Information Revelation in Competing Mechanism Games

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Abstract

We consider multiple-principal multiple-agent games of incomplete information. In this context, we show that restricting principals to propose truth-telling direct mechanisms is problematic: the best reply of a single principal to a given array of offers proposed by his rivals cannot always be represented through truth-telling mechanisms. We then show that if contracts are exclusive, truth-telling direct mechanisms are able to characterize best replies. This provides a rationale for the use of truth-telling direct mechanisms in the situation in which each agent can contract with at most one principal, that has been postulated in most economic applications.

Key words: Incomplete information, multiple principals, multiple agents, information revelation.
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1 Introduction

This paper contributes to the analysis of competing mechanism games with incomplete information. More specifically, we consider a scenario in which a number of principals simultaneously design incentive contracts in the presence of agents who are privately informed about their characteristics. This class of strategic interactions has provided a natural framework to represent the relationship between competition and incentives under incomplete information.

The situation in which only one agent is considered (common agency) has become a reference set-up to formalize competition in financial markets (Attar, Mariotti, and Salanie (2009), Biais, Martimort, and Rochet (2000)), conflicts between regulators (Calzolari (2004), Laffont and Pouyet (2004)), and to revisit classical issues in oligopoly theory (d'Aspremont and Ferreira (2009), Stole (2005)). Building on the seminal contribution of McAfee (1993), multiple-principal, multiple-agent games have been developed to represent markets in which buyers can choose between different auctions, as it is the case in many online tradings (Ellison, Fudenberg, and Mobius (2004), Peters and Severinov (1997)). Vertical restraints contexts and competition among hierarchies under asymmetric information have also been modeled as games with several principals and agents.[1]

Despite the increased economic relevance of competing mechanism models, there is still little consensus on their general predictive power. The recent contributions of Yamashita (2009), Peters and Szentes (2009), and Peters and Valverde (2009) provide different instances of a Folk Theorem: if no restriction is put on the set of available mechanisms, then a very large number of allocations can be supported as Perfect Bayesian equilibria of a competing mechanism game in which players play pure strategy.[2] One natural way to overcome such indeterminacy has been to restrict attention to some specific classes of mechanisms. Following the traditional approaches to oligopolistic screening, most economic models of competing mechanisms restricted attention to a class of simple communication mechanisms. That is, principals commit to message-contingent decisions which induce agents to truthfully reveal their initial private information:

> *[...] the literature on competing mechanisms [McAfee (1993), Peters and Severinov (1997)] has restricted principals to direct mechanisms in which agents report only private information about their preferences*, (Epstein and Peters (1999), p.121).

With some abuse of language, these simple mechanisms have been called direct revelation mechanisms.

[1] See Gal-Or (1997) for a survey of the early contributions in this area and Martimort and Piccolo (2009) for a recent application.

[2] These contributions often postulate the presence of at least two agents; Martimort-Stole(2003) suggest a similar result in a class of common agency models of incomplete information.

Although such a restriction involves a loss of generality even in single agent environments,[3] applied models of common agency have identified a rationale for making use of these mechanisms. At any pure strategy equilibrium, every single competitor behaves as a monopolist facing a single agent whose reservation utility is endogenous. The optimal behavior of such a monopolist can therefore be characterized through direct revelation mechanisms. Given this property, and provided that the agent's indirect utility function satisfies some regularity conditions, several features of equilibrium allocations can be derived.[4] On a more theoretical ground, Theorem 2 in Peters (2003) establishes that pure strategy equilibria of common agency games in which principals offer direct revelation mechanisms are robust to unilateral deviations towards any arbitrary indirect mechanism. That is, the corresponding outcomes will be supported in a pure strategy equilibrium of the game in which principals can make use of general communication mechanisms.

The present work investigates the rationale of such a restriction in multiple-principal multiple-agent models of incomplete information. The presence of multiple agents introduces additional strategic effects: from the viewpoint of a single principal, the messages sent to his rivals can be interpreted as hidden actions. He could therefore gain by introducing some uncertainty over his decisions, having the agents correlate on their message choices. It can be shown, however, that this uncertainty cannot always be reproduced through truth-telling direct mechanisms. We develop this idea in an example showing that, in a two-principal two-agent game of incomplete information, if Principal 2 proposes a truth-telling direct mechanism, then Principal 1 has a strict incentive to propose an untruthful one. That is, there is an equilibrium of the game in which principals are restricted to truth-telling direct mechanisms that does not survive to unilateral deviations towards more sophisticated communication schemes.

We reinterpret the result of the example in the light of the Myerson (1982) construction. Given the profile of mechanisms proposed by his opponents, a single principal is facing the same situation as if he was interacting with several agents who have private information and take some non-contractible actions, i.e. the messages they send to the other principals. In order to take this endogenous element of moral hazard into account, we assign a "virtual" game to each single principal. In the virtual game, the principal is facing both incomplete information and moral hazard, and the mechanisms offered by his opponents are part of the environment.

[3]The existence of equilibrium outcomes that can be supported through arbitrary (indirect) communication mechanisms but not through direct revelation ones has been acknowledged as a failure of the Revelation Principle in common agency games (Martimort and Stole (2002), and Peters (2001)).

[4]This methodology has been intensively used in most applications of common agency models with incomplete information to Finance (Biais, Martimort, and Rochet (2000), Khalil, Martimort, and Parigi (2007)), Public Economics and Regulation Theory (Bond and Gresik (1996), Calzolari (2004), Laffont and Pouyet (2004), Olsen and Osmudsen (2001)), and to Political Economy (Martimort and Semenov (2008)). A general exposition of the methodology is provided in Martimort and Stole (2003) and Martimort and Stole (2009).

Using this tool, we are able to characterize the best reply of any given principal using Myerson's methodology: the optimal mechanism consists in a "grand contract" in which every agent is honest (she truthfully reports her type) and obedient (she follows the recommendation of the principal over her hidden actions). If the grand contract designed by every single principal corresponds to the mechanism he selected at equilibrium in the original game (in other words, if recommendations are useless in the virtual game), the considered equilibrium will be robust to the introduction of complex communication schemes.

Our intuitions crucially exploit the presence of contractual externalities among principals, which suggests that the way in which agents' participation decisions are modeled plays a fundamental role. We hence move to consider the situation in which agents can participate with at most one principal, and we show that in such a scenario the best reply of every single principal can always be characterized through truth-telling direct mechanisms. This indeed suggests a rationale for the restriction to revelation mechanisms that has been postulated in most applied models with multiple principals and multiple agents under incomplete information, in which exclusivity of trades is typically assumed. The assumption of exclusive contracting is however not sufficient to guarantee that the restriction to direct revelation mechanisms involves no loss of generality: using indirect communication schemes, principals can induce equilibrium outcomes that cannot be supported by revelation mechanisms. We show this by means of a second example.

The remaining of the paper is organized as follows. Section 2 introduces the general model. Section 3 develops an example of a multiple-principal multiple-agent setting with non-exclusive negotiations, and Section 4 discusses robustness of equilibria in multi-principal multi-agent games within this class. Section 5 deals with the case of exclusive contracting, and Section 6 concludes.

2 Model

We refer to a scenario in which several principals (indexed by $j \in \mathcal{J} = \{1,...,J\}$) are contracting with several agents (indexed by $i \in I = \{1,...,I\}$). Each agent has private information about her preferences. The information available to agent i is represented by a type $\omega^i \in \Omega^i$. We denote $\omega \in \Omega = \times_{i \in I} \Omega^i$ a state of the world. Principals have common beliefs on the probability distribution of ω, and we take F to be the corresponding distribution function. Each principal $j \in \mathcal{J}$ communicates with the agents by means of a private message $m^i_j \in M^i_j$ that he receives from each agent i. Principal j can make his decisions $y_j \in Y_j$ contingent on the array of messages $m_j = (m^1_j, m^2_j, ..., m^I_j)$ he receives. Final allocations are determined by the multilateral contracts that principals independently sign with agents. More formally, we say that a mechanism proposed by principal j is the measurable mapping $\gamma_j = \underset{i \in I}{\times} M_j^i \to \Delta(Y_j)$, with $M_j = \times_i M_j^i$ is the relevant set of messages and $\Delta(Y_j)$ denotes the set of probability distributions over Y_j. We also let Γ_j be the set of mechanisms available to principal j, and we denote $\Gamma = \underset{j \in \mathcal{J}}{\times} \Gamma_j$. All relevant sets are taken to be compact and measurable; in addition, each message space M_j^i is assumed to satisfy the standard size restriction $\left| M_j^i \right| \geq \left| \Omega^i \right|$ for all i and j.[5] We also assume that every message space M_j^i is sufficiently rich to include the element $\{\emptyset\}$ corresponding to the information "agent i does not participate with principal j", which allows to incorporate the agents' participation decisions in a simple way.

Principal j's payoff is given by $v_j : Y \times \Omega \to \mathbb{R}_+$, and $u^i : Y \times \Omega \to \mathbb{R}_+$ is the payoff to agent i, with $Y = \underset{j \in \mathcal{J}}{\times} Y_j$. For a given array of principals' decisions $\tilde{y} = (\tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_J) \in \underset{j \in \mathcal{J}}{\times} \Delta(Y_j)$, the state contingent utilities of agent i and principal j are:

$$u^i(\tilde{y}, \omega) = \int_{Y_1} \int_{Y_2} ... \int_{Y_J} u^i(y, \omega)\, d\tilde{y}_1 d\tilde{y}_2 ... d\tilde{y}_J \quad \text{and} \quad v_j(\tilde{y}, \omega) = \int_{Y_1} \int_{Y_2} ... \int_{Y_J} v_i(y, \omega)\, d\tilde{y}_1 d\tilde{y}_2 ... d\tilde{y}_J$$

The competing mechanism game relative to Γ begins when principals simultaneously commit to a mechanism. Having observed the array of offered mechanisms $(\gamma_1, \gamma_2, ..., \gamma_J) \in \Gamma$, agents simultaneously send a message to each of the principals. In the final step, payoffs realize. In this incomplete information game, a strategy for agent i is the measurable mapping $\lambda^i : \Gamma \times \Omega^i \to \Delta(M^i)$, and a (pure) strategy for principal j is given by a mechanism γ_j. Every profile of agents' strategies $\lambda = (\lambda^1, \lambda^2, ..., \lambda^I)$ induces a probability distribution over principals decisions; given the array of mechanisms $\gamma = (\gamma_1, \gamma_2, ..., \gamma_J)$, we take this distribution to be $\beta_\lambda(\gamma, \omega)$. Thus, players' utilities corresponding to the array $\lambda = (\lambda^1, \lambda^2, ..., \lambda^I)$ are given by:

$$U^i(\lambda, \gamma, \omega) = \int_{Im(\gamma)} u^i(\tilde{y}, \omega)\, d\beta_\lambda(\gamma, \omega) \quad \text{and} \quad V_j(\lambda, \gamma, \omega) = \int_{Im(\gamma)} v_j(\tilde{y}, \omega)\, d\beta_\lambda(\gamma, \omega)$$

for each agent i and for each principal j, respectively. We typically say that the strategy profile $\lambda = (\lambda^1, \lambda^2, ..., \lambda^I)$ constitutes a continuation equilibrium relative to Γ if for every $i \in I$, for every $\gamma \in \Gamma$, and for every $\lambda^{i\prime}$, one gets:

$$U^i(\lambda, \gamma, \omega) \geq U^i(\lambda^{i\prime}, \lambda^{-i}, \gamma_1(\lambda^{i\prime}, \lambda^{-i}), \gamma_2(\lambda^{i\prime}, \lambda^{-i}), ..., \gamma_J(\lambda^{i\prime}, \lambda^{-i}), \omega).$$

[5]Observe that we treat the message spaces available to every single principal as given; an alternative route to define a mechanism would be to let each principal j choosing the array of message spaces $(M_j^i)_{i \in I}$, as it is done in Peters (2001). This alternative formulation can be introduced in the model without affecting our results.

We follow standard analyses of competing mechanism games with incomplete information and focus on Perfect Bayesian Equilibrium (PBE) as the relevant solution concept.

In the following, we will often consider a situation in which principals are restricted to offer truth-telling direct mechanisms. A mechanism available to principal j is said to be direct if agents can only communicate their type to principal j, i.e. if $M_j^i = \Omega^i$ for every (i, j). We take $\tilde{\gamma}_j : \times_{i \in I} \Omega^i \to \Delta(Y_j)$ to be the corresponding decision rule, and we let Γ_j^D be the set of direct mechanisms available to principal j. For a given profile of mechanisms $\gamma_{-j} \in \Gamma_{-j}$, we then say that $\tilde{\gamma}_j \in \Gamma_j^D \subseteq \Gamma_j$ is a truth-telling direct mechanism if the array $(\tilde{\gamma}_j, \gamma_{-j})$ induces a continuation equilibrium in which agents truthfully reveal their type to principal j.

3 Incentive Compatibility and Competing Mechanisms

This section emphasizes, by means of an example, that if principals compete in the presence of more than one agent under incomplete information, the restriction to truth-telling direct mechanisms involves a severe loss of generality. That is, there exist equilibrium outcomes of the game in which principals are restricted to truth-telling direct mechanisms that cannot be supported at equilibrium if principals can arbitrarily select their mechanisms.

Participation of the agents in this example is not an issue, we assume that agents participate with both principals.

Example 1 Let $I = J = 2$. Denote the principals as P1 and P2, and the agents as A1 and A2. The allocation spaces are $Y_1 = \{y_{11}, y_{12}, y_{13}, y_{14}\}$ for P1, and $Y_2 = \{y_{21}, y_{22}, y_{23}, y_{24}\}$ for P2. Each agent can be of two types, and $\Omega^1 = \Omega^2 = \{\omega^1, \omega^2\}$. Agents' types are perfectly correlated, thus the two states of the world are $\omega = \{(\omega^1, \omega^1), (\omega^2, \omega^2)\}$; in addition, we take $prob(\omega^1, \omega^1) = prob(\omega^2, \omega^2) = 0.5$. The payoffs of the game are represented in the two tables below. In each table, the first payoff is that of P1, who chooses the row, the second payoff is that of P2, who chooses the column, and the last two payoffs are those of A1 and A2, respectively. The first table shows the payoffs corresponding to the state (ω^1, ω^1):

	y_{21}	y_{22}	y_{23}	y_{24}
y_{11}	$(a, 0, -1, -1)$	$(a, 0, 0, 0)$	$(a, 0, 0, 9)$	$(16, 1, 15, 10)$
y_{12}	$(a, 0, 11, 0)$	$(a, 0, -1, -1)$	$(16+\varepsilon, 0, 10, 15)$	$(a, 0, 0, 0)$
y_{13}	$(a, 0, 0, 0)$	$(16+\varepsilon, 0, 10, 15)$	$(a, 0, -1, -1)$	$(a, 0, 11, 0)$
y_{14}	$(16, 0, 15, 10)$	$(a, 0, 0, 9)$	$(a, 0, 0, 0)$	$(a, 0, -1, -1)$

Table 1: Players payoffs in state (ω^1, ω^1)

The second table shows the payoffs corresponding to the state (ω^2, ω^2):

	y_{21}	y_{22}	y_{23}	y_{24}
y_{11}	$(a,0,-1,-1)$	$(a,0,0,0)$	$(a,0,0,9)$	$(16,0,15,10)$
y_{12}	$(a,0,11,0)$	$(a,0,-1,-1)$	$(16+\varepsilon,0,10,15)$	$(a,0,0,0)$
y_{13}	$(a,0,0,0)$	$(16+\varepsilon,0,10,15)$	$(a,0,-1,-1)$	$(a,0,11,0)$
y_{14}	$(16,1,15,10)$	$(a,0,0,9)$	$(a,0,0,0)$	$(a,0,-1,-1)$

Table 2: Players payoffs in state (ω^2,ω^2)

Observe that only P2's preferences are state-dependent. That is, incomplete information is neither affecting agents' behavior nor P1's one. In addition, $\varepsilon \in \mathbb{R}_+$ is taken to be arbitrarily small. We develop our argument via a series of claims.

Claim 1 *Suppose both principals are restricted to offering truth-telling direct mechanisms. Then, the profile of expected payoffs* $(16,1,15,10)$ *can be supported in an equilibrium in which principals play a pure strategy.*

Proof.

Consider the following behaviors. P1 selects the decision rule $\tilde{\gamma}_1$:

$$\tilde{\gamma}_1(m_1^1,m_1^2) = \begin{cases} y_{11} & \text{if he receives the message } \omega^1 \text{ from both A1 and A2} \\ y_{13} & \text{if he receives } \omega^1 \text{ from A1 and } \omega^2 \text{ from A2} \\ y_{12} & \text{if he receives } \omega^2 \text{ from A1 and } \omega^1 \text{ from A2} \\ y_{14} & \text{if he receives the message } \omega^2 \text{ from both A1 and A2} \end{cases}$$

P2 selects the decision rule $\tilde{\gamma}_2$:

$$\tilde{\gamma}_2(m_2^1,m_2^2) = \begin{cases} y_{24} & \text{if he receives the message } \omega^1 \text{ from both A1 and A2} \\ y_{23} & \text{if he receives } \omega^1 \text{ from A1 and } \omega^2 \text{ from A2} \\ y_{22} & \text{if he receives } \omega^2 \text{ from A1 and } \omega^1 \text{ from A2} \\ y_{21} & \text{if he receives the message } \omega^2 \text{ from both A1 and A2} \end{cases}$$

Given the mechanisms $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$, agents play a continuation game over messages. In this example, the agents' payoffs associated to this message game are independent of whether (ω^1,ω^1) or (ω^2,ω^2) realizes. They are represented in the following table where the first element in each cell denotes the payoff to A1 who chooses the row, and the second element denotes the payoff to A2.

	(ω^1,ω^2)	(ω^1,ω^1)	(ω^2,ω^2)	(ω^2,ω^1)
(ω^1,ω^2)	$(-1,-1)$	$(0,0)$	$(0,0)$	$(10,15)$
(ω^1,ω^1)	$(0,9)$	$(15,10)$	$(-1,-1)$	$(11,0)$
(ω^2,ω^2)	$(11,0)$	$(-1,-1)$	$(15,10)$	$(0,9)$
(ω^2,ω^1)	$(10,15)$	$(0,0)$	$(0,0)$	$(-1,-1)$

Table 3: Agents payoffs in the continuation game induced by $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$

Then, one can check that the following strategies constitute a continuation equilibrium:

- if (ω^1,ω^1) realizes, then both A1 and A2 truthfully reveal ω^1 to P1 and P2; that is: $m_1^1 = m_2^1 = \omega^1$ and $m_1^2 = m_2^2 = \omega^1$;
- if (ω^2,ω^2) realizes, then both A1 and A2 truthfully reveal ω^2 to P1 and P2; that is: $m_1^1 = m_2^1 = \omega^2$ and $m_1^2 = m_2^2 = \omega^2$.

These behaviors induce the profile of expected payoffs $(16,1,15,10)$. We now show that none of the principals has a unilateral incentive to deviate towards an alternative truth-telling direct mechanism. Since P2 is getting his maximal payoff of 1, one should only consider deviations of P1.

Observe that since agents' preferences are type-independent, deviations to a truth-telling direct mechanism may play an additional role only through their effects on the message-contingent decisions selected by P2. In this respect, we notice that, at the stage of communicating with P1, both agents are fully informed of the state of the world, given perfect correlation of types. Since principals are restricted to truth-telling direct mechanisms, each agent is perfectly informed of the report sent by the other one on the equilibrium path. This indeed guarantees that any P1's decision induced by some deviation γ_1' will be observed by both agents. That is, by proposing an arbitrary truth-telling direct mechanism, P1 cannot introduce any additional uncertainty with respect to the situation in which a (eventually stochastic) take-it or leave-it offer is proposed.

Consider now the case in which P1 deviates proposing a single take-it or leave-it offer to agents, irrespective of the received messages. We take this deviation to be y_1'; if y_1' is a deterministic offer, four alternative continuation games can be induced. Following a deviation towards $y_1' \in \{y_{11}, y_{12}, y_{13}, y_{14}\}$, the agents strategically set the messages to be sent to P2 who sticks to his equilibrium mechanism $\tilde{\gamma}_2$. The corresponding agents' payoffs are represented in the four tables below.

$y'_1 = y_{11}$

	$m_2^2 = \omega^2$	$m_2^2 = \omega^1$
$m_1^2 = \omega^2$	$(-1,-1)$	$(0,0)$
$m_1^2 = \omega^1$	$(0,9)$	$(15,10)$

$y'_1 = y_{12}$

	ω^2	ω^1
ω^2	$(11,0)$	$(-1,-1)$
ω^1	$(10,15)$	$(0,0)$

$y'_1 = y_{13}$

	ω^2	ω^1
ω^2	$(0,0)$	$(10,15)$
ω^1	$(-1,-1)$	$(11,0)$

$y'_1 = y_{14}$

	ω^2	ω^1
ω^2	$(15,10)$	$(0,9)$
ω^1	$(0,0)$	$(-1,-1)$

We argue that there is no continuation game yielding a payoff strictly greater than 16 to P1. More precisely,

1. If $y'_1 = y_{11}$, the agents' continuation game admits only one Nash equilibrium: both A1 and A2 report ω^1 to P2. The corresponding payoff to P1 is 16.

2. If $y'_1 = y_{12}$, the agents' continuation game admits only one Nash equilibrium: both A1 and A2 report ω^2 to P2. The corresponding payoff to P1 is a.

3. If $y'_1 = y_{13}$, the agents' continuation game admits only one Nash equilibrium: both A1 and A2 report ω^1 to P2. The corresponding P1 payoff is a.

4. If $y'_1 = y_{14}$, the agents' continuation game admits only one Nash equilibrium: both A1 and A2 report ω^2 to P2. The corresponding payoff to P1 is 16.

Suppose now that P1 deviates toward a stochastic take-it or leave-it offer (i.e. a randomization over $\{y_{11}, y_{12}, y_{13}, y_{14}\}$), and consider the continuation game induced by such a stochastic offer. One can check that if agents coordinate on a pure strategy equilibrium, then a can always be chosen low enough to guarantee that P1 cannot achieve a payoff strictly greater than 16. The only way for P1 to gain is therefore to have agents playing a mixed strategy in the continuation game. Let δ_{1k}, with $k = 1,2,3,4$ be the probability to play y_{1k} in the randomization selected by P1. For a deviation to be profitable one should have $(\delta_{12} + \delta_{13}) > 0$, otherwise P1 will not achieve a payoff greater than 16. Once again, it can be checked that, in any of such mixed strategy equilibria, the level of a can be fixed to be small enough to make the deviation unprofitable.[6]

[6]Observe that when the probability $\delta_{12} + \delta_{13}$ approaches zero, agents will play a pure strategy equilibrium yielding a payoff strictly smaller than 16 to P1.

This suggests that P1 has no incentive to deviate towards any truth-telling direct mechanism.
■

Claim 2 *Suppose P2 keeps playing the same truth-telling direct mechanism $\tilde{\gamma}_2$. Then, P1 can profitably deviate to an alternative direct mechanism attaining a payoff strictly higher than 16. Following this deviation both agents will have an incentive to lie with a strictly positive probability.*

Proof. Consider the following direct mechanism for P1:

$$\tilde{\gamma}_1'(m_1^1, m_1^2) = \begin{cases} y_{11} & \text{if he receives the message } \omega^1 \text{ from both A1 and A2} \\ y_{12} & \text{if he receives } \omega^1 \text{ from A1 and } \omega^2 \text{ from A2} \\ y_{13} & \text{if he receives } \omega^2 \text{ from A1 and } \omega^1 \text{ from A2} \\ y_{14} & \text{if he receives the message } \omega^2 \text{ from both A1 and A2} \end{cases}$$

The new array of mechanisms $\{\tilde{\gamma}_1', \tilde{\gamma}_2\}$ induces the following message game among agents:

	(ω^1,ω^2)	(ω^1,ω^1)	(ω^2,ω^2)	(ω^2,ω^1)
(ω^1,ω^2)	$(-1,-1)$	$(0,0)$	$(11,0)$	$(-1,-1)$
(ω^1,ω^1)	$(0,9)$	$(15,10)$	$(10,15)$	$(0,0)$
(ω^2,ω^2)	$(0,0)$	$(10,15)$	$(15,10)$	$(0,9)$
(ω^2,ω^1)	$(-1,-1)$	$(11,0)$	$(0,0)$	$(-1,-1)$

Table 4: Agents' payoffs in the continuation game induced by $\{\tilde{\gamma}_1', \tilde{\gamma}_2\}$.

Observe that, for both agents the strategies (ω^1,ω^2) and (ω^2,ω^1) are strictly dominated, hence we can eliminate them. In the reduced game, there is no equilibrium in pure strategies. The unique mixed-strategy Nash equilibrium prescribes that:

- Agent 1 mixes between (ω^1,ω^1) and (ω^2,ω^2), with equal probabilities.
- Agent 2 mixes between (ω^1,ω^1) and (ω^2,ω^2), with equal probabilities.

Simple computations show that these strategies give to principal P1 a payoff strictly greater than 16. With some strictly positive probability, the mechanism offered by P1 induces the agents to send contradicting information to the principals: one agent will send the message ω^1, while the other sends the message ω^2. Hence, the mechanism offered by P1 does not induce truthful information revelation. ■

The example suggests that if P1 introduces some uncertainty over the decisions he is effectively selecting, by inducing a mixed-strategy equilibrium in the agents' continuation game, he can get a higher payoff. The peculiarity of the example is that this uncertainty cannot be mapped into a truth-telling direct mechanism hence, given the mechanisms proposed by his opponents, a single principal can strictly gain through non-truthful mechanisms. The presence of multiple agents is crucial to get the result. In a single-agent environment, provided that all principals are playing a pure strategy, uncertainty is fully revealed at equilibrium: the best reply to any array of mechanisms can always be formulated as a truth-telling direct mechanism.[7]

One should also observe that in the equilibrium constructed in Claim 1, agents' choices are optimal from the point of view of every principal, both on and off the equilibrium path, following any principals' unilateral deviation. That is, the equilibrium supported by truth-telling direct mechanisms satisfies the requirement of strong robustness (Peters (2001)), which guarantees that our arguments do not rely on equilibrium multiplicity in the agents' continuation game.

4 Robust Equilibria

In multi-principal multi-agent contexts of incomplete information, each deviating principal is facing several agents who send him a message and take some payoff relevant, non-contractible actions (in our Example 1, from the point of view of P1, these are the messages that agents send to P2). This induces an endogenous element of moral hazard. It is hence legitimate to ask whether there is a set of mechanisms allowing principals to send private recommendations to agents, and to control their correlated behaviors, as in the standard analysis of Myerson (1982).[8] One would then be tempted to say that every equilibrium outcome induced by such a class of direct mechanisms will survive the introduction of more complex communication schemes.

We deal with this issue in the present section.[9]

It should first be noticed that the framework we consider is an extensive form game, in which all principals simultaneously propose a mechanism to the agents, and all agents, having observed the array of offered mechanisms, simultaneously send a private message to each of the competing principals. By construction, it is impossible for a given principal j to send recommendations to the agents over the messages they should send to his opponents. Once the recommendations would be received, agents have already chosen their non-contractible actions, i.e. the messages

[7] Theorem 2 in Peters (2003) formalizes this intuition.

[8] The need to explicitly incorporate principals' recommendations in the definition of a simple mechanism has been documented by Attar, Campioni, Piaser, and Rajan (2010) with reference to a pure moral hazard context with several principals and agents.

[9] We thank an anonymous referee and the editor for pointing us in the direction of this comparison.

sent to the opponent principals. This suggests that these messages are not formally equivalent to the agents' hidden actions (efforts) originally considered in the Myerson (1982) set-up.

The remaining of this section will indeed use Myerson (1982) construction to derive a simple characterization of equilibria supported by truth-telling direct mechanisms which survive the introduction of more complex communication schemes.

To this extent, we introduce a virtual game, in which we analyze the optimal behavior of a single principal within a standard moral hazard framework. Consider first the game Γ^D in which principals are constrained to use truth-telling direct mechanisms, and let $\left((\tilde{\gamma}_j)_{j\in\mathcal{J}},(\tilde{\lambda}^i)_{i\in I}\right)$ be the strategy profile corresponding to a well chosen equilibrium.

We then define the virtual j-game as a single principal game, in which principal j is facing I agents. We consider the other principals as non strategic players who stick to the mechanisms $\tilde{\gamma}_{-j}$. In this game, each agent i has both private information and the possibility to take the non-contractible actions $c^i = \left(c^i_1,\ldots,c^i_{j-1},c^i_{j+1},\ldots,c^i_J\right) \in \left(\Omega^i\right)^{J-1}$, which correspond to the messages she sends to (inactive) principals other than j. From the point of view of any principal $l\in\mathcal{J}$, $l\neq j$, we can define $\tilde{c}_l = \left(c^1_l,\ldots,c^I_l\right)\in\Omega$, as the array of actions that each agent i chooses for him. When considering together all (inactive) principals other than j, we then define $\tilde{c}_{-j}$, which is the array of all $\tilde{c}_l$ for every $l\in\mathcal{J}$, $l\neq j$, that is $\tilde{c}_{-j} = (\tilde{c}_1,\ldots,\tilde{c}_{j-1},\tilde{c}_{j+1},\ldots,\tilde{c}_J)$. Finally, we also denote $\tilde{\gamma}_{-j}(\tilde{c}_{-j}) = [\tilde{\gamma}_1(\tilde{c}_1),\ldots,\tilde{\gamma}_{j-1}(\tilde{c}_{j-1}),\tilde{\gamma}_{j+1}(\tilde{c}_{j+1}),\ldots,\tilde{\gamma}_J(\tilde{c}_J)]$ the decisions induced by the mechanisms offered by principals other than j, given the messages they receive from all agents. In a virtual j-game, principal j has a full control of the structure of communication; the extensive form of the game coincides with that developed by Myerson (1982) for principal-agents problems:

- Principal j receives one private message from any of the agents;
- Given these messages, he privately sends to each agent a recommendation on the actions that she should take, and selects some decision.

The Revelation Principle guarantees that, when looking for the optimal behavior of principal j, one can restrict attention to mechanisms which induce agents to be honest, truthfully reporting their types, and obedient, following the recommendations sent by principal j. Mathematically, any such mechanism is a mapping associating to each array of reported types a joint probability distribution over actions $\tilde{c}_{-j}$ and decisions y_j. We denote this mapping G_j, and we let $G_j(\tilde{c}_{-j},y_j\,|\omega)$ be the probability of sending the recommendations $\tilde{c}_{-j}$ and taking the decision y_j given the array of reports ω. Finally, we let G^*_j be an optimal mechanism for principal j. In the virtual j-game, the expected payoff to principal j when proposing the mechanism G^*_j is:

$$\int_{\Omega}\int_{\Omega^{J-1}}\int_{Y_j} v\left[y_j,\gamma_{-j}\left(\tilde{c}_{-j}\right),\omega\right] G_j^*\left(\tilde{c}_{-j},y_j\left|\omega\right.\right) F\left(\omega\right) dy_j d\tilde{c}_{-j} d\omega. \tag{1}$$

The following result generalizes Peters (2003)'s Theorem 2.

Lemma 1 *Let $\left(\tilde{\gamma},\tilde{\lambda}\right)$ be an equilibrium of the game Γ^D, and let $\left(\bar{V}_j\right)_{j\in J}$ be the corresponding principals' payoffs. If for every principal j*

$$\int_{\Omega}\int_{\Omega^{J-1}}\int_{Y_j} v\left[y_j,\gamma_{-j}\left(\tilde{c}_{-j}\right),\omega\right] G_j^*\left(\tilde{c}_{-j},y_j\left|\omega\right.\right) F\left(\omega\right) d\omega d\tilde{c}_{-j} dy_j = \overline{V}_j, \tag{2}$$

then none of the principals has an incentive to unilaterally deviate towards any arbitrary communication mechanism.

Proof. We prove the statement by contradiction. Take $\tilde{\lambda}=\left(\tilde{\lambda}^1,\tilde{\lambda}^2,\ldots,\tilde{\lambda}^I\right)$ to be the continuation equilibrium relative to the equilibrium mechanisms $\tilde{\gamma}=(\tilde{\gamma}_1,\tilde{\gamma}_2,\ldots,\tilde{\gamma}_J)\in\Gamma^D$, with $\Gamma^D=\times_{j\in\mathcal{J}}\Gamma_j^D$. As in the proofs of Theorem 1 and 2 in Peters (2003), we have to extend the continuation equilibrium $\tilde{\lambda}$ to all profile of mechanisms in Γ so to support the original equilibrium. We take $\bar{\lambda}=\left(\bar{\lambda}^i(\gamma,\omega),\bar{\lambda}^{-i}(\gamma,\omega)\right)$ to be the continuation equilibrium on such extended set. In particular, we let

$$\bar{\lambda}^i(\gamma,\omega)=\tilde{\lambda}^i(\gamma,\omega)\quad\forall i,\forall\omega,\text{ and }\forall\gamma\in\Gamma^D\subseteq\Gamma$$

that is, each type of every agent sends exactly the same array of messages when she is offered the same profile of truth-telling direct mechanisms in the "enlarged" game in which all communication mechanisms are feasible. Suppose now that principal j deviates towards the mechanism $\gamma_j'\notin\Gamma_j^D$, and let $\left(\lambda^i(\gamma_j',\tilde{\gamma}_{-j},\omega),\lambda^{-i}(\gamma_j',\tilde{\gamma}_{-j},\omega)\right)$ be a continuation equilibrium induced by such a deviation. The corresponding probability distribution over principals' decisions is $\beta_\lambda(\gamma_j',\tilde{\gamma}_{-j},\omega)$, for every ω. Let V_j' be the payoff to principal j following the deviation; since the deviation is profitable, one has $V_j'>\bar{V}_j$.

We then consider the virtual j-game, and argue that principal j can always achieve the payoff V_j' by asking each agent to report her type and recommending to each agent the messages to be sent to his opponents. In this game, principal j effectively acts as a mechanism designer, taking as given the (truth-telling direct) mechanisms offered by his opponents. It is a best reply for each agent to be truthful and obedient to principal j. Standard arguments guarantee that one can always construct a mechanism G_j such that $Im(G_j)=Im\left(\gamma_j'(\lambda^i,\lambda^{-i},\omega)\right)$. In particular, this mechanism induces a joint probability distribution over the decisions y_j and the recommended actions $\tilde{c}_{-j}$ which is the same as that generated in the continuation equilibrium induced by $(\gamma_j',\tilde{\gamma}_{-j})$. That is, given the array $\tilde{\gamma}_{-j}$, there is a $G_j\left(y_j,c_{-j},|\omega\right)$ such that:

$$G_j(y_j, c_{-j}|\omega) = \beta_{\bar{\lambda}}(\gamma_j', \tilde{\gamma}_{-j}, \omega) \quad .$$

The corresponding payoff to principal j is given by:

$$V_j' = \int_\Omega \int_{Im(\gamma_j', \tilde{\gamma}_{-j})} v_j(y_j, y_{-j}, \omega)\, \beta_{\bar{\lambda}}(\gamma_j', \tilde{\gamma}_{-j}, \omega) dF(\omega) = \int_\Omega \int_{\Omega^{J-1}} \int_{Y_j} v_j(y_j, \tilde{\gamma}_{-j}(\tilde{c}_{-j}), \omega)\, dG_j(y_j, \tilde{c}_{-j}|\omega) dF(\omega)$$

From the definition of G_j^* one gets

$$\int_\Omega \int_{\Omega^{J-1}} \int_{Y_j} v_j(y_j, \tilde{\gamma}_{-j}(\tilde{c}_{-j}), \omega)\, dG_j^*(y_j, \tilde{c}_{-j}|\omega) dF(\omega) \geq V_j' > \bar{V}_j$$

which contradicts (2).

■

The intuition for Lemma 1 is as follows. If a single principal could achieve a strictly higher payoff by using a complex communication mechanism, then the same payoff could always be obtained in the corresponding virtual game in which he fully controls agents' behaviors by recommending them the messages to be sent to his opponents.

The reasoning developed in our Example 1 can be interpreted in the light of the above results. In particular, the profitable deviation $\tilde{\gamma}_1'(m_1^1, m_1^2)$ can be reproduced by a revealing mechanism in the virtual game of P1. Given P2's equilibrium mechanism, let's suppose that P1 has the possibility to send private recommendations to the agents over the messages that they send to P2. Then, P1 can offer the mechanism G_1 defined as: $\quad \forall (\omega', \omega'') \in \{\omega^1, \omega^2\}^2$,

$$G_1(\omega, y|\omega', \omega'') = \begin{cases} G_1\left(\omega^1, \omega^1, y_{11}\,|\omega', \omega''\right) = \frac{1}{4} \\ G_1\left(\omega^1, \omega^2, y_{12}\,|\omega', \omega''\right) = \frac{1}{4} \\ G_1\left(\omega^2, \omega^1, y_{13}\,|\omega', \omega''\right) = \frac{1}{4} \\ G_1\left(\omega^2, \omega^2, y_{14}\,|\omega', \omega''\right) = \frac{1}{4} \end{cases}$$

For the agents, their messages to P1 do not affect the mechanism, hence the mechanism G_1 is type revealing (agents are honest). Now, if A1 receives the recommendation ω^1 from P1, he knows that with probability $1/2$ A2 has received the recommendation ω^1 and that P1 will take the decision y_{11}, and that with identical probability A2 has received the recommendation ω^2 and that P1 will take the decision y_{12}. Hence, it is optimal for him to report ω^1 to P2. Similarly, it can be checked that all recommendations are incentive compatible and that mechanism G_1 gives to P1 a payoff strictly greater than 16.

One should however notice that Lemma 1 only provides sufficient conditions for an equilibrium of the game Γ^D to survive the introduction of more sophisticated mechanisms. To clarify this point, suppose that, following a unilateral deviation to some arbitrary mechanism γ_j', the corresponding continuation game exhibits multiple equilibria, with agents coordinating on the most unfavorable one for the deviating principal j. In such a situation, the original equilibrium may survive even though (2) is violated. That is, our analysis allows one to fully characterize those robust equilibria in which agents play the best continuation equilibrium from the point of view of each principal j both on and off the equilibrium path following any unilateral deviation to some mechanism $\gamma_j' \in \Gamma_j$.[10]

5 Multiple-Principal, Multiple-Agent Games with Exclusivity

Based on applied literature, we examine here the situation in which agents are restricted to participate with at most one principal. We refer to these settings as multiple-principal multiple-agent games with exclusivity, since exclusivity clauses are imposed from the outset. In this framework, we provide a result on the (robustness) structure of equilibria of the game in which principals are restricted to use truth-telling direct mechanisms.

At any equilibrium in which principals play a pure strategy, there is no unilateral incentive to deviate towards alternative communication mechanisms. That is, the outcomes supported by truth-telling direct mechanisms will be also supported at equilibrium in the game with unrestricted mechanisms. In these frameworks, pure strategy equilibria of truth-telling direct mechanisms are robust.

However, truth-telling direct mechanisms do not provide a complete description of the set of equilibria of multi-principal multi-agent games with exclusivity, even if one restricts attention to equilibria with players playing pure strategies. A simple way to represent a multiple-principal multiple-agent game with exclusivity within our modeling framework is to assume that now the message set $M^i = \times_{j\in\mathcal{J}} M^i_j$ available to each agent i only includes arrays with at most one element different from $\{\emptyset\}$, i.e. $m^i = \left\{\emptyset, ..., \emptyset, m^i_j, \emptyset, ..., \emptyset\right\}$ if agent i participates with principal j. In this context, pure strategy equilibria of games in which principals are restricted to truth-telling direct mechanisms survive to the introduction of more complex mechanisms.

Lemma 2 *Consider the class of incomplete information multiple-principal multiple-agent games with exclusivity. Let $(\tilde{\gamma}, \tilde{\lambda})$ be a PBE of the game in which principals are restricted to use truth-telling direct mechanisms. Then, the corresponding equilibrium outcome can be supported as a*

[10] The competing mechanism literature has extensively looked at such a refinement, which is referred to as strongly robust equilibrium. See Han (2007) for a recent application of this notion to analyze the robustness of take-it or leave-it offers schemes in competing mechanism models of complete information.

pure strategy PBE of the game in which principals can offer arbitrary communication mechanisms.

Proof. We take $\tilde{\lambda} = \left(\tilde{\lambda}^1, \tilde{\lambda}^2, \ldots, \tilde{\lambda}^I\right)$ to be the continuation equilibrium relative to the equilibrium mechanisms $\tilde{\gamma} = (\tilde{\gamma}_1, \tilde{\gamma}_2, \ldots, \tilde{\gamma}_J) \in \Gamma^D$, with $\Gamma^D = \times_{j \in \mathcal{J}} \Gamma_j^D$. As in the proofs of Theorem 1 and 2 in Peters (2003), we have to extend the continuation equilibrium $\tilde{\lambda}$ to all profile of mechanisms in Γ so to support the original equilibrium. We take $\bar{\lambda} = \left(\bar{\lambda}^i(\gamma, \omega), \bar{\lambda}^{-i}(\gamma, \omega)\right)$ to be the continuation equilibrium on such extended set. In particular, we let

$$\bar{\lambda}^i(\gamma, \omega) = \tilde{\lambda}^i(\gamma, \omega) \quad \forall i, \forall \omega, \text{ and } \forall \gamma \in \Gamma^D \subseteq \Gamma$$

that is, each type of every agent selects exactly the same participation decisions (i.e. sends exactly the same array of messages) if offered with a collection of truth-telling direct mechanisms in the "enlarged" game where all communication mechanisms are feasible. Suppose now that principal j deviates towards the mechanism $\gamma_j' \notin \Gamma_j^D$, and let $\left(\lambda^i(\gamma_j', \tilde{\gamma}_{-j}, \omega), \lambda^{-i}(\gamma_j', \tilde{\gamma}_{-j}, \omega)\right)$ be a continuation equilibrium induced by such a deviation. In this continuation equilibrium a subset $\tilde{\Omega}^i \subseteq \Omega^i$ of each agent's types will be participating with principal j with a strictly positive probability. With respect to these types, principal j effectively acts as a single mechanism designer, since the messages sent to his rivals are not payoff relevant to him. Standard arguments guarantee that one can construct a truth-telling direct mechanism $\tilde{\gamma}_j'$ embedding all decisions induced by the agents' continuation equilibrium on the mechanism γ_j'; that is $Im(\tilde{\gamma}_j') = Im\left(\gamma_j'(\lambda^i, \lambda^{-i}, \omega)\right)$. In addition, given the array $\tilde{\gamma}_{-j}$, the same probability distribution over principals' decisions can be generated. That is,

$$\beta_{\tilde{\lambda}}(\tilde{\gamma}_j', \tilde{\gamma}_{-j}, \omega) = \beta_{\lambda}(\gamma_j', \tilde{\gamma}_{-j}, \omega) \quad \forall \omega \in \times_i \tilde{\Omega}^i.$$

It follows that:

$$\int_{\Omega} \int_{Im(\gamma_j', \tilde{\gamma}_{-j})} v_j(\tilde{y}, \omega)\, d\beta_{\lambda}(\gamma_j', \tilde{\gamma}_{-j}, \omega) dF = \int_{\Omega} \int_{Im(\tilde{\gamma}_j', \tilde{\gamma}_{-j})} v_j(\tilde{y}, \omega)\, d\beta_{\lambda}(\tilde{\gamma}_j', \tilde{\gamma}_{-j}, \omega) dF$$

which guarantees that the unilateral deviation towards γ_j' cannot be profitable since:

$$\int_{\Omega} \int_{Im(\tilde{\gamma}_j', \tilde{\gamma}_{-j})} v_j(\tilde{y}, \omega)\, d\beta_{\lambda}(\tilde{\gamma}_j', \tilde{\gamma}_{-j}, \omega) dF \leq \int_{\Omega} \int_{Im(\tilde{\gamma})} v_j(\tilde{y}, \omega)\, d\beta_{\tilde{\lambda}}(\tilde{\gamma}, \omega) dF$$

■

The intuition for the result is that, in games with exclusivity, the uncertainty generated at any (mixed strategy) continuation equilibrium of the agents' message game can always be reproduced through truth-telling direct mechanisms. This indeed provides some foundation for the restriction to truth-telling direct mechanisms that has typically been postulated in most economic applications of multi-principal multi-agent games with incomplete information.[11]

The introduction of exclusivity clauses, however, is not sufficient to guarantee that this restriction involves no loss of generality.

We provide an example of a game with multiple principals and multiple agents exhibiting a payoff profile that can be supported at equilibrium by indirect communication mechanisms but not through truth-telling direct ones. A first result in a multiple-principal, multiple-agent game of exclusivity has been provided by Peck (1997).[12]

Example 2 Let $I = J = 2$ and $\Omega^1 = \Omega^2 = \{\omega\}$. In addition, take $Y_1 = \{y_{11}, y_{12}\}$ and $Y_2 = \{y_{21}, y_{22}\}$. The actions that each agent can take are simply to accept or reject the allocation proposed by the principals. The game is exclusive, hence each agent can at most accept one of the proposals.

The payoffs in the following tables represent the utilities of the two principals and those of the two agents, i.e. (V_1, V_2, U^1, U^2), respectively.

For the sake of the example, payoffs associated to agents' participation decisions are such that if both agents do not participate in the game, then every player gets a payoff 0. If A2 and A1 accept the proposal of P2, payoffs are $(0,0,0,2)$ whatever the decision of the principals. If A2 and A1 accept the proposal of P1, then everybody gets 0.

If A1 participates with P2 and A2 participates with P1, payoffs are described by the following matrix:

	y_{21}	y_{22}
y_{11}	$(1,1,1,1)$	$(1,2,1,1)$
y_{12}	$(-1,-1,1,1)$	$(-1,-1,1,1)$

If A1 (resp. A2) participates with P1 and A2 (resp. A1) does not participate, payoffs are those given in the matrix above, except for A2 (resp. A1) and P2 who get 0 in any case.

If A1 participates with P1 and A2 participates with P2, payoffs are:

[11] Consider, as an example, the literature on competing auctions where the restriction to direct revelation mechanisms has been typically introduced, starting from the original work of McAfee (1993).

[12] The example indeed formalizes an intuition originally suggested in Martimort (1996).

	y_{21}	y_{22}
y_{11}	$(1,1,2,2)$	$(1,2,2,2)$
y_{12}	$(-1,-1,2,2)$	$(-1,-1,3,2)$

If A1 (resp. A2) participates with P2 and A2 (resp. A1) does not participate, payoff are those given in the matrix above, adjusted for A2 (resp. A1) and P1 getting 0 in any case.

We first consider the situation in which principals are restricted to truth-telling direct mechanisms that, in this simple setting, coincide with mechanisms conditional on participation, only. We argue that the decision profile:

- P1 proposes y_{11} and P2 proposes y_{21} with probability one,
- A1 and A2 accept the offer of P1 and P2, respectively

cannot be supported at equilibrium when principals use truth-telling direct mechanisms. Indeed, if principals were offering participation contingent mechanisms that implement y_{11} and y_{21}, P2 would have and incentive to deviate toward the take-it or leave-it offer y_{22}. Whatever the truthful direct mechanism proposed by P1, it is always a better option for A2 to participate with P2 than not to participate at all or than to participate with P1: in any continuation game, participating with P2 is a strictly dominant strategy. It also clear that there is no equilibrium in which both agents participate with P2: A1 prefers to get a strictly positive payoff rather than 0. Hence, the restriction to truthful direct mechanisms does not allow to sustain at equilibrium the payoffs $(1,1,2,2)$.

Nonetheless, we argue that the same decision profile can be supported at equilibrium if principals can make use of indirect communication mechanisms. Let the relevant message spaces be $M^1 = \{m_1, m_2\}$ and $M^2 = \{m\}$. That is, only A1 effectively communicates with principals. Consider the following decision rules: P1 selects

$$\gamma_1^* = \begin{cases} y_{11} & \text{if he receives the message } m_1 \text{ or no message from A1} \\ y_{12} & \text{if he receives the message } m_2 \text{ from A1} \end{cases}$$

and P2 makes the take-it or leave-it offer y_{21}. The corresponding agents' continuation game admits one Nash Equilibrium where A1 sends the message m_1 to P1 and participates with P1, and A2 participates with P2. The associated payoff profile is $(1,1,2,2)$.

Since P1 is earning his maximal payoff, to argue that this strategy profile constitutes an equilibrium, one only needs to verify that P2 has no unilateral incentive to deviate. Now, if P2 deviates proposing the deterministic offer y_{22}, the unique equilibrium in the agents' game has A1 participating with P1 sending him the message m_2, while A2 participates with P2. The payoff to the deviating principal P2 will be -1. One should observe that P2 cannot deviate through stochastic take-it or leave-it offers either. If $\alpha \in (0,1)$ is the probability that is assigned to the decision y_{21} in any randomization of P2, whatever α is, it is a best reply for A1 to send the message m_2. Finally, our argument straightforwardly extends to any direct of indirect mechanism: as soon as A1 anticipate that P2 will implement the decision y_{22}, he sends the message m_2 to P1.

Our argument makes a crucial use of the presence of multiple agents. In common agency environments, the restriction to exclusivity of contracts is sufficient to guarantee that principals have no incentive to extract additional information from the single agent beyond type revelation. In multiple-agents frameworks, instead there is room for a principal to correlate among agents. This example shares with Peck (1997) the result that in multiple-principal, multiple-agent games of exclusivity asking the agents only to report their initial type may be restrictive.[13] From our point of view, additional equilibria arise even if one restricts attention to a situation where principals play pure strategies at equilibrium. The additional equilibrium is not sustained by means of randomization on principals' mechanisms as in Peck (1997), but by asking the agents to enrich the communication including details of the principals' mechanisms.

6 Conclusions

This paper contributes to analyze multi-principal multi-agent games of incomplete information, both in case of exclusive and non-exclusive negotiation. We show that when the framework is non-exclusive, i.e. each agent can simultaneously negotiate with several principals, limiting principals to use truth-telling direct mechanisms can be restrictive. In the presence of several agents, a principal tries to exploit correlation among agents' behaviors relative to his opponents in his own interest and can profitably induce (untruthful) random behaviors.

In the case of exclusive contracting, equilibria characterized through truth-telling direct mechanisms are robust to extension to more complex communication schemes. Assuming exclusive contracting may hence provide a rationale for restricting to truth-telling direct mechanisms in economic applications. Such a restriction, however, always involves a loss of generality. Even if contracts are exclusive, the use of complex mechanisms may induce additional equilibrium outcomes by generating new threats in the agents' continuation game.

[13] The example indeed formalizes an intuition originally suggested in Martimort (1996).

References

ATTAR, A., E. CAMPIONI, G. PIASER, AND U. RAJAN (2010): "On Multiple Principal, Multiple Agent Models of Moral Hazard," *Games and Economic Behavior*, 68(1), 376–380.

ATTAR, A., T. MARIOTTI, AND F. SALANIE (2009): "Nonexclusive competition in the market for lemons," Mimeo, Toulouse School of Economics.

BIAIS, B., D. MARTIMORT, AND J.-C. ROCHET (2000): "Competing mechanisms in a common value environment," *Econometrica*, 78(4), 799–837.

BOND, E., AND T. GRESIK (1996): "Regulation of Multinational Firms with Two Active Governments: A Common Agency Approach," *Journal of Public Economics*, 59 (1), 33–53.

CALZOLARI, G. (2004): "Incentive Regulation of Multinational Enterprises," *International Economic Review*, 45(1), 257–282.

D'ASPREMONT, C., AND R. D. S. FERREIRA (2009): "Price-quantity competition with varying toughness," *Games and Economic Behavior*, 65(1), 62–82.

ELLISON, G., D. FUDENBERG, AND M. MOBIUS (2004): "Competing auctions," *Journal of the European Economic Association,*, 2(1), 30–66.

EPSTEIN, L. G., AND M. PETERS (1999): "A revelation principle for competing mechanisms," *Journal of Economic Theory*, 88(1), 119–160.

GAL-OR, E. (1997): "Multiprincipal Agency Relationships as Implied by Product Market Competition," *Journal of Economics and Management Strategy*, 6(2), 235–256.

HAN, S. (2007): "Strongly Robust Equilibrium and Competing Mechanism Games," *Journal of Economic Theory*, 137(1), 610–626.

KHALIL, F., D. MARTIMORT, AND B. PARIGI (2007): "Monitoring a Common Agent," *Journal of Economic Theory*, 135(1), 35–67.

LAFFONT, J.-J., AND J. POUYET (2004): "The Subsidiarity Bias in Regulation," *Journal of Public Economics*, 88(1), 255–283.

MARTIMORT, D. (1996): "Exclusive dealing, common agency and multiprincipals incentive theory," *RAND Journal of Economics*, 27(1), 1–31.

MARTIMORT, D., AND S. PICCOLO (2009): "The Strategic Value of Quantity Forcing Contracts," *American Economic Journal: Microeconomics*, forthcoming.

MARTIMORT, D., AND A. SEMENOV (2008): "Ideological Uncertainty and Lobbying Competition," *Journal of Public Economics*, 92(3-4), 456–481.

MARTIMORT, D., AND L. A. STOLE (2002): "The revelation and delegation principles in common agency games," *Econometrica*, 70(4), 1659–1673.

——— (2003): "Contractual externalities and common agency equilibria," *Advances in Theoretical Economics*, 3(1).

——— (2009): "Market participation under delegated and intrinsic common agency games," *Rand Journal of Economics*, 40(1), 78–102.

MCAFEE, P. (1993): "Mechanism design by competing sellers," *Econometrica*, 61(6), 1281–1312.

MYERSON, R. B. (1982): "Optimal coordination mechanisms in generalized principal-agent problems," *Journal of Mathematical Economics*, 10, 67–81.

OLSEN, T., AND P. OSMUDSEN (2001): "Strategic Tax Competition: Implications of National Ownership," *Journal of Public Economics*, 81(2), 253–2877.

PECK, J. (1997): "A note on competing mechanisms and the revelation principle," mimeo Ohio State University.

PETERS, M. (2001): "Common Agency and the Revelation Principle," *Econometrica*, 69(5), 1349–1372.

——— (2003): "Negotiation and take-it-or-leave-it in common agency," *Journal of Economic Theory*, 111(1), 88–109.

PETERS, M., AND S. SEVERINOV (1997): "Competition Between Sellers Who Offer Auctions Instead of Prices,," *Journal of Economic Theory*, 75(1), 141–179.

PETERS, M., AND B. SZENTES (2009): "Definable and Contractible Contracts," Mimeo, University of British Columbia.

PETERS, M., AND C. VALVERDE (2009): "A Folk Theorem for Competing Mechanisms," Mimeo, University of British Columbia.

STOLE, L. (2005): "Price discrimination in competitive environments," in *Handbook of Industrial Organization*, ed. by M. Armstrong, and R. Porter. Amsterdam: North Holland.

YAMASHITA, T. (2009): "Mechanism Games with Multiple Principals and Three or More Agents," *Econometrica*, forthcoming.