Arm's length relationships without moral hazard*

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Abstract

I show that cutting the flow of information between a principal and an agent can increase the power of the incentives of the agent to reveal private information.

1 Introduction

Williamson (1985) considered the dilemma faced by a firm deciding to acquiring a supplier. If it does, it has better inside information on the cost of production and more authority on the management of the "upstream firm". On the other hand the acquisition weakens the "power" of the incentives faced by the supplier. For Riordan (1990), by acquiring its supplier, the downstream firm takes control of its information and management; it obtains better information but at the same time looses the ability to measure objectively the performance of the management of its supplier, whose incentives to provide efforts are weakened. The improvement in information flows alleviates an adverse selection problem but creates a moral hazard problem. The same theme has been visited by a number of other papers in the literature: Prat (2005), for instance, summarized the conclusions of Crémer (1995), Dewatripont, Jewitt and Tirole (1999) and Holmström (1999) in the following way: "In these three instances, transparency is bad for discipline (the agent works less) but it is good for sorting (it is easier to identify agent type)."

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In this paper, I point out that limiting the information flow between the agent and the principal can mitigate problems of adverse selection as well as problems of moral hazard. In order to do so, I study a model where a principal must enter into a "Baron-Myerson" type relationship with an agent chosen in a large pool. In each period, the cost parameter of the agent is drawn from a distribution that depends on his "type" or "quality". In the second period, the principal must decide whether to rehire her first period agent, or to pick another one. If she has obtained positive information about the first period agent, she will, of course, rehire him, and will hire another one otherwise. This model is similar to the model in Crémer (1995), with moral hazard replaced by adverse selection.[1]

We will compare the optimal contract from the viewpoint of the principal under two information structures. Under the "direct information" structure, during the first period the principal observes the quality of the agent. Ex-post, she finds it in her best self interest to rehire him if he turns out to be of good quality, whatever his first period cost parameter. In the "arm's length" information structure, she only observes the reported cost of the agent. Theorem 1 shows under which circumstances arm's length information structures can dominate direct information structures.

In the first period, the principal would like to acquire information about the quality of the agent and information about his cost parameter. Direct information is always better at providing information about the quality, but can be worse at providing information about the first period cost parameter. As usual in adverse selection models, the binding revelation constraint is the constraint on the low cost agent, who would like to pretend that he has a high cost. With arm's length relationships, he has more incentives to announce a low cost parameter as this signals a high quality, and therefore increases his probability of being rehired in the second period, which will lead to extra rents. Therefore, the incentives to announce a low cost are more powerful with arm's length relationships, for very much the same reasons that the incentives to provide more effort are higher in a moral hazard model.

2 Model and results

2.1 The static building block

The basic building block of our model is a simple Baron-Myerson (Baron and Myerson (1982)) framework with two possible levels of marginal cost, as studied, for instance, by Laffont and Martimort (2001). The payoff, or profit, of the principal is $V(q) - t$ where q is the quantity produced and t the monetary transfer from the principal to the

[1]As the referee as pointed out, the literature contains other examples of parallelism between moral hazard and adverse selection models. For instance, the ratchet effect has been studied in pure adverse selection frameworks (Weitzman, 1980; Freixas, Guesnerie and Tirole, 1985; Laffont and Tirole, 1988) and in frameworks with a moral hazard component (Meyer and Vickers, 1997). See also Milgrom (1987)

agent. The utility of the agent is $t-\theta q$ where β takes value θ_1 with probability p_1 and value $\theta_2>\theta_1$ with probability $p_2=1-p_1$. Using the revelation principle, and with standard notation, the problem of the principal is written as follows:

$$\max\ p_1(V(q_1)-t_1)+p_2(V(q_2)-t_2) \tag{1}$$
$$\text{subject to } t_1-\theta_1 q_1 \geq 0, \tag{$\mathscr{P}_1^{\text{BM}}$}$$
$$t_2-\theta_2 q_2 \geq 0, \tag{$\mathscr{P}_2^{\text{BM}}$}$$
$$t_1-\theta_1 q_1 \geq t_2-\theta_1 q_2, \tag{$\mathscr{I}_1^{\text{BM}}$}$$
$$t_2-\theta_2 q_2 \geq t_1-\theta_2 q_1. \tag{$\mathscr{I}_2^{\text{BM}}$}$$

At equilibrium, only constraints ($\mathscr{P}_2^{\text{BM}}$) and ($\mathscr{I}_1^{\text{BM}}$) are binding. Solving for t_1 and t_2, and substituting in (1), the expected profit of the principal is

$$S(p_1) \stackrel{\text{def}}{=} \max_{q_1,q_2} p_1\Big(V(q_1)-\theta_1 q_1-(\theta_2-\theta_1)q_2\Big)+(1-p_1)\Big(V(q_2)-\theta_2 q_2\Big). \tag{2}$$

A straightforward application of the envelope theorem shows that the function S is increasing in p_1.

We will also need to define the expected rent of an agent who does not yet know his quality: imagine that an agent is suddenly called by the principal, who tells him to come and examine the project to determine what his cost will be (he will sign the contract only after knowing this cost). Then, because is rent is equal to 0 if his cost parameter is θ_2, his expected rent is

$$R(p_1) \stackrel{\text{def}}{=} p_1(t_1-\theta_1 q_1), \tag{3}$$

with, of course, q_1 and t_1 taken to be equal to their optimal values in the problem of the principal.

2.2 Two period dynamics

We now embed our simple static Baron-Myerson model in a two period model. There is large pool of ex-ante undistinguishable agents. Each of them can be either a "good quality" agent or a "bad quality" agent:

➢ In each period, a good quality agent has a probability π_1 that his cost parameter is θ_1 and a probability $\pi_2=1-\pi_1$ that it is θ_2. The cost parameters are drawn independently in each period.

➢ On the other hand, bad quality agents always have a cost parameter equal to θ_2.

A proportion γ_1 of agents are good quality agents, and at the outset, neither the principal, nor the agent know anything about either the quality of the agent or his cost parameter. Therefore, the unconditional probability that the cost parameter of a randomly chosen agent is θ_1 is

$$p_1 \stackrel{\text{def}}{=} \gamma_1 \times \pi_1.$$

In the first period, the principal picks an agent randomly and offers him a contract. Then, the agent examines at no cost the project and learns his first period cost parameter, but *he does not learn his quality*; he either refuses the contract or accepts it, in which case the contract is executed. I assume that there is "not enough time" for the principal to offer the contract to another agent in case she picked a bad quality agent — she is stuck with the first agent.

In the second period, given the information that she has acquired in the first period, the principal either offers a new contract to the first period agent, or picks another agent. In either case, the continuation game is similar to the game in the first period.

In Crémer (1995), I assumed that the principal could offer a two period contract, but imposed a renegotiation proofness constraint. It is much simpler to assume that the principal can only offer one period contracts; it does not change the economic insights, and I believe, but am not entirely sure, that it does not change the results at all.

We now look at a two period problem in which the principal may or may not have information at the end of the first period. We first consider the case where the principal has direct information, that is where she learns the quality of the agent at the end of the first period.

2.3 The principal has direct information about the quality of the agent

At the end of period the principal learns the quality of the first period agent: she will rehire him if, and only if, he is of good quality. When she rehires the agent, she will assign a probability π_1 to the fact that the cost parameter is θ_1; therefore her second-period payoff will be $S(\pi_1)$ and the second period rent of the agent is $R(\pi_1)$.

The decision to rehire the agent does not depend on his first period actions. Hence, the revelation principle holds and the optimal first period contract is the solution of

$$\max\ p_1(V(q_1) - t_1) + p_2(V(q_2) - t_2) \tag{4}$$

$$\text{subject to } t_1 + R(\pi_1) - \theta_1 q_1 \geq 0, \tag{$\mathscr{P}_1^{\mathrm{di}}$}$$

$$t_2 + \frac{\gamma_1(1-\pi_1)}{1-\gamma_1\pi_1} R(\pi_1) - \theta_2 q_2 \geq 0, \tag{$\mathscr{P}_2^{\mathrm{di}}$}$$

$$t_1 - \theta_1 q_1 \geq t_2 - \theta_1 q_2, \tag{$\mathscr{I}_1^{\mathrm{di}}$}$$

$$t_2 - \theta_2 q_2 \geq t_1 - \theta_2 q_1. \tag{$\mathscr{I}_2^{\mathrm{di}}$}$$

The Incentive Compatibility constraints are the same as in ($\mathscr{I}_1^{\mathrm{BM}}$) and ($\mathscr{I}_2^{\mathrm{BM}}$): the first period announcements by the agent does not affect the second period. On the other hand, the Participation constraints are modified. Equation ($\mathscr{P}_1^{\mathrm{di}}$) stems from the fact that if his cost parameter is equal to θ_1, the agent knows that he is a good quality agent. He therefore knows that he will be rehired and obtain rents equal to $R(\pi_1)$ in the second period (we are assuming that the discount rate is equal to 0 for the agents and for the principal). On the other hand, the agent whose cost parameter is θ_2 assigns a probability equal to

$$\frac{\gamma_1 \times (1-\pi_1)}{\gamma_1 \times (1-\pi_1) + (1-\gamma_1) \times 1} = \frac{\gamma_1 (1-\pi_1)}{1-\gamma_1 \pi_1}$$

to the fact that he is of good quality. If he is, he will be rehired; if he is of bad quality, he will not be rehired. Hence, the Participation constraint ($\mathscr{P}_2^{\mathrm{di}}$).

We can use the same solution technique as in the one period model. First, it is easy to check that if ($\mathscr{P}_2^{\mathrm{di}}$) and ($\mathscr{I}_1^{\mathrm{di}}$) both hold, so does ($\mathscr{P}_1^{\mathrm{di}}$). We then drop the constraint ($\mathscr{I}_2^{\mathrm{di}}$) and verify later that the solution which we find satisfies that constraint. Constraints ($\mathscr{P}_2^{\mathrm{di}}$) and ($\mathscr{I}_1^{\mathrm{di}}$) must be strictly satisfied, which enables us to compute t_1 and t_2. Substituting in the objective function, the principal chooses q_1 and q_2 by maximizing

$$p_1 \left(V(q_1) - \theta_1 q_2 - \theta_1 q_1\right) + p_2 \left(V(q_2) - \theta_2 q_2\right) + \frac{\gamma_1 (1-\pi_1)}{1-\gamma_1 \pi_1} R(\pi_1).$$

Therefore the optimal q_1 and q_2 are the same as in the one period model;[2] the principal's first period profit is

$$S(p_1) + \frac{\gamma_1 (1-\pi_1)}{1-\gamma_1 \pi_1} R(\pi_1), \tag{5}$$

and that her total profit over both periods is

$$\begin{aligned} S(p_1) + \frac{\gamma_1 (1-\pi_1)}{1-\gamma_1 \pi_1} R(\pi_1) + \gamma_1 S(\pi_1) + (1-\gamma_1) S(p_1) \\ = (2-\gamma_1) S(p_1) + \gamma_1 S(\pi_1) + \frac{\gamma_1 (1-\pi_1)}{1-\gamma_1 \pi_1} R(\pi_1). \end{aligned} \tag{6}$$

2.4 Profits under arm's length information structure

With an arm's length relationship structure, the principal bases her decision to retain the first period agent on the basis of his choices. One could imagine that pooling or semi-pooling contracts would be optimal, but we will restrict ourselves to fully discriminating contracts where the principal offers in the first period two transfer-output pairs in such a way that the agent chooses (t_i, q_i) when his cost parameter is θ_i. Therefore the agent will be rehired whenever he chooses (t_1, q_1), as the principal will know that he is of good quality. He will not be rehired if he chooses (t_2, q_2): through Bayesian updating, the principal assigns a probability strictly smaller than γ_1 to the fact that he is of high quality.

Although I have not proved it, I believe that separating first period contracts are optimal for the principal, at least for a large set of parameters. It is true that the "ratchet

[2]This implies $q_1 > q_2$ at the optimum, and therefore, by the same reasoning as in the one period case that ($\mathscr{I}_2^{\mathrm{di}}$) is satisfied.

effect" literature[3] has stressed the fact that in many cases when a principal cannot commit to a long term contract with an agent, she will offer a first period contract which leads to bunching: both types of the agent will be offered the same output - transfer pair. The reason is clear: because the agent knows that in the second period the principal will use her knowledge of his quality to reduce his rent, it is very difficult for the principal to extract the information. In the model of this paper, two effects counteract this tendency to bunching. First, because there is imperfect correlation between the cost parameters in both periods, the agent will receive some rents in the second period, and hence will have fewer reasons to hide his quality. Second, the principal has the opportunity to hire another agent in the second period, in case she obtains negative information about the first period agent, and hence has stronger incentives to offer separating contracts.

Even if it were true that separating first period contracts were not optimal, the main result of this paper, Theorem 1 would still hold: the proof establishes that there exists a separating arm's length contract which is better than the best direct information contract; a fortiori, a superior pooling or semi-pooling contract would also be better.

The optimal separating first period contract is the solution of

$$\max\; p_1(V(q_1)-t_1)+p_2(V(q_2)-t_2) \tag{7}$$

$$\text{subject to } t_1+R(\pi_1)-\theta_1 q_1 \geq 0, \tag{$\mathscr{P}_1^{\text{al}}$}$$

$$t_2-\theta_2 q_2 \geq 0, \tag{$\mathscr{P}_2^{\text{al}}$}$$

$$t_1+R(\pi_1)-\theta_1 q_1 \geq t_2-\theta_1 q_2, \tag{$\mathscr{I}_1^{\text{al}}$}$$

$$t_2-\theta_2 q_2 \geq t_1-\theta_2 q_1+\frac{\gamma_1(1-\pi_1)}{p_2}R(\pi_1). \tag{$\mathscr{I}_2^{\text{al}}$}$$

The term $R(\pi_1)$ in the left hand sides of equations ($\mathscr{P}_1^{\text{al}}$) and ($\mathscr{I}_1^{\text{al}}$) is the expected second period rent of an agent who truthfully reports that his first period cost parameter is θ_1. The term $\gamma_1(1-\pi_1)R(\pi_1)/p_2$ in the right hand side of ($\mathscr{I}_2^{\text{al}}$) is the expected second period rent of an agent whose first period cost parameter is equal to θ_2 if he is rehired in the second period when the principal believes that he is of good quality: with probability $(\gamma_1(1-\pi_1))/p_2$ he is indeed of high quality and obtains rent $R(\pi_1)$. Otherwise, he is of bad quality. In this case, his second period cost parameter is θ_2 with probability 1, and he obtains no rent.

Obviously, ($\mathscr{P}_2^{\text{al}}$) and ($\mathscr{I}_1^{\text{al}}$) imply ($\mathscr{I}_1^{\text{al}}$). As usual, we will solve the problem by dropping ($\mathscr{I}_2^{\text{al}}$), and it is possible to check later that it is fulfilled by the solution that we identify. The two remaining constraints must be met strictly and substituting in (7) we obtain

$$p_1\left(V(q_1)-\theta_1 q_1-(\theta_2-\theta_1)q_2\right)+p_2\left(V(q_2)-\theta_2 q_2\right)+p_1 R(\pi_1).$$

[3] See, for instance, Laffont and Tirole (1988) and the literature quoted therein.

Therefore, the first period quantities will be the same as in the one period model, and the first period profit of the principal is

$$S(p_1) + p_1 R(\pi_1). \tag{8}$$

Because the first period agent is rehired if and only if his first period cost parameter is θ_1, the two period profit of the principal is

$$S(p_1)+p_1R(\pi_1)+p_1S(\pi_1)+(1-p_1)S(p_1) = (2-p_1)S(p_1)+p_1S(\pi_1)+p_1R(\pi_1). \tag{9}$$

2.5 Comparing direct information and arm's length relationships

We now turn to the main aim of this paper, comparing the two structures of information. We begin by focusing our attention on the first period payoffs before turning to the topic of main interest, the total payoffs over both periods.

From equations (5) and (8), the first period profit of the principal is larger under arm's length relationships than under direct information if and only if $\gamma_1 \leq (2\pi_1 - 1)/{\pi_1}^2$. Therefore, an increase in γ_1 favors direct information while an increase in π_1 favors arm's length relationships (because the function $(2x-1)/x^2$ in increasing for $x \in (1/2, 1]$). Details are messy,[4] but two basic effects are at play. In the case of direct information, the relevant constraint is the participation constraint of agents with high first period cost parameters; when γ_1 is larger, they assign a larger probability to the fact that they are unlucky high quality agents. In the case of arm's length relationships, the relevant constraint is the incentive constraint of agents with low first period cost parameters; the larger π_1, the more likely they are to be rehired. It is quite striking that arm's length relationship do not necessarily provide stronger first period incentives.

The second period payoff of the principal is obviously greater with direct information; we now compare her aggregate payoff over two periods to see if this effect always dominates an eventual lower first profit payoff. From (6) and (9), the principal will prefer *not* to obtain information about the quality of the agent in the first period, if and only if

$$\begin{aligned}(2-p_1)S(p_1)+p_1S(\pi_1)+p_1R(\pi_1) &\geq (2-\gamma_1)S(p_1)+\gamma_1S(\pi_1)+\frac{\gamma_1(1-\pi_1)}{1-\gamma_1\pi_1}R(\pi_1)\\ \iff \left(\pi_1 - \frac{1-\pi_1}{1-\gamma_1\pi_1}\right)R(\pi_1) &\geq (1-\pi_1)(S(\pi_1)-S(\gamma_1\pi_1)). \end{aligned} \tag{10}$$

When $\gamma_1 = 1$, the left hand side of this equation is equal to $(\pi_1 - 1)R(\pi_1) < 0$ while the left hand side is equal to 0: hence if there are many good quality agents, the direct

[4]We are really interested in understanding the relative changes in $\gamma_1(1-\pi_1)/(1-\gamma_1\pi_1)$ and $p_1 = \pi_1\gamma_1$ as γ_1 and π_1 vary. When γ_1 increases, the first quantity increase "more"; when π_1 increases, p_1 increases while the fraction decreases.

information information structure is better than the arm's length structure. When γ_1 decreases, both the left and right hand sides decrease, but I am enable to conduct a general comparative statics exercise.[5] To examine the problem further, let us turn to a family of examples, where we set $V(q) = 2\sqrt{q}$ and $\theta_1 = 1$. This yields $q_1 = 1$, $q_2 = (1-p_1)^2/(\theta_2 - p_1)^2$, $S(p_1) = (1-(2-\theta_2)p_1)/(\theta_2 - p_1)$ and

$$R(p_1) = p_1(1-p_1)^2 \frac{\theta_2 - 1}{(\theta_2 - p_1)^2},$$

which enable us to prove the following theorem.

Theorem 1. *There exist values of the parameters such that the arm's length relationship information structure yields a larger profit for the principal than the direct information information structure.*

Proof. Simply substitute $\pi_1 = 0.8$, $\theta_2 = 1.2$, and $\gamma_1 = 0.5$ to show that the payoff of the principal under arm's length relationships is equal to 1.736 while her payoff under direct information is equal to 1.731667. □

Contrary to the case with moral hazard treated in Crémer (1995), there is no easy way to derive comparative statics results. Within the family of examples which we have just described, numerical computations show that an increase in θ_2 favors direct information. Figure 1 presents the results of numerical computations and shows quite clearly that there is no simple comparative statics. For instance, the left panel of the figure shows that for given values of θ_2 and γ_1, direct information can be optimal for small and large values of π_1, but not for intermediate values. Given the results of Drugov (2009), which shows how complicated the comparative statics are in the simplest Baron-Myerson model, this is not surprising.

3 Conclusion

We have been able to show that limiting the information flow can alleviate adverse selection problems, very much in the same way as it can alleviate moral hazard problems. However, this effect seems to be more important when there is moral hazard, as intuition would suggest. First, contrary to what happens with moral hazard, first period payoff is not necessarily larger with arm's length relationship than with direct information. Second, the numerical investigation found rather small sets of parameters for which arm's length relationships dominated. This does not imply that the result is without practical importance: it points out that the loss of objective information linked to vertical separation can, at least in part, be compensated by greater incentives to reveal private information.

[5] This seems to be extremely difficult, as, for instance, $R(\pi_1)$ is not monotone in π_1. (See Drugov (2009) for a discussion of the difficulties of comparative statics in the standard Baron-Myerson problem.)

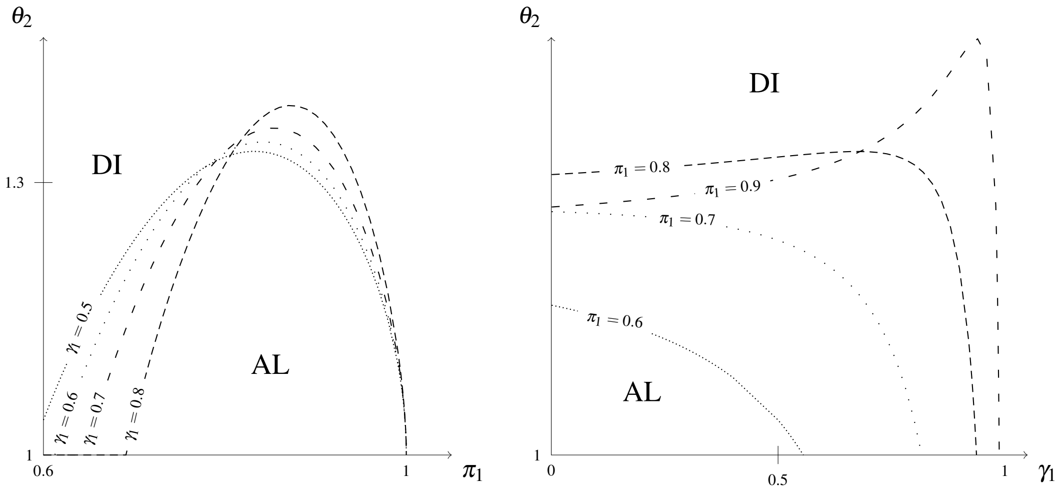

Figure 1: These two families of graphs present the comparative statics of the choice between arm's length relationship and direct information. On the left hand side panel, I represent how this choice is affected for different values of γ_1 by changes in π_1: for each π_1 the graphs represent the limit value of θ_2 for which arm's length relationships will be chosen. Below the curves, arm's length relationships yield a higher payoff to the principal. Above the curves, direct information is better. The right hand side panel shows the same data, keeping π_1 fixed and varying γ_1.

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