Competing Mechanisms, Exclusive Clauses and the Revelation Principle

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Abstract

We consider multiple-principal multiple-agent games of incomplete information in which each agent can at most participate with one principal. In such contexts, we show that the restriction to direct truthful mechanisms involves a loss of generality, even if one only focuses on pure strategy equilibria. However, the traditional Revelation Principle retains its power in games with a single agent.

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1 Introduction

It is an established finding that the standard restriction to direct truthful mechanisms involves a loss of generality whenever several principals compete through mechanisms in the presence of agents who have private information on their characteristics. This result has been acknowledged as a failure of the Revelation Principle in competing mechanism games, and it has been documented in a number of game-theoretic examples. The examples (see e.g. Peters (2001) and Martimort and Stole (2002)) typically focus on a single agent setting and postulate that the agent participates with many principals at a time. Equilibria are typically supported by having each single principal using sophisticated (indirect) communication mechanisms to control the agent's out-of-equilibrium behavior in such a way to create a threat preventing unilateral deviations by his opponents. In these situations, direct revelation mechanisms might not flexible enough to reproduce the same threats.

One should however observe that most economic applications of competing mechanism models consider a scenario where every agent can participate with at most one principal. That is, exclusive clauses can be enforced at no cost. In games with several agents, such an assumption has for example been postulated in the analysis of multiple auctions (McAfee (1993), Peters and

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Severinov (1997), and Moldovanu, Selab, and Shi (2008)) as well as in the literature on competing hierarchies (see Caillaud, Jullien, and Picard (2000) and Martimort and Piccolo (2010) among many others). In single agent contexts, it is worth remarking that many recent approaches to oligopolistic screening in financial markets restrict the agent to participate with at most one principal, in line with the canonical Rothschild and Stiglitz (1976) analysis.[1]

A common feature to all these approaches to competing mechanisms is that principals are restricted to make use of simple direct revelation mechanisms. That is, their decisions are made contingent on agents' truthfully revealing their private information.[2] It seems therefore meaningful to ask to what extent such a restriction involves a loss of generality, taking also into account that this issue has received very little attention in the analysis of competing mechanism models of exclusivity.[3]

The present work argues that one can safely assume that principals make use of direct truthful mechanisms as long as only one agent is considered (exclusive agency). In addition, we show by means of an example that, in competing mechanism games of exclusivity and at least two agents, there are equilibrium outcomes that can be supported through arbitrary communication mechanisms, but not through direct truthful ones. Importantly, this holds true even if one restricts attention to pure strategy equilibria. The result suggests that additional research is needed to identify the properties of equilibrium contracts in competing mechanism games of exclusivity, even if the analysis is limited to pure strategy equilibria, as it is typically done in applications.

The remaining of the paper is organized as follows. In Section 2 we develop a multiple-principal multiple-agent model of incomplete information where exclusive clauses are imposed from the outset. In Section 3 we show that the restriction to direct truthful mechanisms is with no loss of generality as long as only one agent is considered. In Section 4 we provide an example showing that that in competing mechanism models of exclusivity with at least two agents there exist equilibrium outcomes that can be supported through arbitrary communication mechanisms but not through direct truthful ones. Section 5 concludes.

2 The Model

We refer to a scenario where several principals (indexed by $j \in \mathcal{J} = \{1, ..., J\}$) contract with several agents (indexed by $i \in I = \{1, ..., I\}$). Each agent has private information about her preferences. The information available to agent i is represented by a type $\omega^i \in \Omega^i$. We denote $\omega \in \Omega = \times_{i \in I} \Omega^i$ a state of the world. Principals have common beliefs on the probability distribution of ω, and F is the corresponding distribution function.

[1] See Freixas and Rochet (1997) and Rees and Wambach (2008) for an in-depth discussion of the applications to competition among banks and among insurance providers, respectively.

[2] Although it has now become customary to denote such type-revealing mechanisms as direct revelation mechanisms (see for example Epstein and Peters (1999), p.121), they must be distinguished from the standard direct mechanisms described in the traditional single principal analysis of Myerson (1982).

[3] Observe that we will not consider here more general attempts at modeling exclusivity of contracting, as it is for instance done in the literature on contractible contracts initiated by Epstein and Peters (1999). Indeed, the approach followed in this paper is motivated by those economic applications of multiple-principal multiple-agent games which postulate that each of the agents can at most deal with one principal.

Each principal j may take an action $x_j \in X_j$. Agents only take participation decisions;[4] each agent i cannot participate with more than one principal. Specifically, we take $A^i = \{a^i = (a^i_1, a^i_2, ..., a^i_J) \in \{Y,N\}^{\sharp J} : a^i_j = \{Y\}$ for at most one $j\}$ to be the set of possible participation choices for each agent i, with $a^i \in A^i = (Y,N,N,N,...)$ indicating that she participates with principal 1. Each principal fully observes the set of agents who participate with him. Thus, the measurable mapping $c_j : \{Y,N\}^{\sharp I} \to \Delta(X_j)$, where $\Delta(X_j)$ denotes the set of probability distributions over X_j, is an incentive scheme available to principal j. In addition, we take C_j to be the set of all c_j mappings. Every principal communicates with the agents by means of a private message that he receives from each agent participating with him. To make explicit the relationship between the agents' participation choices and the structure of communication, we assume that every message space M^i_j is sufficiently rich to include the element $\emptyset$ corresponding to the information "*agent i does not communicate with principal j*". In addition, each space M^i_j is taken to satisfy the standard size restriction $\sharp M^i_j \geq \sharp\left(\Omega^i \cup \{\emptyset\}\right)$ for all i and j. Principal j can make his decisions contingent on the array of messages $m_j = (m^1_j, m^2_j, ..., m^I_j)$ he receives. Final allocations are determined by the multilateral contracts that principals independently sign with agents. More formally, we say that a mechanism proposed by principal j is the measurable mapping $\gamma_j = \underset{i \in I}{\times} M^i_j \to \Delta(C_j)$, where $M_j = \underset{i \in I}{\times} M^i_j$ is the relevant set of messages that can be sent to principal j. Finally, Γ_j is the set of mechanisms available to principal j, and we denote $\Gamma = \underset{j \in \mathcal{J}}{\times} \Gamma_j$. To keep the analysis simple, all relevant sets are taken to be finite.

Principal j's payoff is given by $v_j : X \times A \times \Omega \to \mathbb{R}_+$, and $u^i : X \times A \times \Omega \to \mathbb{R}_+$ is the payoff to agent i, where $X = \underset{j \in \mathcal{J}}{\times} X_j$ and $A = \underset{i \in I}{\times} A^i$. For a given array of agents' actions $a = (a^1, a^2, ..., a^I)$ and of principals' decisions $x = (x_1, x_2, ..., x_J)$, the state contingent utilities of agent i and principal j are $u^i(x, a, \omega)$, and $v_j(x, a, \omega)$, respectively.

The competing mechanism game relative to Γ begins when principals simultaneously commit to a mechanism. Having observed the array of offered mechanisms $(\gamma_1, \gamma_2, ..., \gamma_J) \in \Gamma$, and given their own type, agents simultaneously send a message to each of the principals. In the final step, payoffs realize. In this incomplete information game, a strategy for each type of any agent i associates to every profile of offered mechanisms a joint decision in terms of participation and communication. More precisely, we take $m^i_j \in M^i_j$ to be the message she sends to principal j and we denote $M^i = \underset{i \in I}{\times} M^i_j$. We also let $S^i = \{s^i \in M^i \times A^i : m^i_j = \emptyset$ if $a^i_j = \{N\}\}$ be the set of joint participation and communication choices for agent i. Thus, agent i's strategy is the measurable mapping $\lambda^i : \Gamma \times \Omega^i \to \Delta\left(S^i\right)$, and a (pure) strategy for principal j is given by a mechanism γ_j.

Following standard analyses of competing mechanism games, we focus on Perfect Bayesian Equilibrium (PBE) as the relevant solution concept.

We will typically consider a situation where principals are restricted to offer direct *truthful* mechanisms. A mechanism available to principal j is said to be direct if any agent who participates with principal j can only communicate her type to him, i.e. if $M^i_j = \Omega^i \cup \{\emptyset\}$ for every (i, j). We take $\tilde{\gamma}_j : \underset{i \in I}{\times} \left(\Omega^i \cup \{\emptyset\}\right) \to \Delta\left(C_j\right)$ to be the corresponding direct mechanism, and we let

[4] The model can straightforwardly be extended to accommodate any arbitrary set of agents' actions.

Γ_j^D be the set of direct mechanisms available to principal j. For a given profile of mechanisms $\gamma_{-j} \in \Gamma_{-j}$, we then say that $\tilde{\gamma}_j \in \Gamma_j^D \subseteq \Gamma_j$ is a direct truthful mechanism if the array $\left(\tilde{\gamma}_j, \gamma_{-j}\right)$ induces a continuation equilibrium where agents who participate with principal j truthfully reveal him their type.

If $\Gamma = \left(\Gamma_1, \Gamma_2, \ldots, \Gamma_J\right)$ is the set of mechanisms available to each of the principals, we take G^Γ to be the corresponding multiple-principal multiple-agent game, and we denote $\tilde{G}$ the game where principals are restricted to use direct mechanisms.

3 Single Agent

In this section, we show that the restriction to direct truthful mechanisms is with no loss of generality in single agent games with exclusivity clauses.

Lemma 1 *Take any arbitrary game G^Γ and let $J = 1$. Then, every equilibrium outcome of G^Γ is an equilibrium outcome of $\tilde{G}$ where all principals who participate with at least one one type of the agent make use of direct truthful mechanisms.*

Proof.

Let $\gamma = (\gamma_1, \gamma_2, ..., \gamma_J)$ be the offered mechanisms and λ be the agent's strategy in some equilibrium of a given game G^Γ.

Suppose first that the agent plays a pure strategy at equilibrium. For each principal j, we construct the direct mechanism $\tilde{\gamma}_j\,[\omega]$ in the following way:

1. For each ω such that $a_j(\omega) = \{Y\}$, we let $\tilde{\gamma}_j\,[\omega] = \gamma_j\,[m_j\,(\omega)]$, where $m_j(\omega)$ is the message sent by the agent of type ω to principal j at equilibrium.

2. For each ω such that $a_j(\omega) = \{N\}$, we take $\tilde{m}_j\,(\omega) \in \arg\max_{m_j \in M_j} \left\{U(\lambda, \gamma, \omega) : a_j(\omega) = \{Y\}\right\}$ to be any message that this type ω would find optimal to send to principal j if she indeed participated with him, and we let $\tilde{\gamma}_j\,[\omega] = \gamma_j\,[\tilde{m}_j\,(\omega)]$.

3. Finally, we let $\tilde{\gamma}_j(\emptyset) = \gamma_j(\emptyset)$.

Consider now the game $\tilde{G}$, and suppose that principals offer the array of direct mechanisms $(\tilde{\gamma}_1, \tilde{\gamma}_2, ..., \tilde{\gamma}_J)$. By construction, the agent's best reply can always be defined in such a way to guarantee that players will achieve their equilibrium utilities $u(\gamma(m), a, \omega)$, and $v_j(\gamma(m), a, \omega)$ for $j = 1, 2, .., J$. We now argue that none of the principals has a unilateral deviation in the game $\tilde{G}$. Suppose, by contradiction, that principal j could gain by deviating to the direct truthful mechanism $\tilde{\gamma}'_j$.

Since $\sharp M_j \geq \sharp\left(\Omega^i \cup \{\emptyset\}\right)$ for each j, we make use of the surjective mapping $\phi_j : M_j \to \Omega$ to construct the following indirect mechanism γ'_j:

$$\gamma'_j(m_j) \quad = \quad \begin{cases} \tilde{\gamma}'_j\,(\phi_j\,(m_j)) & \text{for } m_j \neq \emptyset \\ \tilde{\gamma}'_j\,(\emptyset) & \text{for } m_j = \emptyset \end{cases}$$

If $\tilde{\gamma}'_j$ is a profitable deviation for principal j, then at least one type $\omega \in \Omega$ of the agent must be willing to participate with principal j. That his, her corresponding payoff must be strictly greater than what she could achieve by participating with some other principal $j' \neq j$, or by not participating at all. With reference to any such ω, one hence gets:

$$u\left[\tilde{\gamma}'_j(\omega), \tilde{\gamma}_{-j}(\emptyset), a, \omega\right] > \max\ \left\{u\left[\tilde{\gamma}'_j(\emptyset), \tilde{\gamma}_{-j}(\omega), a, \omega\right], u\left[\tilde{\gamma}'_j(\emptyset), \tilde{\gamma}_{-j}(\emptyset), a, \omega\right]\right\} \quad (1)$$

where $\tilde{\gamma}_{-j}(\omega)$ indicates that type ω of the agent participates with some principal different from j truthfully revealing her true type. Observe now that, given the construction developed in steps 1-3, the payoff available to type ω when she does not participate with the deviating principal j only depends on the offered mechanisms $\tilde{\gamma}_{-j}$. This payoff, which is represented in the right hand side of (1), therefore coincides with the one she can achieve by not participating with principal j in the original game G^{Γ} given the equilibrium offers $(\gamma_1, \gamma_2, ..., \gamma_J)$. Considering this larger game, and making use of the indirect mechanism $\gamma'_j(m_j)$, one therefore gets

$$u\left[\gamma'_j(m_j), \gamma_{-j}(\emptyset), a, \omega\right] > \max\ \left\{u\left[\gamma'_j(\emptyset), \gamma_{-j}(m_{-j}), a, \omega\right], u\left[\gamma'_j(\emptyset), \gamma_{-j}(\emptyset), a, \omega\right]\right\} \quad (2)$$

which contradicts the assumption that $(\gamma_1, \gamma_2, ..., \gamma_J)$ are the (indirect) mechanisms selected at equilibrium.

One can straightforwardly show that the argument extends to mixed strategy equilibria. A similar reasoning indeed guarantees that indirect mechanisms can be transformed into direct truthful mechanisms even if players randomize over their relevant choices.

■

4 Multiple-Agent: an Example

We present here an example of a two-principal, two-agent model exhibiting the following feature: there exists an equilibrium outcome of a game where indirect communication mechanisms are allowed that cannot be sustained in the simpler situation where principals are restricted to direct truthful mechanisms.

Let $I = J = 2$ and $\Omega^1 = \Omega^2 = \{\omega\}$. In addition, take $X_1 = \{x_{11}, x_{12}\}$ and $X_2 = \{x_{21}, x_{22}\}$. The actions available to each of the agents are to accept or reject the allocations proposed by each of the principals. The payoffs in the following tables represent the utilities of the two principals and those of the two agents.

The example is constructed in such a way that if both agents do not participate in the game, then every player gets a payoff of 0.

If both agents participate with P1, payoffs are:

	x_{21}	x_{22}
x_{11}	$(4,0,2,2)$	$(4,0,2,2)$
x_{12}	$(2,0,4,4)$	$(2,0,4,4)$

If both agents participate with P2, payoffs are:

	x_{21}	x_{22}
x_{11}	$(0,4,3,3)$	$(0,0,0,0)$
x_{12}	$(0,4,3,3)$	$(0,0,0,0)$

If only A1 participates with P2 (with A2 participating with P1 or not participating at all), payoffs are:

	x_{21}	x_{22}
x_{11}	$(0,4,3,0)$	$(0,0,0,0)$
x_{12}	$(0,4,3,0)$	$(0,0,0,0)$

If only A2 participates with P2 (with A1 participating with P1 or not participating at all), payoffs are:

	x_{21}	x_{22}
x_{11}	$(0,4,0,3)$	$(0,0,0,0)$
x_{12}	$(0,4,0,3)$	$(0,0,0,0)$

Finally, if A1 participates with only one principal and A2 does not participate at all, every player gets a payoff of 0.

We first consider the situation where principals are allowed to offer arbitrary communication mechanisms. In particular, we let $M_j^i = M = \{m_1, m_2, \emptyset\}$ for $i, j = 1, 2$ be the set of messages that each agent can send to any of the principals. We argue that the decisions:

- P1 offers x_{11} and P2 offers x_{22} with probability one,
- Both A1 and A2 accept the offer of P1 and refuse the offer of P2,

and the associated payoff profile $(4,0,2,2)$, can be supported at equilibrium.

Let P1 play the mechanism

$$\gamma_1(m_1^1, m_1^2, a) = \begin{cases} x_{11} & \text{if he receives the message } m_1 \text{ from at least one agent} \\ & \text{and both A1 and A2 participate with him} \\ x_{12} & \text{otherwise} \end{cases}$$

and let P2 play the mechanism γ_2: "*for every profile of participation decisions and messages,* x_{22} *is selected*".

Given these mechanisms, agents play a continuation game over messages and participation decisions which payoffs are represented in the following matrix

	YNm_1	YNm_2	NYm_1	NYm_2	NN
YNm_1	$(2,2)$	$(2,2)$	$(0,0)$	$(0,0)$	$(0,0)$
YNm_2	$(2,2)$	$(4,4)$	$(0,0)$	$(0,0)$	$(0,0)$
NYm_1	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
NYm_2	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
NN	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$

where the first payoff is that of A1, the second one is that of A2, and the action $YNm_1(NYm_1)$ stands for the choice of participating with P1(P2) sending him the message m_1.[5] It is straightforward to check that it is an equilibrium for both agents to participate with P1 and to send him the message m_1.

To show that there are no unilateral deviations for principals, it is enough to look at P2, since P1 is earning his maximal payoff of 4. For P2 to have a profitable deviation, he must induce at least one agent to participate with him. However, the agents' equilibrium strategies can be constructed in such a way that, following any of such deviations, both agents do participate with P1 sending him the message m_2, which yields a payoff of 4 to each of them. P2 is therefore left with a payoff of 0, that makes the deviation not profitable.

In a next step, we consider the situation where principals are restricted to direct truthful mechanisms which, in this simple setting, correspond to mechanisms where a principal associates one decision to any array of participation choices that agents are taking with him.

To show that the same profile of decisions cannot be supported at equilibrium in this restricted game, consider first the behavior of P1. Whatever his equilibrium strategy is, he must select the decision x_{11} if both A1 and A2 participate with him out of equilibrium. Similarly, any equilibrium strategy for P2 implies that he must play x_{22} whenever both A1 and A2 participate with P1. Suppose now that P2 plays the direct mechanism "*whoever participates with me, I will take the decision x_{21}*". Choosing this mechanisms turns out to be a profitable deviation for P2. Indeed, the the agents' payoff in the continuation game induced by such a deviation are given by:

	YN	NY	NN
YN	$(2,2)$	$(0,0)$	$(0,0)$
NY	$(3,0)$	$(3,3)$	$(3,0)$
NN	$(0,0)$	$(0,3)$	$(0,0)$

This game admits only one Nash equilibrium, with both agents choosing to participate with P2. It follows that the payoff to P2 will be 4, which makes the deviation profitable.

The example exploits an intuition similar to that used by Yamashita (2010) to establish a Folk Theorem result in competing mechanism games where agents participation choices are not

[5] For simplicity reasons, we do not make explicit the fact that an agent sends the $\emptyset$ message to any of the principals she does not participate with.

restricted. Following a unilateral deviation by P2, agents coordinate on a continuation equilibrium that is worst for the deviator. In the corresponding direct mechanism game, however, the equilibrium mechanism of P1 provides agents with a reduced number of options; specifically, this implies that the threats used to sustain the payoff profile $(4,0,2,2)$ are not available anymore.

5 Conclusion

This paper contributes to the analysis of multiple-principal multiple-agent games of incomplete information in all situations where exclusive clauses are imposed from the outset. Despite their prominent role in most economic models of competing mechanisms, little attention has been devoted to the theoretical investigation of such contexts. We argue that the traditional Revelation Principle fully applies in games with only one agent (exclusive agency). When at least two agents are considered, however, additional strategic effects arise. As a consequence, there exist equilibrium outcomes which can be supported through arbitrary communication mechanisms, but not by direct and truthful ones. This suggests that the equilibrium characterization developed in economic models of exclusive competition with several agents may crucially be affected by the mechanisms that are made available to competitors. Thus, identifying and motivating specific restrictions on contracting assumptions appears to be an important task for future research in this area.

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