Costly State Verification and Debt Contracts

A critical resume

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Abstract

This paper presents a critical review of the role of the *Costly State Verification* framework in financial contracting.

1 Introduction

These notes provide an analysis of the most classical environment used to account for the widespread use of debt contracts in loan relationships. The basic feature of a standard debt contract relies on the borrower's promise to offer a repayment constant over states, with the bank being allowed to seize the whole cash flow when the repayment cannot be guaranteed. The empirical relevance of such a contractual form is widely accepted: Harris and Raviv (1992) provide a very detailed exposition of the most relevant findings. Now, when the lender and the borrower have symmetric information over projects (i.e. cash flows are observable), risk sharing alone cannot predict the prevalence of debt contracts[1]. Asymmetric information has been introduced in the lender borrower relationship in several ways, from the pure adverse selection case in the seminal Stiglitz and Weiss (1981) paper to the classic hidden action moral hazard (Innes, 1990).

We'll develop the so-called Costly State Verification (CSV) paradigm, since according to our view it offers the most clear and articulated basis to understand the role of both debt and intermediaries in financial contracting[2].

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[1]See, among many others, Freixas and Rochet (1997) pp. 93 - 96.

[2]Actually, the standard moral hazard framework (Holmstrom, 1979) cannot support per se the optimality of debt contracts unless an external monotonicity constraint on the repayment structure is imposed. This argument has been clarified in several recent studies (Innes, 1990, 1993, Dionne and Viala, 1994) whose basic result turns out to be that without imposing any sort of monotonicity constraint, the optimal repayment function R can be characterized as follows, :

$R(y) = 0$	for	$y \geq y*$	$y \in [A, B] \subseteq R_+$
$R(y) = y$	for	$y < y*$	

As a starting point, it is useful to remark that the CSV structure deals with situations where it is assumed symmetric information *at the time of contracting*: inside this environment CSV identifies the case where agent's actions can be observed but the contingencies under which they were taken cannot. In the particular terminology introduced by Arrow and used in the popular Hart and Holmstrom (1987) resume we've a *moral hazard with hidden information*, or in other words an $ex - post\ private\ information$ (Hart and Holmstrom, 1987, pp. 75-77). Of course, the focus on this structure will restrict both the solution techniques and the nature of the implementation problem.

In the CSV framework a precise need for banking activity emerges: banks turn out to be essential in reducing monitoring costs (using the famous Diamond's expression they perform a *delegated monitoring* activity). Hence, the discussion will be organized as follows: Sec.2 will present the basic scheme, as it was developed in the classical Gale and Hellwig (1985) paper[3]; Sect. 3 and 4 will examine two relevant extensions of the *one period* contracting problem, namely the introduction of stochastic auditing rules and of *ex ante private information*. Sect. 5 will deal with repeated interaction, while Sect. 6 will present the recent work of Krasa and Villamil attempting to generalize the contracting problem when commitment itself is a strategic variable. Sect. 7 will examine the role of collateral as a potential alternative to verification costs. The last section will come back to the basic scheme focusing on the role of financial intermediaries.

2 The Basic Scheme

The context is very simple: an entrepreneur who is running a firm owns a project which requires an initial investment l at time zero and gives random return at time one. The borrower wants to undertake the project but he has to rely on the lender to get external finance; both agents are conceived as risk neutral and their interaction takes place in a competitive capital market. Therefore, $i > 0$ will be the given return that should be guaranteed to the lender.

Once the project is undertaken, only the borrower can observe the returns at no cost, that is we have *ex post private information*. Borrowers are assumed to report their private information through a message. Returns can be verified once auditing is performed and verification costs are a function of the current firm's assets.

where y is the random return of the project. This is clearly different from a debt-like contract, since it provides incentives for efforts giving to the agent maximal payoff ($R(y) = 0$) when the result is *good* and maximal penalty ($R(y) = y$) when it's bad: we have a live-or-die contract. Debt can be obtained if we require the repayment to be a nondecreasing function of the return (Innes, 1990, p.33).

[3] It is well known that Gale and Hellwig paper is in fact just a reformulation of the Townsend (1979) work.

2.1 Agents and Technology

There is a single investment project with a fixed outlay K. The realizations from the project are uncertain: the set of states of nature is taken to be R_+ and $H : R_+ \to [0,1]$ is the relevant distribution function. Letting $l \in R_+$ to be the investment in the risky project, the returns from investment l in state s will be defined by $f(s,l)$.

Assumption 1: $f : R_+ \times R_+ \to R_+$ is a twice continuously differentiable function on the interior of its domain and continuous at the boundary. Moreover:

- $f(0,l) = f(s,0) = 0$;
- $\frac{\partial}{\partial l} f(s,l) > 0 \qquad \frac{\partial^2}{\partial l^2} f(s,l) < 0 \qquad \frac{\partial^2}{\partial l \partial s} f(s,l) > 0$

Auditing costs are defined by the function $c(s,l)$.

Assumption 2: $c : R_+ \times R_+ \to R_+$ is a twice continuously differentiable function on the interior of its domain and continuous at the boundary. Moreover:
$\frac{\partial}{\partial l} c(s,l) \geq 0 \qquad \frac{\partial^2}{\partial l^2} c(s,l) \geq 0 \qquad \frac{\partial}{\partial s} c(s,l) \geq 0.$

2.2 The Contractual Problem

The total amount to be financed is assumed to be l. The principal has all the bargaining power and she commits to a particular mechanism involving the agent.

In the Gale-Hellwig (GH) framework the general mechanism is given by (l, M, C_0, C_1, W, B) where $M \subseteq R_+$ is the borrower's message space, $C_0 \geq 0$ is the investor's contribution (that is usually interpreted as the amount of equity issued by the borrower), $C_1(m)$, and $W(s,m)$ are respectively the repayment to the lender and the borrower's wealth expressed as a function of the declared state m. Finally, $B(m) : M \to \{0,1\}$ is the function defining the *auditing region*, with $B = 1$ identifying the set of states where auditing is actually taking place[4]. Given the structure of such a mechanism with precommitment, Bayes-Nash equilibrium is chosen as a suitable solution concept.

The first best analysis is straightforward: the efficient level of investment, say l^*, solves the following:

$$l^* = \arg\max_{l \geq 0} E[f(s,l) - (i+1)l] \tag{1}$$

[4]It should be noted that we're not including in the description of the general mechanism the principal's strategies "accept or refuse the mechanism", just for the sake of simplicity.

Unicity of the solution is guaranteed by Assumption 1. Now, if $A_0 \geq 0$ defines the borrower's initial assets, in order to eliminate trivial solutions (that is, focusing on non-autarkic allocations) we should also have:

Assumption 3: $l^* \in (0, \infty)$ and $W = A_0 < l^*$

We can write down the entrepreneur's wealth $W(s,m)$: for any m such that $B(m) = 0$, we have:

$$W(s,m) = f(s,l) + (1+i)(A_0 - C_0) - C_1(m) \tag{2}$$

Considering the asymmetric information case, we will properly define the optimal contracting problem[5]. It can be stated as follows: find an array (l, C_0, C_1, W, B) maximizing borrower's expected utility under lender's zero profit (Individual Rationality constraint) and Incentive Compatibility constraints[6]. We can drop out W from the definition of the optimal contract with easy manipulations. Then, the principal-agent problem would look like:

$$\max_{l,C_0,C_1,B} E[f - (i+1)l - cB] \qquad (3i)$$

$s.t.$

$$EC_1 = (1+i)(l + R_0 - C_0) \qquad (3ii)$$

$$C_1 \leq f - cB + (1+i)(A_0 - C_0) \qquad (3iii)$$

$$l \geq 0 \quad C_0 \in [0, A_0] \qquad (3iv)$$

$$(l, C_0, C_1, B) \quad is \quad IC \qquad (3v)$$

The role of constraints $(3ii)$ and $(3iii)$ is such that the former stays for a zero profit condition for the lender (written as an inequality) while the latter is a feasibility condition. We can now introduce the notion of Standard Debt Contract (SDC henceforth):

Definition 1 *A contract* (l, C_0, C_1, B) *is a SDC iff:*

[5]From now on, we'll simplify notation omitting to specify the state dependence of the relevant variables.

[6]A contract is defined Incentive Compatible in the present framework iff:
- there exists a constant R_1 such that $C_1(m) = R_1$ whenever $B(m) = 0$
- for all (s,m) s.t. $B(m) = 0, B(s) = 1$ and $W(s,m) \geq 0$, then $R_1 - C_1(m) \geq c(s,l)$

In other words, if the true state is observable and the borrower has an incentive to misreport, then the repayment to the lender should be higher under auditing. Also notice that incentive compatibility is forcing repayment under no-auditing to be constant.

A more formal argument is provided in Townsend (1979).

$$\begin{array}{lr} \textit{for some } R_1, \quad (1-B)(C_1 - R_1) = 0 & \textit{(fixed repayment)} \\ B = 1 \Leftrightarrow f < R_1 & \textit{(bankruptcy decision)} \\ BC_1 = B(f - c) & \textit{(maximum recovery)} \end{array}$$

It should be noted that in the definition above the set of states such that $B = 1$ is identified with bankruptcy, while $f - c$ is the part of revenues that is recoverable from the investor under bankruptcy.

In what follows we'll sketch the necessary steps to show how the SDC contract emerges as the optimal mechanism in the class of feasible contracts, that is contracts satisfying $(3ii) - (3v)$

step I: Maximum equity participation

Without loss of generality we can set $C_0 = A_0$

Assume (l, C_0, C_1, B) is an optimal contract that solves the principal-agent problem set up in P2. If we replace it with a new contract (l, C_0', C_1', B) where

$C_0' = A_0 \qquad C_1' = C_1 - (1+i)(C_0 - A_0)$

the objective function and the relevant constraints are unchanged. In fact:

$E(C_1') = (1+i)(l - C_0') \Leftrightarrow E(C_1) - (1+i)(C_0 - A_0) = (1+i)(l - A_0)$ and

$C_1' \leq f - cB + (1+i)(C_0' - A_0) \Leftrightarrow C_1 - (1+i)(C_0 - A_0) \leq f - cB$

Notice that, as a byproduct of step I, the degree of equity participation turns out to be indeterminate at equilibrium[7].

step II: Uniform repayment

$C_1(s, l) = C_1(s', l) = R_1$ for all $s, s' \notin B$ is guaranteed by IC

step III: Bankruptcy rule

If (l, C_1, B) solves the principal-agent problem, then $B = \{s : f(s,l) < R_1\}$ where $R_1 = C_1(s)$ for $s \notin B$.

If $f(s,l) < R_1$ then $C_1(s,l) \leq f(s,l) - cB \leq f(s,l) < R_1$ and therefore $C_1(s,l) < R_1$.

But, if $C_1(s,l) < R_1$, is it that $s \in B : f(s,l) < R_1$?

Assume by contradiction that $C_1(s,l) \geq R_1$ could be compatible with $s \in B : f(s,l) < R_1$. The IC constraint will imply $C_1(s,l) \leq R_1$. There can be two cases:

a) if $C_1(s,l) = R_1$, it is possible to define a new contract $B' = B \backslash \{s\}$ that will be strictly preferred to the initial one, generating a contradiction.

b) if $C_1(s,l) < R_1$, then we can define a new array $\left[C_1'(.), B'\right]$ with $B' = B \backslash \{s\}$ and $C_1' = \left\{ \begin{array}{lll} R_1 - d & \text{for} & s = s' \\ C_1(s') - d & \text{for} & s = s' \end{array} \right\}$

where d is a linear function of $[R_1 - C_1(s.l)]$ that increases the objective function and satisfies the IR and IC constraints.

[7]Gale and Hellwig, 1985, p. 654.

Therefore, the so called bankruptcy rule implies that if the array $\left[l, C_1'(.), B'\right]$ solves the pricipal-agent problem, then $B = \{s : f(s,l) < R_1\}$ defines the region where verification will take place as the one where bankruptcy will occurr.

step IV: Maximum recovery
$C_1(s,l) = f(s,l) - c \quad \text{for all } s \in B$

We obtain this result solving the Kuhn Tucker program. We won't report here the whole derivation. It suffices to notice that if $C_1(s,l) < f(s,l) - c$ on some subset of B, then it would be possible to raise $C_1(s.l)$ in that subset and to reduce R_1 so to keep unaltered feasibility conditions and to give an higher payoff to borrowers. Thus, IR and LL are binding and the lender appropriates all the available assets of the firm in bankruptcy states.

step V: Characterization of R

If $(l, C_1 B)$ is an optimal contract, then $\exists\, R_1\ s.t.\ C_1(s.l) = \min\{f(s,l), R_1\}$, $B = \{s : f(s,l) < R_1\}$ and (l, R_1) solves

$$\max_{l,R_1}\ E_s\{\max[f(s,l) - R_1], 0\} - \textstyle\int_B c dH(s)$$
$$s.t.$$
$$E_s\{\min[f(s,l), R_1]\} = \bar{u}$$

Thus, any optimal contract can be meaningfully represented by a *SDC* with *maximum equity participation*, that is by the array (l, R_1). Now, by previous assumptions we can write $R_1 \geq 0$[8] and, without loss of generality, we can also assume $R_1 \leq \sup\{f(s,l) \mid H(s) < 1\}$ for any optimal contract. The existence of upper and lower bounds for R_1 together with Assumption 2 ensure us about the existence of a state γ s.t. $f(\gamma, l) = R_1$. It follows that we can alternatively identify a contract with the pair (l, γ), where γ is defined to be the *bankruptcy point*, in the sense that bankruptcy occurs if and only if $s < \gamma$[9].

It's important to remark that we didn't introduce here any sort of risk sharing consideration, given that we dealt with *risk neutral* agents. Now, allowing for a risk averse borrower together with a risk neutral lender still enables us to find a sort of SDC at equilibrium. GH can show that under these conditions bankruptcy states are associated to borrower's detention of a positive and constant amount of the asset, that constitutes a form of insurance

[8] Assume not: then incentive compatibility implies that $C_1 < 0$ and the zero-profit condition implies $l - W < 0$. It would follow that $l = l^*$ contradicting Assumption 3.

[9] Notice that γ is unique whenever $l > 0$, while if $l = 0$ (implying $W = 0$) by Assumption 1 such a γ does not exist.

(risk sharing) against non favorable states[10]. Hence the present notes won't deal explicitly with the issue of borrower's risk sharing and we'll refer both to risk averse and risk neutral borrowers.

In this brief resume of the GH incentive problem, we did not discuss the features of the second best level of investment l^{**}: the authors show that with a positive probability of bankruptcy and an increasing cost of bankruptcy with respect to investment, the Pareto constrained level of investment is strictly lower than l^{*}[11].

3 Stochastic auditing rules

The first relevant extension of the basic CSV framework we'll consider is the allowance for random auditing, following Mookherjee and Png (MP) (1989). The basic finding we will show is that under stochastic verification the optimal contract may not exhibit the SDC feature, since in any nontrivial optimal incentive scheme is not possible to identify a bankruptcy point. Moreover, when auditing is not performed, repayments turn out not to be constant over states. We'll just slightly modify our previous setting: in order to focus on the optimal contracts properties, we will consider a fixed investment level that generates n possible levels of realized income $Y_1...Y_n$ with the correspondent probabilities $\lambda_1...\lambda_n$. As before transfers C_i $(i = 1, 2, ...n)$ from the agent to the principal depend on the agent's report on the realized state Y_i, but now the report Y_i is audited with probability p_i at the cost c_i; if there has been untruthful report of state l when i occurred, the agent must pay a penalty F_{li}. Finally, the analysis introduced here will deal with a risk-neutral principal and a risk-averse agent, whose preferences have a VNM representation with underlying concave and strictly increasing Bernoulli utility functions $U(.)$. We leave aside the additional decision on the optimal level of investment in the risky project in order to isolate the problem of designing the optimal mechanism.

Therefore, the relevant *direct* mechanism is here represented by the array (p_i, C_i, F_{ii}) and the contractual problem can be written as:

[10] A very precise analysis of this framework is developed in Garino and Simmons (1995). Their main finding is that at equilibrium the marginal utility of resources held by the borrower under bankruptcy should be equal to the expected marginal utility of consumption in non bankruptcy states.

Introducing an additional incentive constraint that the borrower doesn't destroy resources before verification takes place (like in Innes, 1990) they are able to figure out that the optimal level of detention under bankruptcy is equal to the lowest borrower's consumption under $B = 0$.

[11] It's also important to mention that GH characterize their result as a form of credit rationing, where the size of the loan is the rationed variable. In such a perspective, credit rationing is no more originated by indivisibility of investment projects, as in the classical Stigliz and Weiss (1981) work.

Problem 2

$$\max_{p_i, C_i, F_{ii}} \sum_{i=1}^{n} \lambda_i [p_i U(Y_i - C_i - F_{ii}) + (1 - p_i) U(Y_i - C_i)]$$

s.t.

$$V_i \equiv p_i U(Y_i - C_i - F_{ii}) + (1-p_i) U(Y_i - C_i) \geq (1-p_h) U(Y_i - C_h) \quad \forall i, h \quad (4)$$

$$\sum_{i=1}^{n} \lambda_i [C_i - p_i(c_i - F_{ii})] \geq R \quad (5)$$

$$Y_i - C_i - F_{ii} \geq 0, \quad Y_i - C_i \geq 0, \quad 1 \geq p_i \geq 0 \quad \forall i \quad (6)$$

where the objective of the principal is to choose a probability structure, a transfer and a penalty maximizing agent's expected utility under the incentive constraint (4) that guarantees agent's truthful report, the participation constraint (5) that gives the principal a minimum payoff (R) and the usual limited liability (6) constraints for both parties to avoid negative consumption.

The first result at the optimum is that IR constraint has to be binding, otherwise it would be possible to define a new contract that increases the agent's expected utility, reducing the penalty F_{ii}, without affecting the principal's payoff, contradicting the optimality of the old solution. Then, it is shown that if the agent's report is verified and discovered to be truthful, she must not be punished, thus $F_{ii} \leq 0$. The main results, however, are stated in the following proposition:

Proposition 3 *(i) Optimal schemes exist.*

(ii) All income reports that are audited must be audited randomly- that is, $p_i < 1$ for all i- in any optimal scheme that provides the agent with positive consumption in every income realization.

(iii) Every optimal scheme has the property that if the agent's report is audited and verified to be truthful, the agent must be rewarded; that is, $F_{ii} < 0$ whenever $1 > p_i > 0$[12].

This proposition deserves some comments since it is at the heart of the discussion on stochastic verification. The proof of part *(ii)* is given assuming that there exist some income returns that are always verified, i.e. $p_i = 1$ for some i, and then noticing that in such cases, it would always be profitable to slightly reduce these p_i without changing the direction of the IC inequality but strictly diminishing the expected cost of auditing in IR. Thus any candidate mechanism with $p_i = 1$ for some i would not be optimal.

Up to this point only two components of the optimal mechanism have been characterized, it remains to describe the relationship between the optimal transfers and the optimal audit probabilities, which can be derived by discussing the following:

[12] See MP, 1989, p.406, Proposition 1, b) and c).

Proposition 4 *In any optimal scheme: reports corresponding to the highest transfer will not be audited; all other reports must be audited with positive probability; reports corresponding to higher transfers will be audited with equal or lower probability, if $C_k > C_l$ and $1 > p_l$, then $p_k < p_l$.*

An analysis of the propositions above confirms that the SDC structure is fragile with respect to random auditing: the first proposition actually denies the existence of a treshold bankruptcy point. The second one induces the optimal repayment to the principal to be a function of the reported income returns, given that the probability of auditing is everywhere lower than one: as a consequence, since in the optimal incentive scheme the probability that the report will not be audited is never zero, the agent will not pay a constant transfer across all states of solvency.

Now to gain a broader perspective on the basic CSV environment, we will take into account the interaction between ex ante and ex post private information.

4 Optimal contracting under ex ante and ex post private information

There have been relatively few works suggesting a generalization of the CSV framework to allow for ex ante private information. The first attempt has probably been provided by Williamson (1987) who built on the Stiglitz-Weiss intuitions, but the most clear analysis is in our view the one recently suggested by Choe (1998), who formally introduces a mechanism design set up[13]. We'll therefore briefly discuss his main results.

In order to focus on the characteristics of the optimal contract, we'll start from GH framework assuming that both the investment level and the auditing cost c_1 are fixed. Ex ante private information is represented by the random variable $X : \Omega \longrightarrow \{\theta_1, \theta_2\}$ and it is obviously correlated with $Y : \Omega \longrightarrow R$, which describes the returns from the project: the conditioned distribution function will be given by F_i[14]. To fully characterize the lender-borrower relationship, a general mechanism is needed.

Definition 5 *A mechanism is a collection (D,S,C,F,M,B,R) where each element represents:*

D={0,1} *the borrower's decision of accepting (1) or rejecting (0) the mechanism*

[13]Another preliminary attempt in this direction has been provided by Innes who found that: "With a constant positive cost c of verifying/monitoring ex post firm profit and no *monotonic contract* constraint, the presence of an ex-ante informational asimmetry reinforces Townsend's motivation for debt contracting as long as c is sufficiently large" (Innes, 1993, p.39).

[14]F_i is the conditional distribution of Y given $X = \theta_i$. The unconditional distribution of Y and the conditional F_1 and F_2 have the same support $[y_1, y_2] \subset R_+$.

S the borrower's space of signals
C: S→{0,1} the decision of accepting (1) or rejecting (0) the project
F: {0,1} the lender's decision of financing (1) or not (0) the project
M the borrower's space of messages (reports)
B: S×M→{0,1} the (deterministic) auditing region
R: S×M×B(.,.)×Y→R the transfer to the lender

In this framework all the necessary conditions to apply the Revelation Principle are satisfied, therefore without loss of generality we can substitute the borrower's type space Θ for the signal space S and the output realization space Y for the more general space of messages M. An important assumption for all the following considerations is:

Assumption 4:F_1dominates F_2 in the sense of first order stochastic dominance; moreover $\int ydF_1 > K+c_1$, $\int ydF_2 < K$ and $p\int ydF_1+(1-p)\int ydF_2 \geq K + c_1$

The role of this assumption is to ensure that ex ante private information is in some sense valuable. In other words, the borrower gets higher profits undertaking the project only when θ_1 is observed rather than undertaking it regardless of the observed signal. In the first case the borrower gets $\int ydF_1 - K$, while in the second one $p\int ydF_1 + (1-p)\int ydF_2 - K$. By Assumption 4 the latter is greater than the former and given the asymmetry of information, this outcome identifies the optimal Pareto-constrained allocation (OPC) we want to implement.

Two general classes of mechanisms are of interest here: one where the lender chooses his strategy over F after the borrower has sent his signal on ex ante private information and the other where the lender moves before receiving that message. These two cases define two mechanisms without precommitment (MNP) and with precommitment (MWP), respectively.

Different classes of mechanisms are identified by alternative specification of the lender's strategies: $f : S \to F$ in the mechanism without precommitment (MNP) and $f \in F$ is the uncontingent strategy under commitment (MWP)[15]. Correspondingly different solution concepts are adopted: for the MNP the relevant concept is Perfect Bayesian equilibrium, which is suitable for signaling games; while in the MWP, when the lender moves before any private information is communicated and independently of any such revelation, so that a complete information game is played, the (Bayes) Nash equilibrium implementation is chosen.

Some restrictions will have to be imposed on the principal's expected utility maximization problem in order to guarantee the optimality of the candidate solution. The participation constraint (or IR) and the usual limited liability (LL) constraint are stated below:

[15]Notice that the borrower's strategies remain the same in the two cases: (d, r', r'') where $d \in D$, $r' : \Theta \to S$ and $r'' : \Theta \times X \to M$.

Condition 6 *(IR) A mechanism is individually rational if the equilibrium expected payoffs for both agents are nonnegative*

Condition 7 *(LL) A mechanism satisfies limited liability if, for all* $(s, m) \in S \times M$ *and* $x \in X$*, there is* $v \in [y_1, y_2]$ *such that* $R(s, m, B(s, m), y) \leq B(s, m)(y - c_1) + [1 - B(s, m)]v$ *if* $C(s) = 1$ *and* $R(s, m, B(s, m), y) \leq K$ *if* $C(s) = 0$.

In addition, a specific condition for non-triviality of the project choice decision, applies:

Condition 8 *For any mechanism,* $C(s) = 1$ *for all* $s \in S_1$ *for some proper subset* $S_1 \subset S$.

Because of IR, the borrower will always want to accept the mechanism, i.e. $d = 1$, and the lender will always finance the project in case of precommitment, i.e. $f = 1$, therefore the MNP will be identified by the array (S, C, F, M, B, R) excluding D and a MWP by (S, C, M, B, R) where both D and F have been excluded. However, given the structure of the problem it is clear that there is an advantage for both the lender and the borrower to play the game with precommitment. If the lender could take the financing decision after receiving the borrower's signal and if he can correctly infer the type of borrower from the received signal, then he will never finance a project which belongs to type θ_2[16] while the borrower will always prefer the project to be realized. In MNP it is not optimal for the borrower to send a signal which allows the lender to separate between types: only pooling equilibria can thus be sustained and the OPC allocation can never be implemented[17]. In order to reach the OPC allocation, therefore, we will consider MWP and within this class of mechanisms, only those supporting separating equilibria will be taken into consideration.

Lemma 9 *For any mechanism with precommitment that has a pooling equilibrium in which the project is chosen, there exists a mechanism with precommitment that has a separating equilibrium and dominates it.*

Applying the Revelation Principle (Myerson, 1979) we can restrict our attention within the class of mechanisms with precommitment to those direct mechanisms that allow for truthful implementation, in other words we restrict to incentive compatible (IC) mechanisms.

[16] In case he did it, he would get a negative payoff by Assumption and LL.

[17] This result is stated in Choe's paper, p.244, as Lemma1: *Let* (S, C, F, M, B, R) *be a mechanism without precommitment (MNP) such that* $C(s) = 1$ *for all* $s \in S_1$ *for some* $S_1 \subset S$ *and let* $(r_1, r_2, f; \mu)$ *be its equilibrium where* μ *is the lender's posterior belief about* θ*. Then the only equilibrium at which the project is undertaken has* $r_1(\theta_1) = r_1(\theta_2) = s_1$ *for some* $s_1 \in S_1$.

Lemma 10 *For any mechanism with precommitment that has a separating equilibrium, there exists a direct mechanism with* $S = \Theta$, $M = Y$ *which has an equilibrium in which the borrower truthfully reports the observed information signal as well as the return from the project. The equilibrium payoffs for both agents in the direct mechanism are the same as the equilibrium payoffs of the original mechanisms.*

Given the two sources of asymmetry of information, there will have to be two different IC constraints: one for the truthful report of the information signal and another one for the report of the project return. The restriction to direct mechanisms, in addition, allows us to describe the MWP with a triple (B, R, r) where B represents the verification region, R the transfer to the lender and r, with $r \leq K$ is defined as $R(\theta_2, ., ., .) = r$.

The incentive compatibility constraints (IC) are now specified as:

(IC_1) for truthful report of the information signal:

$$\int \{y - R[y, B(y), y] - c_1 B(y)\} dF_1 \geq K - r \geq \int \{y - R[y, B(y), y] - c_1 B(y)\} dF_2 \tag{7}$$

(IC_2) for truthful report of the return on the project[18]:

there exists $v \in [y_1, y_2]$ such that $\forall y, y'$

$R[y', B(y'), y] = v$ if $B(y') = 0$

$R[y, B(y), y] + c_1 \leq v$ if $B(y) = 1, B(y') = 0$ and $y \geq R[y', B(y'), y]$,

that is untruthful revelation is potentially profitable.

Lemma 11 *Having defined the mechanism as a triple* (B, R, r) *we have to redefine the corresponding IR and LL constraints:*

(IR_B) $p \int \{y - R[y, B(y), y] - c_1 B(y)\} dF_1 + (1-p)(K - r) \geq 0$ *for the borrower*

(IR_L) $p \int R[y, B(y), y] dF_1 + (1-p)r - K \geq 0$ *for the lender*

(LL) $\forall y, y'$ $R[y, B(y), y'] \leq [1 - B(y)]v + B(y)(y' - c_1)$ *for some* $v \in [y_1, y_2]$

The optimal mechanism is obtained solving the lender's maximization problem under the incentive compatibility constraints (IC_1 and IC_2), the IR and LL constraints, that have been adapted to the actual context; it is represented by a triple (B, R, r) and it is optimal in the sense that is does not exists an alternative array (B', R', r') that satisfies the same set of constraints and Pareto dominates the candidate.

Given this structure, the IR_B is implied by LL. This is clearly an extension of the original GH framework and Choe's result consists in proving that whenever the project is undertaken.

[18] This is the same definition of incentive compatible mechanism given by Gale and Hellwig.

The optimal mechanism has the SDC features, in particular:

$$
\begin{array}{llll}
R[y', B(y'), y] = y - c_1 & \text{and} & B(y') = 1 & \text{if } y' \leq v \\
R[y', B(y'), y] = v & \text{and} & B(y') = 0 & \text{if } y' > v
\end{array}
$$

Notice that any such mechanism having the SDC form satisfies LL and IC_2. It is clear then, that an optimal mechanism (B, R, r) can be generally classified as a standard debt contract whenever r can support (B, R) to meet the additional IC_1 constraint.

A mechanism with a standard debt contract can be represented by the pair (v, r). The following proposition[19] states that such a mechanism is feasible:

Proposition 12 *There exists a pair* $(v, r) \in [y_1, y_2] \times [0, K]$ *satisfying both* IC_1 *and* IR_L

Given the existence of a mechanism with SDC, the next step is to show the optimality of such a mechanism. The argument suggested by Choe still relies on the property that a SDC minimizes the auditing region while the existence of an r satisfying IC_1 is basically guaranteed by Assumption 4.

Proposition 13 *If (B,R,r) is a mechanism with precommitment satisfying IC, IR and LL then there exists a mechanism with a standard debt contract that satisfies the same constraints and dominates it.*

Together with optimality of debt contracts, looking also at managerial relationships this setting prescribes that a positive compensation should be guaranteed to the manager even when a project is not undertaken. This type of managerial compensation can be interpreted as a *golden parachute* giving the manager an incentive to make an efficient use of ex-ante private information.

5 Optimal debt contracts in dynamic context

At this point, it's worth noticing that the CSV framework could not cope with additional requirements that most corporate bonds include, such as coupon payments and/or sinking funds payments and options[20]. On the other hand, the optimality results on debt contracts were obtained in a static framework, where the relationship between entrepreneur and investor was plagued by an asymmetry of information, in the form of *moral hazard with hidden information*[21], so that the contract could not be made contingent upon realized cash flows unless some auditing cost had been paid.

[19] See Choe, p.248

[20] Coupon payments are interest payments made on a regular basis by a firm to its bondholders. Sinking funds requirements usually see the firm repurchasing or retiring a portion of bond issues each year starting before maturity. Options give the right to buy or sell a certain amount of the underlying asset at a prespecified price and time.

[21] We're still using this expression following Hart and Holmstrom (1987), given that all the contributions we've been discussing here share this methodological distinction.

This section will explicitly deal with the main attempts of representing a repeated interaction. We start referring to Webb's (1992) work, where the basic distinction between *short term* and *long term* contracts is introduced: "As suggested by Hart and Holmstrom (1987) we argue that long-term contracts derive from an inability to costlessly verify contingencies. In particular, if output realizations of projects are not costlessly verifiable, a long-term contract may then be used to induce truthful revelations that cannot be supported by a sequence of short-term contracts. This leads to a saving of verification costs"[22].

Webb argues that a sequence of two *short term* standard debt contracts, that is the natural extension of the GH result, cannot be the solution of the two period incentive problem. It is, in fact, Pareto dominated by a contract with contingent repayments in the first period.

The main differences with the basic one stage environment are the following: each entrepreneur can select a project in each of the two periods $t = 1, 2$; the project requires an initial investment k_t and yields random return y_t. Before the start of the first period, a long term contract can be signed. The entrepreneur is free to sign contracts with competitive outside lenders at any point. If a sequence of standard debt contracts is selected, then there exists a threshold repayment level R_t which the investor repays when she's solvent and identifies the bankruptcy region in any period. Now, a long-term contract implying contingent first period repayments together with debt in the second period improves upon the previous sequence. The reason is that the amount the entrepreneur will borrow in the second period is inversely related to his first period net assets; therefore, since the second period contract is contingent upon first period reported states, having variable repayments in the first period can reduce the risk of bankruptcy.

It should be remarked that Webb's result strongly relies upon the assumption that the relevant IR constraints are recursive, that is they can be written period by period. Actually, long term contracts require a weaker IR constraint to hold: it imposes the lender to earn non-negative expected profits across states. There exists an explicit improvement when such a weaker IR holds: transferring utility across states is a further source of reduction of the probability of bankruptcy. A numerical example along these lines has been recently provided by Snyder (2001). On the other hand, Webb's setup refers to a two period contracting problem where investment is undertaken twice. Under those circumstances, the introduction of two *individual rationality* constraints is *de facto* a sufficient condition for perfect competition in financial markets. That is, free entry is guaranteed in any period.

We believe that the approach proposed by Chang (1990) can overcome these ambiguities. He proposed a dynamic version of the classical GH framework, where the interaction between borrower and lender is modeled as a two stage game and the results can actually be interpreted in terms of coupon or

[22] Webb, 1992, p.1113.

sinking fund requirements. The mechanism design problem is still solved by the standard debt contract.

The most interesting results on debt contracting are obtained under the assumption that the verification cost function is not decreasing in the firm's cash flow, in such a case verification can only occur if the date one cash flow is below a certain critical level. This level can be interpreted as a coupon payment, in the sense that it tests the financial health of the firm[23].

If the verification cost function is strictly increasing in the value of the firm's asset and if the firm commits not to pay dividends at time one, an increase in the first period payment can reduce the retained earnings carried over at date two, and hence reduce the expected bankruptcy costs. A way to achieve this is to give the firm an option to repay more in period one when its date one cash flow is high, reducing accordingly the amount of repayment carried to period two, so as to avoid misreporting. This feature corresponds to a call option in bond contracts.

In what follows, we will provide a detailed explanation of the main assumptions and results.

The entrepreneur needs funds to finance an investment of one at time zero, which provides random returns y_1 at time one and y_2 at time two. y_t is distributed over an interval $[0, H)$ according to a distribution function F_t for $t = 1, 2$[24]. The random variables y_1 and y_2 are by assumption independent. Both agents are risk neutral and have time additive utility functions, the lenders operate on a competitive market and the riskless interest rate is assumed to be $i = 0$. The realization of the random return can be observed in each period only by the entrepreneur, who is in charge to report to the lender the realized cash flow, here arises the issue to induce truthful reporting.

The crucial assumptions used by Chang are the followings:

i) the framework is the CSV therefore the contract has to be made contingent on the reported cash flows, unless a verification cost is borne, in which case it can be contingent to the realizations;

ii) the verification function $b_t(x_t)$ for $t = 1, 2$ is non decreasing and smooth; x_t represents the total value of the asset of the firm at time t when verification takes place [25].

---

[23] We've to remark that in the CSV framework the verification region, i.e. the interval of reported cash flows for which verification takes place, corresponds to the firm going bankrupt.

[24] It is assumed that $H_t = H$ and that $L_t = 0$ for $t = 1, 2$. In additon, the indipendence among the realizations of project's returns rules out every possibility to analyze the persistence or the correlation among random effects.

[25] Since the firm has no initial resources $x_1 = y_1$, i.e. in the first period firm's asset coincide with the project's cash flow. In the second period $x_2 = y_2 + (y_1 - p_1)$, where $(y_1 - p_1)$ represents the possible retained earnings from previous period.

It is also assumed that the distribution function F_t is twice continuosly differentiable and with a strictly positive density function f_t for $t = 1, 2$ and the hazard rate for y_2, $\frac{f_2(y_2)}{1-F_2(y_2)}$ is increasing, but these are mainly technical; and some covenenats are imposed on firm's behaviour. The firm cannot distribute dividends before the total liabilities to the lender are repaid and cannot raise additional investment funds after period one realization has

Here the GH results cannot be directly applied because of the repetition, thus if it were possible to restore the basic scheme, it would also be possible to look for the same qualitative results. In this context a contract is simply a list $K = [D_1(y_1), P_1(y_1), D_2(y_1, y_2), P_2(y_1, y_2)]$, which prescribes a repayment and a verification decision for each period as function of the reported realizations for y_1 and y_2. In particular, $D_t(y_t, y_s)$ represents the probability that verification takes place given the announcement(s) y_t (and y_s), it is a binary variable since no randomization is allowed and it takes value one when bankruptcy occurs; $P_t(y_t, y_s)$ is the payment to the lender at t given the announcement(s) y_t (and y_s).

We'll apply backward induction: for every possible realization of y_1 the problem to be solved in period two is exactly the same as in the GH formulation, since after period two there is no continuation and the traditional results hold. In this framework, the required repayment schedule when $D_1(y_1) = 0$ is defined as:

$P_2(y_1, y_2) = R_2(y_1)$ when $D_2(y_1, y_2) = 0$[26]

and since the firm is insolvent if she cannot repay the required constant fraction of debt,

$D_2(y_1, y_2) = 1$ if and only if $[y_1 - P_1(y_1)] + y_2 < R_2(y_1)$

thus $P_2(y_1, y_2) = [y_1 - P_1(y_1)] + y_2 < R_2(y_1)$.

Of course, when $D_1(y_1) = 1$ we correspondingly define $p_2(y_1, y_2)$ and $r_2(y_1)$.

Once the repayment schedules $R_2(y_1)$ and $r_2(y_1)$ are known, what remains to analyze is the set of constraints on $D_1(y_1)$, $P_1(y_1), p_1(y_1), R_2(y_1), r_2(y_1)$ that induce the right incentives for the firm to truthful report. Since both agents are risk neutral, the optimal contract is the one that minimizes the expected verification costs, for a given expected payment to the lender. The usual limited liability (LL) constraints which do not allow negative consumption to the borrower, $P_1(y_1) \leq y_1$and $p_1(y_1) \leq y_1$, the participation constraint (IR) for the lender and the incentive compatible (IC) constraints for the entrepreneur have to be specified.

The issue of truthful reporting is relevant only when we deal with realizations that are not observable, i.e. they belong to the non verification region, in those cases the firm must be induced to tell the truth, when the realizations fall into the verification region instead, a misreporting firm could immediately be detected and punished. Given two realizations x_1, y_1 in the support of y_1, such that $D_1(y_1) = D_1(x_1) = 0$, if it were possible for the borrower to misreport the true realization x_1, i.e. if $y_1 \geq P_1(x_1)$, Incentive Compatibility

occurred. In the last part of the paper, the optimality of these two assumption is also discussed.

[26]We will follow the paper notation in writing the repayment function with capital letters when there is no verification and in small letters when verification occurs. Thus, $P_1(y_1)$ represents the payment to be given to the investor in case of no verification, $D_1(y_1) = 0$; while $p_1(y_1)$ indicates the repayment when verification occurs, $D_1(y_1) = 1$. The same holds for $P_2(y_1, y_2)$ and $p_2(y_1, y_2)$.

would imply (IC_1)[27]:

$$R_2(y_1) + P_1(y_1) \leq R_2(x_1) + P_1(x_1) \tag{(IC_1)}$$

For the case when x_1, y_1 are in the support of y_1 and $D_1(x_1) = 0$ but $D_1(y_1) = 1$ and if $y_1 \geq P_1(x_1)$, the relevant constraint would be (IC_2):

$$r_2(y_1) + p_1(y_1) \leq R_2(x_1) + P_1(x_1) \tag{(IC_2)}$$

Finally the IR constraint for the lender is given by

$$\int D_1(y_1)\{p_1(y_1) - b_1(y_1) + \int p_2(y_1, y_2)dF_2\}dF_1 +$$

$$+ \int [1 - D_1(y_1)]\{P_1(y_1) + \int P_2(y_1, y_2)dF_2\}dF_1 \geq 1 \tag{(IR)}$$

Since both parties are in this principal-agent setup risk neutral, the maximization of the entrepreneur's expected payoff subject to the IC_1, IC_2, IR is equivalent to the minimization of the expected cost of verification for a given repayment to the investor under the same constraints. Thus, the contract will determine the array $(D_1, P_1, p_1, R_2, r_2)$ that solves

$$\min \int D_1(y_1)\{b_1(y_1) + \int b_2(y_2 + (y_1 - p_1))dF_2\}dF_1 + \int [1 - D_1(y_1)] \int b_2(y_2 + y_1 - P_1)dF_2\}dF_1 \tag{8}$$

$s.t.$

$$(LL), (IC_1), (IC_2), (IR) \quad and \quad D_1(y_1) = \{\}01, D_2(y_1, y_2) = \{\}01 \tag{9}$$

The problem is solved using optimal control theory, having observed that IC_2 cannot be binding at the optimum[28]. The results obtained depend crucially on the assumption on the verification technology: if it is increasing in the available assets of the firm then it is suboptimal for the firm to repay below the possibility in the first period, because the undistributed revenues would increase the probability of bankruptcy in the second stage. It follows that the optimal first period repayment exhausts all currently realized returns, whenever the verification costs are strictly increasing[29].

[27] If we define as total liability the sum of period one and period two repayments to the investor, $R_2(.) + P_1(.)$, then we can notice that it is not increasing in y_1 in fact, if $y_1 \geq x_1$, since $y_1 \geq P_1(x_1)$ and $x_1 \geq P_1(x_1)$ for the LL constraint, it would also be $y_1 \geq x_1 \geq P_1(x_1)$.

[28] If it were in fact, it would always be possible to substitute the optimal contract prescribing verification with a non verifiable one, leaving unchanged the utility of the investor and increasing the utility of the borrower, who should not be pay the verification costs with this new contract.

[29] In more rigorous terms, if $b_2(.)$ is strictly increasing, then $P_1(y_1) = y_1$, provided that $y_1 - P_1(y_1) < R_2(y_1)$ i.e. that date one realized return cannot repay the total liability of the firm. If $y_1 > P_1(y_1) + R_2(y_1)$ there is no defalut risk at all at time two, thus the firm

Once the asymmetry of information has been overcome, the date two net liability $r_2 - (y_1 - p_1)$ should be set independently of y_1 at the first best level, i.e. at the level where the marginal benefit of increasing the date two required payment (r_2) equals the marginal cost of such an increase[30].

At the optimal contract the total liability of the firm is constant all over the non verified region, as prescribed by the incentive compatibility constraint. This result is similar to the one obtained in most other models of asymmetric information, where the IC constraint is shown to be binding at the optimum since otherwise the optimal candidate contract could be improved by moving toward the first best[31].Therefore, by the combination of the two preceding statements it follows that for a contract to be optimal it has to be that the total liability is constantly equal to M in the non verified region and the date two net liability is a constant k in the verified region.

The final step to obtain an optimal *debt contract* is to show that there exists a critical level of realized cash flow in period one, below which verification will always occur and the firm will go bankrupt[32]. This result is the combination of optimality and incentive compatibility, together with increasing verification costs[33].

In terms of interpretation, we know that with the optimal contract, verification will not occur at date one when $y_1 \geq m$ and the total liability given by $P_1(y_1) + R_2(y_1)$ will be equal to a constant M. If verification cost is not strictly increasing, the division among P_1 and R_2 is undetermined, P_1 will only have to satisfy LL constraint but there are many possible optimal schedules solving the same problem. Therefore, the requirement of $b_2(.)$ being strictly increasing is needed in order to select only one possible solution within this multiplicity, if $b_2(.)$ is strictly increasing in fact, the optimal repayment in period one is given by $P_1(y_1) = y_1$ for $y_1 \in [m, M]$[34].

Thus the optimal contract, when $b_2(.)$ is strictly increasing, can be de-

can decide whether to retain some period one returns or not, the distribution of repayments among stages is irrelevant. If the verification technology is constant, there is no problem of distribution of payments over stages, since bankruptcy costs are independent of firm's assets.

[30] This is stated in Proposition 4 in the original Chang paper in the following terms: *For the optimal contract, $p_1(y_1) + r_2(y_1) - y_1$ is a constant k. Furthermore, if $b_2(.)$ is strictly increasing, $p_1(y_1) = y_1$ and $r_2(y_1) = k$.*

[31] This result corresponds to Proposition 5 of Chang's paper: *For the optimal contract, $P_1(y_1) + R_2(y_1) = M$ for all y_1 for which $D_1(y_1) = 0$. That is, the incentive compatibility constraint $IC_1$is binding.*

[32] Chang's Proposition 6 states: *For the optimal contract, a verification occurs (if it occurs at all) at date one if and only if the reported y_1 is below some critical level m. Formally, $D_1(y_1) = 1$ if and only if $y_1 \leq m$, where $m \in [0, H]$.*

[33] All the described results are directly derived from the interpretation of the first order conditions of the optimal control problem into which the original minimization problem has been transformed. The proof of existence of a solution is given using Cesari's theorem on control problems.

[34] It is worth noting that $P_1 = y_1$ for $M > y_1 \geq m$ is not a mandatory payment schedule, in the sense that for any $y_1 > m$ the firm can always report m hence paying m in period one and retain $y_1 - m$ for date two. This is the call option contained in the contract.

scribed as follows:

- when $y_1 > P_1(y_1) + R_2(y_1)$ all debt is repaid at date $t = 1$;
- when $m < y_1 \leq P_1(y_1) + R_2(y_1) = M$ then $D_1(y_1) = 0$ and the borrower pays $P_1(y_1) = y_1$ and $R_2(y_1) = M - y_1$;
- when $y_1 \leq m$ then $D_1(y_1) = 1$ audit takes place and $p_1(y_1) = y_1$ and $r_2(y_1) = y_2 = k < R_2(y_1)$.

The optimal contract is therefore a standard debt contract with the additional features of a coupon and a call option, in fact the firm is required to repurchase a minimum amount of its liability before maturity (m) and it has the option to pay up to M in the intermediate date, the higher the repayment in the first period the smaller the repayment in the second period, thereby reducing the risk of bankruptcy.

6 Contracts without commitment

A relevant issue still deserves attention: whenever limited commitment is introduced, even if debt turns out to be optimal, agents have an incentive to renegotiate the original agreement once the lender knows the true state. Krasa and Villamil (2000) provide a further generalization of the set-up giving an answer to the issue of limited commitiment and of the dominance of stochastic over deterministic auditing rules. Their setup allows to define stochastic contracts or deterministic contracts as an equilibrium property of the relevant mechanism they introduce, in other words the standard debt contract is the optimal incentive structure in a deterministic environment when commitment is limited, on the other hand stochastic contracts are optimal when the lender fully commits to his initial proposal.

Let's consider an interaction between two risk-neutral agents, where standard assumptions on technology and preferences hold, which takes place over three periods: at date one nature chooses the project outcome in the set $\{y_1, ...y_n\}$, at the date two the borrower makes a voluntary payment which is interpreted as a signal of the realization by the lender, in the third period the lender chooses whether to enforce or not a penalty payment from the borrower. His decision will be taken after having updated his beliefs on the realized outcome according to Bayes' rule, here is the room for renegotiation. Enforcement is costlessly provided by a court[35], whose technology will be specified later on.

The first departure from the basic framework can be found in the description of the contract: instead of defining an array (M, C_1, B), a contract will be defined by $(V, F, \sigma_1, \sigma_2)$, where V is the set of voluntary payments, F is

[35] KV are assuming that the output of the project is observable but not verifiable, in the sense that the enterpreneur can hide a part of the realized project return.

the penalty payment, σ_1 and σ_2 are the associated probabilities. If the enforcement is represented by the binary variable $e = \{0, 1\}$ and defining c as the fixed cost of enforcement, agents payoffs are defined as:

$\pi_B(y, v, e) = y - v - e[F(x, v) + c]$

$\pi_L(y, v, e) = v + e[F(x, v) - c]$ with $x = \max\{y - v - \overline{y}, 0\}$

Perfect Bayes Nash Equilibrium (PBNE) is the appropriate solution concept in this case to complete the description of the mechanism[36].

Optimal Deterministic Contracts

To define an optimal contract, we also have to consider the event of renegotiation after the voluntary payment has been made, in such a case the lender could redefine new terms of contract with respect to what was agreed in the initial period. Since we here allow for limited commitment, both parties have to agree upon the renegotiation for it to take place[37]. Thus, the optimal problem to be solved in the initial period will have to be integrated by a second optimization problem that would give the continuation contract, once the lender's beliefs about the truly realized return will have been upgraded. Thus, in such a naturally dynamic environment, the revelation principle fails to apply since there exist future opportunities to modify the initial agreement.

In order to solve the renegotiation issue, a time consistency constraint will have to be imposed on the initial problem to guarantee that all possible incentives for altering the initial contract are foreseen in the first negotiation[38]. Thus the optimal contract will maximize the investor's expected utility under the usual IR constraint, LL or feasible payments requirement, the PBNE *in lieu* of IC condition and time consistency:

[36]We find useful to refer to the definition used by KV in order to be able to refer to the same conditions when stating theorems and giving proofs.

Definition 14 *A collection of strategies σ_1, σ_2 and beliefs β, β' constitute a Perfect Bayesian Nash Equilibrium if and only if:*

(i) $\sigma_1 \in \Sigma_1$ maximizes $E_{\sigma_1,\sigma_2}\pi_B(y, v, e)$ for every y

(ii) $\sigma_2 \in \Sigma_2$ maximizes $\sum_{y \in Y} \beta'(v; y) E_{\sigma_2}\pi_L(y, v, e)$ for every v

(iii) β' is derived using Bayes' rule whenever possible

[37]Different degree of commitment imply different bargaining power distribution. In case of full commitment, there is no possibility of renegotiation, thus the lender and the borrower committ to the allocation decided in the first period. In case of no commitment, each party has the possibility to break the original agreement after the information has been revealed. In case of limited commitment, both parties have to agree upon a new contractual distribution of the surplus. For more on this, see for example Salanie (1997), chap. 6.

[38]If time consistency constraint is imposed, agents cannot recontract in future periods to increase their expected payoffs, see Dewatripont (1989). In other terms, the time consistency constraint implies that in a future period of renegotiation agents will choose $v' = v, F' = F(x, v), \sigma_2' = \sigma_2$ where v', F', σ_2' is the continuation contract and v, F, σ_2 is the initial contract.

Problem 15 *At $t = 0$ choose $\sigma_1(y; v), \sigma_2(v; e), V, F$ to maximize*

$$\sum_y \beta(y) E_{\sigma_1,\sigma_2} \pi_L(y, v, e) \tag{10}$$

$$s.t. \sum_{y \in Y} \beta E_{\sigma_1,\sigma_2} \pi_B(y, v, e) \geq \overline{U} \tag{11}$$

$$v \in [0, y] \text{ and } F(.) \in [0, x] \tag{12}$$

$$\sigma_1(y; v), \sigma_2(v; e), \beta, \beta' \text{ is a PBNE at } t = 1 \tag{13}$$

$$v, F, \sigma_2(v; e) \text{ is time consistent} \tag{14}$$

It is worth remarking that the last constraint can be interpreted as a sequential efficiency constraint to eliminates ex-post inefficiencies. However, it introduces further restrictions with respect to the full commitment setup, generating a potential loss of ex-ante efficiency. Moreover, equation __ replaces the usual IC constraint.

Now, the new formulation of the contractual problem allows to redefine the optimality of debt contract, this set of constraints in fact does not exclude a priori the possibility of having stochastic contracts. Theorem 1 in KV shows that under some restrictions on minimal payoffs[39], debt is optimal in a limited commitment environment with possible stochastic contracts. It can be stated in the following way:

Theorem 16 *Assume there exists a simple debt contract which satisfies the conditions (*) and (**) (in the footnote) and which gives the borrower the reservation utility $\overline{U}$ then this contract solves Problem 19.*

In order to prove this theorem KV introduce three propositions showing first of all that time consistency implies a deterministic σ_2. Secondly, a one to one relationship between contracts in the CSV framework and in the KV model is proved to exist, expected payoffs turn out to be the same as well[40]. A debt contract in KV setup is optimal because it minimizes information revelation (debt is *informationally minimal*). The first conclusion is that the KV debt outcome turns out to be ex-post efficient and optimal even when random monitoring is allowed, extending therefore the CSV analysis.

[39] These restrictions are:

$$(\overline{y} - c) \in (0, x_0) \tag{(*)}$$

$$x_0 < \sum_{y<y^*} (y - c)\beta(y \mid y < y^*) \tag{(**)}$$

we are, in other words, imposing restrictions on parameters determining lender's and borrower's minimal payoffs from enforcement activity.

[40] Without loss of generality, the analysis can be restricted to deterministic σ_1. See, Appendix of KV for details.

Optimal Stochastic Contracts

In the same way, the optimality of stochastic contracts turns out to be an equilibrium outcome when we consider full commitment mechanisms. In this case, agents commit to the initial contract and have no opportunity to alter their decisions at any point in time. The time consistency constraint is therefore irrelevant here and the subgame perfect Nash equilibrium concept allows to solve the sequential game. The new problem to be solved is:

Problem 17 *At $t = 0$ choose $\sigma_1(y; v), \sigma_2(v; e), V, F$ to maximize:*

$$\sum_{y} \beta(y) E_{\sigma_1,\sigma_2} \pi_L(y, v, e) \tag{15}$$

$$s.t. \qquad \sum_{y \in Y} \beta E_{\sigma_1,\sigma_2} \pi_B(y, v, e) \geq \overline{U} \tag{16}$$

$$v \in [0, y] \text{ and } F(.) \in [0, x] \tag{17}$$

$$\textit{point (i) of PBNE definition} \tag{18}$$

KV show that Problem 21 is equivalent to the original problem defined by Gale-Hellwig and elaborated by Mookherjee-Png, if we consider that the monitoring probability in the GH can be identified with the enforcement probability in the model with commitment. The following theorem holds:

Theorem 18 *Problem 15 and 17 are equivalent. Stochastic contracts are optimal.*

The most important conclusion that can be derived by the KV general framework is that the relevance of debt contracts emerges in environments where renegotiation is possible as in credit and loan contracts, while stochastic contracts emerge when there is the institutional possibility of full-commitment to auditing activity (public authorities or insurance companies).

7 Collateralized debt contracts

The main feature of the CSV framework is the idea that the ex post asymmetry of information can be eliminated at some cost: the cost of verification. This was originally interpreted by Townsend (1979) and GH (1985) as a pecuniary cost in terms of utility that could depend on states realizations and on the amount invested, no matter who had to bear it. It is very interesting to notice that the literature on collateralized debt has recently provided an interpretation of the role of collateral as a potential substitute of verification cost inside the hidden information moral hazard framework: we will briefly present the basic framework proposed by Lacker (1998) in the specification with a perfectly divisible collateral good. The approach formalizes the notion that collateral requirements act as repayment incentives, in the sense that the borrower will have to surrender his collateral in the event that payment cannot be made as promised[41]. The crucial condition for the optimality of debt contract is that the borrower and the lender have different relative valuations of the collateral good, in particular it is required that the borrower values this good more than the lender.

The model is very simple: there are two dates $t = 1, 2$, two agents $j = b, l$ and two goods $i = 1, 2$ where good 1 is the payment good and good 2 is the collateral good. Let's assume, without loss of generality, that the risk-averse borrower (agent b) is endowed with a random quantity of good 2 $(\widetilde{\theta})$ and owns $k > 0$ units of good 1, while the risk-neutral lender (agent l) has a known and constant endowment e of good 1. Agents meet in the first period and contract upon the repayment due in exchange for a loan advance, in the second period the random variable realizes and agent b observes the realization: he can truthfully report it to agent l or lie and claim a lower θ. The lender cannot observe the realized θ but she wants to induce truthful revelation from the borrower. Here is the role of the collateral: the contract prescribes that if the promised repayment cannot be made, the lender can appropriate the collateral of the borrower. Since the borrower values the collateral good more than the lender, the optimal contract will minimize the expected value of the collateral transfer subject to incentive compatibility and individual rationality constraints. More precisely, a contract is defined as a pair of functions of the states $[y_1(\theta), y_2(\theta)]$ that represent the transfers of good 1 and 2, respectively. The optimal contract can be found solving the following constrained expected utility maximization problem:

Problem 19

$$\max_{y_1(\theta),y_2(\theta)} \int \left[u_1(\theta - y_1(\theta)) + u_2(k - y_2(\theta))\right] f(\theta) d\theta$$

$$s.t.\ u_1(\theta - y_1(\theta)) + u_2(k - y_2(\theta)) \geq u_1(\theta - y_1(\theta')) + u_2(k - y_2(\theta')) \tag{19}$$

$$\forall (\theta, \theta') \in \Omega \times \Omega \quad s.t. \quad \theta' < \theta \tag{20}$$

$$\int \left[e + y_1(\theta) + \mu_l y_2(\theta)\right] f(\theta) d\theta \geq \overline{v_l} \tag{21}$$

$$y_1(\theta) \in [-e, \theta] \tag{22}$$

$$y_2(\theta) \in [0, k] \tag{23}$$

[41] With this respect the collateral good has the same role as the "non-pecuniary penalties" in the CSV model developed by Diamond (1984). He introduced the possibility that even if it could not be possible for the lender to verify due to the extremely high verification costs, there existed some "non-pecuniary penalties", in terms of loss of reputation of the entrepreneur for example, that could induce the borrower to truthfully reveal the realized state. This item will be discussed in the following section.

where we have assumed that u_i with $i = 1, 2$ are the continuous, strictly increasing and concave utility functions of the borrower for good 1 and 2; that the lender is a risk-neutral agent and μ_l is her marginal rate of substitution between collateral and payment goods; $f(\theta)$ is the density function for the random variable $\widetilde{\theta}$ which is defined on the support $\Omega = [\theta_0, \theta_1]$. The constraints (20)-(23) are the incentive compatibility, the individual rationality for the lender and the limited liabilities constraints, respectively.

Definition 20 *A* collateralized debt contract *is defined as a pair* $[y_1^*(\theta, R), y_2^*(\theta, R)]$ *such that:*

$$y_1^*(\theta, R) = \min[\theta, R] \qquad \forall\theta \in [\theta_0, \theta_1]$$
$$y_2^*(\theta, R) = 0 \qquad \forall\theta \in [R, \theta_1]$$
$$y_2^*(\theta, R) = k - \phi_2 [u_2(k) - u_1'(0)(R - \theta)] \qquad \forall\theta \in [\theta_0, R)$$

where R is the contractual repayment of good 1. Now, it is clear that this contract implies, for realizations of θ greater than R, a constant transfer of good 1, R, and none of good 2, while if $\theta < R$ then the borrower will have to transfer all of good 1 and part of good 2. It is also evident that the solution to the examined maximization problem will provide the minimum amount of transfer in collateral admissible under incentive and participation constraints.

The main proposition for the optimality of collateralized debt states the following:

Proposition 21 *If a collateralized debt contract R satisfies IR with equality and*

$$\mu_l < \mu_b^*(\theta) - \rho^*(\theta)\mu_b^*(\theta)\phi(\theta), \quad \forall\theta \in [\theta_0, \theta_1] \tag{24}$$

then R is the unique optimal contract.

When the borrower is risk averse[42], the second term on the right hand side of the previous inequality is positive, since $\rho^*(\theta)$ in the coefficient of absolute risk aversion of the borrower for good 1, optimality of the debt contract requires that the gap between the borrower's and the lender's valuations of the collateral to be greater than $\rho^*(\theta)\mu_b^*(\theta)\phi(\theta)$, which represents the improvement in risk-sharing that can be obtained increasing the collateral payment for state θ while reducing the payment of good 1. In other words, Proposition 21 can be interpreted as an implication of introducing a lower bound on $\mu_b^*(\theta) - \mu_l$, that is the gap between the relative valuations of the collateral to the two agents. Since such a bound depends on the coefficient of absolute risk aversion, the proposition shows that the smaller $\rho^*(\theta)$, the smaller the effect of incentive constraint and the smaller the gain of giving the borrower less collateral. When the condition on marginal rates of substitutions does

[42] In the special case when the borrower is also risk neutral, the relevant proposition simply requires that $\mu_l < \mu_b$, i.e. that the borrower simply values the collateral good more highly than the lender. The optimal contract therefore is the one that minimizes the expected deadweight loss due to the transfer of the collateral from the high-value user to the low-value user (Lacker, 1998, p.15).

not hold, in each state there exists an allocation at which the marginal rates of substitution of the borrower and the lender are the same and the contract resembles a riskless debt. The incentive compatibility constraint binds and the value of the repayment is constant over states.

8 Debt contracts and financial intermediation

Up to now we tried to argue that the CSV framework is the most appropriate way to analyze the optimality of debt contracts. Many important contributes tried to evidence how the basic predictions of CSV can be compatible with the existence of financial intermediaries, too. Such intermediaries perform different activities: "[...] (1) they issue securities which have payoff characteristics which are different from those of the securities they hold (2) they write debt contracts with borrowers (3) they hold diversified portfolio (4) they process information. Also, they ration credit in equilibrium which some would characterize as an empirical fact, as do Stiglitz and Weiss (1981)" (Williamson, 1986, p.161).

In what follows the main results obtained by Williamson (1986) are summarized. The basic aim of Williamson's work is to show that an equilibrium allocation where financial intermediation activity is performed Pareto dominates the direct lending allocation, under the assumption of so called *large scale investment project.* The reason for a financial intermediary to exist is to eliminate the duplication of monitoring activity which would emerge in case of direct lending.

More formally, he assumes that every lender is endowed with one unit of consumption good and thus the fixed outlay K to start an investment can only be provided by K lenders. The first step is to consider the usual CSV environment when financial intermediation is not allowed. The optimal allocation implemented by the standard debt contract will guarantee the following expected utility for the direct lender:

$$U_D(R) = \int_0^R y\ f(y)\ dy + R(1 - F(R)) - cF(R) \tag{25}$$

where $y \in [0, \overline{y}]$, is the random return from the project, $f(.)$ and $F(.)$ are the density and distribution function of y , respectively, c is the monitoring cost and R is the usual fixed payment in the debt contract. The second step is to introduce financial intermediaries. It should be noticed that intermediaries are here represented by lenders themselves who write identical contracts with each entrepreneur. Each intermediary aims at maximizing his VNM utility function issuing financial claims to other lenders and lending K units of the unique consumption good to every entrepreneur she is contracting with[43]. If

[43]This way of characterizing financial intermediaries is quite different from the traditional Diamond (1984) one, where intermediaries are conceived as specific agents performing the classical delegated monitoring activity. It should be also clear that Williamson is not dealing with coalitions of intermediaries either.

an intermediary is contracting with m borrowers, then the amount $mK - 1$ will be raised from other lenders and she will herself participate with her endowment. Under some regularity conditions on the relevant distribution functions, any of the m optimal contracts signed between the intermediary and the m borrowers is a standard debt contract. It can be shown (Williamson, 1986, p.170-171) that the constant repayment R received by borrower j ($j = 1, ...m$) solves the following problem:

Problem 22

$$max_R K \int_R^{\overline{y}} y_j f(y_j) dy_j$$

$$s.t. \int_0^R y_j f(y_j) dy_j + R(1 - F(R)) - (c/K)F(R) = r \tag{26}$$

$$U(R) = \int_0^R y f(y) dy + R(1 - F(R)) - (c/K)F(R) \tag{27}$$

In this problem, we let r to be the riskless interest rate paid on the capital market. It turns out that the lender's expected utility will be $U(R)$ which is clearly greater than $U_D(R)$ and leaves unchanged the utility of the borrower. In other words, at any equilibrium (R^*, r^*, q^*)[44] direct lending is dominated by financial intermediation: lenders who act as intermediaries are somehow producing information in a more efficient way. This is clearly a positive result for the existence of financial intermediaries[45].

It should be remarked that the most popular work on financial intermediation under asymmetric information is probably the one due to Diamond (1984), developed inside a CSV structure. As well known, the main departure from Williamson's framework stays in the introduction of nonpecuniary bankruptcy penalties which replace the auditing activity in inducing truthful revelation of the realized state; they are *deadweight losses* at social level since the lender cannot appropriate them. However, the optimality of debt obtained by Diamond is not robust to the introduction of borrower's risk aversion,as Hellwig has recently shown: "The nonlinearity of the borrower's utility function implies that the nonpecuniary bankruptcy penalty that is required to discourage the borrower from underreporting his ability to pay will itself be given by a nonlinear function of the amount of underreporting. [...] An optimal incentive compatible contract will typically not take the form of

[44]Notice that r^* and q^* are the market riskless interest rate and the aggregate loan quantity at equilibrium, respectively, and $\overline{R^*}$ is the constant repayment in the debt contract. An equilibrium with intermediation is defined by a triple $(\overline{R^*}, r^*, q^*)$ which satisfies:

(i) $\overline{R^*}$ solves Problem 23

(ii) $q^* = \alpha \int_{\underline{t}}^{r^*} h(t)dt$

(iii) either (a) $q^* = (1 - \alpha)K$

or (b) $q^* < (1 - \alpha)K$ and $1 - F(\overline{R^*}) - (c/K)f(\overline{R^*}) = 0$.

[45]It's important to remark that we're not dealing here with the issue of credit rationing, that is a central part of Williamson's analysis.

a Standard Debt Contract"[46]. Actually, this result provides a very strong argument in favour of the standard CSV framework as the only foundation for the coexistence of debt and intermediation[47].

In a partially different set up Boyd and Prescott (1986) derive a role for intermediaries as multi-agents coalitions who perform *delegated screening*. They refer to an economy populated by infinite consumers-borrowers that can be divided in two classes according to their investment opportunities. They can be informed on the return prospects using their labor endowment in the project evaluation activity rather than in the productive sector. The equilibrium allocation, that satisfies the core property, can be associated with the existence of large coalitions of borrowers who evaluate investment projects[48]. The most remunerative projects are financed first. Boyd and Prescott show that the equilibrium allocation of such an economy Pareto dominates the equilibrium allocation of another economy where a competitive securities market is formally introduced. Moreover, the presence of asymmetric information turns out to be a necessary condition for the possibility of such an ordering.

9 Some conclusive remarks

The main objective of our discussion has been to isolate the conditions under which the fact that it is costly for external investors to observe investment returns implies that it is optimal for external finance to be obtained issuing debt. With this respect Krasa and Villamil identify the more general set of conditions necessary for optimality.

Moreover, under some definite circumstances it is also optimal for debt issues to be intermediated by institutions frequently identified with banks. The work of Hellwig (2001) provides us with a careful restatement of the sources of coexistence between debt and intermediation.

In the light of these results, it seems that two sharply distinct streams of research can farther be explored.

The first one deals with a generalization of the logical structure we've been describing up to now[49] through the introduction of multi-lender analysis, that is a potential requisite for decentralizing the contractual allocation. We just mention here the very recent work by Khalil, Martimort and Parigi (2001), that builds on the *common agency* literature[50]: one of the most relevant predictions of this work is to recover optimality of debt contracts as a consequence of principals competing both in transfers and in monitoring.

[46] Hellwig, 2001, p. 417.

[47] It should be also stressed that the *collateralized debt* approach presented in the last section is unaffected by Helwig's remark.

[48] Any of these coalitions is assumed to have no market power.

[49] It should also be noted that the analysis developed here is confined to the *complete* contracts paradigm.

[50] In particular, they qualify their work as an extension of the Bernheim and Whinston (1986) classical paper on common agency.

The second one is definitively more related to macroeconomic issues. Given that under this asymmetric information structure Modigliani-Miller's theorem fails to hold (Townsend, 1988), there is room for the CSV framework to besed to analyze the interaction between business cycle fluctuations and credit market frictions, emphasizing the modifications in the financial contracts' structure as salient features of credit cycles[51].

[51] Actually, these intuitions are summarized in several different constructions. A very well known one aims at the renewal of I. Fisher's *debt-deflation* channel relying on limited commitment, as in the Kyiotaki-Moore (1997) approach, while a somehow less popular one is offered by the analysis of deterministic competitive economies where the presence of asymmetric information in the lender-borrower relationship is responsible for endogenous business fluctuations. The introduction of financial constraints is generally considered as an enrichment and a generalization of the more traditional endogenous business cycle framework. An important work in this line, for example, has been recently provided by Suarez and Sussman (1997): assuming that the lender borrower relationship is subject to moral hazard, liquidity effects turn out to be the source of endogenous fluctuations in an economy where financing takes place through both debt and internal revenues. Given that firms' effort to subscribe good project is a decreasing function of the fraction of debt-financed investment, cycles may take place because of the dependence of internal liquidity on prices.

For analyses developed in stochastic environments useful references are: Azariadis and Smith (1998), Bernanke and Gertler (1989), Boyd and Smith (1998).

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