Strategic Liquidity Supply and Security Design*

Bruno Biais† Thomas Mariotti‡

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Abstract

We study how securities and issuance mechanisms can be designed to mitigate the adverse impact of market imperfections on liquidity. In our model, asset owners seek to obtain liquidity by selling claims contingent on privately observed future cash-flows. Liquidity suppliers can be competitive or strategic. In the optimal trading mechanism associated to an arbitrary given security, issuers with low cash-flows sell their entire holdings of the security, while issuers with high cash-flows are typically excluded from trade. By designing the security optimally, issuers can avoid exclusion altogether. We show that the optimal security is debt. Because of its low informational sensitivity, debt mitigates the adverse selection problem. Furthermore, by pooling all issuers with high cash-flows, debt also reduces the ability of a monopolistic liquidity supplier to exclude them from trade in order to better extract rents from issuers with lower cash-flows.

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†Université de Toulouse and CEPR. Correspondence address: IDEI, Manufacture des Tabacs, 21 Allée de Brienne, 31000 Toulouse, France. Email: `biais@cict.fr`.

‡Université de Toulouse, LSE and CEPR. Correspondence address: IDEI, Manufacture des Tabacs, 21 Allée de Brienne, 31000 Toulouse, France. Email: `mariotti@cict.fr`.

1. INTRODUCTION

While corporate finance offers insights in the design of optimal securities (see for instance Townsend (1979), Gale and Hellwig (1985), Allen and Gale (1988), Harris and Raviv (1989)), market microstructure analyzes how different trading mechanisms can offer variable degrees of liquidity, emphasizing the impact of adverse selection and strategic behavior (see for instance Glosten (1989), Biais, Glosten and Spatt (2003)). Borrowing from these two approaches, we study how securities and markets can be designed to mitigate these market imperfections, and thus enhance the liquidity and efficiency of the issuance and trading processes.

To motivate our analysis, consider the owner of random future cash-flows, hereafter called the *issuer*. Because she is impatient or faces current liquidity needs, the issuer would like to sell claims on these cash-flows to more patient or less constrained investors, hereafter called the *liquidity suppliers*. This fits the case of the owner-manager of a firm who applies for a loan from a bank, using trade receivables as collateral. Alternatively, to raise cash, the owner-manager could float a security issue, relying on the intermediation services of an investment bank. Similarly, a firm or a financial intermediary may wish to securitize part of her assets. Finally, consider the case of a firm in which a private equity fund has invested. Private equity funds are impatient since, by design, they have limited horizons. To allow the fund to withdraw cash from the firm, the latter distributes a special dividend, which she finances by applying for a loan, or by floating a security issue.

In all these examples, the extent to which gains from trades can be achieved through the issuance process is subject to two limitations. A first obstacle to trade is that the issuer is likely to have private information about future cash-flows. As emphasized by Leland and Pyle (1977), Myers and Majluf (1984), or DeMarzo and Duffie (1999), this creates an adverse selection problem, and reduces the liquidity of the market for claims contingent on these cash-flows. A second obstacle to trade is the market power of financial intermediaries. Loans are often provided by banks with whom firms have an outstanding relationship (Petersen and Rajan (1994)). For instance, Detragiache, Garella and Guiso (2000) report that 45% of small businesses in the US have relationships with a single bank. This can generate situations of informational monopoly in which the bank can extract rents from the firm (Sharpe (1990), Rajan (1992), von Thadden (1992)). When, instead of applying for a bank loan, the issuer turns to the market to raise funds, she often relies on the intermediation services of financial institutions. Securities issues are typically managed by syndicates of underwriting banks. As argued by Chen and Ritter (2000), the structure of this industry is not very conducive to competitive behavior, because repeated relationships between syndicate members raise the possibility of implicit collusion. Furthermore, the underwriting industry is highly concentrated. For example, Brealey and Myers (2000, §15.2, Table 15.1) report that the six largest underwriters (Merrill Lynch, Salomon Smith Barney, Morgan Stanley, Goldman Sachs, Lehman Brothers, and JP Morgan) managed 68% of the securities issues for the year 1997. Along with the major role of large banks, niche positions are sometimes occupied by specialized financial institutions. Benveniste, Busaba and Wilhelm (2002) report for instance that, out of fifteen trucking-industry IPOs completed between 1990 and 1994, nine were managed by a single bank (Alex. Brown). In such situations, financial intermediaries are likely to enjoy market power.

The goal of the present paper is to analyze the security design and issuance process in presence of market power and adverse selection. In particular, we endeavor to shed light on the following issues: (i) Through what channels does market power affect liquidity ? How does it exacerbate the lemons problem induced by adverse selection? (ii) How do issuers react to the market power of financial intermediaries? How can they mitigate the illiquidity it induces? Does this alter qualitatively the type of security that they issue?

The timing of our security design and trading game is in line with the shelf registration process that is used in practice to issue securities. The issuer first designs her security, such as straight debt, equity or convertible. Then, at a later stage, she actually issues the security as needs for funds arise. Meanwhile, she acquires critical private information about the realization of future cash-flows. Thus, while the security is designed under homogeneous information, it is traded under asymmetric information, as in DeMarzo and Duffie (1999). In line with their model, we consider a risk-neutral framework, in which gains from trade stem from the difference between the issuer's and the liquidity suppliers' rates of time preference. However, we depart from their model in two crucial ways. First, we analyze the consequences of the market power of liquidity suppliers, while they consider competitive liquidity suppliers earning zero expected profits. Second, we take a mechanism design approach to characterize the optimal trading mechanism, while they study a specific trading mechanism, namely a signalling game similar to Kyle (1985). Therefore, both the design of the security and that of the issuance process are endogenous in our analysis.

For a given security designed by the issuer, a trading mechanism consists of a menu of contracts, specifying a transfer for each fraction of the security sold.[1] Once she has received her private information, the issuer selects from this menu the trade size that maximizes her expected utility. To assess the impact of market power on the issuance process, and to disentangle the consequences of market power from the implications of the screening nature of our trading game, we compare two polar cases on the frontier of interim efficient allocations. Supposing at first and for clarity that the issuer faces a single representative liquidity supplier, we distinguish the *competitive case*, in which the issuer has all the bargaining power in designing the trading mechanism, from the *monopolistic case*, in which the liquidity supplier has all the bargaining power in designing the trading mechanism. A key observation is that, in these two treatments, the agents can commit to the trading mechanism before private information is observed and the fraction of the security to be offered is chosen. Because of this additional commitment power, optimal allocations differ from those arising in the separating equilibrium of the signalling game considered by DeMarzo and Duffie (1999), even when, as in the signalling game, the seller has all the bargaining power.

These two ideal polar cases aim at representing two different market structures for the supply of liquidity. While the monopolistic case corresponds to situations in which a privileged liquidity supplier can prevent competition from other liquidity suppliers, or several liquidity suppliers manage to implicitly collude to extract rents from the issuer, the competitive case corresponds to situations in which relationships between the issuer and the liquidity suppliers are non-exclusive and implicit collusion between liquidity suppliers is difficult to enforce. In line with this interpretation, and justifying our terminology, we show that if the traded security is optimal from the issuer's viewpoint, the optimal trading mechanism arising in the competitive case can be decentralized as a Nash equilibrium of an oligopolistic screening

[1] This encompasses the case in which liquidity suppliers posts fixed prices, as in the description of securities issues offered by Brealey and Myers (2000, §15.4). This is also consistent with the IPO process in which investors place offers to buy shares at certain prices (Hanley and Wilhelm (1995), Cornelli and Goldreich (2001), Biais and Faugeron-Crouzet (2002)).

game in which several liquidity suppliers compete in non-exclusive transfer schedules. We shall see below that the non-exclusivity assumption is crucial for this result.

As discussed above, monopolistic liquidity supply is to be expected in many instances. However, competitive liquidity supply is not only theoretically relevant as a benchmark, but also empirically. Firms, especially when large or mature, often have ongoing relationships with multiple banks (Detragiache, Garella and Guiso (2000), Machauer and Weber (2000)). In those cases, contracting is not exclusive, and firms may be able to play banks off against each other. For example, firms can contract with several banks or other financial institutions to refinance commercial paper. Similarly, financial intermediaries who securitize part of their assets typically deal with many investors, and large private equity funds can market bonds to a broad population of investors to raise cash in order to finance their special dividends. In all these cases, the issuer is likely to have significant bargaining power in the trading process, and investors compete in non-exclusive contracts to offer liquidity.

For a given security, and irrespective of the allocation of bargaining power between the issuer and the liquidity supplier, the optimal trading mechanism has the following properties. The worse the private signal of the issuer, the more she is eager to sell the security, and the greater her trade. In line with Akerlof (1970), issuers with large future cash-flows are those who suffer the most from this adverse selection problem. Because of the linearity of the mechanism design problem, the solution has an all-or-nothing feature, reflecting partial market breakdown. Issuers with cash-flows above a certain threshold are entirely excluded from trade, while issuers with cash-flows below this threshold sell 100% of their holdings of the security. Accordingly, the optimal trading mechanism is a fixed-price mechanism, in which the liquidity supplier stands ready to buy any fraction of the security at a constant unit price.[2] Cross-subsidization occurs at equilibrium, as the liquidity supplier makes losses when trading with issuers whose cash-flows are low, while he earns profit when trading with issuers whose cash-flows are high, but not so high as to entail exclusion.

Reflecting the adverse selection problem, and the screening nature of our trading game, the endogenous cost function of the liquidity supplier takes the form of lower-tail conditional expectations, as in Glosten (1994) or Biais, Martimort and Rochet (2000). This implies that, as long as a non-empty set of issuer types is excluded from the market, the price at which the liquidity supplier is ready to purchase any amount of the security is strictly lower than the unconditional expectation of the value of the security. This spread is analogous to the informational component of the small trade spread arising in screening models of market microstructure (Glosten (1994)), and in line with event studies documenting a drop in the market perception of the value of a firm when sales of securities are announced (see for instance Mikkelson and Partch (1986)). While the qualitative features of the market outcome are the same in the competitive and the monopolistic cases, the fraction of issuer types excluded from trade is greater with a monopolistic liquidity supplier. In line with the classical IO paradigm, a monopolist liquidity supplier prefers to reduce the volume of trade by excluding more issuer types, in order to extract greater rents from the remaining

[2] Strictly speaking, this is true in the competitive case only for a restricted class of securities. In Subsection 3.2, we provide a sufficient regularity condition on securities such that a fixed-price mechanism is indeed optimal in this case. We then derive in Subsection 4.2 the optimal regular security from the issuer's viewpoint. We show however in Subsection 5.1 that the unrestricted optimal security from the issuer's viewpoint is indeed regular, so that this restriction does not entail any loss of generality. No such assumption is required in the monopolistic case. See Samuelson (1984) for a discussion of a related point in a general model of bargaining under asymmetric information.

ones. Thus, on top of the discount reflecting the informational content of trade, a monopolist liquidity supplier charges an additional discount reflecting his market power. As a result, and in contrast to the competitive case, the price at which the security is sold is below the expectation of its value conditional on sale occurring. This underpricing corresponds to the underwriter spread.

These results differ markedly from those obtained by DeMarzo and Duffie (1999), namely that (i) infinitesimal trades have an infinitesimal impact on prices, (ii) issuers sell a fraction of the security (which is interpreted as collateralization or tranching), and (iii) the good types are not entirely excluded from the market. This contrast can be traced back to the difference in trading mechanisms. In that respect, it should be noted that the separating equilibrium allocation obtained by DeMarzo and Duffie (1999) is implementable in our screening game, although it is not interim efficient. By contrast, the optimal mechanism in the competitive case provides the upper bound on the expected gains of trade that can be achieved given the agents' preferences and information.

While adverse selection and market power induce inefficiencies, the issuer can design the security to mitigate these imperfections and increase the gains from trade. In both the competitive and the monopolistic cases, we find that the optimal security is a standard debt contract. This result reflects two phenomena.

First, as in Myers and Majluf (1984) and DeMarzo and Duffie (1999), debt mitigates the adverse selection problem, by making the payoff of the security less sensitive to the high cash-flow realizations.[3] Note however that, for the optimal security and the optimal trading mechanism, the outcome of the trading game stands in stark contrast to that in DeMarzo and Duffie (1999). While in the latter high issuer types signal themselves by keeping some of the security, in the former all issuer types sell 100% of the security. Indeed, the face value of the optimal debt contract is set such that even the best types of the issuer are willing to sell. This raises the price that the liquidity supplier is willing to pay for the security, which raises in turn the ex-ante expected utility of the issuer.

Second, and this is a distinctive contribution of our analysis, debt can be used by the issuer to mitigate the adverse consequences of market power on the gains from trade. To maximize his profits, the monopolistic liquidity supplier seeks to reduce the rents earned by the issuers with low cash-flows, by making it costly for them to mimic issuers with high cash-flows. When the payoff of the security increases smoothly with the cash-flow, as with equity, this is achieved by excluding high issuer types from trade. This is no longer the case in the case of debt, provided the face value is not too high. Indeed, with a debt contract, the payoff of the security is the same for all issuers with cash-flows above the debt service. Hence the liquidity supplier must either include them all, or exclude them all from trade. Since the latter is quite costly, as it implies losing a large fraction of the most profitable issuers, he prefers to price the security in such a way that all issuer types participate to trade. By choosing the face value of debt optimally, the issuer can therefore limit the extent to which the liquidity supplier can exert price discrimination against her. As a result, the underwriter spread on the optimal debt contract from the issuer's viewpoint is lower than it would be with an equity contract.

[3] Axelson (2002) also shows that debt can be optimal because of its low informational sensitivity, but our framework differs from his. While we assume that the issuer has private information about the value of the security, he considers the opposite case in which the liquidity suppliers observe private signals. See also Inderst and Müller (2002) for a model of the impact of credit risk analysis on the design of securities.

A key feature of our model is that, both in the competitive and the monopoly cases, the optimal design of the security enables the issuer to avoid exclusion altogether. Therefore, there is no informational content of trade, and the expectation of the value of the security conditional on sale occurring is equal to its unconditional value. This low informational content of the sale of debt securities is in line with the results of several empirical studies (Dann and Mikkelson (1984), Eckbo (1986), Mikkelson and Partch (1986)).

We mentioned above that, if the security traded is optimal from the issuer's viewpoint, the optimal trading mechanism arising in the competitive case can be decentralized as a Nash equilibrium of an oligopolistic screening game in which several liquidity suppliers compete in non-exclusive schedules for the security. Intuitively, this Bertrand outcome obtains because trades convey no information as all issuer types sell 100% of their holdings of the security, and liquidity suppliers quote unit prices equal to the unconditional expected value of the security, so that they break even on average. If instead competition among liquidity suppliers were exclusive, this fully pooling outcome would not be compatible with equilibrium, as in competitive models of insurance under adverse selection (Rothschild and Stiglitz (1976)). Indeed, it would be destabilized by cream-skimming strategies, whereby liquidity suppliers would offer to buy smaller amounts at a higher unit price to attract issuers with high cash-flows. Neither is it true in general that the separating equilibrium arising in the signalling game of DeMarzo and Duffie (1999) is an equilibrium of the game in which liquidity suppliers compete in exclusive schedules for the security. Note however that exclusive competition may not be easy to implement in practice, as it would require the mechanisms offered by each liquidity supplier to be contingent on the mechanisms offered by his competitors.

Our results are robust to relaxing some of our modelling assumptions. In most of the paper, we suppose that the issuer initially designs one security, and is then restricted to that security.[4] We also suppose that both the security payoff and the residual claim of the issuer are increasing in the final cash-flow. This rules out mechanisms which have been shown to be optimal in other security design contexts, such as the "live-or-die" contracts obtained by Innes (1990) under moral hazard. We show that our analysis is robust to these limitations. Specifically, we consider the situation in which, in the competitive case, the issuer initially designs a menu of securities, among which she will be able to choose at the trading stage. We find that the equilibrium allocations arising in this more general setting are exactly the same as those arising in our basic model. Furthermore, we show that in this context no monotonicity assumptions are needed to obtain the optimality of debt.

The paper is organized as follows. The model is described in Section 2. In Section 3, we analyze the design of the issuance process. In Section 4, we turn to the security design problem. Extensions of our analysis are presented in Section 5. Section 6 concludes. All proofs are in the Appendix.

2. THE BASIC MODEL

Our setting is in line with DeMarzo and Duffie (1999). We extend their model along two dimensions. First, we consider arbitrary trading mechanisms. Second, we allow for market power on the part of the liquidity supplier.

[4] This differs from Nachman and Noe (1994), who assume that the security is designed by the issuer after she has observed her private information, and thus conveys a signal of the profitability of her assets.

2.1. The extensive form of the game

Agents. There are two agents, an issuer and a liquidity supplier. The issuer owns assets that generate a future cash-flow X distributed according to a continuously differentiable c.d.f. G with positive density g on a compact interval $\mathcal{X} = [\underline{x}, \overline{x}]$ of $\mathbb{R}_{++}$. Both agents are risk-neutral and the market interest rate is normalized to zero. A key assumption of the model is that the issuer is relatively impatient: she discounts future cash-flows at a higher rate than the liquidity supplier. We denote by $\delta \in (0, 1)$ the discount factor of the issuer, while the discount factor of the liquidity supplier is normalized to one. Since the ex-ante private value of the assets for the issuer, $\delta E(X)$, is lower than the value they have for the liquidity supplier, $E(X)$, there are gains from transferring these assets from the former to the latter in exchange for cash. The extent to which these gains can be achieved is however limited by the private information held by the issuer at the trading stage.

Security design. In order to raise cash, the issuer designs a limited liability security backed by her assets. In general, the payoff F of this security may depend on any ex-post contractible information. For simplicity, we assume that F can only be made contingent on the realized cash-flow X, so that there exists a measurable mapping $\varphi : \mathcal{X} \to \mathbb{R}_+$ such that $F = \varphi(X)$. We impose the following limited liability condition:

(LL) $0 \leq \varphi \leq \mathrm{Id}_{\mathcal{X}}$,

where $\mathrm{Id}_{\mathcal{X}}$ is the identity function on $\mathcal{X}$. Moreover, as Harris and Raviv (1989) or Nachman and Noe (1994), we restrict in a first step the set of admissible securities by requiring that both the payoffs to the liquidity supplier and to the issuer be non-decreasing in the cash-flow:

(M1) φ is non-decreasing on $\mathcal{X}$,

(M2) $\mathrm{Id}_{\mathcal{X}} - \varphi$ is non-decreasing on $\mathcal{X}$.

We denote by $\Phi = \{\varphi : \mathcal{X} \to \mathbb{R}_+ |$ (LL), (M1) and (M2) hold$\}$ the set of admissible payoff functions for the securities. For any admissible security F with payoff function φ, we denote by $\mathcal{F} = \varphi(\mathcal{X}) = [\underline{f}, \overline{f}]$ the set of payoffs associated to F.

Timing and information structure. There are two periods, and five stages. The sequence of events in the first period is as follows:

(i) The issuer designs a security F;

(ii) A transfer schedule $T : [0, 1] \to \mathbb{R}_+$ is designed for the sale of any fraction $q \in [0, 1]$ of the security;

(iii) The issuer privately learns the realization of the cash-flow X;

(iv) The issuer trades a fraction q of the security, for which she obtains a transfer $T(q)$.

Note that since $T(0) \geq 0$ by assumption, the ex-post participation constraint of the issuer is necessarily satisfied at stage (iv) of the game. Finally, in the second period,

(v) The cash-flow X is publicly realized and any remaining consumption takes place.

We shall return in detail to the question of who designs the trading mechanism at stage (ii). Meanwhile, two features of this extensive form are worth emphasizing. First, the issuer has perfect advance knowledge of the cash-flow at the interim stage (iii). This differs from DeMarzo and Duffie (1999), who allow for noisy private signals. Second, the issuer designs her security before she learns the realization of the cash-flow. Therefore, unlike in Nachman and Noe (1994), the choice of a security cannot be used by the issuer as a signal of the profitability of her assets. The assumption that, at stage (i), the issuer designs a single security, rather than a menu of securities, will be relaxed in Section 5.

2.2. Comparison with DeMarzo and Duffie (1999)

The first main difference between our model and the setting considered by DeMarzo and Duffie (1999) is that we take an alternative approach to modelling the trading game. They consider a signalling game, whereby the issuer, after receiving her private information, chooses to sell a fraction q of the security, and competitive liquidity suppliers react to this quantity by quoting prices. In contrast, we take a mechanism design approach. A transfer schedule is a price-quantity menu $\{(T(q), q) \,|\, q \in [0,1]\}$, from which the issuer selects her optimal trade. This menu of trades, designed before the issuer receives her private information, can be interpreted as a screening mechanism. Note that a concave transfer schedule T amounts to a sequence of limit orders, as in Biais, Martimort and Rochet (2000).

Since it is incentive compatible, the allocation that arises in the separating equilibrium considered by DeMarzo and Duffie (1999) is implementable by a transfer schedule. However, it is not interim efficient, as established in Section 3. This is because our timing assumptions yield agents the ability to commit to a price-quantity menu before private information is observed and the fraction of the security to be offered is chosen. This in turn allows to engineer cross-subsidization across the different types of the issuer. Indeed, in the equilibrium of our screening model, the liquidity supplier will earn profits when trading with issuers whose cash-flows are high, while he will make losses when trading with issuers whose cash-flows are low. Cross-subsidization of the bad types by the good types also takes place in the screening games analyzed by Glosten (1994) and Biais, Martimort and Rochet (2000).

The second difference is that, whereas competitive liquidity supply is warranted in the context of DeMarzo and Duffie's (1999) signalling model, we allow for strategic liquidity supply. Specifically, we shall consider two polar cases, corresponding to the extremities of the frontier of interim efficient allocations. In the *competitive case*, the issuer has all the bargaining power at stage (ii), and she designs a transfer schedule to maximize her expected utility, subject to the participation constraint of the liquidity supplier. In the *monopolistic case*, the liquidity supplier has all the bargaining power at stage (ii), and he designs a transfer schedule to maximize his expected profit.

By comparing the allocations arising in the competitive and in the monopolistic cases, we shed light on the consequences of market power for market liquidity. In that respect, the competitive case provides a useful benchmark, in which the ex-ante gains from trade are maximized subject to incentive and participation constraints. As we will see in Section 5, this corresponds to a situation in which several liquidity suppliers compete in non-exclusive schedules for the security, provided the latter is optimal from the issuer's viewpoint.

2.3. Institutional motivation

We now discuss several interpretations of the model, stressing in each case which allocation of bargaining power among the agents is more realistic, and pointing out the circumstances in which the assumption of non-exclusive competition is relevant.

Bank loans. In the first interpretation of the model, the issuer is a firm, and the liquidity supplier is a bank, who extends her a loan secured by future cash-flows. A typical situation in which this occurs is when a firm has extended trade credit to her customers, and pledges the corresponding trade receivables as collateral for a loan. In case of default, the bank can collect the receivables to repay the debt (Brealey and Myers (2000, §32.4)). Relative bargaining power typically depends on the strength of banking relationships, and on the degree of verifiability of information acquired by banks during these relationships. For small or young firms, loans are often provided by a single bank with which the firm has close ties (Petersen and Rajan (1994)). This privileged lender has an incentive to generate significant not transferable information about the firm to prevent other lenders to compete on a par with him and thereby gain monopoly power (Sharpe (1990), Rajan (1992), von Thadden (1992)). Larger or more mature firms may have multiple non-exclusive banking relationships, and may be able to play banks off against each other (Machauer and Weber (2000)).

Shelf registration. In the second interpretation of the model, the issuer is a firm who makes a general cash offer to the public, and the liquidity supplier is an underwriter, or a syndicate of underwriters, who in turn resells the issue to other investors, such as insurance companies or pension funds. In anticipation of future liquidity needs, the firm designs the security and goes through the shelf registration process (Brealey and Myers (2000, §15.4)). As pointed out by DeMarzo and Duffie (1999), significant lags may occur between the design of the security and its actual issuance, during which the issuer can acquire key information about the value of the security. Relative bargaining power depends on how many underwriters there are, and on the degree of collusion in the underwriting market. Large security issues are often split, and sold to the public via a book-building procedure. Several underwriters typically manage the issue, so that competition is non-exclusive. But this in itself may not prevent underwriters to maintain anticompetitive bid-ask spreads (Chen and Ritter (2000)).

Securitization. In the third interpretation of the model, the issuer is herself a financial intermediary, such as an underwriter or a commercial bank, and the liquidity suppliers are investors, such as insurance companies or pension funds. As pointed out by DeMarzo and Duffie (1999), underwriters are likely to have superior ability to price complex securities such as collateralized mortgage obligations. Financial intermediaries can also establish conduits through which trade receivables from different originators are transformed into tradeable securities such as asset-backed commercial paper. Finally, commercial banks can obtain regulatory relief by securitizing some of the loans they have extended to high creditworthy borrowers. In all these cases, the financial intermediary typically trades with several investors, so that competition is non-exclusive.

Private equity. In the last interpretation of the model, the issuer is a firm in which a private equity fund has invested, and the liquidity suppliers are investors, such as mutual funds or hedge funds. Private equity funds are impatient by design, since they have limited horizons. To allow them to withdraw cash from the firm in which they have invested, special dividends are distributed, which are usually financed through bank loans or bond issues.

The managers of large private equity funds play a key role in the negotiation of these issues, as they have the expertise to market these bonds to a broad population of investors, and they are likely to have significant bargaining power in the trading process. In these issues, investors compete in non-exclusive contracts to offer liquidity to the firm.

2.4. *Implementable trading mechanisms*

At stage (iv) of the game, given a security F and a transfer schedule T, the issuer trades a fraction q of the security with the liquidity supplier, conditional on her private information. Since she has perfect advance knowledge of the realization x of the future cash-flow X at this interim stage, and since the payoff of F is only contingent on X, she also perfectly knows the realization $f = \varphi(x)$ of F. Her utility is $T(q) + \delta(x - fq)$, while the profit of the liquidity supplier is $fq - T(q)$. Thus, the type of the issuer is completely summarized by f, and the set of possible types for the issuer is the interval $\mathcal{F}$.

The lemons problem. An issuer with type f prefers to trade a fraction q of the security, rather than not to trade at all and consume x in the second period, if and only if:

$$f \leq \frac{T(q)}{\delta q}.$$

For a given unit price $T(q)/q$ of the security and a given discount factor δ, the willingness to trade reveals a relatively low type. This underscores the adverse selection problem arising in our model, which is very much in line with Akerlof's (1970) lemons problem.

Incentive compatibility. From the Revelation Principle, any allocation achieved via a transfer schedule T can also be achieved via a truthful direct mechanism $(\tau, q) : \mathcal{F} \to \mathbb{R} \times [0, 1]$ that stipulates a transfer and a traded fraction of the security as a function of the issuer's report of her type. Incentive compatibility requires that:

$$f \in \arg\max_{\hat{f} \in \mathcal{F}} \left\{ \tau(\hat{f}) - \delta f q(\hat{f}) \right\}; \quad f \in \mathcal{F}. \tag{1}$$

We denote by U the corresponding informational rent:

$$U(f) = \tau(f) - \delta f q(f); \quad f \in \mathcal{F}. \tag{2}$$

U is analogous to the informational rent of a regulated firm with privately observed marginal cost δf, as in Baron and Myerson (1982). Following Mirrlees (1971), we take the dual approach and characterize the set of pairs (U, q) that correspond to incentive compatible mechanisms.

Lemma 1 *A pair (U, q) is incentive compatible if and only if:*

(i) U is convex on $\mathcal{F}$;

(ii) $\dot{U} = -\delta q$ except on a subset of $\mathcal{F}$ of Lebesgue measure zero.

Lemma 1 simply reflects the fact that U is the upper envelope of a family of affine and decreasing functions of f. Convexity of U together with $\dot{U} = -\delta q$ implies the following important property.

Lemma 2 *In any incentive compatible allocation, the fraction q of the security sold by the issuer is non-increasing in the security payoff f, and consequently in the cash-flow x.*

The intuition is in line with Akerlof (1970). Issuers with high future cash-flows are relatively less eager to trade at a given price than issuers with low future cash-flows. That the latter are always eager to trade depresses the price, which makes issuers with high cash-flows even less eager to trade. This can lead to a partial market breakdown, in which issuers with high cash-flows obtain no gains from trade. Lemmas 1 and 2, and their intuition are similar to Proposition 1 in DeMarzo and Duffie (1999).

Ex-post rationality. Besides the incentive compatibility constraints (1), an implementable trading mechanism must also satisfy the ex-post participation constraint of the issuer. Since the issuer has always the option not to trade, and since in this case she cannot be compelled to pay anything to the liquidity supplier, the issuer's informational rent U must always be non-negative. Since U is non-increasing by Lemma 1, this is equivalent to:

$$U(\overline{f}) \geq 0. \tag{3}$$

2.5. The expected utilities of the agents

Given a security F with payoff function φ, and an incentive compatible trading mechanism (τ, q), the expected profit of the liquidity supplier is:

$$\int_{\mathcal{F}} [f q(f) - \tau(f)] \, dG^{\varphi}(f), \tag{4}$$

where G^{φ} is the c.d.f. of $F = \varphi(X)$, and $dG^{\varphi} = dG \circ \varphi^{-1}$ is the corresponding probability measure on $\mathcal{F}$. Similarly, the expected rent of the issuer is:

$$\int_{\mathcal{F}} [\tau(f) - \delta f q(f)] \, dG^{\varphi}(f). \tag{5}$$

Adding (4) and (5), we obtain the expected gains from trade:

$$(1 - \delta) \int_{\mathcal{F}} f q(f) \, dG^{\varphi}(f). \tag{6}$$

Thus, the expected gains from trade are an increasing function of the difference between the discount rate of the liquidity supplier and that of the issuer, and of the fraction of the security transferred from the issuer to the liquidity supplier.

2.6. Ex-ante efficiency

As a benchmark, consider the case where a benevolent social planner chooses a trading mechanism so as to maximize social welfare. Following Holmström and Myerson (1983), efficiency is defined at an ex-ante stage, that is, before the issuer learns the value of the future cash-flow. Thus an ex-ante efficient mechanism maximizes:

$$\int_{\mathcal{F}} [\tau(f) - \delta f q(f)] \, dG^{\varphi}(f),$$

subject to the participation constraint of the liquidity supplier:

$$\int_{\mathcal{F}} [f q(f) - \tau(f)] \, dG^{\varphi}(f) \geq \pi,$$

for some $\pi \geq 0$. Solving this problem is immediate. The participation constraint of the liquidity supplier is binding, and the entire issue is traded, $q = 1$. It follows then from (6) that the gains from trade are maximized by a pure equity contract, $\varphi = \mathrm{Id}_{\mathcal{X}}$.

3. LIQUIDITY SUPPLY

In this section, we fix a security F with payoff function φ, and we characterize the optimal trading mechanisms for F in both the competitive and the monopolistic cases.

3.1. Competitive versus monopolistic liquidity supply

The competitive case. Suppose first that the issuer has all the bargaining power at stage (ii). Her problem is to find a trading mechanism (τ^c_F, q^c_F) that maximizes her expected rent (5), subject to the incentive compatibility conditions (1), the ex-post participation constraint of the issuer (3), and the participation constraint of the liquidity supplier, that (4) be non-negative. Since the expected profit of the liquidity supplier is equal to the expected gains from trade minus the expected rent of the issuer, this problem consists in finding a pair (U^c_F, q^c_F) that solves:

$$\max\left\{\int_{\mathcal{F}} U(f)\, dG^{\varphi}(f)\right\}, \tag{7}$$

subject to (3) and:

$$\int_{\mathcal{F}} [(1-\delta) f q(f) - U(f)]\, dG^{\varphi}(f) \geq 0, \tag{8}$$

where the maximum in (7) is with respect to all incentive compatible pairs (U, q) characterized in Lemma 1. It is immediate that (8) must be binding in an optimal mechanism. Thus the liquidity supplier makes zero profit on average, and maximizing the expected rent of the issuer amounts to maximizing the expected gains from trade. It should be noted that, since the ex-ante efficient allocation $q = 1$ is clearly incentive feasible, the key difference between this problem and the design of an ex-ante efficient mechanism lies in the ex-post participation constraint of the issuer.

The monopolistic case. Suppose next that the liquidity supplier has all the bargaining power at stage (ii). His problem is to find a trading mechanism (τ^m_F, q^m_F) that maximizes his expected profit (4), subject to the incentive compatibility conditions (1) and the ex-post participation constraint of the issuer (3). Since the expected profit of the liquidity supplier is equal to the expected gains from trade minus the expected rent of the issuer, this problem consists in finding a pair (U^m_F, q^m_F) that solves:

$$\max\left\{\int_{\mathcal{F}} [(1-\delta) f q(f) - U(f)]\, dG^{\varphi}(f)\right\}, \tag{9}$$

subject to (3), where the maximum in (9) is with respect to all incentive compatible pairs (U, q) characterized in Lemma 1. Since giving up rents is costly for the liquidity supplier, it is immediate that (3) must be binding in an optimal mechanism. Thus an issuer with the highest security value $\overline{f}$ earns no rent.

Lemma 3 *Optimal trading mechanisms exist in both the competitive and the monopolistic cases.*

3.2. The optimal trading mechanisms

We are now ready to characterize the optimal trading mechanisms. We shall start with the monopolistic case, which is conceptually simpler.

The monopolistic case. For any subset A of $\mathcal{F}$, let 1_A be the indicator function of A, that is $1_A(f) = 1$ if $f \in A$ and $1_A(f) = 0$ otherwise. The optimal trading mechanism (τ_F^m, q_F^m) offered by the liquidity supplier when the security F is issued is then as follows.

Proposition 1 *There exists a threshold f_F^m such that $q_F^m = 1_{[\underline{f}, f_F^m]}$ and $\tau_F^m = \delta f_F^m 1_{[\underline{f}, f_F^m]}$.*

The structure of the optimal trading mechanism is very simple: issuers with types below a threshold sell 100% of the security, while issuers with types above this threshold do not trade at all. Since the payoff function of the security is non-decreasing in the cash-flow, this implies that issuers with low future cash-flows obtain large gains from trade, while issuers with high future cash-flows can be excluded from the market, and obtain no gains from trade. Transfers are then pinned down by the ex-post participation constraint of the marginal issuer, $U(f_F^m) = \tau_F^m - \delta f_F^m = 0$.

The all-or-nothing feature of the optimal trading mechanism stands in stark contrast to the separating equilibrium of the signalling game studied by DeMarzo and Duffie (1999), in which the fraction of the security sold by the issuer decreases smoothly with her type. The discontinuity of optimal allocations results from the screening nature of the trading game, combined with the linearity of preferences and constraints with respect to the traded fraction of the security. These properties imply that the liquidity supplier maximizes a linear functional on the convex space of non-increasing functions q from $\mathcal{F}$ to $[0, 1]$. Accordingly, the optimal quantity schedule q_F^m is an extreme point of this space, and thus of the form described in Proposition 1. The proof provided in the Appendix establishes this point in a more constructive way, by extending the standard methods of mechanism design to the case of a type distribution G^φ generated by an arbitrary payoff function φ. In particular, G^φ may exhibit atoms, as for instance if F is a debt contract, with payoff function $\varphi = \min\{\mathrm{Id}_{\mathcal{X}}, d\}$ for some $d < \overline{x}$. A key technical tool is the formula of integration by parts for functions of bounded variation (Dellacherie and Meyer (1982, Theorem VI.90)).

In the separating equilibrium of the signalling game analyzed by DeMarzo and Duffie (1999), the unit price of the security is a decreasing function of the fraction of the security sold by the issuer. By contrast, the optimal trading mechanism derived in Proposition 1 is a fixed-price mechanism, and thus the corresponding transfer schedule is linear. By analogy with the monopoly pricing model of Riley and Zeckhauser (1983), it never pays to haggle over the price of the security. In practice, such a mechanism can be implemented with a limit order to buy, or bid price, posted by the liquidity supplier, at which he stands ready to buy any fraction of the security. We can also interpret this arrangement as the option, for the issuer, to sell her security at a predetermined price (Copeland and Galai (1983)).

The competitive case. Things are more complex when the issuer has all the bargaining power at the trading stage. Indeed, the participation constraint of the liquidity supplier has to be satisfied only in expectation. As in the monopoly pricing model of Samuelson (1984), in which the seller holds private information about the gains from trade at the interim stage, this may cause fixed-price mechanisms to be suboptimal.[5] In what follows, we shall focus on

[5]Samuelson (1984) gives an example of a common-value bargaining model under asymmetric information in which the optimal trading mechanism from the informed seller's point of view is a two-step mechanism.

situations in which such mechanisms are nevertheless optimal, which amounts to an implicit restriction on the cash-flow distribution and on admissible securities. We shall verify ex-post that this restriction is innocuous when the security issued is optimal from the issuer's viewpoint, see Subsection 5.1. A sufficient condition under which a fixed-price mechanism is optimal can be expressed in terms of the following mapping:

$$H^{\varphi}(f) = \int_{[\underline{f}, f]} \phi \, dG^{\varphi}(\phi) - \delta G^{\varphi}(f) f; \qquad f \in \mathcal{F}.$$

For any $f \in \mathcal{F}$, $H^{\varphi}(f)$ represents the expected profit of the liquidity supplier if the security is traded at a fixed price δf. Two cases can occur. If $H^{\varphi}(\overline{f}) \geq 0$, then the entire issue can be traded, $q_F^c = 1$, at a price equal to the unconditional expectation of the payoff of the security, $\tau_F^c = \int_{\mathcal{F}} f \, dG^{\varphi}(f)$. If $H^{\varphi}(\overline{f}) < 0$, then there exists a maximal threshold f_F^c such that $H^{\varphi}(f_F^c) = 0$. We will say that the security F is *regular* if $H^{\varphi}(\overline{f}) \geq 0$ or if:

$$J^{\varphi}(f) = \frac{\displaystyle\int_{(f, f_F^c]} \phi \, dG^{\varphi}(f)}{H^{\varphi}(f)}$$

is a non-increasing function of f. This condition can be readily verified in an example, for instance if X is uniformly distributed on $\mathcal{X}$ and the security issued is equity, $\varphi = \mathrm{Id}_{\mathcal{X}}$. The optimal trading mechanism (τ_F^c, q_F^c) offered by the issuer when a regular security F is issued is then as follows.

Proposition 2 *If F is regular, $q_F^c = 1_{[\underline{f}, f_F^c]}$ and $\tau_F^c = \delta f_F^c 1_{[\underline{f}, f_F^c]}$. Moreover, $f_F^c \geq f_F^m$.*

As the exclusion threshold is greater in the competitive than in the monopolistic case, $f_F^c \geq f_F^m$, more gains from trade are achieved in the former than in the latter. As a result, the *informational spread*, that is the difference between the unconditional expected value of the security and the expected value of the security conditional on trade occurring, is greater in the monopolistic than in the competitive case:

$$E(F) - E(F | F \leq f_F^m) \geq E(F) - E(F | F \leq f_F^c).$$

The intuition is that the monopolistic liquidity supplier trades off the benefits of a high volume of trade against the incentive cost of inducing the issuer to report truthfully low values of the security. This rent-efficiency trade-off is less acute when the issuer has all the bargaining power, since the rent extraction motive is absent. In that respect, it should be noted that the allocations characterized in Proposition 1 and 2 only represent two particular interim efficient allocations, in which all the weight is put respectively on the liquidity supplier's and on the issuer's utility. It is not difficult to check that, given a regular security, any interior interim efficient allocation exhibits the same structure as the competitive and monopolistic allocations, with an exclusion threshold between f_F^m and f_F^c.

Given the simplicity of the optimal trading mechanisms, one should expect standard price theory to apply. This is indeed the case, provided some additional conditions are fulfilled. In the competitive case, the participation constraint of the liquidity supplier is binding, and thus, provided F is a regular security, the price at which the security is sold to him is a lower-tail expectation of F,

$$E(F | F \leq f_F^c) = \frac{\displaystyle\int_{[\underline{f}, f_F^c]} f \, dG^{\varphi}(f)}{G^{\varphi}(f_F^c)},$$

as in Akerlof (1970) or Glosten (1994). Whenever $f_F^c < \overline{f}$, issuers with types above f_F^c opt out from trading. The bid price and the exclusion threshold are then pinned down by the ex-post participation constraint of the issuer:

$$\delta f_F^c = E(F|F \leq f_F^c).$$

In line with standard price theory, the valuation of the marginal issuer for the security is equal to the price of the security. A larger mass of low cash-flow issuers lowers the equilibrium price, widens the informational spread $E(F) - E(F|F \leq f_F^c)$, and, in turn, increases further the mass of high cash-flow issuers who are deterred from trading. This is reminiscent of screening models of market microstructure, in which the small trade spread maps into the set of investors who are excluded from trade (Glosten (1989, 1994), Biais, Martimort and Rochet (2000)).

In the monopolistic case, the exclusion threshold f_F^m is chosen so as to maximize the expected profit of the liquidity supplier:

$$f_F^m \in \arg\max_{f \in \mathcal{F}} \{G^\varphi(f)[E(F|F \leq f) - \delta f]\}.$$

Whenever $f_F^m < \overline{f}$, issuers with types above f_F^m opt out from trading. If G^φ is continuously differentiable with positive density g^φ on $\mathcal{F}$ (which is quite restrictive as in excludes for instance debt contracts), and if the first-order approach to the above problem is valid (which is the case for instance if G^φ satisfies condition (10) below), the exclusion threshold is then pinned down by the marginal condition of the liquidity supplier:

$$(1-\delta) f_F^m g^\varphi(f_F^m) = \delta G^\varphi(f_F^m).$$

In line with standard price theory, the expected marginal gain from trade from increasing the exclusion threshold is equal to the expected marginal price impact. From the liquidity supplier's viewpoint, this optimally trades off the benefit of a larger market, from which fewer issuers would be excluded, with the benefit of a smaller market, from which more issuers would be excluded, thereby allowing to extract more rents from the remaining ones.

4. SECURITY DESIGN

We now turn to the security design stage. In both the competitive and the monopolistic cases, the problem of the issuer is to choose a security that maximizes her expected utility, rationally anticipating the equilibrium price at which she will be able to sell this security. For simplicity we assume hereafter that:

$$\left\{ x \in \mathcal{X} \mid \frac{(1-\delta)x}{\delta} - \frac{G(x)}{g(x)} \geq 0 \right\} \text{ is an interval.} \tag{10}$$

Note that as $\mathcal{X} \subset \mathbb{R}_{++}$ by assumption, the worst cash-flow realization $\underline{x}$ belongs to this interval. A sufficient condition is that the mapping $x \mapsto (1-\delta)x/\delta - G(x)/g(x)$ be monotone on $\mathcal{X}$, as when for instance G is the uniform distribution. Condition (10) ensures that one may neglect the constraint that U be convex when solving for the optimal trading mechanism. This enables us to focus on the first-order condition of the mechanism design problem, while warranting that the second-order condition holds.

4.1. Equity and debt

To build some intuition about the security design problem, we first compare the cases of equity and debt. To ease the exposition, we shall focus in Proposition 3 and 4 on fixed-price mechanisms for the trading of securities in the competitive case.[6]

Equity. If the issuer designs a pure equity contract, $\varphi = \mathrm{Id}_{\mathcal{X}}$, the optimal exclusion thresholds f_E^c and f_E^m are as stated in the next proposition.

Proposition 3 *If the issuer designs an equity contract, then:*

(i) In the competitive case, f_E^c is the largest f such that:

$$\int_{\underline{x}}^{f} xg(x)\,dx - \delta G(f)f \geq 0.$$

(ii) In the monopolistic case, f_E^m is the largest f such that:

$$\frac{(1-\delta)f}{\delta} - \frac{G(f)}{g(f)} \geq 0.$$

The intuition is the following. In the competitive case, the issuer determines whether issuers with cash-flow f should be excluded from the market by comparing the expectation of the cash-flow conditional on it being less than f, $\int_{\underline{x}}^{f} xg(x)\,dx/G(f)$, to the valuation δf of an issuer with cash-flow f. In the monopolistic case, the liquidity supplier determines whether issuers with cash-flow f should be excluded from the market by comparing the expected marginal gains from trade that can be achieved with these agents, $(1-\delta)fg(f)$, to the expected marginal rent $\delta G(f)$ issuers with cash-flows below f must be left with. Note that if $(1-\delta)\overline{x}/\delta - 1/g(\overline{x}) \geq 0$, no exclusion occurs and the entire equity issue can be sold at a price $\delta\overline{x}$. This is more likely to happen the smaller is δ, and thus the more impatient the issuer is. For instance, if X is uniformly distributed on $\mathcal{X}$, this occurs as soon as $\delta \leq \overline{x}/(2\overline{x} - \underline{x})$. In that case, the optimal security from the issuer's viewpoint is equity.

Debt. If the issuer designs a debt contract with face value d, $\varphi = \min\{\mathrm{Id}_{\mathcal{X}}, d\}$, the optimal exclusion thresholds f_D^c and f_D^m are as stated in the next proposition.

Proposition 4 *If the issuer designs a debt contract with face value d, then:*

(i) In the competitive case, $f_D^c = d$ if $d \leq d^c$ and $f_D^c = f_E^c$ otherwise, where d^c is the largest d such that:

$$\int_{\underline{x}}^{d} fg(f)\,df + [1 - G(d)]d - \delta d \geq 0.$$

In particular, there is no exclusion whenever $d \leq d^c$.

[6] In the case of equity, one can simply assume that the distribution G of the cash-flow X is such that equity is a regular security, so that a fixed-price mechanism is optimal in the competitive case whenever equity is issued. Things are more complex in the case of debt. Indeed, if the face value of debt is too high, $d > d^c$ in the notation of Proposition 4, debt is an irregular security and it is not clear that a fixed-price mechanism is optimal in the competitive case. However, this is of little concern because we shall eventually prove that debt with face value d^c is an optimal security from the issuer's viewpoint, see Subsection 5.1.

(ii) In the monopolistic case, $f_D^m = d$ if $d \leq d^m$ and $f_D^m = f_E^m$ otherwise, where d^m is the largest d such that:

$$\int_{\underline{x}}^{d} fg(f)\, df + [1 - G(d)]d - \delta d - \int_{\underline{x}}^{f_E^m} (f - \delta f_E^m) g(f)\, df \geq 0.$$

In particular, there is no exclusion whenever $d \leq d^m$.

The intuition is the following. In the competitive case, d^c is the highest face value of debt such that the ex-post participation condition of the issuer (3) is consistent with the participation constraint of the liquidity supplier, that (4) be non-negative, *conditional on the whole issue being traded.* If the face value of debt is too high, $d > d^c$, an optimal fixed-price mechanism requires that issuers with cash-flows above f_E^c be excluded from the market. This effectively converts the debt contract into an equity contract, since $\varphi(x) = x$ for all the issuers with cash-flows $x \leq f_E^c$ who participate in the market. By contrast, if $d \leq d^c$, the whole issue can be sold at a price equal to the unconditional value of the debt issued, $\int_{\underline{x}}^{d} fg(f)\, df + [1 - G(d)]d$. In that case, all the types of the issuer sell their security, and thus achieve positive gains from trade.

In the monopolistic case, d^m is the highest face value of debt such that the liquidity supplier obtains a higher expected profit, *conditional on the whole issue being traded*, than under an equity contract. When the issuer designs a debt contract with face value d, the liquidity supplier has the option to exclude the upper-tail of the payoff distribution by setting a price strictly below δd for the security. If d is larger than d^m, it is indeed optimal for the liquidity supplier to do so. The exclusion threshold is then optimally set at f_E^m, so that, just as the issuer in the competitive case, the liquidity supplier is effectively converting a debt contract into an equity contract. On the other hand, if $d \leq d^m$, then the liquidity supplier prefers to allow the whole issue to be traded, at a price δd.

Remark. In the model of DeMarzo and Duffie (1999), the interpretation of $\min\{\mathrm{Id}_{\mathcal{X}}, d\}$ as a standard debt contract requires the unsold fraction of the security to be off the balance sheet of the issuer at the time of default. This does not arise in our setting since, when the issuer trades, the security is entirely transferred to the liquidity supplier.

Proposition 4 allows us to compare debt and equity from the issuer's viewpoint at stage (i) of the game. In the competitive case, the ex-ante utility of the issuer if a debt contract with face value $d \in [\underline{x}, d^c]$ is issued is given by:

$$(1 - \delta)\left\{\int_{\underline{x}}^{d} fg(f)\, df + [1 - G(d)]d\right\}. \tag{11}$$

Similarly, in the monopolistic case, the ex-ante utility of the issuer if a debt contract with face value $d \in [\underline{x}, d^m]$ is issued is given by:

$$\delta \int_{\underline{x}}^{d} (d - f) g(f)\, df. \tag{12}$$

Since both (11) and (12) are strictly increasing in d on the relevant ranges, the optimal debt contract is reached for a face value d^c in the competitive case and d^m in the monopolistic case.[7] Of course, since the support of the cash-flow distribution is bounded above by $\overline{x}$, equity is just a particular case of debt with a face value equal to $\overline{x}$. Hence an optimal debt contract always weakly dominates a pure equity contract from the issuer's viewpoint. The following proposition delineates the circumstances in which debt strictly dominates equity.

Proposition 5 *In both the competitive and the monopolistic cases, a pure equity contract is strictly dominated by the optimal debt contract from the issuer's viewpoint if and only if it entails exclusion at the trading stage.*

The same argument goes for both the competitive and monopolistic cases. Recall that for each $i \in \{c, m\}$, d^i is defined as the maximal face value for debt such that there is no exclusion in an optimal trading mechanism. If $f_E^i = \overline{x}$, a pure equity contract entails no exclusion, and therefore $d^i = \overline{x}$ as well. In both case, the first-best allocation is reached. If $f_E^i < \overline{x}$, a debt contract with face value f_E^i is equivalent, from the issuer's viewpoint, to a pure equity contract. Indeed, with the former, the whole issue can be entirely sold, but issuers with cash-flows above f_E^i earn no rent. Similarly, with the latter, only issuers with cash-flows up to f_E^i are able to sell their equity. As the price of the security is δf_E^i in both cases, the issuer is ex-ante indifferent between the two contracts. However, it is easy to check that $d^i > f_E^i$ precisely when $f_E^i < \overline{x}$. Hence, the issuer can increase his ex-ante utility (11) or (12) by raising the face value of debt from f_E^i to d^i. It is easy to check that $d^m < d^c$ whenever $f_E^m < \overline{x}$, reflecting the market power of the liquidity supplier in the monopolistic case.

Technically, designing a debt contract with face value d instead of an equity contract amounts to pool all issuers with cash-flows in $[d, \overline{x}]$, thereby generating an atom at d in the distribution of types. Since the liquidity supplier is making profits when trading with issuers with high security payoffs, this relaxes his participation constraint (in the competitive case), and reduces his incentive to restrict the volume of trade (in the monopolistic case), provided that d is not high enough to jeopardize the incentives of the issuer. The latter is thus able to get a better price δd for her security and thereby to increase her ex-ante utility.

Underwriter spreads. We shall call *underwriter spread* the mark-up between the expected value of the security conditional on trade occurring and the price at which it is purchased by the liquidity supplier. This terminology is consistent with the case in which the security is purchased by a liquidity supplier who then resells it to investors on a secondary market at a price equal to its expected value conditional on trade occurring on the primary market. Of course, in the competitive case, the underwriter spread should be zero irrespective of the security traded. In the monopolistic case, the underwriter spread on debt with face value d^m, S_D^m, is smaller than the underwriter spread on equity, S_E^m, because the liquidity supplier is indifferent between the two contracts by definition of d^m. Formally, if $f_E^m < \overline{x}$, one has:

$$S_D^m = \int_{\underline{x}}^{d^m} f g(f)\, df + [1 - G(d^m)]d^m - \delta d^m = \int_{\underline{x}}^{f_E^m} f g(f)\, df - \delta G(f_E^m) f_E^m$$

$$< \frac{\displaystyle\int_{\underline{x}}^{f_E^m} f g(f)\, df}{G(f_E^m)} - \delta f_E^m = S_E^m.$$

[7] Recall however that we restrict ourselves to fixed-price mechanisms in the competitive case. While this is without loss of generality whenever $d \leq d^c$, this is not so for $d > d^c$, unless d is close to $\overline{x}$. But, as pointed out above, this restriction has no bearing on our final results.

The intuition is that, by designing a debt contract optimally, the issuer is able to reduce the extent to which the liquidity supplier can extract rents from her. This ranking of the spreads is consistent with empirical evidence showing that underwriter spreads on equity are significantly larger than those prevailing for debt issues (Brealey and Myers (2000, §15.4, Table 15.2)). One could alternatively interpret this fact as evidence that there is greater competition among underwriters on debt than on equity markets. However, our results suggest that even if the same degree of competition prevails on both markets, the difference in spreads can simply reflect the fact that the face value of debt is optimally chosen by the issuer to counter the monopoly power of the liquidity supplier.

In that respect, it should be noted that this prediction would be reversed if the face value of debt were chosen by the liquidity supplier rather than by the issuer. To show this, observe that if the liquidity supplier selected the face value of debt, he would simply maximize the underwriter spread on debt:

$$S_D(d) = \int_{\underline{x}}^{d} fg(f)\,df + [1 - G(d)]d - \delta d; \qquad d \in \mathcal{X}. \tag{13}$$

It is easy to check that the maximum value of S_D corresponds to a face value $G^{-1}(1-\delta) < d^m$. To show that this results in a higher underwriter spread than under equity, note that setting d in (13) equal to the optimal exclusion threshold f_E^m under equity yields:

$$S_D(f_E^m) = G(f_E^m)\,\frac{\int_{\underline{x}}^{f_E^m} fg(f)\,df}{G(f_E^m)} + [1 - G(f_E^m)]f_E^m - \delta f_E^m > \frac{\int_{\underline{x}}^{f_E^m} fg(f)\,df}{G(f_E^m)} - \delta f_E^m = S_E^m.$$

This can be interpreted as follows. By capping the payoff of the security, debt reduces adverse selection, and thus facilitates trade. The issuer and the liquidity supplier take advantage of this feature in different ways, reflecting their different perceptions of the rent-efficiency trade-off. When the issuer designs the debt contract, she sets the face value of debt at the maximal level compatible with no exclusion at the trading stage, to generate the largest possible expected gains from trade. By contrast, if the liquidity supplier designs the debt contract, he sets the face value level at a lower level to extract more rents from the issuer, at the cost of lowering the expected gains from trade.

4.2. Debt as the optimal security

To begin with, we establish properties of an optimal security that hold irrespective of the allocation of bargaining power between the issuer and the liquidity supplier. The optimality of debt then requires a separate argument in the competitive and monopolistic cases. We emphasize that we restrict attention to regular securities in the competitive case, and we shall use the terminology "(regular) security" to indicate that this restriction is made in the competitive case only. We will establish in Subsection 5.1 that this restriction does not entail any loss of generality.

Preliminaries. Risk-free cash-flows are not subject to adverse selection. Hence, in line with Myers and Majluf (1984), it is always weakly optimal for the issuer to sell these, in order to maximize trade and thus the gains from trade. That is, it is weakly optimal to design a security to yield at least the worst possible realization of the cash-flow, $\underline{x}$.

Lemma 4 *If F is an optimal security from the issuer's viewpoint, then $\varphi(\underline{x}) = \underline{x}$ without loss of generality.*

It follows from Proposition 4 that, in both the competitive and the monopolistic cases, the optimal debt contract is such that the whole issue is entirely traded between the two agents, and therefore no exclusion occurs. The following proposition, which is key to our results, shows that the same holds true more generally in respect of any optimal (regular) security.

Proposition 6 *If F is an optimal (regular) security from the issuer's viewpoint, no exclusion occurs at the trading stage.*

In other words, given an optimal (regular) security, all the types of the issuer entirely sell their holdings of the security to the liquidity supplier at the trading stage. The proof of this result generalizes the argument used when comparing debt and equity. Specifically, given any (regular) security F that is conducive to exclusion at the trading stage, the issuer can design an alternative security $\overline{F}$ whose payoff function function is the same as that of security F, except that it is capped at a level equal to the exclusion threshold under F (in the competitive case), or slightly above that level (in the monopolistic case). Introducing such a flat payoff at the top of the cash-flow distribution ensures that one can find a price for this alternative security $\overline{F}$ under which all the types of the issuer are ready to trade. This can be seen as follows.

In the competitive case, the price of $\overline{F}$ can be set equal to its expected value, $E(\overline{F})$, since the highest type of the issuer under $\overline{F}$ is f^c_F, and:

$$E(\overline{F}) > E(F|F \leq f^c_F) = \delta f^c_F. \tag{14}$$

In turn, since the liquidity supplier rationally anticipates that all the types of the issuer are ready to sell, he will accept to buy $\overline{F}$ at a price equal to its expected value. Note that (14) implies that the price of $\overline{F}$ is strictly greater than the price of F, so that $\overline{F}$ strictly dominates F from the issuer's viewpoint.

In the monopolistic case, let $f^m_F + \varepsilon$ be the highest type of the issuer under $\overline{F}$, for some $\varepsilon > 0$. Then all the types of the issuer are ready to sell at a price $\delta(f^m_F + \varepsilon)$. Provided ε is close enough to zero, one has:

$$E(\overline{F}) - \delta(f^m_F + \varepsilon) > E(F|F \leq f^m_F) - \delta f^m_F. \tag{15}$$

This implies that the liquidity supplier strictly prefers to buy $\overline{F}$ at a price $\delta(f^m_F + \varepsilon)$, rather than excluding the types above f^m_F. From (15), this is because he rationally anticipates that all the types of the issuer are ready to sell at this price, including a positive mass of issuers with high security payoffs between f^m_F and $f^m_F + \varepsilon$. Since the price of $\overline{F}$ is strictly greater than the price of F, $\overline{F}$ strictly dominates F from the issuer's viewpoint. Intuitively, by capping the payoff of the security at a slightly higher level than the exclusion threshold under F, the issuer is able to obtain a better price for her security, while reducing the extent of price discrimination that the liquidity supplier can exercise against her.

Proposition 6 emphasizes the difference between the optimal trading mechanisms and the separating equilibrium of the signalling model studied by DeMarzo and Duffie (1999). In their model, as in Leland and Pyle (1977), the issuer can credibly signal the quality of his assets only by retaining part of the cash-flow generated by these assets. For an arbitrarily chosen (regular) security, the analogue of this phenomenon in our screening model is the possibility of excluding the high types of the issuer. From the issuer's viewpoint, this way of signalling the quality of her assets is however very costly, and she is better off designing her security to avoid exclusion altogether. As a consequence, the market for an optimal (regular) security is perfectly liquid, and the informational spread is eliminated.

The competitive case. In the competitive case, the problem of the issuer at stage (i) of the game is to design a regular security that maximizes the expected gains from trade, anticipating the price which she will set for this security at stage (ii). Proposition 5 simplifies this problem by mandating to focus on securities that entail no exclusion at the trading stage. Thus the problem of the issuer is to find a security payoff function φ^c that solves:

$$\max_{\varphi \in \Phi} \left\{ (1-\delta) \int_{\mathcal{X}} \varphi(x) g(x)\, dx \right\}, \tag{16}$$

subject to the no-exclusion constraint:

$$\int_{\mathcal{X}} \varphi(x) g(x)\, dx \geq \delta \varphi(\overline{x}). \tag{17}$$

The no-exclusion constraint (17) is an ex-post participation constraint for the issuer, requiring that the price $\int_{\mathcal{X}} \varphi(x) g(x)\, dx$ of the security be greater than its present value for all types of the issuer, including the issuer with the greatest possible cash-flow, $\overline{x}$. Note that in formulating the issuer's problem as (16)–(17), we have implicitly taken into account the participation constraint of the liquidity supplier, which must be binding at the optimum. Let us form the Lagrangian:

$$L^c(\varphi, \lambda) = (1-\delta) \int_{\mathcal{X}} \varphi(x) g(x)\, dx + \lambda \left[\int_{\mathcal{X}} \varphi(x) g(x)\, dx - \delta \varphi(\overline{x}) \right],$$

where λ is the Lagrange multiplier associated to (17). The limited liability and monotonicity conditions (LL), (M1) and (M2) imply that any $\varphi \in \Phi$ is absolutely continuous. It follows that the derivative $\dot{\varphi}$ is well-defined except on a subset of $\mathcal{X}$ of Lebesgue measure zero, with $0 \leq \dot{\varphi} \leq 1$ and $\varphi(x) = \int_{\underline{x}}^{x} \dot{\varphi}(\xi)\, d\xi$ for all $x \in \mathcal{X}$. Hence, integrating by parts, we get:

$$L^c(\varphi, \lambda) = -(1+\lambda-\delta) \int_{\mathcal{X}} G(x) \dot{\varphi}(x)\, dx + (1-\delta)(1+\lambda) \varphi(\overline{x}).$$

The maximization of $L^c(\varphi, \lambda)$ with respect to φ can thus be treated as a standard optimal control problem. We then have the following result.

Proposition 7 *In the competitive case, the debt contract with face value d^c is an optimal regular security from the issuer's viewpoint.*

The intuition is that a debt contract trades off in an optimal way two conflicting objectives. On the one hand, it is efficient to transfer as much cash-flow as possible from the issuer to the liquidity supplier. On the other hand, the lemons problem limits the extent to which this can be done. By imposing a cap on the security payoff, a debt contract minimizes this adverse selection cost, in line with Myers and Majluf's (1984) pecking-order hypothesis.

Proposition 7 derives the optimal regular security from the issuer's viewpoint. This leaves open the possibility that a non regular security conducive to partial exclusion at the trading stage could lead to greater expected gains from trade. We will however establish in Subsection 5.1 that debt with face value d^c is optimal in a larger class of contracts than the one considered so far, and therefore represents an optimal security in our basic model, without restrictions on the set of admissible securities.

The monopolistic case. In the monopolistic case, the problem of the issuer at stage (i) of the game is to design a security that maximizes her expected utility, anticipating the price which the liquidity supplier will set for this security at stage (ii). Proposition 5 simplifies this problem by mandating to focus on securities that entail no exclusion at the trading stage. Thus the problem of the issuer is to find a security payoff function φ^m that solves:

$$\sup_{\varphi \in \Phi} \left\{ \delta \int_{\mathcal{X}} [\varphi(\overline{x}) - \varphi(x)] g(x)\, dx \right\}, \tag{18}$$

subject to the no-exclusion constraint:

$$\int_{\mathcal{X}} [\varphi(x) - \delta\varphi(\overline{x})] g(x)\, dx \geq \int_{\underline{x}}^{\tilde{x}} [\varphi(x) - \delta\varphi(\tilde{x})] g(x)\, dx; \qquad \tilde{x} \in \mathcal{X}. \tag{19}$$

The no-exclusion constraint (19) can be interpreted in analogy with a principal-agent problem. The principal is the issuer, who designs the security, while the agent is the liquidity supplier, whose decision is whether or not to exclude issuers with cash-flows above $\tilde{x}$, for any value of $\tilde{x} \in \mathcal{X}$. Note that in formulating the issuer's problem as (18)–(19), we have implicitly taken into account the fact that the liquidity supplier can exclude issuers with cash-flows above $\tilde{x}$ by setting a price equal to $\delta\varphi(\tilde{x})$ for the security. Let us form the Lagrangian:

$$\begin{aligned} L^m(\varphi, \Lambda) \;=\; & \delta \int_{\mathcal{X}} [\varphi(\overline{x}) - \varphi(x)] g(x)\, dx \\ & + \int_{\mathcal{X}} \left\{ \int_{\mathcal{X}} [\varphi(x) - \delta\varphi(\overline{x})] g(x)\, dx - \int_{\underline{x}}^{\tilde{x}} [\varphi(x) - \delta\varphi(\tilde{x})] g(x)\, dx \right\} d\Lambda(\tilde{x}), \end{aligned}$$

where Λ is the Lagrange multiplier associated to (19). It is a distribution function on $\mathcal{X}$, that is, a non-decreasing and right-continuous function such that $\Lambda(\underline{x}) = 0$. The following lemma provides a sufficient condition for an optimal security.

Lemma 5 *Let $\varphi \in \Phi$, and Λ be a distribution function on $\mathcal{X}$ such that:*

$$\int_{\mathcal{X}} \left\{ \int_{\mathcal{X}} [\varphi(x) - \delta\varphi(\overline{x})] g(x)\, dx - \int_{\underline{x}}^{\tilde{x}} [\varphi(x) - \delta\varphi(\tilde{x})] g(x)\, dx \right\} d\Lambda(\tilde{x}) = 0$$

and:

$$L^m(\varphi, \Lambda) \geq L^m(\tilde{\varphi}, \Lambda); \qquad \tilde{\varphi} \in \Phi.$$

Then φ solves (18)–(19).

To prove the optimality of debt, we proceed as follows. The optimal debt contract from the issuer's viewpoint has face value d^m. Given this contract, the only point besides $\tilde{x} = \overline{x}$ at which the no-exclusion constraint (19) is binding is at $\tilde{x} = f_E^m$. This suggests that Λ is a point mass at f_E^m, that is, a mapping of the form $\Lambda_\lambda = \lambda 1_{[f_E^m, \overline{x}]}$ for some $\lambda > 0$. For this choice of Λ, the Lagrangian can be rewritten as:

$$\begin{aligned} L^m(\varphi, \Lambda_\lambda) \,=\, & (1-\lambda)\,\delta \int_{\mathcal{X}} [\varphi(\overline{x}) - \varphi(x)] g(x)\,dx \\ & + \lambda \left\{ \delta \int_{\underline{x}}^{f_E^m} [\varphi(f_E^m) - \varphi(x)] g(x)\,dx + (1-\delta) \int_{f_E^m}^{\overline{x}} \varphi(x) g(x)\,dx \right\}. \end{aligned}$$

Integrating by parts, we get:

$$L^m(\varphi, \Lambda_\lambda) = (1-\lambda)\,\delta \int_{\mathcal{X}} G(x)\dot{\varphi}(x)\,dx + \lambda \left[\delta \int_{\underline{x}}^{f_E^m} G(x)\dot{\varphi}(x)\,dx + (1-\delta) \int_{f_E^m}^{\overline{x}} \varphi(x) g(x)\,dx \right].$$

Now let $\varphi^m = \min\{\mathrm{Id}_{\mathcal{X}}, d^m\}$. By definition of d^m, one has, for any $\lambda > 0$:

$$\begin{aligned} & \int_{\mathcal{X}} \left\{ \int_{\mathcal{X}} [\varphi^m(x) - \delta\varphi^m(\overline{x})] g(x)\,dx - \int_{\underline{x}}^{\tilde{x}} [\varphi^m(x) - \delta\varphi^m(\tilde{x})] g(x)\,dx \right\} d\Lambda_\lambda(\tilde{x}) \\ & \quad = \lambda \left\{ \int_{\mathcal{X}} [\varphi^m(x) - \delta\varphi^m(\overline{x})] g(x)\,dx - \int_{\underline{x}}^{f_E^m} [\varphi^m(x) - \delta\varphi^m(f_E^m)] g(x)\,dx \right\} \qquad (20) \\ & \quad = 0. \end{aligned}$$

Thus, by Lemma 5, φ^m solves (18)–(19) if for some $\lambda^m > 0$,

$$L^m(\varphi^m, \Lambda_{\lambda^m}) \geq L^m(\tilde{\varphi}, \Lambda_{\lambda^m}); \qquad \tilde{\varphi} \in \Phi.$$

It is easy to see that, whenever $\lambda \leq 1$, $L^m(\varphi, \Lambda_\lambda)$ is maximized by a pure equity contract, $\varphi = \mathrm{Id}_{\mathcal{X}}$. But, unless $f_E^m = \overline{x}$, such a contract violates the no-exclusion constraint, and Lemma 4 therefore does not apply. This means that, if the first-best is not reached, one must have $\lambda > 1$ at the optimum. Intuitively, the shadow cost of the no-exclusion constraint must be high enough for debt to be an optimal security. One has the following result.

Proposition 8 *In the monopolistic case, the debt contract with face value d^m is an optimal security from the issuer's viewpoint.*

Just as in the competitive case, a debt contract is optimal from the issuer's viewpoint. The market power of the liquidity supplier, together with his inability to discriminate perfectly between the different types of the issuer, leads to a smaller face value of debt than in the competitive case.

A key feature of our model is that the optimal (regular) security is risky irrespective of the allocation of bargaining power. This informational sensitivity of the optimal (regular) security stands in stark contrast to the results of DeMarzo and Duffie (1999). Indeed, in their model, if the issuer observes a perfectly informative signal about the realization of the cash-flow, the optimal (regular) security payoff does not depend on her private information and is identically equal to the lowest possible realization of the cash-flow, $\underline{x}$. Again, this reflects the difference between our mechanism design approach and their signalling game approach. It should also be noted that, in the monopolistic case, the issuer has a strong incentive to issue an informationally sensitive security, since the liquidity supplier would otherwise be able to extract all her rent.

5. EXTENSIONS

We now investigate three extensions of our basic model. First, we prove that the assumption that the issuer designs a single security is without loss of generality in our framework. Next, we consider two oligopolistic models of liquidity supply, corresponding respectively to the case in which the issuer can trade with multiple liquidity suppliers, and to the case in which he is restricted to trade with a single liquidity supplier.

5.1. *Menus of securities*

So far, we have assumed that the issuer designs a single security at stage (i) of the game. More generally, the issuer could design a menu of securities ex-ante, from which she would select which one to trade at the interim stage. Without loss of generality, a menu of securities can be described by a payoff function $(x, \hat{x}) \mapsto \psi(x, \hat{x})$, whereby, contingent on a report $\hat{x}$ made by the issuer at the interim stage, a security with payoff function $\psi(\cdot, \hat{x})$ is traded. For instance, one could imagine that, if $\hat{x}$ belongs to some subset D of $\mathcal{X}$, the security is a debt contract, while if $\hat{x}$ belongs to the complement of D in $\mathcal{X}$, the security is an equity contract. We impose limited liability, that is, $\psi(\cdot, \hat{x})$ satisfies (LL) for each $\hat{x} \in \mathcal{X}$. By contrast, we do not impose monotonicity conditions on menus of securities.

From the Revelation Principle, any allocation achieved at the trading stage can also be achieved via a truthful direct mechanism $(\tau, q) : \mathcal{X} \to \mathbb{R} \times [0, 1]$ that stipulates a transfer and a traded fraction of the security as a function of the issuer's report of her type. Incentive compatibility requires that:

$$x \in \arg\max_{\hat{x} \in \mathcal{X}} \{\tau(\hat{x}) - \delta\psi(x, \hat{x})q(\hat{x})\}; \quad x \in \mathcal{X}. \tag{21}$$

Besides the incentive compatibility constraints (21), an implementable trading mechanism must also satisfy the ex-post participation constraint of the issuer:

$$\tau(x) - \delta\psi(x, x)q(x) \geq 0; \quad x \in \mathcal{X}, \tag{22}$$

and the participation constraint of the liquidity supplier:

$$\int_{\mathcal{X}} [\psi(x, x)q(x) - \tau(x)]g(x)\, dx \geq 0. \tag{23}$$

We now characterize the menu of securities and the corresponding trading mechanism that maximize the expected gains from trade. This amounts to finding a triple (ψ, τ, q) that solves:

$$\max \left\{ (1 - \delta) \int_{\mathcal{X}} \psi(x, x)q(x)g(x)\, dx \right\}, \tag{24}$$

subject to (21)–(23).

One can simplify this problem as follows. First, there is no loss of generality in setting $\psi(x, \hat{x}) = x$ for $\hat{x} \neq x$, as this can only relax the incentive compatibility constraints of the issuer. The intuition is that it is optimal to trade equity out of the equilibrium path, which represents the maximum punishment that can be inflicted on the issuer. Second, there is no loss in generality in setting $q(\hat{x}) = 1$ for all $\hat{x} \in \mathcal{X}$. Indeed, one can replace $\psi(x, x)$ with $\tilde{\psi}(x, x) = \psi(x, x)q(x)$, and the incentive compatibility constraints are again relaxed by trading the maximum fraction of the security, $\tilde{q} = 1$.

It remains only to determine $\varphi(x) = \psi(x, x)$ for each $x \in \mathcal{X}$, as well as the optimal transfer function τ. Assuming without loss of generality that τ is upper-semicontinuous, let $\overline{\tau} = \max_{x \in \mathcal{X}} \tau(x)$. The constraints (21)–(23) can be rewritten as:

$$\tau(x) - \delta\varphi(x) \geq \overline{\tau} - \delta x; \quad x \in \mathcal{X}, \tag{25}$$

$$\tau(x) - \delta\varphi(x) \geq 0; \quad x \in \mathcal{X}, \tag{26}$$

$$\int_{\mathcal{X}} [\varphi(x) - \tau(x)] g(x)\, dx \geq 0. \tag{27}$$

It is clear from (25)–(26) that, for each $x \in \mathcal{X}$, one at least of these constraints must be binding. Specifically, the incentive compatibility constraint (25) is binding if and only if $x \leq \overline{\tau}/\delta$, and the participation constraint (26) is binding if and only if $x \geq \overline{\tau}/\delta$. The only value of x at which both constraints are binding is $\overline{\tau}/\delta$.

We now prove that the optimal security is the debt contract with face value d^c. Specifically, we show that any pair (φ, τ) that satisfies constraints (25)–(27) is dominated by a debt contract with face value $\overline{\tau}/\delta$ and a constant transfer $\overline{\tau}$ which also satisfy these constraints. The argument is twofold.

Suppose first that $\varphi(x) < x$ for a set of values of $x \leq \overline{\tau}/\delta$ of positive measure, and consider an alternative security payoff function $\tilde{\varphi}$ that coincides with φ for $x \geq \overline{\tau}/\delta$, and that satisfies $\tilde{\varphi}(x) = x$ for $x \leq \overline{\tau}/\delta$. Modify correspondingly τ for $x \leq \overline{\tau}/\delta$ by setting $\tilde{\tau} = \overline{\tau}$ on this interval of values of x. It is immediate to check that (25)–(26) are satisfied by $(\tilde{\varphi}, \tilde{\tau})$. Consider now (27). Since it is satisfied by (φ, τ), we have:

$$\int_{\mathcal{X}} \varphi(x) g(x)\, dx \geq \int_{\mathcal{X}} \tau(x) g(x)\, dx = \int_{\underline{x}}^{\frac{\overline{\tau}}{\delta}} [\overline{\tau} - \delta x + \delta\varphi(x)] g(x)\, dx + \int_{\frac{\overline{\tau}}{\delta}}^{\overline{x}} \tau(x)\, dx,$$

where the equality follows from the fact that (25) is binding for $x \leq \overline{\tau}/\delta$. Therefore, as $x \geq \varphi(x)$, we obtain that:

$$\begin{aligned}
\int_{\mathcal{X}} \tilde{\varphi}(x) g(x)\, dx &= \int_{\underline{x}}^{\frac{\overline{\tau}}{\delta}} x g(x)\, dx + \int_{\frac{\overline{\tau}}{\delta}}^{\overline{x}} \varphi(x) g(x)\, dx \\
&\geq (1 - \delta) \int_{\underline{x}}^{\frac{\overline{\tau}}{\delta}} \varphi(x) g(x)\, dx + \delta \int_{\underline{x}}^{\frac{\overline{\tau}}{\delta}} x g(x)\, dx + \int_{\frac{\overline{\tau}}{\delta}}^{\overline{x}} \varphi(x) g(x)\, dx \\
&\geq \overline{\tau}\, G\left(\frac{\overline{\tau}}{\delta}\right) + \int_{\frac{\overline{\tau}}{\delta}}^{\overline{x}} \tau(x) g(x)\, dx \\
&= \int_{\mathcal{X}} \tilde{\tau}(x) g(x)\, dx,
\end{aligned}$$

which implies that (27) is satisfied by $(\tilde{\varphi}, \tilde{\tau})$. Since $(\tilde{\varphi}, \tilde{\tau})$ clearly dominates (φ, τ), it follows that an optimal contract must be such that $\varphi(x) = x$ and $\tau(x) = \overline{\tau}$ for all $x \leq \overline{\tau}/\delta$.

Consider now the values of x above $\overline{\tau}/\delta$. We know that the participation constraint (26) is binding for such x. Therefore one must have $\varphi(x) \leq \overline{\tau}/\delta$ by definition of $\overline{\tau}$. Suppose that $\varphi(x) < \overline{\tau}/\delta$ for a set of values of $x \geq \overline{\tau}/\delta$ of positive measure, and consider an alternative security payoff function that coincides with φ for $x \le \overline{\tau}/\delta$, and that satisfies $\tilde{\varphi}(x) = \overline{\tau}/\delta$ for $x \ge \overline{\tau}/\delta$. Modify correspondingly τ for $x \ge \overline{\tau}/\delta$ by setting $\tilde{\tau} = \overline{\tau}$ on this interval of values of x. It is immediate to check that (25)–(26) are satisfied by $(\tilde{\varphi}, \tilde{\tau})$. Consider now (27). Since it is satisfied by (φ, τ) and there is no loss of generality in assuming that $\tau(x) = \overline{\tau}$ for $x \le \overline{\tau}/\delta$ by our earlier result, we have:

$$\int_{\mathcal{X}} \varphi(x) g(x)\, dx \ge \int_{\mathcal{X}} \tau(x)\, g(x)\, dx = \overline{\tau}\, G\left(\frac{\overline{\tau}}{\delta}\right) + \delta \int_{\frac{\overline{\tau}}{\delta}}^{\overline{x}} \varphi(x)\, g(x)\, dx,$$

where the equality follows from the fact that (26) is binding for $x \ge \overline{\tau}/\delta$. Therefore, as $\overline{\tau}/\delta \ge \varphi(x)$ and there is no loss of generality in assuming that $\varphi(x) = x$ for $x \le \overline{\tau}/\delta$ by our earlier result, we obtain that:

$$\begin{aligned}
\int_{\mathcal{X}} \tilde{\varphi}(x) g(x)\, dx &= \int_{\underline{x}}^{\frac{\overline{\tau}}{\delta}} x g(x)\, dx + \left[1 - G\left(\frac{\overline{\tau}}{\delta}\right)\right] \frac{\overline{\tau}}{\delta} \\
&\ge \int_{\underline{x}}^{\frac{\overline{\tau}}{\delta}} x g(x)\, dx + (1-\delta) \int_{\frac{\overline{\tau}}{\delta}}^{\overline{x}} \varphi(x) g(x)\, dx + \left[1 - G\left(\frac{\overline{\tau}}{\delta}\right)\right] \overline{\tau} \\
&\ge \overline{\tau}\, G\left(\frac{\overline{\tau}}{\delta}\right) + \left[1 - G\left(\frac{\overline{\tau}}{\delta}\right)\right] \overline{\tau} \\
&= \overline{\tau},
\end{aligned}$$

which implies that (27) is satisfied by $(\tilde{\varphi}, \tilde{\tau})$. Since $(\tilde{\varphi}, \tilde{\tau})$ clearly dominates (φ, τ), it follows that an optimal contract must be such that $\varphi(x) = \overline{\tau}/\delta$ and $\tau(x) = \overline{\tau}$ for all $x \ge \overline{\tau}/\delta$. Combining this with our earlier result, we obtain that φ is the payoff function of a debt contract with face value $\overline{\tau}/\delta$. The largest face value compatible with (27) corresponds to the largest $\overline{\tau}$ such that:

$$\int_{\underline{x}}^{\frac{\overline{\tau}}{\delta}} x g(x)\, dx + \left[1 - G\left(\frac{\overline{\tau}}{\delta}\right)\right] \frac{\overline{\tau}}{\delta} - \overline{\tau} \ge 0,$$

and is therefore equal to d^c. Hence, we have the following result.

Proposition 9 *If the issuer can design a menu of securities from which she selects which one to trade at the interim stage, and the liquidity supplier is competitive, the traded security and the equilibrium allocation are the same as in the basic model.*

Two observations are in order. First, recall that Proposition 7 established that a contract with face value d^c is optimal in the class of regular securities, that is, in particular, for which a fixed-price trading mechanism is optimal. Since we have in this section enlarged the class of possible contracts, Proposition 9 implies that the restriction to regular securities was without loss of generality, provided the issued security is optimal from the issuer's viewpoint. Second, the derivation of Proposition 9 does not require any restrictions on securities besides limited liability. It follows that the monotonicity constraints (M1) and (M2) are imposed without loss of generality in the basic model. In particular, "live-or-die" contracts, under which the issuer transfers the whole cash-flow to the liquidity supplier when it falls below a certain threshold, and nothing otherwise, are never optimal, in contrast to Innes (1990).

5.2. *Oligopolistic screening with non-exclusive dealing*

The competitive case has been defined as a situation in which the issuer has all the bargaining power, and can therefore commit to a price for the security she issues. We now show that, whenever the issuer designs a security such that the whole issue can be traded at the interim stage, the competitive allocation can be decentralized as the outcome of an oligopolistic screening game in which several liquidity suppliers compete in non-exclusive schedules for the security. In line with previous models of multi-principal mechanism design (Stole (1991), Martimort (1992), Biais, Martimort and Rochet (2000)), this can be seen as a situation in which no principal can contract on the quantities that are sold to his competitors. As we argued in the Introduction and in Subsection 2.3, this assumption is often relevant for the sale of securities.

Specifically, suppose there are $I \geq 2$ liquidity suppliers. The timing of the trading game is similar to that of Section 2. Formally, steps (ii) and (iv) are replaced by:

(ii') The I liquidity suppliers simultaneously post transfer schedules $T_i : [0,1] \to \mathbb{R}$, $i \in I$, for the sale of any fraction $q_i \in [0,1]$ of the security;

(iv') If the issuer accepts the transfer schedules $\{T_j\}_{j \in J}$, $J \subset I$, she trades fractions $\{q_j\}_{j \in J}$ of the security, $\sum_{j \in J} q_j \leq 1$, for which she obtains transfers $\{T_j(q_j)\}_{j \in J}$.

For simplicity, we shall assume that the issuer designs a security F that entails no exclusion in the competitive case, and trades at a price $E(F)$. For instance, this holds whenever the issuer designs the optimal debt contract with face value d^c. No attempt is made to give a full characterization of the equilibria of the induced trading game. Instead, we construct an equilibrium that decentralizes the competitive allocation. In this equilibrium, each liquidity supplier stands ready to buy any fraction of the security at the competitive price:

$$T_i(q_i) = E(F)q_i; \quad q_i \in [0,1],\ i \in I.$$

To check that this constitutes an equilibrium, suppose that all liquidity suppliers but one offer this schedule, while the remaining liquidity supplier offers a schedule $\tilde{T}$. Then, whatever her type, and whatever the fraction q of the security she decides to sell to the deviating liquidity supplier, the issuer will sell the remaining fraction $1-q$ to the other liquidity suppliers. Indeed, the price they offer is higher than her retention cost, as, by assumption, the security F entails no exclusion in the competitive case. That is, the issuer solves:

$$\max_{q \in [0,1]} \left\{ \tilde{T}(q) + E(F)(1-q) \right\},$$

independently of her type. We can therefore assume that the deviating liquidity supplier faces the same type distribution than the other liquidity suppliers, and that all types of the issuer sell the same fraction q to him. If this is so, then the only way that the deviating liquidity supplier can attract the issuer is by offering a transfer $\tilde{T}(q) \geq E(F)q$, thereby obtaining a profit at most equal to what he would get by offering the competitive schedule.

In line with standard price theory, Bertrand competition results in an efficient allocation. The non-exclusivity clause is crucial, since it ensures that no cream-skimming deviation is possible whenever other liquidity suppliers offer the competitive schedule. In contrast to other models of competition in mechanisms such as Biais, Martimort and Rochet (2000), this result does not hinge on a large number of liquidity suppliers. This is because, due to the linearity of preferences, the optimal mechanism is a fixed-price mechanism.

5.3. Oligopolistic screening with exclusive dealing

It is interesting to contrast the previous decentralization result with what happens if the issuer can only deal with one liquidity supplier at the trading stage, so that competition occurs only at the mechanism offering stage. This could occur for instance if a firm issues debt, but the banks she can deal with can prevent her from selling this debt to other financial intermediaries through special financing covenants. Formally, step (iv') is replaced by:

(iv") If the issuer accepts the transfer schedule T_i, $i \in I$, she trades a fraction q_i of the security, for which she obtains a transfer $T_i(q_i)$.

For simplicity, we shall assume that the issuer designs the optimal debt contract with face value d^c, and that $d^c < \overline{x}$, so that the first-best is not reached. If there were an equilibrium that decentralized the competitive allocation, the issuer would sell the whole issue to one of the liquidity suppliers for a price δd^c, and each liquidity supplier would receive zero profit. Now suppose that one liquidity supplier deviates and offers to buy a fraction $q < 1$ of the debt against a transfer $\delta d^c q + \varepsilon$, for some $\varepsilon > 0$. This offer will only attract the types:

$$f > d^c - \frac{\varepsilon}{\delta(1-q)}$$

of the issuer, thereby securing a profit:

$$\int_{d^c - \frac{\varepsilon}{\delta(1-q)}}^{d^c} [(f - \delta d^c)q - \varepsilon] g(f)\, df + [1 - G(d^c)][(1-\delta)d^c q - \varepsilon],$$

which is strictly positive provided ε is close enough to zero. Hence, under exclusive dealing, there exists no equilibrium that decentralizes the competitive allocation, because a cream-skimming deviation is profitable whenever other liquidity suppliers behave competitively. This offers high types of the issuer the opportunity to signal themselves by restricting the fraction of the security they trade and bearing some retention cost, as in the signalling models of Leland and Pyle (1977) or DeMarzo and Duffie (1999). In line with competitive models of insurance under adverse selection, fully pooling outcomes do not survive the possibility of offering exclusive contracts (Rothschild and Stiglitz (1976)).

A full characterization of the equilibria of the exclusive trading game is beyond the scope of this paper. Instead, we discuss some robust features of pure-strategy symmetric equilibria, whenever such equilibria exist. To do this, consider an arbitrary security F with payoff function φ, and let $T : [0,1] \to \mathbb{R}_+$ and $q : \mathcal{F} \to [0,1]$ be respectively the transfer schedule and the traded fraction of the security as a function of the type of the issuer in a symmetric equilibrium of the trading game. Thus, in particular,

$$q(f) \in \arg\max_{q \in [0,1]} \{T(q) - \delta f q\}; \qquad f \in \mathcal{F}.$$

Clearly, the expected profit of each liquidity supplier must be equal to zero. More specifically, each liquidity supplier must make zero profit conditional on any fraction of the security traded in equilibrium. Indeed, suppose that the event $\mathcal{F}^+ = \{T(q(F)) > E(F|q(F))q(F)\}$ has positive measure under the distribution of F. Then a liquidity supplier could strictly gain by free-riding and offering just that part of the schedule that is profitable. It must be therefore the case that $T(q(F)) = E(F|q(F))q(F)$ almost surely. In a Walrasian way, the unit price paid by the liquidity suppliers on a given fraction of the security is equal to the expected value of the security conditional on this fraction being traded.

This raises the natural question of the existence of a fully separating equilibrium, in which $E(F|q(F)) = F$ almost surely. Such an equilibrium, if it exists, must coincide with the unique separating equilibrium derived by DeMarzo and Duffie (1999), with:

$$T(q) = \underline{f}\, q^{\delta}; \quad q \in [0,1], \tag{28}$$

and:

$$q(f) = \left(\frac{\underline{f}}{f}\right)^{\frac{1}{1-\delta}}; \quad f \in \mathcal{F}. \tag{29}$$

A key feature of this equilibrium, however, is that it depends only on the support of the type distribution. This is consistent with the fact that, in the signalling context, the issuer first choses to trade a fraction of the security, and liquidity suppliers react by bidding for this fraction. In the screening context, however, the timing is reversed: liquidity suppliers first offer transfer schedules, and only then does the issuer select his trade. This implies that the equilibrium will generally depend on the type distribution.

For instance, suppose that all liquidity suppliers but one offer the schedule (28), and that the remaining liquidity supplier offers a linear schedule $\tilde{T}(q) = \tilde{t}q$, for some $\tilde{t}$ slightly greater than $\underline{f}$. Using (29), one can check that $\tilde{T}$ attracts the types $f \leq \tilde{f}$ of the issuer, where the marginal type is implicitly defined by:

$$\tilde{t} - \delta\tilde{f} = (1-\delta)\tilde{f}\left(\frac{\underline{f}}{\tilde{f}}\right)^{\frac{1}{1-\delta}}.$$

The types $f \leq \tilde{f}$ of the issuer sell a quantity $q(f) = 1$ to the deviating liquidity supplier, thereby securing him an expected profit $G^{\varphi}(\tilde{f})[E(F|F \leq \tilde{f}) - \tilde{t}]$. In particular, the profit on the marginal type, $\tilde{f} - \tilde{t}$, is strictly positive, as $\tilde{f} > \underline{f}$ and:

$$\tilde{f} - \tilde{t} = (1-\delta)\tilde{f}\left[1 - \left(\frac{\underline{f}}{\tilde{f}}\right)^{\frac{1}{1-\delta}}\right].$$

Thus, if the lower-tail expectation $E(F|F \leq \tilde{f})$ is close enough to $\tilde{f}$, this deviation will secure a strictly positive profit. In general, we cannot expect the fully separating equilibrium of DeMarzo and Duffie's (1999) signalling model to be an equilibrium of the exclusive trading game. This is again in line with the potential instability of fully separating outcomes in competitive models of insurance under adverse selection (Rothschild and Stiglitz (1976)).

6. CONCLUSION

This paper studies the optimal design of securities and issuance mechanisms when privately informed asset owners demand liquidity from investors or financial intermediaries. We extend the insightful recent paper by DeMarzo and Duffie (1999). While they focus on a specific trading game, we take a mechanism design approach to characterize both the optimal security and the optimal trading mechanism. In this context we analyze two polar cases, corresponding to different allocations of bargaining power between the issuer and the liquidity supplier at the stage the trading mechanism is designed. This allows us to analyze the consequences of the market power of liquidity suppliers for trading, security design, and gains from trade.

Our theoretical analysis applies to a variety of situations. The issuer can be the owner-manager of a firm who sells future cash-flows or refinance accounts receivables by applying for bank loans or issuing securities. She can be a financial intermediary, establishing conduits through which trade receivables from different originators can be securitized and floated on the market. Or she can be a private equity fund, withdrawing cash from the firm in which she had invested, and financing this special dividend with bank loans or bond issues.

Ideally, the issuer would like to entirely sell future cash-flows to outside investors by issuing equity. The extent to which these gains from trade can be achieved is however restricted by adverse selection and the market power of liquidity suppliers. If these imperfections are limited, the issuer can still issue equity, but when adverse selection is severe and the liquidity supplier strategic, she finds it optimal to issue debt. In line with Myers and Majluf (1984), the intuition in the competitive case is that debt is the least information sensitive security, and thus minimizes the consequences of adverse selection. In the monopolistic case, debt is also an effective instrument to counter the market power of the liquidity supplier.

The market power of liquidity suppliers varies across firms, sectors and countries, and is affected by country-specific regulations and entry in the financial sector. Small firms unable to pay the fixed cost of access to financial markets, and having established relationships with one bank only, have very little bargaining power. By contrast, large financial intermediaries can market their securities to a broad population of investors, and are likely to have significant bargaining power. In this context, competition among investors is typically non-exclusive, in the sense that no investor can buy a fraction of the securities offered under the condition that the issuer will not sell the remaining fraction to other investors. We show that, for the optimal security, the optimal trading mechanism arising when the issuer has all the bargaining power can be decentralized as a Nash equilibrium of an oligopolistic screening game in which several liquidity suppliers compete in non-exclusive schedules for the security.

Finally, our theoretical analysis yields novel empirical implications on the consequences of market power in the financial sector.

(i) While equity issues would enable the issuer to fully exploit the gains from trade in perfect markets, they are costly under conditions of adverse selection and market power. Hence, the issuer will choose to issue equity only if the costs induced by market imperfections are relatively small compared to the gains from trade. This is more likely to arise in the competitive than in the monopoly case, since the costs induced by market imperfections are lower in the former than in the latter. Hence our theoretical analysis predicts that the prevalence of debt issues, relative to equity issues, should be greater when liquidity suppliers have market power than when they are competitive.

(ii) While the above empirical implication bears on the weight of debt relative to other sources of external financing, our theoretical analysis also delivers implications with respect to the amount of debt relative to internal financing. In our model the face value of debt is greater in the competitive than in the monopolistic case. This could be tested for example by studying across countries if, other things equal, there is a link between the ratio of debt to GDP and the degree of competition in the financial sector.

APPENDIX

Proof of Lemma 1. Part (i) follows from the fact that U is the supremum of a family of affine functions of f. As a convex function, U is differentiable except on a set of Lebesgue measure zero. Part (ii) then follows from the Envelope Theorem. $\|$

Proof of Lemma 3. Fix an admissible security F. Using the incentive constraints for types $f, \tilde{f} \in \mathcal{F}$ of the issuer, it is easy to check that:

$$-\delta(\tilde{f}-f)q(f) \leq U(\tilde{f}) - U(f) \leq -\delta(\tilde{f}-f)q(\tilde{f}); \quad (f,\tilde{f}) \in \mathcal{F}^2. \tag{A.1}$$

Since q is bounded, this implies that U satisfies a Lipschitz condition on $\mathcal{F}$, and thus is absolutely continuous on this interval. Since, by Lemma 1, $\dot{U} = -\delta q$ except on a set of Lebesgue measure zero, we obtain that:

$$U(f) = \delta \int_f^{\overline{f}} q(\phi)\, d\phi + U(\overline{f}); \qquad f \in \mathcal{F}. \tag{A.2}$$

We now examine each case in turn.

(i) Consider first the competitive case. The issuer solves (7) subject to (8), and to the further constraints that U be non-negative and convex, with derivative given by $\dot{U} = -\delta q$ except on a set of Lebesgue measure zero, and that q map $\mathcal{F}$ into $[0,1]$. Since the objective function in (7) is continuous in U on the space of continuous real-valued functions on $\mathcal{F}$ endowed with the sup norm, we need only to check that the space $\mathcal{U}^c$ of admissible rent profiles U is compact in this topology. Clearly, $\mathcal{U}^c$ is bounded, and since (A.1) implies that $|U(f) - U(\tilde{f})| \leq \delta|f - \tilde{f}|$ for any $U \in \mathcal{U}^c$ and $(f,\tilde{f}) \in \mathcal{F}^2$, it is equicontinuous. Hence, by Ascoli's Theorem, $\mathcal{U}^c$ is relatively compact. Let $\{U^n\}$ be a sequence in $\mathcal{U}^c$ converging uniformly to a function U. From Rockafellar (1970, Theorem 10.8), U is convex. Moreover, the equicontinuity condition on $\mathcal{U}^c$ implies that the derivative of U, whenever defined, belongs to $[-\delta, 0]$, as required. We thus only need to prove that (8) is preserved in the limit. Direct inspection of (8) shows that there is no loss of generality in assuming that q^n is left-continuous for each $n \geq 0$, and we can thus rewrite (8) as:

$$\int_{\mathcal{F}} U^n(f)\, dG^{\varphi}(f) + \frac{1-\delta}{\delta} \int_{\mathcal{F}} f\dot{U}^{n-}(f)\, dG^{\varphi}(f) \leq 0, \tag{A.3}$$

where $\dot{U}^{n-}$ is the left-derivative of U^n. The first term on the left-hand side of this inequality converges to $\int_{\mathcal{F}} U(f)\, dG^{\varphi}(f)$ by uniform convergence of the sequence $\{U^n\}$. For the second term, one has:

$$\int_{\mathcal{F}} f\dot{U}^{-}(f)\, dG^{\varphi}(f) \leq \int_{\mathcal{F}} f \liminf_{n\to\infty}\{\dot{U}^{n-}(f)\}\, dG^{\varphi}(f) \leq \liminf_{n\to\infty}\left\{\int_{\mathcal{F}} f\dot{U}^{n-}(f)\, dG^{\varphi}(f)\right\}, \tag{A.4}$$

where the first inequality follows from Rockafellar (1970, Theorem 24.5), and the second from Fatou's Lemma. Note that we have used the fact that $\dot{U}^{-} = -U'(\cdot,-1)$, that is, the left-derivative of U is the opposite of its derivative in the direction -1. Together with (A.3), (A.4) implies that (8) holds in the limit, and thus $\mathcal{U}^c$ is compact as it contains all its limit points.

(ii) Consider next the monopolistic case. The liquidity supplier solves (9) subject to the constraints that U be non-negative and convex, with derivative given by $\dot{U} = -\delta q$ except on a set of Lebesgue measure zero, and that q map $\mathcal{F}$ into $[0,1]$. Proceeding as in (i), it is easy to check that the space $\mathcal{U}^m$ of admissible rent profiles U is compact in the space of continuous real-valued functions on $\mathcal{F}$ endowed with the sup norm. We thus need only to check that the objective function in (9) is upper-semicontinuous in U in this topology. Let $\{U^n\}$ be a sequence in $\mathcal{U}^m$ converging uniformly to a function U. Direct inspection of (9) shows that there is no loss of generality in assuming that q^n is left-continuous for each $n \geq 0$, and we can thus rewrite the objective function in (9) as:

$$-\frac{1-\delta}{\delta}\int_{\mathcal{F}} f\dot{U}^{n-}(f)\,dG^{\varphi}(f) - \int_{\mathcal{F}} U^n(f)\,dG^{\varphi}(f).$$

By uniform convergence of the sequence $\{U^n\}$, the second term of this expression converges to $\int_{\mathcal{F}} U(f)\,dG^{\varphi}(f)$. For the first term, one has, by (A.4):

$$-\frac{1-\delta}{\delta}\int_{\mathcal{F}} f\dot{U}^{-}(f)\,dG^{\varphi}(f) \geq \limsup_{n\to\infty}\left\{-\frac{1-\delta}{\delta}\int_{\mathcal{F}} f\dot{U}^{n-}(f)\,dG^{\varphi}(f)\right\},$$

which implies that the objective function in (9) is upper-semicontinuous in U, as claimed. $\|$

Proof of Proposition 1. Clearly, the participation constraint (3) of the issuer must be binding at the optimum. Inserting (A.2) in (9) yields the following expression for the profit of the liquidity supplier:

$$\int_{\mathcal{F}}\left[(1-\delta)fq(f) - \delta\int_f^{\overline{f}} q(\phi)\,d\phi\right] dG^{\varphi}(f). \tag{A.5}$$

We can simplify this expression as follows. First, let us introduce a mapping H^{φ} as follows:

$$H^{\varphi}(f) = (1-\delta)\int_{[\underline{f},f]} \phi\,dG^{\varphi}(\phi) - \delta\int_{\underline{f}}^{f} G^{\varphi-}(\phi)\,d\phi; \qquad f \in \mathcal{F}.$$

H^{φ} is right-continuous and of bounded variation, and thus induces a measure dH^{φ} on the σ-algebra of Borel subsets of $\mathcal{F}$. Let μ be the Lebesgue measure. By construction, the measure dH^{φ} coincides with the measure $(1-\delta)\mathrm{Id}_{\mathcal{F}}\,dG^{\varphi} - \delta G^{\varphi-}d\mu$ on the algebra generated by the intervals $[\underline{f}, f]$, $f \in \mathcal{F}$, and therefore, by the Monotone Class Theorem, on the σ-algebra of Borel subsets of $\mathcal{F}$. Next, note that G^{φ} is a distribution function, and thus a right-continuous function of bounded variation. Let $G^{\varphi-}$ be the left-continuous regularization of G^{φ}, which satisfies $G^{\varphi-}(\underline{f}) = 0$ by convention. From Dellacherie and Meyer (1982, Theorem VI.90), we can therefore integrate (A.5) by parts, and we find that the liquidity supplier's problem is to maximize:

$$(1-\delta)\int_{\mathcal{F}} fq(f)\,dG^{\varphi}(f) - \delta\int_{\mathcal{F}} G^{\varphi-}(f)q(f)\,df = \int_{\mathcal{F}} q(f)\,dH^{\varphi}(f)$$

subject to the constraints that q be non-increasing on $\mathcal{F}$ and map $\mathcal{F}$ into $[0,1]$. Integrating by parts again, we find that:

$$\int_{\mathcal{F}} q(f)\,dH^{\varphi}(f) = H^{\varphi}(\overline{f})q^{+}(\overline{f}) - H^{\varphi}(\underline{f}) + \int_{\mathcal{F}} H^{\varphi}(f)\,d(1-q^{+})(f), \tag{A.6}$$

where q^{+} is the right-continuous regularization of q. Since q is non-increasing, $1-q^{+}$ can be interpreted as the distribution of a probability measure on $\mathcal{F}$. Therefore the maximum in (A.6) is obtained by putting all the weight of this measure on some maximum f_F^m of H^{φ}, which is well-defined since H^{φ} is upper-semicontinuous. Hence $q_F^m = 1_{[\underline{f}, f_F^m]}$, and the transfer τ_F^m is obtained by noting that $U(f_F^m) = \tau_F^m - \delta f_F^m = 0$. $\|$

Proof of Proposition 2. Integrating by parts, it is easy to verify that the function H^{φ} given in the text coincides with that given in the proof of Proposition 1. Since H^{φ} is upper-semicontinuous, the set $A_F^c = \{f \in \mathcal{F} \mid H(f) \geq 0\}$ is compact, and thus has a maximal element, f_F^c. Moreover, H^{φ} has no downward jumps, so that $f_F^c = \overline{f}$ or $H^{\varphi}(f_F^c) = 0$. In the former case, it is immediate to check that

$q_F^c = 1$ and $\tau_F^c = \int_{\mathcal{F}} f\, dG^\varphi(f)$. In the latter case, we show that $q_F^c = 1_{[\underline{f}, f_F^c]}$. Clearly, the participation constraint (8) of the liquidity supplier must be binding at the optimum. Inserting (A.2) in (8) and integrating by parts yields:

$$U(\overline{f}) = (1-\delta) \int_{\mathcal{F}} f q(f)\, dG^\varphi(f) - \delta \int_{\mathcal{F}} G^{\varphi-}(f) q(f)\, df. \tag{A.7}$$

The issuer's problem is to maximize the gains from trade (6), subject to her participation constraint (3) and to the further constraints that q be non-increasing on $\mathcal{F}$ with values in $[0,1]$. Taking into account (A.7), the Lagrangian for this problem is:

$$(1-\delta)(1+\lambda) \int_{\mathcal{F}} f q(f)\, dG^\varphi(f) - \delta\lambda \int_{\mathcal{F}} G^{\varphi-}(f) q(f)\, df$$

For any $\lambda > 0$, it is easy to show along the lines of the proof of Proposition 1 that the problem of maximizing this Lagrangian with respect to q non-increasing on $\mathcal{F}$ with values in $[0,1]$ has a solution of the form $q^\lambda = 1_{[\underline{f}, f^\lambda]}$. What must be shown is that there exists some $\lambda > 0$ such that $f^\lambda = f_F^c$. Equivalently, one must show that for some $\lambda > 0$, the Lagrangian:

$$L^\varphi(f,\lambda) = (1-\delta)(1+\lambda) \int_{[\underline{f}, f]} \phi\, dG^\varphi(\phi) - \delta\lambda \int_{\underline{f}}^{f} G^{\varphi-}(\phi)\, d\phi$$

attains its maximum at f_F^c. By assumption, J^φ is non-increasing on $\mathcal{F}$. Under the assumption that $f_F^c < \overline{f}$, one has $H^\varphi(\overline{f}) < 0$ so that $J^\varphi(\overline{f})$ is well-defined and strictly positive. Moreover, as $H^\varphi(f) = \int_{[\underline{f}, f]} \phi\, dG^\varphi(\phi) - \delta G^\varphi(f) f$ and $\delta < 1$, $H^\varphi(f)$ is strictly positive in a right-neighborhood of $\underline{f}$, and therefore on $(\underline{f}, f_F^c)$ since otherwise J^φ could not be non-increasing on $\mathcal{F}$. Hence, $\lambda_F^c = (1-\delta) \lim_{f \downarrow f_F^c} \{J^\varphi(f)\}$ is well-defined and strictly positive. For any $f \in (\underline{f}, f_F^c)$, one has:

$$(1-\delta) J^\varphi(f) = \frac{(1-\delta) \displaystyle\int_{(f, f_F^c]} \phi\, dG^\varphi(\phi)}{H^\varphi(f)} \geq \lambda_F^c, \tag{A.8}$$

so that, as H^φ is strictly positive on $(\underline{f}, f_F^c)$,

$$(1-\delta) \int_{(f, f_F^c]} \phi\, dG^\varphi(\phi) \geq (1-\delta)\lambda_F^c \int_{[\underline{f}, f]} \phi\, dG^\varphi(\phi) - \delta\lambda_F^c \int_{\underline{f}}^{f} G^{\varphi-}(\phi)\, d\phi. \tag{A.9}$$

Adding $(1-\delta) \int_{[\underline{f}, f]} \phi\, dG^\varphi(\phi)$ to both sides of this inequality, and using the fact that $H^\varphi(f_F^c) = 0$, it is easy to check that $L^\varphi(f_F^c, \lambda_F^c) \geq L^\varphi(f, \lambda_F^c)$ for any $f \in (\underline{f}, f_F^c)$. For $f = \underline{f}$, the same reasoning can be applied if dG^φ has an atom at $\underline{f}$, for then $H^\varphi(\underline{f}) > 0$. If dG^φ has no atom at $\underline{f}$, one has $L^\varphi(f_F^c, \lambda_F^c) > 0 = L^\varphi(\underline{f}, \lambda_F^c)$. Last, for $f \in (f_F^c, \overline{f}]$, inequality (A.8) is reversed but, as H^φ is strictly negative on $(\underline{f}, f_F^c)$, (A.9) still holds and the same reasoning applies. Overall, $L^\varphi(f, \lambda_F^c)$ attains its maximum at f_F^c, and the result follows. That $f_F^c \geq f_F^m$ follows from $H^\varphi(f_F^m) \geq H^\varphi(f_F^c)$ together with the definition of f_F^c. $\|$

Proof of Proposition 3. (i) Consider first the competitive case. To focus on an interesting case, assume that $f_E^c < \overline{x}$. The result then simply follows from saturating the participation constraint of the liquidity supplier, given the restriction to fixed-price mechanisms. Note that such a mechanism is indeed optimal if equity is a regular security.

(ii) Consider next the monopolistic case. Proceeding as in the proof of Proposition 1, the objective function of the liquidity supplier can be expressed, after an integration by parts, as:

$$\int_{\mathcal{X}} \left[(1-\delta) f q(f) - \delta \int_f^{\overline{x}} q(\phi)\, d\phi \right] g(f)\, df = \int_{\mathcal{X}} \left[(1-\delta) f - \delta \frac{G(f)}{g(f)} \right] q(f) g(f)\, df.$$

Pointwise maximizing with respect to q implies that $q = 1$ on the set of $f \in \mathcal{X}$ such that $(1 - \delta)f/\delta \geq G(f)/g(f)$, and $q = 0$ elsewhere. Condition (10) says that this set is an interval $[\underline{f}, f_E^m]$. Hence the associated rent $U(f) = \delta(f_E^m - f)1_{[\underline{f}, f_E^m]}(f)$ is convex in f, which implies the result. $\|$

Proof of Proposition 4. (i) Consider first the competitive case. To focus on an interesting case, assume that $f_E^c < \overline{x}$. We focus on fixed-price mechanisms. If $d < f_E^c$, then the issuer does not want to exclude any type $f \in [\underline{x}, d]$ when debt with face value d is issued, for otherwise the exclusion threshold f_E^c would not be optimal when equity is issued. Suppose now that $d = f_E^c$. The supremum of what the issuer can obtain by excluding some types below f_E^c is:

$$(1 - \delta) \int_{\underline{f}}^{f_E^c} f g(f)\, df.$$

As $f_E^c < \overline{x}$ and thus $G(f_E^c) < 1$, this is strictly less than what she obtains if she does not exclude any type and binds the participation constraint of the liquidity supplier, that is:

$$(1 - \delta) \left\{ \int_{\underline{f}}^{f_E^c} f g(f)\, df + [1 - G(f_E^c)] f_E^c \right\}.$$

It follows by continuity that for any $d \in [f_E^c, d^c]$, the issuer will not exclude any type when debt with face value d is issued, while still preserving the participation constraint of the liquidity supplier. If debt with face value $d > d^c$ is issued, the optimal exclusion threshold is f_E^c as when equity is issued.

(ii) Consider next the monopolistic case. To focus on an interesting case, assume that $f_E^m < \overline{x}$. If $d < f_E^m$, then the liquidity supplier does not want to exclude any type $f \in [\underline{x}, d]$ when debt with face value d is issued, for otherwise the exclusion threshold f_E^m would not be optimal when equity is issued. Suppose now that $d = f_E^m$. From (10) and the characterization of f_E^m provided in Proposition 3, the supremum of what the liquidity supplier can obtain by excluding some types below f_E^m is:

$$\int_{\underline{x}}^{f_E^m} f g(f)\, df - \delta G(f_E^m) f_E^m.$$

As $f_E^m < \overline{x}$ and thus $G(f_E^m) < 1$, and since $\delta < 1$, this is strictly less than what he obtains if he does not exclude any type, that is:

$$\int_{\underline{x}}^{f_E^m} f g(f)\, df + [1 - G(f_E^m)] f_E^m - \delta f_E^m.$$

It follows by continuity that for any $d \in [f_E^m, d^m]$, the liquidity supplier will not exclude any type when debt with face value d is issued. If debt with face value $d > d^m$ is issued, the optimal exclusion threshold is f_E^m as when equity is issued. $\|$

Proof of Lemma 4. (i) In the competitive case, this follows at once from the fact that the issuer maximizes the gains from trade.

(ii) In the monopolistic case, let us suppose that F is an optimal security such that $\varphi(\underline{x}) < \underline{x}$. Then, given (M2), there exists $\varepsilon > 0$ such that $\varphi(x) < x - \varepsilon$ for all $x \in \mathcal{X}$. Consider the security F_ε defined by $\varphi_\varepsilon = \varphi + \varepsilon$. Given this new security, the liquidity supplier chooses an exclusion threshold f_ε so as to maximize his expected profit:

$$\int_{[\underline{f}+\varepsilon, f_\varepsilon]} (f - \delta f_\varepsilon)\, dG^{\varphi_\varepsilon}(f) = \int_{[\underline{f}, f_\varepsilon - \varepsilon]} [f - \delta(f_\varepsilon - \varepsilon)]\, dG^{\varphi}(f) + G^{\varphi}(f_\varepsilon - \varepsilon)(1 - \delta)\varepsilon, \tag{A.10}$$

where used was made of the fact that $G^{\varphi_\varepsilon}(f+\varepsilon) = G^{\varphi}(f)$ for each $f \in \mathcal{F}$. The first term on the right-hand side of (A.10) is maximized by setting $f_\varepsilon = f_F^m + \varepsilon$, where f_F^m is the optimal exclusion threshold for F. Since the second term on the right-hand side of (A.10) is non-decreasing in f_ε, this implies that the optimal exclusion threshold $f_{F_\varepsilon}^m$ for F_ε is greater or equal than $f_F^m + \varepsilon$. It is then easy to check that the expected rent of the issuer is at least as large under F_ε than under F. $\|$

Proof of Proposition 6. (i) Consider first the competitive case. Suppose that F is an optimal regular security sold at a price δf_F^c for which $f_F^c < \overline{f}$. Consider the security $\overline{F}$ defined by $\overline{\varphi} = \min\{\varphi, f_F^c\}$. One then has:

$$E(\overline{F}) = G^{\varphi}(f_F^c)E(F|F \leq f_F^c) + [1 - G^{\varphi}(f^c)]f_F^c > \delta f_F^c,$$

where the strict inequality follows from the fact that $E(F|F \leq f_F^c) = \delta f_F^c$ and $G^{\varphi}(f_F^c) < 1$ as $f_F^c < \overline{f}$. It follows that if security $\overline{F}$ is traded, all the types of the issuer are ready to trade. Anticipating this, the liquidity supplier is ready to pay a strictly higher price, $E(\overline{F})$, than under the original security F. It follows that the expected rent of the issuer is strictly higher under $\overline{F}$ than under F. Hence F cannot be an optimal regular security from the issuer's viewpoint, a contradiction.

(ii) Consider next the monopolistic case. Suppose that F is an optimal security such that the liquidity supplier excludes the types above $f_F^m < \overline{f}$, thereby obtaining a profit:

$$\int_{[\underline{f}, f_F^m]} (f - \delta f_F^m)\, dG^{\varphi}(f).$$

Consider the security $\overline{F}$ defined by $\overline{\varphi} = \min\{\varphi, f_F^m + \varepsilon\}$ for some small $\varepsilon > 0$. If the liquidity supplier decides to exclude some types below $f_F^m + \varepsilon$ under $\overline{F}$, the optimal way to do so is to set $f_{\overline{F}}^m = f_F^m$, for a profit equal to that he obtains under F. By contrast, if he does not exclude any type under $\overline{F}$, the liquidity supplier obtains, for ε close enough to zero:

$$\begin{aligned}\int_{[\underline{f}, f_F^m+\varepsilon]} [f - \delta(f_F^m + \varepsilon)]\, dG^{\overline{\varphi}}(f) &= \int_{[\underline{f}, f_F^m+\varepsilon)} [f - \delta(f_F^m + \varepsilon)]\, dG^{\varphi}(f) \\ &\quad + [1 - G^{\varphi -}(f_F^m + \varepsilon)](1-\delta)(f_F^m + \varepsilon) \\ &> \int_{[\underline{f}, f_F^m]} (f - \delta f_F^m)\, dG^{\varphi}(f),\end{aligned}$$

where the strict inequality follows from the fact that:

$$1 - \lim_{\varepsilon \downarrow 0} \left\{G^{\varphi -}(f_F^m + \varepsilon)\right\} = 1 - G^{\varphi}(f_F^m) > 0$$

as $f_F^m < \overline{f}$, φ is continuous, and g is positive on $\mathcal{X}$. Thus the liquidity supplier is strictly better off not excluding any type under $\overline{F}$, provided ε is close enough to zero. It is then immediate to check that the expected rent of the issuer under $\overline{F}$,

$$\delta \int_{[\underline{f}, f_F^m+\varepsilon]} (f_F^m + \varepsilon - f)\, dG^{\overline{\varphi}}(f) = \delta \int_{[\underline{f}, f_F^m+\varepsilon]} (f_F^m + \varepsilon - f)\, dG^{\varphi}(f),$$

is strictly larger than under F. Hence F cannot be an optimal security, a contradiction. $\|$

Proof of Proposition 7. For any $\lambda \geq 0$, we study the problem of maximizing $L^c(\varphi, \lambda)$ with respect to $\varphi \in \Phi$. Restricting first the set of admissible values for φ to functions in Φ that are piecewise continuously differentiable, we treat this as an optimal control problem with state variable φ and control variable $\dot{\varphi}$, with the additional constraint that $0 \leq \dot{\varphi} \leq 1$. The Hamiltonian can be written as:

$$H^c_\lambda(x, \varphi, \dot{\varphi}, p) = -(1 + \lambda - \delta)G(x)\dot{\varphi} + p\dot{\varphi},$$

where p is the co-state variable. By Pontryagin's Maximum Principle, a necessary condition for $(\varphi^c_\lambda, \dot{\varphi}^c_\lambda)$ to be optimal is that, for all $x \in \mathcal{X}$, $\dot{\varphi}^c_\lambda(x)$ maximizes $H^c_\lambda(x, \varphi^c_\lambda(x), p^c_\lambda(x), \cdot)$ for some piecewise continuously differentiable function p^c_λ that satisfies the Hamilton-Jacobi equation:

$$\dot{p}^c_\lambda(x) = -\frac{\partial H^c_\lambda}{\partial \varphi}(x, \varphi^c_\lambda(x), \dot{\varphi}^c_\lambda(x), p^c_\lambda(x)) = 0$$

at all points of continuity of $\dot{\varphi}^c_\lambda$. Since the boundary $\overline{x}$ is free, the transversality condition yields $p^c_\lambda(\overline{x}) = (1 - \delta)(1 + \lambda)$, so that p^c_λ is constant and equal to $(1 - \delta)(1 + \lambda)$. Substituting this back into H^c_λ, we find that a candidate optimal control is:

$$\dot{\varphi}^c_\lambda = 1_{\{x \in \mathcal{X} \mid -(1+\lambda-\delta)G(x)+(1-\delta)(1+\lambda) \geq 0\}}.$$

As H^c_λ is linear in $(\varphi, \dot{\varphi})$, Mangasarian's sufficiency conditions are satisfied, so $\dot{\varphi}^c_\lambda$ is indeed an optimal control, and φ^c_λ maximizes $L^c(\varphi, \lambda)$ on the space of functions in Φ that are piecewise continuously differentiable. By the Stone-Weierstrass Theorem, this space is dense in Φ endowed with the sup norm. Moreover, $L^c(\varphi, \lambda)$ is continuous in φ in this topology. Hence φ^c_λ maximizes $L^c(\varphi, \lambda)$ on Φ. Since the mapping $x \mapsto -(1 + \lambda - \delta)G(x) + (1 - \delta)(1 + \lambda)$ is decreasing as $\delta < 1$ and $\lambda \geq 0$, φ^c_λ corresponds to the debt contract with face value d^c_λ given by $-(1 + \lambda - \delta)G(d^c_\lambda) + (1 - \delta)(1 + \lambda) = 0$. Now, let d^c be defined as in Proposition 4. It is immediate to verify that:

$$d^c > \arg\max_{d \in \mathcal{X}} \left\{ \int_{\underline{x}}^{d} f g(f)\, df + [1 - G(d)]d - \delta d \right\} = G^{-1}(1 - \delta),$$

from which it follows that:

$$\lambda^c = \frac{(1 - \delta)[1 - G(d^c)]}{G(d^c) - (1 - \delta)}$$

is well-defined and non-negative. Let $\varphi^c = \min\{\mathrm{Id}_{\mathcal{X}}, d^c\}$. Two cases may arise. If $f^c_E = d^c = \overline{x}$, then $\lambda^c = 0$ and the first-best is reached. If $f^c_E < d^c < \overline{x}$, then $\lambda^c > 0$ and $\int_{\mathcal{X}} \varphi^c(x)\, g(x)\, dx = \delta\varphi^c(\overline{x})$, so that (17) is binding at φ^c. Since, by construction, φ^c maximizes $L^c(\varphi, \lambda^c)$, one has:

$$(1 - \delta) \int_{\mathcal{X}} \varphi^c(x) g(x)\, dx = L^c(\varphi^c, \lambda^c) \geq L^c(\varphi, \lambda^c) \geq (1 - \delta) \int_{\mathcal{X}} \varphi(x) g(x)\, dx$$

for any $\varphi \in \Phi$ that satisfies (17), and thus the debt contract with face value d^c is an optimal regular security from the issuer's viewpoint. $\|$

Proof of Lemma 5. Suppose that $\tilde{\varphi} \in \Phi$ guarantees the issuer a higher payoff than φ,

$$\delta \int_{\mathcal{X}} [\tilde{\varphi}(\overline{x}) - \tilde{\varphi}(x)]\, g(x)\, dx > \delta \int_{\mathcal{X}} [\varphi(\overline{x}) - \varphi(x)]\, g(x)\, dx, \tag{A.11}$$

while satisfying the no-exclusion constraint:

$$\int_{\mathcal{X}} [\tilde{\varphi}(x) - \delta\tilde{\varphi}(\overline{x})]\, g(x)\, dx \geq \int_{\underline{x}}^{\tilde{x}} [\tilde{\varphi}(x) - \delta\tilde{\varphi}(\tilde{x})]\, g(x)\, dx; \qquad \tilde{x} \in \mathcal{X}.$$

As Λ defines a positive measure on $\mathcal{X}$, we have:

$$\int_{\mathcal{X}} \left\{ \int_{\mathcal{X}} [\tilde{\varphi}(x) - \delta\tilde{\varphi}(\overline{x})]\, g(x)\, dx - \int_{\underline{x}}^{\tilde{x}} [\tilde{\varphi}(x) - \delta\tilde{\varphi}(\tilde{x})]\, g(x)\, dx \right\} d\Lambda(\tilde{x}) \geq 0. \tag{A.12}$$

But then, since:

$$\int_{\mathcal{X}} \left\{ \int_{\mathcal{X}} [\varphi(x) - \delta\varphi(\overline{x})]\, g(x)\, dx - \int_{\underline{x}}^{\tilde{x}} [\varphi(x) - \delta\varphi(\tilde{x})]\, g(x)\, dx \right\} d\Lambda(\tilde{x}) = 0,$$

(A.11) and (A.12) imply that $L^m(\tilde{\varphi}, \Lambda) > L^m(\varphi, \Lambda)$, a contradiction. $\|$

Proof of Proposition 8. For any fixed $\lambda \geq 1$, we study the problem of maximizing $L^m(\varphi, \Lambda_\lambda)$ with respect to $\varphi \in \Phi$. Rearranging the expression for $L^m(\varphi, \Lambda_\lambda)$ yields:

$$L^m(\varphi, \Lambda_\lambda) = \delta \int_{\underline{x}}^{f_E^m} G(x)\dot{\varphi}(x)\, dx + \int_{f_E^m}^{\overline{x}} [(1-\lambda)\delta G(x)\dot{\varphi}(x) + \lambda(1-\delta)g(x)\varphi(x)]\, dx.$$

Since $\lambda > 0$ and $\delta < 1$, it is clear that it is optimal to set $\varphi(x) = x$ for each $x \in [\underline{x}, f_E^m]$. We are thus left with the problem of maximizing:

$$\int_{f_E^m}^{\overline{x}} [(1-\lambda)\delta G(x)\dot{\varphi}(x) + \lambda(1-\delta)g(x)\varphi(x)]\, dx$$

with respect to $\varphi \in \Phi$. Restricting first the set of admissible values for φ to functions in Φ that are piecewise continuously differentiable, we treat this as an optimal control problem with state variable φ and control variable $\dot{\varphi}$, with the additional constraint that $0 \leq \dot{\varphi} \leq 1$. The Hamiltonian can be written as:

$$H_\lambda^m(x, \varphi, \dot{\varphi}, p) = (1-\lambda)\delta G(x)\dot{\varphi} + \lambda(1-\delta)g(x)\varphi + p\dot{\varphi},$$

where p is the co-state variable. By Pontryagin's Maximum Principle, a necessary condition for $(\varphi_\lambda^m, \dot{\varphi}_\lambda^m)$ to be optimal is that, for all $x \in \mathcal{X}$, $\dot{\varphi}_\lambda^m(x)$ maximizes $H_\lambda^m(x, \varphi_\lambda^m(x), p_\lambda^m(x), \cdot)$ for some piecewise continuously differentiable function p_λ^m that satisfies the Hamilton-Jacobi equation:

$$\dot{p}_\lambda^m(x) = -\frac{\partial H_\lambda^m}{\partial \varphi}(x, \varphi_\lambda^m(x), \dot{\varphi}_\lambda^m(x), p_\lambda^m(x)) = -\lambda(1-\delta)g(x)$$

at all points of continuity of $\dot{\varphi}_\lambda^m$. Since the boundary $\overline{x}$ is free, the transversality condition yields $p_\lambda^m(\overline{x}) = 0$, so that $p_\lambda^m = \lambda(1-\delta)(1-G)$ on $[f_E^m, \overline{x}]$. Substituting this back into H_λ^m, we find that a candidate optimal control on $[f_E^m, \overline{x}]$ is:

$$\dot{\varphi}_\lambda^m = 1_{\{x \in [f_E^m, \overline{x}] \mid (\delta - \lambda)G(x) + \lambda(1-\delta) \geq 0\}}.$$

As H_λ^m is linear in $(\varphi, \dot{\varphi})$, Mangasarian's sufficiency conditions are satisfied, so $\dot{\varphi}_\lambda^m$ is indeed an optimal control and, by an argument similar to that in Proposition 7, φ_λ^m maximizes $L^m(\varphi, \Lambda_\lambda)$ on Φ. Since the mapping $x \mapsto (\delta - \lambda)G(x) + \lambda(1-\delta)$ is decreasing as $\lambda \geq 1 > \delta$, φ_λ^m corresponds to the debt contract with face value $\max\{d_\lambda^m, f_E^m\}$, where d_λ^m is given by $(\delta - \lambda)G(d_\lambda^m) + \lambda(1-\delta) = 0$. Now, let d^m be defined as in Proposition 4. It is immediate to verify that:

$$d^m > \arg\max_{d \in \mathcal{X}} \left\{ \int_{\underline{x}}^{d} f g(f)\, df + [1 - G(d)]d - \delta d \right\} = G^{-1}(1-\delta),$$

from which it follows that:

$$\lambda^m = \frac{\delta G(d^m)}{G(d^m) - (1-\delta)}$$

is well-defined and greater or equal than one. Let $\varphi^m = \min\{\mathrm{Id}_{\mathcal{X}}, d^m\}$. Two cases may arise. If $f_E^m = d^m = \overline{x}$, then $\lambda^m = 1$ and the first-best is reached. If $f_E^m < d^m < \overline{x}$, then $\lambda^m > 1$. Since (20) holds and, by construction, φ^m maximizes $L^m(\varphi, \Lambda_{\lambda^m})$, Lemma 5 implies that the debt contract with face value d^m is an optimal security from the issuer's viewpoint. $\|$

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