Voting over type and generosity of a pension system when some individuals are myopic: Technical appendix

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This appendix contains a presentation of the results obtained with the two extreme possibilities in the family of CES utility functions: the case of perfect substitution between consumptions in the two period of life ($\varepsilon = 1$) and the case of no substitution at all ($\varepsilon = -\infty$).

1 Utility function with perfect substitution across periods

In that case, we have that

$$u(c, d, \ell) = w(1-\tau)\ell - s - \ell^2/2 + s + p.$$

It is easy to see that saving plays no role in that case. Without any role for saving, we now show that the distinction between myopic and far-sighted agents vanishes, in the sense that they have exactly the same preferences, for any given level of productivity w.

In the case of a Beveridgean system, the equilibrium labor supply of both types of agents is the same. Since no one needs to save to equalize marginal utility of consumption across periods, the most preferred contribution rate is given by

$$\tau^*(\theta, 0) = \frac{1-\theta}{2-\theta}. \tag{1}$$

In the Bismarckian case, far-sighted agents are indifferent between any contribution rates lower than 1/4. The main difference with the logarithmic utility case comes from myopic individuals, who now dislike any positive tax rate: forced saving has lost all appeal at the voting stage while keeping the disadvantage of inducing myopics to work less than would be optimal. The majority chosen level of the Bismarckian contribution rate is thus zero.

When voting over the type of pension system, all individuals behave as far-sighted individuals who save did in the logarithmic utility case. The individual who is indifferent between the two systems is given by

$$\tilde{\theta}_R = \frac{2 - 2\tau^V(0,\lambda)}{2 - \tau^V(0,\lambda)},\ 0 < \tau^V(0,\lambda) \leq 1/4, \tag{2}$$

with all $\theta < \tilde{\theta}_R$ preferring Beveridge to Bismarck. We can then proceed as in the logarithmic utility case to show that a majority always prefers Beveridge to Bismarck.

Result 1 *In a society composed of myopic and far-sighted agents with a utility function showing perfect substitution across periods and a positively skewed distribution of abilities, the majority voting equilibrium pension is Beveridgean with a contribution rate equal to* $(1 - \theta^{med})/(2 - \theta^{med})$.

2 Utility function with no substitution across periods

In that case, we have that

$$u(c, d, \ell) = \min[w(1-\tau)\ell - s - \ell^2/2, s + p].$$

We first look at individuals' most preferred contribution rate in both systems before turning to majority voting.

2.1 Individuals' most preferred contribution rate under the Beveridgean system

The myopic individual's most preferred τ equalizes his consumption level across the two periods given that he is not saving and that his labor supply is given by $\ell_i^* = w_i(1-\tau)$. We obtain that

$$\frac{w_i^2(1-\tau^*(\theta,0))^2}{2} = \tau^*(\theta,0)(1-\tau^*(\theta,0))Ew^2,$$

which gives

$$\tau^*(\theta,0) = \frac{\theta}{2+\theta}.$$

The intuition for why the most preferred contribution rate increases with ability is clear: a higher ability individual needs to pay a higher fraction of his first period income in order for the net-of-tax and transfer income to be equalized across the two periods.

Far-sighted individuals who do not save because of credit constraints are in the same position as myopic agents: they use the pension system to equalize consumption across periods. They thus share the same $\tau^*(\theta, 0)$ as the myopic individuals. Far-sighted individuals who save rather choose the contribution rate that maximizes their total income over the two periods, using individual savings to equalize consumption across income. They thus behave as in the logarithmic utility case and their most preferred τ is given by equation (1). It is easy to see that individual with $\theta < 2/3$ do not save while individuals with $\theta > 2/3$ do save at their most preferred contribution rate.

2.2 Individuals' most preferred contribution rate under the Bismarckian system

The myopic individual's most preferred τ equalizes his consumption level across the two periods given that he is not saving and that his labor supply is $w_i (1 - \tau)$. We then obtain

$$\frac{w_i^2 (1 - \tau^*(\theta, 1))^2}{2} = \tau^*(\theta, 1) (1 - \tau^*(\theta, 1)) w_i^2,$$

which gives

$$\tau^*(\theta, 1) = \frac{1}{3}.$$

The most preferred contribution rate is then the same for all myopic individuals.

As for far-sighted individuals, the analysis is the same as in the logarithmic utility case: they are indifferent between any two contribution rates lower than 1/4 (since for any such rates public saving perfectly crowds out private saving) and dislike any rate greater than 1/4:

$$\tau^*(\theta, 1) \in [0, \frac{1}{4}].$$

We now turn to the study of majority voting equilibria. We first look at the majority voting equilibrium contribution rate in a Beveridgean system, before turning to the Bismarckian system. Finally, we look at the result of a vote between Beveridge and Bismarck as a function of the proportion of myopics in society.

2.3 Majority voting equilibrium contribution rate in a Beveridgean system

The most preferred Beveridgean contribution rate increases monotonically with ability for myopic individuals, but it first increases then decreases for far-sighted individuals (see Figure 1 panel B in the paper). If far-sighted are numerous enough (i.e., if λ is small enough), we have a majority voting equilibrium of the type "ends-against-the-middle", where low ability myopics and far-sighted together with high ability far-sighted would like a lower contribution rate (the first group because the contribution rate is high enough for their second period consumption to be higher than their first period's, the second group because they would rather save privately), while middle abilities myopics and far-sighted would like a higher contribution rate. Formally, the majority voting equilibrium contribution rate is given by

$$\begin{cases} \tau^V(0,\lambda) = \frac{\theta^-}{2+\theta^-} = \frac{1-\theta_R^+}{2-\theta_R^+}, \\ F(\theta^-) + (1-\lambda)(1-F(\theta_R^+)) = 1/2, \end{cases}$$

where θ^- is the median voter among the myopic and far-sighted credit constrained individuals while θ_R^+ is the median individual among the non credit constrained far-sighted. The second equation guarantees that these individuals are median, in the sense that half the population would like a higher contribution rate. It is clear that we have $0 < \tau^V(0,\lambda) < 1/4$, and that the majority equilibrium tax rate is weakly increasing in λ.

On the other hand, if the myopics are numerous enough, they may be able to push the Beveridgean rate above 25%. In that case, the majority voting equilibrium is not of an "ends-against-the middle" kind anymore, but rather we have that all far-sighted together with the myopics with $\theta < \theta^-$ would like a lower rate while myopics with $\theta > \theta^-$ would like a bigger contribution rate. θ^- is then obtained from

$$\lambda(1 - F(\theta^-)) = 1/2.$$

It will prove handy to compute the utility levels reached by different groups of individuals when the Beveridgean system is adopted. First, the myopic and far-sighted individuals with $\theta < \theta^-$ would rather prefer a lower tax rate, because the majority chosen one lowers their first period consumption level below their second period one. Hence,

$$U^{Be}(\theta) = c = \frac{\left(1 - \tau^V(0,\lambda)\right)^2}{2} w_i^2$$

for any individual (far-sighted or myopic) with $\theta < \theta^-$. Second, the far-sighted with $\theta > \theta^-$ do save in order to equalize their consumption levels across periods.

Their utility is then

$$U^{Be}(\theta) = c = d = \frac{\left(1-\tau^V(0,\lambda)\right)^2}{4} w_i^2 + \frac{\tau^V(0,\lambda)(1-\tau^V(0,\lambda))}{2} Ew^2.$$

Finally, a myopic individual with $\theta > \theta^-$ does not save although his first period utility is higher than his second period's. His utility is then

$$U^{Be}(\theta) = d = \tau^V(0,\lambda)(1-\tau^V(0,\lambda))Ew^2.$$

2.4 Majority voting equilibrium contribution rate in a Bismarckian system

The majority voting equilibrium contribution rate in a Bismarckian system is 1/4 if $\lambda \leq 1/2$ and 1/3 otherwise. The utility level attained by all myopics is

$$U^{Bi} = \begin{cases} d = \frac{3}{16} w_i^2 \text{ if } \tau^V(1,\lambda) = 1/4, \\ c = d = \frac{2}{9} w_i^2 \text{ if } \tau^V(1,\lambda) = 1/3. \end{cases}$$

As for far-sighted, their utility is

$$U^{Bi} = c^{Bi} = d = \frac{w_i^2}{4} \text{ if } \tau^V(1,\lambda) = 1/4.$$

On the other hand, if $\tau > 1/4$, their first period utility becomes lower than their second period's. This has an impact on their labor supply, since they only contemplate the impact of any labor supply decision on the first period only. In other words, they behave like the myopic agents do and decrease their optimal labor supply to $l_i = w_i(1-\tau)$. In that case,

$$U^{Bi} = c = d = \frac{2}{9} w_i^2 \text{ if } \tau^V(1,\lambda) = 1/3.$$

2.5 Voting over Beveridge vs Bismarck

The result of the vote will depend on whether myopics or far-sighted can impose their view on the Bismarckian contribution rate. We start with the situation where far-sighted are more numerous ($\lambda \leq 1/2$) before turning to the other possibility.

2.5.1 A majority of far-sighted: $\lambda \leq 1/2$

We start with the preferences of far-sighted for the two systems before turning to the myopics. We then analyze which among the two systems gets a majority of votes, as a function of the proportion of myopics.

We first show that all far-sighted who do not save (i.e. with $\theta < \theta^-$) prefer Beveridge to Bismarck, i.e. that

$$\begin{aligned} \frac{\left(1-\tau^V(0,\lambda)\right)^2}{2} w_i^2 &\geq \frac{w_i^2}{4}, \\ &\Leftrightarrow \tau^V(0,\lambda) \leq 1-\sqrt{1/2} \end{aligned}$$

which is always true given that $\tau^V(0,\lambda) \leq 1/4$.

For far-sighted with $\theta > \theta^-$, the analysis is exactly the same as in the case of a logarithmic utility function since they equalize consumption across the two periods. The threshold θ saver who is indifferent between Beveridge and Bismarck, denoted by $\tilde{\theta}_R$, is given by (2).

As for myopics, we first show that all the individuals with $\theta < \theta^-$ (i.e., those who would like a lower Beveridgean tax rate) prefer Beveridge to Bismarck:

$$\begin{aligned} \frac{\left(1-\tau^V(0,\lambda)\right)^2}{2} w_i^2 &\geq \frac{3}{16} w_i^2, \\ &\Leftrightarrow \tau^V(0,\lambda) \leq 1-\sqrt{3/8}, \end{aligned}$$

which is always true given that $\tau^V(0,\lambda) \leq 1/4$. Observe that myopics with $\theta < \theta^-$ differ only from far-sighted of the same productivity by the utility level they get in the Bismarckian system. This utility is smaller for the myopics since they don't work enough in their first period of life. Given that all far-sighted with low ability already prefer Beveridge to Bismarck, it is no surprise that myopics share the same preference.

Myopics for which $\theta > \theta^-$ prefer Beveridge to Bismarck if

$$\tau^V(0,\lambda)(1-\tau^V(0,\lambda))Ew^2 \geq \frac{3}{16} w_i^2,$$

i.e., if their ability is lower than a threshold

$$\tilde{\theta}_M = \frac{16}{3} \tau^V(0,\lambda) \left(1-\tau^V(0,\lambda)\right).$$

This threshold is always lower than 1 (given that $\tau^V(0,\lambda) < 1/4$ with a majority of far-sighted) and increases with $\tau^V(0,\lambda)$ for the reasons mentioned in the logarithmic utility case.

We can now look at the result of a majority vote over α, given λ. If $\lambda = 0$, there is no myopic individual and all far-sighted individuals with $\theta < \tilde{\theta}_R$ prefer Beveridge to Bismarck, as in the logarithmic utility case. However, we cannot simply reproduce the analysis contained in the logarithmic utility case to prove that they constitute a majority, because of the ends-against-the-middle property of the majority voting equilibrium Beveridgean contribution rate: $\tau^V(0,\lambda)$ is not the most-preferred τ of the median-endowed individual anymore, and one cannot simply compare $\tilde{\theta}_R$ with θ^{med}. Rather, we have to compare $\tilde{\theta}_R$ with θ_R^+. Since, at equilibrium, $F(\theta^-) + (1 - F(\theta_R^+)) = 1/2$, it follows that $1 - F(\theta_R^+) \leq 1/2$. The analysis contained in the logarithmic utility case shows that the curve which depicts $\tilde{\theta}_R$ (labeled R in Figure 2 of the paper) is everywhere to the right of the curve on which θ_R^+ is situated. Hence,$1 - F(\tilde{\theta}_R) < 1 - F(\theta_R^+) \leq 1/2$, which means that $F(\tilde{\theta}_R) > 1/2$ and that Beveridge is preferred by a majority.

We now look at what happens as the proportion of myopic individuals increases. As λ increases, $\tau^V(0,\lambda)$ weakly increases, which increases the political support for Beveridge among the myopics but decreases it among the far-sighted. As in the logarithmic utility case, a majority of the population may support Bismarck if there are enough myopics in the population. Finally, observe that $\tau^V(0,1/2) < 1/4$, because all far-sighted plus many low ability myopics prefer a lower-than-25% Beveridgean contribution rate.

We now look at what happens when the proportion of myopics is pushed above one half.

2.5.2 A majority of myopics: $\lambda > 1/2$

We start with the preferences of far-sighted for the two systems before turning to the myopics. We then analyze which among the two systems gets a majority of votes, as a function of the proportion of myopics.

We first show that all far-sighted who do not save with the Beveridgean system (i.e. with $\theta < \theta^-$) prefer Beveridge to Bismarck, i.e. that

$$\frac{\left(1 - \tau^V(0,\lambda)\right)^2}{2} w_i^2 \geq \frac{2}{9} w_i^2, \tag{3}$$

where the right hand side gives the utility for far-sighted under the Bismarckian system. This inequality is always true given that $\tau^V(0,\lambda) \leq 1/3$. The intuition for this result is as follows: in both systems, these far-sighted are credit constrained. In that case, they offer the same amount of labor as a myopic individual of the same ability. They prefer Beveridge to Bismarck because the former offers them a lower contribution rate and thus increases their first-period consumption compared to the latter.

For far-sighted with $\theta > \theta^-$, the analysis is the same as in the case of a logarithmic utility function (since they equalize consumption across the two periods), except that they get a lower utility with the Bismarckian system. The threshold

θ saver who is indifferent between Beveridge and Bismarck is obtained from

$$\frac{\left(1-\tau^V(0,\lambda)\right)^2}{4}w_i^2+\frac{\tau^V(0,\lambda)(1-\tau^V(0,\lambda))}{2}Ew^2=\frac{2}{9}w_i^2.$$

If $\tau^V(0,\lambda)$ is sufficiently low (lower than 0.057), all far-sighted prefer Beveridge to Bismarck. Above that threshold, the value of θ of the far-sighted who is indifferent is decreasing in $\tau^V(0,\lambda)$, and tends to 1 as $\tau^V(0,\lambda)$ tends towards 1/3. In other terms, there is always a majority of far-sighted who prefer Beveridge to Bismarck.

Observe that myopics with $\theta<\theta^-$ do not differ from far-sighted of the same productivity here, since they have the same labor supply and are both credit constrained in the two systems. We then obtain that all such myopics prefer Beveridge to Bismarck.

Myopics for which $\theta>\theta^-$ prefer Beveridge to Bismarck if

$$\tau^V(0,\lambda)(1-\tau^V(0,\lambda))Ew^2\geq\frac{2}{9}w_i^2,$$

where the right hand side, giving their utility under the Bismarckian system, is greater than in the corresponding case when $\lambda<1/2$. The threshold ability level is

$$\tilde{\theta}_M=\frac{9}{2}\tau^V(0,\lambda)\left(1-\tau^V(0,\lambda)\right),$$

which is then lower than its corresponding value when $\lambda<1/2$, for a given value of $\tau^V(0,\lambda)$. This threshold is always lower than 1 (given that $\tau^V(0,\lambda)<1/3$ with a majority of myopics) and increases with $\tau^V(0,\lambda)$ for the reasons mentioned in the logarithmic utility case.

We can now look at the result of a majority vote over α, given $\lambda\geq1/2$. If $\lambda=1/2$, there is no discontinuity in the majority voting equilibrium Beveridgean contribution rate, which is the one most preferred by far-sighted and myopics who are just credit-constrained ($\theta=\theta^-$) and far-sighted with ability $\theta>\theta^-$, who are saving. On the other hand, there is a discontinuity in the majority voting equilibrium Bismarckian rate, which jumps from 1/4 to 1/3 as myopics become a majority. As myopics get their most preferred tax rate in the Bismarckian system, their utility makes a discontinuous upward jump in that case, and there is a discontinuous downward jump in the proportion of myopics preferring Beveridge to Bismarck. The opposite reaction occurs for far-sighted: they are now all worse off in the Bismarckian system (because forced to save too much), and a strictly higher proportion of them prefers Beveridge. The total proportion in the population voting in favor of Beveridge changes discontinuously as one crosses the $\lambda=1/2$ line, but whether the proportion increases or decreases, and whether the majority result changes or not depends on the distribution of abilities.

As λ increases further, $\tau^V(0,\lambda)$ increases which increases the support for Beveridge among the myopics and decreases it for the far-sighted. The net impact on the proportion of votes for Beveridge is indeterminate. As noted in section 2.3, it is possible that, when the proportion of myopics reaches a certain threshold, the majority voting game on the Beveridgean contribution rate loses its "ends-against-the-middle" property, with all far-sighted preferring a lower tax rate. This does not generate any discontinuity in either equilibrium contribution rate or in the proportion of votes in favor of one system. Finally, as λ gets close enough to one, we can prove that the majority chosen system is the Beveridgean one. The decisive voter is then a myopic agent who just equalizes his first and second period consumption in the Beveridgean system. Equation (3) shows that he most prefers Beveridge to Bismarck, and so does a majority of voters. The intuition for this is straightforward: this myopic individual gets his most preferred contribution rate in both systems, but in the Beveridgean system he also benefits from the redistribution embedded in this system. Equation (3) shows that for all myopics with a lower ability, the redistributive benefit from Beveridge outweighs the fact that the Beveridgean tax rate is higher than their most preferred one, and that they end up being credit constrained.

We then obtain the following result.

Result 2 *Assume that society is composed of a fraction λ $(1-\lambda)$ of myopic (far-sighted) agents, that both kinds of agents have the same* $\min[c,d]$ *utility function and the same positively skewed distribution of abilities. Then a majority of voters prefer a Beveridgean social security system if $\lambda \in \{0,1\}$. If both types of agents coexist, it may be the case that a majority of voters prefer Bismarck to Beveridge. Moreover, there is a discontinuity in the political support for either system as myopics become the majority in society.*