now examine the entrepreneur's choice between licensing and excluding competitors when a patent is granted. This is described by the following result.

Lemma 10 *When VC finance is obtained and a patent is granted, the choice between licensing and excluding competitors is determined as follows.*

(i) if $H_2 > G \geq C_2$*, the patent is used to exclude competitors;*

(ii) if $C_2 > G \geq H_2$*, the patent is licensed;*

(iii) if $G \geq \max\{C_2, H_2\}$*, there is a cutoff value* $L^{\#}$*, where* $L^{\#} > \hat{L} \equiv \frac{(1-k^2)[I-(1-\beta)\pi G]}{\pi[\beta + (1-\beta)k^2]}$*, such that the patent is licensed when* $L > L^{\#}$ *and used to exclude competitors otherwise.*

Proof. See Appendix. ■

4.3 The decision to seek patent protection

It is clear from our analysis so far that the decision to seek patent protection differs depending on whether the entrepreneur is financed by a venture capitalist or a non-VC investor. In particular, we have shown that:

(i) it is always optimal to apply for patent protection under VC finance;

(ii) it can be optimal not to apply for patent protection under non-VC finance. This will be the case if, and only if, the expected benefit from applying, due to the ability to protect the innovation from expropriation if the patent is approved, is lower than the expected cost, due to information disclosure.

This difference means that, holding the probability of a successful innovation constant (here exogenously equal to π), we should expect to see a greater propensity to patent among VC-funded firms. Interestingly, this is not due to entrepreneurs' fear of being expropriated by their VC investors: the result in our model is driven instead by the reluctance of non-VC-funded firms to apply for patent protection when there is sufficient uncertainty over the outcome of the application, combined with information disclosure that makes expropriation by competitors more likely if the patent application is unsuccessful. This reluctance is not shared by VC-funded firms, since they can rely on the venture capitalist to transfer knowledge profitably when the patent application is rejected, pre-emptying subsequent expropriation by competitors. Moreover, the venture capitalists' expected gains from such transfers are taken into account at the contracting stage, relaxing financing constraints so that entrepreneurs who would otherwise be denied funding can obtain the external finance needed to undertake their projects. This result is consistent with the finding by Mollica and Zingales (2007) that venture capital firms tend to increase both patents *and* the number of new businesses.

4.4 Robustness and extensions

Our results for patentable innovations were obtained, for tractability as well as ease of exposition, under the assumption that the VC could only engage in *ex-post* knowledge transfer, or equivalently that the cost C of *ex-ante* knowledge transfer was very high. Allowing for a lower cost C modifies our results on the patenting decision and the financing decision in two ways. First, if the private benefit G is low, VC finance may nevertheless be preferred, with the optimal VC contract designed to induce knowledge transfer ex ante, but not ex post. In this case, the decision to apply for a patent under VC finance is based on the same trade-off as under non-VC finance, namely the trade-off between protection against expropriation by competitors if the patent is granted, and a higher probability of expropriation by competitors if the patent is not granted, because of information disclosure. Second, for higher values of G, VC finance may be preferred with contracts inducing both forms of knowledge transfer. In this case, the patenting decision under VC finance remains the same as above; i.e. it remains optimal to always apply for patent protection following a successful innovation.

Another possible extension of our analysis is to consider the case where $k \geqslant 1$. Transferring knowledge to other firms in this case leaves the entrepreneur's probability of success unaffected, or better still, it increases his chances of success. This case is not without practical interest: for example, there can be circumstances when transmitting private knowledge to other firms helps to generate new complementary products and services and profitable opportunities. Financing and patenting decisions then become very straightforward: the entrepreneur will always prefer VC finance, and under VC finance it will always be optimal to apply for a patent following the development of a successful innovation (as long as $L \geqslant G$).

5 Conclusions

This paper has studied the role of venture capitalists as knowledge intermediaries. We focused exclusively on this role because it has been under-researched until now, and yet the limited empirical evidence available so far suggests it is important. Indeed, we view our model as a first step towards understanding its implications for financing constraints and new business creation, for innovation, and for product market competition, leading to promising empirical research.

There is also much theoretical analysis of venture capitalists to be done in the future, notably to explore the interaction of knowledge transfer with other roles, and the implications for the wider economy.

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6 Appendix

6.1 Proof of Lemma 1

Proof. From PC_{VC} and IC_{VC} *ex ante*, we can see that $\frac{C}{\tau} \geq ke^{VN}(R - R_e^{VN}) + G \geq \frac{I}{\pi}$. It holds only when $\frac{C}{\tau} \geq \frac{I}{\pi}$.

From IC_{VC} *ex post* and PC_{VC}, we have

$$G \geq (\omega - k)e^{VN}(R - R_e^{VN})$$
$$e^{VN}(R - R_e^{VN}) \geq \frac{\frac{I}{\pi} - G}{k}$$

Therefore, we have $G \geq \frac{\omega - k}{k}(\frac{I}{\pi} - G)$. Rearrange the above inequality, we have $G \geq \frac{\omega-k}{\omega}\frac{I}{\pi}$.

If $G \geq \frac{I}{\pi}$, PC_{VC} can be satisfied easily by setting $R_e^{VN} = R$, which maximizes the expected payoffs to the entrepreneur. When the inequality holds strictly, the VC will make an additional payment F ex ante, beyond I, so that the participation constraint holds as an equality; i.e. $\pi G = I + F$ because VC is competitive in the market.

If $G < \frac{I}{\pi}$, as the participation constraint will be binding in optimum, the optimal contract, R_e^{VN}, is determined by the largest root of the following equation: $\pi[k^2 R_e^{VN}(R - R_e^{VN}) + G] = I$. The condition for the range of I must be satisfied: $\pi[\frac{k^2R^2}{4} + G] \geq I$. ■

6.2 Proof of Lemma 2

Proof. To induce VC to transfer knowledge ex post, it implies that

$$G \geq (\omega - k)kR_e^{VN}(R - R_e^{VN}) \tag{1}$$

(from IC_{VC} *ex post*). By rewriting PC_{VC} and IC_{VC} *ex ante*, we have

$$kR_e^{VN}(R - R_e^{VN}) \geq \frac{1}{k}[\frac{I + C}{\pi + \tau} - G]$$
$$kR_e^{VN}(R - R_e^{VN}) \geq \frac{1}{k}[\frac{C}{\tau} - G]$$

Combining these two inequalities, we have

$$kR_e^{VN}(R - R_e^{VN}) \geq \frac{1}{k}[\max\{\frac{C}{\tau}, \frac{I + C}{\pi + \tau}\} - G] \tag{2}$$

Substitute the above inequality into the expression (1), we have

$$G \geq \frac{\omega - k}{\omega}\max\{\frac{C}{\tau}, \frac{I + C}{\pi + \tau}\}.$$

That's to say, when $G < \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, it is not possible to induce the VC to transfer knowledge ex post.

From expression (2), it's easy to find out that when $G \geq \max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, the optimal contract is $R_e^{VN} = R$. In this case, IC_{VC} *ex post* is always satisfied. PC_{VC} and IC_{VC} *ex ante* are also satisfied as inequality (2) holds as well.

When $\max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\} > G \geq \frac{\omega-k}{\omega} \max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$, IC_{VC} *ex post* is always satisfied. Expression (2) must be binding, which implies that either PC_{VC} or IC_{VC} *ex post* will be binding in optimum, depending on the relative size between $\frac{I+C}{\pi+\tau}$ and $\frac{C}{\tau}$ (If$\frac{I+C}{\pi+\tau} > \frac{C}{\tau}$, then PC_{VC} will be binding; and vice versa.). The optimal contract is the largest root of (2) when (2) holds in equality.

If $\frac{I+C}{\pi+\tau} < \frac{C}{\tau}$, such that the participation constraint of VC is slack, then VC would pay an extra fee ex ante, F, to the entrepreneur such that $(\pi+\tau)[ke^{VN}(R - R_e^{VN}) + G] = I + C + F$. VCs always earn zero expected rents as they are competitive.

The participation constraint under which VC will invest in the project could be rewritten as:

$$R_e^{VN}(R - R_e^{VN}) \geq \frac{1}{k^2}\{\frac{I+C}{\pi+\tau} - G\}$$

which entails that the problem has a solution iff

$$(\pi+\tau)[\frac{1}{4}k^2R^2 + G] \geq I + C. \blacksquare$$

$\blacksquare$

6.3 Proof of Lemma 3

Proof. Similar to the Proof of Lemma 2, from PC_{VC} and IC_{VC} *ex ante* , we have

$$\omega^2 R_e^{VN}(R - R_e^{VN}) \geq \max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\} \tag{3}$$

From IC_{VC} *ex post*, we have

$$G \leq (\omega - k)\omega R_e^{VN}(R - R_e^{VN}) \tag{4}$$

If expression (4) holds with inequality, then at optimum, expression (3) must hold with equality, i.e. $\omega^2 R_e^{VN}(R - R_e^{VN}) = \max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$. Then substitute it into expression (4), we have $G < \frac{\omega-k}{\omega} \max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$. In this case, either PC_{VC} or IC_{VC} *ex ante* is binding, depending on the relative size between $\frac{C}{\tau}$ and $\frac{I+C}{\pi+\tau}$ (If $\frac{C}{\tau} > \frac{I+C}{\pi+\tau}$, IC_{VC} *ex ante* is binding and PC_{VC} is slack; vice versa.) When the VC participation constraint is slack, then VC would pay an extra fee ex ante, F, to the entrepreneur such that $(\pi+\tau)\omega e^{VN}(R - R_e^{VN}) = I + C + F$. VCs always earn zero expected rents as they are competitive. The optimal contract is the largest root of the following equation: $\omega^2 R_e^{VN}(R - R_e^{VN}) = \max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$. The problem has a solution only when $\frac{1}{4}\omega^2 R^2 \geq max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ as $R_e^{VN}(R - R_e^{VN}) \leq \frac{1}{4}R^2$.

If expression (4) holds with equality, subsititute it into expression (3), it implies that $G \geq \frac{\omega-k}{\omega}\max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$. The optimal contract is the largest root of the following equation:

$$G = (\omega - k)\omega R_e^{VN}(R - R_e^{VN}).$$

The problem has a solution only when $G \leq \frac{1}{4}(\omega - k)\omega R^2$. ■

6.4 Proof of Lemma 4

Proof. Condition PC_{VC} and IC_{VC} *ex ante* implies that $\frac{C}{\tau} > \omega e^{NN}(R - R_e^{NN}) \geq \frac{I}{\pi}$. It holds only when $\frac{C}{\tau} > \frac{I}{\pi}$.

If IC_{VC} *ex post* holds with inequality, it implies that $G < (\omega - k)\omega R_e^{NN}(R - R_e^{NN})$. Then at optimum, R_e^{NN} should be as large as possible, which implies that $\omega e^{NN}(R - R_e^{NN})$ should be as small as possible. Therefore, PC_{VC} is binding while IC_{VC} *ex ante* is slack at optimum. Substitute $\omega e^{NN}(R - R_e^{NN}) = \frac{I}{\pi}$ into IC_{VC} *ex post*, we have $G < \frac{\omega-k}{\omega}\frac{I}{\pi}$. In short, we can say that when $G < \frac{\omega-k}{\omega}\frac{I}{\pi}$, the optimal contract exists, which is the largest root of $\pi\omega^2 R_e^{NN}(R - R_e^{NN}) = I$. The condition for the range of I must be satisfied: $\frac{\omega^2 R^2}{4} \geq \frac{I}{\pi}$.

If IC_{VC} *ex post* holds with equality, it implies that $G = (\omega - k)\omega R_e^{NN}(R - R_e^{NN})$. Substitute it into PC_{VC}, we have $G \geq \frac{\omega-k}{\omega}$. In this case, the optimal contract is the largest value such that IC_{VC} *ex post* is binding. The condition for the range of G must be satisfied: $G \leq \frac{1}{4}(\omega - k)\omega R^2$. In this case, if PC_{VC} is slack, then VC would pay an ex ante fee to the entrepreneur such that his expected rent is zero, similar to the above situations. ■

6.5 Proof of Lemma 5

Proof. By plugging in IC_e into the objective function and PC_i, the optimization problem for the entrepreneur when facing non-VC investor and patent application without license can be rewritten as:

$$\max_{R_e^P, R_e^R} \quad \pi[\frac{\beta}{2}(R_e^P)^2 + \frac{1-\beta}{2}z^2(R_e^R)^2]$$
$$s.t. \quad \pi[\beta R_e^P(R - R_e^P) + (1-\beta)z^2 R_e^R(R - R_e^R)] \geq I \quad (PC_i)$$

The Lagrangian function could expressed in this form:

$$L = \pi[\frac{\beta}{2}(R_e^P)^2 + \frac{1-\beta}{2}z^2(R_e^R)^2] + \lambda\{\pi[\beta R_e^P(R - R_e^P) + (1-\beta)z^2R_e^R(R - R_e^R)] - I\}$$

The first order conditions are:

$$\frac{\partial L}{\partial R_e^P} = \pi\beta R_e^P + \lambda\pi\beta(R - 2R_e^P) = 0 \tag{5}$$

$$\frac{\partial L}{\partial R_e^R} = \pi(1-\beta)z^2R_e^R + \lambda\pi(1-\beta)z^2(R - 2R_e^R) = 0 \tag{6}$$

$$\frac{\partial L}{\partial \lambda} = \pi[\beta R_e^P(R - R_e^P) + (1-\beta)z^2R_e^R(R - R_e^R)] - I = 0 \tag{7}$$

Equation (5) is simply the participation constraint PC_i. Divide equation (**??**) by (**??**), we have

$$\frac{\beta R_e^P}{(1-\beta)z^2R_e^R} = \frac{\beta(R - R_e^P)}{(1-\beta)z^2(R - R_e^R)} \tag{8}$$

Equation (8) finally gives us

$$R_e^P = R_e^R = \hat{R} \tag{9}$$

Combine (9) and (5), the participation constraint of non-VC investor can be simplified as

$$\pi[\beta + (1-\beta)z^2]\hat{R}(R - \hat{R})] = I \tag{10}$$

The largest root of equation (10) is the optimal payment to the entrepreneur when facing non-VC and patent protection without license.

As we all know that $\hat{R} \in [0, R]$, then $\hat{R}(R - \hat{R}) \leq \frac{R^2}{4}$. And from (10), we have $I \leq \frac{R^2}{4}\pi[\beta + (1-\beta)z^2]$. ■

6.6 Proof of Lemma 6

Proof. The Lagrangien function for the problem (P2) can be written as

$$L = \beta k^2(R_e^L)^2 + (1-\beta)(zR_e^R)^2 + \lambda\{\frac{I}{\pi} - \beta[L + k^2R_e^L(R - R_e^L)] - (1-\beta)z^2R_e^R(R - R_e^R)\}$$

The first order conditions give us:

$$\frac{\partial L}{\partial R_e^L} = 2\beta k^2 R_e^L - \lambda\beta k^2 (R - 2R_e^L) = 0 \tag{11}$$

$$\frac{\partial L}{\partial R_e^R} = 2(1-\beta) z^2 R_e^R - \lambda(1-\beta) z^2 (R - 2R_e^R) = 0 \tag{12}$$

$$\frac{\partial L}{\partial \lambda} = \frac{I}{\pi} - \beta L - \beta k^2 R_e^L (R - R_e^L) - (1-\beta) z^2 R_e^R (R - R_e^R) \le 0 \tag{13}$$

1. When $\pi\beta L \ge I \Longleftrightarrow L \ge \frac{I}{\pi\beta}$;
   Then it's possible to set $R_e^L = R_e^R = R$ and still satisfy the investor's participation constraint; If the condition holds as a strictly inequality, the investor can provide additional capital ex ante above I, i.e., $L = \frac{I+Z}{\pi\beta}$, where Z denotes the difference between the initial investment and the willingness to fund of VC as VC market is competitive. Therefore, investor's PC will always be binding and the initial investment becomes $I + Z$.

2. When $\pi\beta L < I \Longleftrightarrow L < \frac{I}{\pi\beta}$;
   The investor's participation constraint is binding. Interior solutions for R_e^L and R_e^R satisfy:

$$\frac{2\beta k^2 R_e^L}{2(1-\beta) z^2 R_e^R} = \frac{\lambda\beta k^2 (R - 2R_e^L)}{\lambda(1-\beta) z^2 (R - 2R_e^R)} \Longrightarrow R_e^L = R_e^R$$

   Let $R_e^L = R_e^R \equiv \hat{\hat{R}}$, the problem becomes

$$\max \quad \hat{\hat{R}}$$
$$s.t.\ \pi(\beta k^2 + (1-\beta) z^2) \hat{\hat{R}} (R - \hat{\hat{R}}) = I - \pi\beta L$$

So it has a solution iff

$$\frac{\pi R^2}{4} (\beta k^2 + (1-\beta) z^2) \ge I - \pi\beta L$$

If this condition holds, the optimal contract is $R_e^L = R_e^R = \hat{\hat{R}} \ge \frac{1}{2}R$, where $\hat{\hat{R}}$ is the largest root of $(\beta k^2 + (1-\beta) z^2) \hat{\hat{R}} (R - \hat{\hat{R}}) = \frac{I}{\pi} - \beta L$. ■

6.7 Proof of Lemma 7

Proof. Suppose at optimum, IC_{VC} is always satisfied. Since PC_{VC} must be binding, the similar routine of Lagrangian function as in Proof of Lemma 5 gives us, at optimum, $R_e^{VP} = R_e^{VR} \equiv \hat{\hat{R}}$.

Then $\hat{\hat{R}}(R-\hat{\hat{R}}) = \frac{\frac{I}{\pi}-(1-\beta)G}{\beta+(1-\beta)k^2}$. Plug it into IC_{VC}, we have $[\beta+(1-\beta)k^2]G \geq (z-k)k[\frac{I}{\pi}-(1-\beta)G]$ $\Longrightarrow [\beta+(1-\beta)k^2+(1-\beta)(z-k)k]G \geq (z-k)k\frac{I}{\pi}$.

Therefore, we could discuss optimal contract by the following cases:

1. If $\pi(1-\beta)G \geq I$, it's possible to set $R_e^{VP} = R_e^{VR} = R$ and it satisfies PC_{VC} and IC_{VC}. However, due to our assumption that $G < \frac{I}{\pi}$, we will ignore this case in our analysis.

2. If $\pi(1-\beta)G < I$, and $G[\beta+(1-\beta)zk] \geq (z-k)k\frac{I}{\pi}$ ($\Longrightarrow G \geq \frac{k(z-k)}{\beta+(1-\beta)zk}\frac{I}{\pi}$) The optimal contract is $R_e^{VP} = R_e^{VR} = \hat{\hat{R}}$, where $\hat{\hat{R}}$ is largest root of $\pi\{(1-\beta)G+[\beta+(1-\beta)k^2]\hat{\hat{R}}(R-\hat{\hat{R}})\} = I$

3. If $\frac{z-k}{(1-\beta)z}[\frac{I}{\pi}-\frac{1}{4}\beta R^2] \leq G < \frac{k(z-k)}{\beta+(1-\beta)kz}\frac{I}{\pi}$, the PC_{VC} and IC_{VC} are both binding. The optimal contract is $\hat{R}_e^{VP}$ and $\hat{R}_e^{VR}$, where $\hat{R}_e^{VR}$ is the largest root of $G = (z-k)k\hat{R}_e^{VR}(R-\hat{R}_e^{VR})$. And given $\hat{R}_e^{VR}$, $\hat{R}_e^{VP}$ is the largest root of $\pi\{\beta\hat{R}_e^{VP}(R-\hat{R}_e^{VP})+(1-\beta)[G+k^2\hat{R}_e^{VR}(R-\hat{R}_e^{VR})]\} = I$. It's easy to see that $\hat{R}_e^{VR} \geq \hat{R}_e^{VP}$ since when G become smaller than $\frac{k(z-k)}{\beta+(1-\beta)kz}\frac{I}{\pi}$, we must give the entrepreneur higher share of return in case of patent rejection such that the IC_{VC} could be easily satisfied. Therefore, $\hat{R}_e^{VR} \geq \hat{R}_e^{VP}$.

4. If $G < \frac{z-k}{(1-\beta)z}[\frac{I}{\pi}-\frac{1}{4}\beta R^2]$, it's not possible to induce the VC to participate and expropriate. Because if the maximum possible level of R_e^{VP}, $\frac{R}{2}$, together with the maximum feasible level of R_e^{VR} that satisfies the IV_{VC}, are not sufficient to satisfy the PC_{VC}, i.e., if

$$\begin{aligned}\beta\hat{R}_e^{VP}(R-\hat{R}_e^{VP})+(1-\beta)[G+k^2\hat{R}_e^{VR}(R-\hat{R}_e^{VR})] &< \frac{I}{\pi}\\ \Rightarrow \beta\frac{R^2}{4}+(1-\beta)[G+k^2\frac{G}{(z-k)k} &< \frac{I}{\pi}\\ \Rightarrow G &< \frac{z-k}{(1-\beta)z}[\frac{I}{\pi}-\frac{1}{4}\beta R^2]\end{aligned}$$

   In this case, the optimal contract is the same as the non-VC case.

■

6.8 Proof of Lemma 8

Proof.

1. Suppose $\pi[\beta L+(1-\beta)G] \geq I$, then we can set $R_e^{VL} = R_e^{VR} = R$, PC_{VC} and IC_{VC} are all satisfied. Note that we assume that $L < \frac{I}{\pi}$, $G < \frac{I}{\pi}$, therefore, this case is ruled out.

2. If $\pi[\beta L + (1-\beta)G] < I$, then PC_{VC} will be binding. Suppose first the IC_{VC} is slack, the problem gives:

$$L = \frac{2}{\pi}k^2[\beta(R_e^{VL})^2 + (1-\beta)(R_e^{VR})^2]$$
$$+\lambda\{\frac{I}{\pi} - \beta[L + k^2 R_e^{VL}(R - R_e^{VL})] - (1-\beta)[G + k^2 R_e^{VR}(R - R_e^{VR})]\}$$

For interior solution, we have

$$\frac{\partial L}{\partial R_e^{VL}} = \pi k^2 \beta R_e^{VL} - \lambda\beta k^2(R - 2R_e^{VL}) = 0$$
$$\frac{\partial L}{\partial R_e^{VR}} = \pi k^2(1-\beta)R_e^{VR} - \lambda(1-\beta)k^2(R - 2R_e^{VR}) = 0$$
$$\Longrightarrow \frac{\beta R_e^{VL}}{(1-\beta)R_e^{VR}} = \frac{\beta(R - 2R_e^{VL})}{(1-\beta)(R - 2R_e^{VR})}$$
$$\Longrightarrow R_e^{VL} = R_e^{VR} = \hat{\hat{R}}^V$$

the problem becomes

$$\max \quad \hat{\hat{R}}^V$$
$$s.t. \;\; \beta L + (1-\beta)G + k^2\hat{\hat{R}}^V(R - \hat{\hat{R}}^V) \geq \frac{I}{\pi}$$
$$\Longrightarrow k^2\hat{\hat{R}}^V(R - \hat{\hat{R}}^V) \geq \frac{I}{\pi} - \beta L - (1-\beta)G$$

So it has a solution iff $\frac{k^2}{4}R^2 \geq \frac{I}{\pi} - \beta L - (1-\beta)G$. If this condition holds, and IC_{VC} is satisfied, the optimal contract is $R_e^{VL} = R_e^{VR} = \hat{\hat{R}}^V$, where $\hat{\hat{R}}^V$ is the largest root of $k^2\hat{\hat{R}}^V(R - \hat{\hat{R}}^V) = \frac{I}{\pi} - \beta L - (1-\beta)G$, plug it into IC_{VC}, we have $G \geq \frac{z-k}{k}[\frac{I}{\pi} - \beta L - (1-\beta)G]$, i.e., $G[1 + \frac{z-k}{k}(1-\beta)] \geq \frac{z-k}{k}[\frac{I}{\pi} - \beta L]$, $\Longrightarrow G \geq \frac{z-k}{\beta k + (1-\beta)z}[\frac{I}{\pi} - \beta L]$.

3. If $G < \frac{z-k}{\beta k + (1-\beta)z}[\frac{I}{\pi} - \beta L]$, IC_{VC} and PC_{VC} will be binding. The optimal contract will be $\hat{\hat{R}}_e^{VR}, \hat{\hat{R}}_e^{VL}$, s.t. $\hat{\hat{R}}_e^{VR}$ is the largest root of $G = (z-k)kR_e^{VR}(R - R_e^{VR})$, $\hat{\hat{R}}_e^{VL}$ is the largest root of $\beta L + (1-\beta)G + k^2[\beta R_e^{VL}(R - R_e^{VL}) + (1-\beta)\hat{\hat{R}}_e^{VR}(R - \hat{\hat{R}}_e^{VR})] = \frac{I}{\pi}$ given $\hat{\hat{R}}_e^{VR}$. It's easy to see that $\hat{\hat{R}}_e^{VR} \geq \hat{\hat{R}}_e^{VL}$ since when G become smaller than $\frac{z-k}{\beta k + (1-\beta)z}[\frac{I}{\pi} - \beta L]$, we must give the entrepreneur higher share of return in case of patent rejection such that the IC_{VC} could be easily satisfied. Therefore, $\hat{\hat{R}}_e^{VR} \geq \hat{\hat{R}}_e^{VL}$.

4. Finally, it's not possible to induce the VC to participate and expropriate if the maximum possible level of R_e^{VL}, $\frac{1}{2}R$, together with the maximum feasible level of R_e^{VR} that satisfies

the IC_{VC}, are not together sufficient to satisfy the PC_{VC}, i.e., if

$$\beta L + (1-\beta)G + k^2[\frac{\beta R^2}{4} + (1-\beta)\frac{G}{k(z-k)}] < \frac{I}{\pi}$$
$$\Longrightarrow (1-\beta)G[1 + \frac{k^2}{k(z-k)}] < \frac{I}{\pi} - \beta L - \frac{\beta k^2 R^2}{4}$$
$$\Longrightarrow G < \frac{z-k}{(1-\beta)z}[\frac{I}{\pi} - \beta L - \frac{\beta k^2 R^2}{4}].$$

■

6.9 Proof of Lemma 9

The proof is straightforward. We are interested in the case where VC finance is chosen. If no patent application is made, the VC is induced to transfer knowledge ex post. If a patent application is made, it is always possible to do at least as well by licensing when the patent is granted (since $L \geqslant G$) and by inducing the VC to transfer knowledge when the patent is not granted.

6.10 Proof of Lemma 10

Proof. We focus on the case where VC finance is obtained; i.e. it is preferred to non-VC finance. This implies $G \geq C_2$ and/or $G \geq H_2$. To see this, note that when the patent is used to exclude competitors, non-VC finance is preferred for $G < C_2$: specifically, for $G < C_3$ it is not possible to induce the VC to participate and transfer knowledge (hence, there is no difference between VC and non-VC finance), while for $C_2 > G \geq C_3$, non-VC finance is preferred.[23]

Similarly, when the patent is licensed, non-VC finance is preferred for $G < H_2$: specifically, for $G < H_3$ it is not possible to induce the VC to participate and transfer knowledge (hence, there is no difference between VC and non-VC finance), while for $H_2 > G \geq H_3$ the VC incentive constraint is binding, implying that non-VC finance is preferred. Clearly then if $G \geq C_2$ and $G < H_2$ there will be no licensing under VC finance; similarly, if $G < C_2$ and $G \geq H_2$ the patent will not be used to exclude competitors under VC finance.

[23] Consider problem P7, $C_2 > G \geq C_3$. Let the solution be S, V, where S is given by $G = (z-k)kS(R-S)$, and then V is given by $\beta V(R-V)+(1-\beta)[k^2 S(R-S)+G] = \frac{I}{\pi}$. These two conditions imply $\beta V(R-V)+(1-\beta)zkS(R-S) = \frac{I}{\pi}$. The participation constraint for non-VC finance can be written as $\beta V(R-V) + (1-\beta)z^2 S(R-S) \geqslant \frac{I}{\pi}$, implying that for the same values of S and V (the ones that solve problem P7) the constraint is slack, since $z^2 > zk$. Moreover, the expected utility for the VC contract is $U^{VC} = \pi\{\beta\frac{V^2}{2} + (1-\beta)\frac{k^2 S^2}{2}\}$, while for the non-VC contract it is $U^{NVC} = \pi\{\beta\frac{V^2}{2} + (1-\beta)\frac{z^2 S^2}{2}\}$. Since $z^2 > k^2$, we have $U^{NVC} > U^{VC}$.

When $G \geq \max\{C_2, H_2\}$, the optimal contracts are the largest root of the following equations,

$$\begin{aligned} for\ patent\ and\ no\ license &: \quad [\beta + (1-\beta)k^2]\hat{\hat{R}}(R - \hat{\hat{R}}) = \frac{I}{\pi} - (1-\beta)G \\ for\ patent\ and\ license &: \quad k^2\hat{\hat{R}}^V(R - \hat{\hat{R}}^V) = \frac{I}{\pi} - \beta L - (1-\beta)G \end{aligned}$$

Therefore, when $\frac{\frac{I}{\pi}-(1-\beta)G}{\beta+(1\beta)k^2} = \frac{\frac{I}{\pi}-\beta L-(1-\beta)G}{k^2}$, the optimal contracts with and without licensing provide the same share of final returns to the entrepreneur, that is, $L = \frac{(1-k^2)[I-(1-\beta)\pi G]}{\pi[\beta+(1-\beta)k^2]} \equiv \hat{L}$.

The condition $G \geq \max\{C_2, H_2\}$ implies that

$$G \geq \frac{k(z-k)}{\beta + (1-\beta)kz}\frac{I}{\pi} \tag{14}$$

$$G \geq \frac{z-k}{\beta k + (1-\beta)z}(\frac{I}{\pi} - \beta L), \tag{15}$$

where $L = \hat{L}$. Inequality (14) implies that

$$G \geq \frac{z-k}{\beta k + (1-\beta)z}\frac{I}{\pi} - \frac{(z-k)\beta}{\beta k + (1-\beta)z}\frac{(1-k^2)[\frac{I}{\pi} - (1-\beta)G]}{\beta + (1-\beta)k^2},$$

which gives us

$$G \geq \frac{(z-k)k^2}{[\beta k + (1-\beta)z][\beta + (1-\beta)k^2] - (z-k)\beta(1-k^2)(1-\beta)}\frac{I}{\pi} \tag{16}$$

Therefore, as long as

$$\begin{aligned} G &\geq \max\{C_2, H_2(L = \hat{L})\} \\ &= \max\{\frac{k(z-k)}{\beta + (1-\beta)kz}\frac{I}{\pi}, \frac{(z-k)k^2}{[\beta k + (1-\beta)z][\beta + (1-\beta)k^2] - (z-k)\beta(1-k^2)(1-\beta)}\frac{I}{\pi}\}, \end{aligned}$$

i.e., $G \in \Phi$, where $\Phi = [\max\{C_2, H_2(L = \hat{L})\}, +\infty)$ then when $L = \hat{L}$, the optimal contracts with and without licensing provide the same share of final returns to the entrepreneur, while the licensing contract implies a lower success probability, thus the contract without licensing is preferred.

Then we have

$$U_P(G \;\; \geq \;\; \max\{C_2, H_2(L = \hat{L})\}) > U_{PL}(G \geq \max\{C_2, H_2(L = \hat{L})\})$$

Cases	Contract Equation	Condition for I, G
I. Outward knowledge transfer	$k^2 R_e^{VN}(R - R_e^{VN}) + G = \frac{I}{\pi}$ (e) $R_e^{VN} = R$ (r)	$\frac{I}{\pi} > G \geq \frac{\omega - k}{\omega}\frac{I}{\pi}$ (o) $G \geq \frac{I}{\pi}$ (m)
II. Inward and outward knowledge transfer	$k^2 R_e^{VN}(R - R_e^{VN}) + G = max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ (a) $R_e^{VN} = R$ (b)	$max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\} > G \geq \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ $G \geq max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$
III. Inward knowledge transfer	$\omega^2 R_e^{VN}(R - R_e^{VN}) = max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ (c) $(\omega - k)\omega R_e^{VN}(R - R_e^{VN}) = G$ (d)	$G < \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ $G \geq \frac{\omega-k}{\omega} max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$
IV. No knowledge transfer	$\omega^2 R_e^{NN}(R - R_e^{NN}) = \frac{I}{\pi}$ (f) $(\omega - k)\omega R_e^{NN}(R - R_e^{NN}) = G$ (g)	$G < \frac{\omega-k}{\omega}\frac{I}{\pi}$ (p) $G \geq \frac{\omega-k}{\omega}\frac{I}{\pi}$ (q)

Cases	Condition for I, G	Utility
I.Outward knowledge transfer	$\pi[\frac{1}{4}k^2R^2 + G] \geq I \& \frac{I}{\pi} \leq \frac{C}{\tau}$ (k) $\frac{I}{\pi} \leq \frac{C}{\tau}$ (n)	$U_1 = \frac{\pi}{2}k^2(R_e^{VN})^2$
II. Inward and outward knowledge transfer	$(\pi + \tau)[\frac{1}{4}k^2R^2 + G] \geq I + C$ (i)	$U_2 = \frac{(\pi+\tau)}{2}k^2(R_e^{VN})^2$
III. Inward knowledge transfer	$\frac{1}{4}\omega R^2 \geq max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\}$ (j) $G \leq \frac{1}{4}(\omega - k)\omega R^2$	$U_3 = \frac{(\pi+\tau)}{2}\omega^2(R_e^{VN})^2$
IV. No knowledge transfer	$\frac{\pi\omega^2R^2}{4} \geq I$ & $\frac{I}{\pi} \leq \frac{C}{\tau}$ (l) $G \leq \frac{1}{4}(\omega - k)\omega R^2$ & $\frac{I}{\pi} \leq \frac{C}{\tau}$ (s)	$U_4 = \frac{\pi\omega^2}{2}(R_e^{NN})^2$

However, $\frac{\partial U_{PL}}{\partial L} > 0$, $\frac{\partial U_P}{\partial L} = 0$. Therefore, there exists a cutoff value $L^{\#}$, where $L^{\#} > \hat{L}$, such that when $L > L^{\#}$, $U_{PL} > U_P$. ■

6.11 Proof of Proposition 1

Proof. The optimal contract for VC-investor with non-patentable knowledge in different cases are listed in the above table. For simplification, in the following, we call case "Outward knowledge transfer", "Inward and outward knowledge transfer", "Inward knowledge transfer", and "No knowledge transfer" as Case I, Case II, Case III, Case IV respectively.

Then it's easy to see that:

1. when $\frac{C}{\tau} < \frac{I}{\pi}$ $(i.e., \frac{c}{\tau} < \frac{I+C}{\pi+\tau})$, $\Rightarrow max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\} = \frac{I+C}{\pi+\tau}$, Case I & IV will not happen since condition (k), (n), (l) and (s) are violated.

    - when $\frac{I}{\pi} > G \geq \frac{I+C}{\pi+\tau}$, VC will choose between case II(b) (inward and outward knowledge transfer) and case III(d) (inward knowledge transfer);

      The entrepreneur will chooose case II(b) over case III(d) iff $\triangle U_{23} = U_2 - U_3 \geq 0$.

That's to say,

$$\triangle U_{23} = \frac{(\pi+\tau)}{2}[k^2R^2 - \omega^2(R_e^{VN})^2] \geq 0, \tag{17}$$

where R_e^{VN} are the optimal contracts determined by equation (d).

The expression (17) implies that only when $R_e^{VN} \leq \frac{k}{\omega}R$, case II(b) is favorable over case III(d). That means

$$\frac{R + \sqrt{R^2 - \frac{4G}{(\omega-k)\omega}}}{2} \leq \frac{k}{\omega}R.$$

Rearrange the above inequality, it gives us

$$\begin{aligned} R^2 - \frac{4G}{(\omega-k)\omega} &\leq (\frac{2k}{\omega} - 1)^2 R^2 \\ \frac{4G}{(\omega-k)\omega R^2} &\geq \frac{4k}{\omega}\frac{k-\omega}{\omega} \\ G &\geq \frac{k(\omega-k)^2R^2}{\omega}. \end{aligned}$$

Therefore, for $\frac{I}{\pi} > G \geq \max\{\frac{I+C}{\pi+\tau}, \frac{k}{\omega}(\omega-k)^2R^2\}$, VC will choose case II(b).

If $\frac{I+C}{\pi+\tau} \geq \frac{k}{\omega}(\omega-k)^2R^2$, then for $\frac{I}{\pi} > G \geq \frac{I+C}{\pi+\tau}$, VC prefers inward and outward knowledge transfer.

If $\frac{I+C}{\pi+\tau} < \frac{k}{\omega}(\omega-k)^2R^2$, then for $\frac{I}{\pi} > G \geq \frac{k}{\omega}(\omega-k)^2R^2$, VC prefers inward and outward knowledge transfer. For $\frac{k}{\omega}(\omega-k)^2R^2 > G \geq \frac{I+C}{\pi+\tau}$, VC prefers inward knowledge transfer.

- when $\frac{I+C}{\pi+\tau} > G \geq \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, VC will choose between case II (a) and case III (d).

  At point $G = \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, $R_e^{VN}(R - R_e^{VN}) = \frac{1}{\omega k}\frac{I+C}{\pi+\tau} > R_e^{VN}(R - R_e^{VN}) = \frac{1}{\omega^2}\frac{I+C}{\pi+\tau}$, $\triangle U_{23} = U_2 - U_3 < 0$;

  When $G = \frac{I+C}{\pi+\tau}$, we have two cases to consider according to the above results:

  - if $\frac{I+C}{\pi+\tau} \geq \frac{k}{\omega}(\omega-k)^2R^2$, $\triangle U_{23} > 0$ at point $G = \frac{I+C}{\pi+\tau}$. Therefore, there exists a cutoff value G^*, $\frac{I+C}{\pi+\tau} > G^* > \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, such that $\triangle U_{23}(G^*) = 0$. For any $\frac{I+C}{\pi+\tau} > G > G^*$, VC will choose inward and outward knowledge transfer; $G^* > G > \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, VC will choose inward knowledge transfer.
  - $\frac{I+C}{\pi+\tau} < \frac{k}{\omega}(\omega-k)^2R^2$, $\triangle U_{23} < 0$ at point $G = \frac{I+C}{\pi+\tau}$. VC will choose inward knowledge transfer for the whole interval $\frac{I+C}{\pi+\tau} > G \geq \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$.

Therefore, we can say, when $\frac{C}{\tau} < \frac{I}{\pi}$, VC will always transfer knowledge ex ante. When $\frac{I+C}{\pi+\tau} \geq \frac{k}{\omega}(\omega-k)^2R^2$, there exists a cutoff value G^*, where $\frac{I+C}{\pi+\tau} > G^* > \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$,(When

$\frac{I+C}{\pi+\tau} < \frac{k}{\omega}(\omega-k)^2R^2$, there exists a cutoff value $\ddot{G}$, where $\ddot{G} = \frac{k}{\omega}(\omega-k)^2R^2 > \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$) such that VC is indifferent between transfer knowledge ex post or not,

  - when $G > G^*(\ddot{G})$, VC will choose advice and transfer knowledge;
  - when $G < G^*(\ddot{G})$, VC will choose advice without knowledge transfer.

- when $G < \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, VC will choose to inward knowledge transfer (Case III) as (c) suggested.

2. when $\frac{C}{\tau} > \frac{I}{\pi}$ $(i.e., \frac{C}{\tau} > \frac{I+C}{\pi+\tau})$, $\Rightarrow max\{\frac{C}{\tau}, \frac{I+C}{\pi+\tau}\} = \frac{C}{\tau}$

   - when $G \geq \frac{\omega-k}{\omega}\frac{C}{\tau}$, VC will choose among case I, case II, case III(d) and case IV(g) (we will study the conditions of each choice in the following Situation 1).
   - when $\frac{\omega-k}{\omega}\frac{I}{\pi} < G < \frac{\omega-k}{\omega}\frac{C}{\tau}$, VC will choose among case I, case III(c) and case IV(g) (we will study the conditions of each choice in the following Situation 2).
   - when $G < \frac{\omega-k}{\omega}\frac{I}{\pi}$, VC will choose among case III(c) and case IV(f) (we will study the conditions of each choice in the following Situation 3).

Now in the following, we will study the three situations:

- Situation 1: when $\frac{C}{\tau} > \frac{I}{\pi}$ & $G \geq \frac{\omega-k}{\omega}\frac{C}{\tau}$, case IV(g) is dominated by case III(d), as the binding incentive constraint of VC give the same final return to entrepreneur while with advice ex ante, the innovation success probability increases by τ. We know from the above derivation that when $G = \frac{\omega-k}{\omega}\frac{C}{\tau}$, $\triangle U_{23} = U_2 - U_3 < 0$; and when $G = \frac{C}{\tau}$,

  $\triangle U_{23} = U_2 - U_3 = \frac{\pi+\tau}{2}[k^2R^2 - \omega^2(R_e^{VN})^2]$, where $R_e^{VN}(R - R_e^{VN}) = \frac{G}{(\omega-k)\omega}$; Similar to the above derivation when $\frac{C}{\tau} < \frac{I}{\pi}$, as long as $G \geq \frac{k(\omega-k)^2R^2}{\omega}$, i.e. $\frac{C}{\tau} \geq \frac{k(\omega-k)^2R^2}{\omega}$, $U_{23} > 0$. Therefore, there exists a cutoff value $G^{**}(\frac{C}{\tau} > G^{**} > \frac{\omega-k}{\omega}\frac{C}{\tau})$, when $G > G^{**}$, case II is preferable to case III(d), while when $\frac{\omega-k}{\omega}\frac{C}{\tau} \leq G < G^{**}$, case III(d) is favorable to case II.

  Therefore, for $G > G^{**}$, we need to compare the expected revenue between case II and case I;

  - when $G \geq \frac{C}{\tau}$, $R_e^{VN} = R$ in both case II and case I. But $U_2 > U_1$ as the probability of innovation success increases in case II.
  - when $G^{**} < G < \frac{C}{\tau}$, VC will choose case II(a) over case I(e) iff $\triangle U = U_2 - U_1 \geq 0$. That's to say,

$$\triangle U = \frac{(\pi+\tau)}{2}k^2(R_e^{VN})^2 - \frac{\pi}{2}k^2(R_e^{VN})^2 \geq 0, \tag{18}$$

where R_e^{VN} & R_e^{VN} are the optimal contracts determined by equation (a) and (e) respectively.

(a) can be rewritten as:

$$(R_e^{VN})^2 = RR_e^{VN} - \frac{\frac{C}{\tau} - G}{k^2} \tag{19}$$

and (c) could be rewritten as:

$$(R_e^{VN})^2 = RR_e^{VN} - \frac{\frac{I}{\pi} - G}{k^2} \tag{20}$$

It's easy to see that as long as $\frac{C}{\tau}$ is smaller enough, such that close to $\frac{I}{\pi}$, then the optimal contracts in case I and case II converge. Then case II is favorable to case I as with inward knowledge transfer ex ante, the probability of innovation success increases by τ.

Therefore, we could say that, when $G > G^{**}$, the optimal contract will always induce the VC to transfer knowledge ex post. For $\frac{C}{\tau}$ below a cutoff value, the contract will also induce the VC to transfer knowledge ex ante.

Similarly, when $\frac{\omega-k}{\omega}\frac{C}{\tau} \leq G < G^{**}$, VC will choose case III(d) over case I(e) iff $\triangle U = U_3 - U_1 \geq 0$. That's to say,

$$\triangle U_{31} = \frac{\pi+\tau}{2}\omega^2(R_e^{VN})^2 - \frac{\pi}{2}k^2(R_e^{VN})^2 \geq 0,$$

where $R_e^{VN}(R - R_e^{VN}) = \frac{G}{\omega(\omega-k)}$, *and* $R_e^{VN}(R - R_e^{VN}) = \frac{\frac{I}{\pi}-G}{k^2}$. It's clear to see that $\frac{d\triangle U_{31}}{dG} < 0$, $\frac{d\triangle U_{31}}{dI} > 0$, $\frac{d\triangle U_{31}}{d\tau} > 0$, $\frac{d\triangle U_{31}}{d\mu} > 0$, $\frac{d\triangle U_{31}}{dC} = 0$.

- Situation 2: when $\frac{C}{\tau} > \frac{I}{\pi}$ & $\frac{\omega-k}{\omega}\frac{I}{\pi} \leq G < \frac{\omega-k}{\omega}\frac{C}{\tau}$, we can find that at point $G = \frac{\omega-k}{\omega}\frac{I}{\pi}$, case I(e) is dominated by case IV(g) since $R_e^{NN}(R - R_e^{NN}) = \frac{G}{\omega(\omega-k)} < R_e^{VN}(R - R_e^{VN}) = \frac{\frac{I}{\pi}-G}{k^2}(\Rightarrow R_e^{VN} < R_e^{NN})$. At point $G = \frac{\omega-k}{\omega}\frac{C}{\tau}$, $\triangle U_{14} = \frac{\pi}{2}k^2(R_e^{VN})^2 - \frac{\pi}{2}\omega^2(R_e^{NN})^2$, where $R_e^{VN}(R - R_e^{VN}) = \frac{\frac{I}{\pi}-G}{k^2}$, $R_e^{NN}(R - R_e^{NN}) = \frac{G}{\omega(\omega-k)}$. And we know that $\frac{d\triangle U_{14}}{dG} < 0$, therefore, for the parameters that satisfies $\triangle U_{14}(G = \frac{\omega-k}{\omega}\frac{C}{\tau}) > 0$, there must exists a point G^{***}, where $G^{***} < \frac{\omega-k}{\omega}\frac{C}{\tau}$, for $G < G^{***}$, case IV(g) is preferred, while for $\frac{\omega-k}{\omega}\frac{C}{\tau} > G > G^{***}$, case I(e) is preferred.

  For $\frac{\omega-k}{\omega}\frac{C}{\tau} > G > G^{***}$, VC will choose inward knowledge transfer iff $\triangle U_{23} = U_2 - U_3 \geq 0$.

$$\triangle U_{23} = \frac{(\pi+\tau)}{2}\omega^2(R_e^{VN})^2 - \frac{\pi}{2}k^2(R_e^{VN})^2 \geq 0, \tag{21}$$

where $R_e^{VN}(R - R_e^{VN}) = \frac{C}{\tau\omega^2}$, $R_e^{VN}(R - R_e^{VN}) = \frac{\frac{I}{\pi} - G}{k^2}$. Therefore, $\frac{d\triangle U_{23}}{dG} > 0$, $\frac{d\triangle U_{23}}{d\tau} > 0$, $\frac{d\triangle U_{23}}{dC} > 0$, $\frac{d\triangle U_{23}}{d\mu} < 0$, $\frac{d\triangle U_{23}}{dI} > 0$.

Combining the interval $\frac{\omega-k}{\omega}\frac{C}{\tau} > G > G^{***}$ and $\frac{\omega-k}{\omega}\frac{C}{\tau} < G < G^{**}$, we could say that, when $G^{**} > G > G^{***}$, the optimal contract will either induce knowledge transfer ex ante or induce knowledge transfer ex post, depending on the magnitude of the parameters, $\frac{C}{\tau}$, $G,\ k\ and\ w$.

For $\frac{\omega-k}{\omega}\frac{I}{\pi} < G < G^{***}$, VC will choose knowledge transfer ex ante (case III) rather than no knowledge transfer (case IV) iff $\triangle U_{34} = U_3 - U_4 \geq 0$.

$$\triangle U_{34} = \frac{(\pi+\tau)}{2}\omega^2(R_e^{VN})^2 - \frac{\pi}{2}\omega^2(R_e^{NN})^2 \geq 0, \tag{22}$$

where $R_e^{VN}(R - R_e^{VN}) = \frac{C}{\tau\omega^2}$, $R_e^{NN}(R - R_e^{NN}) = \frac{G}{\omega(\omega-k)}$. Therefore, $\frac{d\triangle U_{34}}{dG} > 0$, $\frac{d\triangle U_{34}}{d\tau} > 0$, $\frac{d\triangle U_{34}}{dC} < 0$.

- Situation 3: when $\frac{C}{\tau} > \frac{I}{\pi}$ & $G < \frac{\omega-k}{\omega}\frac{I}{\pi}$, $R_e^{NN} > R_e^{VN}$ as $\frac{G}{\omega(\omega-k)} < \frac{\omega-k}{\omega}\frac{I}{\pi}\frac{1}{\omega(\omega-k)} = \frac{1}{\omega^2}\frac{I}{\pi} < \frac{1}{\omega^2}\frac{C}{\tau}$. However U_3 may not be less than U_4 since inward knowledge transfer increases the success probability of innovation development by τ.

  VC will choose to inward knowledge transfer rather than do nothing iff $\triangle U_{34} = U_3 - U_4 \geq 0$.

$$\triangle U_{34} = \frac{(\pi+\tau)}{2}\omega^2(R_e^{VN})^2 - \frac{\pi}{2}\omega^2(R_e^{NN})^2 \geq 0, \tag{23}$$

where R_e^{VN} & R_e^{NN} are the optimal contracts determined by equation (c) and (g) respectively. There must exist a series of parameter set $\Theta^{**} = \{I^{**},\ C^{**},\ \tau^{**},\ \pi^{**}\}$ such that $U_3 = U_4$. Θ^{**} is determined by letting equation (23) hold with equality. Similarly, we could calculate the derivatives of $\triangle U_{34}$ on different parameters, and we have

$$\begin{aligned}
\frac{d\triangle U_{34}}{dI} &= 0 - \pi\omega^2 R_e^{NN}\frac{\partial R_e^{NN}}{\partial I} > 0 \\
\frac{d\triangle U_{34}}{dC} &= (\pi+\tau)\omega^2 R_e^{VN}\frac{\partial R_e^{VN}}{\partial C} < 0 \\
\frac{d\triangle U_{34}}{d\tau} &= \frac{\omega^2(R_e^{VN})^2}{2} + (\pi+\tau)\omega^2 R_e^{VN}\frac{\partial R_e^{VN}}{\partial \tau} > 0 \\
\frac{d\triangle U_{34}}{d\pi} &= \frac{1}{2}[\omega^2(R_e^{VN})^2 - \omega^2(R_e^{NN})^2] - \pi\omega^2 R_e^{NN}\frac{\partial R_e^{NN}}{\partial \pi} < 0
\end{aligned}$$

That's to say, combining the intervals $\frac{\omega-k}{\omega}\frac{I}{\pi} < G < G^{***}$ and $G < \frac{\omega-k}{\omega}\frac{I}{\pi}$, i.e. when $G < G^{***}$, the optimal contract will never induce knowledge transfer ex post. It will induce knowledge transfer ex ante for $\frac{C}{\tau}$ below a threshold value.

To determine the relative size of G^* and G^{**}, remember that both of which are the cutoff value such that $U_2 = U_1$, each corresponding to different case: $\frac{C}{\tau} < (>) \frac{I}{\pi}$. Therefore, G^* satisfies that

$$\triangle U_{12} = \frac{\pi+\tau}{2} k^2 (\frac{R + \sqrt{R - 4\frac{\frac{I+C}{\pi+\tau} - G^*}{k^2}}}{2})^2 - \frac{\pi+\tau}{2}\omega^2(\frac{R + \sqrt{R - 4\frac{G^*}{(\omega-k)\omega}}}{2})^2 = 0$$

while G^{**} satisfies that

$$\triangle U_{12} = \frac{\pi+\tau}{2} k^2 (\frac{R + \sqrt{R - 4\frac{\frac{C}{\tau} - G^{**}}{k^2}}}{2})^2 - \frac{\pi+\tau}{2}\omega^2(\frac{R + \sqrt{R - 4\frac{G^{**}}{(\omega-k)\omega}}}{2})^2 = 0$$

Since $\frac{d\triangle U_{12}}{dG} > 0$, while increasing $\frac{C}{\tau}(or\ \frac{I+C}{\pi+\tau})$ will decrease $\triangle U_{12}$, therefore, to keep $\triangle U_{12}$ unchanged, increasing $\frac{C}{\tau}(or\ \frac{I+C}{\pi+\tau})$ requires increase G. Suppose $\frac{I}{\pi}$ remain the same in both case($\frac{C}{\tau} < (>)\frac{I}{\pi}$), then if $\frac{C}{\tau} > \frac{I}{\pi}$, it implies $\frac{I+C}{\pi+\tau} < \frac{C}{\tau}$, therefore, $G^{**} > G^*$; if $\frac{C}{\tau} < \frac{I}{\pi}$, it implies $\frac{I+C}{\pi+\tau} > \frac{C}{\tau}$, therefore, $G^{**} < G^*$.

■

6.12 Proof of Proposition 2

Proof. To investigate the condition under which the VC investor is preferred over non-VC investor, we need to consider different situations:

- When $\frac{C}{\tau} < \frac{I}{\pi}$,

  1. when $G \leq \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, VC will choose between case III and case IV. Basically, case IV is equivalent to case O (non-VC case). Therefore, we could omit the comparision of case IV and case O. Let's compare case III with case O only. Similar to the above case, the benefit from case III investor over case O can be denoted as:

$$\triangle U_{30} = U_3 - U_0 = \frac{(\pi+\tau)}{2}\omega^2(R_e^{VN})^2 - \frac{\pi}{2}\omega^2(R_e^N)^2$$
$$= \frac{(\pi+\tau)}{2}\omega^2(\frac{R + \sqrt{R^2 - 4\frac{I+C}{(\pi+\tau)\omega^2}}}{2})^2 - \frac{\pi}{2}\omega^2(\frac{R + \sqrt{R^2 - 4\frac{I}{\pi\omega^2}}}{2})^2$$

     It's easy to see that $R_e^{VN} > R_e^N$ because of $\frac{I+C}{\pi+\tau} < \frac{I}{\pi}$. Therefore, $\triangle U_{30} > 0$, VC will be favorable than non-VC. And the optimal contract induces VC to choose knowledge transfer ex ante.

  2. when $G > \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$,

- if $G^* > G > \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, from Proposition 1, we know that VC will choose knowledge transfer ex ante with binding IC_{VC} *ex post* (case III(d)); Additionally, the expected payoff of the entrepreneur in this case is inferior to that in case III(c) when $G < \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$.(For Proof, please see the footnote.) [24] And U_3 is decreasing in G.
- if $G > G^*$, VC will choose knowledge transfer ex ante and ex post (case II). It's clear to see that the expected payoff of the entrepreneur, U_2 is increasing in G.

Since we know that at point $G = \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, VC is favorable to non-VC, therefore,

- if $U_3(G = G^*) < U_0$ & $U_2(G = \frac{I+C}{\pi+\tau}) > U_0$[25], there exists two cutoff values $G^{\#}$, $G^{\#\#}$, where $G^* > G^{\#} > \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, $G^* < G^{\#\#} < \frac{I+C}{\pi+\tau}$, such that entrepreneur is indifferent between VC and non-VC at point $G^{\#}$ *and* $G^{\#\#}$, when $G < G^{\#}$, VC with knowledge transfer ex ante is favorable, when $G^{\#\#} > G > G^{\#}$, the entrepreneur will choose non-VC. when $G > G^{\#\#}$, the entrepreneur will choose VC with knowledge transfer ex ante and ex post.
- If $U_3(G = G^*) < U_0$ & $U_2(G = \frac{I+C}{\pi+\tau}) < U_0$, as U_2 increases with G when $G > G^*$, we can expect that there exists a cutoff value, $G^{\#\#}$, where $G^{\#\#} > \frac{I+C}{\pi+\tau}$, such that when $G^{\#\#} > G > G^{\#}$, entrepreneur will choose non-VC only; when $G > G^{\#\#}$, the entrepreneur will choose VC with knowledge transfer ex ante and ex post.
- If $U_3(G = G^*) > U_0$, then entrepreneur will always choose VC.

The conditions $U_3(G = G^*) < U_0$, $U_2(G = \frac{I+C}{\pi+\tau}) > U_0$, etc., defines the parameters(τ, μ, C) range. We will discuss it by intervals.

Therefore, in summary, we have

- when $G \leqslant G^{\#}$, VC finance is preferred. The optimal VC contract induces (only) knowledge transfer ex ante.
- when $G^* \geqslant G > G^{\#}$, the entrepreneur chooses between VC finance with (only) knowledge transfer ex ante and non-VC finance. If $\triangle U_{30}(G = G^*) = U_3 - U_0 > 0$, the entrepreneur chooses VC with knowledge transfer ex ante. As we know that, the benefit from VC investment comparing to non-VC investment can be expressed

[24] Although in case III(d), the VC participation constraint is slack, and therefore, the optimal contract entails an ex ante fee paid by VC to the entrepreneur. It still can't affect the result that with binding IC_{VC} *ex post*, entrepreneur's utility is undermined, which could be proved easily. Suppose the optimal contract at $G = \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$ is denoted as $\bar{X}$ where PC_{VC} is binding, i.e. $\omega^2\bar{X}(R-\bar{X}) = \frac{I+C}{\pi+\tau}$. Now if G increases, then the optimal contract must be decreased, denoted as $\tilde{X}$, such that IC_{VC} *ex post* must be binding, i.e., $G = (\omega-k)\omega\tilde{X}(R-\tilde{X})$. And the participation constraint changes as $\omega^2\tilde{X}(R-\tilde{X}) = \frac{I+C}{\pi+\tau} + F$. The utility of the entrepreneur changes as $\tilde{U_3} = \frac{\pi+\tau}{2}\omega^2\tilde{X}^2 + F = \frac{\pi+\tau}{2}\omega^2\tilde{X}^2 + \omega^2\tilde{X}(R-\tilde{X}) - \frac{I+C}{\pi+\tau} = \frac{\pi+\tau}{2}\omega^2\tilde{X}^2 + \omega^2\tilde{X}(R-\tilde{X}) - \omega^2\bar{X}(R-\bar{X})$. Comparing to the utility at point $G = \frac{\omega-k}{\omega}\frac{I+C}{\pi+\tau}$, $\bar{U}_3 = \frac{\pi+\tau}{2}\omega^2\bar{X}^2$. It's easy to see that $\bar{U}_3 - \tilde{U_3} > 0$.

[25] Note that at point $G = \frac{I+C}{\pi+\tau}$, $U_3 = U_2$

as

$$\triangle U_{30}(G = G^*) = U_3 - U_0 = \frac{\pi + \tau}{2}\omega^2(R_e^{VN})^2 - \frac{\pi}{2}\omega^2(R_e^N)^2$$

where $R_e^{VN}(R - R_e^{VN}) = \frac{I+C}{(\pi+\tau)\omega^2} = \frac{G}{(\omega-k)\omega}$, $R_e^N(R - R_e^N) = \frac{I}{\pi\omega^2}$. It's easy to see that higher τ and lower $C/G/\mu$ favoring VC finance.

- when $G^{\#\#} \geqslant G > G^*$, the entrepreneur chooses between non-VC finance and VC finance with knowledge transfer ex ante and ex post. If $\triangle U_{20}(G = G^*) = U_2 - U_0 = \frac{\pi+\tau}{2}k^2(R_e^{VN})^2 - \frac{\pi}{2}\omega^2(R_e^N)^2 > 0$, i.e. with higher $\tau/G/\mu$ and lower C, the entrepreneur chooses VC with knowledge transfer ex ante.
- when $G > G^{\#\#}$, VC finance is preferred. The optimal VC contract induces knowledge transfer ex ante and ex post.

- When $\frac{C}{\tau} > \frac{I}{\pi}$, when $G < \frac{\omega-k}{\omega}\frac{I}{\pi}$, as we see from Proposition 1 that in this case, VC will choose between no knowledge transfer(Case IV) and knowledge transfer ex ante(case III). Case IV is equivalent to case O (non-VC investor). When $G^{***} > G > \frac{\omega-k}{\omega}\frac{I}{\pi}$, VC might also choose case IV with binding IC ex post, which implies that as G increases, the final returns goes to the entrepreneur decreases, so does the expected payoff, until the point G^{***}, where VC is indifferent between knowledge transfer ex post (case I) and no knowledge transfer (case IV). Therefore, we know that $U_4(G = G^{***}) < U_4(G < \frac{\omega-k}{\omega}\frac{I}{\pi}) = U_0$. For $G > G^{***}$, case IV is dominated by case I and the expected payoffs to the entrepreneur from case I is increasing with G.

  If $U_1(G = G^{**}) < U_0$, then there exists a cutoff value $G^{\#\#\#}$, where $G^{\#\#\#} > G^{**}$, and $U_3(G = G^{\#\#\#}) = U_0$, such that when $G > G^{\#\#\#}$, Case I is favorable to non-VC case, otherwise, vice versa. Therefore, when $G < G^{**}$, the entrepreneur choose between VC with knowledge transfer ex ante(case III) and non-VC; when $G^{\#\#\#} > G > G^{**}$, the entrepreneur choose between VC with knowledge transfer ex ante & ex post (case II) and non-VC, when $G > G^{\#\#\#}$, the entrepreneur choose VC, and VC will choose between knowledge transfer ex ante & ex post and knowledge transfer ex post.

  If $U_1(G = G^{**}) > U_0$, then there exists a cutoff value $G^{\#\#\#\#}$, where $G^{**} > G^{\#\#\#\#} > G^{***}$, and $U_1(G = G^{\#\#\#\#}) = U_0$, such that when $G > G^{\#\#\#\#}$, Case I is favorable to non-VC case, otherwise, vice versa. Therefore, when $G < G^{\#\#\#\#}$, the entrepreneur choose between VC with knowledge transfer ex ante(case III) and non-VC; when $G^{**} > G > G^{\#\#\#\#}$, the entrepreneur choose VC, and VC will choose between knowledge transfer ex ante and ex post; when $G > G^{**}$, the entrepreneur will choose VC, and VC will choose between knowledge transfer ex ante & ex post and knowledge transfer ex post.

There exists a value μ^*, such that $U_1(G = G^{**}) = U_0$, since $\frac{d\triangle U_{10}}{d\mu} > 0$, it implies that when $U_1(G = G^{**}) < U_0$, $\mu < \mu^*$.

Therefore, if $\mu < \mu^*$,

- when $G < G^{**}$, the entrepreneur's utility difference from choosing VC with knowledge transfer ex ante (case III) and non-VC (case O) can be expressed as:

  when $G < \frac{\omega - k}{\omega}\frac{c}{\tau}$, for case III, VC's IC will not be binding, therefore,

$$\triangle U_{30} = U_3 - U_0 = \frac{\pi + \tau}{2}\omega^2 (R_e^{VN})^2 - \frac{\pi}{2}\omega^2 (R_e^N)^2$$
$$= \frac{\pi}{2}k^2 (\frac{R + \sqrt{R^2 - 4\frac{C}{\tau\omega^2}}}{2})^2 - \frac{\pi}{2}\omega^2 (\frac{R + \sqrt{R^2 - 4\frac{I}{\pi\omega^2}}}{2})^2$$

  It's easy to see that $\frac{\partial \triangle U_{30}}{\partial C} < 0$, $\frac{\partial \triangle U_{30}}{\partial \tau} > 0$. With higher τ and lower C, VC will be favorable than non-VC.

  when $G > \frac{\omega - k}{\omega}\frac{c}{\tau}$, for case III, VC's IC will be binding, therefore,

$$\triangle U_{30} = U_3 - U_0 = \frac{\pi + \tau}{2}\omega^2 (R_e^{VN})^2 - \frac{\pi}{2}\omega^2 (R_e^N)^2$$
$$= \frac{\pi}{2}k^2 (\frac{R + \sqrt{R^2 - 4\frac{G}{\omega(\omega - k)}}}{2})^2 - \frac{\pi}{2}\omega^2 (\frac{R + \sqrt{R^2 - 4\frac{I}{\pi\omega^2}}}{2})^2$$

  It's easy to see that $\frac{\partial \triangle U_{30}}{\partial G} < 0$. With lower G, VC will be favorable than non-VC.

- when $G^{\#\#\#} > G > G^{**}$, the entrepreneur's utility difference from choosing VC with knowledge transfer ex ante & ex post (case II) and non-VC (case O) is

$$\triangle U_{20} = U_2 - U_0 = \frac{\pi + \tau}{2}k^2 (R_e^{VN})^2 - \frac{\pi}{2}\omega^2 (R_e^N)^2$$
$$= \frac{\pi + \tau}{2}k^2 (\frac{R + \sqrt{R^2 - 4\frac{\frac{C}{\tau} - G}{k^2}}}{2})^2 - \frac{\pi}{2}\omega^2 (\frac{R + \sqrt{R^2 - 4\frac{I}{\pi\omega^2}}}{2})^2$$

  It's easy to see that $\frac{\partial \triangle U_{20}}{\partial C} < 0$, $\frac{\partial \triangle U_{20}}{\partial \tau} > 0$, and $\frac{\partial \triangle U_{20}}{\partial G} > 0$,. With higher τ, G and lower C, VC will be favorable than non-VC.

- when $G > G^{\#\#\#}$, the entrepreneur's utility difference from choosing VC's knowledge transfer ex ante& ex post (case II) and knowledge transfer ex post (case I) can be expressed as below:

$$\triangle U_{21} = U_2 - U_1 = \frac{\pi+\tau}{2}k^2(R_e^{VN})^2 - \frac{\pi}{2}k^2(R_e^{VN})^2$$
$$= \frac{\pi+\tau}{2}k^2\left(\frac{R+\sqrt{R^2-4\frac{\frac{C}{\tau}-G}{k^2}}}{2}\right)^2 - \frac{\pi}{2}k^2\left(\frac{R+\sqrt{R^2-4\frac{\frac{I}{\pi}-G}{k^2}}}{2}\right)^2$$

It's easy to see that $\frac{\partial\triangle U_{21}}{\partial C} < 0$, $\frac{\partial\triangle U_{21}}{\partial\tau} > 0$. With higher τ and lower C, VC will choose knowledge transfer ex ante & ex post.

If $\mu \geq \mu^*$,

- when $G < G^{\#\#\#\#}$,
  * when $G^{\#\#\#\#} < \frac{\omega-k}{\omega}\frac{C}{\tau}$, the entrepreneur's utility difference from choosing VC with knowledge transfer ex ante (case III(c)) and non-VC (case O) can be expressed as below:

$$\triangle U_{30} = U_3 - U_0 = \frac{\pi+\tau}{2}\omega^2(R_e^{VN})^2 - \frac{\pi}{2}\omega^2(R_e^{N})^2$$
$$= \frac{\pi+\tau}{2}\omega^2\left(\frac{R+\sqrt{R^2-4\frac{\frac{C}{\tau}}{\omega^2}}}{2}\right)^2 - \frac{\pi}{2}\omega^2\left(\frac{R+\sqrt{R^2-4\frac{I}{\pi\omega^2}}}{2}\right)^2$$

    It's easy to see that $\frac{\partial\triangle U_{30}}{\partial C} < 0$, $\frac{\partial\triangle U_{30}}{\partial\tau} > 0$. With higher τ, and lower C, VC will be favorable than non-VC.
  * when $G^{\#\#\#\#} > \frac{\omega-k}{\omega}\frac{C}{\tau}$,
    then when $G^{\#\#\#\#} > G > \frac{\omega-k}{\omega}\frac{C}{\tau}$,
    the entrepreneur's utility difference from choosing VC with knowledge transfer ex ante (case III(d)) and non-VC (case O) can be expressed as below:

$$\triangle U_{30} = U_3 - U_0 = \frac{\pi+\tau}{2}\omega^2(R_e^{VN})^2 - \frac{\pi}{2}\omega^2(R_e^{N})^2$$
$$= \frac{\pi+\tau}{2}\omega^2\left(\frac{R+\sqrt{R^2-4\frac{G}{\omega(\omega-k)}}}{2}\right)^2 - \frac{\pi}{2}\omega^2\left(\frac{R+\sqrt{R^2-4\frac{I}{\pi\omega^2}}}{2}\right)^2$$

    It's easy to see that $\frac{\partial\triangle U_{30}}{\partial\tau} > 0$, $\frac{\partial\triangle U_{30}}{\partial G} < 0$. With higher τ and lower G, VC will be favorable than non-VC.
    when $G < \frac{\omega-k}{\omega}\frac{C}{\tau}$,
    the entrepreneur's utility difference from choosing VC with knowledge ex ante

(case III(c)) and non-VC (case O) can be expressed as below:

$$\triangle U_{30} = U_3 - U_0 = \frac{\pi+\tau}{2}\omega^2(R_e^{VN})^2 - \frac{\pi}{2}\omega^2(R_e^N)^2$$
$$= \frac{\pi+\tau}{2}\omega^2(\frac{R+\sqrt{R^2-4\frac{\frac{C}{\tau}}{\omega^2}}}{2})^2 - \frac{\pi}{2}\omega^2(\frac{R+\sqrt{R^2-4\frac{I}{\pi\omega^2}}}{2})^2$$

It's easy to see that $\frac{\partial \triangle U_{30}}{\partial C} < 0$, $\frac{\partial \triangle U_{30}}{\partial \tau} > 0$. With higher τ and lower C, VC will be favorable than non-VC.

– when $G^{**} > G > G^{\#\#\#\#}$,

when $G^{\#\#\#\#} > \frac{\omega-k}{\omega}\frac{C}{\tau}$, the entrepreneur's utility difference from choosing VC knowledge transfer ex ante (IC is binding) and knowledge transfer ex post can be expressed below:

$$\triangle U_{31} = U_3 - U_1 = \frac{\pi+\tau}{2}\omega^2(R_e^{VN})^2 - \frac{\pi}{2}k^2(R_e^{VN})^2$$
$$= \frac{\pi+\tau}{2}\omega^2(\frac{R+\sqrt{R^2-4\frac{G}{\omega(\omega-k)}}}{2})^2 - \frac{\pi}{2}k^2(\frac{R+\sqrt{R^2-4\frac{\frac{I}{\pi}-G}{k^2}}}{2})^2$$

It's easy to see that $\frac{\partial \triangle U_{31}}{\partial G} < 0$, $\frac{\partial \triangle U_{31}}{\partial \tau} > 0$. With higher τ, and lower G, VC with advice will be favorable than VC with knowledge transfer.

when $G^{\#\#\#\#} < \frac{\omega-k}{\omega}\frac{C}{\tau}$,

then when $G > \frac{\omega-k}{\omega}\frac{C}{\tau}$, the results is the same as the above case: With higher τ, and lower G, VC with knowledge transfer ex ante will be favorable than VC with knowledge transfer ex post.

then when $G^{\#\#\#\#} < G < \frac{\omega-k}{\omega}\frac{C}{\tau}$, the entrepreneur's utility difference from choosing VC with knowledge transfer ex ante (IC isn't binding) and VC with knowledge transfer ex post can be expressed below:

$$\triangle U_{31} = U_3 - U_1 = \frac{\pi+\tau}{2}\omega^2(R_e^{VN})^2 - \frac{\pi}{2}k^2(R_e^{VN})^2$$
$$= \frac{\pi+\tau}{2}\omega^2(\frac{R+\sqrt{R^2-4\frac{C}{\tau\omega^2}}}{2})^2 - \frac{\pi}{2}k^2(\frac{R+\sqrt{R^2-4\frac{\frac{I}{\pi}-G}{k^2}}}{2})^2$$

It's easy to see that $\frac{\partial \triangle U_{31}}{\partial G} < 0$, $\frac{\partial \triangle U_{31}}{\partial \tau} > 0$. With higher τ, and lower G, VC with advice will be favorable than VC with knowledge transfer.

– when $G > G^{**}$, the entrepreneur's utility difference from choosing VC's knowledge transfer ex ante & ex post (case II) andknowledge transfer ex post (case I) can be expressed as below:

$$\triangle U_{21} = U_2 - U_1 = \frac{\pi+\tau}{2}k^2(R_e^{VN})^2 - \frac{\pi}{2}k^2(R_e^{VN})^2$$
$$= \frac{\pi+\tau}{2}k^2(\frac{R+\sqrt{R^2-4\frac{\frac{C}{\tau}-G}{k^2}}}{2})^2 - \frac{\pi}{2}k^2(\frac{R+\sqrt{R^2-4\frac{\frac{I}{\pi}-G}{k^2}}}{2})^2$$

It's easy to see that $\frac{\partial \triangle U_{21}}{\partial C} < 0$, $\frac{\partial \triangle U_{21}}{\partial \tau} > 0$. With higher τ and lower C, VC will choose knowledge transfer ex ante & ex post.

■

6.13 Proof of proposition 3

Proof. The optimal contracts for not to patent, patent without licensing, and patent with licensing are respectively the largest root of the following equations:

$$\pi\omega^2 R_e^N(R - R_e^N) = I \tag{24}$$
$$\pi[\beta + (1-\beta)z^2]\hat{R}(R - \hat{R}) = I \tag{25}$$
$$\pi[\beta k^2 + (1-\beta)z^2]\hat{\hat{R}}(R - \hat{\hat{R}}) = I - \pi\beta L \tag{26}$$

The expected profit of entrepreneur for not to patent, patent without licensing, and patent with licensing can be expressed respectively as:

$$U_{NP} = \frac{\pi}{2}\omega^2(R_e^N)^2 \tag{27}$$
$$U_P = \frac{\pi}{2}[\beta + (1-\beta)z^2]\hat{R}^2 \tag{28}$$
$$U_{PL} = \frac{\pi}{2}[\beta k^2 + (1-\beta)z^2]\hat{\hat{R}}^2 \tag{29}$$

From (24) and (25), we can see that as long as $\omega^2 < \beta + (1-\beta)z^2$, $\hat{R}$ is larger than R_e^N. In this situation, (27) and (28) tell us that patent without licensing is strictly favorable than not to patent.

Therefore, when $\omega^2 < \beta + (1-\beta)z^2$, not to patent is dominated and can be ignored, we only need to focus on the two cases patent without licensing and patent with licensing: When $\hat{\hat{R}} = \hat{R}$, subtracting these two equations in both sides gives us $(1-k^2)\pi\beta\hat{R}(R-\hat{R}) = \pi\beta L$. From equation (25), we know that $\hat{R}(R-\hat{R}) = \frac{I}{\pi[\beta+(1-\beta)z^2]}$, therefore, it gives us

$$L^* = \frac{(1-k^2)I}{\pi[\beta + (1-\beta)z^2]}$$

The utility from patenting without licensing is given as $U_P = \frac{\pi}{2}[\beta + (1-\beta)z^2]\hat{R}^2$, while the utility from patenting with licensing is given as $U_{PL} = \frac{\pi}{2}[\beta k^2 + (1-\beta)z^2]\hat{\hat{R}}^2$. Since $k < 1$, if licensing is preferable, it must be $\hat{\hat{R}} > \hat{R}$. We can see from (26) that $\hat{\hat{R}}$ is monotonically increasing with L until $L = \frac{I}{\pi\beta}$. Therefore, if $U_{PL}(L = \frac{I}{\pi\beta}) < U_P$, patent without licensing will always be preferred to patent with licensing; Otherwise, there exists a cutoff value, L^N , under which $U_{PL}(L = L^N) = U_P$, and when $L > L^N$, $U_{PL} > U_P$. Define $a = [\beta + (1-\beta)z^2]$, $b = [\beta k^2 + (1-\beta)z^2]$, when $L = L^N$,

$$U_{PL} = \frac{\pi}{2}b\hat{\hat{R}}^2 = U_P = \frac{\pi}{2}a\hat{R}^2 \tag{30}$$

Equation (25) and (26) can be rewritten as

$$\pi aR\hat{R} - \pi a\hat{R}^2 = I \tag{31}$$

$$\pi bR\hat{\hat{R}} - \pi b\hat{\hat{R}}^2 = I - \pi\beta L^N \tag{32}$$

Plug equation (30) in to the above equations, and subtract them from both sides, we have $a\hat{R} - b\hat{\hat{R}} = \frac{\beta L}{R}$. Then $\hat{\hat{R}} = \frac{a}{b}\hat{R} - \frac{\beta L^N}{bR}$, where $\hat{R}$ and $\hat{\hat{R}}$ are the largest root of equation (31) and (32).

When $\omega^2 > \beta + (1-\beta)z^2$, patent without license is dominated by not to patent, and can be ignored, we only need to focus on the two cases not to patent and patent with licensing:

Similar to the above situation, we can see from (26) that $\hat{\hat{R}}$ is monotonically increasing with L until $L = \frac{I}{\pi\beta}$. Therefore, if $U_{PL}(L = \frac{I}{\pi\beta}) < U_{NP}$, not to patent will always be preferred to patent with licensing; Otherwise, there exists a cutoff value, L^P , under which $U_{PL}(L = L^P) = U_{NP}$, and when $L > L^P$, $U_{PL} > U_{NP}$.

Define $c = \omega^2$, $b = [\beta k^2 + (1-\beta)z^2]$, when $L = L^P$,

$$U_{PL} = \frac{\pi}{2}b\hat{\hat{R}}^2 = U_{NP} = \frac{\pi}{2}c(R_e^N)^2 \tag{33}$$

Equation (24) and (26) can be rewritten as

$$\pi cRR_e^N - \pi c(R_e^N)^2 = I \tag{34}$$

$$\pi bR\hat{\hat{R}} - \pi b\hat{\hat{R}}^2 = I - \pi\beta L^P \tag{35}$$

Plug equation (33) in to the above equations, and subtract them from both sides, we have $a\hat{R} - b\hat{\hat{R}} = \frac{\beta L}{R}$. Then $\hat{\hat{R}} = \frac{c}{b}R_e^N - \frac{\beta L^P}{bR}$, where R_e^N and $\hat{\hat{R}}$ are the largest root of equation (35) and (34). Since $c > b$ & $R_e^N > \hat{R}$, therefore $L^P > L^N$. ■