Product Specification, Multi-product Screening and Bundling: the Case of Pay TV

by

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November 1999 (revised)

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Abstract

The paper asks how a for-profit cable or satellite operator allocates a fixed channel capacity to different program types and how the different channels are bundled and priced. It also addresses the question how channel allocation and bundling decisions made by a for-profit firm differ from the decisions a welfare-maximizing firm would make. It also examines the effect on profits and welfare of two regulatory constraints that limit the operator's choices in regard to the number of distinct bundles that may be offered to subscribers.

Keywords

television, audiovisual, self-selection, product specification, bundling.

JEL Codes

L 12, L 15, L 82

Acknowledgements

We are grateful to Bruno Jullien and the participants to the microeconomics workshop at the University of Montreal for helpful discussions on an earlier draft of the paper. We also wish to thank, with the usual disclaimers, two anonymous referees and a coeditor of the Journal for their useful comments.

Introduction

Program selection is one of the more extensively studied problems in television. More often than not, analysis is inspired by allegations that programming lacks diversity or claims that the profit incentive induces excessive production of programs with mass-appeal at the expense of features targeted at minority tastes. Steiner (1952), has shown that the total number of channels would likely be larger under competition than under monopoly, but that the latter would yield greater diversity in programming.[1] However, subsequent work by Rothenberg (1962), Beebe (1977), Spence and Owen (1977), Wildman and Owen (1985), Owen and Wildman (1992) has established that Steiner's results are very sensitive to assumptions in regard to viewers' willingness to watch second-to-most preferred programs, to channel capacity, and to the skewness of the distribution of viewers' preferences.[2]

A common feature of the aforementioned models is that variety of content is captured by the number of program categories. More recent work goes a step further as it investigates the interaction between program diversity and the specification of individual programs. Waterman (1992) e.g., examines the outcome of monopolistic competition when individual operators, each supplying one channel, decide on the quality of their programming. Papandrea (1997) presents a model in which viewers have different preferences in regard to program specification and where the size of the audience is determined by specification and price. He concludes that competition does to a greater extent than monopoly duplicate programs with wide appeal.

Program specification is also the main focus of this paper which addresses the following questions: How does a profit-maximizing firm - cable or satellite operator - allocate channel capacity to different program types? Second, what is the composition of the bundles or packages offered to subscribers? Because subscribers do not as a rule pay for individual channels but for a bundle of channels, two questions arise: (a) What is the number of distinct packages or bundles a profit maximizing cable or satellite operator will let subscribers choose from? and, (b) what is the optimal composition of each bundle, i.e., how many channels devoted to each type of programming should the different bundles contain?

Bundling is customarily viewed as a technique used to extract from consumers surplus that cannot be extracted when offering goods separately.[3] A newer strand of literature

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[1] The intuition behind this result is as follows: A monopolist supplying a number of channels will add one channel when its contribution to total advertising revenue exceeds the cost of distributing programs through that channel. A competitive firm by contrast will supply an additional channel when the advertising revenue accruing to that channel is larger than the cost of programming and broadcasting. Also, when deciding whether a new channel should be targeted at a minority taste rather than to the same mass market as existing channels, the competitive entrant is more likely to opt for the mass market than the monopolist. Indeed, from the perspective of the latter, one more product with mass appeal does not enhance the size of the audience. A competitive firm by contrast may find that attracting a slice of the mass market by providing yet another product with mass appeal is more profitable than catering to a an audience whose preferred programs are not fancied by most viewers.

[2] Owen and Wildman (1992) contains an analytical discussion of these models and of related work.

[3] Pure bundling refers to a situation where the package is offered for sale but individual products in the package are not. Mixed bundling describes a situation where consumers are given the option of buying the package as well as the option of buying the individual products that make up the package.

emphasizes the role of bundling as a means to deter entry or induce rivals to compete less aggressively.[4] This paper stresses the traditional role but it goes a step further, throwing a bridge between bundling and product specification. To be sure, the literature on bundling disregards specification issues as it explores under what circumstances selling two products as a single package yields higher profits than selling them separately. Product specification models on the other hand, do not examine how offering a firm the option to bundle affects its decisions in regard to product characteristics. In this paper by contrast, the primary focus is precisely on the interaction between bundling and product specification decisions.

In the specific context of television, bundling has been examined by Chae (1992) who has looked at economies of scope in the distribution technology. He has examined in particular how production and distribution costs jointly determine the firm's decision whether to offer a single channel as opposed to two channels, and, in the latter case whether to bundle the two channels. This paper is different in that decisions in regard of bundling derive solely from consumer preferences. In fact, cost considerations are ignored altogether in order to underscore more sharply the role played by preferences. As well, and in contrast to a literature inclined to assume that viewers have a single most-preferred program type, this paper postulates that the audience consists of individuals who appreciate variety in programming for its own sake. However, members of the audience do differ from each other in regard to the utility they draw from particular combinations of program types.

Section II states the basic assumptions underlying the model. Section III formally presents the problem of a monopolistic cable or satellite operator facing asymmetric information about viewers' preferences. Section IV establishes how the number of bundles and their composition depend on the distribution of consumer preferences. It shows in particular that when distinct bundles are offered, one of them is fully contained in the other. It also shows that when two bundles are offered, prices and bundle compositions are chosen so as to extract the entire surplus from all subscribers. This is in contrast to a standard result that states that some surplus is left to the buyers with the highest willingness to pay.[5] Section V explains why the optimal capacity allocation depends not only on the preferences of the viewers with the highest willingness to pay, but also on the proportion of the audience made up of such viewers. This finding is also in sharp contrast to the standard result according to which the specification of the product targeted at the group with the highest willingness to pay depends only on the preferences of that group. Section VI compares choices made by a for-profit firm with those of a welfare-maximizing firm that may operate under a minimum revenue constraint. It shows that the for-profit firm supplies too little of the type of programming most favored by the "average" viewer. The welfare consequences of regulatory constraints that require the operator to give all subscribers access to all channels or, that prohibit the sale of distinct bundles are ambiguous and Section VII explores how they depend on the distribution of preferences. Concluding remarks are found in section VIII.

The conditions which would make pure or mixed bundling an effective method for such extraction are examined in Stigler (1968), Adams and Yellen (1976), Schmalensee (1984), McAfee et al. (1989) and Salinger (1995).

[4]see Whinston (1990) , Carbajo *et al*. (1990), Chen (1997).

[5]See e.g. Maskin and Riley (1984), Besanko *et al*. (1988), Corts (1995).

II. Notation and basic assumptions

There is a single cable or satellite operator. All programming distributed by that firm belongs to one of the following types: sports and documentaries. The number of available channels, referred to as capacity, is technologically given and is denoted *x*. Each channel is devoted exclusively to sports or documentaries. The operator must determine the number of channels to be dedicated to sports - denoted *s* - and the number of channels dedicated to documentaries - denoted *c*. The distribution and acquisition cost of both sports and documentary programs is zero.[6]

The utility that the viewer indexed q derives from access to s sports and c documentary channels is $V(q\,;s,c) = q\,u(s) + (1-q)u(c)$. The index $q \in (0,1)$ captures the viewer's intensity of preference for sports relative to documentaries. The function $u(.)$ with $u(0) = 0$, is strictly increasing and concave.

Let $s(q\,,x)$ and $c(q\,,x)$ denote the solution to $max_{s,c}\,V(q\,;s,c)$ subject to $s + c \leq x$ and define $F(q\,,x) \equiv V(q\,;s(q\,,x),c(q\,,x))$. The following can now be shown:

Lemma 1: $\quad i)\ \dfrac{\partial s(q\,,x)}{\partial q} > 0$ *for all x and q*

$$ii)\ sign\ \frac{\partial F(q\,,x)}{\partial q} = sign\left[q - \frac{1}{2}\right] \text{ for all } x \text{ and } q$$

Proof: To prove i), differentiate the first order condition $\dfrac{u'(s*)}{u'(c*)} = \dfrac{u'(s*)}{u'(x-s*)} = \dfrac{1-q}{q}$ to yield $\dfrac{\partial s*}{\partial q} = -\dfrac{u'(s*) + u'(x-s*)}{q\,u''(s*) + (1-q)u''(x-s*)} > 0$. To prove part ii) that states that $F(q\,,\mathrm{x})$ attains a minimum at $q = 1/2$ note that from the above first order condition it follows that $s* = c* = x/2$ when $q = 1/2$. Also, because $u'(\,.\,)$>0, one has $\dfrac{\partial^2 F(q,x)}{\partial q^2} = [u'(s*) + u'(x-s*)]\dfrac{\partial s*}{\partial q} > 0$, so that the second order condition for a minimum is satisfied as well.

All members of the audience belong to one or the other of two classes, labeled 1 and 2. Class 1 members value sports programs more than class 2 members, i.e. $q_1 > q_2$. The proportion of viewers belonging to class 1 is denoted a. The firm is assumed to know the parameters q_1, q_2 and a but to have no information about the class to which an individual subscriber belongs. Therefore, it cannot engage in first-order price discrimination. It may, however, offer program combinations – referred to as bundles – which result in self-selection by subscribers. Technically this is achieved by

[6] This is a simplifying assumption that makes it easier to underscore the role of preferences in shaping the profit maximizing solution.

providing subscribers with unscrambling devices encoded to give them access to a particular bundle. [7]

Figures 1a and 1b display the utility derived by class 1 and class 2 viewers from a bundle of size *x*, when the number of sports channels increases and the number of documentary channels falls by an equal amount.[8] Figure *1a* illustrates the case $1/2 > q_1 > q_2$, and Figure *1b* the case $q_1 > 1/2 > q_2$.[9] Shorthand notation for $V(q_i ; s_i, x - s_i)$ is V_i.

[insert Figures 1a and 1b]

When both classes have access to the same bundle, class 1 derives higher (lower) utility from access than class 2 when the bundle contains more (fewer) sports channels than documentary channels. As well, for any bundle of given size, $sign\ d[V_2 - V_1]/ds = sign[q_2 - q_1]$. The implication is that the vertical distance between V_2 and V_1 falls as *s* increases from *0* to *x/2* Also reflected in Figure 1, is the fact that the utility derived from any bundle containing an equal number of sports and documentary channels is independent of q .

The underpinnings of the utility function are as follows: At any point in time, subscribers will watch at most one channel. The benefit from having access to more channels of a particular type - documentaries e.g. - derives from (a) a better chance that at any time some channel will show a documentary program on a topic that the subscriber is interested in; (b) a better chance that at any time, some documentary channel will show a program of sufficient quality to make it worthwhile to set aside couch time for television. Thus the function *u(c)* reflects the probability that a collection of *c* documentary channels will at any time carry at least one program the subscriber enjoys watching. When $s < x/2 < c$, the chances of being supplied with at least one program that meets the required standard, are better for documentaries than for sports. This explains why class 2 members derive a higher utility than class 1 members from *any* bundle having more documentary than sports channels. It explains in particular why, for the particular case $1/2 > q_1 > q_2$, class 2 members derive a higher utility than class 1 members even when the allocation of capacity maximizes the utility of the latter.

Note that $V(q, x/2, x/2) = u(x/2) = F(1/2, x)$ for all x and all q. This can be interpreted as follows: When a bundle provides an equal chance that a sports program and a documentary program will meet the minimum quality standard, access to that bundle yields a level of utility that does not depend on the relative preference for sports or documentaries. Any disparities in willingness to pay for access are then determined solely by the gaps in the preference for sports relative to documentaries.[10]

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[7] The limiting case where a viewer is given an unscrambling device that does not give access to even a single channel, captures an outcome where that viewer does not subscribe to pay-TV.

[8] i.e. $c = x - s$.

[9] The case $q_1 > q_2 > ½$ is not displayed because it is mirror image of Figure 1a.

[10] A possibility one may consider is that the two groups also differ in their preference for television in general. This can be captured by specifying that $V(q_i, s_i, c_i) = q_i u(s_i) + (1 - q_i) u(c_i) - v_i, i = \{1,2\}$, where $v_1 - v_2$ denotes the gap in their appreciation for television in general. The following sections will assume $v_1 = v_2 = 0$. However, care will be taken to point out which results depend on this assumption.

III. Statement of the problem

Profit maximization requires that the service provider decide on the following: (a) How to allocate a given channel capacity across sports and documentary channels; (b) whether to offer a single package to both classes of viewers or, target a different package at each class; (c) when offering two packages, how many sport channels and how many documentary channels to place in each package; (d) how to set the price either of the single package or, of the two packages.

The number of channels devoted to sports and documentaries in the package targeted at class i consumers is denoted s_i and c_i respectively, and the price of the package targeted at viewers belonging to class i is denoted p_i. The operator's objective is to maximize profits subject to several categories of constraints. First, there are participation constraints [shown as (1) and (2) below], which state that consumers in each subscribing class must obtain non-negative surplus[11]

$$q_1 u(s_1) + (1-q_1)u(c_1) \geq p_1 \qquad m_1 \qquad (1)$$

$$q_2 u(s_2) + (1-q_2)u(c_2) \geq p_2 \qquad m_2 \qquad (2)$$

Second, there are self-selection constraints – (3) and (4) below – stating that whenever distinct bundles are offered, consumers in each class must derive a higher surplus from the bundle targeted at them than from the bundle targeted at the other class.

$$q_1 u(s_1) + (1-q_1)u(c_1) - p_1 \geq q_1 u(s_2) + (1-q_1)u(c_2) - p_2 \qquad l_1 \qquad (3)$$

$$q_2 u(s_2) + (1-q_2)u(c_2) - p_2 \geq q_2 u(s_1) + (1-q_2)u(c_1) - p_1 \qquad l_2 \qquad (4)$$

Finally, technical constraints stating that the total number of channels subscribed to cannot exceed the channel capacity [see (9) below] and, that the number of channels of a particular type contained in a bundle, cannot exceed the total number of channels devoted to that type of programming [see (5) – (8) below].

$$s_1 \leq s \qquad g_{s_1} \qquad (5)$$
$$s_2 \leq s \qquad g_{s_2} \qquad (6)$$
$$c_1 \leq c \qquad g_{c_1} \qquad (7)$$
$$c_2 \leq c \qquad y_{c_2} \qquad (8)$$
$$s + c \leq x \qquad b \qquad (9)$$

The problem of the firm is to maximize the profit function $P = a\, p_1 + (1-a)p_2$ with respect to $p_1, p_2, s_1, s_2, c_1, c_2$, s and c, subject to constraints (1) – (9).[12]

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[11] The m 's which appear next to constraints (1) and (2) as well as the l 's, g 's and the b shown next to the constraints (3) – (9), are the Lagrange multipliers used in the proof of the Appendix.

[12] Note that when the two classes do not derive the same utility from a bundle that contains the same number of sports and documentary channels, the participation constraints have a different form. e.g.

This problem presents itself as multi-product screening problem since the utility of each subscriber depends on two variables, s_i and c_i, [13]. It will become clear in sections below that the very specific characteristics of cable and satellite TV, captured by the constraints (5)-(9), facilitate the search for a solution. One specific attribute is a limited number of channels. The latter rules out the possibility of giving both classes access to *x* channels and yet differentiate between bundles. It remains possible though to target a different bundle at each class because the decoders can be programmed to deny access to certain channels.

IV. Bundle composition for a given capacity allocation

Let $\{s, c\}$ denote the allocation of capacity between channel types and let $[s_i^*, c_i^*]$ denote the composition of the optimal bundle targeted at subscriber *i.*[14] Characterization of the maximum is carried out stepwise.[15] The first result (Lemma 2) is that the firm targets a bundle that contains all channels devoted to a specific type of programming at the class of viewers most inclined towards that type. Because $\theta_1 > \theta_2$ this means:

Lemma 2: $s_1^* = s$ *and* $c_2^* = c$

To understand this result suppose that two distinct bundles were offered and that $s_1 < s$. If, the participation constraint of class 1 were binding, then increasing the number of sports channels in the bundle targeted at class 1 by a small amount ds_1, allows the seller to increase the price charged to class 1 by an amount equal to $q_1 u'(s_1) ds_1$. Doing so does not, however, bring about a switch by class 2 subscribers to the bundle targeted at class 1 because the utility class 2 viewers derive from the bundle targeted at class 1 only increases by an amount $q_2 u'(s_1)\, ds_1$, i.e. it increases by a smaller amount than the increase in price of the bundle targeted at class 1. If on the other hand, the participation constraint of class 1 is *not* binding, then its self-selection constraint must be binding and so must the participation constraint of class 2. The implication is that $p_1 = V(q_1; s_1, c_1) - V(q_1; s_2, c_2) + V(q_2, s_2, c_2)$. Again profits are increasing in with s_1 since p_1 increases with s_1 while p_2 remains unaffected. The implication is that a profit maximizing 2-bundle cannot have $s_1 < s$. It is clear as well that $s_1 = s_2 < s$ cannot hold for an optimal 1-bundle offer since increasing $s_1 = s_2$

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when the reservation value of class *i* is $v_i > 0$ (see footnote 10) the participation constraint of class *i* reads $q_i u(s_i) + (1 - q_i) u(c_i) - p_i \geq v_i$ The self-selection and technical constraints remain unchanged.

[13] Generally, the analytical treatment of multidimensional screening problems requires additional assumptions on the structure of preferences, e.g. to reduce it to a one dimensional problem as in Corts (1995) or, to restore the monotonicity of optimal contracts as in Matthews and Moore (1987) or to assume that the agent's preferences satisfy the single-crossing property for each product (Armstrong and Rochet 1999). Our model is based on a particular specification of preferences that follows the Armstrong-Rochet's additively separable definition. Moreover, our hypothesis of a perfect negative correlation between the parameters that characterize preference for the two types of programs *(q, 1-q)* allows to generate simple equilibria.

[14] Accolades are used to denote capacity allocations; square brackets to denote bundle compositions.

[15] Explicit proofs are relegated to the Appendix.

would allow an increase in the price charged to both classes. By similar reasoning one shows $c_2^* = c$.

The second step involves the elimination of outcomes in which *both* groups subscribe to bundles that contain less than the total number of channels devoted to programming of the type towards which they are less inclined than the other group. Formally,

Lemma 3: *i) if* $c_1^* < c$ *then* $s_2^* = s$; *ii) if* $s_2^* < s$ *then* $c_1^* = c$

This Lemma states that if one class is offered fewer than the total number of channels belonging to one type, the other class is offered all the channels belonging to the other type. To understand this result, consider a hypothetical solution where $c_1 < c$ and $s_2 < s$. It is clear that the only reason a profit-maximizing firm would choose to target at class 1 a bundle containing fewer than the total number of documentary channels, is that doing so would allow it to charge a higher price for the bundle targeted at class 2 without inducing a switch by class 2 viewers to the bundle aimed at class 1. However, when $s_2 < s$, there exists a way to increase p_2 that is less costly in terms of forgone profit contribution by class 1 subscribers. It is simply to increase s_2. The upshot is that a solution where $c_1 < c$ will only be chosen when $s_2^* = s$. A similar reasoning shows the second part of the Lemma.

Jointly Lemmas 2 and 3 imply that at least one class will subscribe to a bundle that contains all the channels. The next Lemma shows that when two distinct bundles are offered, the smaller one will contain an equal number of sports and documentary channels.

Lemma 4: *i) if* $c_1^* < c$, *then* $c_1^* = s$ *ii) if* $s_2^* < s$, *then* $s_2^* = c$

Consider part i) and take as a starting point the offer of the bundle [s, c] at the price $V(q_1, s, c)$. If $V(q_2; s, c) < V(q_1; s, c)$ only class 1 subscribes and one never has $c_1 < c$.
If $V(q_2; s, c) > V(q_1; s, c)$ both classes subscribe; the participation constraint of class 1 is binding whereas that of class 2 is not. Examine now the consequences of targeting the same bundle at class 2, while targeting at class 1 a bundle that contains the same number of sports channels but fewer documentary channels. With two bundles on offer, self-selection constraints become relevant. Because class 1 subscribers were initially on their participation constraint, the price of the smaller bundle must be set below the initial price charged for the bundle [s,c] in order to prompt class 1 members to subscribe to the smaller bundle. Specifically, the amount by which price must be lowered is $dp_1 = (1-q_1)u'(c_1)dc_1$.

How do these changes in c_1 and p_1 affect the maximum price that can be charged to class 2? The answer follows from the condition that states that the self-selection constraint of class 2 is binding. The latter implies $dp_2 = [(1-q_1)-(1-q_2)]u'(c_1)dc_1$. Because $q_1 > q_2$, it must follow that a lower c_1 is accompanied by a higher p_2. The effect on profits is $d\Pi = a\, dp_1 + (1-a)dp_2 = (q_v - q_1)u'(c_1)dc_1$, where $q_v \equiv 1-(1-a)(1-q_2)$. A first implication is that lowering c_1 below c - i.e. offering two distinct bundles rather than a single bundle - can only be profit maximizing when $q_v < q_1$. Otherwise profits are higher under a single bundle offer.

Considering the case $q_v < q_1$, there remains the question by how much c_1 should be lower than c. In this regard, note that reducing c_1 while maintaining class 2 subscribers on their self-selection constraint shrinks the surplus of class 2 subscribers because $dp_2/dc_1 < 0$. Indeed, the composition of the bundle they purchase remains unchanged while p_2 increases. The smallest feasible c_1 is attained when the participation constraint of class 2 becomes binding. This happens when $c_1 = s$. Indeed, when the composition of the small bundle is (s, s) and $p_1=u(s)$, both class 1 and class 2 derive zero surplus from subscribing to the smaller bundle.[16] Therefore, the optimal p_2 is that which removes the entire surplus from class 2 members when they subscribe to the bundle $[s, c]$. [17] The proof of part ii) is based on the difference $q_w - q_2$, where $q_w \equiv a q_1$ but otherwise proceeds along identical lines.

From Lemmas 2-4, it follows that for any capacity allocation $\{s, c\}$ the candidate bundles $[s_i^*, c_i^*]$ are:

a) For $c > s$: Two solutions

a1) if $q_v < q_1$ then $s_1^* = s_2^* = s = c_1^* < c_2^* = c$
implying $p_1 = u(s)$ and $p_2 = q_2 u(s) + (1-q_2)u(c)$

a2) if $q_1 \le q_v$ then $s_1^* = s_2^* = s$ and $c_1^* = c_2^* = c$
implying $p_1 = p_2 = q_1 u(s) + (1-q_1)u(c)$

b) For $c < s$: Two solutions

b1) if $q_w > q_2$, then $s_1^* = s > c_1^* = c_2^* = c = s_2^*$
implying $p_2 = u(c)$ and $p_1 = q_1 u(s) + (1-q_1)u(c)$

b2) if $q_2 \ge q_w$, then $s_1^* = s_2^* = s$ and $c_1^* = c_2^* = c$
implying $p_1 = p_2 = q_2 u(s) + (1-q_2)u(c)$

c) For $c=s$: One solution
$s_1^* = s_2^* = s$ and $c_1^* = c_2^* = c$ implying $p_1 = p_2 = u(x/2)$

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[16] Recall that a bundle containing an equal number of sports and documentary channels yields the same utility to both classes

[17] When v_1-v_2 is not zero, it remains true that all surplus is taken from class 2 viewers. However, the way to do it is no longer $c_1 = s$. Now c_1 must satisfy $(q_1 - q_2)[u(s) - u(c_1)] = v_2 - v_1$. The latter is obtained when the participation constraints of both class and the self-selection constraint of class 2 are binding. Note that $s > (<) c_1$ when $v_2 > (<) v_1$.

V. The allocation of channel capacity

The results derived so far are valid for any capacity allocation between sports and documentary channels. Yet to be answered is the question how capacity is allocated optimally among channel types. In this regard, it is obvious that one chooses $s < x/2 < c$ when $q_2 < q_1 < 1/2$ regardless of whether one or two bundles are offered and one sets $s > x/2 > c$ when $1/2 < q_2 < q_1$. The case $q_2 < ½ < q_1$ is less clear because *a priori* the optimal solution could be $s > c$, $s < c$ or $s = c$. The optimal number of sport and documentary channels as a function of q_1, q_2 and a are given below.

Proposition 1

i) *For $q_2 < q_1 < 1/2$, the optimal channel allocation has $s^* < c^* = x - s^*$ with*

$$u'(s^*)/u'(c^*) = \begin{cases} \dfrac{1-q_1}{q_1} & \text{for } q_v \geq q_1 \quad \text{(1-bundle)} \\[2ex] \dfrac{1-q_v}{q_v} & \text{otherwise} \quad \text{(2-bundle)} \end{cases}$$

ii) *For $1/2 < q_2 < q_1$, the optimal channel allocation has $s^* > c^* = x - s^*$ with*

$$u'(s^*)/u'(c^*) = \begin{cases} \dfrac{1-q_2}{q_2} & \text{for } q_w \leq q_2 \quad \text{(1-bundle)} \\[2ex] \dfrac{1-q_w}{q_w} & \text{otherwise} \quad \text{(2-bundle)} \end{cases}$$

iii) *For $q_2 < 1/2 < q_1$, the optimal channel allocation satisfies*

$$\frac{u'(s^*)}{u'(c^*)} = \begin{cases} \dfrac{1-q_w}{q_w} \quad \text{for } q_w > \dfrac{1}{2} & Þ \quad s^* > c^* \text{ (2- bundle)} \\[2ex] 1 \quad \text{for } q_v > \dfrac{1}{2} > q_w & Þ \quad s^* = c^* \text{ (1-bundle)} \\[2ex] \dfrac{1-q_v}{q_v} \quad \text{for } q_v < \dfrac{1}{2} & Þ \quad s^* < c^* \text{ (2-bundle)} \end{cases}$$

Proof : see appendix

To gain an intuitive grasp of Proposition 1, note that for any capacity allocation where $s<c$, the profits derived from a 2-bundle offer are equal to the profits gained from offering the bundle [s, c] to a “virtual” class that includes all viewers and has preference intensity $q_v=1-(1-a)(1-q_2)$. The utility a virtual viewer derives from the bundle [s, x-s] is displayed in Figure 2 for the case $q_v < q_1$ as $V_v = V(q_v, s, x-s)$.

[insert Figure 2]

The condition $q_v < q_1$ insures that $V_v > V_1$ for all $s < x/2$ and, therefore, the profit maximizing channel allocation is that which maximizes V_v. Note that the optimal channel allocation is a function of the preference parameter of class 2 *and* of the proportion of viewers represented by that class; it does not depend on the preference parameter of class 1[18]. By contrast, when $q_1 < q_v$ one has, $V_v < V_1$ for all $s<x/2$ and profit maximization dictates a 1-bundle offer specified to maximize class 1 utility.

Similarly, when $s > c$, the profits from an optimal 2-bundle offer are equal to the profits from offering the bundle [s, c] to the "virtual" class with preference parameter $q_w = aq_1$. A 2-bundle offer is optimal when $q_w > q_2$.

It is useful at this point to contrast the solution obtained here with the solutions that emerge from standard screening models.[19] Two differences are striking. First, that in this model both participation constraints are binding whenever 2 bundles are offered. Second, that a 2-bundle offer always entails distortions at the bottom *and* at the top.

The model presented here differs from the standard screening model in two significant ways: (1) It is multidimensional; (2) the limited channel capacity x adds a technical constraint to the problem. To understand the importance of the latter consider the consequences of removing the technical constraint.[20] With that constraint gone, the optimal solution for $q_2 < q_1 < ½$, is to target at class 2 a bundle that maximizes the utility of that class; i.e. the solution requires that one set the number of sports channels equal to s^2 (see Figure 2) and, that one target at class 1 a bundle that contains a number of sports channels larger than s^1. This solution displays the standard result; i.e. the participation constraint is binding for the class with the lowest willingness to pay and the self-selection constraint is binding for the other class. The latter also enjoys positive surplus.[21]

Now bring back the technical constraint. At this point it is impossible to offer two distinct bundles containing x channels. If one maintained a capacity allocation that maximizes the utility of class 2, - i.e. $\{s^2, x\text{-}s^2\}$ – and made an optimal 2-bundle offer, total profit would be $V(q_v, s^2, x\text{-}s^2)$ (point A in Figure 2). This profit would be the weighted average of $u(s^2)$ i.e. the profit from class 1, and $V(q_2, s^2, x\text{-}s^2)$, i.e. the profit derived from class 2, with weights a and $1\text{-}a$ respectively. Since the function V_2 is horizontal at $s=s^2$ whereas $u(.)$ has positive slope, the weighted average of the slopes of these functions - which is the slope of V_v - is positive at $s=s^2$. This explains why it is optimal to "distort" the bundle targeted at class 2. The distortion is optimally chosen when the allocation of capacity is $\{s^v, x\text{-} s^v\}$, i.e. where the weighted sum of the slopes of V_2 and $u(s)$ is zero.

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[18] It can be noted that V_v converges to V_2 when a tends to zero.

[19] Such as e.g. Maskin and Riley (1984).

[20] One could imagine a cable-distributor who controls two systems, each with x channels and constrained to selling a single system per subscriber.

[21] This is hardly surprising because, for a given capacity totally allocated to a subscriber, the problem is one-dimensional and one has $\partial^2 V / \partial s \partial q > 0$, that is the model satisfies the standard single crossing property.

Thus, profit maximization requires a distortion at the top in order to gain additional profits from the class at the bottom. The bundle targeted at the bottom class is distorted as well since it contains fewer channels than are available on the system.[22]

To gain a clearer picture of the results enunciated by Proposition 1, consider Figures 3.a and 3.b that show the equilibria associated with different a ,q_1 and q_2. Looking at Figure 3.a the following is readily apparent:

[Insert Figures 3a and 3b]

1) When q_1 and q_2 are adjacent (areas I.a, I.b and I.c), a single bundle is optimal.

2) Although the capacity allocation and the number of bundles depend on q_1 and q_2 they are not necessarily affected by small changes in these parameter values. Specifically, neither profits nor specifications depend on q_1 (q_2) in area I.a (I.b). The reason for this "local " insensitivity is that product specification and prices depend only on the preferences of the group whose participation constraint is binding. The horizontal lines shown in area I.a of figure 3.a are therefore iso-program as well as iso-profit lines. Similarly, the vertical lines in areas II and I.b are iso-profit and iso-program lines. Within the area I.c, neither profits nor capacity allocation are affected by small changes in q_1 or q_2 .

3) As the size of the majority group increases, area II grows at the expense of areas I.c and I.a. Similarly, area II shrinks as a tends towards 1/2. The upshot is that the firm is more likely to offer two bundles when the majority is strong and the gap in preferences is large

4) Profits are lowest for parameter values in the area I.c where they are equal to *u(x/2)*. Within area II, profits increase and programming contains more sports as one moves to the right. Similarly, profits increase and content tends towards more sports as one moves upward inside I.a. Programming contains more documentaries and profits increase as one moves to the left inside I.b.

VI. Bundling and the allocation of capacity by a welfare-maximizing firm

How do capacity allocation and bundling decisions by a for-profit firm differ from those made by a welfare-maximizing firm? The problem of the latter is to maximize (10) below, subject to the constraints (5)-(9)

$$W = aV(q_1;s_1,c_1)+(1-a)V(q_2;s_2,c_2) \qquad (10)$$

When the welfare maximizer is under the obligation to generate a minimum amount of revenue -possibly to cover costs- the constraints (1)–(4) and (11) below also apply.

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[22] This policy is not unlike the strategy of differentiation that calls for the creation of a low quality variety, possibly at additional cost, in order to allow in price discrimination. (see Deneckere and MacAfee (1996).

$$a\,p_1 + (1\text{-}a)p_2 \geq K \qquad \text{where } K \geq 0 \qquad (11)$$

It is helpful to start with the case where (11) is not binding. In this case, welfare maximization requires that all viewers be given access to all channels i.e. $s_1= s_2 =s$ and $c_1=c_2=c$ and $s+c=x$. The objective function is therefore $W(s,c)=q_m u(s)+(1-q_m)u(x-s)$, where $q_m = aq_1 + (1-a)q_2$ denotes the average preference of the public for sports. Clearly, the optimal capacity allocation - denoted (s^m, c^m) - must satisfy the first order condition $u'(s^m) / u'(x-s^m) = (1-q_m) / q_m$. Also, $W(s^m, c^m) = V(q_m ; s^m, c^m) = F(q_m , x)$. This implies:

Proposition II

The number of channels that a private monopoly allocates to sports is too large (too small) from a welfare point of view when the average viewer has a weaker (stronger) preference for sports than for documentaries, i.e. when $q_m <(>)½$.

Proof: It is straightforward to show that

$$\min(q_1,q_v) > q_m > \max(q_2,q_w) \qquad (12)$$

Consider the case $q_m<½$, a condition that is always met when $q_2 <q_1 <½$.[23] By virtue of Proposition 1, $q_2 <q_1 <½$, entails $s^* = \min(s^1, s^v)< c^*$. Hence, $s^*> s^m$ by virtue of (12) and Lemma 1(i). The condition $q_m <½$, is also met when $q_2 <½ <q_1$ and a is not too large.[24] Recall from Proposition 1 that when $q_2 <½ <q_1$ the for-profit firm may choose either $s^*<c^*$ or $s^*\geq c^*$. However, $s^*>c^*$ is possible only when $q_w >½$, a condition that cannot hold when $q_m <½$ .If it held, (12) could not be true. Therefore, one must only deal with the case $s^*\leq c^*$ which implies $s^* = \min(x/2, s^v)$. But then, it follows again from (12) and Lemma 1(i) that $s^*> s^m$.

The standard argument is that there will be an excess supply of the type of program preferred by the majority group. In our model such claim mean**s** that the number of sports channels would be too large if $q_1>1/2$ and $a >1/2$. But Proposition II shows that the question whether or not there is oversupply of particular type of program depends only on the sign of q_m -½. This means in particular that when the majority group prefers sports to documentaries the number of sports channel will be *smaller* than optimal when the average viewer also prefers sports.

Examine now the case where (11) is binding and $q_m <½$, i.e. the case where $K>V(q_1 ;s^m, c^m)$.[25] As $q_m < \min(q_1 ,q_v)$, one has $W(s, c) > \max\{V(q_1 ; s, c), V(q_v ; s, c)\}$ for all $s<x/2$ and $s^m < \min(s^1 ,s^v)$.[26] There are two sub-cases to examine.

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[23] The proof for the case $q_m >½$ proceeds along similar lines.

[24] The condition is $a <((1/2)-q_2)/(q_1 -q_2)$.

[25] The case $q_m > ½$ will not be dealt with since it is analyzed in the same way.

Sub-case i) $q_1 < q_v$

Because $V(q_1; s, c) > V(q_v; s, c)$, the 1-bundle offer yields higher revenue for any capacity allocation than any 2-bundle offer based on that same allocation. The implication is that the only adjustment that lets the firm meet the revenue constraint is one that increases the number of sports channels at the expense of documentary channels. Such adjustment lowers welfare. The constrained maximization has a solution only as long as $K \leq F(q_1, x) = V(q_1, s^1, x - s^1)$.[27]

Sub-case ii) $q_1 > q_v$

Figure 4 displays V_1, V_2, V_n and W as functions of s when $c=x-s$. This is the more interesting case because two different adjustments allow the firm to meet the revenue constraint. The first adjustment is the same as above, i.e. a change in channel allocation; the second adjustment is a switch to a 2-bundle regime.[28]

[Insert Figure 4]

Because two adjustments are now possible, the following questions arise : Is one of them always less costly in terms of welfare that must be surrendered to generate the required revenue? If not, how does one determine the combination of adjustments that minimizes the loss in welfare?

Consider first the reallocation of capacity. For any $s<s^1$, welfare forfeited per dollar of increased revenue is $(dW/dK)^a = [q_m u'(s) - (1-q_m)u'(x-s)] / [q_1 u'(s) - (1-q_1)u'(x-s)]$.[29] As to the welfare cost of the switch to a 2-bundle regime where c_1 is reduced while the capacity allocation remains constant, the amount of welfare forgone per dollar of increased revenue is $(dW/dK)^b = a(1-q_1)/(q_v - q_1)$[30]. The latter is independent of s whereas $(dW/dK)^a$ is decreasing in s.[31] For $s = s^v$ where $V(q_v; s, x-s)$ attains a maximum, one has $(dW/dK)^a = (dW/dK)^b$. Also, $sign\,[(dW/dK)^a - (dW/dK)^b] = sign\,(s^v - s)$. Therefore one concludes that:

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[26] Recall that s^v satisfies the condition $u'(s^v) / u'(x- s^v) = (1- q_v) / q_v$ while s^1 satisfies $u'(s^1)/u'(x-s^1) = (1-q_1)/q_1$.

[27] To see the latter, it is sufficient to show that a higher revenue cannot be attained by selling to class 2 alone i.e. that $(1-a)F(q_2, x) < F(q_1,x)$. Defining s^2 = argmax $V(q_2; s, x-s)$ it is easy to show that $F(q_1,x) > V(q_1; s^2, x- s^2) > V(q_v; s^2, x- s^2) = (1-a) V(q_2; s^2, x- s^2) + a\, u(s^2) > (1-a) V(q_2; s^2, x- s^2) = (1-a) F(q_2, x)$.

[28] Recall from the proof of lemma 4 that lowering the number of documentary channels included in the bundle targeted at class 1 increases revenue by increasing the price that can be charged for the large bundle targeted at class 2.

[29] To see this, note that $dK = [q_1 u'(s)-(1-q_1)u'(x-s)]ds$ and that $dW = [q_m u'(s)-(1-q_m)u'(x-s)]ds$.

[30] In this case, because $p_2 = V(q_2; s_2, c_2) - V(q_2; s_1, c_1) + V(q_1; s_1, c_1)$, one has $dK = a\,dp_1 + (1-a)dp_2 = \{a(1-q_1) + (1-a)[(1-q_1)-(1-q_2)]\}u'(c_1)dc_1 = [q_v - q_1]u'(c_1)dc_1$ and $dW = a(1-q_1)u'(c_1)dc_1$.

[31] The sign of $d(dW/dK)^a/ds$ is the same as that of $[q_m u''(s) + (1-q_m)u''(x-s)]dV(q_1; s, x-s)/ds - [q_1 u''(s) + (1-q_1)u''(x-s)]\, dV(q_m; s, x-s)/ds$. As $u'' < 0$, and $dV(q_1; s, x-s) > 0 > dV(q_m; s, x-s)$ for $s \in\,]s^m, s^1[$, one has $d(dW/dK)^a/ds < 0$.

For $K \in \left[V(q_1, s^m, x-s^m), V(q_1, s^v, x-s^v)\right]$ the firm maintains the 1-bundle offer and meets the revenue constraint by increasing the number of channels devoted to sports.

For $K \in \left[V(q_1, s^v, x-s^v), V(q_v, s^v, x-s^v)\right]$ the firm maintains the capacity allocation at $\{s^v, x\text{-}s^v\}$ and lowers c_1 as K increases. When K equals the lower bound of the interval, welfare is $V(q_m; s^v, x\text{-} s^v)$ - point D in Figure 4 – and revenue is $V(q_1; s^v, x\text{-} s^v)$ - point C in Figure 4. As K increases and the firm responds by lowering c_1, welfare falls while revenue increases. When K equals the upper bound of the interval, both welfare and revenue are equal to each other. They are given by the distance between point E in Figure 4 and the horizontal axis. Point E is the solution chosen by the profit- maximizing firm. Finally, note that there is no feasible solution when $K > V(q_v, s^v, x-s^v)$

So far the effects of imposing (11) have been examined only for the case $q_2 < q_2 < ½$. But, as pointed out earlier, $q_m < ½$ may also hold when $q_2 < ½ < q_1$. By virtue of Proposition I, one knows that when $q_2 < ½ < q_1$ the profit maximizing firm may, depending on parameter values, offer a single bundle having $s=c=x/2$, two bundles with $s>c$ if $q_w > ½$ or two bundles with $s<c$ if $q_v < ½$. Note though that by virtue of (12), $q_m < ½$ entails $q_w < ½$. Therefore the argument unfolds as before, except that the choice now is between having two bundles designed to maximize the utility of the virtual viewer q_v and a single bundle with $s=c=x/2$ and yielding $u(x/2)$. Findings are summarized in the following proposition.

Proposition III

When $q_m < ½$ a welfare maximizing firm subject to a minimum revenue requirement K chooses the following capacity allocations and bundle compositions:

a) If $K \le V(q_1, s^m, x\text{-}s^m)$ the capacity allocation is $\{s^m, x\text{-}s^m\}$ and a single bundle containing all channels is offered to both classes.

b) If $K > V(q_1, s^m, x\text{-}s^m)$ there are two cases to consider

If $q_1 \le q_v$ then

for $K \le V(q_1; s^1, x\text{-}s^1)$ capacity allocation is $\{s(q_1, K), x\text{-}s(q_1, K)\}$with $s(q_1, K)=\arg[V(q_1; s, x\text{-}s)=K]$; one bundle containing more than the welfare maximizing number of sports channels is sold to both classes.

for $K > \max[V(q_1; s^1, x\text{-}s^1), u(x/2)]$ there is no solution.

If $q_1 > q_v$ then

for $K \leq V(q_1;\, s^v,\, x\text{-}s^v)$*, capacity allocation is the same as above, i.e.*$\{s(q_1,K),\, x\text{-}\, s(q_1,K)\}$*; one bundle containing more than the welfare maximizing number of sports channels is sold to both classes.*

for $V(q_1;\, s^v,\, x\text{-}s^v) < K \leq V(q_v;\, s^v,\, x\text{-}s^v)$ *the capacity allocation is* $\{s^v,\, x\text{-}s^v\}$*. Two bundles are offered. The large bundle with composition* $[s^v,\, x\text{-}s^v]$ *is targeted at class 2; the small bundle targeted at class 1 has composition* $[s^v,\, c_1(q_v,K)]$ *with* $c_1(q_v,K) = \arg_{c1}[q_1 u(s^v)+(1-a)(1-q_2)[u(c^v)-u(c_1)]=K]$*.*[32]

for $K> V(q_v;s^v,x\text{-}s^v)$ *there is no solution.*

These findings can be set against Chae (1992) where the bundling and pricing choices of a private monopolist are compared to those of a welfare-maximizing firm. Chae assumes a continuum of viewer types and the existence of one or two channels with *a priori* specified content.[33] Although his results are driven primarily by differences in production and distribution costs relative to reservation prices, whereas ours depend solely on parameters of the distribution of preferences across viewer classes, the two papers yield results that, in some regards, can be compared. Chae finds that a welfare maximizer engages in pure bundling, a for-profit firm prefers mixed bundling, and the profit- constrained welfare maximizer may choose either option. We find that the non-constrained welfare maximizer offers a single bundle, i.e. a choice that translates into a single price as for pure bundling. The 2-bundle solution chosen by the profit-maximizing firm for some parameter values entails distinct prices as is the case for mixed bundling. We also find that the revenue-constrained welfare maximizer may offer one or two bundles depending on the severity of the revenue constraint i.e. the solution is, as it were, intermediate between that chosen by a for-profit firm and that chosen by a non-constrained welfare maximizer.

VII. Regulation of the conditions of access

Consider now the effects on profits and welfare of the following regulatory constraints: (1) the obligation to make all channels accessible to both groups; (2) the prohibition to sell a bundle containing fewer channels than the total available on the system. Again we limit the analysis to the particular case where $q_2 < q_1 < 1/2$. To establish a baseline, we examine first how profits and welfare evolve as functions of α in the non-constrained equilibrium. In this regard, we note that for all

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[32] When the participation constraint is binding for class 1 and the self-selection constraint is binding for class 2, the budget equation reads $V(q_1;s_1,c_1)+(1-a)[V(q_2;s_2,c_2)-V(q_2;s_1,c_1)]=K$. As class 2 receives the large bundle [s^v, x-s^v] and class 1 receives the small bundle [s^v, c_1], we obtain the relation in the text.

[33] Chae addresses the following questions: (1) How is the choice between pure component pricing (i.e each channel is priced separately) and bundled pricing affected by production and distribution cost? (2) How do costs determine whether the firm provides programming for both channels, for one channel only (if so for which channel) or, no programming at all? In this paper by contrast, there are only two viewer classes but the firm can choose a capacity allocation from a continuum of possibilities. In terms of Chae's model, this comes close to an endogenization of channel content.

$a \in [(q_1 - q_2)/(1-q_2), 1]$, profit maximization dictates that a single bundle be sold to all consumers at the price $F(q_1, x)$.[34] Whereas profits remain constant for all a 's in that interval, welfare declines with a because it includes the surplus of class 2 subscribers – equal to $V[q_2; s^1, c^1] - F(q_1, x)$. Figure 5 displays profits and welfare for $a \in [(q_1 - q_2)/(1-q_2), 1]$ as segments BA and CA.

[Insert Figure 5]

For $a \in (0, (q_1 - q_2)/(1-q_2))$, the for-profit firm offers two bundles and sets prices that leave no surplus to either class 1 or class 2 subscribers.[35] Hence profits coincide with welfare. They are given by $F(q_v, x)$ and shown as DB in Figure 5. Lemma 1 and $\partial q_v / \partial a > 0$ imply that DB is downward sloping.[36] The discontinuity in the welfare function at $a = (q_1 - q_2)/(1-q_2)$ is attributable to the fact that the size of the bundle targeted at class 1 suddenly drops at this point.

Regulation 1: All viewers must have access to all channels

Imposing the obligation to grant all subscribers access to all channels is tantamount to setting a price ceiling $F(q_1, x)$ when $q_2 < q_1 < 1/2$, except if a =0. Profits are also $F(q_1, x)$ and are shown as AH in Figure 5. It is obvious from Figure 5 that profits are not affected by the constraint when a is sufficiently large; they are lowered by it when a is small.

Under the regulation, welfare is $aF(q_1, x) + (1-a)V(q_2; s^1, c^1)$ and is shown as segment GA in Figure 5. Because welfare tends to $V(q_2; s^1, c^1) < F(q_2, x)$ as α tends to zero, it must be true that DB and GC intersect at $a > 0$. The implication is that imposing the regulation is welfare increasing or decreasing depending on the proportion of viewers in each class. The regulation is more likely to increase welfare when the proportion of viewers who absent the constraint would subscribe to the smaller bundle is larger.

Regulation 2: No bundle may contain fewer channels than is available on the system.

Under this regulation, the question is whether the firm is better off selling to both groups and at a price $p_1 = F(q_1, x)$ the bundle specified to maximize the utility of class 1 viewers, or whether the firm does better by selling to class 2 only and at a price $p_2 = F(q_2, x)$ the bundle specified to maximize class 2 utility. Figure 6 displays the profits from of the two options as segments HA [profits $F(q_1, x)$] and DR [profits $(1-a)F(q_2, x)$] respectively.

[Insert Figure 6]

[34] See area Ib in figures 3a and 3b.
[35] We are in area III in figure3b.
[36] Recall that $q_v \equiv 1 - (1-a)(1-q_2)$

The former (latter) yields higher profits for $a > (<) 1-[F(q_1,x)/F(q_2,x)]$. It is evident from Figure 6 that profits are strictly lower under the regulation when $a \in (0, (q_1 - q_2)/(1-q_2))$, and that they are unchanged for α outside that interval.
In regard to welfare, the impact of the regulation is ambiguous. For $a < 1-[F(q_1,x)/F(q_2,x)]$ welfare equals profits when the constraint is binding and, profits themselves are lower than in the absence of regulation. Imposing the regulation must therefore be welfare decreasing.[37]
For $a \in [1 - F(q_1)/F(q_2), (q_1 - q_2)/(1-q_2)]$ welfare under the regulation is given by JC in Figure 6. It is higher than welfare in the absence of regulation, given by SB.

The reason regulation may either increase or decrease welfare is as follows: For a slightly smaller than $(q_1 - q_2)/(1-q_2)$, the bundle offered under regulation to class 2 is only faintly different from the bundle targeted at that class in the absence of regulation. Hence, the effect of regulation is essentially to force the operator to set a price at which group 1 purchases the larger bundle. This adjustment is welfare increasing. However, a lower price becomes more costly when a becomes larger. This explains why the firm prefers to target only class 2 as soon as a drops below $1-[F(q_1,x)/F(q_2,x)]$. For smaller a, imposing the constraint affects welfare in two ways: (i) it changes channel allocation; (2) it excludes class 1which in the absence of the constraint purchases the smaller bundle.[38]

VIII. Conclusion

This paper has examined pricing, capacity allocation and bundle composition decisions by a monopolistic cable or satellite operator. Using a model that assumes two classes of viewers and two programming types, it has established that whenever two distinct bundles are offered to subscribers, one bundle will contain all the channels supplied by the distributor. The implication is that a subset of channels will be made accessible to all subscribers. This is in conformity with the practice under which a basic service is paid for by all subscribers, while additional channels are purchased by some subscribers. A second result is that a two-bundle offer is more likely to yield higher profits than a single bundle offer when: (1) the disparity in consumer preferences is larger; (2) the size of the group having the strongest preference for the type of programming favored by the average consumer is larger relative to the size of the other group. A third finding is that when two bundles are offered, prices and bundle compositions are chosen so as to remove all consumer surplus from subscribers. This is in contrast to the standard result that has surplus left to those with the highest willingness to pay. Also in contrast to that literature, the

[37] Recall that in the latter case welfare is given by DS.
[38] The constraints investigated in this section apply to the conditions of access and the number of bundles offered. This is in contrast to Chae (1992) who looks at the consequences of imposing either a pure-bundling or a pure-components restriction on prices. Chae finds that the pure bundling restriction -which is close to our regulation 2 because it forces the firm to become a single product producer - can be welfare decreasing or increasing. In his model, the bundling restriction increases total surplus unless it induces the monopolist to produce fewer channels. In Chae, welfare and consumer surplus decrease when the pure bundling restriction induces the firm to produce fewer channels. In our set-up welfare falls when the constraint induces the firm to withdraw from making an offer to one class of viewers.

paper finds that the specification of the product sold to the group with the highest willingness to pay is not chosen to maximize the utility of that group. It is chosen to maximize a utility that depends on the preferences of that group and on the group's share of the total audience.

The paper also addresses the question how the choices made by a for-profit firm depart from those made by a welfare maximizer. It finds that whenever the average viewer has a stronger preference towards a particular type of program, a profit-maximizing monopolist allocates fewer channels to that type than is optimal. The number of channels allocated to the type preferred by the majority group will not be excessive unless the average viewer has stronger preference for programs of the other type. In this sense, the model presented in this paper fails to lend support for the claim that private markets lead to insufficient programming targeted at minority tastes.

The paper has also established that a welfare maximizer subject to a mildly restrictive revenue constraint will offer a bundle containing all channels to all viewers. However, the channel allocation to a particular content will differ from that chosen by a welfare-maximizing firm that is not subject to such constraint. As the minimum required revenue increases, the channel allocation chosen by the constrained welfare-maximizing firm comes closer to the allocation that maximizes the utility of the class with the lowest willingness to pay. As the constraint becomes even more restrictive, a threshold may be crossed where the welfare maximizer switches to a 2-bundle regime. If such point is reached, further tightening of the constraint is met by reducing the size of the smaller bundle. This increases revenue but lowers welfare. The switch to a 2-bundle offer occurs only when the distribution of preference parameters is such as to induce a for-profit firm to offer two bundles. When this condition is satisfied, the allocation of capacity by the welfare-maximizing firm that offers two bundles is the same as that of the for-profit firm.

The effects of two regulatory constraints have also been explored. First the obligation to give all potential subscribers access to all channels. The welfare effect of such requirement is ambiguous. It does not affect welfare when parameter values dictate to an unregulated firm to offer a single bundle. It increases welfare if absent the regulation, preferences and relative group sizes dictate a channel allocation only slightly different from that chosen by the regulated firm. However, when the group with the highest willingness to pay for a particular type of program is a strong majority, imposing the constraint is welfare decreasing. The second constraint this paper has examined is a prohibition to sell bundles that include less than the total number of channels on the system. The paper has shown that such prohibition increases welfare when it brings about a switch from a two-bundle offer to a single bundle offer targeted at all potential subscribers. However, this comes about only when the group that would have purchased the larger bundle in the absence of the constraint is a relatively weak majority. However, as the proportion of viewers belonging to that group grows larger, it becomes less costly to satisfy the constraint by limiting sales to the group with the highest willingness to pay. If so, imposing the constraint lowers welfare.

Appendix

Since the objective function of the monopolist is concave and the feasible set convex, first order conditions describe a global maximum. With L denoting the Lagrange function, these conditions read

$$\frac{\partial L}{\partial p_1} = a - m_1 - l_1 + l_2 = 0 \tag{A1}$$

$$\frac{\partial L}{\partial p_2} = (1-a) - m_2 + l_1 - l_2 = 0 \tag{A2}$$

$$\frac{\partial L}{\partial s_1} = u'(s_1)[q_1(l_1 + m_1) - l_2 q_2] - g_{s_1} = 0 \tag{A3}$$

$$\frac{\partial L}{\partial c_1} = u'(c_1)[(1-q_1)(l_1 + m_1) - l_2(1-q_2)] - g_{c_1} = 0 \tag{A4}$$

$$\frac{\partial L}{\partial s_2} = u'(s_2)[q_2(l_2 + m_2) - l_1 q_1] - g_{s_2} = 0 \tag{A5}$$

$$\frac{\partial L}{\partial c_2} = u'(c_2)[(1-q_2)(l_2 + m_2) - l_1(1-q_1)] - g_{c_2} = 0 \tag{A6}$$

$$\frac{\partial L}{\partial s} = g_{s_1} + g_{s_2} - b = 0 \quad \text{and} \quad \frac{\partial L}{\partial c} = g_{c_1} + g_{c_2} - b = 0 \tag{A7}$$

plus the constraints (1)-(9) in the text.[39]

Lemma 2: Under profit maximization, $s_1^* = s$ and $c_2^* = c$

Proof: Note first that $s_1^* < s$ implies $g_{s_1} = 0$ which, by virtue of (A3) requires $q_1(l_1 + m_1) - l_2 q_2 = 0$. This implies $q_2(l_1 + m_1) - l_2 q_2 < 0$ since $q_1 > q_2$, contradicting (A1) since $a > 0$. Hence, it must be true that $s_1^* = s$. A similar series of steps establishes $c_2^* = c$.

Lemma 3: i) if $c_1^* < c$ then $s_2^* = s$; ii) if $s_2^* < s$ then $c_1^* = c$

Proof: If both inequalities held, (A4) and (A5) would imply $(1-q_1)(l_1 + m_1) = l_2(1-q_2)$ and $q_2(m_2 + l_2) = l_1 q_1$. In view of (A1) and (A2) these equalities could be rewritten $(1-q_1)(a + l_2) = l_2(1-q_2)$ and $q_2(1-a+l_1) = l_1 q_1$. Since the latter would entail $l_1 > 0$ and $l_2 > 0$, conditions (3) and (4) in the text would have to be binding. But then, summation of (3) and (4), and Lemma 2 would imply $(q_1 - q_2)\{[u(s) - u(s_2^*)] + [u(c) - u(c_1^*)]\}$=0. This equality, however, cannot be true for both $s_2^* < s$ and $c_1^* < c$, since $q_1 > q_2$.

Lemma 4: i) if $c_1^* < c$, then $c_1^* = s$; ii) if $s_2^* < s$, then $s_2^* = c$

Proof: Consider part (ii).[40] When $s_2^* < s$, one has $g_{s_2} = 0$. By virtue of (A5), we obtain $q_2(l_2 + m_2) = l_1 q_1$ which entails that one of the following must hold:

[39] Also, each multiplier is non-negative and the product of each constraint with its multiplier is zero.

[40] The proof of part (i) involves going through the same steps as for part (ii) and will not be shown.

(i) $l_1 > 0$ and $l_2 > 0$ and/or $m_2 > 0$

(ii) $l_1 = l_2 = m_2 = 0$.

Since (A2) rules out (ii), one can immediately turn to the implications of (i). In regard of (i), note that when $l_1 > 0$, then (3) in the text, by virtue of Lemma's 2 and 3, yields
$$q_1[u(s) - u(s_2)] = p_1 - p_2 \qquad \text{(A8)}$$

Now suppose $l_2 > 0$, a condition which in view of (4) and by virtue of Lemma's 2 and 3 entails
$$q_2[u(s) - u(s_2)] = p_1 - p_2 \qquad \text{(A9)}$$

Clearly, (A8) and (A9) contradict each other. The implication is that $l_1 > 0$ must entail $l_2 = 0$ and $m_2 > 0$. But, when $m_2 > 0$ holds, condition (2) reads
$$p_2 = q_2 u(s_2) + (1 - q_2)u(c) \qquad \text{(A10)}$$
Note, that upon use of (A10), condition (A8) can be rewritten
$$p_1 \quad q\ u(s_2) + (1 - q_2)u(c) \quad q\ [u(s) - u(s_2)] \qquad \text{(A11)}$$
and (A10) and (A11) jointly imply
$$\frac{d\Pi}{ds_2} = a\frac{dp_1}{ds_2} + (1-a)\frac{dp_2}{ds_2} = [q_2 - aq_1]u'(s_2)$$
The latter indicates that s_2 is to be chosen either at the upper bound or at the lower bound of its feasible interval, depending on the sign of $q_2 - aq_1$. When $q_2 > aq_1$, profit maximization dictates that s_2 be as large as possible, i.e. $s_2^* = s$. It dictates that s_2 be as small as possible when $q_2 < aq_1$. Since conditions (A11), (1) in the text and the inequality $q_1 > q_2$ imply $s_2 \geq c$, it follows that $s_2^* = c$ when $q_2 < aq_1$.[41]

Proof of Proposition 1: The proof contains two parts. In the first part it is shown that any optimally chosen {s*, c*} satisfies.
$$\frac{u'(s^*)}{u'(c^*)} = \frac{1 - m_1 q_1 - m_2 q_2}{m_1 q_1 + m_2 q_2} \qquad \text{(A12)}$$
and
$$m_1 + m_2 = 1 \qquad \text{(A13)}$$

In the second part, this result is used to determine the allocations of capacity associated with different values of parameters a ,q_1 and q_2.

a) Note first that (A13) follows from the summation of (A1) and (A2). In regard of (A12) note that (A7) entails $g_{s_1} + g_{s_2} = g_{c_1} + g_{c_2}$. Therefore, summation of (A3) with (A5) and summation of (A4) with (A6) yield

$$\begin{aligned} &u'(s)[q_1(l_1 + m_1) - l_2 q_2] + u'(s_2)[q_2(l_2 + m_2) - l_1 q_1] \\ &= u'(c_1)[(1-q_1)(l_1 + m_1) - l_2(1-q_2)] + u'(c)[(1-q_2)(l_2 + m_2) - l_1(1-q_1)] \quad \text{(A14)} \end{aligned}$$

[41] Clearly, this outcome is possible only when $c < s$.

Consider now the following two possibilities: i) $s_2^* = s$, and ii) $s_2^* < s$. When $s_2^* = s$, the left-hand-side of (A14) can be rewritten as $u'(s)[m_1q_1 + m_2q_2]$. When $s_2^* < s$, one has $g_{s_2} = 0$ and, since the latter implies $q_2(l_2 + m_2) - l_1q_1 = 0$ one obtains $u'(s)[q_1(l_1 + m_1) - l_2q_2] = u'(s)[m_1q_1 + m_2q_2]$. Thus, the left-hand-side of (A14) is the same regardless of whether s_2^* is smaller or equal to s. By following a series of similar steps one shows that the right-hand-side of (A14) is always equal to $u'(c)[1 - m_1q_1 - m_2q_2]$. This completes the proof of (A12).

b) Consider now the case $q_2 < q_1 < 1/2$, one has $m_1q_1 + m_2q_2 < \frac{1}{2}$ by virtue of (A13). Since $u'' < 0$, the latter implies $s^* < c^*$ [see (A12)]. The question to be answered now is under what conditions the latter entails an offer of two bundles. Because the optimal capacity allocation entails more documentary than sports channels, it is the sign of $q_v - q_1$ that determines the number of bundles.[42] When $q_v \geq q_1$ a single bundle is offered and therefore, profits are maximized by choosing the allocation that yields the highest willingness to pay by the class having the lower utility whenever $s < c$, i.e. class 1. The optimal allocation $(s^1, x- s^1)$ must satisfy the first order condition $q_1 u'(s)-(1-q_1)u'(x-s)=0$.[43] By contrast, when $q_v < q_1$, profit maximization entails a 2-bundle offer with profits $\Pi = a\, p_1+(1-a)p_2=[1-(1-a)(1-q_2)]u(s)+(1-a)(1-q_2)u(c)$. The optimal allocation $(s^v, x- s^v)$ is given by the first order condition $[1-(1-a)(1-q_2)]u'(s)- (1-a)(1-q_2)u'(x-s) = 0$. Since $q_v =1-(1-a)(1-q_2)$, this completes the proof of part i) of proposition 1. The proof of part ii) proceeds along similar lines.

Consider now part iii) of the proposition. Since V_1 is an increasing function of s for $s < x/2$ and V_2 is a decreasing function of s for $s > x/2$ (see figure 1b), it must be true that an optimal 1-bundle offer has $s = c = x/2$. The question is how the profits yielded by the latter bundle, i.e. $u(x/2)=F(½, x)$ compares to the profits derived from an optimal 2-bundle offer. Look first at the candidate 2-bundle offer where $s<c$. The highest profits one can derive from such 2-bundle offer is $F(q_v, x)$ with $q_v =1-(1-a)(1-q_2)$ and lemma 1 has established that $F(q_v, x) – F(½, x) > 0$ when $q_v < ½$.. When the optimal 2-bundle yields the higher profits, the profit function has the same form as for the case $q_2 < q_1 < 1/2$. Therefore, the allocation of capacity must satisfy the same first order condition as for the 2-bundle offer in part i). Look now at a candidate 2-bundle solution where $s > c$. By virtue of Lemma 1, profits from this 2-bundle offer are higher than those from a 1- bundle offer when $q_w > ½$. When this condition is met, the allocation of channels capacity must satisfy the same first order condition as under the 2-bundle offer of part ii) of the proposition.

[42] See summary of candidate solutions at the end of section IV.

[43] It should be noted for further reference that when $a > ½$ a single bundle is offered for all $q_2 < q_1 < 1/2$. Similarly, $a < ½$ insures that a single bundle is offered for all $1/2 < q_2 < q_1$.

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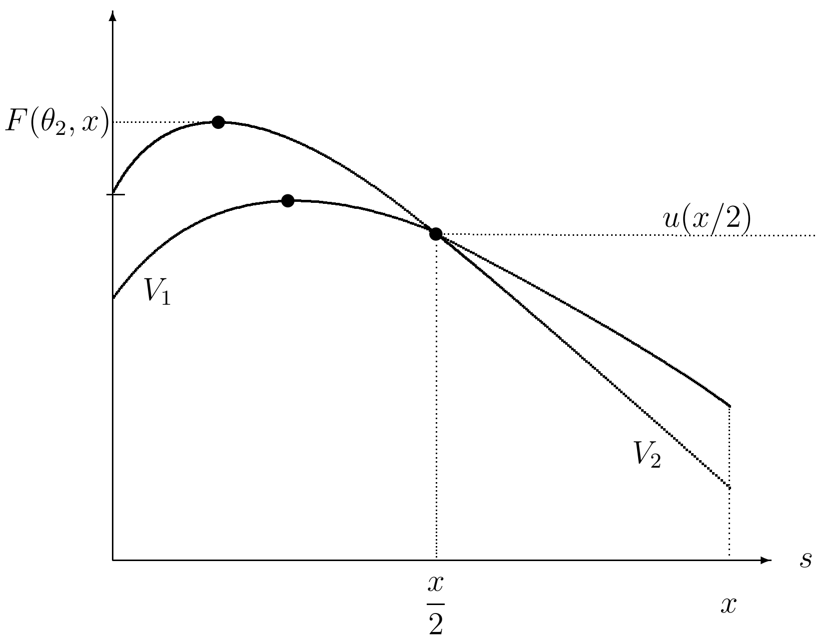

Figure $\theta_2 < \theta_1 < \frac{1}{2}$

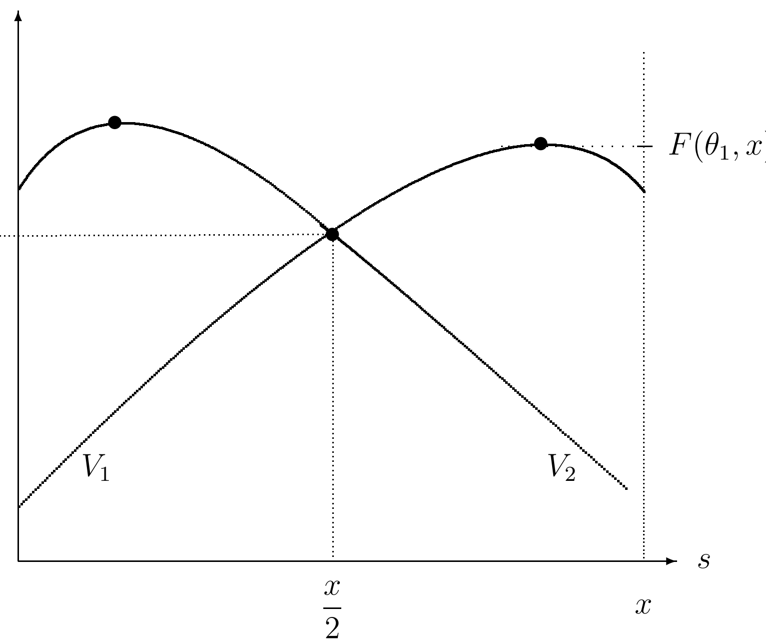

Figure $\theta_2 < \frac{1}{2} < \theta_1$

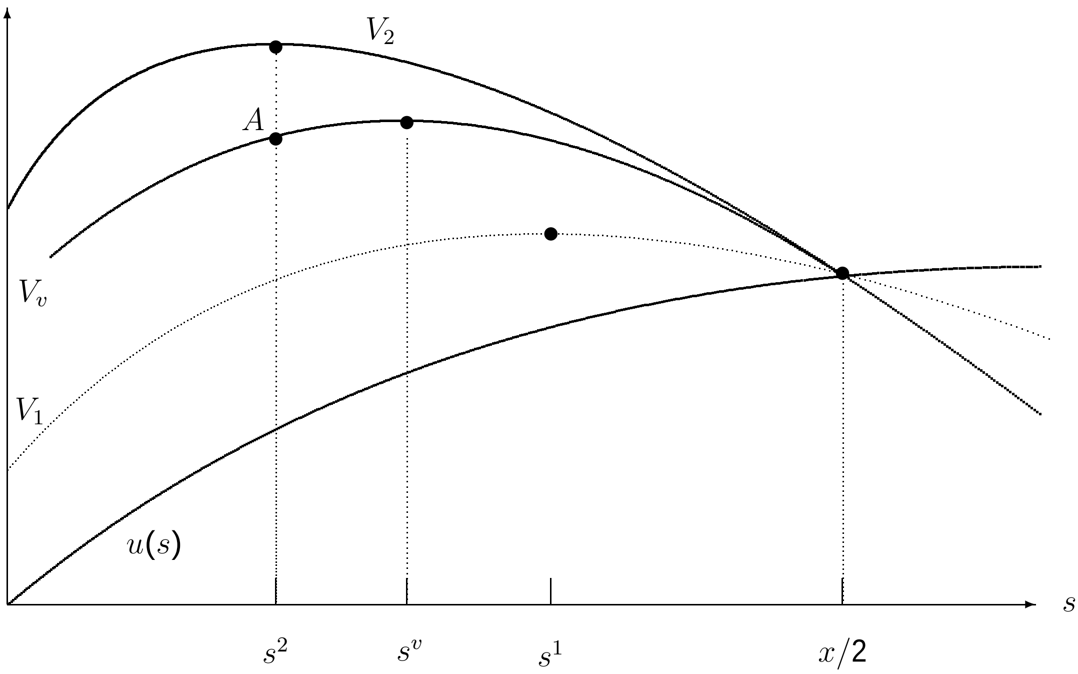

Figure 2: Actual and virtual viewers

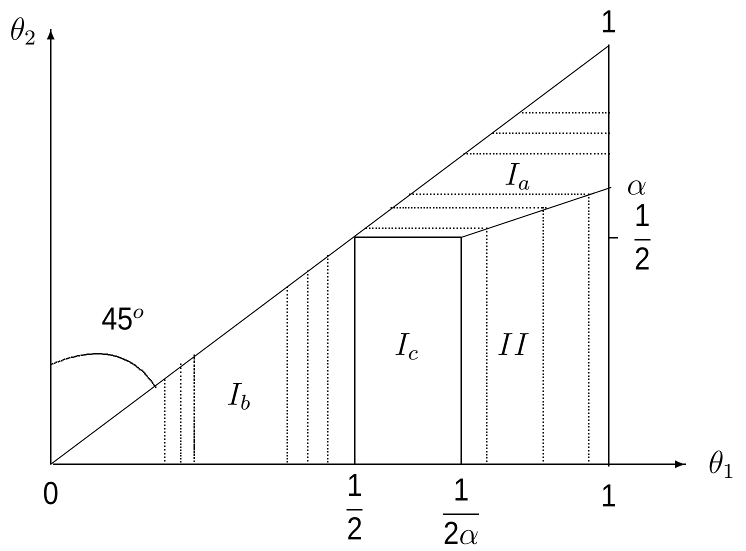

Figure 3a : Equilibrium configurations for $\alpha > 1/2$

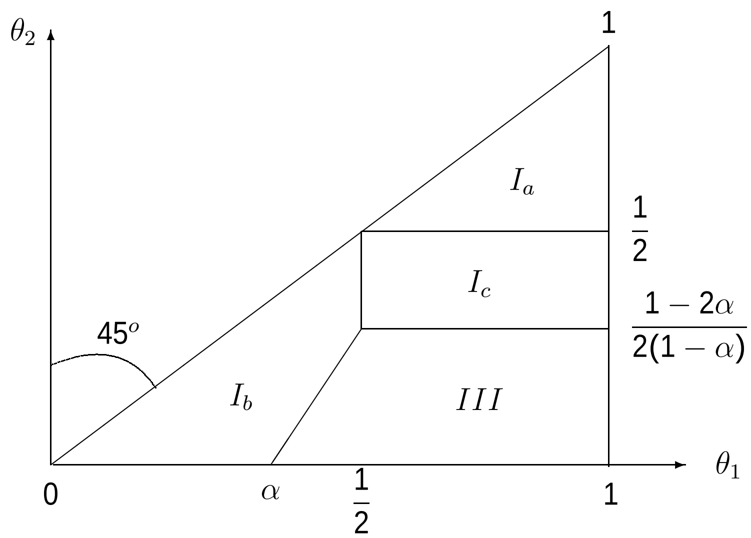

Figure 3b : Equilibrium configurations for $\alpha < 1/2$

I_a : $s_1 = s_2 = s > x/2 > c = c_1 = c_2$ and $p_1 = p_2 = \theta_2 u(s) + (1-\theta_2)u(c)$

I_b : $s_1 = s_2 = s < x/2 < c = c_1 = c_2$ and $p_1 = p_2 = \theta_1 u(s) + (1-\theta_1)u(c)$

I_c : $s_1 = s_2 = s = x/2 = c = c_1 = c_2$ and $p_1 = p_2 = u(x/2)$

II : $s_1 = s > x/2 > c = c_1 = c_2 = s_2$ and $p_1 = \theta_1 u(s) + (1-\theta_1)u(c)$ and $p_2 = u(c)$

III : $c_1 = s_1 = s_2 = s < x/2 < c = c_2$ and $p_1 = u(s)$ and $p_2 = \theta_2 u(s) + (1-\theta_2)u(c)$

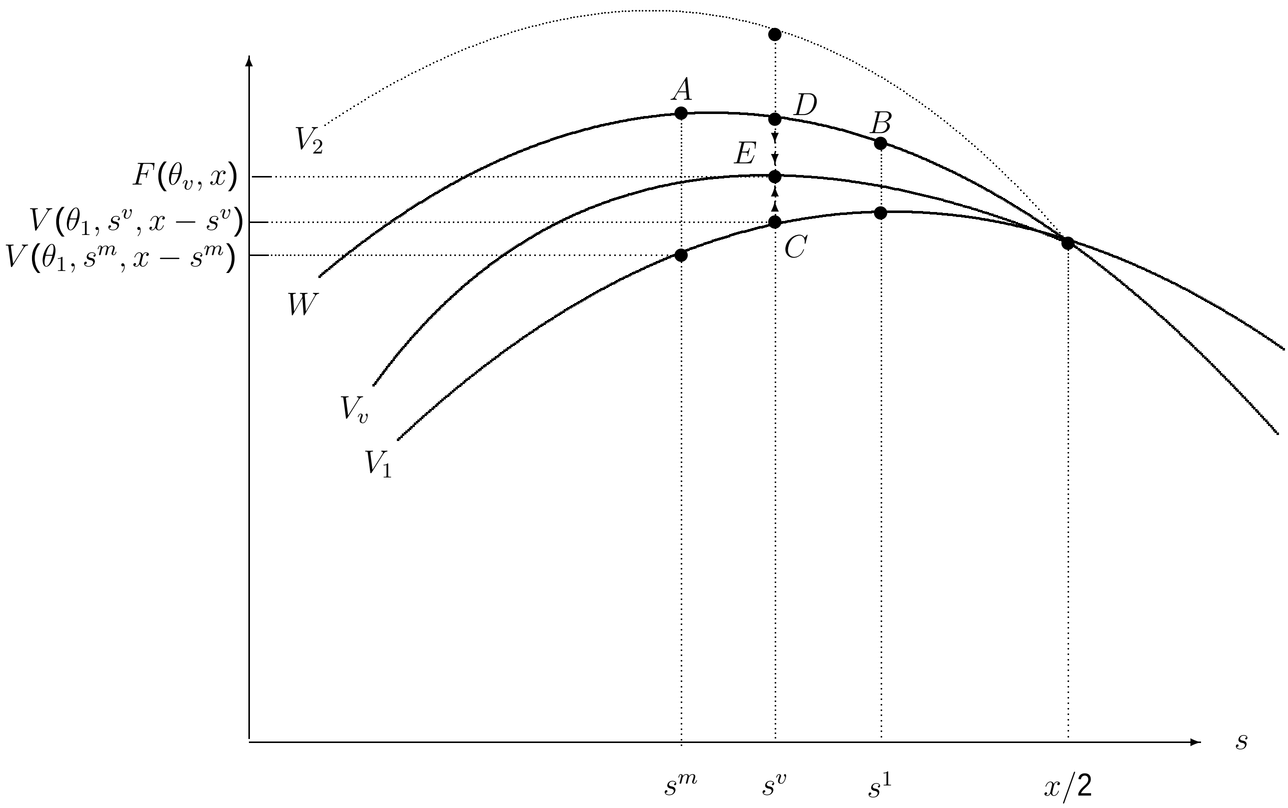

Figure 4: The effect of a budget constraint on welfare

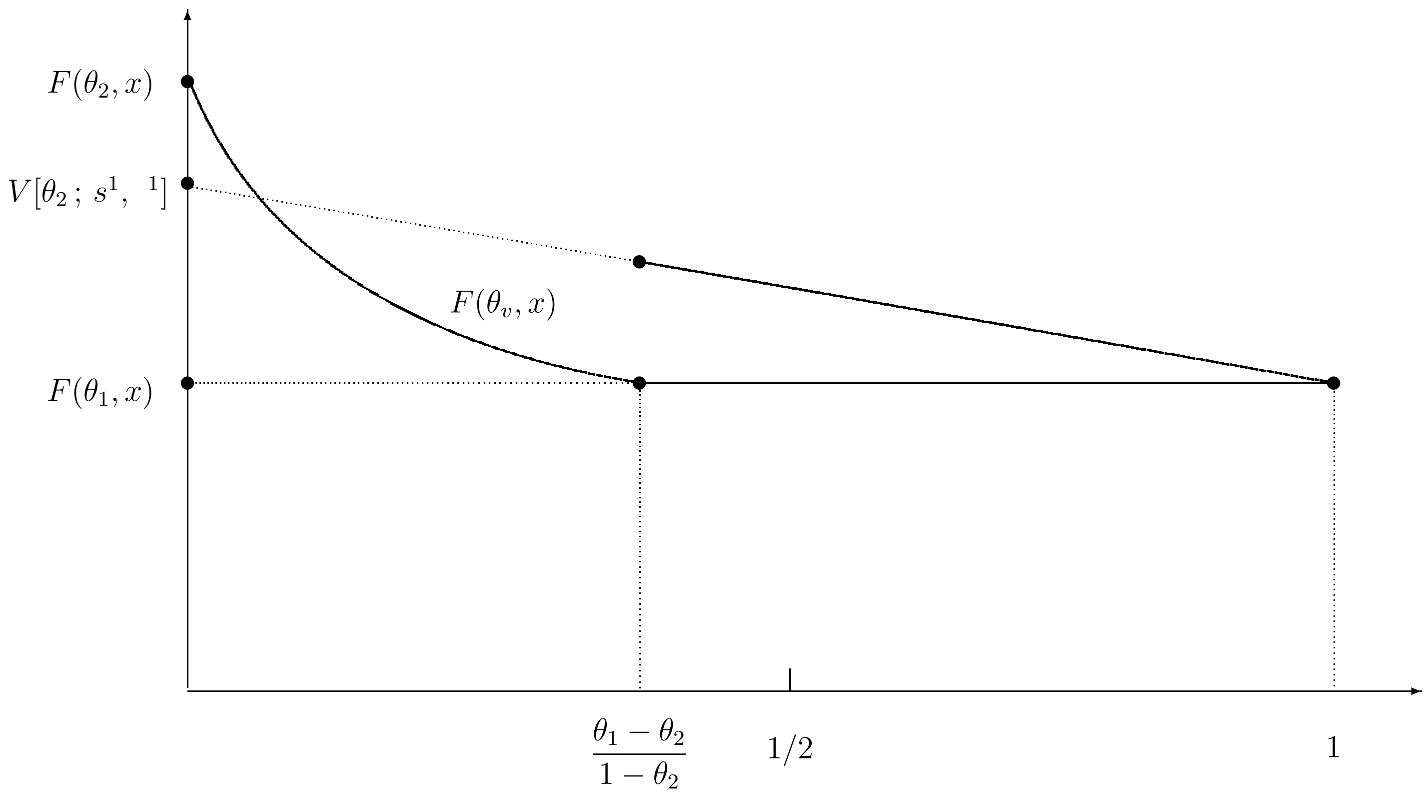

Figure 5 : Profit and welfare when the monopolist is obligated to give both classes access to all channels

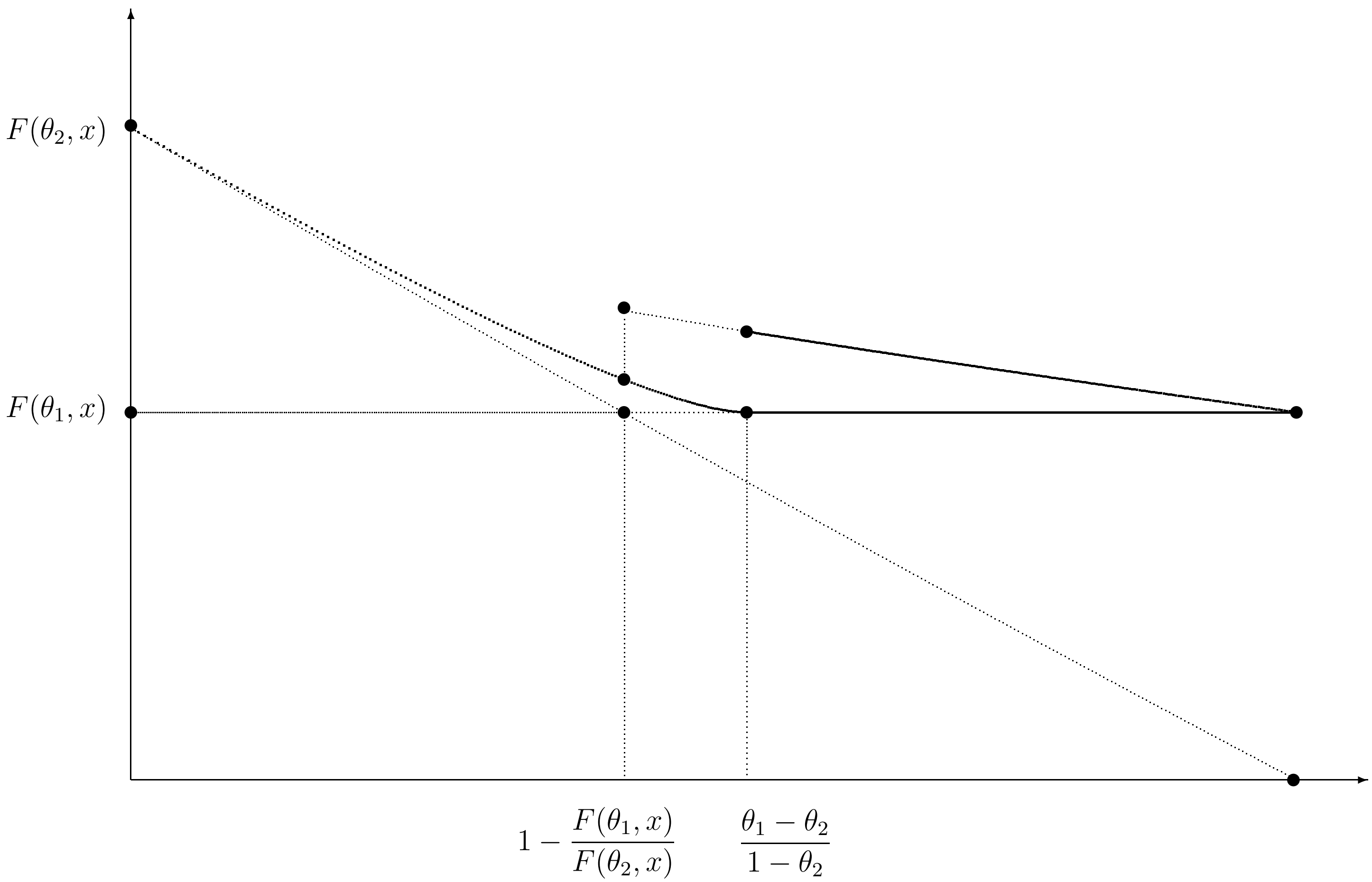

Figure 6 : Profit and welfare when the monopolist is not allowed
to sell two different bundles