Environmental taxation in open economies: unilateralism or partial harmonization

Helmuth Cremer

University of Toulouse
(IDEI and GREMAQ)
Toulouse, France

Firouz Gahvari*

Department of Economics
University of Illinois at Urbana-Champaign
Urbana, IL 61801, USA

February 2003

*We thank Hormozd Gahvari for help in computations.

Abstract

This paper studies whether, in the presence of a global negative externality, economic integration will necessarily lower environmental quality and the provision of public goods. It shows that it is possible for the tax competition to help environment in that it may induce firms to adopt less polluting technologies. This occurs because emission taxes may in fact increase as the economy opens up, despite the fact that the rule for setting emission taxes remain unaffected. Moreover, tax competition may also increase public goods provision although it leads to the introduction of a negative term in the rule that determines commodity taxes. The paper also examines the efficacy of partial tax harmonization policies. It shows that harmonization of output taxes (above their unrestricted Nash equilibrium values) leads to the adoption of cleaner technologies and to improvements in the overall quality of the environment and welfare under some circumstances, and to dirtier technologies and reductions in the quality of the environment and welfare under other. On the other hand, harmonizing of emission taxes above their Nash equilibrium values appear to always lead to improvements in the environment and welfare via adoption of cleaner technologies.

JEL classification: H21; H23; H73; H87; F15

Keywords: Environmental taxation; global externality; tax competition; partial harmonization; polluting technology; environmental quality.

1 Introduction

There are two international dimensions to environmental policies each raising a specific type of question. First, when pollution is worldwide (e.g., emissions of greenhouse gases like CO_2) we are effectively dealing with a global (and essentially pure) public good, namely environmental quality.[1] Different countries contribute to this public good or, more precisely, they contribute towards its degradation through their emissions. As long as there is no supranational government, one has a framework which resembles that of the voluntary provision of public goods. The problem here is that individual countries do not have the right incentives to take the welfare of the other countries into consideration. Their cost/benefit calculus does not account for the full cost of the emissions imposed on the rest of the world. Consequently, one can expect (non-cooperative) national policies to lead to an "excessive" level of emissions..[2]

The remedies that countries adopt to combat pollution introduce a second international dimension of their own. For example, when France (unilaterally) taxes domestic producers in order to entice them to cut their emissions, the price of domestic products will increase and consumers may turn to imported substitutes. This effect is of course neither intended nor (in general) positive for the French economy. In addition, it mitigates the environmental benefits of the policy since the (non taxed) foreign producers can be expected to use dirtier technologies. This is indicative of the fact that unilateral environmental policies, regardless of the global or local character of pollution, are not immune to the phenomenon of "fiscal competition". When tax bases are mobile, the capacity of an individual country to levy taxes is reduced. This problem arises for most forms of taxation including for environmental levies. The fiscal competition aspect may

[1] As measured for instance by the negative of the stock of CO_2 in the atmosphere.

[2] There is a vast literature on the inefficiency of non-cooperative provision of public goods; see, e.g., Cornes and Sandler (1996). In the context of transboundary emission, see Silva and Caplan (1997), and Caplan and Silva (1999). They examine the roles of federal and regional governments in combating pollution abstracting from tax competition.

then tempt an individual country to cut its environmental taxes in order to enhance the competitiveness of its economy (as long as there is some degree of mobility of goods and/or factors).[3]

The two dimensions of international environmental policies have thus far been studied independently of one another. Each dimension alone suggests a reason for the environmental quality to be inefficiently low. One may then be tempted to argue that when the two elements are put together, they can only reinforce one another. However, the inefficiencies due to global nature of externalities and tax competition are not simply "additive". We present a model where the two problems are accounted for simultaneously, thus providing a single framework to study the complexities that are brought about by their interaction. In our model, emissions vary with the level of output *and* the polluting technology employed. This feature captures the different roles that output and emission taxes may play in financing government expenditures and combating emissions—the two instruments do not collapse to one. This provides an appropriate framework to study a number of questions regarding the design of environmental policies. Does economic integrations (and the potential for tax competition it induces) necessarily lead to a decline in environmental quality? And if so, how should one go about remedying this problem? Will there be a targeting of tax instruments with output taxes used for financing public goods and tax competition, and emission taxes solely for combating emissions? Of particular importance, in this respect, is the implications of a policy of "partial harmonization"; that is, harmonization in only one instrument. We shall examine, in particular, if harmonization of output taxes (intended to avoid tax competition) are "neutralized" by an adjustment of other taxes, like those that are directly imposed on emissions.[4]

[3] The tax competition have recently been surveyed by Cremer *et al.* (1996), Wilson (1999), Wellisch (2000) and Haufler (2001). For specific applications to environmental issues, see Oates (2001).

[4] Cremer and Gahvari (2000) have examined the partial harmonization question in the context of tax evasion. See Keen (1989) and Kanbur and Keen (1993) for a general discussion of harmonization.

Another dimension of the interaction between the two sources of inefficiency, i.e. tax competition and the presence of a global externality, manifests itself in the properties of the equilibrium level of public goods supply. How robust is the traditional picture of the countries ending up with an inefficiently low level of provision? To get an intuitive feel for this, observe that the existence of a global externality implies that the initial (i.e. prior to economic integration) equilibrium level of local public goods is itself inefficient even in comparison to the second-best outcome.[5] Put differently, cooperation between the countries will improve the outcome despite their economies being closed. An additional complication is that it is no longer clear whether one has "too little" or "too much" public goods. When the economy opens up, the interplay between emission and output taxes may increase (rather than decrease) the provision of public goods. Either way, depending on an initially over- or under-provided level of public goods, the induced change may entail an efficiency gain.

Certain aspects of the questions we are raising, have been studied in the literature. There is a huge literature on environmental dumping which compares cooperative and non-cooperative outcomes under trade; see Ulph (1997) for a survey. However, as a rule, this literature does not distinguish between emission and output taxes, ignores the question of the public good provision, and is cast in terms of competition between imperfectly competitive firms. In yet another approach, Antweiler *et al.* (2001) do distinguish between scale of output and the intensity of polluting technologies in determining emissions, but their concern is not tax competition and public good provision.[6]

We consider a simple model of commodity tax competition. There are two identical

[5]The second-best outcome is one that is constrained only by the availability of tax instruments.

[6]There are numerous other trade models. Copeland and Taylor (1995), for example, motivate trade through income differences and show that trade worsens the environment by making rich countries specialize in production of clean goods and poor countries (with less stringent regulations) in dirty goods. They generalize their setup in Antweiler *et al.* (2001) by including factor abundance in determining trade. There are also papers that study the impact of trade on environmental *resources*; see, e.g., Chichilinisky (1994) and Karp *et al.*(2001) who build models of North-South trade and motivate trade through differences in property rights.

countries whose inhabitants consume three goods: one publicly-provided (locally) and two privately-provided goods. The publicly-provided good is nonpolluting. One of the privately-provided good is the numeraire good which is also nonpolluting. The other privately-provided good is polluting. Every consumer has an endowment of the numeraire good, some of which he consumes, spending the rest to purchase the polluting good and pay taxes. Production technologies are identical in both countries. The publicly-provided good is produced at a constant average and marginal cost.

Pollution (CO_2, SO_2, etc.) is global and a by-product of production. The polluting good may be produced in different ways. Each procedure entails a different resource cost and a different emission level. Emissions are beneficial in that a higher level of emission reduces the private (per unit) production costs of polluting goods. That is, the production costs of polluting goods are negatively correlated to their emissions. This is to capture the fact that technologies which cut emissions are more expensive to employ. Firms producing the polluting good operate in a competitive environment. The good is produced by an industry that is comprised of a fixed but sufficiently large number of identical firms. It is produced, for a given unit cost of production, by a linear technology subject to constant returns to scale.

Each country provides the publicly-provided good to its own residents only. The polluting good is produced and consumed in both countries. Prior to economic integration, there is no trade between the two countries. Upon integration, residents of each country will be able to purchase the polluting good from the foreign as well as the home country. While the physical characteristics of the home- and foreign-produced goods are identical, consumers have a preference for purchasing the home-produced goods. We model this by assuming that consumers experience a certain disutility when they consume one unit of the foreign-produced good. The extent of the disutility differs across consumers. Individuals have otherwise identical quasi-linear preferences.

There are two (distortionary) tax instruments: commodity *and* emission taxes.

These are “origin-based”. Thus, each country levies a certain tax on each unit of the (polluting) consumption good that its firms produce and sell (regardless of where the purchasers come from). Second, to combat pollution, the country imposes another tax per unit of emissions on (home) firms.

Within this framework, we characterize second-best commodity and emission tax rates. Next, we characterize the equilibrium values of commodity and emission taxes in closed and open economies. We show that the formula for emission tax remains the same in closed and open economies. On the other hand, the equilibrium value of the commodity tax changes and includes a negative term due to tax competition. The targeting principle applies; emission taxes are used only for the purpose of combating emissions and commodity taxes for tax competition. We argue that the reason for this is that emission taxes do not add extra “power” to commodity taxes as far as tax competition is concerned. There is thus no reason to distort their Pigouvian role. We show that the resulting output and emission tax rates are “too little” in that marginally increasing them enhances welfare. We nevertheless show that economic integration may in fact encourage the firms to adopt less polluting technologies and in this regard help the environment. Turning to the expenditures on public goods, we show that it is possible for them to go up and that the induced change may bring their level closer to their second-best value. Finally, we show that partially harmonizing commodity taxes (above their unrestricted Nash equilibrium value) can potentially hurt as well as improve the overall quality of the environment and welfare. Specifically, if the policy leads to an increase in emission taxes, it will necessarily imply a switch to less polluting technologies, an improved environmental quality and enhanced welfare. On the other hand, harmonizing of emission taxes above their Nash equilibrium values appear to always lead to improvements in the environment and welfare via adoption of cleaner technologies.

2 The model

Consider two identical countries, A and B, whose inhabitants consume three goods: one publicly-provided (locally) and two privately-provided goods. The publicly-provided good, G, is nonpolluting. One of the privately-provided good is the numeraire good which is also nonpolluting. The other privately-provided good, x, is polluting. Every consumer has an endowment of m units of the numeraire good, some of which he consumes, spending the rest to purchase the polluting good and pay his taxes. Production technologies are identical in both countries. The publicly-provided good is produced at a constant average and marginal cost which we can normalize at one.

Pollution is global and a by-product of production. The polluting good may be produced in different ways. Each procedure entails a different resource cost and a different emission level.[7] Specifically, assume that the resource cost of producing one unit of output $C(e_i)$, where e_i $(i = A, B)$ denotes emission per unit of output in country i, is a continuously differentiable, decreasing and convex function of e_i.[8] Firms producing the polluting good operate in a competitive environment. The good is produced by an industry that is comprised of a fixed but sufficiently large number of identical firms. It is produced, for a given $C(e_i)$, also by a linear technology subject to constant returns to scale.

Each country provides the publicly-provided good to its own residents only. The polluting good is produced and consumed in both countries. Prior to economic integration, there is no trade between the two countries. Upon integration, citizens of each country will be able to purchase the polluting good from the foreign as well as the home country. While the physical characteristics of the home- and foreign-produced goods are

[7] This models situations where a polluting good may be produced through different production techniques, or using different polluting inputs where each particular input entails a different emission level. Different abatement techniques also imply that a unit of polluting good is associated with different emission levels.

[8] More precisely the assumption is that $C'(.) < 0$ for all e_i up to some limit $\bar{e}$, and that $C'(\bar{e}) = 0$, $i = A, B$.

identical, consumers have a preference for purchasing the home-produced goods. Let θ denote the inhabitants of A and B, with $|\theta|$ determining θ's disutility when consuming one unit of the foreign-produced good. Assume that θ is uniformly distributed over [-1, 1], with a negative θ indicating a resident of B and a positive θ a resident of A. Normalize the population size in each country at one.

Consumers have quasi-linear preferences. Denote the utility level of a person in $j = A, B$ who purchases the polluting good produced in $i = A, B$ by u_i^j and his consumption of the polluting good by x_i^j. All consumers who buy from i, regardless of their country of origin, face the same consumer price for x.[9] Denote this price by p_i, the level of publicly-provided good in country i by G_i, and the global emission level by E. We have:

$$\begin{cases} u_j^j = & m - p_j x_j^j + h(x_j^j) + \phi(G_j) - \varphi(E), \\ u_i^j = & m - p_i x_i^j + h(x_i^j) - \delta|\theta| x_i^j + \phi(G_j) - \varphi(E), \qquad \text{with } j \neq i, \end{cases} \tag{1}$$

where $\delta > 0$ is a "dislike index". Note that as $\delta \to \infty$, one never purchases the foreign-produced good regardless of the price. We will also assume that $h(.)$ and $\phi(.)$ are continuously differentiable, increasing and (strictly) concave functions of their argument while $\varphi(.)$ is continuously differentiable, increasing and convex; that is, $h'(.) > 0, h''(.) < 0, \phi'(.) > 0, \phi''(.) < 0$ and $\varphi'(.) > 0, \varphi''(.) \geq 0$.

When a resident of country j buys the home-produced good, his net cost of purchasing one unit of the good is simply its consumer price, p_j. On the other hand, when he buys the foreign-produced good, he incurs, per unit, a net (utility) cost of $p_i + \delta|\theta|$. In either case, the number of units the consumer buys corresponds to that which maximizes his utility. Thus assuming that m is sufficiently large so that the consumer chooses an interior solution

$$\begin{cases} h'(x_j^j) = & p_j, \\ h'(x_i^j) = & p_i + \delta|\theta|, \qquad \text{with } j \neq i. \end{cases} \tag{2}$$

[9] All taxes are origin-based.

Inverting these functions, one can write the demand for the polluting good as: $x_j^j = x(p_j)$ and $x_i^j = x(p_i + \delta|\theta|)$. Substituting in (1) then yields $u_j^j = u(p_j, G_j, E)$ and $u_i^j = u(p_i + \delta|\theta|, G_j, E)$.

Denote the "marginal" consumer, i.e. the person who is just indifferent between buying home- or foreign-produced goods, by $\tilde{\theta}$. If the marginal consumer is a resident of A ($\tilde{\theta} > 0$), $u_A^A = u_B^A \Rightarrow u(p_A, G_A, E) = u(p_B + \delta\tilde{\theta}, G_A, E)$. Similarly, if $\tilde{\theta}$ is a resident of B ($\tilde{\theta} < 0$), $u_B^B = u_A^B \Rightarrow u(p_B, G_B, E) = u(p_A - \delta\tilde{\theta}, G_B, E)$. It follows that, regardless of which country sells the good at a higher price,

$$\tilde{\theta} = \frac{p_A - p_B}{\delta}. \tag{3}$$

It then also follows that all individuals to the left of $\tilde{\theta}$ buy the good from country B, and all the individuals to the right of $\tilde{\theta}$ buy the good from country A.

With $C(e_i)$ being the cost of producing one unit of the good in country i and p_i its consumer price, its production and sale will generate $p_i - C(e_i)$ in revenues for the government of i. Assuming the governments of A and B undertake no other expenditures or transfers except for G, which is produced at a fixed unit cost normalized at one, one can easily show that

$$G_A(p_A, e_A; p_B) = \begin{cases} [p_A - C(e_A)]\,(1 - \tilde{\theta})x(p_A) & \text{if } p_A \geq p_B, \\ [p_A - C(e_A)]\;\; x(p_A) + \int_{\tilde{\theta}}^{0} x(p_A - \delta\theta)d\theta\Big] & \text{if } p_A < p_B. \end{cases} \tag{4a}$$

$$G_B(p_B, e_B; p_A) = \begin{cases} [p_B - C(e_B)]\;\; x(p_B) + \int_{0}^{\tilde{\theta}} x(p_B + \delta\theta)d\theta\Big] & \text{if } p_A \geq p_B, \\ [p_B - C(e_B)]\,(1 + \tilde{\theta})x(p_B) & \text{if } p_A < p_B. \end{cases} \tag{4b}$$

In the same way, that total pollution, E, is related to each country's pollution according to

$$E(p_A, p_B, e_A, e_B) = \begin{cases} e_B\;\; x(p_B) + \int_{0}^{\tilde{\theta}} x(p_B + \delta\theta)d\theta\Big] + e_A(1 - \tilde{\theta})x(p_A) & \text{if } p_A \geq p_B, \\ e_B(1 + \tilde{\theta})x(p_B) + e_A\;\; \int_{\tilde{\theta}}^{0} x(p_A - \delta\theta)d\theta + x(p_A)\Big] & \text{if } p_A < p_B. \end{cases} \tag{5}$$

Note that at $p_A = p_B$, $\tilde{\theta} = 0$ so that G_A, G_B and E are continuous at this point. The following Lemma, proved in the Appendix, shows that these functions are in fact differentiable at $p_A = p_B$.

Lemma 1 *The functions $G_A(p_A, e_A; p_B)$, given by (4a), $G_B(p_B, e_B; p_A)$, given by (4b), and $E(p_A, p_B, e_A, e_B)$, given by (5), are continuously differentiable at $p_A = p_B$.*

2.1 Tax instruments

The properties of the equilibrium depend on what tax instruments are feasible. Assume that each country has two (distortionary) tax instruments: commodity *and* emission taxes. These are "origin-based". Thus, country i ($i = A, B$) levies a tax of τ_i on each unit of the (polluting) consumption good that its firms produce and sell (regardless of where the purchasers come from). Second, to combat pollution, the country imposes a tax of t_i per unit of emissions on (home) firms.[10]

A representative firm in country i, regardless of where its purchasers come from, will have to sell its output at the domestic price of p_i and pay domestic taxes τ_i and t_i. Moreover, given the constant returns to scale assumption, the firm's profit maximization problem is simply one of maximizing profits *per unit* of output. That is, the firm chooses e_i to maximize

$$p_i - C(e_i) - t_i e_i - \tau_i.$$

This yields, for $i = A, B$,

$$-C'(e_i) = t_i, \tag{6}$$

where the second-order condition $C''(e_i) > 0$ is satisfied from the convexity of $C(.)$.

[10] We rule out lump-sum taxes. This is in line with the literature on tax competition. However, unlike that literature, allowing for lump-sum taxation here does not make tax competition disappear. This special case is studied in Cremer and Gahvari (2003). See also Cremer and Gahvari (2001) for a thorough discussion of the properties of output taxes versus emission taxes in the context of a closed economy.

Moreover, the zero-profit condition implies that

$$p_i = C(e_i) - C'(e_i)e_i + \tau_i. \tag{7}$$

2.2 Welfare

It is natural, given our setup, to measure the welfare of each country on a utilitarian basis. Denote this utilitarian measure for country i by W_i, $(i = A, B)$. We prove in the Appendix that:

Lemma 2 *Define* $U_i(.)$ *and* $F(.)$ *by*

$$\begin{aligned} U_i(p_i, G_i, E) &\equiv \int_0^1 [m - p_i x(p_i) + h(x(p_i)) + \phi(G_i) - \varphi(E)]\, d\theta, \\ &= m - p_i x(p_i) + h(x(p_i)) + \phi(G_i) - \varphi(E), \quad i = A, B. \end{aligned} \tag{8}$$

$$\frac{1}{\delta}F(p + \delta\theta) \equiv \int [h(x(p + \delta\theta)) - (p + \delta\theta)x(p + \delta\theta)]\, d\theta. \tag{9}$$

We have: (i)

$$W_A = \begin{cases} U_A - \tilde{\theta}\,[h(x(p_A)) - p_A x(p_A)] + \frac{1}{\delta}\,[F(p_A) - F(p_B)] & \text{if } p_A \geq p_B, \\ U_A & \text{if } p_A < p_B. \end{cases} \tag{10a}$$

$$W_B = \begin{cases} U_B & \text{if } p_A \geq p_B, \\ U_B + \tilde{\theta}\,[h(x(p_B)) - p_B x(p_B)] - \frac{1}{\delta}\,[F(p_A) - F(p_B)] & \text{if } p_A < p_B. \end{cases} \tag{10b}$$

where G_i *and* E *are given by equations (4a)–(4b) and (5).*

(ii) $W_A(p_A, G_A, E; p_B)$ *and* $W_B(p_B, G_B, E; p_A)$ *are continuously differentiable at* $p_A = p_B$.

Note that the middle expression in the right-hand side of (10a) [when $p_A \geq p_B$] measures the consumer surplus that country A does not get when some of its residents do not buy the home-produced good (at p_A). The last expression in the right-hand side, on the other hand, indicates the surplus attained by buying from B. It is plain that the last expression dominates the second so that the net change in consumer surplus is positive.[11] The same interpretation applies to (10b) and residents of B.

[11] These are the people with a $\theta \in [0, \tilde{\theta})$ for whom the net cost of buying one unit of the good from B is $p_B + \delta\theta < p_A$.

2.3 Optimal benchmark

Denote the welfare of each country at *a symmetric allocation* by W_i^S, $i = A, B$. On the basis of Lemma 2, one can write this as

$$W_i^S(p_i, G_i, E) = m - p_i x(p_i) + h(x(p_i)) + \phi(G_i) - \varphi(E). \tag{11}$$

Similarly, from (4a)–(4b), i's budget constraint at a symmetric allocation is

$$G_i = [p_i - C(e_i)]\, x(p_i). \tag{12}$$

Assume that the two countries cooperate fully in their fiscal policies. That is, they do not engage in tax competition *and* set their emission taxes while taking the welfare of the citizen of *both* countries into account. Optimal symmetric allocations are found through maximization of W_i^S subject to (12) and

$$E = 2e_i x(p_i). \tag{13}$$

The fiscal instruments in this optimization are, as observed earlier, τ_i and t_i. We prove in the Appendix that

Proposition 1 *Assume that countries set their environmental policies cooperatively. Denote the absolute value of the elasticity of demand for the polluting good in country i, $i = A, B$, by $\varepsilon_i \equiv -x'(p_i)p_i/x(p_i)$. The optimal symmetric allocations, and the supporting prices and tax instruments, are characterized by equations (2), (6), (7), (12), (13), and*

$$\frac{\tau_i}{p_i} = \frac{\phi'(G_i) - 1}{\phi'(G_i)\,\varepsilon_i}, \tag{14a}$$

$$-C'(e_i) = \frac{2\varphi'(E)}{\phi'(G_i)}. \tag{14b}$$

Observe that equation (14a) reflects the well-known "elasticity rule" of optimal commodity taxes: The higher is ε, the smaller will be the required tax. The equation has two implications for public good provision in the second-best. In arriving at these implications, we will assume that $\tau_i > 0$; this being the most likely outcome in practice.[12]

3 Closed borders

This section examines the properties of the equilibrium if the borders are closed (assuming that the government chooses the values of its fiscal instruments optimally). When there is no trade, everyone buys the home-produced good. The government's budget constraint in each country will thus be represented by equation (12). Turning to aggregate emissions, setting $\tilde{\theta}$ in (5) equal to zero yields

$$E = e_A x(p_A) + e_B x(p_B). \tag{15}$$

The government of i chooses τ_i and t_i to maximize U_i as specified in equation (8). In doing this, it takes the tax instruments of the other country, and thus its aggregate emissions, as fixed. Note that despite the economy being closed, there still exists a "strategic" interaction between the two countries through E. We model this interaction à la Nash. Solving this problem yields the equilibrium allocation, price and the tax rates in country i. Note that the problem facing each country is not the same as the "optimal benchmark" problem studied earlier. The two problems differ with respect to the optimal level of global emissions. Here, in the absence of coordination with respect to the environmental policy, each country considers the damage to its own citizens only when it determines its emission policy. We have

Proposition 2 *The symmetric equilibrium allocations, and the supporting prices and taxes, in a closed economy are characterized (for $i = A, B$) by equations (2), (6), (7),*

[12]In principle, it is possible for τ_i to be negative. This possibility arises *if* preferences for G are sufficiently "weak" so as the revenues from emission taxes exceed the amount required to finance the desired level of G_i. Under this scenario, the extra revenues will have to be returned through a subsidy on x_i (a negative τ_i). This possibility is of course extremely unlikely to occur in practice and we shall rule it out in the paper.

(12), (15), and

$$\frac{\tau_i}{p_i} = \frac{\phi'(G_i) - 1}{\phi'(G_i)\,\varepsilon_i}, \tag{16a}$$

$$-C'(e_i) = \frac{\varphi'(E)}{\phi'(G_i)}. \tag{16b}$$

Condition (16a) is identical to condition (14a) that characterized the second-best determination of τ_i. This should not be surprising. With closed borders, there is no tax competition between the countries. Hence the optimal tax rule for setting τ_i remains unaffected. On the other hand, the rule for setting emission taxes now differs from the optimal benchmark case. Condition $-C'(e_i) = \varphi'(E)/\phi'(G_i)$ in (16b) replaces condition $-C'(e_i) = 2\varphi'(E)/\phi'(G_i)$ of the second-best. Thus, the environmental tax is set at one half the full marginal social damage of emissions. This reflects our earlier observation that each country, when determining its emissions policy, considers the damage to its own citizens only. Note that the 1/2 factor corresponds to the relative size of the country to total global population. Apart from this factor, the Pigouvian formula remains unaffected.

3.1 Rule versus level: perfectly inelastic demand

With the environmental tax being "set" at less than marginal social damage of emissions under the closed-economy solution, and equal to it at the second best, one might think that the closed-economy emission *levels* (e_i or E) will exceed their second-best values. This is not guaranteed; however. This is another facet of the "rule" versus "level" question originally addressed by Atkinson and Stern (1974) in the context of public goods supply with distortionary taxes. The ambiguity arises here because $x(p_i)$ and G_i take different values under the second-best and the closed-economy solutions. To understand the significance of theses changes, consider a special case where the demand for the polluting good is perfectly inelastic. Under this circumstance, and when the economy is closed, τ_i acts as a lump-sum tax. Consequently, G_i will be set at its first-best value as characterized by

$$\phi'(G_i) = 1. \tag{17}$$

This condition replaces (16a). Of course, with a perfectly inelastic demand, the second-best condition (14a) is also replaced by $\phi'(G_i) = 1$. Hence the closed-economy value of G_i remains the same as its second-best (equal to first-best) value. Condition $-C'(e_i) = \varphi'(2e_i)$, and the convexity of $C(e)$ and $\varphi(E)$, then imply that the closed-economy values of both e_i and E exceed their second-best values.

4 Open borders

With opening of the borders, the citizens of one country may find it advantageous to buy from the other country. Whether a particular individual would do that or not, depends on his distaste for the foreign-produced goods as explained in Section 2. This possibility has an important implication for a country's potential public revenues. When the borders are closed, the "tax base" (number of taxpayers) is the population size and is thus fixed. When borders open, the tax base becomes endogenous varying with the size of the price differentials between the two countries. The government of each country will then be able to affect it by the choice of its tax rates. This introduces an additional dimension to the strategic interaction between the countries. As in the previous Section, and following the tax competition literature, we model the interaction between the two countries using the Nash equilibrium concept.

Each country chooses its tax rates t_i and τ_i to maximize its social welfare function while treating the values of the other country's tax instruments as given. This yields the best-reply functions of each country (to the other country's choice of values for its tax instruments). One can then determine the properties of the symmetric equilibrium of the Nash game in tax instruments through solving these best-reply function. We have the following result which we prove in the Appendix.

Proposition 3 *The symmetric open-economy equilibrium allocations, and the supporting prices and taxes, in an open economy are characterized (for $i = A, B$) by equations (2), (6), (7), (12), (15), and*

$$\frac{\tau_i}{p_i} = \frac{\phi'(G_i) - 1}{\phi'(G_i)\,\varepsilon_i} - \frac{G_i}{\delta\,\varepsilon_i x(p_i)}, \tag{18a}$$

$$-C'(e_i) = \frac{\varphi'(E)}{\phi'(G_i)}. \tag{18b}$$

Comparison of (16a) and (18a) is revealing. It indicates that fiscal competition changes the closed-economy *rule* for setting the optimal commodity tax rate on x_i by $-G_i/\delta\varepsilon_i\, x(p_i)$. This reflects the negative impact of tax competition on τ_i. On the other hand, the *rule* for setting the optimal emission tax remains unaffected; see equations (16b) and (18b). The marginal social damage of emissions (to a resident of i) *is evaluated by its government* by the same Pigouvian rule whether the economy is closed or open. This suggests a targeting of tax instruments: use commodity taxes for tax competition and reserve emission taxes for the purpose of combating emissions only.

The intuition for the commodity tax characterization is best seen in terms of the familiar "fiscal externality" arguments. As with models of tax competition in the absence of emissions, an increase in the commodity tax of the home country affects the welfare of the foreign country's residents through a a tax-base effect (positive), and a private consumption externality (a negative externality on foreign country residents who buy from the home country).[13] Global emissions introduce a third source of externality. The increase in the home-country's tax increases the price of the polluting good and reduces its consumption. Consequently, aggregate emissions fall, benefiting the foreign country residents as well. The last term in (18a) reflects the combined effect of the three sources of externalities. On the other hand, the emission tax rate characterization remains unaffected (as compared to the closed economy case), simply because changing

[13] See, among others, Mintz and Tulkens (1986). Lockwood (2001) has termed these externalities "consumer price spillovers".

it cannot increase the effectiveness of tax competition (as attained through competition in commodity taxes) but distorts the production decisions. Clearly, that a country faces no restriction in using τ_i for the purpose of tax competition plays a crucial role in this.

The next interesting question relates to the equilibrium *levels* of emissions and public goods. Specifically, one wants to know how they compare with their values in the absence of trade. Of course, the welfare properties of these changes are also of paramount interest.

4.1 Pollution technologies, emissions, public goods and welfare

First, note that except for τ_i, identical equations characterize the equilibria of the closed and open economies. These are equations (6), (12), (13) and (18b). The τ_i itself, is given by equation (16a) in a closed economy and equation (18a) in an open economy. Next, observe that as the value of δ increases, the optimal choice of τ_i in the open economy converges to the optimal choice of τ_i in the closed economy so that the two equilibria converge. Consequently, if one can determine how the equilibrium values of the various variables in the system change as δ changes, one may be able to compare their values under the two equilibria.

We show in the Appendix that the induced changes in the open-economy equilibrium values of t_i, e_i, p_i, G_i, E and W_i^S, as δ increases, are of ambiguous signs. Using numerical examples, we are able to show that it is indeed possible for e_i to increase as well as decrease with δ. This suggests that international trade may encourage the firms to adopt cleaner technologies under some circumstances and more polluting technologies under others. We also show that the expenditures on public goods may change in either direction.[14] On the other hand, all our numerical examples indicate that opening up the economy leads to an increase in aggregate emissions and a reduction in overall

[14] In fact, there is a systematic relationship between e_i and G_i. They always move in opposite directions provided that the elasticity of demand and the marginal social damage of emissions are constant.

welfare (whether e increases or declines). This occurs despite the fact that the algebraic expressions for $dE/d\delta$ and $dW_i^S/d\delta$ are of ambiguous signs.

Finally, while we cannot make any clearcut statements about the relative levels of welfare at the open- and closed-economy equilibria, we have a result for the marginal changes from the open-economy equilibrium. We show in the Appendix that

$$\frac{\partial W_i^S(\tau^N, t^N)}{\partial \tau_i} = x(p_i)\phi'(G_i)\varepsilon_i \left[\frac{p_i - C(e_i)}{\delta \varepsilon_i} + \frac{e_i}{p_i}\frac{\varphi'(E)}{\phi'(G_i)}\right] > 0, \tag{19a}$$

$$\frac{\partial W_i^S(\tau^N, t^N)}{\partial t_i} = e_i x(p_i)\phi'(G_i)\varepsilon_i \left[\frac{p_i - C(e_i)}{\delta \varepsilon_i} + \frac{e_i}{p_i}\frac{\varphi'(E)}{\phi'(G_i)}\right] + \frac{x(p_i)\varphi'(E)}{C''(e_i)} > 0, \tag{19b}$$

where (τ^N, t^N) denotes the symmetric Nash equilibrium value of τ_i and t_i, and all the variables in above are evaluated at this point. This result highlights the usual "less-than-optimal" property of Nash equilibrium values. Note, however, that this does not mean that *both* τ^N and t^N are necessarily lower than their corresponding second-best values. A sufficient condition for this is for W_i^S to be strictly concave in τ_i and t_i and that $\partial^2 W_i^S(\tau^N, t^N)/\partial \tau_i \partial t_i \geq 0$.

4.2 Perfectly inelastic demand

To understand the reasons why economic integration has ambiguous effects on the environmental quality, public goods supply and welfare, consider again the special case where the demand for the polluting good is perfectly inelastic. Under this circumstance, tax revenues can change only at the "extensive margin"; that is, through a change in the number of buyers. There will be no change at the "intensive margin" as everyone always buys the same amount of the good. We will see below that in the absence of this latter effect, economic integration affects the polluting technology, quality of the environment, public goods supply and aggregate welfare unambiguously.

With a perfectly inelastic demand, one can easily show that condition

$$\phi'(G_i) = \frac{\delta}{\delta - G_i} > 1, \tag{20}$$

replaces (18a) of the open economy. It follows from (20), and the concavity of $\phi(.)$, that G_i will be "under-provided" in the open-economy (relative to its closed-economy value which is first-best).

Turning to emissions, we have, from (16b) and (18b),

$$-C'(e_i) = \varphi'(2e_i), \tag{21}$$

if the economy is closed, and

$$-C'(e_i) = \frac{\varphi'(2e_i)}{\phi'(G_i)} < \varphi'(2e_i), \tag{22}$$

if the economy is open. It then immediately follows from (21)–(22), and the convexity of $C(.)$ and $\varphi(.)$, that the open-economy equilibrium value of e_i exceeds its closed-economy value. Note also that with $E = 2e_i$, E and e_i always move together. Hence aggregate emissions are also higher at the open-economy equilibrium.

Finally, as far as overall welfare is concerned, rewrite W_i^S as

$$W^S = m - C(e) + \phi(G) - G - \varphi(2e), \tag{23}$$

where we have dropped the i subscript for ease in notation. One can easily show that at the open-economy solution, $\partial W^S/\partial e < 0$ and $\partial W^S/\partial G > 0$. This, the fact that the open-economy value of e is greater and of G smaller than their closed-economy values, and the strict concavity of W^S in (e, G), implies that overall welfare is smaller at the open-economy equilibrium as compared to the closed-economy equilibrium.

We summarize the results of this subsection in the following proposition and then illustrate them using numerical examples.

Proposition 4 *(i) Economic integration may encourage the firms to adopt less as well as more polluting technologies.*

(ii) Economic integration may induce a country to increase or to decrease its expenditures on public goods.

(iii) Let (τ^N, t^N) denote the symmetric Nash equilibrium value of τ_i and t_i. Marginal increases in τ^N and t^N are welfare enhancing.

(iv) Assume that the demand for polluting goods is perfectly inelastic. Then, economic integration leads to the adoption of more polluting technologies, a worsening of the overall environmental quality, a decline in expenditure on public goods and a reduction in overall welfare.

4.3 Numerical illustrations

Consider the following specifications for the unit production cost of x, and the various components of a resident of i's preferences:

$$C(e) = \frac{b(1-e)^2}{2}, \tag{24a}$$

$$\phi(G) = a\ln(G), \tag{24b}$$

$$\varphi(E) = \varphi E, \tag{24c}$$

$$h(x) = \frac{x^{1-1/\varepsilon}}{1-1/\varepsilon}, \tag{24d}$$

Note that (24d) implies that the demand for x is equal to

$$x = p^{-\varepsilon}, \tag{25}$$

with a constant elasticity of demand equal to ε (in absolute value).

Given these specifications, equations (14a)–(14b), that [along with (2), (6), (7), (12), (13)] characterize the second-best solution, assume the following functional forms

$$\frac{\tau}{p} = \frac{a-G}{a\varepsilon}, \tag{26a}$$

$$1 - e = \frac{2\varphi G}{a}. \tag{26b}$$

In the closed-economy case, we have equations (16a)–(16b). As with (14a), equation (16a) takes the form of (26a) continues to hold. Corresponding to equation (16b), we will have

$$1 - e = \frac{\varphi G}{a}. \tag{27}$$

In the open-economy case, we have equations (18a)–(18b). Equation (18a) will take the form of

$$\frac{\tau}{p} = \frac{a - G}{a\varepsilon} - \frac{G}{\delta \varepsilon p^{-\varepsilon}}. \tag{28}$$

Equation (18b) is, as with (16b), represented by equation (27).

Table 1 shows the second-best, closed- and open-economy solution values for τ, t, e, E, G, W^S, p and $x(p)$. They are based on two different values for ε: 1 and 2 (assuming $m = 4$, $b = 1, a = 10, \varphi = 0.25$ and $\delta = 1$). Observe that the closed-economy solutions for e and E exceed their second-best values (for both $\varepsilon = 1, 2$). The same is true for G.

What happens to these variables as the economy opens up varies in the two cases considered. When $\varepsilon = 1$, τ decreases sharply and t just a little bit. With a lower t, e increases making production somewhat more polluting. This negative impact on the quality of environment is magnified through an increase in $x(p)$ (brought about by a reduction in p, itself caused by the lowering of τ and t). Given the initially excessive level of E, this additional increase works to reduce welfare. At the same time, the reduction in the tax rates lowers the equilibrium value of G. However, this is not a bad outcome given that G was excessive to begin with (relative to its second-best value). In fact, the open-economy equilibrium value of G mirrors its second-best level.

When $\varepsilon = 2$, things are a bit different. Whereas τ decreases, as the economy opens up, t increases. The result is a reduction in e. That is, tax competition now makes the firms to switch to less polluting technologies. The reduction in e notwithstanding, E increases lowering welfare. Finally, we observe an increase in G (unlike the case where $\varepsilon = 1$). This moves the equilibrium value of G further away from its second-best level.

Table 1. Second-best, closed- and open-economy solutions

	($\varepsilon = 1$)							
	τ	t	e	E	G	W^S	p	$x(p)$
Second-best	0.4389	0.0499	0.9501	3.8977	0.9974	2.7184	0.4875	2.0512
Closed Economy	0.2222	0.0250	0.9750	7.8989	0.9987	2.4115	0.2469	4.0506
Open Economy	0.0903	0.0249	0.9751	16.9741	0.9973	0.8932	0.1149	8.7041
	($\varepsilon = 2$)							
	τ	t	e	E	G	W^S	p	$x(p)$
Second-best	0.0772	0.1877	0.8123	26.5514	3.7548	14.6352	0.2474	16.3441
Closed Economy	0.0366	0.1406	0.8594	61.3954	5.6236	11.8974	0.1673	35.7195
Open Economy	0.0207	0.1477	0.8523	68.7196	5.9095	10.9352	0.1575	40.3160

5 Commodity tax harmonization

A commonplace result of the tax competition literature is that of the restoration of the first-best allocations through the coordination of fiscal policies particularly a "harmonized" tax policy. In the context of our model, restoring second best requires harmonization of both output *and* emission taxes. In practice, however, such a sweeping coordination is rather difficult to achieve. It is more likely that countries coordinate their policies on a piecemeal basis. Will such "partial" harmonization policies help? Specifically, suppose each country fixes the value of one of its instruments but continues to compete via the other. What can we say about the outcome of this "restricted" competition? In particular, what would be the implication of this for the environment and welfare? In studying this question, and as previously, we model the strategic interaction between the countries using the Nash equilibrium concept. This section discusses harmonization of commodity taxes taking up the harmonization of emission taxes in the next section. We start by adopting the following terminology.

Definition 1 *The countries are said to "harmonize" a policy instrument if they set its value at a common specified level.*

Let the two countries harmonize their commodity tax rates at $\tau = \hat{\tau}$. Each country then chooses the value of its emission tax to maximize the welfare of its citizens. This is done à la Nash assuming that the optimizing country treats the value of the other country's emission tax as given. In this way, one derives each country's best-reply function (to the other country's choice of a value for its emission tax).[15] Finally, solving the best reply functions yield the Nash equilibrium value of the emission tax.

It is easy to show that the Nash equilibrium value of t_i, conditional on $\tau = \hat{\tau}$, is the solution to[16]

$$\left[\frac{\phi'(G_i) - 1}{\phi'(G_i)\,\varepsilon_i} - \frac{p_i - C(e_i)}{\delta\,\varepsilon_i} - \frac{\tau_i}{p_i}\right] + \left[\frac{e_i}{p_i} + \frac{1}{C''(e_i)e_i\varepsilon_i}\right]\left[C'(e_i) + \frac{\varphi'(E)}{\phi'(G_i)}\right] = 0. \tag{29}$$

Denote this solution by $t^N(\hat{\tau})$ and the corresponding solutions for e, E, G, p by $e^N(\hat{\tau})$, $E^N(\hat{\tau}), G^N(\hat{\tau})$ and $p^N(\hat{\tau})$. Further denote all unrestricted Nash equilibrium values by the superscript N ($\tau^N, t^N, e^N, E^N, G^N$ and p^N). It is clear that if one were to harmonize τ at its unrestricted Nash equilibrium value, t and all the other variables will also take their Nash equilibrium values. Equation (29) bears this out: When τ is unrestricted, the first bracketed expression in the right-hand side of (29) will be zero.

5.1 Harmonization and the emission tax rate

The first question we address is the impact of harmonization on the emission tax rate. This plays a central role in determining the implications of harmonization. Assume the countries choose (again à la Nash) the values of their fiscal instruments t_i and τ_i subject to the constraint that $\tau_i \geq \hat{\tau} > \tau^N$. It is plain that at a country's optimum this constraint must be binding (as the removal of the constraint would yield a different optimum in (τ^N, t^N)). This means that at the equilibrium values of $\hat{\tau}$ and $t^N(\hat{\tau})$, a country's welfare would increase if it could lower its value of τ *assuming no response on the part of the other country.* Hence, at $(\hat{\tau}, t^N(\hat{\tau}))$, $\partial U_i/\partial \tau_i < 0$ $(i = A, B)$. It then

[15] The best-reply function for i is given by equation (A41) in the Appendix when τ_i is set at $\hat{\tau}$.

[16] Of course, one must solve (29) in conjunction with equations (2), (6), (7), (12), and (15).

follows, from the expression for $\partial U_i/\partial\tau_i$[17] and equation (29), that at $(\hat{\tau}, e^N(\hat{\tau}))$,

$$t_i = -C'(e_i) < \frac{\varphi'(E)}{\phi'(G_i)}. \tag{30}$$

Condition (30) tells us that the equilibrium emission tax in a country is *smaller* than its marginal social cost to that country (marginal utility loss divided by the shadow cost of public funds). It replaces the condition $t_i = -C'(e_i) = \varphi'(E)/\phi'(G_i)$, i.e. the equality of a country's emission tax to its own marginal social cost, that holds under unrestricted Nash equilibrium solution [see equation (18b)]. The intuition is plain. In absence of output taxes, emission taxes are use for both combating emissions as well as tax competition. The fiscal externalities that, in the absence of harmonization, resulted in a negative term in the formula for the output tax, now have a similar negative impact on the emission tax.

Condition (30) suggests that there is a "tendency" for the emission tax rate to decrease. Nevertheless, the suggested change in *rule*, does not necessarily imply that the *level* of emission tax will in fact decrease. Were G_i and $x(p_i)$ to remain unchanged in the two equilibria, the convexity of $C(.)$ and $\varphi(.)$ would imply that $t^N(\hat{\tau})$ decreases. However, one should not expect that harmonization would leave either G_i or $x(p_i)$ unaffected. Indeed, with $G_i = \tau_i + t_i e_i$, setting τ_i at $\hat{\tau} > \tau^N$ will increase G_i if $t_i e_i$ does not decrease "much". If this happens, $\phi'(G_i)$ will decrease thus increasing the value of $\varphi'(E)/\phi'(G_i)$. This may be reinforced if p_i were to increase as a result of pushing τ_i up, lowering $x(p_i)$ and decreasing E. On the other hand, any decrease in p_i (due to tax competition in t) will increase $x(p_i)$ and possibly E. All these complications imply that condition (30) may call for an increase, and not a reduction, in the equilibrium value of $t^N(\hat{\tau})$. The numerical illustrations at the end of this section show that this is indeed possible.

[17] See equation (A42) in the Appendix.

5.2 Harmonization and the environment

It is plain that if harmonization lowers (increases) $t^N(\hat{\tau})$, then $e^N(\hat{\tau})$ will increase (decrease) and the countries will switch to more (less) polluting technologies. We show that harmonization (at $\hat{\tau} > \tau^N$) can change $t^N(\hat{\tau})$, and thus $e^N(\hat{\tau})$, positively as well as negatively. The numerical examples at the end of this Section illustrate these possibilities.

The more intriguing question is that of the effect of harmonization on the countries' overall environmental quality. To study this issue, observe that, from differentiating $E = 2e_i x(p_i)$ with respect to $\hat{\tau}$, one has

$$\frac{dE}{d\hat{\tau}} = -2x(p)\left[\left(\frac{e^2\,\varepsilon}{p} + \frac{1}{C''(e)}\right)\frac{dt^N(\hat{\tau})}{d\hat{\tau}} + \frac{e\varepsilon}{p}\right], \tag{31}$$

where we have dropped the i subscripts for ease in notation. It is clear from (31) that $dt^N(\hat{\tau})/d\hat{\tau} \geq 0 \Rightarrow dE^N(\hat{\tau})/d\hat{\tau} < 0$. Consequently, $dt^N(\hat{\tau})/d\hat{\tau} \geq 0$ (or $de^N(\hat{\tau})/d\hat{\tau} \leq 0$) is sufficient to ensure that overall environmental quality will improve. In words, if harmonization induces firms to adopt cleaner (or same) technologies, it will also raise the quality of the environment.

On the other hand, if $dt^N(\hat{\tau})/d\hat{\tau} < 0$ ($de^N(\hat{\tau})/d\hat{\tau} > 0$), it is *possible* for E to increase so that environmental quality will deteriorate. Which case prevails depends on the demand and cost structures in the economy. The numerical examples at the end of this section show all the various possible outcomes.[18]

5.3 Harmonization and welfare

The above discussion makes it clear that harmonizing τ_i at $\hat{\tau} > \tau^N$ may not necessarily improve the environment. A related question is whether such a policy is welfare improving (as opposed to environmental quality enhancing). To answer this question, use

[18] It is easy to check that the ambiguity in the sign of $de^N(\hat{\tau})/d\hat{\tau}$ (and with it $dE^N(\hat{\tau})/d\hat{\tau}$) remains even in the face of a perfectly inelastic demand for polluting goods.

(11) to write a country's welfare, at a symmetric Nash equilibrium with $\tau = \hat{\tau}$, as

$$W^S\left(t^N(\hat{\tau}), \hat{\tau}\right) = m - p^N(\hat{\tau})x\left(p^N(\hat{\tau})\right) + h\left(x\left(p^N(\hat{\tau})\right)\right) + \phi\left(G^N(\hat{\tau})\right) - \varphi\left(E^N(\hat{\tau})\right), \quad (32)$$

where we have dropped the i subscript for ease in notation. Differentiate $W^S\left(t^N(\hat{\tau}), \hat{\tau}\right)$ totally with respect to $\hat{\tau}$ to get

$$\frac{dW^S\left(t^N(\hat{\tau}), \hat{\tau}\right)}{d\hat{\tau}} = \frac{\partial W^S\left(t^N(\hat{\tau}), \hat{\tau}\right)}{\partial \hat{\tau}} + \frac{\partial W^S\left(t^N(\hat{\tau}), \hat{\tau}\right)}{\partial t^N(\hat{\tau})} \frac{dt^N(\hat{\tau})}{d\hat{\tau}}. \quad (33)$$

We know from (19a)–(19b) that at $\hat{\tau} = \tau^N$, $\partial W^S/\partial \hat{\tau}$ and $\partial W^S/\partial t^N(\hat{\tau})$ are both positive. It then immediately follows from (33) that a sufficient condition for a policy of harmonizing τ_i at $\hat{\tau}$, which is "just above" τ^N, to be welfare enhancing is that $dt^N(\hat{\tau})/d\hat{\tau} \geq 0$ ($de^N(\hat{\tau})/d\hat{\tau} \leq 0$). In words, if harmonization leads firms to adopt cleaner (or same) technologies, it will necessarily improve overall welfare. Recall that this is also the sufficient condition for enhancement of the environmental quality.

On the other hand, if $dt^N(\hat{\tau})/d\hat{\tau} < 0$ ($de^N(\hat{\tau})/d\hat{\tau} > 0$), $W^S\left(t^N(\hat{\tau}), \hat{\tau}\right)$ may change in either direction when τ_i is harmonized at $\hat{\tau}$. Again, the demand and cost structure in the economy determine if this inequality is satisfied. Note also that welfare and the quality of environment may move in opposite directions as a result of harmonization; see the numerical illustrations.

5.4 Numerical illustrations

Consider again the example of Subsection 4.3. Within this setting, we derive the unrestricted open-economy equilibria for four specific parameter values. Then, in each case, we solve the problem again while setting τ at a value exceeding its unrestricted Nash equilibrium value (by replacing equations (18a)–(18b) with (29)). In all the examples, the values of δ and ε are set equal to one, and the value of m at 4. They differ with respect to the specified values for b, a and ϕ.

In the first three examples, harmonization of τ above its unrestricted Nash equilibrium value lowers the emission tax rate, t. The first example is a case of welfare

Table 2. Commodity tax harmonization

	$(a = 1, b = 2, \varphi = 0.25)$							
	τ	t	e	E	G	W^S	p	$x(p)$
Absent harmonization	-0.0225	0.2328	0.8836	8.9824	0.9311	2.3089	0.1967	5.0828
With harmonization	-0.01	0.2200	0.8900	8.9922	0.9388	2.3086	0.1979	5.0520
	$(a = 1, b = 1, \varphi = 0.25)$							
	τ	t	e	E	G	W^S	p	$x(p)$
Absent harmonization	-0.0069	0.2183	0.7817	8.3366	0.8730	2.4538	0.1875	5.3320
With harmonization	0.0	0.2115	0.7885	8.3375	0.8817	2.4550	0.1891	5.2870
	$(a = 10, b = 1, \varphi = 0.25)$							
	τ	t	e	E	G	W^S	p	$x(p)$
Absent harmonization	0.0903	0.0249	0.9751	16.9741	0.9973	0.8932	0.1149	8.7041
With harmonization	0.10	0.0177	0.9823	16.7088	0.9987	0.9501	0.1176	8.5052
	$(b = 1, a = 1, \varphi = 2.0)$							
	τ	t	e	E	G	W^S	p	$x(p)$
Absent harmonization	0.1821	0.8719	0.1281	0.3801	0.4360	1.8042	0.6739	1.4839
With harmonization	0.19	0.8752	0.1248	0.3658	0.4386	1.8266	0.6822	1.4658

reducing partial harmonization. The reduction in t, and the subsequent increase in e, is accompanied by an increase in E and a reduction in W^S. In the second example, overall environmental quality continues to deteriorate, but welfare increases. In the third example, environmental quality improves despite the increase in e. Welfare increases. In the final example, harmonization of τ increases t. Consequently, the sufficient condition for improvement in the quality of the environment and welfare is satisfied. It is thus no surprise that E declines and W^S increases.

The results of this section are summarized as

Proposition 5 *Assume countries A and B harmonize their commodity tax rates, at $\tau = \hat{\tau}$ which is "just above" its unrestricted Nash equilibrium value, τ^N; they continue to compete in emission taxes. Denoting the (conditional) Nash equilibrium values by superscript N, we have:*

(i) The equilibrium emission tax in each country is smaller *than the marginal social*

cost of emissions to that country. That is,

$$t^N(\hat{\tau}) = -C'(e^N(\hat{\tau})) < \frac{\varphi'(E^N(\hat{\tau}))}{\phi'(G^N(\hat{\tau}))}.$$

(ii) Harmonizing τ_i at $\hat{\tau} > \tau^N$ may increase t_i. This will necessarily imply less polluting technologies, an improved environmental quality and enhanced welfare.

(iii) Harmonizing τ_i at $\hat{\tau} > \tau^N$ may lower t_i. Under this circumstance, the countries will switch to more polluting technologies. This may be accompanied by more as well as less aggregate pollution. Welfare may also increase as well as decrease with the possibility of welfare and environmental quality moving in opposite directions.

6 Emission tax harmonization

Instead of commodity taxes, let the two countries harmonize their emission taxes (at $t = \hat{t}$). Each country then chooses the value of its commodity tax to maximize the welfare of its citizens. As previously, we assume this is done à la Nash. It is easy to show that the Nash equilibrium value of τ_i, conditional on $t = \hat{t}$, is the solution to[19]

$$\frac{\phi'(G_i) - 1}{\phi'(G_i)\,\varepsilon_i} - \frac{p_i - C(e_i)}{\delta\,\varepsilon_i} - \frac{\tau_i}{p_i} + \frac{e_i}{p_i}\left[C'(e_i) + \frac{\varphi'(E)}{\phi'(G_i)}\right] = 0. \tag{34}$$

Denote this solution by $\tau^N(\hat{t})$ and the corresponding solutions for e, E, G, p by $e^N(\hat{t})$, $E^N(\hat{t}), G^N(\hat{t})$ and $p^N(\hat{t})$. Note that when t is unrestricted, the bracketed expression in the left-hand side of (34) will be zero so that τ will also take its unrestricted Nash equilibrium value, τ^N.

6.1 Harmonization and the commodity tax rate

The first question we address is the impact of harmonization on the commodity tax rate. This plays a central role in determining the implications of harmonization. We consider harmonizing t_i above its unrestricted Nash equilibrium. To study this question,

[19] Of course, one must solve (34) in conjunction with equations (2), (6), (7), (12), and (15). Note also that the best-reply function for i is given by equation (A42) in the Appendix when t_i is set at $\hat{t}$.

assume the countries choose the values of their fiscal instruments t_i and τ_i subject to the constraint that $t_i \geq \hat{t} > t^N$. It is plain that at a country's optimum this constraint must be binding (as the removal of the constraint would yield a different optimum in t^N and τ^N). This means that at the equilibrium values of $\hat{t}$ and $\tau^N(\hat{t})$, a country's welfare would increase if it could lower its value of t *assuming no response on the part of the other country.* Hence, at $(\hat{t}, \tau^N(\hat{t}))$, $\partial U_i/\partial t_i < 0$ $(i = A, B)$. This, plus the fact that at this point $\partial U_i/\partial \tau_i = 0$, implies that the bracketed expression in the left-hand side of (34) is negative[20] so that at $(\hat{t}, \tau^N(\hat{t}))$,

$$\frac{\tau_i}{p_i} < \frac{\phi'(G_i) - 1}{\phi'(G_i)\,\varepsilon_i} - \frac{G_i}{\delta\,\varepsilon_i, x(p_i)}, \tag{35}$$

where the right-hand side is the "rule" for setting the commodity tax rate in the absence of harmonization [see equation (18a)]. Condition (35) shows that increasing t_i above its Nash equilibrium value generates a "tendency" to compete more through the commodity tax and lower its value.[21]

6.2 Harmonization and the environment

It is clear that fixing t_i fixes e_i and thus the emission technology. Thus if one were to harmonize the emission tax rate above its unrestricted Nash equilibrium value, the firms will adopt less polluting technologies. What is not clear, however, is the impact of this on E and thus on the overall environmental quality. To see the determining factors, differentiate $E = 2e_i x(p_i)$ with respect to $\hat{t}$. We have

$$\frac{dE}{d\hat{t}} = -\frac{2x(p)}{C''(e)}\left[1 + \frac{eC''(e)\varepsilon}{p}\frac{dp}{d\hat{t}}\right], \tag{36}$$

where we have dropped the i subscripts for ease in notation. It follows from (36) that a sufficient condition for aggregate emissions to decrease is $dp^N(\hat{t})/d\hat{t} \geq 0$.[22] In words, if

[20]See equation (A43) in the Appendix which gives the expression for $\partial U_i/\partial t_i$.

[21]This is again a "rule" question; the equilibrium value of τ_i may in fact increase. We shall see this in our numerical examples.

[22]In light of

$$\frac{dp^N(\hat{t})}{d\hat{t}} = e^N(\hat{t}) + \frac{d\tau^N(\hat{t})}{d\hat{t}},$$

harmonizing the emission tax rate at just above its unrestricted Nash equilibrium value increases the (restricted Nash equilibrium) polluting good price, the overall quality of the environment will improve. This is of course not surprising because in this case the consumption of the polluting good declines also (or remains the same).

Observe also that if $\varepsilon = 0$, $dE/d\hat{t} < 0$ and harmonization will necessarily improve the quality of the environment regardless of its impact on price.

6.3 Harmonization and welfare

That partially harmonizing the emission tax rate, despite encouraging the adoption of less polluting technologies, may lower the environmental quality indicates that overall welfare of the countries may also decline. To address this question formally, use (11) to write a country's welfare, at a symmetric Nash equilibrium with $t = \hat{t}$, as[23]

$$W^S\left(\tau^N(\hat{t}), \hat{t}\right) = m - p^N(\hat{t})x\left(p^N(\hat{t})\right) + h\left(x\left(p^N(\hat{t})\right)\right) + \phi\left(G^N(\hat{t})\right) - \varphi\left(E^N(\hat{t})\right). \quad (37)$$

Differentiate $W^S\left(\tau^N(\hat{t}), \hat{t}\right)$ totally with respect to $\hat{t}$ to get

$$\frac{dW^S\left(\tau^N(\hat{t}), \hat{t}\right)}{d\hat{t}} = \frac{\partial W^S\left(\tau^N(\hat{t}), \hat{t}\right)}{\partial \hat{t}} + \frac{\partial W^S\left(\tau^N(\hat{t}), \hat{t}\right)}{\partial \tau^N(\hat{t})} \frac{d\tau^N(\hat{t})}{d\hat{t}}. \quad (38)$$

Now at $\hat{t} = t^N$, $\partial W^S/\partial \hat{t}$ and $\partial W^S/\partial \tau^N(\hat{t})$ are both positive (see (19a)–(19b)). It then immediately follows from (38) that a sufficient (but not necessary) condition for a policy of harmonizing t_i at "just above" $\hat{t} > t^N$ to be welfare enhancing is $d\tau^N(\hat{t})/d\hat{t} \geq 0$.

Next, substitute for $\partial W^S/\partial \hat{t}$ and $\partial W^S/\partial \tau^N(\hat{t})$ from (19a)–(19b) in (38) and simplify using the relationship $dp^N(\hat{t})/d\hat{t} = e + d\tau^N(\hat{t})/d\hat{t}$. We have, at $\hat{t} = t^N$,

$$\frac{dW^S}{d\hat{t}} = \frac{x(p)\varphi'(E)}{C''(e)} + x(p)\phi'(G)\left[\frac{p - C(e)}{\delta} + \frac{\varepsilon e \varphi'(E)}{p\phi'(G)}\right]\frac{dp^N(\hat{t})}{d\hat{t}}. \quad (39)$$

$d\tau^N(\hat{t})/d\hat{t} \geq 0$ will also be a sufficient condition for this. However, E will decrease even if $d\tau^N(\hat{t})/d\hat{t}$ is negative as long as $dp^N(\hat{t})/d\hat{t} \geq 0$.

[23] We have again dropped the i subscript for ease in notation.

It follows from (39) that a sufficient condition for harmonization to increase welfare is for $dp^N(\hat{t})/d\hat{t} \geq 0$. Recall that this is also the sufficient condition for harmonization to improve the overall environmental quality.

The last piece of the puzzle is to consider the impact of harmonization on $p^N(\hat{t})$. We derive an expression for $dp^N(\hat{t})/d\hat{t}$ in the Appendix indicating that it can take positive as well as negative values (when evaluated at $\hat{t} = t^N$). This is further confirmed by the numerical examples below. Nevertheless, all our numerical examples indicate that harmonization will reduce aggregate emissions and increase welfare regardless of its impact on $p^N(\hat{t})$.

Finally, the Appendix also shows that if the demand for the polluting good is perfectly inelastic, $dp^N(\hat{t})/d\hat{t} > 0$. It follows from (39) that under this circumstance, $dW^S/d\hat{t} > 0$. That is, welfare will necessarily improve.

6.4 Numerical illustrations

Consider again the example of Subsection 4.3. Set $\delta = 1, \varphi = 0.25$ with $a = 1, b = 1$, $\varepsilon = 1$ in one case, and $a = 10, b = 0.4, \varepsilon = 2$ in a second case, and derive the unrestricted open-economy equilibria for each case. Then, solve the problem again while setting t at a value exceeding its unrestricted Nash equilibrium value (by replacing equations (18a)–(18b) with (34)). Observe that $p^N(\hat{t})$ goes up in the first case and down in the second. However, in both cases, $E^N(\hat{t})$ will decrease and $W^S(\hat{t})$ will increase.[24]

The results of this Section are summarized as

Proposition 6 *Assume countries A and B harmonize their emission tax rates, at $t = \hat{t}$ which is "just above" its unrestricted Nash equilibrium value, t^N, but continue to compete in commodity taxes. We have:*

[24] We have tried many other parameter values as well different functional forms for the utility function such as setting $\phi(G) = aG^\alpha$ and $\phi(G) = aG^\alpha/\alpha$ and varying α between zero and one. We cannot generate any example for which either E increases or W^S declines.

Table 3. Emission tax harmonization

	$(a=1, b=1, \varepsilon=1)$							
	t	τ	e	E	G	W^S	p	$x(p)$
Absent harmonization	0.2183	-0.0069	0.7817	8.3366	0.8730	2.4538	0.1875	5.3321
With harmonization	0.2190	-0.0075	0.7810	8.3278	0.8721	2.4549	0.1876	5.3315
	$(a=10, b=0.4, \varepsilon=2)$							
	t	τ	e	E	G	W^S	p	$x(p)$
Absent harmonization	0.1450	0.0215	0.6376	64.9222	5.7980	12.4797	0.1402	50.9096
With harmonization	0.1480	0.0192	0.6300	64.4616	5.7519	12.5325	0.1398	51.1600

(i) The rule for setting commodity taxes now includes an additional negative term in comparison the unrestricted Nash equilibrium case. The tax rate will be the solution to (34).

(ii) If harmonizing t_i at $\hat{t} > t^N$ increases p_i, it will improve the quality of the environment and enhance welfare in country i.

(iii) If the demand for polluting good is perfectly inelastic, partial harmonization of emission taxes will necessarily enhance the quality of the environment and improve welfare.

7 Conclusion

This paper has studied whether, in the presence of a global negative externality, economic integration will necessarily lower environmental quality and the provision of public goods. What has emerged quite clearly is that the inefficiencies due to global externalities and tax competition are not simply additive. Instead, there is a rich and complex interrelationship between them. The paper has shown that tax competition may not necessarily lower emission taxes when a centralized authority has access to both emission and commodity tax instruments. The possibility of higher emission taxes implies that economic integration may very well result in firms' adopting less polluting technologies. The net effect on the overall environmental quality would then depend on what happens to the aggregate consumption of polluting goods.

We pointed out that the existence of a global externality has important implications for the supply of (local) public goods as well. The initial (i.e. prior to economic integration) level may be lower or greater than the optimal level, and that tax competition may increase or decrease it. The induced change may then entail an efficiency gain.

These results have interesting policy implications. First, there is a case to be made for trade to lead to cleaner technologies. Second, the case for global cooperation to combat environmental issues (e.g. Kyoto agreements) does not rest only on whether or not trade hurts the environment. One must also be cognizant of the inefficiencies due to global nature of emissions. Third, one can make a stronger case for economic integration than hitherto recognized. To the extent that economic integration encourages the member countries to harmonize their policies, it is a force for solving the global emissions problem as well.

Secondly, we studied the efficacy of partial tax harmonization policies and their impact on the environment and welfare. Here, we showed that partially harmonizing emission taxes appear to always enhance environmental quality and welfare. This will certainly be the case if the demand for polluting goods is "very inelastic". The effects of commodity tax harmonization can go either way. A policy of partially harmonizing commodity taxes (above their unrestricted Nash equilibrium value) may be helpful in that it may lead to the adoption of technologies that are less polluting, improve the overall quality of the environment, and enhance welfare. On the other hand, such a policy is also potentially damaging. It can induce firms to switch to more polluting technologies, hurt the overall environmental quality, and lower welfare. This suggests that countries will have to be very careful when they do not fully coordinate their policies. Whether or not a particular harmonization policy should be adopted depends on the specifics of the policy, the demand and cost functions, and how much the people care about the environment. Theory alone cannot settle this.

Appendix

Derivation of (4a),(4b), (5): We have

$$G_A = \begin{cases} \int_{\tilde{\theta}}^{1} \left[p_A - C(e_A)\right] x_A^A d\theta & \text{if } p_A \geq p_B, \\ \int_{\tilde{\theta}}^{0} \left[p_A - C(e_A)\right] x_A^B d\theta + \int_0^1 \left[p_A - C(e_A)\right] x_A^A d\theta & \text{if } p_A < p_B, \end{cases} \tag{A1}$$

$$G_B = \begin{cases} \int_{-1}^{0} \left[p_B - C(e_B)\right] x_B^B d\theta + \int_0^{\tilde{\theta}} \left[p_B - C(e_B)\right] x_B^A d\theta & \text{if } p_A \geq p_B, \\ \int_{-1}^{\tilde{\theta}} \left[p_B - C(e_B)\right] x_B^B d\theta & \text{if } p_A < p_B. \end{cases} \tag{A2}$$

And

$$E = \begin{cases} \int_{-1}^{0} e_B x_B^B d\theta + \int_0^{\tilde{\theta}} e_B x_B^A d\theta + \int_{\tilde{\theta}}^{1} e_A x_A^A d\theta & \text{if } p_A \geq p_B, \\ \int_{-1}^{\tilde{\theta}} e_B x_B^B d\theta + \int_{\tilde{\theta}}^{0} e_A x_A^B d\theta + \int_0^1 e_A x_A^A d\theta & \text{if } p_A < p_B. \end{cases} \tag{A3}$$

Simplifying yields equations (4a)–(4b) and (5) in the text.

Proof of Lemma 1: Partially differentiate equations (4a) and (5) with respect to p_A and e_A, and equations (4b) and (5) with respect to p_B and e_B. We have

$$\frac{\partial G_A}{\partial p_A} = \begin{cases} (1-\tilde{\theta})x(p_A) - \frac{1}{\delta}\left[p_A - C(e_A)\right] x(p_A) + \left[p_A - C(e_A)\right](1-\tilde{\theta})x'(p_A) & \text{if } p_A \geq p_B, \\ x(p_A) + \int_{\tilde{\theta}}^{0} x(p_A - \delta\theta)d\theta + \left[p_A - C(e_A)\right]\left[x'(p_A) - \frac{1}{\delta}x(p_A)\right] & \text{if } p_A < p_B. \end{cases} \tag{A4}$$

$$\frac{\partial G_B}{\partial p_B} = \begin{cases} x(p_B) + \int_0^{\tilde{\theta}} x(p_B + \delta\theta)d\theta + \left[p_B - C(e_B)\right]\left[x'(p_B) - \frac{1}{\delta}x(p_B)\right] & \text{if } p_A \geq p_B, \\ (1+\tilde{\theta})x(p_B) - \frac{1}{\delta}\left[p_B - C(e_B)\right] x(p_B) + \left[p_B - C(e_B)\right](1+\tilde{\theta})x'(p_B) & \text{if } p_A < p_B. \end{cases} \tag{A5}$$

$$\frac{\partial E}{\partial p_A} = \begin{cases} \frac{1}{\delta}e_B x(p_B + \delta\tilde{\theta}) - \frac{1}{\delta}e_A x(p_A) + (1-\tilde{\theta})e_A x'(p_A) & \text{if } p_A \geq p_B, \\ \frac{1}{\delta}e_B x(p_B) + e_A\left[x'(p_A) - \frac{1}{\delta}x(p_A)\right] & \text{if } p_A < p_B. \end{cases} \tag{A6}$$

$$\frac{\partial E}{\partial p_B} = \begin{cases} e_B\left[x'(p_B) - \frac{1}{\delta}x(p_B)\right] + \frac{1}{\delta}e_A x(p_A) & \text{if } p_A \geq p_B, \\ -\frac{1}{\delta}e_B x(p_B) + (1+\tilde{\theta})e_B x'(p_B) + \frac{1}{\delta}e_A x(p_A - \delta\tilde{\theta}) & \text{if } p_A < p_B. \end{cases} \tag{A7}$$

$$\frac{\partial G_A}{\partial e_A} = \begin{cases} -C'(e_A)(1-\tilde{\theta})x(p_A) & \text{if } p_A \geq p_B, \\ -C'(e_A)\ \ x(p_A) + \int_{\tilde{\theta}}^{0} x(p_A - \delta\theta)d\theta\Big] & \text{if } p_A < p_B. \end{cases} \tag{A8}$$

$$\frac{\partial G_B}{\partial e_B} = \begin{cases} -C'(e_B)\ \ x(p_B) + \int_0^{\tilde{\theta}} x(p_B + \delta\theta)d\theta\Big] & \text{if } p_A \geq p_B, \\ -C'(e_B)(1+\tilde{\theta})x(p_B) & \text{if } p_A < p_B. \end{cases} \tag{A9}$$

$$\frac{\partial E}{\partial e_A} = \begin{cases} (1-\tilde{\theta})x(p_A) & \text{if } p_A \geq p_B, \\ \int_{\tilde{\theta}}^{0} x(p_A - \delta\theta)d\theta + x(p_A) & \text{if } p_A < p_B. \end{cases} \tag{A10}$$

$$\frac{\partial E}{\partial e_B} = \begin{cases} \int_0^{\tilde{\theta}} x(p_B+\delta\theta)d\theta + x(p_B) & \text{if } p_A \geq p_B, \\ (1+\tilde{\theta})x(p_B) & \text{if } p_A < p_B. \end{cases} \tag{A11}$$

where, in the derivations of (A4)–(A7), we have utilized the following expressions:

$$\begin{aligned} \frac{\partial}{\partial p_A}\int_{\tilde{\theta}}^{0} x(p_A-\delta\theta)d\theta &= \int_{\tilde{\theta}}^{0} x'(p_A-\delta\theta)d\theta - x(p_A-\delta\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial p_A} \\ &= -\frac{1}{\delta}\ x(p_A-\delta\theta)\Big]_{\theta=\tilde{\theta}}^{\theta=0} - \frac{1}{\delta}x(p_A-\delta\tilde{\theta}) = -\frac{1}{\delta}x(p_A), \qquad \text{(A12)} \\ \frac{\partial}{\partial p_B}\int_0^{\tilde{\theta}} x(p_B+\delta\theta)d\theta &= \int_0^{\tilde{\theta}} x'(p_B+\delta\theta)d\theta - x(p_B+\delta\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial p_B} \\ &= \frac{1}{\delta}\ x(p_B+\delta\theta)\Big]_{\theta=0}^{\theta=\tilde{\theta}} - \frac{1}{\delta}x(p_B+\delta\tilde{\theta}) = -\frac{1}{\delta}x(p_B). \qquad \text{(A13)} \end{aligned}$$

Evaluating expressions (A4)–(A11) at $p_A = p_B$ and simplifying, we get an identical expression for the left- and the right-hand derivatives of each of the functions $G_A(.), G_B(.)$ and $E(.)$. They are all continuous and given by, for $i = A, B$,[25]

$$\frac{\partial G_i}{\partial p_i} = x(p_i) - [p_i - C(e_i)]\left[\frac{x(p_i)}{\delta} - x'(p_i)\right], \tag{A14}$$

$$\frac{\partial E}{\partial p_i} = e_i x'(p_i), \tag{A15}$$

$$\frac{\partial G_i}{\partial e_i} = -C'(e_i)x(p_i), \tag{A16}$$

$$\frac{\partial E}{\partial e_i} = x(p_i). \tag{A17}$$

Proof of Lemma 2: To simplify the exposition of the proof, first calculate the following expressions based on the definition of $F(p)$ in (9).

$$\int_0^{\tilde{\theta}} \left[h\left(x(p_B+\delta\theta)\right) - (p_B+\delta\theta)x(p_B+\delta\theta)\right]d\theta = \frac{1}{\delta}\left[F(p_A) - F(p_B)\right], \tag{A18}$$

$$\int_{\tilde{\theta}}^{0} \left[h\left(x(p_A-\delta\theta)\right) - (p_A-\delta\theta)x(p_A-\delta\theta)\right]d\theta = -\frac{1}{\delta}\left[F(p_A) - F(p_B)\right]. \tag{A19}$$

[25]One can easily show that the same properties hold for all other partial derivatives of $G_A(.), G_B(.)$ and $E(.)$.

$$F'(p_i) = h(x(p_i)) - p_i x(p_i), \quad i = A, B. \tag{A20}$$

$$\begin{aligned}\frac{\partial}{\partial p_A}\tilde{\theta}\left[h(x(p_A)) - p_A x(p_A)\right] &= \frac{1}{\delta}\left[h(x(p_A)) - p_A x(p_A)\right] + \tilde{\theta}\big[h'(x(p_A))x'(p_A) \\ &\quad - x(p_A) - p_A x'(p_A)\big] = \frac{1}{\delta}F'(p_A) - \tilde{\theta}x(p_A), \qquad \text{(A21)}\\ \frac{\partial}{\partial p_B}\tilde{\theta}\left[h(x(p_B)) - p_B x(p_B)\right] &= -\frac{1}{\delta}\left[h(x(p_B)) - p_B x(p_B)\right] + \tilde{\theta}\big[h'(x(p_B))x'(p_B) \\ &\quad - x(p_B) - p_B x'(p_B)\big] = -\frac{1}{\delta}F'(p_B) - \tilde{\theta}x(p_B). \qquad \text{(A22)}\end{aligned}$$

Proof of part (i): Consider the case where $p_A \geq p_B$. We have $\tilde{\theta} \geq 0$ and

$$\begin{aligned}W^A = &\int_0^{\tilde{\theta}} \left[m + h(x(p_B + \delta\theta)) - (p_B + \delta\theta)x(p_B + \delta\theta) + \phi(G_A) - \varphi(E)\right] d\theta + \\ &\int_{\tilde{\theta}}^1 \left[m + h(x(p_A)) - p_A x(p_A) + \phi(G_A) - \varphi(E)\right] d\theta, \\ = &\, U_A - \int_0^{\tilde{\theta}} \left[h(x(p_A)) - p_A x(p_A)\right] d\theta + \int_0^{\tilde{\theta}} \left[h(x(p_B + \delta\theta)) - (p_B + \delta\theta)x(p_B + \delta\theta)\right] d\theta, \\ = &\, U_A - \tilde{\theta}\left[h(x(p_A)) - p_A x(p_A)\right] + \frac{1}{\delta}\left[F(p_A) - F(p_B)\right]. \qquad \text{(A23)}\\ W^B = &\int_{-1}^0 \left[m + h(x(p_B)) - p_B x(p_B) + \phi(G_B) - \varphi(E)\right] d\theta, \\ = &\, U_B, \qquad \text{(A24)}\end{aligned}$$

where we have made use of equations (A18)–(A19) and the definition of U_i $(i = A, B)$ in (8).

Similarly, using (A18)–(A19) and the definition of U_i $(i = A, B)$ for the case $p_A < p_B$, we have

$$\begin{aligned}W^A = &\int_0^1 \left[m + h(x(p_A)) - p_A x(p_A) + \phi(G_A) - \varphi(E)\right] d\theta, \\ = &\, U_A. \qquad \text{(A25)}\\ W^B = &\int_{-1}^{\tilde{\theta}} \left[m + h(x(p_B)) - p_B x(p_B) + \phi(G_B) - \varphi(E)\right] d\theta + \\ &\int_{\tilde{\theta}}^0 \left[m + h(x(p_A - \delta\theta)) - (p_A - \delta\theta)x(p_A - \delta\theta) + \phi(G_B) - \varphi(E)\right] d\theta,\end{aligned}$$

$$= U_B - \int_{\tilde{\theta}}^{0} [h(x(p_B)) - p_B x(p_B)]\, d\theta + \int_{\tilde{\theta}}^{0} [h(x(p_A - \delta\theta)) - (p_A - \delta\theta)x(p_A - \delta\theta)]\, d\theta,$$

$$= U_B + \tilde{\theta}\,[h(x(p_B)) - p_B x(p_B)] - \frac{1}{\delta}\,[F(p_A) - F(p_B)]\,. \tag{A26}$$

Proof of part (ii): Differentiating equation (10a) with respect to p_A and equation (10b) with respect to p_B, making use of (A21)–(A22), yields

$$\frac{\partial W_A}{\partial p_A} = \begin{cases} \frac{\partial U_A}{\partial p_A} + \frac{1}{\delta}F'(p_A) - \left[\frac{1}{\delta}F'(p_A) - \tilde{\theta}x(p_A)\right] & \text{if } p_A \geq p_B, \\ \frac{\partial U_A}{\partial p_A} & \text{if } p_A < p_B. \end{cases} \tag{A27}$$

$$\frac{\partial W_B}{\partial p_B} = \begin{cases} \frac{\partial U_B}{\partial p_B} & \text{if } p_A \geq p_B, \\ \frac{\partial U_B}{\partial p_B} + \frac{1}{\delta}F'(p_B) - \frac{1}{\delta}F'(p_B) - \tilde{\theta}x(p_B) & \text{if } p_A < p_B. \end{cases} \tag{A28}$$

The equality of left- and right-hand derivatives result follows immediately from the fact that at $p_A = p_B$, $\tilde{\theta} = 0$.[26]

Proof of Proposition 1: The first-order conditions are:

$$\frac{\partial W_i^S}{\partial \tau_i} = \frac{\partial W_i^S}{\partial p_i} = -x(p_i) + \phi'(G_i)\frac{\partial G_i}{\partial p_i}|_{e_i} - \varphi'(E)\frac{\partial E}{\partial p_i}|_{e_i} = 0, \tag{A29}$$

$$\frac{\partial W_i^S}{\partial t_i} = \frac{\partial W_i^S}{\partial p_i}\frac{\partial p_i}{\partial t_i}|_{\tau_i} + \frac{\partial W_i^S}{\partial t_i}|_{p_i} = \frac{\partial W_i^S}{\partial p_i}\frac{\partial p_i}{\partial t_i}|_{\tau_i} + \phi'(G_i)\frac{\partial G_i}{\partial t_i}|_{p_i} - \varphi'(E)\frac{\partial E}{\partial t_i}|_{p_i} = 0. \tag{A30}$$

Simplifying (A29)–(A30), via differentiation of equations (12)–(13), we have

$$\frac{\partial W_i^S}{\partial \tau_i} = -x(p_i) + \phi'(G_i)\left\{x(p_i) + [p_i - C(e_i)]\, x'(p_i)\right\} - \varphi'(E)2e_i x'(p_i) = 0, \tag{A31}$$

$$\frac{\partial W_i^S}{\partial t_i} = \frac{\partial W_i^S}{\partial p_i}\frac{\partial p_i}{\partial t_i}|_{\tau_i} + \phi'(G_i)\left[-C'(e_i)\frac{\partial e_i}{\partial t_i}x(p_i)\right] - \varphi'(E)\left[2\frac{\partial e_i}{\partial t_i}x(p_i)\right] = 0. \tag{A32}$$

Further algebraic manipulation of (A31)–(A32) simplifies these equations into:

$$\frac{\partial W_i^S}{\partial \tau_i} = x(p_i)\phi'(G_i)\varepsilon_i\left\{\frac{\phi'(G_i) - 1}{\phi'(G_i)\,\varepsilon_i} - \frac{\tau_i}{p_i} + \frac{e_i}{p_i}\left[C'(e_i) + \frac{2\varphi'(E)}{\phi'(G_i)}\right]\right\} = 0, \tag{A33}$$

$$\frac{\partial W_i^S}{\partial t_i} = \frac{\partial W_i^S}{\partial \tau_i}e_i + \frac{\phi'(G_i)x(p_i)}{C''(e_i)}\left[C'(e_i) + \frac{2\varphi'(E)}{\phi'(G_i)}\right] = 0, \tag{A34}$$

where we have substituted ε_i for $-p_i x'(p_i)/x'(p_i)$ and τ_i for $p_i - C(e_i) + C'(e_i)e_i$.

[26] One can easily show that the same properties hold for all other partial derivatives of W_i ($i = A, B$).

First, to prove (14b), set $\partial W_i^S / \partial \tau_i = 0$ in (A34) and simplify.

Second, to prove (14a), set $-C'(e_i) = 2\varphi'(E)/\phi'(G_i)$ in (A33) and simplify.

Proof of Proposition 2: Summarize country i's problem through the Lagrangian

$$\Delta_i = m + h(x(p_i)) - p_i x(p_i) + \phi(G_i) - \varphi(E), \tag{A35}$$

where $E = e_i x(p_i) + e_j x(p_j)$. Thus the difference with the optimization problem of Proposition 1 is only in the treatment of E. The proof will then be identical to the proof of Proposition 1 except that $\varphi'(E)$ replaces $2\varphi'(E)$ everywhere.

Perfectly inelastic demand in the closed economy: First, set $x(p_i) = 1$ in (A35) and differentiate Δ_i with respect to τ_i and t_i (taking the tax parameters of the other country as fixed). Setting $\partial \Delta_i / \partial \tau_i = 0$ gives

$$\phi'(G_i) = 1.$$

Second, consider the relationship

$$-C'(e_i) = k\varphi'(2e_i), \tag{A36}$$

where k is a parameter taking the value of two in the second best and one under the closed-economy solution. Differentiating (A36) with respect to k yields

$$\frac{de_i}{dk} = -\frac{\varphi'(2e_i)}{C''(e_i) + 2k\varphi''(2e_i)} < 0. \tag{A37}$$

This proves the claim that the closed-economy equilibrium values of e_i and E exceed their second-best values.

Proof of Proposition 3: To derive the best-reply functions of each country, differentiate equations (10a)–(10b) with respect to the instrument employed. Thus, let I_i stand for τ_i, t_i or e_i. We have:

$$\frac{\partial W_A}{\partial I_A} = \begin{cases} \frac{\partial U_A}{\partial I_A} + \frac{\partial}{\partial I_A} \; \frac{1}{\delta}\left[F(p_A) - F(p_B)\right] - \tilde{\theta}\left[h(x(p_A)) - p_A x(p_A)\right]\Big] & \text{if } p_A \geq p_B \\ \frac{\partial U_A}{\partial I_A} & \text{if } p_A < p_B. \end{cases} \tag{A38}$$

$$\frac{\partial W_B}{\partial I_B} = \begin{cases} \frac{\partial U_B}{\partial I_B} & \text{if } p_A \geq p_B \\ \frac{\partial U_B}{\partial I_B} + \frac{\partial}{\partial I_B} \quad - \frac{1}{\delta}\left[F(p_A) - F(p_B)\right] + \tilde{\theta}\left[h(x(p_B)) - p_B x(p_B)\right]\Big] & \text{if } p_A < p_B. \end{cases} \tag{A39}$$

The first-order conditions are found by setting the above equations equal to zero. Note, however, that in (A38)–(A39), only $\partial U_i/\partial I_i$ $(i = A, B)$ terms matter. Any additional term will vanish at a symmetric equilibrium.

The first-order conditions for country A are then given by,

$$\frac{\partial U_A}{\partial \tau_A} = \frac{\partial U_A}{\partial p_A} = -x(p_A) + \phi'(G_A)\frac{\partial G_A}{\partial p_A} - \varphi'(E)\frac{\partial E}{\partial p_A} = 0, \tag{A40}$$

$$\begin{aligned} \frac{\partial U_A}{\partial t_A} &= \frac{\partial U_A}{\partial p_A}\frac{\partial p_A}{\partial t_A}|_{\tau_A} + \frac{\partial U_A}{\partial t_A}|_{p_A} \\ &= \frac{\partial U_A}{\partial p_A}\frac{\partial p_A}{\partial t_A}|_{\tau_A} + \phi'(G_A)\frac{\partial G_A}{\partial t_A}|_{p_A} - \varphi'(E)\frac{\partial E}{\partial t_A}|_{p_A} = 0. \end{aligned} \tag{A41}$$

At $p_A = p_B, e_A = e_B$, one can simplify equations (A40)–(A41) by substituting from (A14)–(A15) in (A40) and from (A16)–(A17) in (A41). Same conditions hold for country B and we have:

$$\begin{aligned} \frac{\partial U_i}{\partial \tau_i} &= x(p_i)\phi'(G_i)\varepsilon_i \Bigg\{ \left[\frac{\phi'(G_i) - 1}{\phi'(G_i)\,\varepsilon_i} - \frac{p_i - C(e_i)}{\delta\,\varepsilon_i} - \frac{\tau_i}{p_i}\right] \\ &\quad + \frac{e_i}{p_i}\left[C'(e_i) + \frac{\varphi'(E)}{\phi'(G_i)}\right]\Bigg\} = 0, \end{aligned} \tag{A42}$$

$$\frac{\partial U_i}{\partial t_i} = \frac{\partial U_i}{\partial \tau_i}e_i + \frac{\phi'(G_i)x(p_i)}{C''(e_i)}\left[C'(e_i) + \frac{\varphi'(E)}{\phi'(G_i)}\right] = 0. \tag{A43}$$

Substituting $\partial U_i/\partial \tau_i = 0$ from (A42) into (A43) gives us equation (18b). Setting $-C'(e_i) = \varphi'(E)/\phi'(G_i)$ in (A42) then yields (18a).

Open economy equilibrium and the variation in δ: To simplify the calculations, we assume here that the elasticity of demand and the marginal social damage of emissions are constant. Substitute the optimal value of τ from (18a) in (7). This yields

$$p - \frac{\delta\varepsilon\left[C(e) - C'(e)e\right]}{p} = C(e) + \delta\left[1 - \varepsilon - \frac{1}{\phi'(G)}\right], \tag{A44}$$

where we have dropped the subscript i for simplicity in exposition. Differentiate equations (6), (12), (13), (18b) and (A44) totally with respect to δ.[27] We have

$$\frac{de}{d\delta} = \frac{-1}{C''(e)}\frac{dt}{d\delta}, \tag{A45}$$

$$\frac{dG}{d\delta} = \frac{-\phi'^2(G)}{\phi''(G)\varphi'(E)}\frac{dt}{d\delta}, \tag{A46}$$

$$\frac{dp}{d\delta} = -p\frac{\frac{\phi'^2(G)}{\phi''(G)\varphi'(E)x(p)} + \frac{C'(e)}{C''(e)}}{p - \varepsilon\left[p - C(e)\right]}\frac{dt}{d\delta}, \tag{A47}$$

$$\frac{dE}{d\delta} = \frac{-2x(p)}{C''(e)}\left\{1 - \varepsilon e\frac{C'(e) + \frac{C''(e)\phi'^2(G)}{\phi''(G)\varphi'(E)x(p)}}{p - \varepsilon\left[p - C(e)\right]}\right\}\frac{dt}{d\delta}, \tag{A48}$$

with

$$\frac{dt}{d\delta} = \frac{\left. 1 - \varepsilon\frac{p-C(e)}{p}\right] \frac{p-C(e)}{\delta}}{A + \left. 1 - \varepsilon\frac{p-C(e)}{p}\right] \left. \frac{-\delta\varepsilon e}{p} + \frac{C'(e)}{C''(e)} + \frac{\delta}{\varphi'(E)}\right]}, \tag{A49}$$

where

$$A \equiv \left[\frac{-\phi'^2(G)}{x(p)\phi''(G)\varphi'(E)} - \frac{C'(e)}{C''(e)}\right]\left\{1 + \frac{\delta\varepsilon}{p^2}\left[C(e) - C'(e)e\right]\right\} > 0. \tag{A50}$$

It is clear from (A49) that the sign of $dt/d\delta$ is ambiguous making the signs of all the derivatives in (A45)–(A48) ambiguous as well.

Perfectly inelastic demand in the open economy: First, from (A42)–(A43), we again have $-C'(e_i) = \varphi'(E)/\phi'(G_i)$. Upon substitution in (A42), one gets

$$\phi'(G_i) - 1 - \frac{\phi'(G_i)}{\delta}G_i = 0.$$

Rewriting this, yields (20) in the text.

Second, observe that setting $k = 1$ in equation (A36) yields condition (16b) under the closed-economy, and setting $k = 1/\phi'(G) < 1$ gives condition (18b) under the open economy. It then immediately follows from (A37) that the open-economy equilibrium value of e_i exceeds its closed-economy value.

[27] Details of the derivations can be obtained from the authors on request.

Third, differentiating (23) partially with respect to e_i and G_i yields

$$\frac{\partial W^S}{\partial e_i} = -C'(e_i) - 2\varphi'(E), \tag{A51}$$

$$\frac{\partial W^S}{\partial G_i} = \phi'(G_i) - 1. \tag{A52}$$

Evaluating (A51) at the open-economy equilibrium values results in

$$\frac{\partial W^S}{\partial t_i} = -\big[\phi'(G_i) - 1\big]\frac{\varphi'(E)}{\phi'(G_i)} - \varphi'(E) < 0.$$

Additionally, with $\phi'(G_i) > 1$, we have $\partial W^S/\partial G_i > 0$.

Finally, differentiating (A51)–(A52) with respect to e_i and G_i yields

$$\frac{\partial^2 W^S}{\partial {e_i}^2} = -C''(e_i) - 4\varphi''(E) < 0, \tag{A53}$$

$$\frac{\partial^2 W^S}{\partial {G_i}^2} = \phi''(G_i) < 0, \tag{A54}$$

$$\frac{\partial^2 W^S}{\partial G_i \partial e_i} = 0, \tag{A55}$$

which proves W^S is strictly concave in e_i and G_i.

Proof of $\partial W^S(\tau^N, t^N)/\partial\tau > 0$ and $\partial W^S(\tau^N, t^N)/\partial t > 0$: Compare equations (A33)–(A34) with (A42)–(A43). This reveals that

$$\frac{\partial W_i^S}{\partial \tau_i} = \frac{\partial U_i}{\partial \tau_i} + x(p_i)\phi'(G_i)\varepsilon_i\left[\frac{p_i - C(e_i)}{\delta\varepsilon_i} + \frac{e_i}{p_i}\frac{\varphi'(E)}{\phi'(G_i)}\right], \tag{A56}$$

$$\frac{\partial W_i^S}{\partial t_i} = \frac{\partial U_i}{\partial t_i} + e_i x(p_i)\phi'(G_i)\varepsilon_i\left[\frac{p_i - C(e_i)}{\delta\varepsilon_i} + \frac{e_i}{p_i}\frac{\varphi'(E)}{\phi'(G_i)}\right] + \frac{x(p_i)\varphi'(E)}{C''(e_i)}. \tag{A57}$$

Now at (τ^N, t^N), $\partial U_i/\partial\tau_i = \partial U_i/\partial t_i = 0$. The result follows immediately from the signs of the remaining expressions in (A56)–(A57).

Emission tax harmonization: Totally differentiate the system of equations (6), (7), (12), (13), and (34), which determine $\tau^N(\hat{t}), e^N(\hat{t}), G^N(\hat{t}), E^N(\hat{t})$ and $p^N(\hat{t})$, with respect to $\hat{t}$. Algebraic manipulations of the resulting expressions, evaluated at the open-economy equilibrium values, yield[28]

$$\frac{dp^N(\hat{t})}{d\hat{t}} = -\frac{C'(e)}{C''(e)} \frac{1-\frac{\delta\phi''(G)}{\phi'^2(G)} \quad 1-\frac{\varepsilon e\varphi'(E)}{p}\Big] x(p)}{1+\frac{\delta\varepsilon}{P}\left(1-\frac{\tau}{p}\right) - \frac{\delta\phi''(G)}{\phi'^2(G)} \quad 1-\frac{\varepsilon e\varphi'(E)}{p}\Big] \quad 1-\frac{\varepsilon(p-C(e))}{p}\Big] x(p)}. \tag{A58}$$

At $\varepsilon = 0$, equation (A58) will simplify to

$$\frac{dp^N(\hat{t})}{d\hat{t}} = -\frac{C'(e)}{C''(e)} > 0. \tag{A59}$$

[28]Details of the derivations can be obtained from the authors on request.

References

Antweiler, W., Copeland, B.R. and M.S. Taylor (2001) "Is free trade good for the environment?," *American Economic Review*, **91**, 877–908.

Atkinson, A.B. and N.H.Stern (1974) "Pigou, taxation and public goods," *Review of Economic Studies*, **41**, 119–128.

Caplan, A.J. and E.C.D. Silva (1999) "Federal acid rain games," *Journal of Urban Economics*, **46**, 25–52.

Chichilinisky, G. (1994) "North-south trade and the global environment," *American Economic Review*, **84**, 851–874.

Copeland, B.R. and M.S. Taylor (1995) "Trade and transboundary pollution," *American Economic Review*, **85**, 716-737.

Cremer, H., and F. Gahvari (2001) "Second-best taxation of emissions and polluting goods," *Journal of Public Economics*, **80**, 169–197.

Cremer, H. and F. Gahvari (2003) "Environmental taxation, tax competition and harmonization," mimeo.

Cremer, H., V. Fourgeaud, M. Leite Monteiro, M. Marchand and P. Pestieau (1996) "Mobility and redistribution: A survey of the literature," *Public Finance*, **51**, 325–352.

Kanbur, R. and M. Keen (1993) "Jeux sans frontières: Tax competition and tax coordination when countries differ in size," *American Economic Review*, **83**, 877–892.

Karp, L., Sacheti, S. and J. Zhao (2001) "Common ground between free-traders and environmentalists," *International Economic Review*, **42**, 617–647.

Keen, M.J. (1989) "Pareto-improving indirect tax harmonisation," *European Economic Review*, **33**, 1–12.

Lockwood, B. (2001) "Tax competition and tax co-ordination under destination and original principles: a synthesis," *Journal of Public Economics*, **81**, 279–319.

Mintz, J. and H. Tulkens (1986) "Commodity tax competition between member states of a federation: equilibrium and efficiency," *Journal of Public Economics*, **29**, 133–172.

Oates, W.E. (2001) "A reconsideration of environmental federalism," *Resources for the Future*, Discussion Paper 01–54.

Silva, E.C.D. and A.J. Caplan (1997) "Transboundary pollution control in federal systems," *Journal of Environmental Economics and Management*, **34**, 173–186.

Ulph, A. (1997) "International trade and the environment: A survey of recent economic analysis," in *The International Yearbook of Environmental and Resource Economics 1997/8*, ed. Folmer, H. and T. Tietenberg, Eds.. Gloucester, England: Edward Elgar, 205–242.