Optimal Environmental Taxation and Enforcement Policy

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Abstract

We study the optimal environmental taxation and enforcement policy when (i) the regulator does not know firms abatement costs, (ii) penalties for tax evasion are limited, and (iii) monitoring of pollution is costly. We show that the threat of being audited alter the usual incentives of firms to over-estimate their abatement costs. In particular, depending on firms abatement costs, the optimal policy may involve over or under-deterrence compared to the full information outcome. We then investigate the properties of a pollution standard. We show that this policy is close to an environmental tax once the economic incentives of the enforcement policy of the standard are considered.

Key-words : Environmental Taxation, Law Enforcement, Tax Evasion, Adverse Selection.

JEL Classification : D62, D82, H21, H26, H32

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1 Introduction

Environmental taxes are an important part of the system for regulating pollution in several European countries. At Member state level, there has been a continuing increase in the use of environmental taxes over the last decade, especially in Scandinavia, Austria, Belgium, France, The Netherlands, Germany and United Kingdom (European Environmental Agency, 1996). Following textbook discussions, the main advantages of environmental taxes are first to provide polluters with the correct incentives to internalize external costs which leads to a better integration of economic and environmental policies and, second, to raise revenue which may be used to improve environmental expenditures and/or to reduce the distortions due to taxation on labor, capital and savings (the so-called double dividend).

However, a careful design of environmental taxes should also include an enforcement mechanism to deter non compliance and tax evasion. Monitoring and enforcement may not be the first things that come in mind in this field and indeed, these issues are often ignored by both academic and policy makers when discussing environmental policy reform (Cohen, 1999). This general lack of attention may have negative consequences for environmental quality and for social welfare. Trying to implement stricter regulations than the existing ones may result in increased pollution levels if the agency cannot control firms activities and enforce compliance. Moreover, ignoring monitoring and enforcement costs in the case of a new regulation might lead the public authority to implement costlier policies than the current ones. Consequently, when investigating the optimal regulation of polluting firms, the analysis must include the facts that the monitoring of actual polluters' emissions is costly and, more generally, that the information needed by the regulator is decentralized in the economy (see Lewis, 1996, for a survey on these issues). In particular, if it is realistic to assume that the regulator possesses aggregate information about polluters' abatement costs, it seems doubtful that she can observe the costs of each individual firm. Moreover, many systems for enforcing pollution controls are impeded by legislations that put limitations on the regulator's power to punish non compliance.

In the following, we analyze an environmental taxation when emission levels can be observed through a costly audit, assuming that abatement costs remain private information even if an audit is performed. Given this information structure, we derive and investigate the properties of the optimal tax and enforcement policy.

Our analysis brings out the following points. i/ The optimal tax is different from the Pigovian level because of the second dividend of the environmental tax and of the social cost of monitoring. Both effects induce greater allowances to polluting firms (under-deterrence). This phenomenon is easily understood: With a strictly positive social cost of public funds, due to imperfections of existing tax systems on labor or capital, the regulator is tempted to raise more money using environmental taxes than the Pigovian levels. Moreover, costly enforcement per se entails pollution levels strictly higher than the Pigovian levels in order to reduce the incentive to evade. ii/ The usual rule of equalizing private marginal benefits to social marginal damages is violated further because of complex distortions due to the adverse selection problem, which give rise to firm specific marginal tax rates. From Baron (1985) and Laffont (1994), we know that without monitoring cost, the optimal tax schedule when the agency doesn't know the firms abatement costs induces lower emission levels the higher the firms abatement costs (over-deterrence effect due to the adverse selection problem). With costly enforcement and self-reporting, we show that the agency has to take into account another adverse-selection effect: under-deterrence increases the informational rents of profitable firms but allows to decrease the inspection effort on less efficient firms. The regulator has thus to arbitrate between two effects when designing the taxation policy: an increase of the emission levels for a given firm allows to reduce the inspection efforts on less profitable firms but oblige to decrease the tax collection on more profitable ones. iii/ Since the optimal tax policy is intimately related to the monitoring capabilities of the agency, monitoring and tax policies must be designed jointly. In particular, optimal monitoring efforts of the agency are inversely related to the amounts of tax paid by the firms (and thus, to their emission levels). iv/ Command-and-control instruments are closer to economic instruments once the economic incentives of their enforcement policies are considered. We investigate the properties of a pollution standard, where the firms are allowed to pollute up to a given level (the emission standard) by paying a lump-sum transfer (the license) and where the agency verifies the firms compliance with an uniform inspection probability. We show that the optimal standard policy induces the less efficient firms up to an intermediate profitability type to pollute at the standard in accordance to their licence and consequently these firms are not fined when inspected. But more profitable firms pollute over the quota paying the corresponding penalty when inspected. Consequently, this licence cum fine schedule resembles a taxation scheme. Malik (1992) already observed that an environmental standard policy may be considered as a special case of a taxation policy. A standard policy appears to be a restricted case of a taxation policy because of the “marginal deterrence” induced by the penalty schedule of its enforcement policy. Using a increasing fine schedule allows the agency to introduce efficiency considerations in an otherwise rather inefficient policy.

Although there has been a rapid growth in theoretical and empirical studies on environmental enforcement over the last years (see Cohen, 1999, for a recent survey), only few papers analyze the case of environmental taxes. Swierzbinski (1994) analyses the optimal regulation when abatement costs are private information and when monitoring is costly. The regulator being allowed to reward compliant firms, he shows that the optimal mechanism resembles a deposit-refund system. He assumes also that all firms, whatever their abatement costs, obtain the same pay-off from evasion. We consider alternatively that the (reservation) profit obtained from evading depends on the firm.

In their study on marginal deterrence, Mookherjee and Png (1994) analyse the optimal enforcement of a standard. Our analysis extends this model by considering the possibility of raising money through a licence paid by every active firm. Since they take the opposite point of view that fines are socially costly, it comes from no surprise that their results differ from ours. In particular, they obtain that the marginal expected penalty should be strictly less than the corresponding marginal social harm while we show that the marginal expected penalty for violating the standard can be higher or lower than the marginal damage depending on the efficiency of the firm.

Applications of principal-agent models with audit have been developped in various fields like insurance, income taxation or monopoly regulation. The key differences between our problem and these models are that they usually assume that the maximal penalty depends on the private information of the agents and/or that the regulator can reward honesty. As a result, the incentive constraints of audit models are badly behaved. This fact was first recognized by Baron and Besanko (1984) in their analysis of a monopoly regulation. For example, in most studies on income taxation, the labor supply is assumed to be given and auditing the income allows to know all the agent's private information. The maximum fine can thus be set to the entire benefit the agent extracts from evading, which is type-dependent (see Border and Sobel, 1987 and Chander and Wilde, 1998).[1] However, in our problem,

[1]See Cremer and Gahvari (1996) for a model with endogeneous labor supply where the

the audit does not allow to apraise firm's benefit, which implies that the maximum penalty cannot depend on the firm. Moreover, as it is often the case in practice, we forbid tax rebates in case of compliance. Using standards arguments, we show that the optimal fines are not type-dependent: the fine is either 0 if the firm do not evade or equal to the maximal penalty otherwise. This simple penalty scheme allows to handle the two aspects of the audit problem separately: The inspection/penalty schedule is intended to solve the evasion problem only, and the tax/emissions schedule to solve the mimicking problem. Once it is verified that no firm is induced to evade, the incentive constraints of the remaining adverse-selection problem are well-behaved.

The paper proceeds as follows. Section 2 is devoted to notations, assumptions and the derivation of the optimal policy in the perfect information benchmark case. Section 3 offers the main results concerning the optimal taxation and enforcement policy. In section 4, we study the optimal enforcement of an environmental standard. In section 5, we compare with an example the relative performances of these policies. The last section concludes. Most of proofs are relegated into an appendix.

2 The model

Consider an economy consisting of a continuum of firms with mass unity. Firms differ in a one-dimensional measure of their private benefit of pollution. The profitability parameter θ is distributed over a non-negative interval $\Theta = [\underline{\theta}, \bar{\theta}]$, according to a probability density function g with $g(\theta) > 0$ for all $\theta \in \Theta$. Each firm chooses an emission level q (or abatement effort) which yields the profit $\pi(q, \theta)$ and we normalize the set Θ by assuming that $\partial_\theta \pi > 0$.[2] Without regulation, the individual emission level $q^\circ(\theta)$ satisfies

$$q^\circ(\theta) \in \arg\max_q \pi(q, \theta).$$

We assume that $\partial_{qq}\pi(q, \theta) \leq 0$ and $\partial_{q\theta}\pi(q, \theta) > 0$ for all $q < q^\circ(\theta)$, i.e., that the marginal benefit of pollution decreases with pollution and increases

agent's income is the result of two unobservable variables, ability and labor supply, the latter being discovered through a costly audit.

[2]We denote by $\partial_x f$ the partial derivative of a function $f(\cdot)$ with respect to the variable x.

with the firms' type. This implies that the emissions pattern $q^{\circ}(\cdot)$ is a non-decreasing function of θ. The latter assumption corresponds to the well-known Spence-Mirrlees single crossing condition that simplifies the analysis of the adverse-selection problems. The profit of the type-θ firm without regulation is given by

$$\pi^{\circ}(\theta) \equiv \pi(q^{\circ}(\theta), \theta)$$

which is an increasing function of θ under the assumption $\partial_{\theta}\pi > 0$.

The environmental damage D is supposed to depend on aggregate pollution $Q = \int_{\underline{\theta}}^{\bar{\theta}} q(\theta)g(\theta)d\theta$ according to the relation $D = Qd$ where d is the marginal environmental damage[3]. The regulator's task is thus to design an environmental tax schedule that induces producers to internalize this damage. More precisely, we assume that the regulator's objective is to maximize the expected social welfare given by

$$\int_{\underline{\theta}}^{\bar{\theta}} \{\pi(q(\theta), \theta) - dq(\theta) + \lambda t(\theta)\}g(\theta)d\theta \tag{1}$$

where $\lambda \geq 0$ and $t(\theta)$ is the tax paid by the type-θ firms. The term $\lambda \int_{\underline{\theta}}^{\bar{\theta}} t(\theta)g(\theta)d\theta$ corresponds to an indirect social benefit of an environmental taxation, commonly called the "second dividend": Using tax to correct the externality allows also the government to diminish the tax burden that weights on the rest of the society. This reduction decreases the deadweight losses associated to other existing tax systems, like income taxation, that induce distortions in the economy. λ is a per monetary unit measure of these deadweight losses ($1+\lambda$ is commonly called the shadow cost of public funds). Raising t on a polluting firm with an environmental tax allows the government to diminish other taxations by a same amount and thus induces an indirect social gain equal to λt. As a consequence, the governement wants to raise as much money as possible using an environmental tax to save on costlier tax systems, and thus will try to capture the entire profit of the polluting firms.[4] More precisely, if the regulator were able to observe the firms'

[3]This assumption is made to simplify the algebra while keeping the main insights of our analysis. Extensions to more general damage functions is straightforward.

[4]We implicitly assume that there is no political constraint that impedes the government latitude in the setting of an environmental taxation. Such political constraints may be taken into account by requiring that the firms' profits after the tax be greater than a given level $\bar{\pi} > 0$. Assuming $\bar{\pi} = 0$ allows us to save on notation without changing the qualitative results of the analysis.

emissions directly, it would be possible to implement the perfect information outcome via a tax-emissions schedule $\{t^*(\theta), q^*(\theta), \theta \in \Theta\}$ satisfying

$$\partial_q \pi(q^*(\theta), \theta) = d/(1+\lambda)$$

and

$$t^*(\theta) = \pi(q^*(\theta), \theta).$$

Observe that when $\lambda = 0$, the optimal emission pattern corresponds to the first-best levels, defined by the usual rule of equating the marginal private benefit $\partial_q \pi(q^*(\theta), \theta)$ to the marginal social damage d. Moreover, in that case, there is no reason to charge firms per se, and the optimal policy may be implemented using a Pigovian tax equal to the marginal social damage without worrying about the private information of firms. This is no longer the case when there are distortions in the rest of the economy ($\lambda > 0$). In that case, the regulator desires to raise as much tax revenue as possible using the environmental tax and she has not only to consider the environmental objective of the policy but also its incentive aspect. A direct effect of the distortions is that the regulator is induced to allow firms to over-pollute compared to first-best levels. This increase of emission levels allows to raise more revenue, since the entire benefit of pollution is captured by the agency. However, such a capture would be possible only if the regulator were able to assess the benefit that each firm extracts from polluting and if the firms are not tempted to evade the taxation. Moreover, assuming that the regulator desires to implement a given tax-emissions schedule, it is doubtful that she would be able to check without cost the firms' compliance to the legislation. More generally, the agency has to take into account the informational aspect of the problem when designing its policy. We assume the following information structure. First, the individual profitability is the firm's private information while the regulator knows only the distribution of types. Pollution is not directly observable by the regulator either, but can be discovered through a costly audit. However, the audit cost increases with the number of firms audited. The agency has thus to balance the benefits of an audit with its cost.

Given this information structure, we derive the optimal revelation mechanism and investigate the properties of the optimal tax and audit policies. The process of regulation and inspection is modeled as a three-stage game. First, the regulator chooses a mechanism. Second, the firm reports its type and simultaneously chooses its pollution level. Third, the contract is implemented, that is the firm is monitored with the probability determined by the type's report. If no audit occurs, the firm pays only the tax corresponding to its announcement. In the case of audit, it pays the transfer corresponding to its announcement and to the result of the audit.

More formally, a mechanism for the environmental agency consists of four functions: $q(\tilde{\theta})$, $\mu(\tilde{\theta})$, $t(\tilde{\theta})$ and $f(\tilde{\theta}, q)$, where $\tilde{\theta}$ is the reported type and q is the pollution as revealed by the audit. A firm that has reported an profitability parameter $\tilde{\theta}$ is assigned to an emission level $q(\tilde{\theta})$, pays a tax $t(\tilde{\theta})$ and is audited with probability $\mu(\tilde{\theta})$. In case of audit, it pays a fine $f(\tilde{\theta}, q) \geq 0$ if $q \neq q(\tilde{\theta})$.[5]

A firm may cheat along two ways. First, it may misreport its type while producing the emission level assigned to the reported type. In this case, the firm perfectly mimics another type but does not evade, and the result of the audit gives $q = q(\tilde{\theta})$. This mimicking cannot be discovered by auditing the firm, because the inspection reveals only the level of pollution and not the firm's type and profit level. Secondly, it may evade by choosing an emission level different from the one it is assigned to given its report, i.e., $q \neq q(\tilde{\theta})$. This shirking is detected by auditing the firm.

As the revelation principle applies in our context, we can restrict the search of the optimal mechanism to the set of direct and incentive-compatible mechanisms without loss of generality. The expected profit of a type-θ firm that announces to be of type $\tilde{\theta}$ and that pollutes a level q is given by

$$U(\theta, \tilde{\theta}, q) = \pi(q, \theta) - t(\tilde{\theta}) - \mu(\tilde{\theta}) f(\tilde{\theta}, q) \tag{2}$$

Compared to the perfect information situation, a positive expected profit corresponds to an informational rent for the firm that enjoys it. Let denote by $R(\theta)$ the expected profit of a type-θ firm that truthfully announces its type and pollutes according to the regulator's requirement. The first type of incentive constraints can be written as

$$R(\theta) \equiv U(\theta, \theta, q(\theta)) \geq U(\theta, \tilde{\theta}, q(\tilde{\theta})) \tag{IC1}$$

for all θ and $\tilde{\theta}$ in Θ, where $U(\theta, \tilde{\theta}, q(\tilde{\theta}))$ corresponds to the expected profit of a type-θ firm that perfectly mimics a firm of type $\tilde{\theta}$, i.e., that announces

[5]Since the fine is constrained to be non negative, we forbid tax rebates. Consequently, in accordance to the general practice, the agency cannot reward firms in case of compliance. See Swierzbinski (1994) for an analysis of the incentive properties of tax rebates.

$\tilde{\theta}$, pays the tax $t(\tilde{\theta})$ and pollutes $q(\tilde{\theta})$. The constraints (IC1) insure that the firm doesn't improve its expected profit by choosing the tax-pollution pair designed for another type. The second type of incentive constraints is given by

$$R(\theta) \geq \max_{\tilde{\theta},q} U(\theta, \tilde{\theta}, q) \tag{IC2}$$

for all θ. The difference between (IC1) and (IC2) is that with the latter the firm doesn't constraint itself to mimic another existing type by choosing the pollution level which corresponds to its announcement. Instead, it allows itself to choose any pollution level. If the regulator's mechanism satisfies (IC2), the firm is better off choosing the tax-pollution pair designed for its type than producing any other emission level and trying to pay any other tax amount. Obviously, if the (IC2) constraints are satisfied, so are the (IC1) constraints. Taking into account the (IC2) constraints only is thus sufficient to pursue the analysis. However, it is useful to distinguish between these two sets of constraints as shown in the following.

To satisfy (IC2) the regulator must be able to inflict severe punishments to the firm. However, the fines that the regulator may inflict are usually bounded, i.e.,

$$f(\tilde{\theta}, q) \leq \bar{F} \tag{3}$$

where $\bar{F}$ is the exogenous maximum fine due, for example, to limited firms' liability.[6] It is easily seen that the maximum fine should be applied to any firm that is caught shirking. This does not affect (IC1) constraints and relax

[6]This assumption has been largely discussed in the literature and is founded by several justifications ranging from the limited liability of shareholders (the extension of firms liability to third parties, such as lenders or contractors, has been recently considered, see Boyer and Laffont, 1997) to the functioning of the judicial system (even if prescribed by laws, courts are usually reluctant to enforce penalties that are not reasonably related to the damage). More technically, unlimited liability gives rise to the improbable result that using an arbitrarily large penalty in case of fraud, the agency can deter tax evasion with almost no cost (as pointed out by Border and Sobel, 1987). One possibility often investigated in the literature is to assume that the maximal penalty is limited by the additional profit the firms can extract from polluting. However, in our context, the audit does not allow the agency to appraise the firms' benefits. The maximum fine has thus to be the same for every firms.

(IC2) constraints. Hence, we have $f(\tilde{\theta}, q) = \bar{F}$ whenever $q \neq q(\tilde{\theta})$.[7] The same reasoning applies if the audit reveals that the firm is compliant with the rule: We can relax (IC2) constraints by setting $f(\tilde{\theta}, q(\tilde{\theta})) = 0$ without changing (IC1) constraints. Indeed, with risk neutral firms, all incentives to truthfully report the type can be embedded in the tax level $t(\theta)$.[8] Consequently, the penalty schedule $f(\cdot)$ takes the form

$$f(\theta, q) = \begin{cases} 0 & \text{if } q = q(\theta) \\ \bar{F} & \text{otherwise} \end{cases} \tag{4}$$

We thus have a simple penalty scheme: the payment is either 0 if the firm do not evade or equal to the maximal penalty otherwise. This simplicity allows us to separate the two aspects of the audit problems: the evasion problem is taken care of by the inspection-penalty schedule whereas the inefficient mimicking problem is deterred by the tax-emissions schedule. Indeed, since compliance is not rewarded, the expected profit of a type-θ firm with an incentive-compatible mechanism becomes

$$R(\theta) = \pi(q(\theta), \theta) - t(\theta) \tag{5}$$

which corresponds to the informational rent of a firm when emissions are observable without cost by the principal.

As mentioned above, the audit of the firms entails a cost that diminishes the expected social welfare (1). This cost, by expending the public expenses, diminishes also the indirect benefit of the environmental taxation. Assuming that the cost of auditing a type-θ firm with a probability $\mu(\theta)$ is given by $c\mu(\theta)$, the social welfare (1) becomes[9]

$$W = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \pi(q(\theta), \theta) - dq(\theta) + \lambda t(\theta) - (1 + \lambda) c\mu(\theta) \right\} g(\theta) d\theta \tag{6}$$

This objective is constrained by the participation of the firms, i.e.;

$$R(\theta) \geq 0 \tag{IR}$$

[7] Moreover, we have the so-called Becker's conundrum: Given that the inspection effort is socially costly, it is optimal to increase penalties as far as possible and to minimize the probability of costly auditing.

[8] Formally, any scheme $q(\cdot), t(\cdot), \mu(\cdot),\ f(\cdot)$ with $f(q(\theta), \theta) \geq 0$ may be replaced by a schedule $q(\cdot), \hat{t}(\cdot), \mu(\cdot),\ \hat{f}(\cdot)$ with $\hat{t}(\theta) = t(\theta) + \mu(\theta) f(q(\theta), \theta)$ and $\hat{f}(q(\theta), \theta) = 0$.

[9] The assumption of a constant marginal monitoring cost is made for analytical convenience. Extensions to more general cost functions are straightforward.

which insure that the firms' revenues are at least equal to their profits is they choose not to pollute (normalized to 0, see footnote 4). Finally, the audit probability must satisfy

$$0 \leq \mu(\theta) \leq 1 \tag{7}$$

for all $\theta \in \Theta$. The agency's program may thus be written as

$$\max_{q(\cdot),t(\cdot),f(\cdot),\mu(\cdot),R(\cdot)} W : \text{(IC1), (IC2), (IR), (5), (7)} \tag{I}$$

where W is given by (6), and the right hand sides of (IC1) and (IC2) are deduced from (2) and (4).

3 Analysis

Problem I presents three sets of inequality constraints and cannot be solved directly. We shall proceed by presenting intermediate results (lemmii 1 and 2) to transform this general problem into a simple (although parametric) optimal control program (Program III). The following lemma allows us to simplify the incentive and profit constraints.

Lemma 1 *Assuming a non-decreasing emission schedule $q(\cdot)$ such that $q^\circ(\theta) \geq q(\theta)$ for all $\theta \in \Theta$, the sets of constraints (IC1), (IR) and (IC2) reduce to*

$$\dot{R}(\theta) = \partial_\theta \pi(q(\theta), \theta) \tag{IC1'}$$

$$R(\underline{\theta}) \geq 0 \tag{IR'}$$

and

$$R(\bar{\theta}) \geq \pi^\circ(\bar{\theta}) - K \tag{IC2'}$$

with

$$\mu(\theta) \geq (K - \pi(q(\theta), \theta) + R(\theta))/\bar{F} \tag{8}$$

and where

$$K = \min_\theta \{t(\theta) + \mu(\theta)\bar{F}\} \tag{9}$$

Proof. (IR'), (IC1') and the monotonicity constraint on $q(\cdot)$ are derived using standard arguments (see, e.g., Guesnerie and Laffont, 1984). From (9), we can rewrite the (IC2) constraints as

$$K \geq \pi^{\circ}(\theta) - R(\theta).$$

(IC2') follows from the fact that

$$\begin{aligned}\frac{d}{d\theta}\left[\pi^{\circ}(\theta) - R(\theta)\right] &= \partial_{\theta}\pi(q^{\circ}(\theta), \theta) - \partial_{\theta}\pi(q(\theta), \theta) = \int_{q(\theta)}^{q^{\circ}(\theta)} \partial_{\theta q}\pi(u, \theta) du \\ &\geq 0\end{aligned}$$

since $\partial_{\theta q}\pi \geq 0$ when $q(\theta) \leq q^{\circ}(\theta)$. (9) also implies

$$\begin{aligned} K &\leq t(\theta) + \mu(\theta)\bar{F} \\ &= (\pi(q(\theta), \theta) - R(\theta)) + \mu(\theta)\bar{F}\end{aligned}$$

using (5). Rearranging terms gives (8). ■

By (IC1'), to deter imitation of low profitability firms by high ones, the informational rents have to increase according to the (marginal) advantage in term of profitability at the assigned pollution levels. Condition (IR') states that it suffices that the firm with the lowest type enjoys positive rent to guarantee that all other (more profitable) firms will. (IC1') and (IR'), in addition to a monotonic emission schedule, are reminiscent of the reduced incentive constraints of standard adverse-selection models, as explained in Guesnerie and Laffont (1984). In addition to these no-mimicking conditions, (IC2'), (8) and (9) allow to deter tax evasion. Condition (IC2') states that it suffices to deter the more profitable firm from evading to insure that all other (less profitable) firms will follow the policy requirements. Condition (8) recalls that minimal inspection efforts are necessary to maintain these incentives, whereas (9) defines the minimal expected cost of evading which would be incurred by shirking firms. Observe that thanks to a simple fine schedule, once it is verified that firms are deterred from evading (by designing an audit policy that insures that the most profitable firms will not), we are back to the standard problem of designing a contract in an pure adverse-selection setting: The mimicking incentive constraints are the same as those of a pure contract problem with perfect observability of the emission levels, and usual results on second-order conditions of these models hold.[10]

[10]However, the conditions to obtained a monotonic emission schedule are more stringent than in the case of free observability of emissions as explained below.

Substituting for $t(\cdot)$ using (5), Lemma 1 allows us to transform program I as

$$\begin{array}{ll} \max_{q(\cdot),\mu(\cdot),R(\cdot),K} \int_{\underline{\theta}}^{\bar{\theta}} \left\{(1+\lambda)(\pi(q(\theta),\theta) - c\mu(\theta)) - dq(\theta) - \lambda R(\theta)\right\} g(\theta)d\theta & \\ \text{s.t.} & \\ \dot{R}(\theta) = \partial_\theta \pi(q(\theta),\theta) & \\ R(\bar{\theta}) \geq \pi^\circ(\bar{\theta}) - K & \text{II} \\ R(\underline{\theta}) \geq 0 & \\ \mu(\theta) \geq (K + R(\theta) - \pi(q(\theta),\theta))/\bar{F} & \\ 0 \leq \mu(\theta) \leq 1 & \end{array}$$

where the monotonicity condition on the emission schedule $q(\cdot)$ and the conditions $0 \leq q(\cdot) \leq q^\circ(\cdot)$ are neglected. We thus have to verify that these conditions hold with the emission scheme solution of program II.

This program may be simplified further by observing that the constraints (IR') and (8) are binding at the optimum, as stated formally in the following lemma.

Lemma 2 *At the optimum of program II we have*

(i) $R(\underline{\theta}) = 0$

(ii) $\mu(\theta) = (K + R(\theta) - \pi(q(\theta),\theta))/\bar{F}$ *for all* $\theta \in \Theta$.

(iii) $\mu(\theta) > 0$ *whenever* $q(\theta) < q^\circ(\theta)$

Proof. (i). Assume that $R(\underline{\theta}) > 0$ at the optimum. Using (IC1'), we have

$$R(\theta) = R(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \partial_\theta \pi(q(u),u)du.$$

Since $R(\cdot)$ affects negatively the program's objective, reducing $R(\underline{\theta})$ allows to increase the objective while satisfying the constraints, hence a contradiction. (ii). Assume that (8) is lenient on a non-negligeable subset Θ at the optimum. We may reduce slightly $\mu(\theta)$ on this subset and still satisfy (8). This diminishes the audit cost and thus increases the objective, hence a contradiction. (iii). We have

$$K + R(\theta) - \pi(q(\theta),\theta) \geq K + R(\theta) - \pi^\circ(\theta) \geq 0$$

where the last inequality comes from (IC2). So, unless $q(\theta) = q^\circ(\theta)$, we have $\mu(\theta) > 0$. ■

By (i), the tax collected on the lowest profitable firm corresponds to the entire benefit it obtains from polluting. Since the informational rents are increasing, the other (more profitable) firms benefit from the asymmetric information. As stated point (ii), at the optimum the expected cost of evading is the same whatever the announcement, given by $K = t(\theta) + \mu(\theta)\bar{F}$ for all θ. Since this cost is constant, the optimal tax schedule and inspection rate are negatively related: The lower the tax paid by the firm, the higher the probability it will be inspected. Since $q(\cdot)$ and $t(\cdot)$ are non-decreasing, the inspection rate is thus a non-increasing function of the firms' type. Consequently, the more a firm announces it pollutes, the less likely it is inspected. This can be easily understood: One of the agency's tasks is to deter firms from cheating about their pollution levels. It is tempting for a firm to evade, announcing a low emission level. The agency has thus to increase the probability of inspection for firms paying low taxes to deter such shirking. Finally, by (iii), the agency has to check compliance if it wants to induce lower levels of emission than the selfish ones. Observe that it may be the case that for low types we have $\mu = 1$.

Using (2), we can substitute the right hand side of (8) for μ in program II to obtain

$$\begin{array}{l} \max\limits_{q(\cdot),R(\cdot),K} \int_{\underline{\theta}}^{\bar{\theta}} \left\{(1+\lambda)(\pi(q(\theta),\theta) - c(K + R(\theta) - \pi(q(\theta),\theta))/\bar{F}) - dq(\theta) - \lambda R(\theta)\right\} g(\theta)d\theta \\ \text{s.t.} \\ \dot{R}(\theta) = \partial_\theta \pi(q(\theta),\theta) \\ R(\underline{\theta}) = 0 \\ R(\bar{\theta}) - \pi^\circ(\bar{\theta}) + K \geq 0 \end{array} \qquad \text{III}$$

where we have neglected the constraint $\mu \leq 1$, i.e.

$$R(\theta) - \pi(q(\theta),\theta) + K \leq \bar{F} \tag{10}$$

Program III is a parameterized optimal control problem (the parameter being K), where the informational rent $R(\cdot)$ stands for the state variable and the emission schedule $q(\cdot)$ for the control variable. Observe that since K affects negatively the objective of program III, the last inequality of this program is binding at the optimum. The resolution of program III is given in appendix. This solution solves the agency problem I if the monotonicity constraint on $q(\cdot)$ and the conditions $0 \leq q(\cdot) < q^{\circ}(\cdot)$ and (10) are satisfied. We shall go back to these constraints after having characterized the unconstrained emission schedule given in the following proposition.

Proposition 1 *The emission schedule $\hat{q}(\cdot)$ solution of program III satisfies*

$$\partial_q \pi(\hat{q}(\theta), \theta) = \frac{d}{1+\lambda} - \frac{c}{\bar{F}} \partial_q \pi(\hat{q}(\theta), \theta) + M(\theta) \partial_{\theta q} \pi(\hat{q}(\theta), \theta) \tag{11}$$

where

$$M(\theta) \equiv \frac{\lambda}{1+\lambda} \frac{1-G(\theta)}{g(\theta)} - \frac{c}{\bar{F}} \frac{G(\theta)}{g(\theta)}$$

Proof. See the appendix.

Compared to perfect information levels, the emission schedule defined by (11) involves two additional terms that reflect the incidence of audit costs and the social cost of firms' informational rents. To interpret (11), it is useful to consider the two benchmark cases of a costly audit cum perfect information and a free audit cum asymmetric information. Depicted figure 1 are the emissions schedules corresponding to these different assumptions.

First, assume that the regulator has perfect knowledge of type but that the agency has to audit the firm to discover its emission level. Denote by $e(\cdot)$ the optimal emission schedule in that case. Since it is not worth deterring imitation, the (IC1) constraints do not matter. We have $R(\theta) = 0$, i.e.; $t(\theta) = \pi(e(\theta), \theta)$. However, the agency has still to deter evasion, i.e.; that firms pay the tax amounts corresponding to their type and pollute accordingly. Using (11), the optimal emission schedule satisfies

$$\partial_q \pi(e(\theta), \theta) = \frac{d}{1+\lambda} \frac{1}{1+c/\bar{F}}$$

and thus $e(\cdot) > q^*(\theta)$, i.e.; we have under-deterrence compared to first-best levels. Indeed, to deter any deviation of firms from their assigned emission levels, the agency has to inspect the type-θ firm at a minimal rate $\mu(\theta) = (K - \pi(q(\theta), \theta))/\bar{F}$. Compared to the perfect information level, an increase dq of the type-θ emission level induces a decrease of the inspection cost of $(d/dq)[c\mu] = -\partial_q \pi(q^*(\theta), \theta) c/\bar{F}$. The corresponding marginal social loss is $(1 + \lambda) \partial_q \pi(q^*(\theta), \theta) - d = 0$. The agency will thus increase the emission schedule above the perfect information one.

Second, assume that audit is free ($c/\bar{F}$ is negligeable) but does not reveal the firms' types. We would have the usual adverse-selection quantity schedule (à la Baron and Myerson, 1982, B-M hereafter), given by

$$\partial_q \pi(q^{BM}(\theta), \theta) = \frac{d}{1+\lambda} + \frac{\lambda}{1+\lambda} \frac{1-G(\theta)}{g(\theta)} \partial_{\theta q} \pi(q^{BM}(\theta), \theta)$$

which states that, due to asymmetric information, the optimal emission schedule is lower than the perfect information one except for the higher type of firms $\bar{\theta}$ as depicted figure 1. Indeed, in that case, compared to the perfect information level, the principal has to trade-off the social welfare gain of an increase dq of emissions for the $g(\theta)$ type-θ firms, equal to $(1+\lambda)\partial_q \pi(q^*(\theta), \theta) - d = 0$, to the social losses of decreased tax collections (increased informational rents) on all firms of type higher than θ, that amount to $(1 - G(\theta))\lambda \partial_{\theta q} \pi(q^*(\theta), \theta) > 0$. The optimal schedule solution of this trade-off $q^{BM}(\cdot)$ allows the agency to deter firms of type higher than a given θ to mimic the type-θ firm. Observe that this adverse-selection effect induces marginal tax rates that decrease with the firms' type and that the most efficient firms only has a marginal tax rate equal to the perfect information level.

Back to the general case where the audit is costly and reveals only firm's emission levels, there is an additional distortion that affects the emission schedule, equal to

$$-(1+\lambda)c/\bar{F}G(\theta)\partial_{\theta q}\pi(q(\theta), \theta). \tag{12}$$

The interpretation of this term is the following. First, in addition to the usual mimicking adverse-selection effects produced by an increase dq of the emission level of the type-θ firm, this increase changes indirectly the inspection efforts on higher types. Indeed, to maintain the same expected penalty when reporting a type greater than θ, the agency has to increase the inspection efforts on these firms, since the increase dq induces an increase dR for all firms with type greater than θ and that we have $d\mu/dR = 1/\bar{F}$ for all θ (recall that increased rents correspond to lower tax burdens). This induces an additional marginal social cost equal to

$$(1+\lambda)c/\bar{F}(1 - G(\theta))\partial_{\theta q}\pi(q(\theta), \theta).$$

However, the increase of the type-$\bar{\theta}$ informational rent allows the agency to reduce the expected cost of evading K, since we have $R(\bar{\theta}) = \pi^\circ(\bar{\theta}) - K$

at the optimum, i.e.; these firms must be indifferent between evading and paying their taxes at the optimum. This decrease induces in turn a lower inspection rate for all firms, since $d\mu/dK = 1/\bar{F}$, that amounts to a marginal saving of inspection costs given by

$$(1+\lambda)c/\bar{F}\partial_{\theta q}\pi(q(\theta),\theta).$$

Consequently, the inspection efforts are unaffected for the firms with type higher than θ (the increase of their rents $R(\cdot)$ and the decrease of K let $\mu(\cdot)$ unchanged), but they are reduced for types lower than θ (the decrease of K is not compensated by an increase of their informational rents). To sum up, with costly enforcement and self-reporting, the agency has to take into account two opposite adverse-selection effects: under-deterrence increases the informational rents of the most profitable firms, but greater informational rents allow to decrease the inspection effort on less efficient firms. The result of the agency's tradeoff is an emission schedule that is steeper than the previous ones, as depicted figure 1. Observe that mimicking and shirking adverse-selection effects distort the emission schedule in opposite directions around the emission levels $e(\theta)$ that the regulator would implement if he knew the firms' type taking into account the audit costs. As depicted, there is a unique firm's type θ_s such that these adverse-selection effects annihilate each other, i.e.; $M(\theta_s) = 0$ which implies $\hat{q}(\theta_s) = e(\theta_s)$. More generally, we have the following results:

Corollary 1 *The emission schedule $\hat{q}(\cdot)$ solution of program III satisfies*

(i) *For all $\theta < \theta_s$, $\hat{q}(\theta) < e(\theta)$ and for all $\theta > \theta_s$, $\hat{q}(\theta) > e(\theta)$, where θ_s is given by*

$$\theta_s = G^{-1}\left(\frac{\lambda}{\lambda + (1+\lambda)c/\bar{F}}\right)$$

and we have $\underline{\theta} < \theta_s < \bar{\theta}$, $d\theta_s/d\lambda > 0$ and $d\theta_s/d[c/\bar{F}] < 0$.

(ii) $\hat{q}(\underline{\theta}) < q^*(\underline{\theta})$ *if* $\partial_{\theta q}\pi(q^*(\underline{\theta}),\underline{\theta}) > c/\bar{F}g(\underline{\theta})/\lambda$.

(iii) $\hat{q}(\bar{\theta}) = q^\circ(\bar{\theta})$ *if* $\partial_{\theta q}\pi(q^\circ(\bar{\theta}),\bar{\theta}) \geq g(\underline{\theta})/((1+\lambda)c/\bar{F})$.

Proof. See the appendix.

As explained point (i), the mimicking effect dominates the evasion one for the most inefficient firms. The agency thus designs a tax schedule that induces these firms to under-pollute compared to the schedule $e(\cdot)$. If the marginal profit of pollution increases rapidly with the type at the perfect information emission levels $q^*(\cdot)$, this mimicking effect may induce the agency to enforce pollutions levels below perfect information emission levels for low types as stated point (ii). This is the case figure 1 for all types lower than θ_2, with all firms bellow θ_1 having their emissions reduced to zero. For more profitable firms, the shirking effects is the dominant one, leading the agency to induce higher emission levels than $e(\cdot)$ and even to give up pollution reduction for the higher types if the marginal profit of pollution increases rapidly as revealed point (iii). This case is also depicted figure 1 with all firms with types greater than θ_3 having their emission levels equal to their private optimum $q^\circ(\theta)$.

Finally, if (10) does not bind, that is if the inspection rate μ is inferior to 1 everywhere, the emission schedule $e^*(\cdot)$ solution of program I is given by $e^*(\theta) = \hat{q}(\theta)$ for all $\theta \in \Theta$ (whenever $0 \leq \hat{q}(\theta) \leq q^\circ(\theta)$). To complete the analysis, the last proposition characterizes the opposite case of an inspection rate μ equal to 1 for the low types firms. As stated formally, the entire regulation schedule is affected, with emission levels of every firm larger than in the unconstrained case.

Proposition 2 *If (10) binds on a subset of Θ, the emission schedule $e^*(\cdot)$ solution of program I is such that $e^*(\theta) > \hat{q}(\theta)$ for all $\theta \in \Theta$ (whenever $\hat{q}(\theta) < q^\circ(\theta)$).*

Proof. See the appendix.

It remains to verify that the second-order conditions for an incentive-compatible policy are satisfied. Differentiating (11) gives

$$\begin{aligned} 0 \;=\; & \left[(1+c/\bar{F})\partial_{qq}\pi(\hat{q}(\theta),\theta) - M(\theta)\partial_{\theta qq}\pi(\hat{q}(\theta),\theta)\right]\hat{q}'(\theta) \\ & -M(\theta)\partial_{\theta\theta q}\pi(\hat{q}(\theta),\theta) - \left[M'(\theta) - (1+c/\bar{F})\right]\partial_{\theta q}\pi(\hat{q}(\theta),\theta) \end{aligned} \tag{13}$$

Assuming monotonic inverse hazard rates, i.e.; $d/d\theta[G(\theta)/g(\theta)] > 0$ and $d/d\theta[(1-G(\theta))/g(\theta)]$, $M(\cdot)$ is non-increasing. However, since $M(\underline{\theta}) > 0$ and $M(\bar{\theta}) < 0$, the sign of $\hat{q}'(\theta)$ is ambiguous, even under the usual assumptions on third derivatives of agent's profit function commonly made in standard adverse selection problems. Consequently, the second-order conditions on the emission schedule are more likely to be binding when the emissions are costly to observe than assuming perfect observation of pollution levels.[11]

However, in the particular case of profit functions linear in θ, e.g.; $\pi(q,\theta) = \theta B(q)$ where B is an increasing and concave function of q satisfying $B'(0) = +\infty$, monotonic hazard rates are sufficient to insure an increasing emission schedule. Indeed, in that case we have $\hat{q}(\theta) > 0$ for all θ only if

$$1 + \frac{c}{\bar{F}} > M(\underline{\theta})/\underline{\theta} \tag{14}$$

or equivalently

$$\underline{\theta} g(\underline{\theta}) > \frac{\lambda}{1+\lambda} \frac{1}{1 + c/\bar{F}}$$

that also implies $\hat{q}'(\theta) > 0$ using (13). If (14) is not satisfied, firms with type $\theta < \theta_1$ will choose not to pollute (i.e.; we have $\hat{q}(\theta) = 0$ for all $\theta \leq \theta_1$) , with θ_1 satisfying

$$(1 + c/\bar{F})\theta_1 - M(\theta_1) = 0$$

and the second-order condition is trivially satisfied for all $\theta < \theta_1$.

4 Environmental standard policy

In this section, we investigate the properties of the typical command-and-control policy of a pollution standard enlarged to its enforcement aspects. As we will show, using a increasing fine schedule allows the agency to introduce efficiency considerations in an otherwise rather inefficient policy. The standard policy then appears to be a restricted case of a taxation policy because of the "marginal deterrence" induced by the penalty schedule of its enforcement policy. To make this point more striking, we consider the simplest case of a standard policy where the firms do not report their emission levels to the agency: They simply pay a licence k that allows them to pollute up to an uniform quota z. Since the firms' pollution levels q are not reported to the agency and because the agency cannot distinguish between firms, it

[11]When the monotonicity constraints on the emission schedule bind, the optimal solution entails the bunching of individuals (see Guesnerie and Laffont, 1984, for a formal treatment).

audits all firms with the same inspection rate μ. The task of the agency is thus to decide on a quota level z, a licence k and a fine schedule $f(q)$ (along with a monitoring rate μ) such that $f(q) \geq 0$ whenever the audit reveals an emission level q greater than z. In the following, the standard z will be implicitly defined as the maximal emission level such that the fine is null.

Thanks to the taxation principle, we can use the mechanism-design approach and consider that a uniform standard policy is given by $\{z, k, \mu, F(\tilde{\theta}), q(\tilde{\theta}) : \tilde{\theta} \in \Theta\}$ where $\tilde{\theta}$ is the firm's announcement, $q(\tilde{\theta})$ and $F(\tilde{\theta}) = f(q(\tilde{\theta}))$ two functions of this announcement, and z, k, μ are three constants. The interpretation of this mechanism is the following: The firm reports truthfully its type θ, and pollutes $q(\theta)$ that may be greater or lower than the quota z. Whenever $q(\theta)$ is lower than z, it pays only a lump sum licence k, whereas it is charged an expected amount $k + \mu F(\theta)$ for levels greater than z. The monitoring rate is constrained to be the same for all firms, and we have $F(\tilde{\theta}) = 0$ for all $\tilde{\theta}$ such that $q(\tilde{\theta}) \leq z$.

Observe that inflicting the maximum fine whenever the firms exceed the quota is no longer socially efficient. Indeed, fines proportional to the fraud may induce the more profitable firms to pollute over the standard and thus restore the efficiency of the environmental policy. In that case, the expected fine $\mu F(\tilde{\theta})$ for a pollution $q(\tilde{\theta}) > z$ is close to an environmental tax as discussed in the previous section.[12]

The type-θ firm's expected profit with a quota policy is given by

$$u(\theta, \tilde{\theta}) = \pi(q(\tilde{\theta}), \theta) - k - \mu F(\tilde{\theta}) \tag{15}$$

and the schedule is incentive-compatible if, for all $\theta \in \Theta$,

$$R(\theta) \equiv u(\theta, \theta) \geq u(\theta, \tilde{\theta})$$

and

$$R(\theta) \geq \max_{q} \pi(q, \theta) - k - \mu \bar{F} \tag{16}$$

[12]A standard corresponds to a particular (and constrained) case of a taxation policy. The mechanism proposed is truly stochastic, since the payment depends on the probability of inspection μ. In the unconstrained taxation case, the payment structure is $p(\theta) = t(\theta) + \mu(\theta) f(\theta, q(\theta)) = t(\theta)$, whereas it is given by $p(\theta) = k + \mu F(\theta)$ (for all θ such that $q(\theta) > 0$) in the case of a standard. The mechanism is thus constrained by $t(\theta) = k$ and $\mu(\theta) = \mu$, and economic incentives are given through the expected penalty $\mu F(\theta)$ only.

We also have to take into account the (IR) constraints. However, it may be worthwhile when defining the quota policy to exclude (implicitly) the lower profitability firms by charging a large fee. Indeed, there is an obvious trade-off between the licence charged and the number of firms that are able to pay it. As a consequence, for a given licence k, only firms with type greater than a threshold level $\theta_0 \geq \underline{\theta}$ benefit from polluting, and we have $R(\theta_0) = 0$.

Taking into account the licence and the fine schedule, the social welfare becomes

$$W = \int_{\theta_0}^{\bar{\theta}} \{\pi(q(\theta), \theta) - dq(\theta) + \lambda(k + \mu F(\theta))\} \, g(\theta) d\theta - (1 + \lambda) c\mu$$

Using previous arguments (see lemma 1), an incentive mechanism must satisfy

$$\dot{R}(\theta) = \partial_\theta \pi(q(\theta), \theta) \tag{17}$$

along with a non-decreasing emission schedule $q(\cdot)$ and

$$R(\bar{\theta}) \geq \pi^\circ(\bar{\theta}) - k - \mu \bar{F} \tag{18}$$

which is binding at the optimum of the agency's program and defines μ. The fine is deduced from (15) according to the relation

$$\mu F(\theta) = \pi(q(\theta), \theta) - R(\theta) - k$$

and since $0 \leq F(\cdot) \leq \bar{F}$, we must have

$$0 \leq \pi(q(\theta), \theta) - R(\theta) - k \leq \mu \bar{F} \tag{19}$$

for all θ, where the rightmost inequality is satisfied whenever (16) holds (and thus whenever (18) holds).

The agency's program is thus given by

$$\max_{q(\cdot), R(\cdot), k, \theta_0} \int_{\theta_0}^{\bar{\theta}} \{(1 + \lambda)\pi(q(\theta), \theta) - dq(\theta) - \lambda R(\theta)\} g(\theta) d\theta - (1 + \lambda) \frac{c}{\bar{F}} (\pi^\circ(\bar{\theta}) - R(\bar{\theta}) - k)$$

s.t.

$\dot{R}(\theta) = \partial_\theta \pi(q(\theta), \theta)$

$R(\theta_0) = 0$

$\pi(q(\theta), \theta) - R(\theta) - k \geq 0$

$\pi^\circ(\bar{\theta}) - R(\bar{\theta}) - k \leq \bar{F}$

IV

where $\theta_0 \geq \underline{\theta}$ and where the monotonicity condition on the emission schedule $q(\cdot)$ is neglected and thus must be verified afterward.

Program IV is a simple parametrized optimal control problem, where $R(\cdot)$ stands for the state variable, $q(\cdot)$ for the control variable and k for the parameter. The solution of this program is characterized in the following proposition, assuming that the last constraint is lenient (i.e. $\mu < 1$).

Proposition 3 *Assuming an interior solution in μ, the solution of program IV satisfies*

(i) $k = \pi(z, \theta_0) > 0$.

(ii) $F(\theta) = 0$ *and* $q(\theta) = z < q^{\circ}(\theta_0)$ *for all* $\theta_0 \leq \theta \leq \hat{\theta}$ *with* $\hat{\theta} > \theta_0$.

(iii) z *and* $\hat{\theta}$ *satisfy*

$$\begin{aligned} 0 &= -\lambda(1 - G(\theta_0))\frac{\partial_q \pi(z, \theta_0)}{\partial_q \pi(z, \hat{\theta})} + d\frac{G(\hat{\theta}) - G(\theta_0)}{\partial_q \pi(z, \hat{\theta})} - \int_{\theta_0}^{\hat{\theta}} \frac{\partial_q \pi(z, x)}{\partial_q \pi(z, \hat{\theta})} g(x) dx \\ &\quad -(1 + \lambda)\frac{c}{\bar{F}} + \lambda(1 - G(\hat{\theta})) \\ 0 &= ((1 + \lambda)\partial_q \pi(z, \hat{\theta}) - d) g(\hat{\theta}) + ((1 + \lambda)\frac{c}{\bar{F}} - \lambda(1 - G(\hat{\theta})))\partial_{\theta q} \pi(z, \hat{\theta}) \end{aligned}$$

with $\theta_0 \geq \underline{\theta}$, *and if* $\theta_0 > \underline{\theta}$ *we have*

$$0 = (1 + \lambda)\pi(z, \theta_0) - dz.$$

(iv) *For all* $\theta > \hat{\theta}$, $q(\theta)$ *satisfies*

$$\begin{aligned} \partial_q \pi(q(\theta), \theta) &= \frac{d}{1 + \lambda} - \frac{c}{\bar{F}} \frac{1}{g(\theta)} \partial_{\theta q} \pi(q(\theta), \theta) \\ &\quad + \frac{\lambda}{1 + \lambda} \frac{1 - G(\theta)}{g(\theta)} \partial_{\theta q} \pi(q(\theta), \theta) \end{aligned} \tag{20}$$

Proof. See the appendix.

By (i), the licence corresponds to the benefit that the last profitable firms obtains from polluting at the standard. These firms thus don't benefit from the policy. As revealed by (ii), low profitable firms up to an intermediate profitability type $\hat{\theta}$ pollute at the standard in accordance to their licence and consequently are not fined when inspected. For these firms the quota corresponds to a real constraint compared to the levels they would have chosen without regulation. More profitable firms exceed the standard, the emission levels they choose being given by (20). These levels result from the agency trade-offs between adverse selection costs and audit costs. Indeed, as observed previously, if $\lambda = 0$, the inspection cost induces an increase in the emission levels compared to perfect information ones. On the other hand, if $c/\bar{F}$ is negligeable, the adverse selection effect is the predominant one, and the emission schedule is lower than the perfect information scheme for all but the more profitable firms.

Again, one can easily verify by differentiating (20) that usual assumptions on third derivatives of the profit function and on hazard rates are not sufficient to guarantee an increasing pollution level. The optimal policy may thus involve a bunching of individuals on the subset $[\hat{\theta}, \bar{\theta}]$.

5 An illustrative example

As noted above, a standard policy may be considered as a particular and constrained case of a taxation policy. However, these policies are quite different in practice, and it is useful to illustrate these differences through a simple example. This example will also allows us to assess the value of self-reporting. Let us assume that the firms' types are distributed evenly over $[\bar{\theta} - 1, \bar{\theta}]$, with $\bar{\theta} \geq 1$, and that the profit function is given by $\pi(q, \theta) = \theta B(q)$ with $B(q) = \sqrt{q}$.[13] Under these assumptions, the perfect information emission schedule is

$$q^*(\theta) = [(1 + \lambda)/(2d)]^2 \theta^2.$$

As explained above, with an environmental tax the optimal level of pollution is strictly positive only if $\theta > \theta_1$ given by

$$\theta_1 = \frac{\lambda\bar{\theta} + \underline{\theta}(1 + \lambda)c/\bar{F}}{(1 + \lambda)(1 + 2c/\bar{F}) + \lambda}$$

and every firms with type $\theta \leq \theta_1$ will choose not to pollute. Using (11), the optimal emission schedule for the more profitable firms is given by

$$\hat{q}(\theta) = [(1 + \lambda)/(2d)]^2 \{\theta(1 + c/\bar{F}) + M(\theta)\}^2.$$

[13] Absent a governmental regulation, we would have to assume that there is a maximum level of pollution which would be chosen by all firms.

It is shown in appendix that no firm is excluded under the standard policy (i.e. $\theta_0 = \underline{\theta}$) which means that every firm with type θ less than the threshold type $\hat{\theta}$ given by

$$\hat{\theta} = \underline{\theta} + \sqrt{2\underline{\theta}(1+\lambda)(c/\bar{F})/(1+2\lambda)}$$

pollutes at the standard given by

$$z = [(1+2\lambda)\hat{\theta} + (1+\lambda)c/\bar{F} - \lambda\bar{\theta})]^2/(4d^2).$$

For the more profitable firms, the emission schedule given by (20) reduces to

$$q(\theta) = [(1+2\lambda)\theta + (1+\lambda)c/\bar{F} - \lambda\bar{\theta})]^2/(4d^2).$$

Let us take the following value for the parameters: $d = 1, \underline{\theta} = 1$, $c = 1$, $\bar{F} = 10$ and $\lambda = 0.2$.[14] Figure 2 depicts the emission levels for the different situations (first best, audit cum perfect information, optimal environmental tax, optimal standard). It is shown that the pollution scheme of the optimal tax is mostly above the first best level except for the lowest types. It crosses the pollution scheme e (audit cum perfect information on type) at $\theta_s \simeq 1.625$, i.e.; there is over-deterrence for 62.5 percents of the firms and under-deterrence for 37.5 percents. The pollution quota z is equal to 0.722, the licence to 0.849 and $\hat{\theta}$ to 1.43, which means that 43 percents of the firms are compliant with the standard policy. Compared to the optimal tax, the standard policy implies more (less) pollution for the lowest (highest) types.

The corresponding rents are depicted Fig.3. One can observe that the standard policy leads to the larger extraction of rents by the firms (except for the lowest type). In particular, firms' rents are rapidly increasing with the type under the standard policy, particularly for firms polluting over the quota, even if they have to pay a fine when audited. Optimal inspection probabilities are depicted Fig.4. The lower the type, the higher the inspection probability in the environmental tax case. Indeed, 13 percents of the lowest type firms are inspected, for only 0.5 percent of the highest type. For the standard policy, 5 percents of the total set of firms are inspected. These inspections result in a total cost of audit $c\mathbb{E}_\theta[\mu(\theta)]$ approximately equal to

[14]For these parameters, the emission level for the optimal environmental tax is always strictly positive ($\theta_1 < \underline{\theta}$).

0.075 for the tax policy, while it is only 0.051 for the standard policy. A tax policy thus appears to be more costly than a standard policy to administrate, with 50 percents larger budget devoted to inspection

Social welfare for each type of firms is depicted Fig.5. The total social welfare corresponding to the environmental tax is evaluated to 0.657 while it is 0.650 for the optimal quota. The first best level is about 0.84 and under the Baron-Myerson setting we would obtain 0.763. Note that if the environmental tax policy leads to a higher total welfare than the optimal standard policy, a quota performs better than a tax for the lowest types. Both curves cross at $\theta \simeq 1.55$. Note also that the difference between the social welfare under the tax policy and the social welfare under the standard policy is equal to 0.007, which is approximately 1 percent of the total welfare.

Finally, the taxes and fines under the different policies are depicted Fig.6 and Fig.7. The expected payment for pollution is about 1.11 for the environmental tax policy while it is only 0.89 for the standard policy. One can also observe Fig.7 that we have the largest range of pollution levels under the optimal tax. On the contrary, for the standard policy, the range of pollution levels above the quota is relatively limited. For a given level of pollution, the largest tax levels are the first best levels, and are increasing the more rapidly. Asymmetric information thus contributes to flatten the tax and fines schemes under the different settings. Finally, when audit is either costless (the B-M setting) or costly, the environmental tax is concave with regard to the level of pollution, that is the tax is regressive. On the contrary, with the optimal standard policy, the fine scheme may be convex or progressive.

6 Conclusion

We have studied the optimal environmental taxation and standard policies under asymmetric information with an imperfect and costly audit. Compared to the results of Baron (1985) and Laffont (1994), we have shown that the threat of being audited alter the usual incentives of firms to over-estimate their abatement costs. In particular, depending on firms abatement costs, the optimal policy may involve over or under-deterrence compared to perfect information levels. We also showed that a pollution standard is close to an environmental tax once the economic incentives of the enforcement policy of the standard are considered.

The main policy implications of our analysis are the following. We first showed that environmental quality and fiscal considerations are conflicting objectives. A Pigouvian tax allows to raise some revenues while reducing pollution to its efficient levels. Assuming that the firms do not evade, the agency does not have to be worried about the private information of firms to obtain the double dividend of the policy. However, the efficiency of the environmental policy is severely reduced if the regulator wants primarily to raise tax revenues using an environmental tax. The environmental quality is reduced since the agency gives more allowances to pollute in order to increase the tax base. Moreover, the environmental policy suffers from the usual woes of the other tax systems: Tax evasion and inefficient uses of plants or production facilities to reduce the tax burden. The administration has thus to design an incentive tax schedule to enhance economic efficiency and to perfom costly monitoring activities to enforce the policy. We also demonstrated that monitoring pollution to deter evasion and screening heterogenous firms through taxation are intimately related. From a fiscal viewpoint, environmental taxes are more efficient than standards. Informational rents left to firms are larger under an environmental standard than a taxation policy. Whether this result extends to the case of tradable pollution permits remains an open question and needs further research. However, from an environmental viewpoint, standards appear more efficient. Indeed, as shown in our illustrative example, total pollution may be lower under the optimal standard than under the optimal tax. Moreover, our computations showed that the administrative costs may be greater with a taxation policy than with a standard.

Finally, as usual in most models of audit with commitment, the tax policy analyzed here suffers from a time inconsistency problem. Indeed, no audit is needed ex-post as all firms are compliant with the optimal pollution scheme. In the case where the inspection effort of the agency is not readily verifiable by firms, such a commitment seems unrealistic (for more on this problem, see Khalil, 1997). This is not the case for the command-and-control policy of a pollution standard. Indeed, the monitoring of the firms allows the regulator to raise some revenues in addition to the licences in case of quota violations.

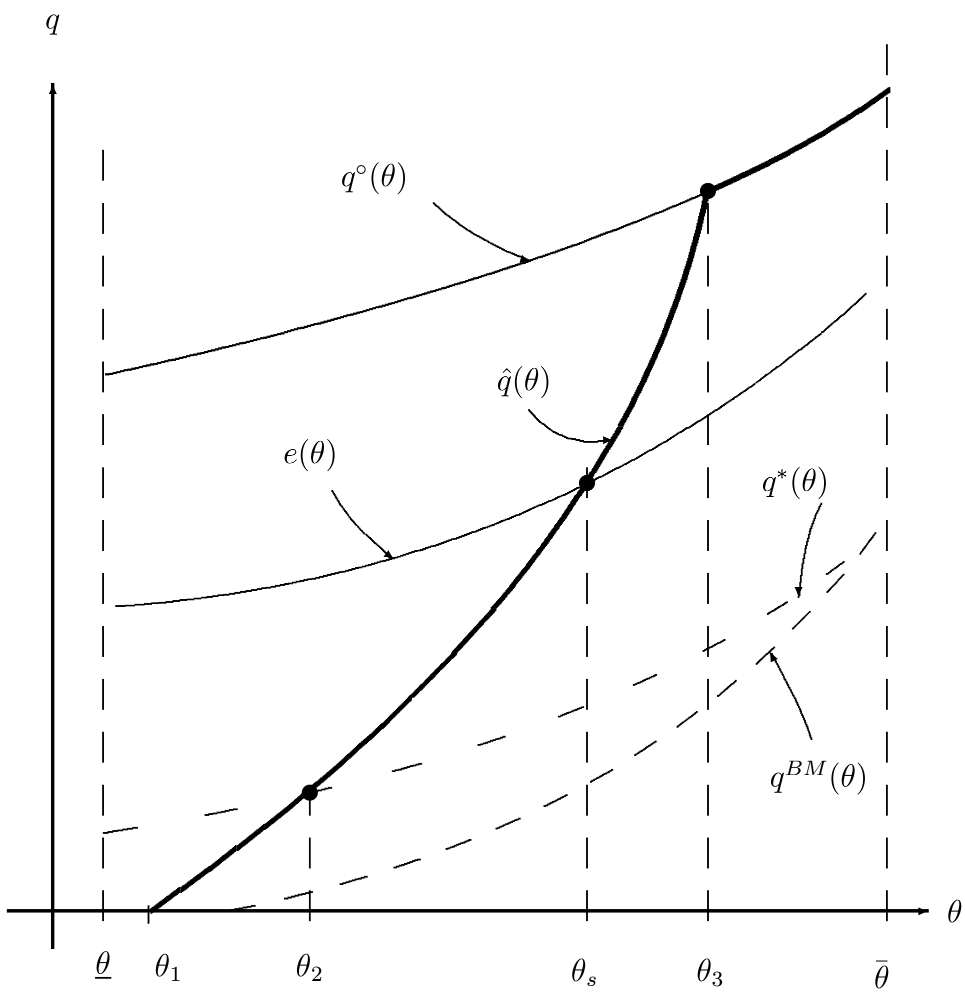

Figure 1: Optimal emission schedule and asymmetric information.

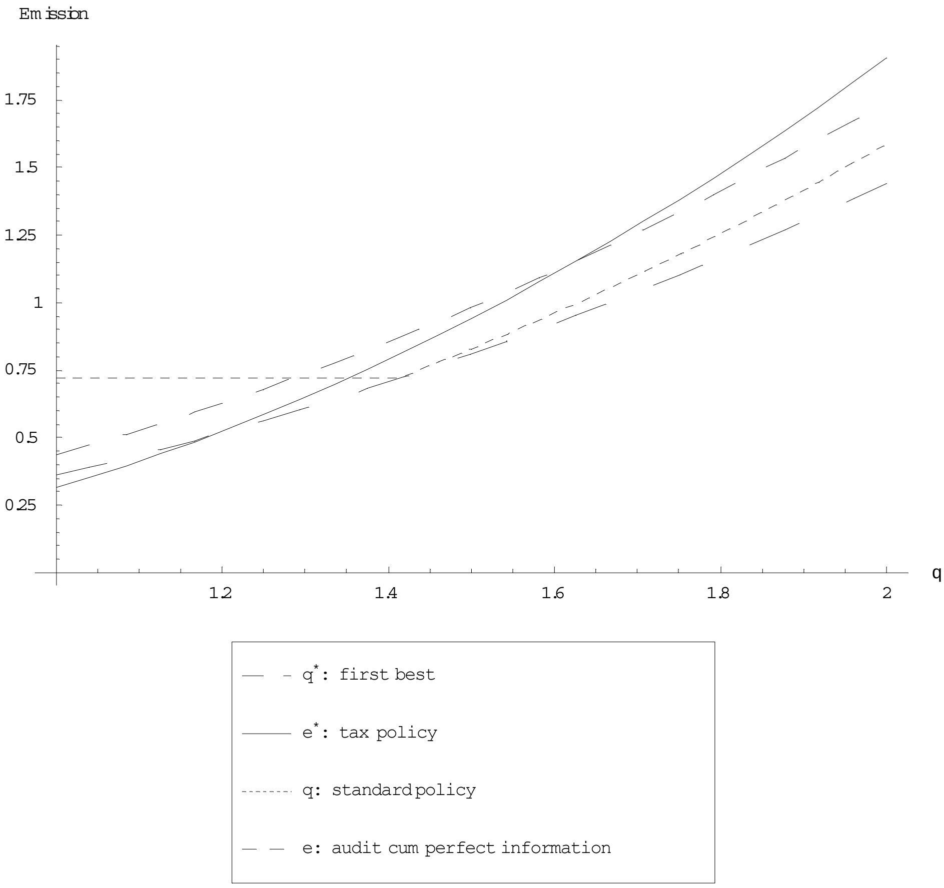

Figure 2: Comparison of emission levels.

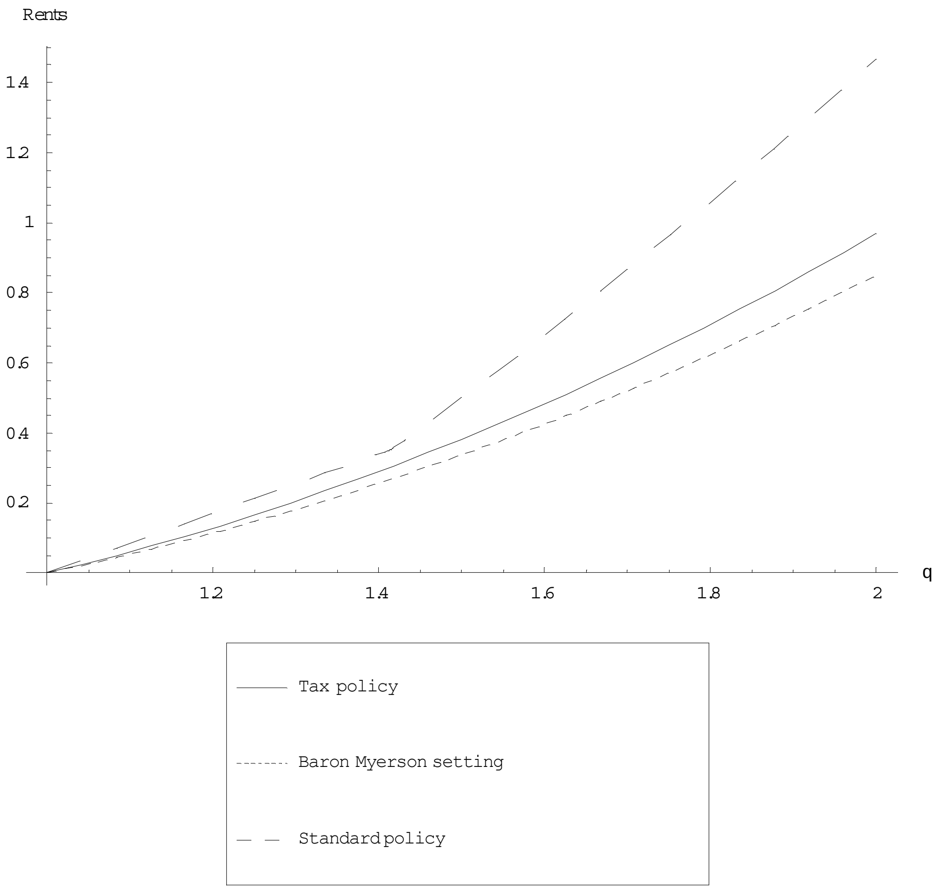

Figure 3: Firms informational rents

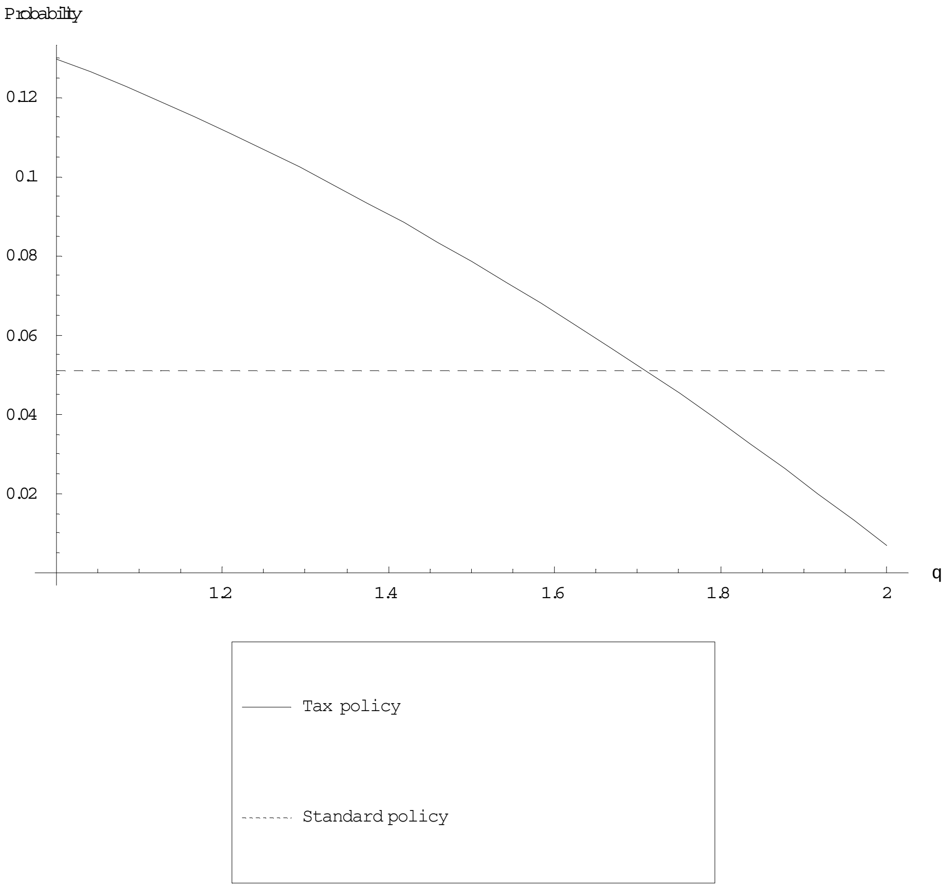

Figure 4: Inspection probabilities

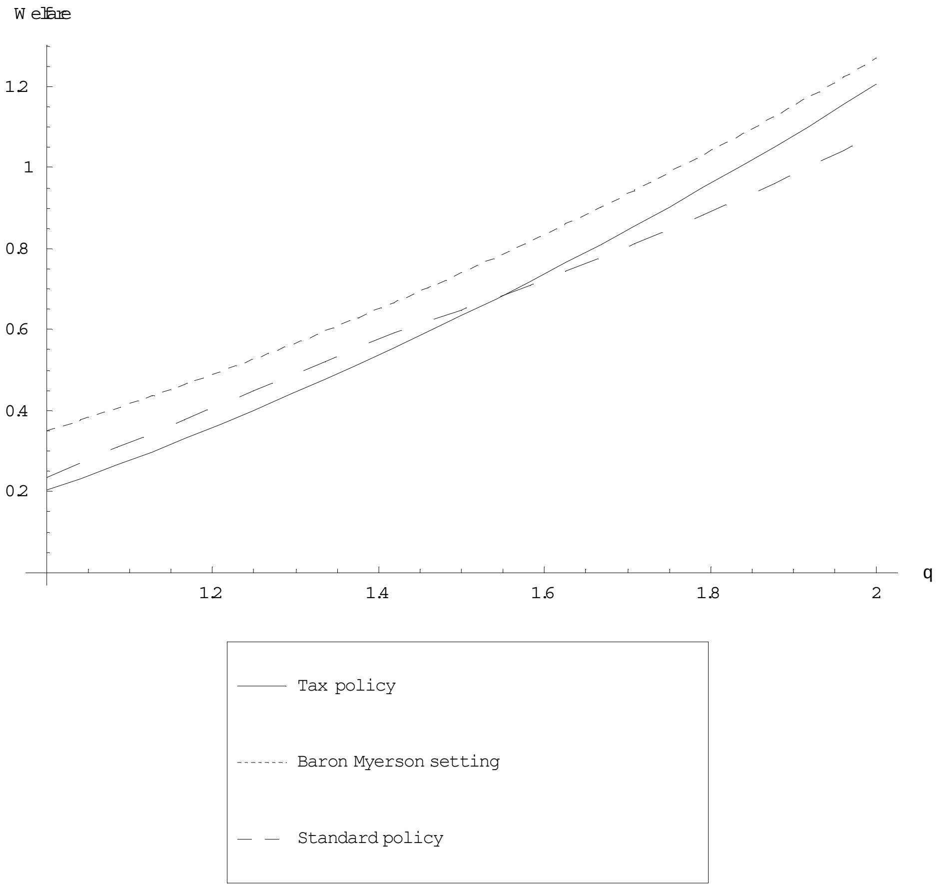

Figure 5: Social welfare.

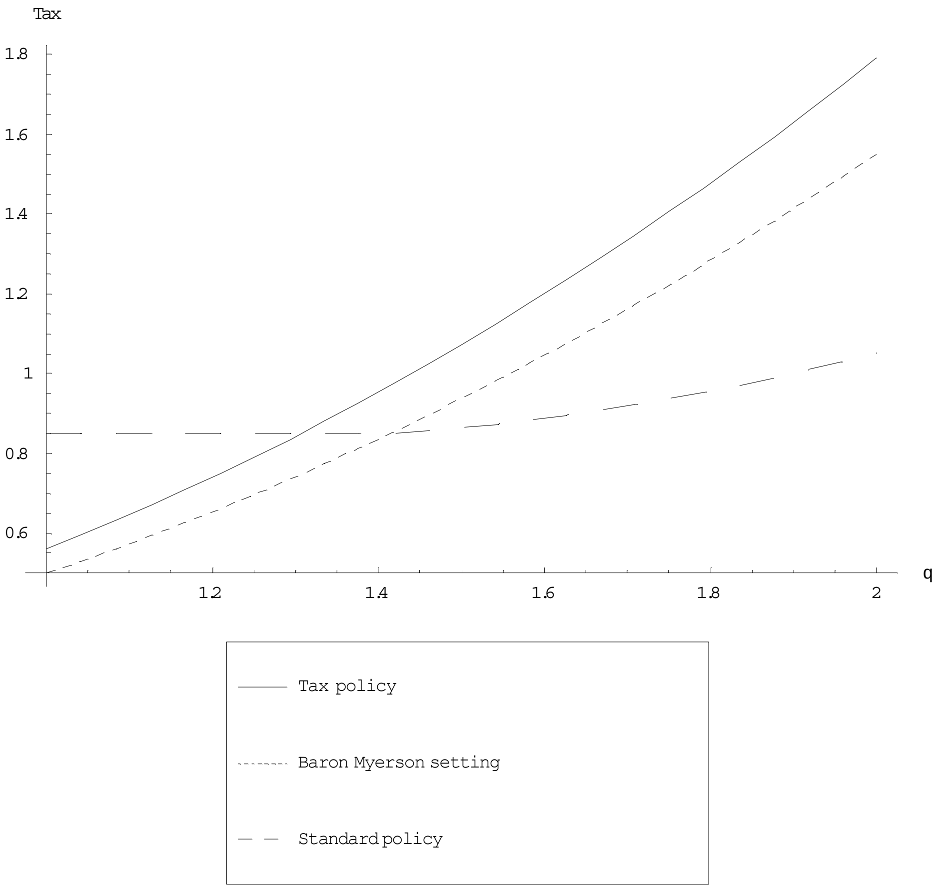

Figure 6: Taxes and expected fines.

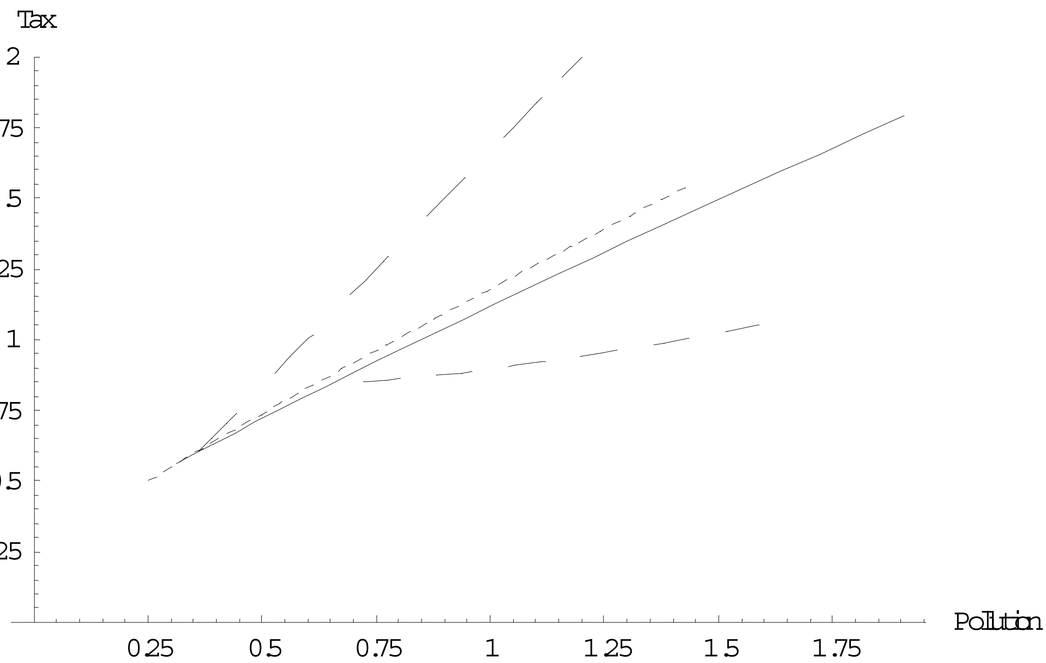

Figure 7: Taxes and fines as functions of the pollution level (big dash: first best, medium dash: expected fine under standard policy, small dash: B-M setting, continuous line: environmental tax).

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Appendix

A Proof of proposition 1

Denoting by $\nu(\cdot)$ the Lagrange multipliers corresponding to (IC1') and by $\tau \geq 0$ the multiplier corresponding (IC2'), the Lagrangian of program III is given by

$$\begin{aligned}\mathcal{L} &= \int_{\underline{\theta}}^{\bar{\theta}} \left\{(1+\lambda)(1+\frac{c}{\bar{F}})\pi(q(\theta),\theta) - dq(\theta) - (\lambda + (1+\lambda)\frac{c}{\bar{F}})R(\theta)\right\} g(\theta)d\theta \\ &\quad -(1+\lambda)K\frac{c}{\bar{F}} + \int_{\underline{\theta}}^{\bar{\theta}} \nu(\theta)(\partial_\theta \pi(q(\theta),\theta) - \dot{R}(\theta))d\theta + \tau(R(\bar{\theta}) - \pi^\circ(\bar{\theta}) + K)\end{aligned}$$

Integrating by parts gives

$$\int_{\underline{\theta}}^{\bar{\theta}} \nu(\theta)\dot{R}(\theta)d\theta = \nu(\bar{\theta})R(\bar{\theta}) - \nu(\underline{\theta})R(\underline{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} \dot{\nu}(\theta)R(\theta)d\theta$$

and the Lagrangian reduces to

$$\mathcal{L} = \int_{\underline{\theta}}^{\bar{\theta}} H(\theta)d\theta - ((1+\lambda)\frac{c}{\bar{F}} - \tau)K - R(\bar{\theta})(\nu(\bar{\theta}) - \tau) - \tau\pi^\circ(\bar{\theta})$$

where

$$\begin{aligned}H &\equiv ((1+\lambda)(1+c/\bar{F})\pi(q(\theta),\theta) - dq(\theta))g(\theta) \\ &\quad +R(\theta)\{\dot{\nu}(\theta) - (\lambda + (1+\lambda)c/\bar{F})g(\theta)\} + \nu(\theta)\partial_\theta\pi(q(\theta),\theta)\end{aligned}$$

Assuming an interior solution, pointwise maximizations give

$$\begin{aligned}\frac{\partial H}{\partial q} &= 0 = g(\theta)((1+\lambda)(1+c/\bar{F})\partial_q\pi(q(\theta),\theta) - d) \\ &\quad +\nu(\theta)\partial_{\theta q}\pi(q(\theta),\theta) \qquad (21)\\ \frac{\partial H}{\partial R} &= \dot{\nu}(\theta) - (\lambda + (1+\lambda)c/\bar{F})g(\theta) = 0 \qquad (22)\end{aligned}$$

with the transversality condition

$$\frac{\partial \mathcal{L}}{\partial R(\bar{\theta})} = \tau - \nu(\bar{\theta}) = 0 \qquad (23)$$

and the first-order condition for K is given by

$$\frac{\partial \mathcal{L}}{\partial K} = \tau - (1+\lambda)\frac{c}{\bar{F}} = 0. \tag{24}$$

Integrating (22) gives

$$\nu(\bar{\theta}) - \nu(\theta) = (\lambda + (1+\lambda)c/\bar{F})(1 - G(\theta))$$

where

$$\nu(\bar{\theta}) = (1+\lambda)c/\bar{F}$$

using (23) and (24). We thus have

$$\begin{aligned} \nu(\theta) &= (1+\lambda)c/\bar{F} - (\lambda + (1+\lambda)c/\bar{F})(1 - G(\theta)) \\ &= (1+\lambda)c/\bar{F}G(\theta) - \lambda(1 - G(\theta)) \end{aligned} \tag{25}$$

Plugging this expression in (21) and rearranging terms gives (11). ■

B Proof of corollary 1

Point (i) is straightforward. We have $\hat{q}(\underline{\theta}) < q^*(\underline{\theta})$ if $(\partial H/\partial q)|_{\theta=\underline{\theta},q=q^*} < 0$ and $\hat{q}(\bar{\theta}) = q^\circ(\bar{\theta})$ if $(\partial H/\partial q)|_{\theta=\bar{\theta},q=q^\circ} \geq 0$. Using (25) and (21),

$$\left.\frac{\partial H}{\partial q}\right|_{\theta=\underline{\theta},q=q^*} = g(\underline{\theta})((1+\lambda)(1+c/\bar{F})\partial_q\pi(q^*(\underline{\theta}),\underline{\theta}) - d) - \lambda\partial_{\theta q}\pi(q^*(\underline{\theta}),\theta)$$

and

$$\left.\frac{\partial H}{\partial q}\right|_{\theta=\bar{\theta},q=q^\circ} = g(\bar{\theta})((1+\lambda)(1+c/\bar{F})\partial_q\pi(q^\circ(\bar{\theta}),\bar{\theta}) - d) + (1+\lambda)c/\bar{F}\partial_{\theta q}\pi(q^\circ(\bar{\theta}),\theta)$$

where $\partial_q\pi(q^*(\underline{\theta}),\underline{\theta}) = d/(1+\lambda)$ and $\partial_q\pi(q^\circ(\bar{\theta}),\bar{\theta}) = 0$ by definition, which gives (ii) and (iii).

C Proof of proposition 2

Denoting by $\gamma(\theta) \geq 0$ the lagrange multiplier corresponding to the constraints (10), the first-order conditions (21)-(24) become

$$\begin{aligned}
\frac{\partial H}{\partial q} &= 0 = g(\theta)((1+\lambda)(1+c/\bar{F})\partial_q \pi(q(\theta),\theta) - d) \qquad (26)\\
&\quad + \rho(\theta)\partial_{\theta q}\pi(q(\theta),\theta) + \gamma(\theta)\partial_q\pi(q(\theta),\theta) \geq 0 \ (q \leq q^\circ)\\
\frac{\partial H}{\partial R} &= \dot{\rho}(\theta) - (\lambda + (1+\lambda)c/\bar{F})g(\theta) - \gamma(\theta) = 0 \qquad (27)\\
\frac{\partial \mathcal{L}}{\partial R(\bar{\theta})} &= \tau - \rho(\bar{\theta}) = 0 \qquad (28)\\
\frac{\partial \mathcal{L}}{\partial K} &= \tau - (1+\lambda)\frac{c}{\bar{F}} - \int_\Theta \gamma(\theta)d\theta = 0. \qquad (29)
\end{aligned}$$

where $\rho(\cdot)$ is the costate variable. Integrating (27) gives

$$\begin{aligned}
\rho(\bar{\theta}) - \rho(\underline{\theta}) &= \lambda + (1+\lambda)c/\bar{F} + \int_\Theta \gamma(\theta)d\theta\\
&= \lambda + \tau\\
&= \lambda + \rho(\bar{\theta})
\end{aligned}$$

using (29) and (28). Consequently, using (25), $\rho(\underline{\theta}) = -\lambda = \nu(\underline{\theta})$ and since

$$\begin{aligned}
\dot{\rho}(\theta) &= (\lambda + (1+\lambda)c/\bar{F})g(\theta) + \gamma(\theta)\\
&\geq (\lambda + (1+\lambda)c/\bar{F})g(\theta)\\
&= \dot{\nu}(\theta)
\end{aligned}$$

by (22), we thus have $\rho(\theta) \geq \nu(\theta)$. Moreover, since $\mu(\cdot)$ is non-increasing, if the constraints (8) bind on a subset of Θ, they bind on an interval $[\underline{\theta}, \theta_1]$ which implies $\rho(\theta) > \nu(\theta)$ for all $\theta > \underline{\theta}$. In that case, whenever $e^*(\theta) < q^\circ(\theta)$, (26) gives

$$\begin{aligned}
\partial_q\pi(e^*(\theta),\theta) &= \frac{d - \rho(\theta)/g(\theta)\partial_{\theta q}\pi(e^*(\theta),\theta)}{(1+\lambda)(1+c/\bar{F})\partial_q\pi(e^*(\theta),\theta) + \gamma(\theta)}\\
&< \frac{d - \nu(\theta)/g(\theta)\partial_{\theta q}\pi(e^*(\theta),\theta)}{(1+\lambda)(1+c/\bar{F})\partial_q\pi(e^*(\theta),\theta)}
\end{aligned}$$

and thus $e^*(\theta) > \hat{q}(\theta)$ whenever $e^*(\theta) < q^\circ(\theta)$. ■

D Proof of proposition 3

Denoting by $\nu(\cdot)$ and $\tau(\cdot)$ the multipliers corresponding to (17) and (19) respectively (with $\tau(\cdot) \geq 0$), the Lagrangian of program IV is given by

$$\mathcal{L} = \int_{\theta_0}^{\bar{\theta}} H(\theta)d\theta - (1+\lambda)\frac{c}{\bar{F}}(\pi^\circ(\bar{\theta}) - R(\bar{\theta}) - k) - \nu(\bar{\theta})R(\bar{\theta}) + \nu(\theta_0)R(\theta_0)$$

where

$$\begin{aligned} H \equiv\ & ((1+\lambda)\pi(q(\theta),\theta) - dq(\theta))g(\theta) + \nu(\theta)\partial_\theta\pi(q(\theta),\theta) \\ & +R(\theta)\{\dot{\nu}(\theta) - \lambda g(\theta)\} + \tau(\theta)(\pi(q(\theta),\theta) - R(\theta) - k) \end{aligned}$$

Pointwise maximizations give

$$\begin{aligned} \frac{\partial H}{\partial q} &= ((1+\lambda)\partial_q\pi(q(\theta),\theta) - d)g(\theta) + \nu(\theta)\partial_{\theta q}\pi(q(\theta),\theta) && (30) \\ &\quad +\tau(\theta)\partial_q\pi(q(\theta),\theta) \geq 0 \ (= 0 \text{ if } q(\theta) < q^\circ(\theta)) \\ \frac{\partial H}{\partial R} &= \dot{\nu}(\theta) - \lambda g(\theta) - \tau(\theta) = 0 && (31) \end{aligned}$$

The first-order conditions for k and θ_0 are given by

$$\frac{\partial \mathcal{L}}{\partial k} = -\int_{\theta_0}^{\bar{\theta}} \tau(\theta)d\theta + (1+\lambda)c/\bar{F} \leq 0 \ (= 0 \text{ if } k > 0) \qquad (32)$$

$$\frac{\partial \mathcal{L}}{\partial \theta_0} = -H(\theta_0) + \dot{\nu}(\theta_0)R(\theta_0) + \nu(\theta_0)\dot{R}(\theta_0) \leq 0 \ (= 0 \text{ if } \theta_0 > \underline{\theta}). \qquad (33)$$

and the transversality conditions by

$$\frac{\partial \mathcal{L}}{\partial R(\bar{\theta})} = -\nu(\bar{\theta}) + (1+\lambda)c/\bar{F} = 0 \qquad (34)$$

and

$$\frac{\partial \mathcal{L}}{\partial R(\theta_0)} = \nu(\theta_0) \leq 0 \ (= 0 \text{ if } R(\theta_0) > 0). \qquad (35)$$

(i) and (ii). Since $c/\bar{F} > 0$ we have, using (32), $k > 0$ and $\tau(\theta) > 0$ for all θ in a non-negligeable subset Θ_1 of Θ. Over Θ_1, we thus have $R(\theta) = \pi(q(\theta), \theta) - k$ which gives $F(\theta) = 0$. Since $F(\cdot)$ is non-decreasing, we must have $\Theta_1 = [\theta_0, \hat{\theta}]$ with $\hat{\theta} > \theta_0$. Moreover, using (17), we must have $(d/d\theta)\pi(q(\theta), \theta) = \partial_\theta \pi(q(\theta), \theta)$ which implies $\partial_q \pi(q(\theta), \theta)\dot{q}(\theta) = 0$ and thus either $q(\theta) = q^\circ(\theta)$ (we then have $\partial_q \pi(q^\circ(\theta), \theta) = 0$) or $q(\theta) = z$ for all $\theta \leq \hat{\theta}$. Assume $q(\theta) = q^\circ(\theta)$. (30) implies

$$\nu(\theta)\partial_{\theta q}\pi(q^\circ(\theta), \theta) > g(\theta) > 0$$

for all $\theta \in [\theta_0, \hat{\theta})$. Integrating (31) over $[\theta, \bar{\theta}]$ and using (34) gives

$$\nu(\theta) = (1 + \lambda)c/\bar{F} - \lambda(1 - G(\theta)) - \int_\theta^{\bar{\theta}} \tau(x)dx \tag{36}$$

and thus, using (32),

$$\begin{aligned} \nu(\theta_0)\partial_{\theta q}\pi(q^\circ(\theta_0), \theta_0) &= \left((1 + \lambda)c/\bar{F} - \lambda(1 - G(\theta_0)) - \int_{\theta_0}^{\bar{\theta}} \tau(\theta)d\theta\right) \partial_{\theta q}\pi(q^\circ(\theta_0), \theta_0) \\ &= -\lambda(1 - G(\theta_0))\partial_{\theta q}\pi(q^\circ(\theta_0), \theta_0) \leq 0 \end{aligned}$$

hence a contradiction. We thus have $q(\theta) = z < q^\circ(\theta_0)$ for all $\theta \in [\theta_0, \hat{\theta}]$. Using (32) and (36), $k > 0$ implies $\nu(\theta_0) = -\lambda(1 - G(\theta_0)) < 0$ and thus $R(\theta_0) = \pi(z, \theta_0) - k = 0$. (33) simplifies to

$$-((1 + \lambda)\pi(z, \theta_0) - dz)g(\theta_0) \leq 0 \;(= 0 \text{ if } \theta_0 > \underline{\theta})$$

and we have $\theta_0 > \underline{\theta}$ if $(1 + \lambda)\pi(z, \underline{\theta}) - dz > 0$.

(iii). Plugging (31) in (30) gives, for all $\theta \in [\theta_0, \hat{\theta})$,

$$\dot{\nu}(\theta) + \nu(\theta)\partial_{\theta q}\pi(z, \theta)/\partial_q \pi(z, \theta) + (1 - d/\partial_q \pi(z, \theta))g(\theta) = 0$$

which is a linear first-order differential equation in $\nu(\cdot)$. The solution is given by $\nu(\theta) = h(\theta)r(\theta)$ where

$$\begin{aligned} \dot{r}(\theta) + r(\theta)\partial_{\theta q}\pi(z, \theta)/\partial_q \pi(z, \theta) &= 0 \\ \dot{h}(\theta)r(\theta) + (1 - d/\partial_q \pi(z, \theta))g(\theta) &= 0 \end{aligned}$$

which gives $r(\theta) = 1/\partial_q \pi(z, \theta)$ and

$$h(\theta) = h(\theta_0) + d[G(\theta) - G(\theta_0)] - \int_{\theta_0}^{\theta} \partial_q \pi(z, x)g(x)dx$$

Using $\nu(\theta_0) = -\lambda(1 - G(\theta_0))$ we have

$$\nu(\theta) = -\lambda(1 - G(\theta_0))\frac{\partial_q\pi(z,\theta_0)}{\partial_q\pi(z,\theta)} + d\frac{G(\theta) - G(\theta_0)}{\partial_q\pi(z,\theta)} - \int_{\theta_0}^{\theta} \frac{\partial_q\pi(z,x)}{\partial_q\pi(z,\theta)}g(x)dx$$

for all $\theta < \hat{\theta}$. By continuity of $\nu(\cdot)$, we have $\nu(\hat{\theta}^{-}) = \nu(\hat{\theta}^{+})$ hence

$$-\lambda(1 - G(\theta_0))\frac{\partial_q\pi(z,\theta_0)}{\partial_q\pi(z,\hat{\theta})} + d\frac{G(\hat{\theta}) - G(\theta_0)}{\partial_q\pi(z,\hat{\theta})} - \int_{\theta_0}^{\hat{\theta}} \frac{\partial_q\pi(z,x)}{\partial_q\pi(z,\hat{\theta})}g(x)dx = (1+\lambda)\frac{c}{\bar{F}} - \lambda(1 - G(\hat{\theta}))$$

Using (30) and (36), we also have

$$((1+\lambda)\partial_q\pi(z,\hat{\theta}) - d)g(\hat{\theta}) + ((1+\lambda)c/\bar{F} - \lambda(1 - G(\hat{\theta})))\partial_{\theta q}\pi(z,\hat{\theta}) = 0$$

(iv) For all $\theta > \hat{\theta}$, (36) gives

$$\nu(\theta) = (1+\lambda)c/\bar{F} - \lambda(1 - G(\theta)).$$

Plugging this expression in (30) gives (20). ■

E Deriving the illustrative example

To determine $\hat{\theta}$ and z we have to solve the following system

$$\begin{aligned} 0 &= -\lambda(\bar{\theta} - \theta_0)\theta_0/\hat{\theta} + d(\hat{\theta} - \theta_0)/\hat{\theta}B'(z) - \int_{\theta_0}^{\hat{\theta}} x/\hat{\theta}dx - (1+\lambda)\frac{c}{\bar{F}} + \lambda(\bar{\theta} - \hat{\theta}) \\ 0 &= ((1+\lambda)\hat{\theta}B'(z) - d) + ((1+\lambda)\frac{c}{\bar{F}} - \lambda(\bar{\theta} - \hat{\theta}))B'(z) \end{aligned}$$

The second equation gives

$$B'(z) = \frac{d}{(1+2\lambda)\hat{\theta} + (1+\lambda)c/\bar{F} - \lambda\bar{\theta}}$$

while the first one reduces to

$$B'(z) = \frac{2d\theta_0}{(1+2\lambda)(\hat{\theta}^2 + \theta_0^2) - 2\lambda\bar{\theta}\theta_0}$$

Equalizing both expressions yields the following second order polynomial in $\hat{\theta}$

$$(1+2\lambda)\hat{\theta}^2 - 2(1+2\lambda)\hat{\theta}\theta_0 + (1+2\lambda)\theta_0^2 - 2\theta_0(1+\lambda)c/\bar{F} = 0$$

which gives

$$\hat{\theta} = \theta_0 + \sqrt{2\theta_0(1+\lambda)c/\bar{F}/(1+2\lambda)}$$

Assuming an interior solution for θ_0, we must have $(1+\lambda)\theta_0 B(z) = dz$ or

$$B'(z) = \frac{d}{1+\lambda}\frac{1}{2\theta_0}$$

assuming $B = \sqrt{}$. Equalizing with the first equation gives

$$2\theta_0(1+\lambda) = (1+2\lambda)\hat{\theta} + (1+\lambda)c/\bar{F} - \lambda\bar{\theta}$$

Replacing $\hat{\theta}$ by its expression and collecting terms, we have to solve

$$\theta_0^2 + 2\lambda\theta_0\left[\bar{\theta} + 2(1+\lambda)c/\bar{F}\right] + \left[(1+\lambda)c/\bar{F} - \lambda\bar{\theta}\right]^2 = 0$$

which admits no positive solution for θ_0. Consequently, $\theta_0 = \underline{\theta}$.