Federal Mandates with Local Agenda Setters[1]

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Abstract

This paper investigates the effect of local monopoly agenda setting on federal standards. Federal standards specify a minimum (or maximum) point in policy space which can be raised (or lowered) by local option. Without local agenda setters, this creates incentives for nonmajoritarian outcomes, with a tendency for policies to be too high (low). Local agenda setters may have incentives to distort these outcomes even further. We demonstrate that federal standards can counterbalance the distortions of local agenda setters.

1 Introduction

Romer and Rosenthal (1979)[1] have proposed a model of decision making, which they apply to local governments, in which a single person (or an homogeneous group) has monopoly control over the agenda: the voters must choose, through referendum, between a proposal made by this "agenda setter" and some exogeneously given "reversion policy". The policy which receives the most votes is implemented.

In their theoretical and applied work, Romer and Rosenthal focus on setups where the reversion policy is determined either exogeneously or on the basis of the past experience of the government body that they consider. However, in reality, the constraints that are imposed on the choices of local governments often stem from mandates imposed by a higher level of government, which, in this paper, we will call the federal level. Our aim is to examine the way in which these mandates will be set in such circumstances, and in particular to provide some preliminary steps towards an analysis of institutions that try to control local monopoly agenda setters by fixing reversion levels centrally.

In order to conduct this analysis, we start from the model of federal mandates introduced in Crémer and Palfrey (2000), where we use a standard spatial model, and where we assume that voting results in median outcomes at both the federal and the local levels, for instance because political parties compete for voters in a Hotelling-like model. We showed that such an arrangement leads to too much central intervention: a mandate, that is a minimum level for the policy, is imposed on all districts and is binding of some of them. More precisely, in that model, even when there is no externality and no reason for federal intervention, the federal mandate is equal to the median of all the preferred policies of all the voters in the confederation.

The Romer-Rosenthal model yields two key insights, both of which are relevant to the question of federal mandates. The first is that non-majoritarian outcomes can easily arise if some one person or group, whose preferences are different from the median, can set the agenda. Second, this agenda power is greater when the reversion point is farther away from the ideal point of the median voter. Because monopoly distortion away from majoritarian outcomes is less when the reversion is close to the median, setting the federal standard by majority rule could presumably mitigate these distortions and

[1] See also Romer and Rosenthal (1992) and Rosenthal (1990).

provide a counterbalance to the monopoly power of local agenda setters. Of course this cannot be done perfectly, since there will only be one federal standard, but many differerent local standards. Nevertheless, an overall standard set by a majoritarian process might, on average, be close to the median voter in each district, and therefore have a beneficial effect.

In this paper, we present a model that helps us explore this intuition about the effect of federal standards. In this model voting takes place in two stages: the federal stage, which determines a mandate in the form of a minimum policy, and the local stage, which determines the specific policies in each district subject to this constraint. Between the two voting stages, there is an agenda setting stage, where the agenda setter in each district proposes an alternative to compete against the outcome of the first stage – the federal mandate.

In effect the federal mandate serves as the status quo point for all districts in the final voting stage. If the federal mandate is below the local median and the agenda setter has an ideal point that is above the local median, then the threat of the reversion to a low status quo provides the agenda setter with leverage to obtain an outcome above the median voter's ideal point. Voters in the first stage will take this into account when voting for the federal standard, so it is possible that voters with high ideal points will want F to be below their ideal point, while voters with low ideal points will want F to be above their ideal point. We analyze this as a three stage game, first solving for the third stage (local vote), then the second stage (agenda setting), and then working back to the first stage (federal vote).

Because F serves as a status quo point in each local district, the induced preferences of voters over F are distorted in systematic ways. Low-demand voters in a district (that is, voters with ideal points at or below their district's median) would like F to be as close as possible to their district's median, in order to minimize the monopoly power of the agenda setter. On the other hand, high demand voters would like the agenda setter to have at least some degree of monopoly power, in order to pull up the district outcome above the district median. Therefore, such voters actually have *double-peaked* preferences, with one peak at their true ideal point, and the other peak at a point just far enough below their district's median so the agenda setter can achieve their true ideal point as the local outcome. In spite of this added complication, there is a majority rule equilibrium at the federal stage. In fact, there are a lot of equilibria.

We fully characterize a subset of these, which we call *locally responsive.*

This characterization is simple and intuitive. Furthermore, we show that there is always a unique equilibrium that is immune to coalitional deviations. That equilibrium corresponds to the median of the maxima of voters' induced utilities over F. Hence, we do find that federal standards may be a way of counterbalancing nonmajoritarian influences, even though some distortions may still arise. To the extent that majoritarian outcomes are inherently valued, this provides a possible normative justification for federal mandates, as a second-best solution.

2 The Model

We consider a confederation composed of an odd number of districts. In each district d, there are N_d voters, where $N_d \geq 3$ is an odd integer.

The policy space is the real line,[2] $\Re$. We assume that every voter i in district d has complete, transitive, and single-peaked[3] preferences over $\Re$, which are represented by a continuous utility function, $U_{id}(X_d)$ which satisfies

$$\lim_{X_d \longrightarrow -\infty} U_{id}(X_d) = \lim_{X_d \longrightarrow +\infty} U_{id}(X_d) = -\infty.$$

The ideal policy of that voter, the maximum of his or her utility function, is $t_{id} \in \Re$. To simplify the exposition, and without loss of generality, we assume that no two voters have the same ideal point (this assumption will be strengthened somewhat by assumption 1).

It is important to note that we are only assuming single peaked preferences. We are not assuming either that the utility functions are symmetric, as would be so-called "tent preferences", or even that they satisfy any type of single crossing property.

We denote the median ideal policy in district d by m_d: it is equal to the median of the t_{id} of the voters (i, d) in district d. Because of our assumption that two voters always have different ideal policies, no two districts have the same median ideal policy.

The position of the preferred policy of a voter compared to the median ideal policy in his or her district will play an important role in what follows.

[2]For some applications, it would be more natural to assume that the policy space is the semi real line $\Re^+$. This would not change the results, but complicate at points an already involved discussion.

[3]In all that follows, a single peaked function will be assumed to be strictly increasing to the left of the peak and strictly decreasing to the right.

If $t_{id} \leq m_d$, voter i is called a *low demand voter* (so that the median voter is a low demand voter), and if $t_{id} > m_d$ then voter i is called a *high demand voter.*

In each district, there is a *local agenda setter*, who would like the policy implemented in the district to be as large as possible. (We could weaken[4] this hypothesis as long as the ideal policy of the agenda setter of district d is greater than m_d.)

Policies in each district are determined as the outcome of the following three stage game.

Stage 1 (referendum stage) Each voter announces a proposed policy, $\widehat{F}_{id}$, and a federal standard, F, is determined by $F = \text{med}\{\widehat{F}_{id}\}$, where the median is taken over the votes of all the voters in all the districts.

Stage 2 (agenda setter stage) In each district, d, the local agenda setter proposes a policy $\widehat{X}_d \geq F$.

Stage 3 (local voting stage) Each voter in each district casts a vote either for or against $\widehat{X}_d$. If a majority of voters in district d vote for $\widehat{X}_d$, then it becomes the local policy in district d. Otherwise, the policy of district d if F.

We look at a subset of the Nash equilibria of this game. The last two stages are dominance solvable, and then we assume that in the first stage, voters adopt strategies that are not dominated given the solution to the second and third stages. We refer to this as an *equilibrium*. We will completely characterize two specially attractive subsets of the set of equilibria.

[4]This would imply that the indirect utility functions defined below have flat parts for large policies, and not change the analysis.

Note also, that we will assume that the agenda setter is not a voter. However, if the preferred policy of the agenda setter is equal to the maximum of the t_{id}s, the results of the analysis are exactly the same as if she is the voter with the greatest t_{id}.

3 Equilibria

3.1 Equilibrium Solution in the Third (Local Voting) Stage

In the last stage, each voter is confronted with a choice between voting for $\widehat{X}_d$ and voting for F. For any such ($\widehat{X}_d$,F) pair, every voter has a dominant strategy to vote for his or her most prefered policy.[5]

3.2 Equilibrium Solution in the Second (Agenda Setting) Stage

In the second stage, the status quo is simply the federal standard, F. Thus, in the second (local) stage of the game, the local agenda setter in district d, will propose the largest possible local standard, $\widehat{X}_d$, subject to the constraint that $\widehat{X}_d$ is preferred to F by a majority of the voters in the district. Without risk of confusion we will sometimes use the notation $\widehat{X}_d$ for the function that links F to the best outcome that the agenda setter can obtain.

As shown in Romer and Rosenthal (1979), there are two cases to consider, depending on F and m_d.

Case 1 : $m_d \leq F$. In this case, the agenda setter has no power at all, and the outcome will be F. Indeed, all the low demand voters, which are a majority in the district, prefer F to any larger policy.

Case 2 : $F < m_d$. The agenda setter chooses the $\widehat{X}_d$ which is the greatest x such that a majority of the voters in district d (weakly) prefer x to F (we assume that if a voter is indifferent between the status quo and the proposal of the agenda setter, he or she votes for the proposal of the agenda setter).

In the simplest case of Euclidean preferences[6], the local standard $\widehat{X}_d$ will be chosen in such a way that the median voter is indifferent between $\widehat{X}_d$ and F. Therefore, $\widehat{X}_d$ will be equal to $m_d + (m_d - F) = 2m_d - F$ and will be decreasing in F. We also use the following generalization of this result to any single peaked utility function:

[5] It is assumed that voters who are indifferent between $\widehat{X}_d$ and F vote for $\widehat{X}_d$

[6] Euclidian preferences can be represented by the utility function $U_{id}(x) = -|t_{id} - x|$.

Lemma 1 *The function* $\widehat{X}_d(F)$ *is decreasing and continuous on* $(-\infty, m_d)$ *and* $\lim_{F \longrightarrow m_d^-} \widehat{X}_d(F) = m_d$.

The proof is presented in the appendix.

We obtain the simple following corollary, which summarizes all the information we will need about the behavior of $\widehat{X}_d$ when F varies:

Corollary 1 *The function* $\widehat{X}_d$ *is continuous, strictly decreasing on* $(-\infty, m_d]$, *with* $\lim\limits_{F \longrightarrow -\infty} \widehat{X}_d(F) = +\infty$, *and strictly increasing on the half-line* $[m_d, +\infty)$ *where we have* $\widehat{X}_d = F$.

3.3 Equilibrium Solution in the First Stage

In this section, we solve for the equilibrium at first stage of the game, where the federal standard is decided. First we derive the induced preferences over federal standards, given the equilibrium behavior in the continuation game. Second, we define some natural refinements of the equilibrium based on weak dominance, coalition proofness, and a new concept called local responsiveness.

3.3.1 Induced preferences over the federal standard (first stage), given the solutions to stages 2 and 3.

We derive the voters' induced preferences over F in the first stage, given the outcome of the subsequent stages that is described by corollary 1. These preferences will be represented by a indirect utility function that we will call V_{id}.

There are two cases to consider, depending on whether a voter's ideal point is above or below their district median.

For a low demand voter, the outcome is always greater than his or her ideal policy, as it is always greater than m_d. Furthermore, for $F \leq m_d$, the outcome is closer to his or her ideal policy, the greater F, whereas for $F \geq m_d$, the outcome is farther away from his or her ideal policy, the greater F. Hence, we obtain the following lemma.

Lemma 2 *The indirect utility function* $V_{id}(F)$ *of a low demand voter* (i, d) *is a continuous singled peaked function, with a peak at* m_d.

Intuitively, the voter wants to restrict the agenda setter's power by imposing a federal standard, but at the same time doesn't want too high a federal standard. The best solution is to choose the smallest federal standard that totally constrain the agenda setter, that is m_d.

There are two values of F such $\widehat{X}_d(F)$ is equal to the preferred policy t_{id} of a high demand voter (i, d). One is $F = t_{id}$, and the other, which we will call t'_{id}, is strictly smaller that m_d and satisfies $\widehat{X}_d(t'_{id}) = t_{id}$. From corollary 1, we easily obtain the following lemma.

Lemma 3 *The indirect utility function $V_{id}(F)$ of a high demand voter (i, d) is continuous, has two local maxima, t'_{id} and t_{id}, and a local minimum, m_d. It is increasing on $(-\infty, t'_{id})$, decreasing on $[t'_{id}, m_d]$, increasing on $[m_d, t_{id}]$ and decreasing on $[t_{id}, +\infty)$.*

In order to simplify the exposition, we make the following assumption, that is satisfied generically:

Assumption 1 *The sets of peaks of the utility function of two voters are distinct, except if the two voters are both low demand voters from the same district.*

This imply, in particular, that all the t_{id}s and the t'_{id}s are different from each other.

The caracterization of the equilibrium of the first stage of the game will rely only on the properties of the indirect utility functions described in lemmas 2 and 3.

3.3.2 Undominated strategies in the referendum stage

We now turn to the characterization of the equilibrium of the first stage voting game where each voter submits a proposed value of F and the median proposal is chosen. As usual in voting games, there are many Nash equilibria of this game, some very unreasonable. We will therefore restrict the sets of equilibria we are considering.

The first natural restriction is to consider only equilibria where the voters choose (weakly undominated) strategies. Given the single peakedness of their utility functions, it is immediate that low demand voters have only one undominated strategy, m_d. The behavior of high demand voters, those for whom $t_{id} > m_d$ is more complicated, but we can still prove that they have

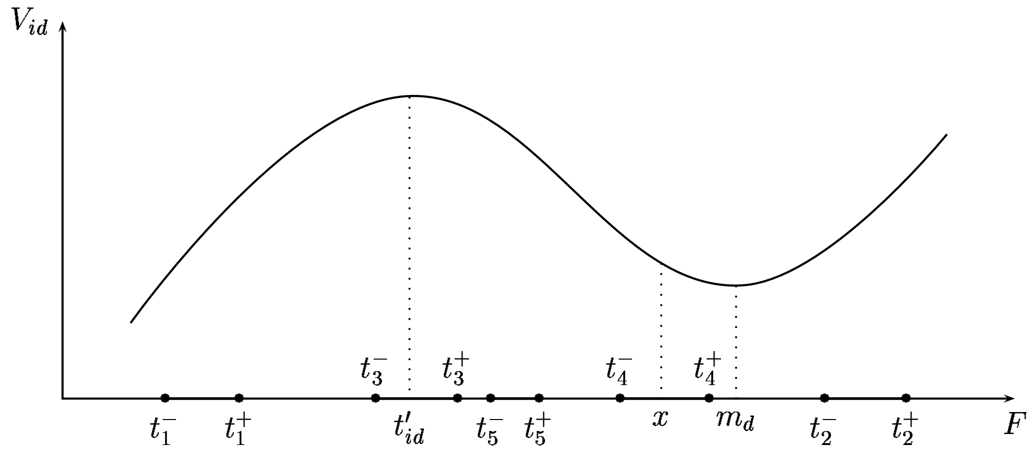

Figure 1: This figure illustrates the proof presented in the that x is dominated by t'_{id}. Whatever the interval $t_i^-, t_i^+]$ in which agent (i, d) can make the outcome vary, he or she is at least as well off, and sometimes strictly better off, voting t'_{id}.

only two undominated strategies, t'_{id} and t_{id}. More precisely, any strategy in $(-\infty, m_d]$ is dominated by t'_{id} and any strategy in $[m_d, +\infty)$ is dominated by t_{id}. To see this consider the situation illustrated in figure 1, where $t'_{id} < x < m_d$; we will show that x is dominated by t'_{id} (the proofs for other configurations of policies are similar). Let us call $[t^-, t^+]$ the interval over which voter (i, d) can make the federal standard move by his vote (i.e., if $t^- \neq t^+$, there are $(N-1)/2$ voters with votes equal to or smaller than t^- and also $(N-1)/2$ voters with votes greater than or equal to t^+).

First, if the intersection of the open interval (t'_{id}, x) and $[t^-, t^+]$ is empty, as for instance with the intervals $[t_1^-, t_1^+]$ or $[t_2^-, t_2^+]$ on figure 1, voting t'_{id} or x yields the same outcome. In other cases, represented by the intervals $[t_3^-, t_3^+]$, $[t_4^-, t_4^+]$ or $[t_5^-, t_5^+]$, voter (i, d) strictly prefers to announce t'_{id} rather than x. Note that m_d is dominated both by t'_{id} and t_{id}.

We state this result formally in the following lemma.

Lemma 4 *A low demand voter (i, d) has only one undominated strategy in the referendum stage, m_d. A high demand voter (i, d) has two undominated strategies, t_{id} and t'_{id}.*

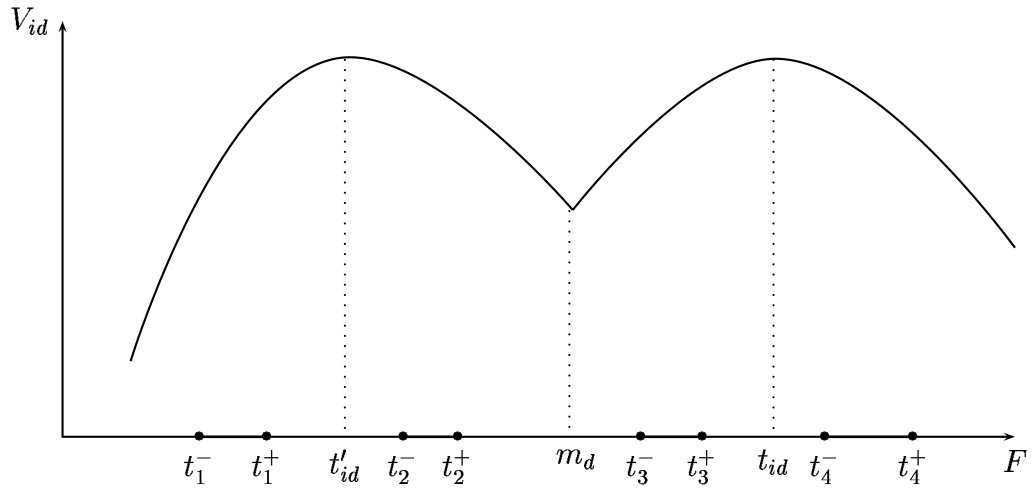

Figure 2: This picture illustrates the meaning of local responsiveness in the text. For instance, when voter (i, d) can obtain any outcome in $[t_1^-, t_1^+]$, he could do so by any of his or her two undominated strategies t'_{id} and t_{id}. Local responsiveness assumes that he or she chooses the "closest" one, in this case t'_{id}.

3.3.3 Locally responsive equilibria: definition

In addition to requiring undominated strategies, we add a second restriction, that the equilibrium be locally responsive. Later in the paper, we provide a complete and simple characterization of all locally responsive equilibria.

Consider a high demand voter, who, taking as given the other votes, must choose his or her vote, and consider the cases where this vote matters, i.e., cases where changing the vote can make the federal standard vary over an interval with a non empty interior. By the rules of the game, the interval on which the voter can make the federal standard vary cannot include any other vote in its interior. Hence, because all the low demand voters in his district vote m_d, the interval cannot include m_d in its interior. Neglecting the case where this interval contains either t_{id} or t'_{id}, we can find ourselves in one of the four situations illustrated in figure 2. If the interval is either $[t_2^-, t_2^+]$ or $[t_3^-, t_3^+]$, the voter has only one optimal undominated strategy, respectively t'_{id} and t_{id}. On the other hand, if the interval is either $[t_1^-, t_1^+]$ or $[t_4^-, t_4^+]$, the voter will be indifferent between voting t'_{id} or t_{id}. Local responsiveness imposes on the voter the choice of the closest undominated strategy, that is

t'_{id} when the interval is $[t_1^-, t_1^+]$ and t_{id} when the interval is $[t_4^-, t_4^+]$. Formally, we need to translate this into a statement about the equilibrium of the game played by the voters.

Definition 1 *An equilibrium whose outcome F^* is* locally responsive *if a high demand voter (i, d) votes t_{id} if $F^* > m_d$ and t'_{id} if $F^* < m_d$.*

3.3.4 Coalition proof locally responsive equilibrium

As we show in the next section, there are many locally responsive equilibrium. However, only one of them is immune to manipulation by small coalitions of voters. Therefore, with the following coalition proof requirement, we obtain a unique equilibrium.

When the equilibrium outcome is m_d, the high demand voters in district d would like to move the outcome either to the right or to the left. Therefore, m_d can be an equilibrium only if none of these high demand voters is pivotal, and we will use this fact in order to characterize locally responsive equilibria in theorem 2.

On the other hand, it may seem reasonable to assume that the high demand voters in district d would try to coordinate to move the equilibrium away from m_d. We will say that a locally responsive equilibrium is coalition proof when they cannot do so, which leads to the following formal definition.

Definition 2 *A locally responsive equilibrium is* coalition proof *if it is stable to coordinated deviations by any coalition of voters for which it is a local minimum of the indirect utility function.*

4 Characterization of Locally Responsive Equilibria

In this section, we characterize locally responsive equilibria and coalition proof locally responsive equilibria, in terms of a set of conditions that are very easy to check. Existence follows immediately.

It will be convenient to use the term candidate to designate potential equilibria, i.e., the set of all peaks of utility functions of the voters. Thus, a *candidate* is a policy x such that either x is equal to m_d for some d, in which case it is called a *district candidate*, or x is equal to t_{id} or to t'_{id} for some high demand voter (i, d), in which case it is called an *individual candidate*.

By construction, the total number of candidates generated by the voters from district d is precisely equal to the number of voters in that district, N_d. Indeed, there are two candidates for every high demand voter, for a total of $2(N_d - 1)/2 = (N_d - 1)$ candidates, and, by assumption 1, they are all different from each other and from m_d. To these $N_d - 1$ candidates, we simply need to add the unique median candidate. As a consequence, using assumption 1 once again, the total number of candidates is N.

To construct the appropriate formula, we begin by assigning a *rank* to every candidate in the following way:

- the left-most candidate (which is necessarily an individual candidate) has *rank* 1;
- every other candidate has a *rank* equal to 1 plus the rank of the candidate immediately to its left.

We will call $r(x)$ the rank of candidate x.

The following theorem is the main result of the paper and proves that the median *candidate* is the unique coalition proof and locally responsive equilibrium outcome of the federal stage.

Theorem 1 *The unique coalition proof locally responsive equilibrium is the candidate x for which $r(x) = (N + 1)/2$.*

The next two results fully characterize all locally responsive equilibria in terms of ranks. The corollary follows immediately, since coalition proofness does not have any bite for individual candidate equilibria. The second result, the characterization of locally responsive equilibria with a district candidate is more complicated and is proved in the next section, following the proof of theorem 1.

Corollary 2 *An individual candidate x is a locally responsive equilibrium if and only if $r(x) = (N + 1)/2$.*

Theorem 2 *A district candidate x which belongs to district d is a locally responsive equilibrium if and only if $r(x) \in [(N - N_d)/2 + 2, (N + N_d)/2 - 1]$.*

In the next section, section 5, we provide a detailed formal proof. In section 6, we illustrate the proof with some examples. We conclude this section by presenting the proof in a specific case, which will provide insights on the reason for the equilibrium.

Begin with the situation is which there is only one district, district 1 with 2001 voters. Although we have assumed at least three districts throughout the paper, the same result holds: the median of the types of district 1, m_1, is the unique equilibrium, which will also be coalition proof (even if all the high demand voters coordinate to vote either to the left or to the right of m_1, the median of the votes will be m_1).

Add now two more districts, each with three voters, all with very low preferred policies (the largest of the t_{i2}s and of the t_{i3}s are both smaller than the smallest of the t_{i1}s). There are now 2007 voters, and hence 2007 candidates. Before continuing the analysis, we need to introduce a tiny extra piece of notation to make the analysis easier. Let us assume that it is agents 1, 2 and 3 in district 1 who have the largest t'_{i1}s, so that for all $i > 3$ we have $t'_{i1} < t'_{31} < t'_{21} < t'_{11} < m_1$. It is easy to check that t'_{31} is the median of the candidates. Indeed, the candidates that are strictly smaller than t'_{31} are the 6 candidates generated by district 2 and 3 and the 1000-3=997 candidates t'_{i1} for $i > 3$, for a total of 1003 candidates. It is also easy to check that t'_{31} is a locally responsive equilibrium, by checking that no voter has an incentive to change his vote: the only voters who could change the outcome by voting for another one of their undominated strategies are the high demand voters $(i, 1)$ of district 1 with $i > 3$. Neither individually, nor collectively, do they have any incentive to do so, as they could only push the outcome to t'_{21}, t'_{11}, or m_d, which are all three worse from their viewpoint.

It is easy to check in this case that m_1 is still a locally responsive equilibrium, for instance if all the high demand voters of district 1 vote for their respective preferred policy t_{i1}, the median of the votes will be m_1 and none of them can modify the equilibrium by changing his own vote. It is more instructive to understand why t'_{21} is not an equilibrium: if it were an equilibrium, at least one of the voters $(i, 1)$ with $i \geq 3$ would have to be voting t_{i1}, but this voter can increase his or her utility by voting t'_{i1} which will move the equilibrium back to t'_{31}, which he or she strictly prefers as we have $t'_{i1} \leq t'_{31} < t'_{21} < m_1$.

As the size of districts 2 and 3 increase, the individual candidate that is an equilibrium moves further and further away from m_1, and is still the only coalition proof equilibrium.

If all the high demand voters of district 1 vote t_{i1}, there are 2001 voters voting for m_1 or for a policy larger m_1. Hence, as the sizes of districts 2 and 3 increase, m_1 remains an equilibrium (albeit not coalition proof) as long as one high demand voter of district 1 cannot move by him or herself the equilibrium to t'_{11}, i.e. as long as the total number of voters in districts 2 and 3 is strictly less than 2000. We can check that this indeed what was predicted by the upper bound for $r(x)$ of theorem 2. If the total number of voters in these two districts is 2000, $r(m_1)$ is equal to 3001, whereas $(N+N_d)/2-1$ is equal to 3000. On the other hand, if the total number of voters in districts 2 and 3 is 1998, $r(m_1)$ is equal to 1999, which is equal to $(N+N_d)/2-1$.

5 Proof of Theorems 1 and 2

5.1 Partitioning the Candidates

Before giving the proofs, we introduce some notation. Suppose that x is a candidate. For any such x, we can partition the voters into four sets which will be used later to describe their strategies if x were the equilibrium outcome of the first stage: if you were told that x is a locally responsive equilibrium and to which one of these sets a voter (i,d) belonged, you would know the position of the vote of the voter with respect to x.

Definition 3 *For any candidate x:*

- *A low demand voter (i,d) belongs to $\mathcal{L}(x)$ (votes to the $\mathcal{L}$eft of x) if and only if $m_d < x$. A high demand voter belongs to $\mathcal{L}(x)$ if and only if either $t_{id} < x$ or $t'_{id} < x < m_d$.*
- *A low demand voter belongs to $\mathcal{R}(x)$ (votes to the $\mathcal{R}$ight of x) iff $x < m_d$. A high demand voter belongs to $\mathcal{R}(x)$ iff either $x < t'_{id}$ or $m_d < x < t_{id}$.*
- *A low demand voter belongs to $\mathcal{E}(x)$ (votes $\mathcal{E}$xactly for x) iff $x = m_d$. A high demand voter belongs to $\mathcal{E}(x)$ iff x is equal to either t_{id} or t'_{id}.*
- *A voter belongs to $\mathcal{LR}(x)$ (votes either to the $\mathcal{L}$eft or the $\mathcal{R}$ight of x) iff he or she is a high demand voter and $x = m_d$.*

The following remark states the relationship between this partition and equilibrium.

Remark 1 *If F^* is a locally responsive equilibrium, all voters in $\mathcal{L}(F^*)$ vote for a federal standard strictly less than F^*, all voters in $\mathcal{R}(F^*)$ vote for a federal standard strictly greater than F^*, all voters in $\mathcal{E}(F^*)$ vote for F^*, and no voter in $\mathcal{LR}(F^*)$ votes for F^*.*

5.2 Example

The intuition behind this candidate partition, and its relation to the characterization theorems, is best illustrated with an example. In this example, there are 3 districts with 5 voters in each district, with voter ideal points given by:

District 1 voter ideal points: $\{1, 4, 7, 9, 20\}$

District 2 voter ideal points: $\{2, 5, 10, 17, 25\}$

District 3 voter ideal points: $\{3, 6, 18, 30, 38\}$

Each voter is assumed to have symmetric preferences around their ideal point. Therefore, the set of candidates in this example is

$$\{-6, -5, -2, 3, 5, 6, 7, 9, 10, 17, 18, 20, 25, 30, 38\}.$$

Table 1 shows how the four different candidate sets $\mathcal{L}(\cdot)$, $\mathcal{R}(\cdot)$, $\mathcal{E}(\cdot)$, and $\mathcal{LR}(\cdot)$ vary with F. The top row of the table lists 15 different possible values of F, which correspond to the 15 different candidates. The next four rows list the number of members of each of the four sets of candidate types, $\mathcal{L}(\cdot)$, $\mathcal{R}(\cdot)$, $\mathcal{E}(\cdot)$, and $\mathcal{LR}(\cdot)$. From the two main theorems, one can identify exactly three locally responsive equilibria. Two correspond to district medians (7 and 10), and one is an individual equilibrium (9). The latter is the median candidate, and therefore is the unique coalition proof equilibrium.

There are a number of observations that can be made about the structure of the $\mathcal{L}(x)$ and $\mathcal{R}(x)$ sets, which can be understood easily by referring to table 1. First, for all values of x that do *not* correspond to a candidate, $|\mathcal{L}(x)| + |\mathcal{R}(x)| = N$. First, as x increases, every time an individual candidate, $\widetilde{x}$, is reached, $|\mathcal{R}(\cdot)|$ decreases by 1, so that $|\mathcal{L}(\widetilde{x})| + |\mathcal{R}(\widetilde{x})| = N - 1$. Immediately to the right of $\widetilde{x}$, $|\mathcal{L}(\cdot)|$ increases by one. This implies that there can be at most one individual candidate.

Second, as we increase F^* in the region $F^* \in (-6, m_1)$, $|\mathcal{L}(F^*)|$ is weakly increasing and $|\mathcal{R}(F^*)|$ is weakly decreasing, as some rightist voters move to

	-6	-5	-2	3	5	6	7*	9**	10*	17	18	20	25	30	38
$\mathcal{L}(x)$	0	1	2	3	4	5	4	7	6	9	8	11	12	13	14
$\mathcal{R}(x)$	14	13	12	11	10	9	6	7	4	5	2	3	2	1	0
$\mathcal{E}(x)$	1	1	1	1	1	1	3	1	3	1	3	1	1	1	1
$\mathcal{LR}(x)$	0	0	0	0	0	0	2	0	2	0	2	0	0	0	0

Table 1: Example. (Note: * denotes locally responsive equilibrium; ** denotes coalition proof locally responsive equilibrium)

$\mathcal{E}(F^*)$ and then to $\mathcal{L}(F^*)$. In any case, it must be that $|\mathcal{L}(F^*)| \leq (N-1)/2$ and $|\mathcal{R}(F^*)| \geq (N+1)/2$ in this region. At exactly the point $F^* = m_1 = 7$ *both* $|\mathcal{L}(F^*)|$ and $|\mathcal{R}(F^*)|$ each decrease by at least 1, since the median voter and all leftist voters in district 1 (who previously had been in $\mathcal{R}(F^*)$) now belong to $\mathcal{E}(m_1)$ and all rightist voters in district 1 (who had previously belonged to belong to $\mathcal{L}(F^*)$) now belong to $\mathcal{LR}(m_1)$. After $F^* = m_1$, $\mathcal{L}(F^*)$ immediately increases, since the median voter and all leftist voters in district 1 move from $\mathcal{E}(m_1)$ to $\mathcal{L}(F^*)$.

Third, as we increase F^* in the region $F^* \in (m_1, m_2)$, we again have $|\mathcal{L}(F^*)|$ is weakly increasing and $|\mathcal{R}(F^*)|$ is weakly decreasing. A locally responsive equilibrium corresponds to any value of F^* in the figure, such that $|\mathcal{L}(F^*)| \leq (N-1)/2$ and $|\mathcal{R}(F^*)| \leq (N-1)/2$.

Fourth, every time a district median, $\widetilde{m_d}$, is reached, $|\mathcal{L}(\cdot)|$ decreases by $(N_d - 1)/2$ while $|\mathcal{R}(\cdot)|$ decreases by $(N_d + 1)/2$. Then, immediately to the right of $\widetilde{x}$, $|\mathcal{L}(\cdot)|$ increases by $(N_d + 1)/2$ and $|\mathcal{R}(\cdot)|$ increases by $(N_d - 1)/2$. Thus, the net effect of passing any candidate is to increase $|\mathcal{L}(\cdot)|$ by 1 and to decrease $|\mathcal{R}(\cdot)|$ by 1.

5.3 Proofs

We know that in the referendum stage, the low demand voters vote m_d, but the high demand voters can vote either for their left or their right peak. The following lemma provides an easy way to recognize whether a high demand voter is using his or her best response to the votes of the others. We will call profile of votes the assignment of a vote to each voter, bearing in mind that we only allow voters to choose an undominated strategy.

Lemma 5 *Let x be the median of a profile of votes, with voter (i,d) voting x_{id}. For any high demand voter (i,d), t'_{id} is a best response to the votes of* *the others if $x < m_d$ and t_{id} is a best response to the votes of the others if $x > m_d$.*

Proof. We prove the result for $x < m_d$. If $x_{id} = t_{id}$ $\mathbf{(i.e., if\ (i, d)\ votes\ t_{id})}$, by voting t'_{id}, voter (i, d) would either not change the outcome or would move it closer to t_{id} without ever "jumping over it". This would increase his or her utility. On the other hand, if $x_{id} = t'_{id}$, voter (i, d) would not improve his or her utility by changing vote to t_{id}. Either the outcome would not change, or the outcome would move away from t'_{id} and towards m_d, without jumping over it as all the low demand voters of district d vote m_d. The same type of argument applies for $x > m_d$. ■

The next lemma follows from Definition 3 and Remark 1..

Lemma 6 *If a candidate F^* is a locally responsive equilibrium, then $|\mathcal{L}(F^*)| \leq (N-1)/2$ and $|\mathcal{R}(F^*)| \leq (N-1)/2$.*

Proof. Suppose that $|\mathcal{L}(F^*)| > (N-1)/2$ and suppose that all voters vote as described in the definition of locally responsive equilibrium. Then the median of the profile of reports will be less than F^*, so F^* is not an equilibrium. A similar argument applies if $|\mathcal{R}(F^*)| > (N-1)/2$. ■

We now prove a lemma which provides a link between the rank of a candidate and, loosely speaking, the number of voters that would vote to its left if it were a responsive equilibrium.

Lemma 7 *Let x be a candidate, d be a district with $x \neq m_d$, and $\rho_d(x)$ the number of candidates generated by district d with rank less than x. Then the number of voters of district d belonging to $\mathcal{L}(x)$ is also equal to $\rho_d(x)$.*

Proof. The candidates generated by district d consist of the two peaks of the high demand voters in d, and the median, m_d.

Suppose first $x > m_d$. The candidates generated by district d which are smaller than x consist of the $(N_d - 1)/2$ left peaks of the high demand voters, of the median m_d, and of those right peaks of high demand voters which are smaller than x. Let K denote the number of such right peaks. Then $\rho_d(x) = (N_d - 1)/2 + 1 + K$. Since $x > m_d$, the set of voters of district d belonging to $\mathcal{L}(x)$ consists of the $(N_d + 1)/2$ low demand voters plus the K high demand voters whose right peaks lie between m_d and x. Thus $|\mathcal{L}(x)| = (N_d + 1)/2 + K = \rho_d(x)$.

Consider now the case $x < m_d$. The candidates to the left of x generated by district d correspond to the left peaks of high demand voters. The number of such candidates is, by definition, $\rho_d(x)$. Each of these candidates corresponds to a unique high demand voter in d who belongs to $\mathcal{L}(x)$. These are the only voters in $\mathcal{L}(x)$ since the low demand voters in d all have peaks at the district median, and the other high demand voters all belong to $\mathcal{R}(x)$. Hence $\left|\mathcal{L}(x)\right| = \rho_d(x)$. ■

Lemma 7 immediately implies the following two corollaries.

Corollary 3 *Let x be an individual candidate. Then $\left|\mathcal{L}(x)\right| = r(x) - 1$, $\left|\mathcal{E}(x)\right| = 1$, $\left|\mathcal{R}(x)\right| = N - r(x)$ and $\left|\mathcal{LR}(x)\right| = 0$.*

Corollary 4 *Let $x = m_d$ be a district candidate. Then $\left|\mathcal{L}(x)\right| = r(x) - 1 - (N_d - 1)/2$, $\left|\mathcal{E}(x)\right| = (N_d + 1)/2$, $\left|\mathcal{R}(x)\right| = N - r(x) - (N_d - 1)/2$, and $\left|\mathcal{LR}(x)\right| = (N_d - 1)/2$.*

Lemma 8 *If x is an individual candidates such that $\left|\mathcal{L}(x)\right| = (N-1)/2$, and $\left|\mathcal{R}(x)\right| = (N-1)/2$, then x is a locally responsive equilibrium*

Proof. We will construct a profile of votes, such that x is the median of this profile, and such that each vote satisfies the conditions that caracterize locally responsive equilibria:

- the voters in $\mathcal{L}(x)$ have at least one peak of their indirect utility function to the left of x, and we associate the peak which is the closest to x;
- we associate to the voters in $\mathcal{R}(x)$ the closest to x of the peak of their indirect utility function which is to the right of x;
- the single voter in $\mathcal{E}(x)$ report x.

The candidate x is the median of this profile. By lemma 5 and definition 3, every vote it is easy to check that this profile satisfies the conditions that characterize locally responsive equilibria. ■

Proof of Theorem 1 The unique coalition proof locally responsive equilibrium is the candidate x for which $r(x) = (N+1)/2$.

Proof. There are two cases to consider. First, let x be an individual candidate. This is the simplest case, because, by the definition of coalition proofness, a locally responsive equilibrium which is an individual candidate is always strategy proof. Thus we only need to show that an individual candidate x is a locally responsive equilibrium if and only if $r(x) = (N+1)/2$.

(if) Suppose that x is an individual candidate with $r(x) = (N+1)/2$. Then the conditions of lemma 8 are satisfied, so x is a locally responsive equilibrium.

(only if) Consider an individual candidate x that is a locally responsive equilibrium. We have $|\mathcal{E}(x)| = 1$, and, by lemma 6, $|\mathcal{L}(x)| \leq (N-1)/2$ and $|\mathcal{R}(x)| \leq (N-1)/2$. As $|\mathcal{L}(x)| + |\mathcal{E}(x)| + |\mathcal{R}(x)| = N$, we must have $|\mathcal{L}(x)| = (N-1)/2$, which proves that x is of rank $(N+1)/2$. This proves the theorem for the case where x is an individual candidate.

Next, consider a district candidate $x = m_d$.

(if) Suppose x has rank $(N+1)/2$. We will show that both $|\mathcal{L}(x)|+|\mathcal{E}(x)|$ and $|\mathcal{E}(x)|+|\mathcal{R}(x)|$ are equal to $(N+1)/2$. Therefore, if all the voters in $\mathcal{L}(x)$ vote to the left of x, all the voters in $\mathcal{E}(x)$ vote x and all the voters in $\mathcal{R}(x)$ vote to the right of x, the high demand voters of district d cannot modify the outcome. Then, it is clear that x satisfies the definition of a coalition proof responsive equilibrium.

The candidates which are strictly smaller than x are the candidates generated by voters of districts $d' \neq d$ with rank less than the rank of x plus the $(N_d - 1)/2$ candidates corresponding to the left peaks of the high demand voters of district d. Because x is of rank $(N+1)/2$, the number of these candidates is also equal to $(N+1)/2 - 1$. Therefore

$$\frac{N+1}{2} - 1 = \sum_{d' \neq d} \rho_{d'}(x) + \frac{N_d - 1}{2},$$

and therefore

$$\sum_{d' \neq d} \rho_{d'}(x) = \frac{N - N_d}{2}.$$

Because no voter in district d belongs to $\mathcal{L}(x)$, by lemma 7,

$$|\mathcal{L}(x)| = \sum_{d' \neq d} \rho_{d'}(x) = \frac{N - N_d}{2},$$

which implies

$$|\mathcal{L}(x)| + |\mathcal{E}(x)| = \frac{N - N_d}{2} + \frac{N_d + 1}{2} = \frac{N+1}{2}.$$

A similar proof shows that $|\mathcal{E}(x)|+|\mathcal{R}(x)|$ is equal to $(N+1)/2$, and therefore we have shown the "if" part of the lemma.

(only if) Consider a coalition proof locally responsive district equilibrium $x = m_d$. For the high demand voters in d not to be able to coordinate to move the equilibrium either to the left or to the right of x, we must have

$$|\mathcal{L}(x)| + |\mathcal{E}(x)| \geq \frac{N+1}{2}$$

and

$$|\mathcal{E}(x)| + |\mathcal{R}(x)| \geq \frac{N+1}{2}.$$

Because $|\mathcal{E}(x)| = (N_d+1)/2$, this implies that both $|\mathcal{L}(x)|$and $|\mathcal{R}(x)|$ are at least equal to $(N-N_d)/2$. Furthermore, no voter in d belong to either $\mathcal{L}(x)$ or $\mathcal{R}(x)$, and therefore $|\mathcal{L}(x)| + |\mathcal{R}(x)| = N - N_d$, which implies

$$|\mathcal{L}(x)| = |\mathcal{R}(x)| = \frac{N-N_d}{2}.$$

Noticing that the rank of x is equal to the number of candidates to its left generated by districts other than d plus $(N_d-1)/2$ plus one and using lemma 7 proves the result. ■

We now turn to the characterization of all locally responsive coalition proof outcomes that are district candidates. We do this through two lemmas.

Proof of Theorem 2 A district candidate x is a locally responsive equilibrium if and only if $r(x) \in [(N-N_d)/2+2, (N+N_d)/2-1]$.

Proof.

(if) Consider first the case $|\mathcal{L}(x)| = |\mathcal{R}(x)|$, which by corollary 4, is equivalent to $r(x) = (N-1)/2$. By lemma 1 we know that x is a locally responsive coalition proof equilibrium, and a fortiori a locally responsive equilibrium.

Assume therefore $|\mathcal{L}(x)| > |\mathcal{R}(x)|$ (a similar proof would yield the result if the inequality were reversed) and consider the following profile of reports: each of the voters in $\mathcal{L}(x)$ reports his closest peak to the left of x, each of the voters in $\mathcal{R}(x)$ reports his closest peak to the right of x, the $\mathcal{E}(x)$ voters (i.e. the low-demand voters in d) report x, each of the voters in $\mathcal{LR}(x)$ report their closest peak to the right of x.

Because $|\mathcal{L}(x)| > |\mathcal{R}(x)|$, corollary 4 implies $r(x) > (N+1)/2$.

The total number of voters whose vote is strictly to the left of x is

$$\left|\mathcal{L}(x)\right| \leq \frac{N+N_d}{2} - 1 - 1 - \frac{N_d - 1}{2} = \frac{N-3}{2} < \frac{N-1}{2},$$

and the total number of votes strictly to the right of x is

$$\left|\mathcal{R}(x)\right| + \left|\mathcal{LR}(x)\right| = N - r(x) < \frac{N-1}{2}.$$

Hence, x is the median of the votes. Furthermore, no voter can by him or her self change the outcome, and hence the result is proved.

(only if) Assume we had $r(x) < (N - N_d)/2 + 1$. By corrolary 4, this implies$\left|\mathcal{R}(x)\right| \geq (N-1)/2$. If x were a locally responsive equilibrium, all the voters in $\mathcal{R}(x)$ would have to vote for a policy strictly greater than x and a single high demand voter in district d could move the median vote away from x, which is a local minimum of his or her utility function, and increase his or her utility. A similar reasoning shows that we cannot have $r(x) > (N + N_d)/2 - 1$. ■

6 Remarks about equilibrium

1. There can be at most one individual candidate equilibrium, but there are cases where no individual candidate equilibrium exists. This is the case if we modify the example of section 5.2 in such a way that the ideal points of voters in district 1 are $\{1, 4, 7, 11, 20\}$ instead of $\{1, 4, 7, 9, 20\}$.

2. In the example of 5.2, since the agenda setter tries to maximize x, the district outcomes will be $\{7, 13, 29\}$ if $F = 7$, $\{9, 11, 27\}$ if $F = 9$, and $\{10, 10, 26\}$ if $F = 10$, compared to $\{14, 20, 36\}$ with no federal standard. Therefore, out of the three locally responsive equilibria, 9 is the Condorcet winner. It is the (strictly) most preferred of the three by 8 of the 15 voters. Each of the three equilibria is strictly preferred to no federal standard at all, by a 2/3 majority (10 of 15) of the voters.

3. Loosely speaking, larger districts are more likely than smaller districts to be district candidate equilibria. In particular, the restriction that the rank of a district median, $m = (N+1)/2 \in [(N - N_d)/2 + 2, (N + N_d)/2 - 1]$ is easier to satisfy the larger is N_d. The reason is that, all else equal, there are fewer members of $\mathcal{L}(m_d)$ and $\mathcal{R}(m_d)$ when N_d is larger, and more members of $\mathcal{E}(m_d)$.

4. The federal standard can be either higher or lower than it would be in a situation of competitive local policy making. In the example of table 1, it is lower in one of the equilibria (7), higher in one of the equilibria, (10), the same for one of the equilibria (9).

7 Conclusions

This paper considers a two stage model of federalist policymaking in a one dimensional spatial model. In the first stage, a mandate is determined at the federation level by majority rule. This mandate places a (minimum) constraint on policy that can be selected in any district. In the second stage, local districts independently decide on their policy: local policies are decided by a monopoly agenda setter, who wishes to maximize the policy outcome.

The presence of the agenda setter in the second stage induces voter preferences for the federal mandate that are double-peaked for high-demand voters. In spite of this complication, we are able to prove existence of majority rule equilibria which satisfy some reasonable selection criteria, and we show that they can be easily identified.

While characterizing these equilibria, we are able to make some very strong testable hypotheses on the way in which votes will be cast. In each district, there should be bunching of the votes of low demand voters. Furthermore, high demand voters of any district for which the median preferred policy is large will support even lower federal standards than the low demand voters in their district.

The results can be extended in a number of directions. First, the federal mandate does not have to be a minimum constraint, but could be either a maximum constraint or a range, consisting of both a minimum and a maximum. This would just involve a relabelling of the model. Second, the agenda setter does not have to be a policy maximizer. For either of these directions of generalization, the exact conditions that would characterize the equilibria would be different, but existence will still hold. A more difficult direction of extension, and one that we are currently working on, allows for externalities across districts, as would be the case in, for example, environmental or education policies (see Crémer and Palfrey, 2002).

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Appendix

In this appendix, we prove that the function $\widehat{X}_d$ is a decreasing function of F for all d. As the identity of the district will be the same thoughout we omit the subscript d when refering to an individual agent. Agent i should be interpreted as agent (i, d) and t_i as t_{id}.

$\widehat{X}_d(F)$ is the largest X such that at least $(N_d + 1)/2$ voters of district d (weakly) prefer X to F.

All high demand voters and the median voter strictly prefer m_d to F. Hence, the set of Xs such that at least $(N_d + 1)/2$ voters of district d prefer X to F is not empty and contains some points strictly greater than m_d. Hence, the function $\widehat{X}_d$ is well defined for $F < m_d$ and $\widehat{X}_d(F) > m_d$ for $F < m_d$ (the fact that $\widehat{X}_d(F)$ is not "equal to $+\infty$" is due to the fact that $\lim_{X_d \longrightarrow +\infty} U_{id}(X_d) = -\infty$).

Because the function U_i is single peaked, $U_i(\widehat{X}_d(F)) \geq U_i(F)$ and the fact that $\widehat{X}_d(F)$ is strictly greater than F imply $F < t_i$. To show that $\widehat{X}_d$ is strictly decreasing, consider $F' < F$. For all i such that $U_i(\widehat{X}_d(F)) \geq U_i(F)$, we have $F' < F < t_i$ and therefore

$$U_i(F') < U_i(F) \leq U_i(\widehat{X}_d(F))$$

and therefore there exists a small enough ε such that for all such i $U_i(F') \leq U_i(\widehat{X}_d(F + \varepsilon))$, which proves the result.

To prove the continuity of $\widehat{X}_d$, consider a monotone sequence $\{F_n\}_{n=1,\dots,+\infty}$ such that $\lim_{n \longrightarrow +\infty} F_n = F$. For each n, there exists an agent $i(n)$ such that $U_{i(n)}(\widehat{X}_d(F_n)) = U_{i(n)}(F_n)$ (otherwise, one could increase $\widehat{X}_d(F_n)$ by at least a small amount and still obtain a majority of votes against F_n). Because the number of agents is finite, there exists an j and a subsequence of $\{F_n\}$ such that for all n in that subsequence $U_j(\widehat{X}_d(F_n)) = U_j(F_n)$. Taking the limit along this subsequence, we obtain $\lim_{n \longrightarrow +\infty} U_j(\widehat{X}_d(F_n)) = U_j(F)$. This implies that along the subsequence $\lim_{n \longrightarrow +\infty} \widehat{X}_d(F_n) = \widehat{X}_d(F)$.Because the sequence is monotone and $\widehat{X}_d$ is also monotone, $\lim_{n \longrightarrow +\infty} \widehat{X}_d(F_n) = \widehat{X}_d(F)$ is also along the whole sequence, which proves the result.

A similar reasoning proves $\lim_{F \longrightarrow m_d^-} \widehat{X}_d(F) = m_d$.