Pricing of delivery services and the emergence of marketplace platforms

Claire Borsenberger, Helmuth Cremer, Denis Joram
and Jean-Marie Lozachmeur

March 11, 2016

1 Introduction

As mail volumes continue to decline, parcel delivery becomes an increasingly important market for postal operators. This market is expanding mainly because of the development of e-commerce. At the same time the e-commerce sector is continuously evolving; right now it has certainly not a steady state and it is hard to predict how this steady state will look like. One of the ongoing trends is the development of marketplaces. [1] A retailer may not just sell its own products; it may also provide a marketplace for other sellers offering a variety of services including delivery. From a parcel delivery operator's perspective marketplaces create a "secondary" delivery market which undermines its ability to differentiate prices. Consequently, the market structure in the e-commerce sector will affects the pricing strategy of the delivery operator. Interestingly, the effect will also go in the opposite direction: the pricing structure of parcel delivery will in part determine the development of marketplaces.

Our paper deals with this interaction. We study the pricing of delivery services and its impact on the market structure in the-commerce sector. Our study builds on Borsenberger *et al.* (2016) which took a first pass at this subject. We generalize Borsenberger *et al.* (2016) in four important directions. First, marketplaces use more sophisticated nonlinear pricing policies when they sell their services to their affiliates. In particular, they can use two part tariffs charging a fixed fee along with a per unit rate. This not only adds a realistic feature to the analysis, but is shown to have a dramatic impact on the equilibrium market structure. Second, we explicitly recognize the fact that marketplace may have some monopsony power. [2] More precisely, the delivery rates charged to the marketplace are no longer set by the delivery operator as a take it or leave it offer. Instead, the contractual arrangements between marketplace and delivery operator are the result of a bargaining process. We consider a bargaining solution, in which the bargaining powers of the parties are determined by their respective bargaining

[1] According to the FEVAD, a French organization of distance sellers, the volume of sales on marketplaces rose by 46% between 2014 and 2015. The market share of marketplaces is estimated at almost 3 billion in 2015, representing 9% of sales of products online.

[2] In France, Amazon's bargaining power is reinforced by its vertical integration strategy into the delivery activity through the acquisition of Colis Privé.

weights. Last but not least, we have completely redesigned the numerical approach so that we can now derive a full solution of a calibrated specification of our model.

The timing of the "full game" consisting of delivery and retail pricing is as follows. In Stage 1 the postal operator sets the single piece rate. In Stage 2, the postal operator and retailer 0 bargain over the delivery rate paid by the marketplace. In stage 3, retailer 0 chooses the per-unit rate and the fixed fee at which it is willing to sell its delivery service to the other retailer. In Stage 4, retailer 1 chooses independent delivery or marketplace delivery. Finally, in Stage 5 the retailers simultaneously choose their prices in either the I or M subgame. As usual we solve this game by backward induction to characterize the subgame perfect Nash equilibrium.

While some analytical results can be obtained for the last stages of the sequential game, a full solution of the delivery operator's problem can only be achieved numerically. These simulations allow us to address a number of interesting questions. In particular we can determine the factors which will favor or impede the creation of marketplaces. We can also study the impact of this dynamic on prices, profits and welfare.

2 Model

The basic setup and the last stages of the game are based on Borsenberger *et al.*(2016). To make this paper self-contained, we shall recall the main features. Consider an electronic retail market with two sellers (e-retailers) located at the endpoints 0 and 1 of a Hotelling line. Consumers are identified by their location $z \in [0, 1]$ with a distribution function $G(z)$ with the density $g(z)$. The Hotelling specification is the simplest way to represent horizontal differentiation. The variable z is not meant to describe a geographical location but rather a parameter characterizing the individuals' preferences across retailers.

The retailers sell a single product which apart from their specific retail services is otherwise homogenous. Its marginal cost, excluding delivery, is constant and denoted by k.

There is a single delivery operator, who charges a rate of r_0 to retailer 0. For the time being bypass is ruled out but will be considered below. Seller 1 can either deliver directly via postal operator at rate r; the general rate which also applies for single piece customers. Alternatively, it can "join" the marketplace and use the delivery services of retailer 0. This affects utility and also the degree of product differentiation. We will consider these two market configurations separately.

2.1 Independent delivery

In this case referred to as subgame I, the utility of consumer z, who buys x units of the good is given by

$$\begin{cases} \alpha u(x) - p_0 x - tz^2 & \text{if the good is sold by firm 0} \\ u(x) - p_1 x - t(1-z)^2 & \text{if the good is sold by firm 1,} \end{cases} \tag{1}$$

where $\alpha \geq 1$. Firm 0 is a "big"

$$v(a,q) = \max_x \quad au(x) - qx. \tag{2}$$

The marginal consumer $\widehat{z}(\alpha, 1, p_0, p_1)$ is defined by

$$v(\alpha, p_0) - t\widehat{z}^2 = v(1, p_1) - t(1-\widehat{z})^2.$$

This consumer is indifferent between buying from retailer 0 or 1. All consumers with a lower value of z will patronize retailer 0; they represent a share of $G[\widehat{z}(\alpha, 1, p_0, p_1)]$ of the total population. The consumers with $z \geq \widehat{z}$, who represent a share of $(1 - G[\widehat{z}(\alpha, 1, p_0, p_1)])$ will buy from seller 1. Solving for $\widehat{z}$ yields

$$\widehat{z}(\alpha, 1, p_0, p_1) = \frac{1}{2} + \frac{v(\alpha, p_0) - v(1, p_1)}{2t}. \tag{3}$$

Aggregate (market) demand for the two products is given by

$$X_0^I(\alpha, 1, p_0, p_1) = x(\alpha, p_0) G[\widehat{z}(\alpha, 1, p_0, p_1)], \tag{4}$$

$$X_1^I(\alpha, 1, p_0, p_1) = x(1, p_1)(1 - G[\widehat{z}(\alpha, 1, p_0, p_1)]). \tag{5}$$

Profits of the e-retailers are given by

$$\pi_0^I(\alpha, 1, p_0, p_1) = (p_0 - k - r_0) X_0^I(\alpha, 1, p_0, p_1), \tag{6}$$

$$\pi_1^I(\alpha, 1, p_0, p_1) = (p_1 - k - r) X_1^I(\alpha, 1, p_0, p_1). \tag{7}$$

The e-retailers simultaneously set their prices and the solution is given by the Nash equilibrium, denoted by the superscript NI. The equilibrium prices are then (p_0^{NI}, p_1^{NI}), equilibrium demands are

$$X_0^{NI} = X_0^I(\alpha, 1, p_0^{NI}, p_1^{NI}), \tag{8}$$

$$X_1^{NI} = X_1^I(\alpha, 1, p_0^{NI}, p_1^{NI}), \tag{9}$$

and equilibrium profits are

$$\pi_0^{NI} = \pi_0^I(\alpha, 1, p_0^{NI}, p_1^{NI}), \tag{10}$$

$$\pi_1^{NI} = \pi_1^I(\alpha, 1, p_0^{NI}, p_1^{NI}). \tag{11}$$

The postal operator's profits are given by

$$\Pi^{NI} = (r_0 - c)X_0^I(\alpha, 1, p_0^{NI}, p_1^{NI}) + (r - c)\left[X_1^I(\alpha, 1, p_0^{NI}, p_1^{NI}) + Y(r)\right] - F \tag{12}$$

where F denotes the delivery operator's fixed cost and where $Y(r)$ is the demand for single piece delivery services (by household and other small firms). Formally we have

$$Y(r) = \arg\max[S(Y) - rY],$$

where $S(Y)$ is the (aggregate) gross surplus of single-piece customers (other than e-retailers).

2.2 Marketplace delivery

In this case, referred to as subgame M, the utility of consumer z, who buys x units of the good is given by

$$\begin{cases} \alpha u(x) - p_0 x - \delta t z^2 & \text{if the good is sold by firm 0} \\ \gamma u(x) - p_1 x - \delta t(1-z)^2 & \text{if the good is sold by firm 1,} \end{cases} \tag{13}$$

where $\alpha \geq 1$, $1 \leq \gamma \leq \alpha$ and $\delta \leq 1$. The parameter δ represents the property that delivery through the marketplace reduces the degree of horizontal product differentiation. It reduces the utility loss customers experience when patronizing a seller whose characteristics differ from their preferred ones. Consequently the goods become closer substitutes and price competition will be more intense. When $\gamma > 1$, marketplace delivery also increases the perceived quality of good 1; the seller now benefits from the reputation and warranties of the marketplace.

Proceeding as above, the marginal consumer is now determined by

$$\widetilde{z}(\alpha,\gamma,p_0,p_1) = \frac{1}{2} + \frac{v(\alpha,p_0) - v(\gamma,p_1)}{2\delta t},$$

and aggregate (market) demand for the two products is

$$X_0^M(\alpha,\gamma,p_0,p_1) = x(\alpha,p_0)G[\widetilde{z}(\alpha,\gamma,p_0,p_1)], \tag{14}$$

$$X_1^M(\alpha,\gamma,p_0,p_1) = x(\gamma,p_1)(1 - G[\widetilde{z}(\alpha,\gamma,p_0,p_1)]). \tag{15}$$

Define the retailers profits gross of the fixed fee as

$$\widehat{\pi}_0^M(\alpha,\gamma,p_0,p_1,s) = (p_0 - k - r_0)X_0^M(\alpha,\gamma,p_0,p_1) + (s - r_0)X_1^M(\alpha,\gamma,p_0,p_1), \tag{16}$$

$$\widehat{\pi}_1^M(\alpha,\gamma,p_0,p_1,s) = (p_1 - k - s)X_1^M(\alpha,\gamma,p_0,p_1), \tag{17}$$

where s is the per unit fee retailer 0 charges to retailer 1.

Denoting the fixed fee by T, profits of the e-retailers are given by

$$\pi_0^M(\alpha,\gamma,p_0,p_1,s,T) = \widehat{\pi}_0^M(\alpha,\gamma,p_0,p_1,s) + T, \tag{18}$$

$$\pi_1^M(\alpha,\gamma,p_0,p_1,s,T) = \widehat{\pi}_1^M(\alpha,\gamma,p_0,p_1,s) - T. \tag{19}$$

As in the case of independent delivery we assume that the e-retailers simultaneously set their prices and that the solution is given by the Nash equilibrium, denoted by the superscript NM. The equilibrium prices are denoted by (p_0^{NM}, p_1^{NM}). Substituting into expressions (14)–(17) yields the equilibrium demands and profit levels, *i.e.* counterparts to expressions (8)–(11).

Finally, the postal operator's profits under marketplace delivery are given by

$$\Pi^{NM} = (r_0 - c)\left[X_0^M(\alpha,\gamma,p_0^{NM},p_1^{NM}) + X_1^M(\alpha,\gamma,p_0^{NM},p_1^{NM})\right] + (r-c)Y(r) - F. \tag{20}$$

Comparing equations (12) and (20) shows that the total sales of *both* retailers are now delivered at the rate r_0. The marketplace thus introduces a secondary market for delivery services which, even in the absence of bypass, restricts the delivery operator's ability to differentiate prices.

2.3 Sequence of decisions

The timing of the "full game" consisting of delivery and retail pricing is as follows. In Stage 1 the postal operator set r, to maximize welfare subject to a minimum profit constraint. While this includes profit maximization as a special case, the profit maximizing solution is also determined separately. In Stage 2, the postal operator and retailer 0 bargain over r_0. In stage 3, retailer 0 chooses s, that is the per unit rate and T, the fixed fee at which it is willing to sell its delivery service to the other retailer. In Stage 4, retailer 1 chooses independent delivery or marketplace delivery. Finally, in Stage 5 the retailers simultaneously choose their prices p_0 and p_1 in either the I or M subgame, which are described in Subsections 2.1 and 2.2 above.

As usual we solve this game by backward induction to characterize the subgame perfect Nash equilibrium. At each stage the players (delivery operator or retailers) anticipate the impact their choices will have on the equilibrium in the subsequent stages. Though highly stylized, our model is too complicated to provide a full analytical solution. However, some analytical results can be obtained and in any event a thorough examination of the various stages is necessary to properly define the numerical solutions we will calculate in Section 4.

3 Equilibrium

We start by studying the last stage of the game. At this point retailer 1 has already decided if it delivers independently or via the marketplace. Consequently, the retailers play subgame I or subgame M. We shall examine them separately.

3.1 Stage 5

3.1.1 Subgame I

At this point r_0 and r are given and s is of no relevance because the retailer has decided not to join the marketplace. The equilibrium of the price game yields the equilibrium prices, $p_0^{NI}(r_0, r)$, $p_1^{NI}(r_0, r)$, and profits, $\pi_0^{NI}(r_0, r)$, $\pi_1^{NI}(r_0, r)$ as functions of the variables set in the earlier stages.

3.1.2 Subgame M

Once again, r_0, r and s are given. The equilibrium of the price game yields $p_0^{NM}(r_0, r, s)$, $p_1^{NM}(r_0, r, s)$ and the profit levels $\pi_0^{NM}(r_0, r, s, T)$ and $\pi_1^{NM}(r_0, r, s, T)$. Observe that s affects the equilibrium prices and profits, while T affects only profits.

3.2 Stage 4

At this stage, retailer 1 will decide whether or not to join the marketplace. To do so it will compare $\pi_1^{NI}(r_0, r)$ and $\pi_1^{NM}(r_0, r, s, T)$. When $\pi_1^{NI}(r_0, r) > \pi_1^{NM}(r_0, r, s, T)$, the retailer will choose independent delivery. Otherwise it will join the marketplace.

3.3 Stage 3

Retailer 0, the potential marketplace, sets s and T and anticipating Stage 4 effectively decides if it wants to induce the other retailer to join the marketplace or not. If the other firm joins, it will solve

$$\max_{s,T} \quad \widehat{\pi}_0^M + T \tag{21}$$

$$\text{s.t.} \quad \widehat{\pi}_1^M - T \geq \pi_1^I \tag{22}$$

which can be written as

$$\max_s [(\widehat{\pi}_0^M + \widehat{\pi}_1^M) - \pi_1^I].$$

Note that π_1^I is a constant in this problem. Consequently this amounts to setting s so as to maximize the sum of profits (total surplus of retailers) and use T to extract all the surplus above π_1^I.

Retailer 0 will find it profitable to induce the other retailer to join the marketplace if and only of

$$\max_s [(\widehat{\pi}_0^M + \widehat{\pi}_1^M) - \pi_1^I] \geq \pi_0^I$$

which is equivalent to

$$\max_s [\widehat{\pi}_0^M + \widehat{\pi}_1^M] \geq \pi_0^I + \pi_1^I; \tag{23}$$

in words, this means that joint profits (sum of profits) are larger in M than in I. This may or may not be true depending on the parameters. The creation of the marketplace has three effects on joint profits.

1. It increases the perceived quality level of the good sold by retailer 1 as long as $\gamma > 1$. This effect is positive (zero if $\gamma = 1$)

2. It decreases the cost of delivering retailer 1's sales, as long as $r_0 < r$. This effect is positive (zero if $r_0 = r$).

3. It decreases the degree of horizontal differentiation and thereby intensifies price competition in Stage 5. This effect is negative (zero if $\delta = 1$).

The only case where analytical results can be obtained is when $\delta = \gamma = 1$. Then, it is plain that condition (23) is always satisfied. As long as $r_0 \leq r$ retailer 0 can then always replicate the I equilibrium under M by setting $s = r$. When $r_0 = r$ (23) then holds as an equally; otherwise it is satisfied with strict inequality. In other words, without a discount, regime M weakly dominates, while it strictly dominates when $r_0 < r$. Furthermore the difference between the joint profits in the two regimes (evaluated at $s = r$) increase as r_0 decreases. Now, of course $s = r$ is not in general retailer 0's optimal strategy, but this makes regime M only more attractive.

One can expect that this result continues to hold when $\gamma > 1$. However, the comparative statics of the price last stage subgames are too complicated to get an unambiguous expression. We shall revisit this question through a numerical example in Section 4. On the other hand, $\delta < 1$ appears to plead in favor of regime M.

Returning to the general case, whether or not condition (23) is satisfied depends, at least potentially on r_0, which is given at Stage 3 but endogenous in Stage 2. So we have to study the comparative statics of condition (23) with respect to r_0. Note that both sides of the expression decrease with r_0, but one can conjecture that the LHS decreases faster (r_0 concerns a larger volume under M than under I). When δ is close to 1 and γ sufficiently large one can expect that the condition holds for all $r_0 \leq r$. In that case M will always emerge. Otherwise, one would expect that there is a critical level $\widetilde{r}_0$ such that $r \leq \widetilde{r}_0$ yields M, while we obtain I otherwise.

Note that the expressions in (23) are functions of (r_0, r); more precisely, the RHS is a function of (r_0, r) and the LHS of r_0. Let us denote by S^M the set of (r_0, r) which yields

$$\max_s [\widehat{\pi}_0^M(r_0) + \widehat{\pi}_1^M(r_0)] \geq \pi_0^I(r_0, r) + \pi_1^I(r_0, r)$$

so that a marketplace equilibrium is induced. Similarly we denote S^I the set of (r_0, r) which yield

$$\max_s [\widehat{\pi}_0^M(r_0) + \widehat{\pi}_1^M(r_0)] \leq \pi_0^I(r_0, r) + \pi_1^I(r_0, r)$$

so that an I equilibrium is induced. Observe that when the condition holds as equality (r_0, r) belongs to both sets.

3.4 Stage 2

The delivery operator and retailer 0 bargain over r_0. We describe the bargaining solution as the level of r_0 which maximizes a weighted sum of the delivery operator and the retailer's respective objectives. At this stage the delivery operator maximizes profits. Consequently, the bargaining problem can be stated as

$$\max_{r_0} \quad N_2 = \eta \Pi(r_0, r) + (1 - \eta)\pi_0(r_0, r), \tag{24}$$

$$\text{s.t.} \quad r_0 \leq r, \tag{25}$$

where *all expressions are evaluated at the induced equilibrium in the subsequent stages,* and where $0 \leq \eta \leq 1$ is the bargaining weight of the delivery operator.[3] We remain agnostic about the specific bargaining process which is used. We assume that the parties can sign binding agreements so that the solution must be on the "contract curve". In other words it is Pareto efficient from the firms' perspectives. This is a property shared by all solution concepts considered in cooperative game theory. The specific point on the contract curve which is adopted depends on the parties' respective bargaining weights η and $(1 - \eta)$. When $\eta = 1$, r_0 is set so as to maximize the delivery operator's profits; the weights of the retailer's profits increase as η increases. The solution is denoted

[3] One can easily show that the solution to (24) is equivalent to that of a generalized Nash bargaining solution given by

$$\max_{r_0} \quad \widetilde{N}_2 = \left[\Pi(r_0, r) - \overline{\Pi}\right]^\gamma \left[\pi_0(r_0, r) - \overline{\pi}\right]^{1-\gamma}, \tag{26}$$

where $\overline{\Pi}$ and $\overline{\pi}$ are the thread points (outside option that is solution when no agreement is achieved), while $0 \leq \gamma \leq 1$ is the bargaining weight of the delivery operator. In other words, any solution to problem (24) be achieved as a solution to problem (26); the reverse is also true. We use (24) because it is easier to solve numerically.

$r_0^*(r)$. Observe that the rigorous writing of these expressions is somewhat tedious. In particular the expressions differ depending on the subgame I or M induced in the last stage.

Observe that we can expect that

$$\frac{\partial \pi_0(r_0, r)}{\partial r_0} \leq 0.$$

The profits of the delivery operator, on the other hand can be increasing or decreasing

$$\frac{\partial \Pi(r_0, r)}{\partial r_0} \gtreqless 0.$$

To solve our problem, we have to distinguish two cases:

- $\partial \Pi(r, r)/\partial r_0 > 0$. In that case profits of the delivery operator are maximized for $r_0^* = r$ (noting that $r_0 > r$ is not possible). In words, it is not profitable for the delivery operator to give a discount to retailer 0. In that case, $r_0^* = r$ is on the contract curve. It is the solution to the bargaining problem when $\eta = 1$ (and possibly in the neighborhood of $\eta = 1$). In words, we get no discount as long as the bargaining weight of the delivery operator is sufficiently large. Otherwise $r_0^* < r$ is possible.

- $\partial \Pi(r_0, r)/\partial r_0 < 0$. Now, the delivery operator can increase its profits by giving a discount to retailer 0. In that case there exists a level $0 < r_0^{pm} < r$, which maximizes the delivery operator's profits. This (namely $r_0^* = r_0^{pm}$) is also the solution to the bargaining problem if $\eta = 1$, that is if the delivery operator has all the bargaining weight and can freely chose r_0 to maximize its profits. Otherwise, when $\eta < 1$, we will get $r_0^* < r_0^{pm}$. In words, because of the retailer's bargaining power, the delivery operator has to concede a discount that exceeds the profit maximizing level.

In either case we expect

$$\frac{\partial r_0^*(r)}{\partial r} \geq 0.$$

3.5 Stage 1

We assume that to set r the delivery operator maximizes either *(i)*profits or *(ii)* welfare as measured by the sum of consumer and producer surplus.

3.5.1 Profit maximization

Now, the delivery operator chooses r to solve

$$\max_{r} \quad \Pi[r_0^*(r), r]. \tag{27}$$

The solution depends on the elasticity of $Y(r)$, particularly if regime M is induced in the subsequent stages.

Since the expressions differ according to the induced subgame in the last stage, we have to write this problem in 2 steps.

First we solve the following two problems

$$P^M = \max_{r} \quad \Pi^{NM}[r_0^*(r), r],$$
$$\text{s.t. } (r_0^*(r), r) \in S^M.$$

$$P^I = \max_{r} \quad \Pi^{NI}[r_0^*(r), r],$$
$$\text{s.t. } (r_0^*(r), r) \in S^I.$$

The solution is then obtained by comparing the profit levels achieved at these two solutions.

3.5.2 Welfare maximization

Define welfare as the sum of consumer and producer surplus

$$W(r_0, r) = CS(r_0, r) + \Pi(r_0, r) + \pi_0(r_0, r) + \pi_1(r_0, r).$$

Then in stage 1 the delivery operator maximizes $W[r_0^*(r), r]$ subject to a minimum profit constraint $\Pi[r_0^*(r), r] \geq \overline{\Pi}$, where the profit requirement $\overline{\Pi}$ is exogenously given.

Denoting $\lambda \geq 0$ the Lagrange multiplier associated with profit constraint, this problem is equivalent to the maximization of

$$W(r_0^*(r), r, \lambda) = CS(r_0^*(r), r) + (1+\lambda)\Pi(r_0^*(r), r) + \pi_0(r_0^*(r), r) + \pi_1(r_0^*(r), r), \quad (28)$$

where the level of λ is exogenously chosen. Observe that setting $\lambda = 0$ yields the unconstrained welfare maximum, while $\lambda \to \infty$ yields the profit maximizing solution as special cases.

As in the profit maximizing case, the specification of all these expressions of course depends on the induced regime I or M. Thus one has to consider the cases where $(r_0^*(r), r) \in S^M$ and where $(r_0^*(r), r) \in S^I$ separately and then compare the achieved level of W.

4 Numerical illustrations

Though stylized, our model is too complex to obtain further analytical results. Observe that we consider individuals with continuous demand functions, which is typically not done in IO models which concentrate on indivisible goods. This makes even the last stages of the game very complicated.[4] Consequently, we resort to numerical illustrations. We first briefly present our numerical approach and the computational strategy used. Then we present the underlying specification and the parameters used in the benchmark case. Then we present and discuss some results starting with the case where the postal operator maximizes profits. Then we turn to the situation where the delivery operator maximizes welfare.

4.1 Numerical strategy

The solution to Stages 3–5 follows exactly the theoretical model. Stage 2 was more challenging because the specification of the objective depends on the regime induced in the subsequent stages. While this is not problematic in the theoretical specification of the game, the numerical solution required some detours. We have considered the two

[4]But even with zero-one individual demands, the full game cannot be solved analytically.

regimes separately. For I we have solved

$$\max \ \eta\Pi^{NI}(r_0, r) + (1-\eta)\,\pi_0^I(r_0, r) \tag{29}$$

$$\text{s.t}: \pi_0^I(r_0, r) + \pi_1^I(r_0, r) \geq \hat{\pi}_0^M(r_0, r) + \widehat{\pi}_1^M(r_0, r) \tag{30}$$

$$\text{s.t}: \ r \geq r_0 \tag{31}$$

which yields $r_0^I(r)$ in 3 regimes: one in which the FOC is simply solved. One in which the participation constraint (30) binds. One in which $r \geq r_0$ binds. Similarly, for M we have analyzed

$$\max \ \eta\Pi^{NM}(r_0, r) + (1-\eta)\,\hat{\pi}_0^M(r_0, r) \tag{32}$$

$$\text{s.t}: \hat{\pi}_0^M(r_0, r) + \hat{\pi}_1^M(r_0, r) \geq \pi_0^I(r_0, r) + \pi_1^I(r_0, r) \tag{33}$$

$$\text{s.t}: \ r \geq r_0 \tag{34}$$

which yields $r_0^M(r)$ in 3 regimes: one in which the FOC is simply solved. One in which the participation constraint (33) binds. One in which $r \geq r_0$ binds. The solution is then determined by evaluating the respective objective functions for the results of the different subcases. This yields $r_0^*(r)$ which is then used to maximize the first stage objective (profit or welfare).

4.2 The benchmark parameters of the model

We make use of the following benchmark values: $k = 10$, $c = 0.5$, $t = 20$,$\alpha = 1.1$, $\gamma = 1$, $\delta = 1$. We further assume linear individual demand functions for the goods sold by firms 0 and 1. These are obtained from quadratic utilities of the form

$$u(\alpha, x) = \alpha\left(bx - ax^2\right),$$

so that

$$x(\alpha, p) = \frac{b}{2a} - \frac{p}{2a\alpha}.$$

which are such that (i) their direct price elasticity is -4 at a consumer price of 10, and (ii) that $x(1, 10) = 10$. This yields

$$a = 0.125 \text{ and } b = 12.5.$$

For the demand $Y(r)$, we assume that its direct price elasticity is -3 at a consumer price of 3 with $Y(3) = 0.5$. This yields

$$Y = \left(\frac{b'}{2a'} - \frac{r}{2a'}\right)$$

with

$$a' = 1 \text{ and } b' = 4.$$

Except for reflecting the stylized facts about market shares and elasticities these parameter values are not meant to represent a realistic calibration. Fixed costs are neglected. They would not change the equilibrium allocation, except that they have to be deducted from the delivery operator's reported profit. We finally assume that the distribution of tastes G is uniform over $[0, 1]$.

4.3 Numerical results: profit maximization

4.3.1 The benchmark case

Table 1 presents the results in the benchmark case for the levels of the delivery operator's bargaining weight in Stage 2: $\eta = 1$, 0.75 and 0.5. The solutions are all in regime M; since $\delta = \gamma = 1$, this is consistent with the theoretical results. We also report the "counterfactual" (CF) in regime I. This represents the equilibrium in subgame I induced by the considered level of r_0 and r. This is an interesting reference even though this subgame is not reached in equilibrium. In particular, the equilibrium profit of retailer 1 in this subgame, $\pi_1^I(r_0, r)$, determines the level of T; see equations (21)–(22). The columns "no M" give the solution when regime M is ruled out in an *ad hoc* way. The delivery operator then faces two independent but competing retailers. Most notations are in line with the theoretical part, except for CSX and CSY which represent the consumer surplus associated with X and Y respectively. Let us start by examining the results obtained for $\eta = 1$. In that case there is effectively no bargaining, and both rates are set by the delivery operator to maximize its profits. Still, the solution implies a discount for retailer 0. This does not come as a surprise. When $\eta = 1$, r is set exclusively to maximize profits extracted from single piece customers; it does not affect the profit the delivery operator can earn from the delivery of good X. Since the demand for X is

regime M	$\eta = 1$	CF I	no M	$\eta = 0.75$	CF I	no M	$\eta = 0.5$	CF I	no M
p_0^*	12.71	12.70	12.74	12.56	12.55	12.59	11.97	11.95	11.95
p_1^*	12.28	12.37	12.18	12.16	12.37	12.13	11.74	12.40	12.05
s^*	2.07	-	-	1.86	-		1.13	-	-
T	0.04	-	-	0.13	-		0.69	-	-
r_0^*	1.83	1.83	1.89	1.56	1.56	1.65	0.55	0.55	0.57
r^*	2.25	2.25	1.89	2.25	2.25	1.81	2.30	2.30	1.67
π_0 ($\hat{\pi}_0$ when M)	2.05	2.00	1.79	2.73	2.68	2.33	6.32	6.38	5.92
π_1 ($\hat{\pi}_1$ when M)	0.07	0.03	0.15	0.15	0.02	0.19	0.70	0.01	0.23
Π	5.02	4.96	5.19	4.88	4.78	5.08	1.80	1.98	2.38
$X_0/(X_0+X_1)$	0.86	0.92	0.80	0.83	0.94	0.81	0.78	0.97	0.88
$X_1/(X_0+X_1)$	0.14	0.08	0.20	0.17	0.06	0.19	0.22	0.03	0.12
$X_0/(X_0+X_1+Y)$	0.65	0.68	0.58	0.65	0.72	0.60	0.67	0.83	0.71
$X_1/(X_0+X_1+Y)$	0.10	0.06	0.14	0.13	0.05	0.14	0.19	0.02	0.10
$Y/(X_0+X_1+Y)$	0.25	0.26	0.28	0.22	0.23	0.26	0.14	0.15	0.19
CSX	-17.36	-17.84	-16.87	-17.09	-18.27	-16.87	-15.40	-20.38	-17.53
CSY	0.76	0.76	1.10	0.76	0.76	1.19	0.71	0.71	1.34
W	-9.44	-10.07	-8.62	-8.55	-10.01	-8.06	-5.85	-11.29	-7.65

Table 1: Simulation results for the benchmark case with $\alpha = 1.1$, $\gamma = \delta = 1$.

more elastic than the demand for Y, the discount is in line with standard third degree price discrimination results. This is of course just a rough intuition because it ignores all "intermediate" effects. For instance, the elasticity of X with respect to r_0 depends on the degree of pass-through which, in turn, depends on the strategic interaction in subgame M. The strategic interaction also explains that $s > r_0$: the marketplace charges a delivery fee that exceeds its marginal cost. This increases the competitor's cost which has two conflicting effects. First, it increases retailer 0's equilibrium profit. Second, it also reduces retailer 1's profit and thus the amount retailer 0 can extract via T.

As far as consumer prices are concerned, we rather surprisingly have $p_0^* > p_1^*$: retailer 0's cost advantage ($r_0 < s$) is not passed through to consumers, quite the opposite. With $\alpha = 1.1 > \gamma = 1$, retailer 0 has a quality advantage of which it takes advantage to achieve a larger market share and a larger profit than the other retailer. This comes on top of its strategic advantage: by setting s it can influence retailer 1's reaction function in the M subgame to achieve a "large" equilibrium price.

If given the delivery rates r_0^* and r_1^*, retailer 1 would stay independent we obtain the outcome described in column CF I. It represents the equilibrium of subgame I, which is *not* played in the equilibrium of the full game. In fact, both consumers and the delivery operator would be worse off in this case and welfare would be lower. Retailer 1 is by construction indifferent (because $\widehat{\pi}_1^M - T = \pi_1^I$), but retailer 0 obviously benefits from the creation of the marketplace; otherwise the equilibrium would not be in regime M.

Column with no M presents the equilibrium if, for whatever reasons, regulatory or others, the creation of a marketplace is not an option. With $\eta = 1$, the constraint $r_0 \leq r$ is then binding. This means that absent of regime I and of bargaining the profit maximizing delivery operator does not give a discount to retailer 0. As a matter of fact if this were possible it would even want to set r_0 above r. For the rest, all parties (including the delivery operator and the other retailer) except of course of retailer 0 would be better off in that case. Consumer surplus and total welfare would also be larger.

The remaining columns present the results when the delivery operator has lower bargaining weights. Not surprisingly, the discount achieved by retailer 0 increases as its bargaining weight increases, and its equilibrium profit follows suit. As expected, the profit of the delivery operator decreases and the reduction in both retailers prices also increases welfare and consumer surplus (from X). Notice that while p_0 decreases it remains larger than p_1 and the price decrease falls short of the decrease in r_0. The single piece delivery rate r, on the other hand, increases as η decreases so that the surplus of single piece customers also goes down. This is because as $\eta < 1$ the single piece delivery rate is no longer solely determined by the single piece market since it also affects actors' profits (via its impact on the counterfactual). When $\eta = 1$ this effect plays no role in the objectives of the first two stages. However, when $\eta < 1$, it does affect the second stage objective which spills over into the first stage choices.

We now consider a number of scenarios obtained by changing one of the parameters; all others remain the same as in the benchmark case.

regime M	$\eta = 1$	CF I	$\eta = 0.5$	CF I
p_0^*	12.86	12.86	12.03	12.02
p_1^*	12.93	12.37	12.24	12.40
s^*	2.26	-	1.13	
T	0.95	-	2.89	
r_0^*	2.10	2.10	0.66	
r^*	2.25	2.25	2.31	
π_0 ($\hat{\pi}_0$ when M)	1.45	1.43	5.70	5.89
π_1 ($\hat{\pi}_1$ when M)	0.98	0.03	2.90	0.01
Π	6.49	4.96	2.48	2.44
$X_0/(X_0 + X_1)$	0.53	0.90	0.56	0.97
$X_1/(X_0 + X_1)$	0.47	0.10	0.44	0.03
$X_0/(X_0 + X_1 + Y)$	0.41	0.64	0.49	0.82
$X_1/(X_0 + X_1 + Y)$	0.37	0.07	0.39	0.02
$Y/(X_0 + X_1 + Y)$	0.22	0.29	0.12	0.16
CSX	-26.01	-17.47	-22.47	-20.18
CSY	0.76	0.76	0.71	0.71
W	-16.31	-10.28	-10.67	-11.11

Table 2: Simulation results with $\gamma = 1.1$.

4.3.2 Case of γ=1.1

In the benchmark case we had $\gamma = 1$. This means that joining the marketplace has no impact on the perceived quality of retailer 1's product. Let us now consider an alternative level of γ and in a way go to the other extreme by setting $\gamma = \alpha = 1.1$. Consequently, when retailer 1 joins the marketplace, it fully benefits from retailer 0's quality advantage. The results are presented Table 2.

Interestingly prices are higher even though products are now less differentiated which one would expect to lead to a more intense price competition. However, quality of retailer 1 is larger than in the benchmark case. This increases the willingness to pay of consumers which via the strategic interaction leads to higher prices in the last stage subgame. Furthermore, delivery rates are larger as both r_0 and s increase. The single piece rate remains unchanged for $\eta = 1$; it continues to be set only with regard to single piece mail demand Y. When $\eta = 0.5$, on the other hand, r affects the second stage objective and it is set at a larger level than in the benchmark case. And this

regime M	$\eta = 1$	CF I	$\eta = 0.5$	CF I
p_0^*	12.70	12.72	11.93	11.94
p_1^*	12.37	12.37	11.98	12.40
s^*	2.24	-	1.59	
T	-0.002	-	0.22	
r_0^*	1.86	1.86	0.53	
r^*	2.25	2.25	2.31	
π_0 ($\hat{\pi}_0$ when M)	2.17	1.95	7.20	6.46
π_1 ($\hat{\pi}_1$ when M)	0.02	0.02	0.23	0.01
Π	5.18	4.97	1.71	1.89
$X_0/(X_0+X_1)$	0.93	0.92	0.89	0.98
$X_1/(X_0+X_1)$	0.07	0.08	0.11	0.02
$X_0/(X_0+X_1+Y)$	0.70	0.68	0.77	0.83
$X_1/(X_0+X_1+Y)$	0.05	0.06	0.1	0.02
$Y/(X_0+X_1+Y)$	0.25	0.26	0.13	0.15
CSX	-11.14	-17.81	-9.43	-20.43
CSY	0.76	0.76	0.71	0.71
W	-3.00	-10.09	0.43	-11.34

Table 3: Simulation results with $\delta = 0.7$.

increase *is* shifted in part to the consumers. So much that we have now $p_1 > p_0$ and that CSX decreases significantly in spite of the increase in quality.[5] Not surprisingly π_0 also increases.[6]

For the rest, given $\gamma = 1.1$, the effect of an increase in the retailer's bargaining weights follows pretty much the same pattern as in the benchmark case.

4.3.3 Case of $\delta = 0.7$

We now return to $\gamma = 1$ but assume that $\delta = 0.7$ rather than 1. Consequently, the degree of horizontal differentiation between retailers, decreases as the marketplace is set up. The results for this case are presented in Table 3.

One would expect that this leads to lower prices and indeed p_0 decreases but p_1 effectively increases. This follows the increase in s; as goods become closer substitutes

[5] Since the utility of retailer 1's customer changes, this welfare comparison has to be interpreted with care. It makes sense if we think of the quality of the product as an argument of the utility function.

[6] Recall that in regime M retailer 0's profits are given by $\widehat{\pi}_0 + T$ and the decrease in $\widehat{\pi}_0$ is more than compensated by the increase in T.

retailer 0 finds it profitable to increase the competitors cost even more than in the benchmark case. Not surprisingly all this combined with retailers quality advantage results in an overwhelming market share for retailer 0. Observe that r_0 also increases but to a lesser extent than s. The delivery operator's profits increase slightly when $\eta = 1$ and decrease when $\eta = 0.5$ (compared to the benchmark situation). In either case, the equilibrium is in regime M. As mentioned in Subsection 3.3 with $\delta < 1$, regime I cannot be ruled out *a priori*. However, as conjectured in the analytical part, the comparative statics of the price equilibrium are indeed nontrivial, since one of the prices increases. So at the end, regime M remains the better choice for operator 1.

Once again the pattern according to which the equilibrium changes with η is similar to that observed in the benchmark case. In particular welfare increases as the delivery operator's market power decreases. This comes of course at the expense of a decrease in the delivery operator's profit, and while r remains large, r_0 gets rather close to marginal cost (which is equal to 0.5). While this appears to be a "good thing", it may in reality be problematic since its profits may not be sufficient to cover fixed costs.

4.4 Numerical results: welfare maximization

In stage 1 we now maximize

$$W(r_0^*(r), r, \lambda) = \beta \left[CS(r_0^*(r), r) + \pi_0(r_0^*(r), r) + \pi_1(r_0^*(r), r)\right] + (1-\beta)\Pi(r_0^*(r), r) \quad (35)$$

with $\beta = 0.4$. This is equivalent to maximizing (28) by defining

$$\beta = \frac{1}{2+\lambda} \quad \text{or} \quad (1-\beta) = \frac{1+\lambda}{2+\lambda}$$

so that $\lambda = 0$ corresponds to $\beta = 0.5$ while $\lambda \to \infty$ yields $\beta = 0$. Observe that $\beta = 0.4$ is equivalent to $\lambda = 0.5$. This is probably too large when λ is seen as a cost of public funds but this interpretation assumes that the deficit of the delivery operator (recall that the profit Π we report does not account for the fixed cost) would be financed by a transfer from the government. When the delivery operator is required to break even and the fixed costs are sufficiently large we can have larger levels of λ. Recall that in stage 2 when setting r_0, the delivery operator continues to maximize profits.

regime M	$\eta = 1$	CF I	no M	$\eta = 0.5$	CF I	no M
p_0^*	12.17	12.13	12.01	11.95	11.91	12.01
p_1^*	11.89	11.61	11.49	11.73	11.89	11.49
s^*	1.38	-		1.10	-	
T	-0.56	-		0.30	-	
r_0^*	0.90	0.90	0.71	0.51	0.51	0.71
r^*	0.90	0.90	0.71	1.39	1.39	0.71
π_0 ($\hat{\pi}_0$ when M)	4.89	4.21	4.79	6.48	5.97	4.79
π_1 ($\hat{\pi}_1$ when M)	0.48	1.04	1.31	0.73	0.43	1.31
Π	2.43	2.61	1.47	1.25	2.01	1.47
$X_0/(X_0 + X_1)$	0.79	0.70	0.69	0.77	0.83	0.69
$X_1/(X_0 + X_1)$	0.21	0.30	0.31	0.23	0.17	0.31
$X_0/(X_0 + X_1 + Y)$	0.58	0.53	0.53	0.62	0.66	0.53
$X_1/(X_0 + X_1 + Y)$	0.16	0.23	0.24	0.18	0.14	0.24
$Y/(X_0 + X_1 + Y)$	0.26	0.24	0.23	0.20	0.20	0.23
CSX	-16.11	-14.63	-14.01	-15.32	-16.44	-14.01
CSY	2.39	2.39	2.70	1.69	1.69	2.70
W	-5.90	-4.37	-3.72	-5.15	-6.32	-3.72

Table 4: Simulation results: welfare maximization in the benchmark case: $\alpha = 1.1$, $\gamma = 1$ and $\delta = 1$.

It is well know that when λ is zero or sufficiently small the solution involves a delivery rate that is set below marginal cost. To be more precise in our setting, r would be set at marginal cost, while r_0 would be subsidized so as to bring the final prices closer to marginal costs. This results of course in a deficit for the delivery operator but with $\lambda = 0$ we implicitly assume that this can be financed from the general budget at no cost. This isn't hardly a realistic scenario to consider and we set λ sufficiently high to avoid this kind of situation. For the rest, we return to the parameters considered in the benchmark scenario in the previous subsection. The results are reported in Table 4.

The results do not offer any major surprises. Delivery rates are lower than in the profit maximizing case.[7] Not surprisingly this is more significant for $\eta = 1$ than for $\eta < 0.5$; in the latter case the delivery operator's market power is in any way already lessened by the bargaining process. The decrease is also more significant for r than for r_0. Under profit maximization regime, the single piece rate was a "pure" monopoly price (or

[7] The constraint $r_0 \leq$ is binding for $\eta = 1$.

close to it); the price setting was not or very little affected by strategic considerations. Accordingly the markup was large and switching to welfare maximization has a dramatic impact. In all scenarios consumer prices of both product are of course lower than in the profit maximizing case, but it appears once again that the decrease in delivery rates is only partially passed through to final consumers. This also explains why both retailers profits are now larger and so are CSX and especially CSY. Finally the delivery operator's profits decrease and, again, the decrease is most significant in the $\eta = 1$ scenario.

To sum up, switching from profit to welfare maximization in the first stage has the most significant effect when $\eta = 1$. When $\eta < 1$, the market power of the delivery operator has already diminished, which brings the rates closer to marginal costs. However, in that case the benefits of the rate reduction are to a large extent caught by the retailers. This effect remains present for a switch from profit to welfare maximization, but it is mitigated.

5 Conclusion

This paper has examined the link between the delivery rates charged by parcel delivery operators and the e-commerce market structure. In reality electronic retail markets are characterized by the growing importance of marketplaces, who offer a variety of services including delivery to its affiliated e-retailers. From a parcel delivery operator's perspective, marketplaces create a secondary delivery market which undermines its ability to differentiate prices. Consequently, the market structure in the e-commerce sector will affect the pricing strategy of the delivery operator. Interestingly, the effect will also go in the opposite direction: the pricing structure of parcel delivery will in part determine the development of marketplaces. More specifically, we developed a model in which two retailers, a big one, 0, and a smaller one, 1, sell a homogenous good online. Retailer 0 may offer its competitor the option to join its marketplace through the payment of a fixed fee along with a per unit rate. Affiliation to the marketplace has several consequences for retailer 1. First, it reduces the degree of differentiation between the products. Second, it increases the perceived quality of retailer 1's product which in turn increases the consumers' willingness to pay and, third, the marketplace consolidates the parcels send by retailers 0 and 1 and could, in theory obtain better pricing conditions from the parcel delivery operator. This last point is reinforced by the fact that it is assumed the marketplace has some monopsony power: the delivery rate set by the parcel delivery operator to the marketplace results from a bargaining process. The timing of the full game is as follows. In Stage 1 the postal operator sets the single piece rate. In Stage 2, the postal operator and retailer 0 bargain over the delivery rate paid by the marketplace. In stage 3, retailer 0 chooses the per-unit rate and the fixed fee at which it is willing to sell its delivery service to the other retailer. In Stage 4, retailer 1 chooses independent delivery or marketplace delivery. Finally, in Stage 5 the retailers simultaneously choose their prices in either the independent or marketplace subgame.

Under these conditions and according to the assumptions made in the numerical simulations, we obtain that a marketplace will always emerge in profit maximizing equilibrium, even if from a social point of view, the welfare would be higher if the creation of a marketplace was not an option. Indeed, the marketplace allows the big e-retailer to make higher profits thanks to the possibility to extract some of its competitor profit through the affiliation fixed fee and to increase its own price. Even if the parcel delivery operator gives a discount to the marketplace, it makes higher profits since through its effect on the quality perceived, the marketplace has a positive impact on demand, increasing the number of parcels to be delivered. Consumers are better off since retailer 1's price is lower in the marketplace regime than in the independent one and the (perceived) quality is higher. And by definition, the small retailer is indifferent between the independent and marketplace regime.

Not surprisingly, the discount achieved by retailer 0 increases as its bargaining weight increases, leading to the increase of its profits and the reduction of that of the delivery operator. The reduction in the delivery rate set to the marketplace is partially passed through to the final price paid by consumers, increasing their surplus and the global welfare. In other words, from a social point of view, it is better to have balanced bargaining powers between the delivery operator and the marketplace.

To sum up, the following two main lessons emerge from our results. First, the equilibrium nearly always implies a discount to the "leading" retailer, even when the profit maximizing operator has all the bargaining power.[8] Second, the delivery operator cannot avoid the emergence of a marketplace even though this decreases its profits (compared to the hypothetical situation where a marketplace is not permitted). This is because the possibility of using a two part tariff for marketplace services, provides the dominant retailer with a tremendous strategic advantage. It can set the variable fee to enhance its competitive position in the subsequent pricing game, while extracting all extra profits via the fixed fee.

The combination of these two effects then leads to discounted rates which get fairly close to marginal cost, especially when the bargain weight of the retailer increases. While this is very good for the marketplace retailer and some of the benefits are passed through to consumers the big looser is not surprisingly the delivery operator who may not be able to cover its fixed cost. Consequently, the growing bargaining power of Amazon's marketplace over parcel delivery operators in several European countries could raise concerns, notably if this prevents the latter to cover their fixed costs and calls for a transfer from the government to finance the provision of the postal universal service.

References

[1] Borsenberger C. (2015), "The concentration phenomenon in e-commerce", in M.A. Crew and T.J. Brennan (eds), *Postal and Delivery Innovation in the Digital Economy*, Edward Elgar, Cheltenham UK, 31–42.

[2] Borsenberger, C., Cremer, H., de Donder, P. and Joram, D. (2016), Differential pricing of delivery services in the e-commerce sector, in: Crew, M. and T. Brennan, eds, *The Future of the Postal Sector in a Digital World,* Springer, 191–213.

[8] And this is true even at the (out of equilibrium) subgames with independent delivery.