Voting on pensions with endogenous retirement age[1]

Georges Casamatta[2], Helmuth Cremer[3] and Pierre Pestieau[4]

November 2001, revised October 2002

[1]This paper has been presented at CES/IFO workshop on pensions, Munich; Public economics workshop, GREQAM, Marseille; IIPF Conference, Linz; Recent advances in political economy, DELTA, Paris; CEPR workshop Dynamic Aspects of Policy Reforms, Vienna and in seminars at Toulouse and the Balearic Islands. We thank all the participants for their helpful comments. In particular, we are grateful to Mathias Kifmann, Eytan Sheshinski, Bertrand Wigniolle and especially to Vincenzo Galasso, for their very insightful discussions.

[2]GREMAQ-CNRS, Université de Toulouse.

[3]GREMAQ and IDEI, Université de Toulouse.

[4]CREPP, Université de Liège and CORE.

Abstract

It is often argued that the observed trend towards early retirement is due mainly to the implicit tax imposed on continued activity of elderly workers. We study the relevance of such a distortion in a political economy model with endogenous age of retirement. The setting is a two-period overlapping generations model. Individuals di¤er in their productivity. In the …rst period they work a …xed amount of time; in the second, they choose when to retire and then receive a ‡at rate pension bene…t. Pensions are …nanced by a payroll tax on earnings in the …rst and in the second period of life. Such a tax is non distortionary in the …rst period; it is in the second period. We allow for some rebating of the second period tax. Individuals vote on the level of the payroll tax given the rebate which can range from 0 (biased system) to 100% (neutral system). We provide su¢ cient conditions for the existence of a voting equilibrium and study its properties. Under these conditions, high tax rates are supported by all the old and by low productivity young individuals. We show that the pivotal voter is a young individual. The number of young individuals who have higher wage than the pivotal voter equals half the total population. Finally, we study the simultaneous determination of the bias and the tax rate through a voting procedure and show that the equilibrium (if any) implies a bias which is always positive and may or not be larger than one.

1 Introduction

Over the last forty years, labor force participation of the elderly has been dramatically decreasing in almost all industrialized countries. Participation rates for men aged 60 to 64 were above 70% in the early 60s; they have fallen to 57% in Sweden and to below 20% in Belgium, France, Italy and the Netherlands by the mid 90s.[1] At the same time, people are living longer and longer. In the European Union, life expectancy at age 65 has increased by more than one year per decade since 1950. This puts an enormous pressure on the …nancial viability of Pay-As-You-Go (PAYG) pension systems and the situation will become even more problematic when the “baby boomers” will come to retirement.

Gruber and Wise (1997) attribute this large decline in the labor force participation to the incentives created by social security systems. Continued work at later ages may be subject to two burdens: the traditional payroll tax which is nowhere age-dependent and forgone bene…ts when social security wealth decreases with the age of retirement. This double burden which Gruber and Wise calls an implicit tax represents an allocative distortion that induces early retirement. As they show it is higher in France, Belgium and Italy than in Japan, Sweden or the US. Average retirement is also quite earlier in these …rst countries than in the second.

Why do we have such an implicit tax on continued activity? On pure e¢ciency grounds, we would like to avoid any distortion in the labor retirement choice of aged workers and let them choose the age of retirement such that the marginal utility of retiring is equal to the worker productivity times marginal utility of consumption. This is the case in the laissez-faire equilibrium when there is no pension system and no taxes. More generally, the pension system could be designed to preserve this …rst-best trade-o¤ and we can then think of it as “neutral”. However, Cremer, et al. (2002) show

[1] A notable exception is Japan.

that when the design of the pension system reflects some redistributive concern and when non-uniform lump-sum transfers are not available (because individual characteristics like productivity or health status are not publicly observable) a neutral system is no longer desirable. The optimal system is then "biased" as it implies an implicit tax on continued activity which creates downward distortions.

In this paper we consider a setting where the optimal second-best pension system also implies a bias. We do characterize this second-best solution as a benchmark, but our main focus is positive rather than normative. We study the implications of a bias on the political support for pension systems. Specifically, we characterize the equilibrium (majority voting) size of the pension system for a given bias in the benefit formula. Finally, we also study the endogenous determination of the bias through a specific voting procedure.

To shed light on these questions, we consider a model where each individual lives two periods. He works one unit of time in the first, pays a proportional tax to the pension system and saves. In the second period he works for some time and then retires. His consumption then is financed by disposable earnings, gross returns of savings and a flat pension benefit. Individuals differ according to age, they are young or old, and according to productivity. In the second period, only a fraction of the payroll tax is imposed on elderly workers, the remaining being implicitly rebated. Given such a fraction which implies a distortion, a bias, on the retirement decision, young and old vote on their preferred payroll tax rate.

We provide sufficient conditions for the existence of a voting equilibrium and study its properties. Under these conditions, high tax rates are supported by all the old and by low productivity young individuals. We show that the pivotal voter is a young individual. The number of young individuals who have higher wage than the pivotal voter equals half the total population. Finally, we study the simultaneous determination of the bias and the tax rate through a voting procedure and show that the equilibrium (if any) implies a bias which is always positive and may or not be larger than one.

Before turning to these positive considerations, we brie‡y discuss the second-best solution where (linear) taxes and the bias are chosen to maximize a utilitarian social welfare function (Section 4). We show that the optimal system is also biased, but that the bias should be less than 100%, at least under some conditions. Intuitively, the optimal implicit tax in the second period is determined very much like Sheshinski's optimal linear income tax; i.e. by balancing redistributive bene…ts against distortions. Redistributive bene…ts arise because the implicit tax is proportional to income while the pension is ‡at. The distortion is related to the decrease in retirement age. Like in Sheshinski's setting the optimal (implicit) tax is positive which implies that a full rebate of the second period tax is not called for. Put di¤erently, there is a bias. However, under realistic assumptions on the range of wage inequalities, the distortion on retirement age leads to a lower tax in the second period than in the …rst one. Consequently, the optimal bias can be expected to be less then 100%.

The e¤ect of the introduction of a pension system on the retirement decision has been studied by Sheshinski (1978) and Crawford and Lilien (1981). These authors argue that introducing a pension system which is actuarially fair (total bene…ts are equal to total contributions) does not a¤ect the retirement decision when there are no borrowing constraints. In this case, private savings are just replaced by public pensions. Introducing borrowing constraints may cause individuals to retire earlier but not later. When the level of public pension contributions are high, forced saving may result. This induces an income e¤ect leading to early retirement (retirement leisure being a normal good). The introduction of a pension system which is not marginally fair[2] leads to a decrease in the price of leisure with respect

[2] Departure from actuarial fairness can be of three types. First, the pension system

to consumption. If the substitution effect dominates the income effect, it induces people to retire earlier.[3] When the system is marginally fair but gives benefits that outweighs contributions, the income effect will imply early retirement. We can thus expect that with a mature unbalanced PAYG system people tend to retire later.

To our knowledge, only two papers deal with the retirement decision in a political economy environment. Lacomba and Lagos (1999) study the problem of a direct vote on the (mandatory) retirement age. More closely related to our study, Conde Ruiz and Galasso (2000) develop a model in which the vote takes place simultaneously on the payroll tax rate and on the decision to introduce or not an early retirement provision. They show that the early retirement provision may be sustained at equilibrium by a coalition of the poor workers, who want to retire early, and old people with incomplete earnings history, who would receive no pension without this provision. This analysis and ours can be considered as complementary. Indeed, we do not investigate the issue of introducing an early retirement age.

2 The model

Individuals live for two periods and they are differentiated according to their wage level per unit of time (productivity). The distribution of productivities has support $[w_-; w_+]$, density function $f(\cdot)$, and cumulative distribution function $F(\cdot)$. We assume that the median productivity, w_m, is lower than the mean, $\overline{w}$. The intertemporal utility function is:

$$U(c; d) = u(c) + \beta u(d),$$

where c is the first period consumption and d is the second period consumption; β is a factor of time preference, which is, by assumption, equal

may redistribute across individuals. Second, the aggregate level of benefits may outweigh the aggregate level of contributions, which is typically the case in a non mature PAYG system. Third the system may not be marginally fair which is the case when there is an implicit tax as defined above.

[3] This takes an extreme form in our model where income effects are assumed away.

to $1/(1+r)$ where r is the interest rate. The utility function, $u(.)$ is increasing and concave: $u'(.) > 0$, $u''(.) < 0$. Moreover, we assume that $\lim_{x \to 0} u'(x) = +\infty$ and that the coefficient of relative risk aversion is lower than 1: $R_r(x) = -xu''(x)/u'(x) \leq 1$. Second period consumption, d, is to be distinguished from overall spending in the second period, x. Overall spending includes consumption plus the monetary disutility of $z \in [0;1]$, which is the fraction of the period the individual continues to work. This variable is interpreted as the retirement age. We assume a quadratic specification for the disutility of work, so that $d = x - \gamma z^2/2$. The parameter γ specifies the intensity of this disutility.

First and second periods are of equal length, normalized to 1. labor supply is assumed to be inelastic in the first period. In the second period, individuals decide which fraction of time, z, they spend working. Observe that with our specific form of the labor disutility function, there are no income effects in labor supply decisions which thus depends only on the relative price of leisure and consumption.

First and second period consumptions for an individual with productivity w are respectively given by:

$$
\begin{aligned}
c &= w(1-\tau) - s \\
x &= s(1+r) + wz(1-\theta\tau) + P,
\end{aligned}
$$

where $\tau \in [0;1]$ is the payroll tax rate and $s \geq 0$ is the amount of savings; P corresponds to the total pension received and, by assumption, does not depend on z. The parameter $\theta \in [0;1]$ measures the bias of the pension system. We define a neutral system as a system that does not distort individual decisions concerning retirement age. In other words, it does not modify the relative price of leisure and consumption, compared to the situation with no pension scheme. In a neutral system, the marginal benefit of working one more year is then w. This is the case in our setting when $\theta = 0$. When $\theta > 0$, the relative price of leisure and consumption becomes $w(1-\theta\tau)$.

Consumption is therefore more expensive and individuals are induced to retire earlier.[4]

Note that P does not depend on w. This means that the pension system considered operates income redistribution across individuals of the same generation. Everyone contributes for an amount proportional to his labor income but the benefit received does not vary across individuals.

3 Individual saving and retirement decisions

In this section, we characterize the savings and retirement decisions of old and young individuals, for given τ, P and θ. We denote $(z^y; s^y)$ the optimal decisions of young individuals, where z^y is the retirement age and s^y is the amount of savings. Decisions concerning savings have been made in the past for old people. Their only decision is to choose when to retire. The optimal retirement decision of an old individual is denoted z^o.

3.1 The old

The program of old individuals is the following:

$$\max_z s(1+r) + wz(1-\theta\tau) + P - \gamma\frac{z^2}{2} \tag{1}$$

subject to

$$0 \leq z \leq 1.$$

The first-order condition for an interior value of z is:

$$w(1-\theta\tau) - \gamma z^o = 0:$$

This leads to

$$z^o = \frac{w(1-\theta\tau)}{\gamma}.$$

[4] A pension system might also induce people to retire earlier when the amount of pension benefit forgone if working one more year is not compensated by a corresponding increase in the pension level. This effect would be taken into account in our model if P were decreasing in z.

In order to ensure that $z^o \leq 1$ for everyone, we assume $\gamma \geq w_+$. All the individuals choose to work in the second period (except when $\theta\tau = 1$). The higher the productivity of an individual, the later he retires: consumption being cheaper for more productive individuals, they choose to work and consume more, provided of course that there is no income effect. On the other hand, increasing the bias of the system or the payroll tax rate increases the price of consumption with respect to leisure and consequently induces people to retire earlier. When $\theta\tau = 0$; there are no distortions so that z^o is equal to w/γ which corresponds to the first best level. Finally, a higher disutility of work yields lower retirement ages.

3.2 The young

The program of young individuals is the following:

$$\max_{z,s} u\left[w(1-\tau) - s\right] + \beta u\left[s(1+r) + wz(1-\theta\tau) + P - \gamma z^2/2\right] \tag{2}$$

subject to

$$0 \leq z \leq 1 \text{ and } 0 \leq s \leq w(1-\tau):$$

The choice of z is the same as for old individuals and we have:

$$z^y = \frac{w(1-\theta\tau)}{\gamma}. \tag{3}$$

This yields

$$d = s(1+r) + \frac{w^2(1-\theta\tau)^2}{2\gamma} + P.$$

Recalling that by assumption $\beta(1+r) = 1$, the first order condition for an interior solution of s is:

$$-u'(c) + u'(d) = 0.$$

Individuals equalize first and second period consumptions (net of the disutility of labor). For individuals choosing an interior solution, we obtain:

$$s^y = \frac{w(1-\tau) - \dfrac{w^2(1-\theta\tau)^2}{2\gamma} - P}{(2+r)}. \tag{4}$$

3.3 Budget constraint

A feasible pension scheme must satisfy the government budget constraint:

$$
\begin{aligned}
N^o \int_{w_-}^{w_+} P f(w)\,dw &= N^y \tau \int_{w_-}^{w_+} w f(w)\,dw + N^o \theta\tau \int_{w_-}^{w_+} wz f(w)\,dw \\
\Leftrightarrow\ P &= (1+n)\,\tau\bar{w} + \theta\tau\bar{y} \\
&= (1+n)\,\tau\bar{w} + \frac{\theta\tau\,(1-\theta\tau)}{\gamma} E\left(w^2\right), \qquad (5)
\end{aligned}
$$

where N^y and N^o are respectively the numbers of young and old individuals, n is the rate of population growth, $y = wz$, $\bar{y} = \int_{w_-}^{w_+} y f(w)\,dw$ and $E\left(w^2\right) = \int_{w_-}^{w_+} w^2 f(w)\,dw$. We assume that

$$
\frac{1+n}{1+r} \leq \sqrt{E(w^2)}/\bar{w}. \qquad (6)
$$

Because $\sqrt{E(w^2)} > \bar{w}$, condition (6) requires that n is not too large compared to r. It is always satisfied when $n \leq r$.

The total pension received by a given individual is the sum of (per capita) tax revenues on first and second period incomes. The tax base in the first period, $(1+n)\,\bar{w}$, is fixed whereas it depends on $\theta\tau$ in the second period. Put differently, taxation only gives rise to distortions on second period income. Differentiating (5) yields:

$$
P'(\tau) = (1+n)\,\bar{w} + \frac{\theta - 2\theta^2\tau}{\gamma} E\left(w^2\right) \qquad (7)
$$

and

$$
P''(\tau) = \frac{-2\theta^2}{\gamma} E\left(w^2\right) \leq 0. \qquad (8)
$$

The budget curve, represented on Figure 1, is concave, always above the line $\tau(1+n)\,\bar{w}$ and P is equal to $\tau(1+n)\,\bar{w}$ when $\tau = 0$ and $\theta\tau = 1$.

4 Optimal solution: first- and second-best

Even though our approach is mainly positive, it is worth looking at the solution chosen by a utilitarian social planner. For simplicity we concentrate in

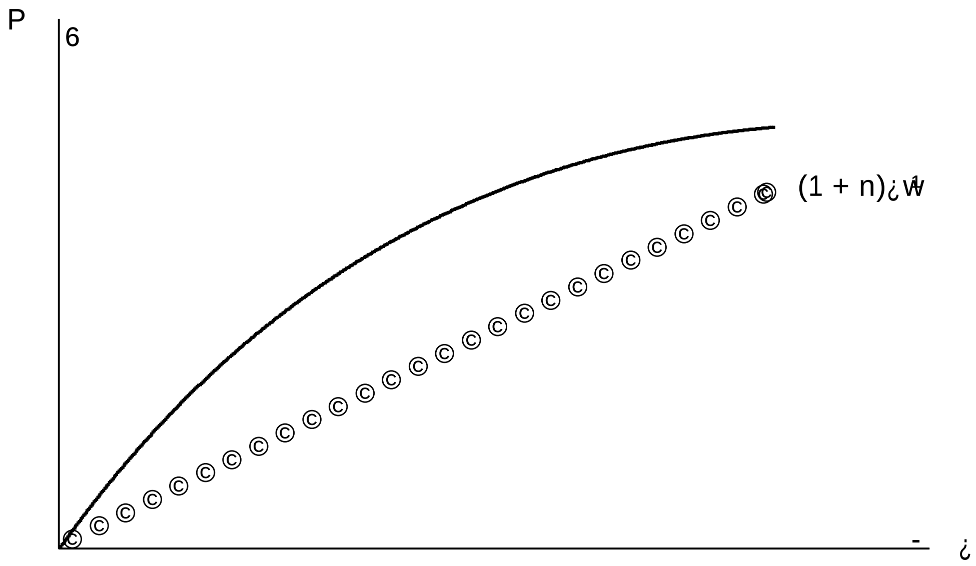


Figure 1: The budget curve

this section on the case where $r = n$. We successively consider the first-best allocation and then the second-best solution achieved with the instruments (and information) available in our setting. These solutions provide interesting benchmarks for the interpretation of the voting equilibria considered below. In particular they allow one to study the rationale (if any) of a distorting tax (biased system) from a normative optimal taxation perspective.

The first-best solution is obtained by solving:

$$\max \int_{w_-}^{w_+} \left[u(c(w)) + \beta u\left(x(w) - \gamma z(w)^2/2 \right) \right] f(w)\, dw$$

subject to

$$\int_{w_-}^{w_+} \left[c(w) + \frac{x(w)}{1+n} - \frac{w}{1+n}(1+n+z(w)) \right] f(w)\, dw = 0.$$

The first order conditions imply:

$$u'(c(w)) = \beta (1+n) u'(d(w)) = \mu \tag{9}$$

and

$$z(w) = \frac{w}{\gamma} \tag{10}$$

where μ is the Lagrange multiplier associated with the resource constraint. These conditions are rather standard and intuitive. Consumption is equalized across individuals and even across periods if $\beta(1+n) = 1$.[5] Labor supply varies across individuals such that the marginal utility of retirement z/γ is equal to the marginal productivity of labor, w. It is plain that this solution cannot be achieved with the instruments considered in our setting.

We now consider the second-best problem that obtains when instruments are limited to the parameters τ, θ and P. Without loss of generality we will introduce a new variable $\varsigma \equiv \theta\tau$ so that the social planner acts as if he determines a tax rate for the first period τ and another one, ς, for the second period.

The optimality problem can now be rewritten:

$$\max_{\tau,\varsigma} \varpi = \int_{w_-}^{w_+} \left[u\left(w(1-\tau) - s^y\right) + \beta u\left(s^y(1+r) + P + wz^y(1-\varsigma) - \frac{\gamma z^{y2}}{2} \right) \right] f(w)\, dw$$

subject to

$$P = (1+n)\tau\bar{w} + \varsigma \int_{w_-}^{w_+} wz(w).$$

Using (3), the utility in the second period is:

$$u\left(s^y(1+r) + \frac{w^2(1-\varsigma)^2}{2\gamma} + (1+n)\tau\bar{w} + \frac{\varsigma(1-\varsigma)}{\gamma} E\left(w^2\right) \right).$$

The first-order conditions are given by:

$$\frac{\partial \varpi}{\partial \tau} = -\int_{w_-}^{w_+} \left[wu'(c) - \beta(1+n)\bar{w}u'(d) \right] f(w)\, dw \qquad (11)$$

$$\frac{\partial \varpi}{\partial \varsigma} = -\int_{w_-}^{w_+} u'(d) \left(\frac{w^2(1-\varsigma)}{\gamma} - \frac{1-2\varsigma}{\gamma} E\left(w^2\right) \right) f(w)\, dw. \qquad (12)$$

To interpret (11), assume first that there is no liquidity constraint (savings can be negative). Then, $c(w) = d(w)$ for all w and we have $\tau = 1$. When there is a liquidity constraint, on the other hand, τ has to be less than 1 to ensure a positive consumption in the first period. However, we continue to have a positive tax. Recall that the tax on the first period income does

[5] With our assumption that $\beta(1+r) = 1$ this holds when $r = n$.

not affect labor supply. The standard linear income tax problem then calls for a one hundred percent tax which is redistributed in a lump sum way. However, here the redistribution takes place in the second period (through the flat pension). When there is no liquidity constraint this is of no relevance and the traditional result goes through. With a liquidity constraint, a 100% tax is not feasible. In addition, a high tax then adversely affects individuals with a low w: they have to pay taxes on an already low first-period income while the lump sum refund (pension) only occurs in the second period.

The condition for σ is equivalent to the standard formula for an optimal linear tax:

$$\sigma = \frac{\int_{w_-}^{w_+} u'(d)\left[w^2 - E\left(w^2\right)\right] f(w)\,dw}{\int_{w_-}^{w_+} u'(d)\,[w^2 - 2E(w^2)]\, f(w)\,dw}$$

where the numerator is the covariance between the marginal utility of $d(w)$ and the square of the productivity levels. This covariance is negative. We thus have:

$$\sigma = \frac{\operatorname{cov}\left(u'(d), w^2\right)}{\operatorname{cov}\left(u'(d), w^2\right) - E(w^2)\int_{w_-}^{w_+} u'(d)\, f(w)\,dw}, \tag{13}$$

namely $0 < \sigma < 1$. Observe that if σ were imposed on w rather than on wz, in other words if there was no distortion, σ would be equal to 1. This is useful for what follows. With a utilitarian objective, it is the liquidity constraint for τ and the labor disincentive for σ which prevent the tax rates to be maximal. These two factors are also crucial in our political economy model.

To sum up, and recalling the definition $\sigma = \theta\tau$ we have shown that the (utilitarian) second-best policy always calls for a positive bias. In the case where there is no liquidity constraint, we can also be sure that $\theta < 1$. Consequently, it is then always optimal to have some rebating of the second period tax. When there is a liquidity constraint, on the other hand, the comparison between τ and σ is in principle ambiguous and the bias can be smaller as well as larger than one. However, for reasonable assumptions on the wage distribution and labor supply elasticities $\tau < \sigma$ (i.e., $\theta > 1$)

will not arise. Intuitively, $\tau < \tilde{\tau}$ could arise if the redistributive benefits of the second period tax (relative to the first period tax) were to outweight the distortion. This requires a highly unequal wage distribution; recall that (with a liquidity constraint) the individuals with the lowest wage tend to be penalized by a high first-period tax.

5 Majority voting equilibrium tax rate

We now turn to the study of the voting equilibrium. For the time being we assume that the bias, θ, is exogenously given so that the policy choice involves a single dimension, namely τ (with P then being automatically determined by the budget constraint (5)).

5.1 Preferred tax rates

Define

$$V^y(\tau;\theta;w) = u[w(1-\tau) - s^y] + \beta u\left[s^y(1+r) + P(\tau;\theta) + w^2(1-\theta\tau)^2/2\gamma\right] \qquad (14)$$

and

$$V^o(\tau;\theta;w) = u\left[s^o(1+r) + P(\tau;\theta) + w^2(1-\theta\tau)^2/2\gamma\right], \qquad (15)$$

which represent the utility levels attained by type w individuals, young or old, for given τ and θ. Preferred tax rates for young and old individuals, denoted respectively τ^y and τ^o, are obtained by solving the following programs:

$$\max_{\tau \in [0;1]} V^i(\tau;\theta;w), \; i = y;o.$$

The following proposition states the properties of the most preferred tax rates.

Proposition 1 (i) Preferred tax rates of young individuals are decreasing with productivity.

(ii) No young individual chooses a corner solution at $¿ = 1$. The preferred tax rate of young individuals with productivity $w \cdot \overline{w}(1 + n) = (1 + r)$ is positive.

(iii) Old individuals choose corner solutions with either $¿^o = 1$ for everyone or alternatively $¿^o = 1$ for the poor old and $¿^o = 0$ for the rich old.

The formal proof is given in Appendix 1. Here we provide a sketch of the main intuition The intuition for the …rst result is as follows. Consider for simplicity the case $µ = 0$. When w increases, …rst period income also increases. Second period consumption being a normal good, it is increased through an increase of the tax rate. On the other hand, the relative price of …rst and second periods consumptions, $\overline{w}(1 + n) = w$ decreases. Put di¤erently, it costs less (in terms of …rst period consumption) to buy one unit of second period consumption for a low productivity individual than for a high one. By this substitution e¤ect, high productivity individuals are induced to buy less second period consumption. For utility functions such that $R_r(:) < 1$, the substitution e¤ect dominates and low productivity individuals want tax rates larger than high productivity individuals. Note that when $R_r(:) = 1$ (logarithmic utility function), income and substitution e¤ects neutralize and preferred tax rates are constant with respect to productivity. Finally, there is a third e¤ect of an increase in w. Because second period income increases with productivity, high productivity individuals raise their …rst period consumption (which is a normal good) by reducing the payroll tax rate. This e¤ect reinforces the second one. As a consequence, preferred tax rates are decreasing with productivity when $R_r(:) \cdot 1$.

The …rst part of point (ii) is obvious. When the tax rate equals one, marginal utility of consumption tends to in…nity and all individuals prefer a smaller tax rate. To illustrate the second part, let us write the …rst-order derivative of a young individual life cycle utility at the point $¿ = 0$ (saving being optimally chosen):

$$\left.\frac{dV^y}{d\tau}\right|_{\tau=0} = -wu'(c) + \beta(1+n)\overline{w}u'(d) + \beta\frac{\mu}{\gamma}\left(E\left(w^2\right) - w^2\right)u'(d)$$
$$= \left(-w + \beta(1+n)\overline{w} + \beta\frac{\mu}{\gamma}\left(E\left(w^2\right) - w^2\right)\right)u'(d),$$

where we have used the fact that saving is positive when $\tau = 0$ which implies that $u'(c) = u'(d)$. A first observation is that, in a neutral system ($\mu = 0$), individuals choose a positive tax rate if and only if $w \leq \beta\overline{w}(1+n) = \overline{w}(1+n)/(1+r)$. This is because with $\mu = 0$ there is no taxation of second period income. Individuals favoring a positive tax rate are those for whom the rate of return of the PAYG system, $(1+n)\overline{w}/w$ is higher than the rate of return of private savings, $1+r$. Furthermore, such individuals do not want to save when the tax equals their most preferred rate. Now, if one introduces a bias in the system, second period incomes are redistributed from individuals with a productivity level higher than $\sqrt{E(w^2)}$ towards individuals with a lower productivity.[6] Therefore, individuals such that $w \leq \overline{w}(1+n)/(1+r)$ continue to favor a positive tax rate but some individuals with a higher productivity also do. It should be noted that unlike in the neutral case, some individuals may now be saving, even when the tax is at their most preferred rate. To understand this, observe that the payroll tax rate serves two objectives: intertemporal consumption smoothing and (second period) income redistribution. An individual may then have some incentive to reduce his preferred tax rate with respect to the neutral case in order to limit tax distortions. In such a case, savings might constitute a useful instrument to transfer resources between first and second periods.

It should be noted that when $\mu > 0$, some young individuals with productivity larger than $\overline{w}(1+n)/(1+r)$ support the pension system (i.e., are in favor of a positive tax rate). Consequently, it appears that the introduction of a bias increases the political support for the pension system.

[6] Second period income (i.e., wz) is less than the average if and only if $w < \sqrt{E(w^2)}$.

The last point of the proposition says that old individuals choose corner solutions for the tax rates. To understand this, differentiate the objective function of the old, (15) with respect to the tax rate, yielding:

$$\frac{dV^o}{d\tau} = \left((1+n)\,\overline{w} + \frac{\theta(1-2\theta\tau)}{\gamma} E\left(w^2\right) - \frac{\theta(1-\theta\tau)}{\gamma} w^2 \right) u'(d) . \qquad (16)$$

When $\theta = 0$; the RHS of (16) is necessarily positive. In a neutral system (and by continuity in a slightly biased system), the welfare of an old individual is an increasing function of the tax rate. Consequently, all old individuals want the tax rate to be as high as possible and choose $\tau = 1$. When θ is increased, one can see, by evaluating the above expression at $\tau = 0$, that old people with productivity $w^2 < E\left(w^2\right) + (1+n)\,\overline{w}\gamma/\theta$ want a positive tax rate. We prove in the appendix that they in fact most prefer $\tau = 1$. Starting from $\tau = 0$, old individuals with a higher productivity dislikes a marginal increase in the tax rate. However this does not mean that their optimal tax rate is 0. Indeed, we show that their objective function may be convex. To see this, evaluate the above expression at $\tau = 1$ when $\theta = 1$: it is positive. When $\theta = 1$ and τ approaches 1, everyone stops working and the old rich do not suffer anymore from the redistribution towards the poor. On the other hand, their pension increases with the tax rate. They thus favor a marginal increase in the tax rate.

5.2 Voting equilibrium

We turn to the determination of the equilibrium payroll tax rate under majority voting. We have proved in Appendix 1 that, for utility functions such that $R_r(\cdot) \leq 1$, preferences of the young over tax rates satisfy the single-crossing condition established by Gans and Smart (1996). This means that we can order young individuals and alternatives in such a way that if an individual prefers the higher of two alternatives, all the individuals ranked to the right of this individual display the same preference. Similarly, one can show that preferences of the old also satisfy the single-crossing property. However, these properties are not sufficient to guarantee existence of a Condorcet winner. This is because when the entire population (young and old jointly) is considered, single-crossing can no longer be established. To overcome this difficulty, we restrict our attention to cases where the utility of the old is monotonically increasing with the tax rate. This is true when the marginal utility of the richest old individual is positive at $\tau = 0$ which, from (16), is the case when $\theta/\gamma \leq (1+n)\,\overline{w}/\left(w_+^2 - E\left(w^2\right)\right)$. This condition is satisfied when the bias parameter, θ, is small enough, or when γ is large enough. With the old preferring a maximum tax rate and the preferred tax of the young decreasing with productivity (Proposition 1, (i)) the construction of the equilibrium is straightforward. Specifically, the pivotal voter is a young individual so that the number of young individuals who have higher wage (and thus want a lower tax) equals half the total population. This is stated formally in the following proposition.

Proposition 2 If $\theta/\gamma \leq (1+n)\,\overline{w}/\left(w_+^2 - E\left(w^2\right)\right)$, a voting equilibrium on τ exists. It is given by $\tau^{mv} = \tau^y(\theta, w_{piv})$, where w_{piv} is the productivity of the pivotal individual which is implicitly determined by the following condition:

$$N^y(1 - F(w_{piv})) = \frac{N^o + N^y}{2}$$
$$\Leftrightarrow \quad F(w_{piv}) = \frac{n}{2(1+n)}. \tag{17}$$

Observe that condition (17) also implies that $N^o + N^y F(w_{piv}) = (N^o + N^y)/2$ so that the young with a lower wage plus the old (i.e., the individuals who want a higher tax) represent also half of the population.

Using Proposition 1 (ii) we can now establish the following properties of the equilibrium tax rate.

Proposition 3 The tax rate τ^{mv} defined by Proposition 2 is positive if

$$w_{piv} < \overline{w}(1+n)/(1+r). \tag{18}$$

Interestingly, condition (18) is necessarily satisfied when $n = r$. To see this observe that $n/(2(1+n)) < 1/2$. Consequently, the pivotal voter has a productivity level below the median level w_m which in turn is below the mean level $\overline{w}$. This argument also makes it clear that condition (18) continues to hold when n is smaller than r, but not "too small". However, as the differential between n and r increases the condition becomes more and more difficult to satisfy. Intuitively, the pivotal individual benefits from the redistribution implied by the PAYG system. Consequently, he favors a positive tax, unless the return of PAYG, namely n, is significantly smaller than the interest rate.

To conclude, it is important to stress that the condition for existence imposed in Proposition 2 is only a sufficient condition. A less stringent (though more complex) condition would be to require that the utility of the richest old be higher at the point $\tau^y(\theta; w_{piv})$ than at $\tau = 0$, that is:

$$
\begin{aligned}
& V^o\left[\tau^y(\theta; w_{piv}); \theta; w_+\right] \geq V^o\left[0; \theta; w_+\right] \\
\Leftrightarrow \quad & \frac{w_+^2\left[1 - \left(1 - \theta\tau^y(\theta; w_{piv})\right)^2\right]}{2\gamma} \\
\leq \quad & (1+n)\,\tau^y(\theta; w_{piv})\,\overline{w} + \frac{\theta\tau^y(\theta; w_{piv})\left(1 - \theta\tau^y(\theta; w_{piv})\right)}{\gamma} E\left(w^2\right) \qquad (19)
\end{aligned}
$$

If this condition is satisfied for the richest old, it is also true for individuals with a lower productivity. It follows that any tax rate lower than $\tau^y(\theta; w_{piv})$ is rejected by the coalition of all the old and the young to the left of w_{piv}. In addition, a tax rate higher than $\tau^y(\theta; w_{piv})$ is (by definition) rejected by the young to the right of w_{piv}. Consequently, as long as condition (19) holds, $\tau^y(\theta; w_{piv})$ continues to be a Condorcet winner even when the preferences of the old are not monotonically increasing.[7]

[7] Condition (19) being violated does not necessarily imply that there is no equilibrium. Specifically, there may be an equilibrium where some old favor a zero tax while other are in favor of $\tau = 1$. The partition of individuals is then more complex and w_{piv} defined by (17) is no longer the pivotal individual.

6 Endogenous bias

In the previous section, we have assumed that θ was exogenously given. We now turn to the political determination of θ. The natural approach would be to determine the pair $(\theta; \tau)$ chosen jointly in a majority vote. However, it is well known that a Condorcet winner is unlikely to exist when the issue space is multidimensional and this model is not an exception. To overcome this difficulty we consider a more restricted voting procedure, introduced by Shepsle (1979), and assume that the parameters θ and τ are chosen simultaneously and independently. The equilibrium (if any) of this procedure is then a pair $\left(\theta^S; \tau^S\right)$ such that the level of each variable is a Condorcet winner given the level of the other variable. In other words, it is given by the intersection of the two "reaction functions" $\tau^{mv}(\theta)$ and $\theta^{mv}(\tau)$ specifying the majority equilibrium level of a variable given the level of the other variable. The function $\tau^{mv}(\theta)$ has been studied in Section 5, at least over some range of values of θ. We now turn to the determination of $\theta^{mv}(\tau)$, the equilibrium bias for a given tax rate.

6.1 Voting equilibrium level of θ for a given level of τ

With τ given, the variables left to determine are θ and P, with of course a single degree of freedom because of the budget constraint. Differentiating (1) and (2) shows that the slope of an indifference curve in the $(\theta; P)$ plane is given by

$$\frac{dP}{d\theta} = \frac{\tau w^2 (1 - \theta\tau)}{\gamma} > 0;$$

for the young as well as for the old individuals. This expression is increasing with w. Consequently, preferences are single-crossing so that a majority voting equilibrium on θ exists.

An individual's most preferred level of θ is obtained by solving:

$$\max_{\theta \in [0;1/\tau]} V^i(\tau; \theta; w), \ i = y; o,$$

This yields

$$\theta^i = \frac{E\left(w^2\right) - w^2}{\tau\left(2E\left(w^2\right) - w^2\right)}, \; i = y, o; \tag{20}$$

and

$$\frac{d\theta^i}{dw} = -\frac{2\tau w E\left(w^2\right)}{\left[\tau\left(2E\left(w^2\right) - w^2\right)\right]^2} < 0; \; i = y, o. \tag{21}$$

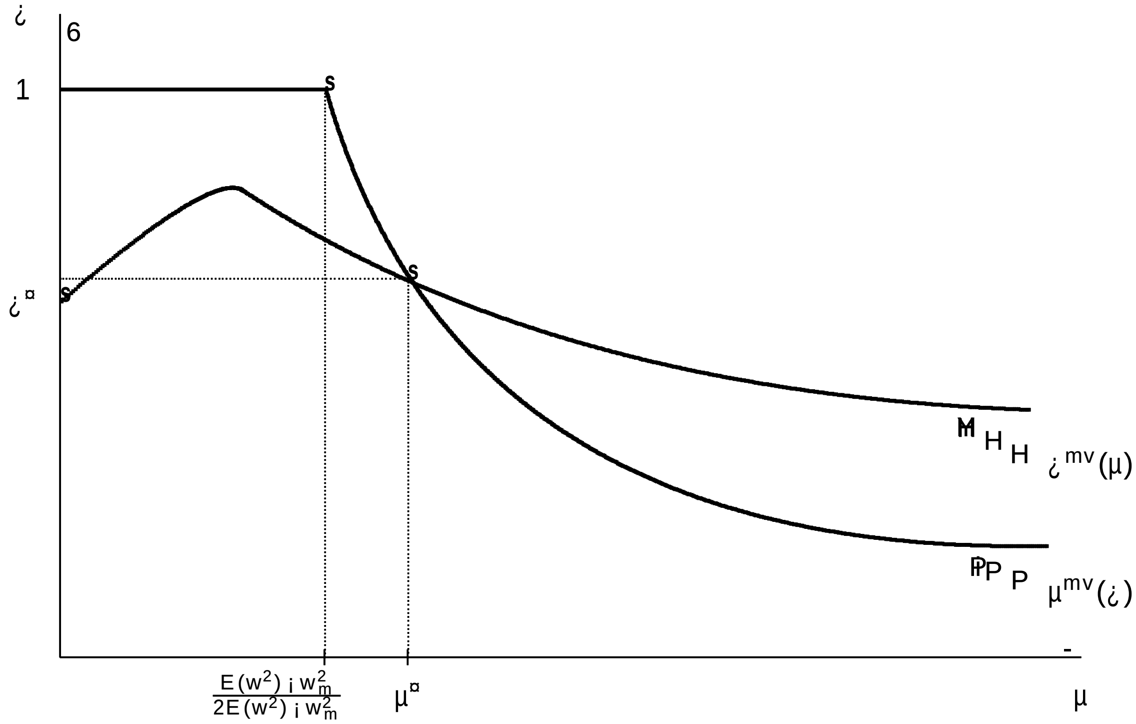
Notice that these expressions apply for old as well as for young individuals. Consequently, optimal levels of θ are decreasing with productivity and do not depend on age. The Condorcet winner is then the level of θ most preferred by the individuals with median productivity and we have:

$$\theta^{mv}(\tau) = \frac{E\left(w^2\right) - w_m^2}{\tau\left(2E\left(w^2\right) - w_m^2\right)} > 0. \tag{22}$$

This hyperbolic expression is represented on Figure 2 for $\tau \in [0, 1]$. It thus appears that the equilibrium bias is positive and decreasing in the tax rate. Intuitively, (22) defines the second period tax rate $\varrho = \theta\tau$, which is optimal from the median voter's perspective. The level of this second period tax does not depend on the first period tax, τ. In other words, when τ changes, the median voters most preferred level of ϱ remains constant and is achieved by adjusting θ.

6.2 Simultaneous voting

Returning to the original problem, namely the determination of the equilibrium pair $\left(\theta^S, \tau^S\right)$, we face the difficulty that $\tau^{mv}(\theta)$, may not be defined for all levels of θ; see Section 5. The simplest way to address this problem is to consider a candidate equilibrium (θ^c, τ^c), given by the intersection of $\theta^{mv}(\tau)$ with the curve $\tau^y(\theta, w_{piv})$, which gives the preferred tax of a young individual with productivity w_{piv} (defined by (17)). Put differently, the candidate equilibrium, (θ^c, τ^c), is defined by the conditions $\theta^c = \theta^{mv}(\tau^c)$, $\tau^c = \tau^y(\theta^c, w_{piv})$; see Figure 2 for an illustration. We show in Appendix D that $\theta^{mv}(\tau)$ and $\tau^y(\theta, w_{piv})$ intersect at least once so that the candidate equilibrium (θ^c, τ^c) exists. This does not imply that a simultaneous,



issue by issue, voting equilibrium, $\left(\theta^S; \tau^S\right)$, exists. However, $(\theta^c; \tau^c)$ is an equilibrium when (sufficient) condition (19) is satisfied, that is when

$$\frac{w_+^2\left[1 - (1 - \theta^c\tau^c)^2\right]}{2\gamma} \leq (1+n)\,\tau^c\overline{w} + \frac{\theta^c\tau^c\,(1 - \theta^c\tau^c)}{\gamma} E\left(w^2\right),$$

where

$$\theta^c\tau^c = \frac{E\left(w^2\right) - w_m^2}{\left(2E\left(w^2\right) - w_m^2\right)}. \tag{23}$$

For simplicity we assume in the remainder of this section that this is effectively the case. Then it is plain that this procedure yields a positive bias $\theta^S > 0$ and an interior solution for the tax rate $0 < \tau^S < 1$. We also have an interior solution for the second period tax $\rho = \theta^S\tau^S$. However, there is no obvious way to determine whether the bias is larger or smaller than one; inspection of expressions only allows one to conclude that both cases appear to be possible. Similarly, the comparison between the second period tax implied by (23) and the socially optimal level defined by (13)

Skewed dist.	Shepsle			Util. Opt.		
	θ	τ	$\theta\tau$	θ	τ	$\theta\tau$
$\varepsilon = 0.2$	0.44	0.82	0.36	0.25	0.30	0.08
$\varepsilon = 0.5$	0.80	0.45	0.36	0.56	0.29	0.16
$\varepsilon = 1$	1.17	0.31	0.36	0.96	0.26	0.25
Unif. dist.	**Shepsle**			**Util. Opt.**		
	θ	τ	$\theta\tau$	θ	τ	$\theta\tau$
$\varepsilon = 0.2$	0.22	0.89	0.19	0.17	0.28	0.05
$\varepsilon = 0.5$	0.40	0.49	0.19	0.42	0.26	0.11
$\varepsilon = 1$	0.64	0.31	0.19	0.97	0.21	0.21

appears to be ambiguous. Not surprisingly, it is then also not possible to compare the equilibrium bias θ^S to the optimal (second-best) bias studied in Section 4. To illustrate the results and to show that the comparisons between equilibrium and optimum are effectively ambiguous we now present some numerical examples.

6.3 Numerical examples

Consider the following specifications for the various components of our model. Productivities are distributed on $[1; 100]$ and we set $\gamma = 100$ and $r = n = 1$. We consider two possible distributions: a distribution skewed to the right with $w_m = 17.59 < \overline{w} = 22.33 < \sqrt{E(w^2)} = 26.82$ and a uniform distribution function with $w_m = \overline{w} = 50.5 < \sqrt{E(w^2)} = 58.03$. The utility function is isoelastic: $u(x) = x^{1-\varepsilon}/(1-\varepsilon)$, where ε is the coefficient of relative risk aversion. We consider three levels of ε: 0.2, 0.5 and 1.

Table 1 presents the simultaneous voting equilibrium $\left(\theta^S; \tau^S\right)$ and the utilitarian (second-best) optimum $(\theta^*; \tau^*)$ for each combination of parameter values. To facilitate interpretations, we also report $\theta\tau = \hat{t}$, i.e., the second period implicit tax rate.

These results are very interesting and confirm a number of points suggested by the analytical expressions. In particular, they show that a bias which is larger than one can effectively arise at the voting equilibrium. Furthermore, we …nd cases where the equilibrium bias exceeds the optimal one, and case where the opposite is true. Similarly, for the second period tax, $\mu^{S}¿^{S} > \mu^{¤}¿^{¤}$ as well as $\mu^{S}¿^{S} < \mu^{¤}¿^{¤}$ can e¤ectively occur.

7 Conclusion

This paper has examined two related questions. First, we have studied the determination of the size of the pension system through the political process when retirement age is endogenous. Speci…cally, each individual chooses his retirement age by taking into account the (given) bene…t formula which may or may not be biased (i.e., impose an implicit tax on continued activity of the elderly). We have shown how the endogenous retirement age decision a¤ects voters preferences over tax rates and how this relationship is a¤ected by the bene…t formula. Overall, it has turned out that the general properties of the voting equilibrium (if any) are not qualitatively di¤erent from the case with exogenous retirement age, see Casamatta et. al (2000).[8] In particular, the feature that high tax rates are supported by all the old and by low productivity young individuals has reemerged within our current setting.

The second question has involved a more signi…cant departure from the existing literature. It concerns the determination of the bene…t formula and, more speci…cally, the bias of the pension system. The challenge we were addressing was to seek explanations for the empirically observed bias in retirement systems. This is a more ambitious problem and we can only claim to have provided some preliminary and partial solutions. We have shown that a biased system is optimal from a utilitarian perspective and that it also tends to emerge from a speci…c political process (simultaneous and independent vote). From that perspective, political economy consideration are not necessary to explain the emergence of a biased system. On the other hand, an extreme bias ($\mu > 1$) though often observed in reality is not

[8] Except that existence may be more problematic.

consistent with welfare maximization.[9] It can, however, emerge from the political process. Speci…cally, we have shown that a very simple majority procedure is su¢cient to obtain such a drastic departure from optimality.

Our setting is rather restrictive and our …ndings have to be quali…ed accordingly. Our main endeavor it to point to possible explanations for the high implicit tax on continues activity. We do not claim that the e¤ects we discuss are always relevant, nor that they provide the unique or even main driving forces. To address these issues additional studies featuring (for instance) more general bene…t formulas and alternative speci…cations of the political process are required. We leave these questions open for future research.

[9] Under plausible assumptions on wage distribution and labor supply (retirement age) elasticity.

References

[1] Casamatta, G., Cremer, H. and P. Pestieau, 2000, "The political economy of social security", Scandinavian Journal of Economics, 102, 503-522.

[2] Conde Ruiz, J.I. and V. Galasso, 2000, "Early retirement", CEPR discussion paper n±2589.

[3] Crawford, V. and D.M. Lilien, 1981, "Social security and the retirement decision", Quarterly Journal of Economics, 95, 505-529.

[4] Cremer, H., J.-M. Lozachmeur and P. Pestieau, 2001, "Social security and variable retirement schemes: an optimal taxation approach", mimeo.

[5] Gans, J.S. and M. Smart, 1996, "Majority voting with single-crossing preferences", Journal of Public Economics, 59, 219-237.

[6] Gruber, J. and D. Wise, 1997, "Social security programs and retirement around the world", NBER working paper 6134.

[7] Lacomba, J.A. and F.M. Lagos, 1999, "Social security and political election on retirement age", mimeo.

[8] Shepsle, K.A., 1979, Institutional arrangements and equilibrium in multidimensional voting models, American Journal of Political Science, 23, 27–59.

[9] Sheshinski, E., 1978, "A model of social security and retirement decisions", Journal of Public Economics, 10, 337-360.

Appendix

A Proof of proposition 1

(i) The equation of an indifference curve is derived by solving:

$$u\left[w(1-\tau)-s^y\right]+\beta u\left[s^y(1+r)+P+w^2(1-\theta\tau)^2/2\gamma\right]=c,$$

where c is a constant. Differentiating this expression, the slope of an indifference curve is

$$\frac{dP}{d\tau}=\frac{wu'(c)+\beta\frac{w^2}{\gamma}\theta(1-\theta\tau)u'(d)}{\beta u'(d)}>0. \tag{24}$$

If savings are positive, $u'(c)=u'(d)$. This leads to

$$\frac{dP}{d\tau}=\frac{w+\beta\frac{w^2}{\gamma}\theta(1-\theta\tau)}{\beta}. \tag{25}$$

Indifference curves are increasing and concave. Moreover, $d^2P/d\tau dw>0$: the slope of indifference curves is increasing with productivity.

If the individual does not want to save, the differentiation of (24) with respect to w leads to

$$\begin{aligned}\frac{d^2P}{d\tau dw} &= \frac{(u'(c)+w(1-\tau)u''(c))\beta u'(d)-\beta wu'(c)\frac{w}{\gamma}(1-\theta\tau)^2u''(d)}{(\beta u'(d))^2}\\ &\quad+\frac{2w}{\gamma}\theta(1-\theta\tau)\\ &= \frac{(u'(c)(1-R_r(c)))\beta u'(d)-\beta wu'(c)\frac{w}{\gamma}(1-\theta\tau)^2u''(d)}{(\beta u'(d))^2}\\ &\quad+\frac{2w}{\gamma}\theta(1-\theta\tau).\end{aligned}$$

If $R_r(.)\le 1$, $d^2P/d\tau dw$ is positive. It follows that the slope of indifference curves is increasing with productivity. This leads to our conclusion that preferred tax rates are decreasing with productivity.

(ii) Differentiating (14) with respect to τ and using (7), we have

$$\begin{aligned}\frac{dV^y}{d\tau} &= -wu'(c)+\beta\left(P'(\tau)-\frac{w^2\theta(1-\theta\tau)}{\gamma}\right)u'(d)\\ &= -wu'(c)+\beta\left((1+n)\overline{w}+\frac{\theta-2\theta^2\tau}{\gamma}E\left(w^2\right)-\frac{w^2\theta(1-\theta\tau)}{\gamma}\right)u'(d).\end{aligned}$$

At $\tau = 1$,

$$\left.\frac{dV^y}{d\tau}\right|_{\tau=1} = -wu'(0) + \beta\left((1+n)\overline{w} + \frac{\theta - 2\theta^2}{\gamma}E\left[w^2\right] - \frac{w^2\theta(1-\theta)}{\gamma}\right) \times u'\left((1+n)\overline{w} + \frac{E\left(w^2\right)\theta(1-\theta)}{\gamma}\right).$$

It is clear that, if $\lim_{x \to 0} u'(x) = +\infty$, $dV^y/d\tau|_{\tau=1} < 0$.

At $\tau = 0$,

$$\left.\frac{dV^y}{d\tau}\right|_{\tau=0} = -wu'(c) + \beta\left((1+n)\overline{w} + \frac{\theta}{\gamma}E\left[w^2\right] - \frac{w^2\theta}{\gamma}\right)u'(d).$$

We argue now that $s^y > 0$ when $\tau = 0$. From (4),

$$s^y|_{\tau=0} = \frac{w - \frac{w^2}{2\gamma} - P(0)}{(2+r)} = \frac{w - \frac{w^2}{2\gamma}}{(2+r)} > 0 \;\Leftrightarrow\; w < 2\gamma.$$

Because $\gamma \geq w_+$, the condition $w < 2\gamma$ is satisfied for any w. It follows that $u'(c) = u'(d)$ and

$$\begin{aligned}
\left.\frac{dV^y}{d\tau}\right|_{\tau=0} &= u'(c)\left(-w + \beta\left((1+n)\overline{w} + \frac{\theta}{\gamma}E\left[w^2\right] - \frac{w^2\theta}{\gamma}\right)\right) \\
&> 0 \\
&\Leftrightarrow\; -w + \beta\left((1+n)\overline{w} + \frac{\theta}{\gamma}E\left[w^2\right] - \frac{w^2\theta}{\gamma}\right) > 0 \\
&\Leftrightarrow\; -w + \frac{1+n}{1+r}\overline{w} + \frac{1}{1+r}\left(\frac{\theta}{\gamma}E\left[w^2\right] - \frac{w^2\theta}{\gamma}\right) > 0.
\end{aligned}$$

If $w < \overline{w}(1+n)/(1+r)$ then, by assumption, $w < \sqrt{E(w^2)}$ and the above expression is positive.

(iii) Indifference curves for old individuals are derived by solving

$$u\left(s(1+r) + P + \frac{w^2(1-\theta\tau)^2}{2\gamma}\right) = c.$$

Differentiating, we have

$$\frac{dP}{d\tau} = \frac{w^2}{\gamma}\theta(1-\theta\tau) > 0$$

and

$$\frac{d^2P}{d\tau^2} = -\frac{w^2\theta^2}{\gamma} < 0.$$

Comparing with (8), we find that the slope of indifference curves decreases more quickly than the slope of the budget curve if and only if $w^2 > 2E\left(w^2\right)$. At $\tau = 0$, the productivity level such that the slope of the indifference curve equals the slope of the budget curve is given by:

$$\frac{w^2}{\gamma}\theta = (1+n)\,\overline{w} + \frac{\theta}{\gamma}E\left(w^2\right)$$
$$\Leftrightarrow \quad w_s^2 = \frac{\gamma}{\theta}(1+n)\,\overline{w} + E\left(w^2\right).$$

Noting that

$$E\left(w^2\right) = \int_{w_-}^{w_+} w^2 f(w)\,dw < w_+ \int_{w_-}^{w_+} w f(w)\,dw = w_+\overline{w}$$

and recalling that $\gamma \geq w_+$,

$$w_s^2 > 2E\left(w^2\right).$$

Following the discussion above, this indifference curve is always below the budget curve. Therefore, individuals with productivity w_s have a preferred tax rate equal to 1. Observing that the slope of indifference curves is increasing with productivity, all individuals with a productivity less than w_s want also a tax rate equal to 1. Individuals with a higher productivity do not choose an interior solution for τ. Indeed, their indifference curves being "more" concave than the budget curve, a point of tangency between an indifference curve and the budget curve corresponds to a minimizing tax rate. The individuals indifferent between $\tau = 0$ and $\tau = 1$ are such that

$$V^o(0; w^o) = V^o(1; w^o)$$
$$\Leftrightarrow \quad u\left(s(1+r) + P(0) + \frac{w^{o2}}{2\gamma}\right) = u\left(s(1+r) + P(1) + \frac{w^{o2}(1-\theta)^2}{2\gamma}\right)$$
$$\Leftrightarrow \quad \frac{w^{o2}}{2\gamma} = (1+n)\,\overline{w} + \frac{\theta(1-\theta)}{\gamma}E\left(w^2\right) + \frac{w^{o2}(1-\theta)^2}{2\gamma}$$
$$\Leftrightarrow \quad w^{o2} = \frac{2\gamma(1+n)\,\overline{w}}{\theta(2-\theta)} + 2\frac{1-\theta}{2-\theta}E\left(w^2\right), \text{ if } \theta \neq 0. \tag{26}$$

Note that this equation does not always have a solution. In particular, when $\theta = 0$, every old individual most prefer a tax rate equal to 1. Observe also that the indifferent old individual has a productivity level higher than $\overline{w}$. Finally, $dw^{o2}/d\theta < 0$.

B Proof of proposition 2

The majority voting tax rate, when positive, is determined by the following first-order condition:

$$-wu'(c) + \beta\left((1+n)\overline{w} + \frac{\theta - 2\theta^2\tau}{\gamma}E\left(w^2\right) - \frac{w^2\theta(1-\theta\tau)}{\gamma}\right)u'(d) = 0,$$

where

$$\begin{aligned} c &= w(1-\tau) - s^y \\ d &= s^y(1+r) + (1+n)\tau\overline{w} + \frac{\theta\tau(1-\theta\tau)}{\gamma}E\left(w^2\right) + \frac{w^2(1-\theta\tau)^2}{2\gamma} \end{aligned}$$

and $w = w_{piv}$.

Differentiating this expression with respect to θ, we obtain

$$\begin{aligned} \frac{d\tau^y(w_{piv})}{d\theta} = & \frac{w\frac{ds^y}{d\theta}u''(c) + \beta\left(\frac{1-4\theta\tau}{\gamma}E\left[w^2\right] - \frac{w^2}{\gamma}(1-2\theta\tau)\right)u'(d)}{-D_\tau} \\ & + \frac{\beta\left((1+n)\overline{w} + \frac{\theta-2\theta^2\tau}{\gamma}E\left[w^2\right] - \frac{w^2\theta(1-\theta\tau)}{\gamma}\right)}{-D_\tau} \\ & \times\left(\frac{ds^y}{d\theta}(1+r) + \frac{\tau-2\theta\tau^2}{\gamma}E\left(w^2\right) - \frac{w^2\tau(1-\theta\tau)}{\gamma}\right)u''(d) \end{aligned}$$

where

$$\begin{aligned} D_\tau = & w^2u''(c) + w\frac{ds^y}{d\theta}u''(c) + \left(\frac{w^2\theta^2 - 2E\left[w^2\right]\theta^2}{\gamma}\right)u'(d) \\ & + \beta\left((1+n)\overline{w} + \frac{\theta-2\theta^2\tau}{\gamma}E\left(w^2\right) - \frac{w^2\theta(1-\theta\tau)}{\gamma}\right)^2u''(d) \\ & + \beta\frac{ds^y}{d\theta}(1+r)\left((1+n)\overline{w} + \frac{\theta-2\theta^2\tau}{\gamma}E\left(w^2\right) - \frac{w^2\theta(1-\theta\tau)}{\gamma}\right)u''(d). \end{aligned}$$

When $\theta = 0$, we know that the pivotal voter does not want to save. Moreover, the "identity" of the pivotal voter does not change when θ stays close to 0. All this leads to

$$D_\tau|_{\theta=0} = w^2u''(c) + \beta((1+n)\overline{w})^2u''(d) < 0$$

and

$$\begin{aligned} \left.\frac{d\tau^{mv}}{d\theta}\right|_{\theta=0} &= \frac{\beta\left(\frac{E\left[w^2\right]-w^2}{\gamma}\right)u'(d) + \beta\tau(1+n)\overline{w}\left(\frac{E\left[w^2\right]-w^2}{\gamma}\right)u''(d)}{-D_\tau} \\ &= \frac{\beta\left(\frac{E\left[w^2\right]-w^2}{\gamma}\right)\left(u'(d)(1-R_r(d)) - \frac{w^2}{2\gamma}u''(d)\right)}{-D_\tau}. \end{aligned}$$

Recalling that $w^2_{piv} < w^2_m \leq \overline{w}^2$, we have that $w^2_{piv} < E\left(w^2\right)$ by Jensen's inequality ($\overline{w}^2 < E\left(w^2\right)$). We can conclude that this expression is positive if $R_r(\cdot) \leq 1$.

C Proof of proposition 3

We differentiate (14) with respect to θ:

$$\frac{dV^y}{d\theta} = \beta\left(\frac{\tau - 2\theta\tau^2}{\gamma}E\left(w^2\right) - \frac{\tau(1-\theta\tau)}{\gamma}w^2\right)u'(d)$$
$$+ \frac{d\tau^{mv}}{d\theta}\left(-wu'(c) + \beta\left((1+n)\overline{w} + \frac{\theta - 2\theta^2\tau}{\gamma}E\left(w^2\right) - \frac{\theta(1-\theta\tau)}{\gamma}w^2\right)u'(d)\right).$$

This becomes at $\theta = 0$

$$\left.\frac{dV^y}{d\theta}\right|_{\theta=0} = \beta\left(\frac{\tau}{\gamma}E\left(w^2\right) - \frac{\tau}{\gamma}w^2\right)u'(d)$$
$$+ \left.\frac{d\tau^{mv}}{d\theta}\right|_{\theta=0}\left(-wu'(c) + \beta(1+n)\overline{w}u'(d)\right).$$

It is clear that all the individuals to the left of the pivotal voter are such that $w^2 < E\left(w^2\right)$. The first term is then positive. Besides, these individuals want a tax rate higher than the median voter. The majority voting tax rate being increasing when the coefficient of relative risk aversion is lower than 1, the second term is also positive. It follows that every individual to the left of the pivotal voter benefits from an increase in θ.

We now turn to old individuals. From (15):

$$\frac{dV^o}{d\theta} = \beta\left(\frac{\tau - 2\theta\tau^2}{\gamma}E\left(w^2\right) - \frac{\tau(1-\theta\tau)}{\gamma}w^2\right)u'(d)$$
$$+\frac{d\tau^{mv}}{d\theta}\beta\left((1+n)\overline{w} + \frac{\theta - 2\theta^2\tau}{\gamma}E\left(w^2\right) - \frac{\theta(1-\theta\tau)}{\gamma}w^2\right)u'(d).$$

Therefore

$$\left.\frac{dV^o}{d\theta}\right|_{\theta=0} = \beta\left(\frac{\tau}{\gamma}E\left(w^2\right) - \frac{\tau}{\gamma}w^2\right)u'(d)$$
$$+ \left.\frac{d\tau^{mv}}{d\theta}\right|_{\theta=0}\beta(1+n)\overline{w}u'(d).$$

This is positive for individuals with productivity $w^2 < E\left(w^2\right)$ as soon as relative risk aversion is lower than 1.

D Proof that the curves $\tau^{mv}(\theta)$ and $\theta^{mv}(\tau)$ intersect at least once

We already know that when $\theta = 0$, $\theta^{mv}(\tau)$ is above $\tau^{mv}(\theta)$. We now proceed to show that the converse is also occurs for some value of θ. This, along with the continuity of the curves proofs the claim.

Let define $(\theta^{sv}; \tau^{sv})$ as the solution of

$$\max_{\theta;\tau} u\left[w_{piv}(1-\tau) - s^y\right] + \beta u\left[s^y(1+r) + P(\tau;\theta) + w_{piv}^2(1-\theta\tau)^2/2\gamma\right]$$

st

$$\theta\tau = \frac{E\left(w^2\right) - w_m^2}{2E\left(w^2\right) - w_m^2}.$$

Substituting the constraint in the objective function, we obtain the following FOC on τ:

$$w_{piv} u'(c) = \beta(1+n) u'(d).$$

Using this condition, we obtain

$$\begin{aligned} \frac{\partial V^y}{\partial \tau}(\tau^{sv}; \theta^{sv}; w_{piv}) &= \frac{\beta\theta}{\gamma}\left[(1-2\theta^{sv}\tau^{sv})E\left(w^2\right) - (1-\theta^{sv}\tau^{sv})w_{piv}^2\right]u'(d) \\ &> 0 \iff w_{piv}^2 < \frac{1-2\theta^{sv}\tau^{sv}}{1-\theta^{sv}\tau^{sv}}E\left(w^2\right). \end{aligned}$$

Observing that $\theta^{sv}\tau^{sv} = w_m^2/E\left(w^2\right)$, this condition becomes $w_{piv} < w_m$, which is always true. We can conclude that at the point $(\theta^{sv}; \tau^{sv})$ which is on the curve $\theta^{mv}(\tau)$, w_{piv} wants to increase the tax rate. Therefore, $\tau^{mv}(\theta)$ is above $\theta^{mv}(\tau)$.