Optimal pricing and price-cap regulation in the postal sector

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Abstract

This paper studies the optimal price structure in the postal sector when worksharing is available (e.g., for collection, sorting and transportation) and when the operator faces a break-even constraint. Users differ in opportunity and cost to engage in worksharing. We determine the optimal worksharing discount and provide sufficient conditions (on demand functions) under which it exceeds the ECPR level. Furthermore, we show that the optimal prices can be implemented through a global price cap imposed on a weighted average of the prices of all products. The appropriate weights are proportional to the market demand (evaluated at optimal prices) of the corresponding products.

JEL classifications: L10; L51; L99

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1 Introduction

The postal network consists of different segments ranging from collection to delivery. We can think of them as activities or inputs in a vertically organized production process. Some postal products like, single-piece letters, rely on the usage of all of these inputs. Other products, however, require only a subset of the potentially available activities. This is true for instance for some industrial mail, either because worksharing has occurred, because it has already been processed by some other operator or simply because some types of clients simply have no demand (no positive willingness to pay) for activities like collection or sorting. These features have important implications for operators and regulators alike. In particular, they are crucial for determining the optimal pricing structure in the industry. Some of the main questions that arise are as follows. How should intermediate goods be priced when they are offered to clients or to competitors? What should be the relationships between discounts and the cost savings associated with the non usage of some segments of the network?

We address these issues by studying the optimal price structure in the postal sector in a setting where users differ in opportunity and cost to engage in worksharing. Furthermore, we show how these optimal prices can be implemented through a global price cap imposed on the historic operator provided that the weights are designed in an appropriate way.

Our model relies on a stylized representation of the postal sector, with two activities (e.g. distribution and a composite activity) and a single operator. There are two types of clients and at least two different products. Households consume single-piece mail which uses the entire network. Business clients may or may not engage in worksharing depending on the price structure.

We start by characterizing the first-best allocation which essentially involves marginal cost pricing. Consequently, differences in prices reflect solely
differences in marginal costs. Then we proceed to a more realistic setting where a fixed network cost has to be covered through the pricing scheme. This problem bears some similarity with a standard Ramsey problem with some added features due to the sector specific modeling of the production process. However, this issue has a much wider scope in the postal sector then in other sectors. In particular, the problem is not confined to an issue of competitors’ access to a bottleneck segment.

The main lesson that emerges is that when the relevant features of the postal sectors are accounted for, regulation through a global price cap appears to be the dominant policy. However, the appropriate design of the formula (and specifically the weights) is crucial. Partial price caps (or price caps involving different “baskets”), on the other hand, do not appear to be justified in the considered setting.

Our model bears some obvious similarity to the recent literature on access pricing in the telecommunications sector; see LaFont and Tirole (2000). As far as the modeling of the postal sector is concerned it also builds on Cremer et al. (1995, 1997 and 2001) and on Crew and Kleindorfer (1992, ch. 2, 3 and 6). The closest predecessors of our analysis are Crew and Kleindorfer (1995) who present a Ramsey pricing approach to worksharing discounts and, more recently, Sherman (2001) who also endogenizes the worksharing decision.\(^1\) However, Sherman considers a simpler setting which separates issues of (choice of) production technology from pricing issues per se. The main addition of our setting is to provide an integrated framework which studies pricing with endogenous technology choice. We also further depart from Sherman’s setting by studying the implementation of optimal pricing

\(^1\)In their “Technical Appendix”, Crew and Kleindorfer (1995) derive optimal first and second best pricing formulas. Formally, our model differs mainly in the cost structure. We also obtain more precise results and further interpretations.
2 Model

The stylized postal network we consider consists of two segments. Segment 1 corresponds to a composite activity including collecting, sorting and transportation. This activity implies a constant marginal cost of $c_1$. Segment 2 is delivery with marginal cost of $c_2$. In addition, there is a fixed cost of $F$. For simplicity we assume that there is a single operator. However, the basic framework we introduce here can easily be adapted to account for the presence of several operators.

There are two types of clients and two goods. Clients of type $h$ (households) consume good $x$ which uses both segments. The marginal cost of $x$ is thus given by $c_1 + c_2$. Clients of type $f$ may or may not use segment 1 of the operator’s network. If they do not use segment 1 they consume good $z$ which implies a marginal cost of $k + c_2$, where $k$ is distributed over $F; k; x$ according to the cumulative distribution $G(k)$ with density $g(k)$. Observe that $c_2$ is the operator’s cost, while $k$ is directly born by the client. Alternatively, they can consume good $x$ for which they pay the same price as households. Without loss of generality we assume that each type of client represents one half of the total population.

Let $S_h(\phi)$ and $S_f(\phi)$ denote the (gross) surplus of the two types of clients as a function of their consumption level. Net surplus is obtained by subtracting total cost: payment to the operator plus cost of activity 1, if applicable.

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2The specification of the access pricing problem in telecommunications which comes closest to our analysis (as far as the formal modeling is concerned) is the one where bypass is possible; see LaFont and Tirole (2000) or Armstrong et al. (1996) for surveys and further references.

3For instance, our formal model does not change if there is a competitive fringe of operators in the upstream market.

4Except for the cost difference $x$ and $z$ are considered as perfect substitutes.

5For simplicity we use surplus as a welfare measure for $s$. From a strict welfare economics point of view, this can be understood as representing the surplus of the consumers who buy the goods produced by $s$ which use postal services as inputs. One can easily show that our shortcut does not involve any loss of generality in the case where all downstream markets are competitive.
cable. We can then define the following demand functions:

\[ x_i(p) = \arg \max_{f \in \mathcal{S}_i} x_i(p) \cdot px; \quad i = h, f; \quad (1a) \]

\[ z(p) = \arg \max_{f \in \mathcal{S}_f} z(p) \cdot pz; \quad (1b) \]

Observe that we have two demand functions for \( x \), one for each type of potential user. Substituting demand functions into net surplus yields the following indirect utility functions:

\[ V_h(p_x) = S_h[x_h(p_x)]; \quad (2a) \]

\[ V_f(p_z; p_x; k) = S_f[z(p_z + k)] \cdot \begin{cases} \frac{1}{2} z(p_z + k) & \text{if } p_z + k \cdot p_x > p_x \\ x_f(p_x) \cdot px & \text{if } p_z + k \geq p_x; \end{cases} \quad (2b) \]

where \( p_x \) and \( p_z \) denote prices. To understand (2b) note that all users of type \( f \) for which \( p_z + k \cdot p_x \) (i.e., when \( k \cdot p_x \geq p_z \)) find it profitable to buy good \( z \) at a level \( z(p_z + k) \). Observe that overall per-unit cost of \( z \) is equal to \( p_z + k \); it is this overall cost rather than just \( p_z \) which determines demand. On the other hand, when \( p_z + k > p_x \), it is cheaper to consume \( x \) (which is otherwise a perfect substitute) and demand is \( x_f(p_x) \).

### 3 First-best optimum

Let us start by considering the first-best allocation, that is the solution which maximizes total surplus (sum of consumer surplus and profits). At this point the operator is not required to break even. We thus implicitly assume that fixed costs can be financed at no efficiency cost through a subsidy financed from the general budget. Such a solution is usually not feasible in practice. Nevertheless it provides us with an interesting benchmark. We consider two formulations of the problem. The first one is direct and intuitive: we optimize with respect to quantities and directly derive the optimal allocation. The second one uses prices as decision variables. It is more complicated in a first-best setting but it will simplify the second-best problem significantly.
3.1 Direct approach

Total surplus can be expressed as follows:

\[ W_1 = S_{x_h}(x_h) (c_1 + c_2) x_h + \]
\[ + \int_{c_1}^{k} [S_{x_f}(z_k)(k + c_2)z_k]g(k) \, dk; \]
\[ + \int_{c_1}^{k} [S_{x_f}(x_f)(c_1 + c_2)x_f]\, g(k) \, dk \, F; \quad (3) \]

where consumers of type \( f \) buy \( z \) when \( k + c_2 \cdot c_1 + c_2 \), i.e. when \( k \cdot c_1 \).

Differentiating (3) with respect to \( x_h, x_f \) and \( z_k \), and rearranging yields the following first-order conditions:

\[ S_{x_h}^0(x_h) = S_{x_f}^0(x_f) = c_1 + c_2; \quad (4a) \]
\[ S_{z_k}^0(z_k) = k + c_2; \quad (4b) \]

Consumer maximizing behavior implies \( S_{x_h}^0(x_h) = S_{x_f}^0(x_f) = p_x \) and \( S_{z_k}^0(z_k) = k + p_z \). Substituting these expressions into (4a)-(4b) yields

\[ p_x = c_1 + c_2; \quad (5a) \]
\[ p_z = c_2; \quad (5b) \]

Expressions (5a)-(5b) do not come as a surprise. They show that the first-best allocation can be decentralized through marginal cost pricing.\(^6\)

This has a number of interesting implications. First, even if we would allow for the price of good \( x \) to differ between \( h \) and \( f \), it would not be desirable (on efficiency grounds) to charge different prices; this is because marginal costs are the same. Second, and most interestingly, the rebate for type \( f \) customers who engage in worksharing (rather than making use of the segment 1 of the operators network) is equal to \( c_1 \), i.e., the marginal cost savings of the operator. This induces efficient worksharing in the sense that all clients with \( k < c_1 \) will not use segment 1 of the operator’s network while all clients with \( k \geq c_1 \) will use this segment. Finally, one can easily verify

\(^6\)Crew and Kleindorfer (1995) have exactly the same result.
that this pricing policy does not allow the operator to break even. More precisely, revenues will cover only marginal cost and the operator will have a deficit corresponding to the level of fixed cost $F$. Consequently, the first-best solution is not feasible if the operator faces a break-even constraint. One then has to adopt a second-best solution where prices are set above marginal cost in order to recover fixed cost. This is studied in Section 4. However, to facilitate the transition to the second-best setting, it is interesting to consider an alternative specification of the first-best problem.

### 3.2 Indirect approach

Alternatively we express total surplus as function of prices (rather than quantities) which then also become our decision variables. The objective function is then given by:

$$\begin{align*}
W_2 &= V_h(p_x) + \int K \frac{Z}{p_x} V_f(p_x; p_z; k) g(k) \, dk \\
&+ [p_x i (c_1 + c_2)] \int X_h(p_x) \\
&+ (p_z i c_2) \int Z (p_z + k) g(k) \, dk \\
&+ [p_x i (c_1 + c_2)] \int X_f(p_x) g(k) \, dk i F:
\end{align*}$$

(6)

Differentiating $W_2$ with respect to $p_x$ and $p_z$ and rearranging yields the marginal cost pricing conditions (5a)–(5b). Not surprisingly, both approaches thus yield the same results. While the direct approach is convenient in a first-best setting it is difficult to handle when a budget constraint is introduced.

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7Recall that marginal costs are constant.
4 Second-best solution

We now turn to the second-best solution which consists in maximizing $W_2$ subject to the operators break even constraint which can be written as:

$$\iota = [p_x i (c_1 + c_2)] x_h (p_x) + (p_x i c_2) z (p_z + k) g(k) dk$$

$$+ [p_x i (c_1 + c_2)] x_f (p_x) g(k) dk i F', 0;$$

Let $L$ be the Lagrangian expression associated with this problem while $\lambda$ is the multiplier of the break-even constraint. We obtain the following first-order conditions:

$$\frac{\partial L}{\partial p_x} = i x_h (p_x) i 1 i G \kappa x_f (p_x)$$

$$+(1 + \lambda) x_h (p_x) + [p_x i (c_1 + c_2)] x_h (p_x)$$

$$+ (1 + \lambda) \sum_{i=1}^{n} (p_x i c_2) z (p_z + k) g \kappa i [p_x i (c_1 + c_2)] x_f (p_x) g \kappa$$

$$+(1 + \lambda) x_f (p_x) 1 i G \kappa$$

$$+(1 + \lambda) [p_x i (c_1 + c_2)] x_f (p_x) 1 i G \kappa = 0$$

(7a)

$$\frac{\partial L}{\partial p_z} = \sum_{k=1}^{\kappa} z (p_z + k) g(k) dk + (1 + \lambda) z (p_z + k) g(k) dk$$

$$+ (1 + \lambda) \sum_{i=1}^{n} (p_z i c_2) z (p_z + k) g \kappa i [p_x i (c_1 + c_2)] x_f (p_x) g \kappa$$

$$+ (1 + \lambda) (p_z i c_2) z0 (p_z + k) g(k) dk = 0;$$

(8)

where $\kappa = p_x i p_z$ is the marginal consumer, i.e., the consumer of type $f$ who is indifferent between buying $z$ and $x$.\(^8\)

4.1 Optimal pricing rules

These conditions can now be used to derive the optimal pricing rules. To facilitate the interpretations it is useful to introduce aggregate (market)

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\(^8\)We assume for simplicity that $\kappa$ is given by an interior solution so that $\kappa < \kappa < \hat{\kappa}$. The possibility of a corner solution can easily be introduced.
demand function for $x$ and $z$:

\[
X(p_x; p_z) = x_h(p_x) + x_f(p_x) \left[ \frac{h}{G} \right] \left[ \frac{k}{g(k)} \right] \Delta k;
\]

(9a)

\[
Z(p_x; p_z) = z(p_z + k) g(k) \Delta k;
\]

(9b)

The derivatives of these demand functions with respect to prices are given by:

\[
\frac{\partial X}{\partial p_x} = x_h(p_x) + x_f(p_x) \left[ \frac{h}{G} \right] \left[ \frac{k}{g(k)} \right] \Delta k;
\]

(10a)

\[
\frac{\partial Z}{\partial p_z} = z(p_z + k) g(k) \Delta k;
\]

(10b)

\[
\frac{\partial X}{\partial p_z} = x_f(p_x) g(k) \Delta k = z(p_z + k) g(k) \Delta k;
\]

(10c)

To understand these expressions, one has to keep in mind that consumers of type $f$ are partitioned between the market areas of the two goods $x$ and $z$, with a marginal consumer at $k$. A variation in a goods own price thus induces adjustments both at the intensive and at the extensive margin. For instance, an increase in $p_x$ affects demand of the $h$ type and the second segment of the $f$ type ($k > k$); this is reflected in the two first terms on the RHS of (10a). The third term measures the demand contraction associated with the shifting of the marginal consumer (increase of $k$). Similarly, the first term on the RHS of (10b) measures the reduction in demand of the first segment of $f$ types (small $k$'s), while the second term reflects the change in the marginal consumer. Observe that cross-price effects are solely associated with effects on the position of the marginal consumer; demand within each segment only depends on the price of the relevant variety of the good. Furthermore, these cross-price effects are symmetric because $x_f(p_x) = z(p_z + k)$, which in turn follows directly from (1a) and (1b). Finally, and not surprisingly, goods $X$ and $Z$ are substitutes in the sense that a price increase for either good, increases demand for the other good.

\footnote{By definition, $p_x + k = p_z$; the two goods (which are otherwise perfect substitutes) are equally costly for the marginal consumer. Consequently his demand is the same whether he demands one good or the other.}
Define the (absolute values of) aggregate demand elasticities:

\[ X = i \frac{\partial X}{\partial p_x} \frac{p_x}{X(p_x; p_z)} \quad \text{and} \quad Z = i \frac{\partial Z}{\partial p_z} \frac{p_z}{Z(p_x; p_z)}, \]  

(11)

and using (9a)-(10c) to simplify (7a)-(8), we obtain the following expressions for the optimal prices:

\[ \frac{p_x}{p_x} - \frac{(c_1 + c_2)}{p_x} = \frac{1}{1 + i} \frac{p_x (c_2)}{p_x} \quad \text{and} \quad \frac{p_z}{p_z} - \frac{c_2}{p_z} = \frac{1}{1 + i} \frac{p_z (c_1 + c_2)}{p_z} \]  

(12a)

\[ \frac{p_x}{p_x} - \frac{(c_1 + c_2)}{p_x} = \frac{1}{1 + i} \frac{p_x (c_2)}{p_x} \quad \text{and} \quad \frac{p_z}{p_z} - \frac{c_2}{p_z} = \frac{1}{1 + i} \frac{p_z (c_1 + c_2)}{p_z} \]  

(12b)

The first term on the RHS of (12a) and (12b) correspond to the simple Ramsey-type inverse elasticity rule. The other terms are more complicated and they reflect the property that demands of x and z are not independent. Consumers of type f may consume x or z and their opportunities for substitution imply that there are cross price effects.

To simplify the expressions further it is useful to introduce some additional notation. Denote the cross-price elasticities by:

\[ X = \frac{p_x (c_1 + c_2)}{p_x} \quad \text{and} \quad Z = \frac{p_z (c_1 + c_2)}{p_z} \]  

(13)

Next introduce the superelasticities, \( X \) and \( Z \) of goods x and z:

\[ X = X \frac{\partial X}{\partial p_x} \frac{p_x}{X(p_x; p_z)} \quad \text{and} \quad Z = Z \frac{\partial Z}{\partial p_z} \frac{p_z}{Z(p_x; p_z)} \]  

(14)

“Superelasticities” are modified elasticities of demand which account for possible substitution or complementarity between goods; see Rohlfs (1979).
and LaFont and Tirole (2000, p. 103). When demand functions are independent, superelasticities are equal to ordinary elasticities. However, when there are cross price effects, superelasticities and ordinary elasticities differ. Most interestingly, using the property that the goods are substitutes, it follows from (11), (13) and (14) that \( ^\prime_X < ^\prime_X \) and \( ^\prime_Z < ^\prime_Z \). Consequently, the superelasticities are smaller than the respective ordinary elasticities.

Rearranging (12a) and (12b) while making use of this notation yields:

\[
\frac{p_x - (c_1 + c_2)}{p_x} = \frac{1}{1 + ^\prime_X} \frac{1}{1 + ^\prime_X} \frac{x''z'' + x''z''x}{x''x''} = \frac{1}{1 + ^\prime_X} \quad \text{(15)}
\]

and

\[
\frac{p_z - c_2}{p_z} = \frac{1}{1 + ^\prime_Z} \frac{x''z'' + z''z''x}{x''x''} = \frac{1}{1 + ^\prime_Z} \quad \text{(16)}
\]

Consequently, once elasticities have been appropriately redefined, we obtain expressions with a familiar Ramsey flavor also for goods \( x \) and \( z \).

### 4.2 Optimal worksharing discounts

We can now use the results of the previous subsection to compare the worksharing discount implied by these pricing rules (i.e., \( p_x - p_z \)) to the operator’s cost savings, \( c_1 \). Using (15) and (16) we obtain:

\[
\frac{p_x - p_z}{p_x} = \frac{1}{1 + ^\prime_X} \frac{p_x}{p_z} = \frac{1}{1 + ^\prime_Z} \quad \text{(17)}
\]

Observe that an application of the ECPR rule would require \( p_x - p_z = c_1 \)\(^{10}\). In words, the discount conceded is equal to the marginal (avoided) cost. Under ECPR, a user’s decision whether or not to engage in worksharing is simply based on the first-best tradeoff. Not surprisingly, equation (17) shows that this rule applies only under special circumstances; in general, one will have \( p_x - p_z > c_1 \). Furthermore, when \( ^\prime_Z > ^\prime_X \), (17) implies that \( p_x - p_z > c_1 \) holds.\(^{11}\) Consequently, if the superelasticity of \( Z \) is larger than

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\(^{10}\) For more details on ECPR see Baumol and Sidak (1994a and 1994b).

\(^{11}\) As long as \( p_x > p_z \), a condition which must necessarily hold to obtain a positive demand for \( Z \).
the superelasticity of $X$ it is optimal to concede a discount which exceeds the operator's cost saving. Observe that while it appears reasonable to assume "$z > "x" this does not necessarily imply "$z > "x". Recall that by their definitions (13) and (14) superelasticities also depend on the cross price effects which can (at least in principle) reverse their ranking.

To overcome this difficulty, we can also assess the worksharing discount by using the more primitive expressions (12a) and (12b). Combining these conditions yields:

$$p_x i p_z i c_1 = \frac{1}{1 + \frac{\mu}{\gamma} \frac{\partial X}{\partial p_z} - \frac{\mu}{\gamma} \frac{\partial Z}{\partial p_z} \frac{\partial X}{\partial p_x} - \frac{\mu}{\gamma} \frac{\partial Z}{\partial p_x}}, 0$$

(18)

where

$$\gamma = 1, \frac{\mu}{\gamma} \frac{\partial X}{\partial p_z} - \frac{\mu}{\gamma} \frac{\partial Z}{\partial p_z} \frac{\partial X}{\partial p_x} - \frac{\mu}{\gamma} \frac{\partial Z}{\partial p_x}$$

Provided that $\gamma$ is positive, which is the case if cross elasticities are lower (in absolute value) than direct elasticities, the discounts should exceed the operator marginal costs savings if

$$p_x \mu \frac{\partial X}{\partial p_z} - \frac{\mu}{\gamma} \frac{\partial Z}{\partial p_z} \frac{\partial X}{\partial p_x} - \frac{\mu}{\gamma} \frac{\partial Z}{\partial p_x}, 0$$

(19)

Using "x < "z and $p_x > p_z$ it follows that a sufficient condition for (19) to hold is that

$$\frac{\partial X}{\partial p_z} = \frac{\partial Z}{\partial p_z} = \frac{\partial X}{\partial p_z}$$

Now because $\frac{\partial X}{\partial p_z} = \frac{\partial Z}{\partial p_z}$ and with $\frac{\partial X}{\partial p_z} < 0$ and $\frac{\partial Z}{\partial p_z} < 0$; this is the case whenever

$$i \frac{\partial X}{\partial p_z} = \frac{\partial Z}{\partial p_z} = 0$$

(20)

To sum up, a sufficient condition for the worksharing discount to exceed the ECPR rule is simply that the slope of the demand for $X$ is smaller (in absolute value) than the slope of the demand for $Z$.12

Stylized evidence

12With "x < "z inequality (20) holds whenever

$$\frac{X}{Z} \frac{p_x}{p_z} < 1$$

11
available on demand in the postal sector suggest that this condition is likely to be satisfied empirically. For instance it holds for price levels and demand functions considered by Mitchel (1999, p.315). Similarly, using data from the French postal sector “back of the envelope” calculations suggest that the slope of the demand for $X$ is about 75% of the slope of the demand for $Z$ (in the relevant price ranges).

To interpret these results and their practical implications, one has to recall that optimal prices are obtained by maximizing total, unweighted, surplus. This means that the regulator cares about efficiency only. If the regulator has redistributive concerns and puts a higher weight on households, the pricing formulas change. One can conjecture that this would result in a lower price for $x$ than in the pure efficiency case. Then, it may become optimal to apply discounts for worksharing which are smaller than the avoided cost $c_1$ (which corresponds to the current practice in the US). To validate this conjecture, a more sophisticated model accounting for distributive objectives is required. We are planning on pursuing this avenue in future research.

In what follows we will use a star to denote the second-best solution derived in this section: $(p_x^*; p_z^*)$.

5 Decentralization and global price caps

So far we have concentrated on the pricing policy that would be chosen by a welfare maximizing (and well-informed) regulator. Let us now examine how this solution can be decentralized through a regulatory policy when the regulator faces a profit-maximizing operator.\(^\text{13}\) In other words, we study how $(p_x^*; p_z^*)$ can be achieved as a solution to the operator's profit maximization problem. It is plain that in the absence of regulation, the (monopoly) operator...
ator will generally not choose the socially optimal policy. Some regulatory intervention is thus necessary to achieve the optimal outcome. The question is then, how "tight" this regulation has to be. Specifically, is it necessary to regulate every single price, or is some more "global" regulation sufficient?

To address these questions, we study the problem of an operator who is subject to a global price cap, i.e., a constraint imposing an upper limit on a weighted average of its prices. Throughout the section we consider price cap formulas under which the weights (of the different prices) are exogenous for the operator.

Using the notation introduced by (9a) and (9b) the operator’s profit can be written as:

\[ \pi (p_x; p_z) = \left[ p_x - (c_1 + c_2) \right] \chi (p_x; p_z) + (p_z - c_2) \zeta (p_x; p_z); \]

(21)

while the global price-cap constraint is given by

\[ \sum_k \omega_k p_k \cdot \hat{p} \]

(22)

where \( \omega_k \) is the weight of good \( k = x; z \).

Let \( L^\omega \) be the Lagrangian expression of the operator’s problem while \( ^1 \) is the multiplier of the constraint (22). The first-order conditions are given by:

\[ \frac{\partial L^\omega}{\partial p_x} = \chi (p_x; p_z) + \left[ p_x - (c_1 + c_2) \right] \frac{\partial \chi (p_x; p_z)}{\partial p_x} + (p_z - c_2) \frac{\partial \zeta (p_x; p_z)}{\partial p_x} \omega_x = 0; \]

(23a)

\[ \frac{\partial L^\omega}{\partial p_z} = \zeta (p_x; p_z) + (p_z - c_2) \frac{\partial \zeta (p_x; p_z)}{\partial p_z} + \left[ p_x - (c_1 + c_2) \right] \frac{\partial \chi (p_x; p_z)}{\partial p_z} \omega_z = 0; \]

(23b)

The decentralization of the second-best solution requires that there exists a value \( ^1 \) of the Lagrange multiplier, such that \( (p_x^\ast; p_z^\ast; ^1 \), solves (23a)–(23b) and (22). For this to be true, the parameters of the price cap formula,

\footnote{Except of course when the maximum achievable profit is equal to zero. In that case, the budget constraint can only be met if profit is maximized. Profit maximization and welfare maximization subject to a break even constraint then yield the same result.}
namely $\zeta_x$, $\zeta_z$ and $p$, must be set appropriately. Comparing (7a)–(8), the expressions determining the second-best solution to (23a)–(23b), while making use of (9a) and (9b) we show that this is the case when

$$
\zeta_x = \frac{X(p_x^x; p_z^z)}{X(p_x^x; p_z^z) + Z(p_x^x; p_z^z)};
$$

and

$$
\zeta_z = \frac{Z(p_x^x; p_z^z)}{X(p_x^x; p_z^z) + Z(p_x^x; p_z^z)};
$$

and

$$
p = \zeta_x p_x^x + \zeta_z p_z^z = \frac{X(p_x^x; p_z^z)}{X(p_x^x; p_z^z) + Z(p_x^x; p_z^z)}p_x^x + \frac{Z(p_x^x; p_z^z)}{X(p_x^x; p_z^z) + Z(p_x^x; p_z^z)}p_z^z.
$$

In words, the appropriate weights are simply equal to the (relative) aggregate demand levels at the second-best solution. Once these weights are determined, one can set $\hat{p}$ such that (22) holds with equality. One readily verifies that the Lagrange multiplier of the price cap constraint is given by

$$
\hat{\rho} = \frac{X(p_x^x; p_z^z) + Z(p_x^x; p_z^z)}{1 + \rho};
$$

where $\rho$ is the Lagrange multiplier of the break-even constraint in the second-best problem.$^{15}$ Observe that there is one degree of freedom in determining the appropriate weights. Prices do not change when $\zeta_x$ and $\zeta_z$ are multiplied by the same (strictly positive) number, provided of course that $\hat{p}$ is adjusted appropriately. We have used this degree of freedom to normalize the weights so that $\zeta_x + \zeta_z = 1$. Alternatively, one could set weights equal to aggregate demand yielding $\zeta_x = X(p_x^x; p_z^z)$ and $\zeta_z = Z(p_x^x; p_z^z)$. In either case, it is important to stress that the weights are constant from the operators perspective. They are based on (second-best) optimal demand levels rather than on actual demand levels.$^{16}$

Finally, recall that the optimal prices which are implemented here are obtained by maximizing the unweighted surplus. As mentioned earlier, pricing

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$^{15}$Evaluated at the second-best optimum.

$^{16}$This is similar to the global price cap policy considered in LaFont and Tirole (1996). Crew and Kleindorfer (1994) have also suggested to include some intermediate (access related) and ..nal services in the same price cap basket.
rules, and thus also the implementing weight would change if distributive objectives were accounted for.\textsuperscript{17}

6 Concluding comments

We have determined the optimal prices of a regulated postal operator when some of his clients may or may not use his entire network. Once elasticities (and superelasticities) are properly redefined to account for the specific structure of demand and technologies, the otherwise complex pricing rules can be rearranged to resemble traditional Ramsey rules. Interpretations then follow along familiar lines. We show that under plausible conditions, worksharing discounts exceed their ECPR level. We also show that optimal prices can be decentralized through a global price cap regulation, provided that its formula is designed properly. In other words, a profit-maximizing operator who is subject to the appropriate price cap constraint will choose the socially optimal prices. This is achieved by imposing a cap on a weighted average of prices where weights are exogenous from the operator's perspective.

Our analysis bears a number of restrictions which will be reconsidered in future research. First, we have only considered the case where the regulator maximizes unweighted surplus. We have conjectured that distributive considerations, may mitigate or reverse some of our results. This is illustrated in a companion paper by considering a regulator's objective function with distributional weights. Second, there is no competition, or at least no imperfect competition in our setting. Clearly this would be an interesting complement.

Last but not least, the explicit introduction of some uncertainty or asymmetric information would make the decentralization issue much more interesting. At this stage, we have not been explicit about the informational

\textsuperscript{17}Distributive objective are considered in a companion paper; see Billette de Villedumeur et al. (2002).
requirements that the implementation of the policies impose on the regu-
lator. This raises two related concerns. The first is the relevance of the
decentralization result. One can indeed argue that the setting of the weights
in the price cap formula requires as much information as the direct control
the prices. This can be overcome by designing a Vogelsang-Finsinger type
mechanism which make explicitly use of passed (accounting) information to
set and adjust the weights— which then eventually converge to the appro-
priate levels.\textsuperscript{18} The second implication is that our argument is not suf-
cient to establish the (strict) dominance of global over partial price caps. We ef-
fectively show that the global price cap is suf cient so that there is no need
to impose tighter controls on prices. However, under perfect information
additional constraints on pricing do not hurt either (as long as they are well
designed of course). The point which is missing in our model is that under
cost or demand uncertainty, giving the operator control of its relative prices
may well proof to be the dominating solution.

\textsuperscript{18}See Vogelsang and Finsinger (1979). These mechanisms and their limitation are also
discussed by Laxont and Tirole (1993, section 2.5.2) and, more recently by Cowan (1998)
References


