PRIVATE CONTRACTS IN TWO-SIDED MARKETS

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Abstract. We study a two-sided market in which a platform connects buyers and sellers, and signs private contracts with sellers. We compare this situation with a two-sided market with public contracts. We find that the platform provider sets positive (negative) royalties to sellers and earns a negative (positive) markup on consumers when contracts are private (public). Thus, private contracting has a significant effect on the price structure. Private contracting leads to lower platform profits, consumer surplus, and social welfare. We study the welfare effects of most-favored-nation and resale-price-maintenance clauses, vertical integration with sellers, and relate our results with the agency model of sales. Our results indicate that giving more market power to a dominant platform may be welfare-enhancing because it eliminates adverse-selection problems arising from information asymmetries between the platform, sellers and buyers.

Keywords: Two-Sided Markets, Platforms, Vertical Relations, Private Contracts, Most-Favored Nation, Resale Price Maintenance, Vertical Integration, Agency Model of Sales.

1. Introduction

Private contracts are common in two-sided markets. For example, Amazon signs private contracts with publishers, Netflix with movie studios, Sony and Nvidia with game developers, Spotify with music studios, HMOs with health-care providers, Google with phone manufacturers, Apple with cellphone carriers, and Intel and Microsoft with computer manufacturers. In this paper, we show that private contracting has a critical impact on the platform's price structure, industry profitability, and social welfare, and that it helps explains many commonly observed features of two-sided markets.

We study a two-stage model of a platform that connects buyers and sellers. Sellers' products may be substitutes or complements. In the first stage, the platform provider chooses the membership or access fees to be paid by buyers and sellers to join the platform, and sets the royalty fees to be paid by sellers for each unit of the good sold to consumers; and then sellers decide whether to accept the two-part-tariff contract offered by the platform provider and consumers decide

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whether to access the platform. In the second stage, sellers post prices, and consumers who have purchased access to the platform choose how much to buy from each seller.

Our aim is to compare different information structures and shed light on how each of them affects equilibrium royalty fees, access prices, platform profits, and welfare. In particular, we compare the *public contracts* case, in which the platform provider's pricing scheme is publicly observable –the standard assumption in the two-sided markets literature (see Caillaud and Jullien, 2003; Rochet and Tirole, 2003; Armstrong, 2006; Hagiu, 2006; Weyl, 2010, for example)– with the *private contracts* case, in which the platform's offer to each seller is observed only by that seller.

When the contract offered to a seller is private, equilibrium behavior depends on how sellers and buyers form beliefs about unobserved variables when observing out-of-equilibrium play. In line with the literature, we assume that consumers form "passive beliefs" (Hart and Tirole, 1990; O'Brien and Shaffer, 1992; Hagiu and Hałaburda, 2014) and that sellers form "wary beliefs" (McAfee and Schwartz, 1994; Rey and Vergé, 2004) when observing an unexpected behavior by the platform. That is, consumers interpret unexpected behavior by the platform as "trembles," and believe that the contractual offers received by sellers remain unchanged. On the other hand, sellers interpret an unexpected offer as intentional behavior, and believe that the platform is choosing the contracts offered to other sellers to maximize its residual profits.[1]

We find that the conclusions drawn from a model of a two-sided market with private contracts stand in stark contrast with those of a model with public contracts. When contracts are public, equilibrium royalty fees are negative and the platform provider's markup on consumers is positive. When contracts are private, on the other hand, royalty fees are positive and the platform provider's markup on consumers is negative. These results fit well with the price patterns observed in many industries in which contracts are private (such as videogames and ebooks).

We also find that private contracting results in lower profit for the platform provider, as well as lower consumer and social surplus, in comparison with the public contracts case. These findings contrast with those of papers studying private contracts in one-sided markets (Hart and Tirole, 1990; O'Brien and Shaffer,

[1]Wary beliefs mitigate the opportunistic behavior of the platform provider relative to having sellers form passive beliefs. It is well-known from Rey and Vergé (2004) that there might exist no equilibria if sellers form passive beliefs in a setting like ours.

1992; McAfee and Schwartz, 1994; Rey and Vergé, 2004), in which private contracting lowers industry profits but increases consumer surplus and welfare.

To understand our results, consider first the public contracts case. The platform captures sellers' profits through the fixed fee, and behaves as an integrated monopolist with two price instruments: sellers' prices and access prices for buyers. Sellers' prices have a double impact on consumer demands: they affect per capita demand for sellers' products, and buyers' demand for platform access. Buyer access prices, on the other hand, affect the demand for platform access, but not per capita demands for sellers' products, so they are a more efficient instrument for "taxing" buyers. As a consequence, the platform provider chooses royalties to induce null seller prices and charges positive access prices to buyers.

Consider now an intermediate case in which sellers observe all contractual offers, but sellers' contracts are unobserved by buyers. In this case, buyers anticipate that the platform will behave opportunistically when setting royalty fees, choosing them to induce collusive pricing by sellers. This hold-up problem faced by consumers lowers the demand for platform access. The platform responds by lowering consumer access prices to compensate the decrease in access demand, but also because seller revenue per consumer increases (given that seller prices increase, each buyer that joins the platform becomes more valuable in terms of the revenues she generates when consuming sellers' products). Using a simple revealed-preference argument, is is straightforward to conclude that the effect of the lower demand dominates the effect of higher prices, and thus the platform's profit decreases as a result of buyers inability to observe sellers' contracts. Consumer surplus also decreases, since fewer buyers join the platform and consumer surplus per buyer is lower (recall that sellers' prices are set at their collusive level).

Finally, consider the private contracts case, in which, in addition to assuming that consumers do not observe sellers' contracts, we assume that each seller observes the contract it is offered, but not the contracts of other sellers. Consider first the case in which sellers' goods are substitutes. In this case, one may be tempted to extrapolate Rey and Vergé's (2004) finding that the platform provider must be worse off (relative to the intermediate case mentioned above), for it loses part of its market power vis-à-vis sellers. Such an extrapolation would be incorrect because it would miss the feedback loops that arise in a two-sided market.

In particular, we find that in a two-sided market framework, decreasing the market power on one side may enhance market power on the other side.[2] Sellers

[2]We define market power as a firm's ability to charge prices above marginal cost.

fear that the platform will behave opportunistically, offering lower royalties to other sellers when they accept their contract. Thus, the royalties that sellers are willing to accept from the platform provider are lower, which implies that royalties and seller prices decrease, in relation to the intermediate case. This decrease in seller prices, in turn, encourages consumers to join the platform. Therefore, the lack of commitment when choosing sellers' royalties acts as a commitment device for inducing lower seller prices, and mitigates the hold-up problem suffered by buyers when they cannot observe sellers' contracts. As a result, the platform provider can charge higher access prices to buyers and still increase the number of buyers that join the platform. These effects dominate the lower revenues per buyer that can be extracted from sellers, and platform profits increase as a result.

In contrast with the substitutes case, when sellers' products are complementary, the platform earns less from sellers (for a given number of consumers), but also attracts fewer consumers, relative to the intermediate case.[3] The loss of market power by the platform provider makes it less capable of internalizing the double marginalization problem faced by sellers (Cournot, 1838), so consumers expect sellers to charge higher prices. The platform becomes less valuable for buyers, and the adverse-selection or hold-up problem becomes more severe. Even though the platform provider charges lower prices to attract consumers, platform sales decrease and the platform provider is harmed by the lower usage of the platform by consumers and the smaller profit appropriated from sellers.

Comparing now the public contracts and private contracts cases, it holds when contracts are private rather than public that consumers fear being taken advantage of by the platform because they cannot observe the actual royalties that the platform will receive from sellers. When sellers' goods are substitutes, the adverse selection problem that consumers face is *mitigated* by the loss of market power that the platform provider bears when it secretly contracts with each seller. Our contribution in this case is to show that the consumers' initial concern is not mitigated enough by this loss in control. The platform provider's profits are therefore smaller when contracts are private rather than public. Consumer surplus and social welfare decrease as well. When sellers' offer complements instead of substitutes, our contribution is to show that the consumers' concern about the platform provider's opportunistic behavior is *accentuated* because it has less control over

[3]To the best of our knowledge, the complements case has not been analyzed by the vertical relations literature dealing with secret contracts.

the double marginalization problem faced by the sellers. Relative to public contracting, private contracts again result in higher royalties, higher prices charged by sellers, lower prices for the platform, lower profitability for the platform provider, and lower consumer and social welfare.

We also study the welfare effects of Most-Favored Nation (MFN) clauses, Resale Price Maintenance (RPM), forward integration with sellers, and of using sellers as agents as in Johnson's (2014) agency model of sales. We find that MFN clauses increases welfare when seller's products are complements, and reduce welfare when seller's products are substitutes, while RPM clauses increase welfare in both cases. The difference is that MFN clauses solve the commitment problem with sellers, but not with consumers; while RPM clauses solve the commitment problem with both sellers and consumers. Forward integration with all sellers can also help prevent the welfare losses from private contracting, but incomplete integration (when the platform integrates with some, but not all sellers) is socially desirable only when seller's products are complements. Finally, using sellers as agents rather than customers also allows the platform to overcome commitment problems, and hence has a beneficial effect on welfare. Our findings suggest that enhancing the platform's market power may be greatly beneficial because it prevents consumers from facing an adverse selection problem that would harm adoption and overall platform profitability.

Our paper contributes to the literature on two-sided markets (Caillaud and Jullien, 2003; Rochet and Tirole, 2003; Armstrong, 2006). To the best of our knowledge, the entire literature assumes that contracts are publicly observable to all parties. The only exception in which one of the two sides does not observe the price charged to the other side is the paper by Hagiu and Halaburda (2014), which examines how price transparency affects market outcomes. Our result that contractual transparency is beneficial because it allows the platform to commit not to trick consumers into purchasing a platform that will have expensive goods sold by sellers is different from Hagiu and Halaburda's (2014) insight because buyers and sellers do not interact in their setting. In fact, if sellers' prices were contractible in our setting, then the platform would reduce the endogenously formed adverse selection and consumers would benefit from it (think of iTunes, for example). In contrast with the two-sided markets literature, we also allow sellers to enjoy market power, so the platform provider shapes their competitive interaction through its election of royalty fees.

Our paper also builds on the literature on vertical relations regulated by secret contracts, with important contributions by Hart and Tirole (1990), O'Brien and Shaffer (1992), McAfee and Schwartz (1994) and Rey and Vergé (2004). Using their terminology, the upstream supplier in our setting has another type of customer with whom downstream firms interact, and such interaction is shaped by the upstream suppliers' decisions. This two-sidedness of the problem implies that there are cross-group network effects, so the issues and results are very different from this literature.

2. The model

We consider a model with 3 firms and a continuum of consumers. Firm 0 is a platform provider and produces a platform good (such as a video console) at a normalized marginal cost of zero. Firms $i = 1, 2$ are sellers of platform-specific products (such as video games). These products can only be used by consumers (e.g., gamers) who buy the platform. Sellers produce at zero marginal cost (again a normalization).

Consumers are uniformly spread on the positive real line and firm 0 is located at the left end. Given a consumer at distance $x \in [0, \infty)$ from the platform provider, consider her utility if she purchases one unit of the product sold by such a firm at price p_0 and purchases $q_i \geq 0$ units of the product sold by firm $i \in \{1, 2\}$ at price of p_i per unit. We assume (see Vives, 2001, for example) that such utility equals

$$U_x(p_0, p_1, q_1, p_2, q_2) = u(p_1, q_1, p_2, q_2) - x - p_0,$$

where

$$u(p_1, q_1, p_2, q_2) = \sum_{i=1}^{2} q_i - \frac{1}{2}\left(\sum_{i=1}^{2} q_i^2 + \theta \sum_{i=1}^{2} \sum_{j=1; j\neq i}^{2} q_i q_j\right) - \sum_{i=1}^{2} p_i q_i.$$

Parameter $\theta \in (-1, 1)$ captures the degree of complementarity/substitution between sellers' products. If $\theta < 0$, goods are complements, with their degree of complementarity decreasing with θ. If $\theta = 0$, goods are independent, whereas $\theta > 0$ implies that goods are substitutes, with their degree of substitutability increasing as θ grows.

We consider the following two-stage model. In the first stage, the platform provider offers contracts to sellers and sets a price p_0 for consumers. Sellers decide whether to accept the contract, and then consumers observe both p_0 and how many sellers have accepted the contract before having to decide whether to buy the platform good. In the second stage, sellers set prices for their products, and consumers decide how many products to buy.

Our timing reflects the fact that consumers use the platform for many periods, during which platform-specific products are continuously being launched. For instance, buyers of a video console often buy it without observing the prices charged for the games they will consume during the lifetime of the console.

A contract between seller $i \in \{1, 2\}$ and the platform provider consists of a fixed fee f_i and a per-unit royalty fee w_i.[4] If seller i accepts the contract and then sells Q_i units to consumers, its total payment to the platform provider is $f_i + w_i Q_i$.

We study several games, that differ in the variables that are observable to players. In Section 3, we study a one-sided market with public contracts. That is, we assume that $p_0 = 0$, and that consumers and sellers observe all contracts before making their decisions. In Section 4, we study a two-sided market with public contracts. This game is analogous to the previous one, except that we allow for $p_0 \neq 0$. Finally, in Section 5, we study a two-sided market with private contracts. We first examine a situation in which it is only consumers who do not observe any of the contracts offered to sellers, so they face an endogenously formed adverse selection problem when deciding whether or not to pay p_0. We then examine a situation in which consumers do not observe any of the contracts offered to sellers, and each seller only observes the contract it is offered by the platform provider. We assume throughout that p_0 is contractible and it is written in the contract offered to any seller.[5]

In Sections 3 and 4, we seek for symmetric subgame perfect equilibria (SPE). In Section 5, we seek for symmetric perfect Bayesian equilibria (PBE) given standard constraints on how off-the-equilibrium-path beliefs are formed.

3. Public contracts in a one-sided market

We start by studying the second stage. After observing p_i $(i = 1, 2)$, consumers who have purchased the platform good decide their demands for the sellers' products. Looking at interior solutions of a consumer's utility maximization problem

[4] Our main results do not depend on fixed fees being available. The proof is available on request.

[5] Even if p_0 is not contractible, reputational concerns may prevent the platform provider from cheating sellers. That p_0 is known by sellers when they have to decide whether to accept contracts is standard in some industries such as videogames (see Hagiu, 2006, for example). If p_0 were chosen after sellers have decided whether to accept the platform's offers, sellers would anticipate a hold-up problem that would harm the platform. Note also that it is in principle easier to contract upon p_0 than upon other seller's fees because sellers will eventually observe p_0, but they may never be able to observe the royalty fees that other sellers are paying.

yields the following *per-capita* demand for the product of seller i:

$$q_i(p_i, p_j) = \frac{1 - \theta - \; p_i + \theta p_j}{1 - \theta^2}. \tag{1}$$

Per-capita consumption does not depend on the distance between the consumer and the platform. Thus, the overall demand for seller i's product is $Q_i(p_i, p_j) = x_0 \, q_i(p_i, p_j)$, where x_0 is the number of consumers who choose to buy the platform good in the first stage. Seller $i \in \{1, 2\}$ solves the following problem given a price p_j by the other seller:

$$\max_{p_i} \; (p_i - w_i) \, Q_i(p_i, p_j) - f_i,$$

where f_i is a cost already sunk and the total number of consumers, x_0, is given from the first stage. Seller i's first-order condition is

$$x_0 \, (1 - \theta - 2 \, p_i + w_i + \theta \, p_j) = 0,$$

so its equilibrium price is

$$p_i(w_i, w_j) = \frac{(2 + \theta)(1 - \theta) + 2 \, w_i + \theta \, w_j}{(2 + \theta)(2 - \theta)}. \tag{2}$$

It readily follows from (1) that each consumer buys

$$q_i(w_i, w_j) = \frac{(1 - \theta)(2 + \theta) - w_i \, (2 - \theta^2) + \theta w_j}{(1 - \theta^2) \, (4 - \theta^2)} \tag{3}$$

units of product i ($i, j = 1, 2, \; i \neq j$).

We now turn to the analysis of the first stage. By symmetry, optimal royalties are such that $w_1 = w_2 = w$. Recall that in the one-sided market case, $p_0 = 0$. In the first stage, given w, the utility of consumer x is

$$U_x^o(w) = \frac{(1 - w)^2}{(1 + \theta)(2 - \theta)^2} - x,$$

where the superscript o refers to the one-sided, public contracts case. This results in a demand for the platform good equal to

$$x_0^o(w) = \frac{(1 - w)^2}{(1 + \theta)(2 - \theta)^2}.$$

Anticipating how play will evolve in the second stage, seller i will accept the contract offered by the platform if and only if $f_i \leq x_0(p_i - w_i) \, q_i$. Thus, the platform provider sets $f_i = x_0 \, (p_i - w_i) \, q_i$, and solves

$$\max_{w} \; x_0^o(w) \left[\sum_{i=1}^{2} p_i(w, w) q_i(w, w) \right]. \tag{4}$$

It is then easy to prove the following result.

Proposition 1. *If firm 0 provides a one-sided platform and contracts are publicly observed by all parties, then the equilibrium royalties equal*

$$w^o = \frac{3\theta - 2}{4},$$

so the equilibrium price charged by seller $i \in \{1, 2\}$ *is*

$$p_i^o = \frac{1}{4},$$

per-capita consumption of each product is

$$q_i^o = \frac{3}{4\,(1+\theta)},$$

and the number of consumers is

$$x_0^o = \frac{9}{16(1+\theta)}.$$

Finally, platform profits are

$$\pi_0^o = \frac{27}{128\,(1+\theta)^2},$$

and consumer surplus is

$$cs^o = \frac{81}{512\,(1+\theta)^2}.$$

Note that $w^o < 0$ for $\theta < 2/3$ and $w^o > 0$ for $\theta > 2/3$. To understand this result, note on the one hand that, given that the platform perfectly predicts second stage prices as a function of royalties, it can solve the problem in expression (4) as if it was choosing prices p_i instead of royalties w_i. The first-order condition with respect to price p_1 is:

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} + p_2 \frac{\partial q_2}{\partial p_1} \right) + \frac{\partial x_0}{\partial p_1} \left(p_1 q_1 + p_2 q_2 \right) = 0.$$

Seller 1, on the other hand, chooses price p_1 according to the following first-order condition:

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} - w_1 \frac{\partial q_1}{\partial p_1} \right) = 0.$$

When choosing a price, seller 1 ignores two effects: the effect a change in p_1 has on the per-capita demand of seller 2, and the effect it has on the number of consumers who join the platform. Note that the first effect is positive or negative depending on whether $\partial q_2 / \partial p_1$ is positive or negative, and the second effect is always negative.

It is straightforward to see that the platform can make seller 1 internalize these two effects by choosing an appropriate royalty fee. In particular, it should choose a royalty fee so that

$$-w_1 \frac{\partial q_1}{\partial p_1} = p_2 \frac{\partial q_2}{\partial p_1} + \frac{\partial x_0}{\partial p_1} \frac{p_1 q_1 + p_2 q_2}{x_0}.$$

When $\theta \leq 0$, the two terms on the right hand side are negative. Thus, the optimal royalty fee is negative. The royalty fee will be positive only if θ is positive and sufficiently large to overcome the negative effect of the change in the number of consumers joining the platform. This is precisely the result in Proposition 1.

4. Public contracts in a two-sided market

We now allow the platform to be priced at $p_0 \neq 0$. Second-stage decisions (for a given number of consumers and pair of royalty fees) are equivalent to those of the previous section (see expressions (2) and (3)). In the first stage, given $w_1 = w_2 = w$ and p_0, the utility of consumer x is

$$U_x^t(w, p_0) = \frac{(1-w)^2}{(1+\theta)(2-\theta)^2} - x - p_0,$$

where the superscript t refers to the two-sided, public contracts case. It follows that the demand for the platform good is

$$x_0^t(w, p_0) = \frac{(1-w)^2}{(1+\theta)(2-\theta)^2} - p_0.$$

As in the previous section, the platform provider sets $f_i = x_0 \left(p_i - w_i\right) q_i$, but now solves

$$\max_{w, p_0} \; x_0^t(w, p_0) \; \left[p_0 + p_1(w,w)\, q_1(w,w) + p_2(w,w)\, q_2(w,w)\right],$$

which leads to the following result.

Proposition 2. *If firm 0 provides a two-sided platform and contracts are publicly observed by all parties, then the equilibrium royalties equal*

$$w^t = -(1-\theta) < 0,$$

and the equilibrium price for the platform equals

$$p_0^t = \frac{1}{2(1+\theta)} > 0.$$

The equilibrium price charged by seller $i \in \{1, 2\}$ is

$$p_i^t = 0,$$

per-capita consumption of each product is

$$q_i^t = \frac{1}{1+\theta},$$

and the number of consumers is

$$x_0^t = \frac{1}{2(1+\theta)}.$$

Finally, platform profits are

$$\pi_0^t = \frac{1}{4(1+\theta)^2},$$

and consumer surplus is

$$cs^t = \frac{1}{8(1+\theta)^2}.$$

Note that, in contrast with the previous case, in this case the optimal royalty fee is always negative, and it goes to zero as θ goes to one. To understand this result, we can proceed in a similar way as before. We start by noting that the first-order condition of the platform with respect to price p_0 is:

$$x_0 + \frac{\partial x_0}{\partial p_0}\left(p_0 + p_1 q_1 + p_2 q_2\right) = 0.$$

If the platform acts as if it was choosing price p_1 instead of royalty fee w_1, it would choose price p_1 according to the following first-order condition:

$$x_0\left(q_1 + p_1\frac{\partial q_1}{\partial p_1} + p_2\frac{\partial q_2}{\partial p_1}\right) + \frac{\partial x_0}{\partial p_1}\left(p_0 + p_1 q_1 + p_2 q_2\right) = 0.$$

The first-order condition with respect to p_0 implies that

$$p_0 + p_1 q_1 + p_2 q_2 = x_0,$$

given that $\partial x_0/\partial p_0 = -1$. Since Roy's identity implies that $\partial x_0/\partial p_1 = -q_1$, the first-order condition becomes:

$$x_0\left(p_1\,\frac{\partial q_1}{\partial p_1} + p_2\,\frac{\partial q_2}{\partial p_1}\right) = 0.$$

In a symmetric equilibrium:

$$x_0\; p_1\left(\frac{\partial q_1}{\partial p_1} + \frac{\partial q_2}{\partial p_1}\right) = 0,$$

so it is optimal to set a royalty that induce sellers to sell their products at a price of zero. Given that seller 1 chooses price p_1 so that

$$x_0\left(q_1 + p_1\,\frac{\partial q_1}{\partial p_1} - w_1\,\frac{\partial q_1}{\partial p_1}\right) = 0,$$

the royalty fee must be negative so that sellers choose prices equal to zero. Finally, note that prices go to marginal cost as $\theta \to 1$ due to pure Bertrand competition, so the royalty converges to zero as products become perfect substitutes. Hence the results in Proposition 2.

5. Private contracts

In this section, we assume contracts between the platform and the sellers are private. Thus, consumers cannot observe any of the contracts offered to sellers, and a seller can only observe the contract it is offered. We will seek for symmetric Perfect Bayesian Equilibria (PBE) given standard constraints on how off-the-equilibrium-path beliefs are formed.[6] In what follows, let p_0^* denote the price charged to consumers by the platform provider in a symmetric PBE. Also, let w^* denote the royalty fee that is offered to seller $i \in \{1, 2\}$ in a symmetric PBE, and f^* the associated fixed fee.

Regarding the formation of out-of-equilibrium beliefs, note that, upon observing any $p_0 \neq p_0^*$, rational consumers would realize that such a deviation affects sellers' profits and potentially their incentives to enter the market (this happens when $p_0 > p_0^*$). They should therefore conclude that a price deviation must be accompanied by a change in the fixed fee and/or a change in the royalty fee offered to each seller. We will look at equilibria in which consumers rationalize any price deviation by conjecturing that there was no deviation in the royalty fee offered to each seller; hence, consumers believe upon observing $p_0 \neq p_0^*$ that the platform is simply adjusting the fixed fee offered to each seller just to make it break-even given w^*. These beliefs are in the spirit of "passive beliefs" (Hart and Tirole, 1990), but they require some rationability by consumers. In particular, when consumers observe a price deviation, they acknowledge that this should have had an impact on the sellers' willingness to accept the contract, and they reason that the absence of such an impact must be due to a change in the fixed fee offered to each seller. We refer to this weak form of passive beliefs held by consumers as "weakly passive beliefs."[7] Note that the main implication of such belief formation

[6]No asymmetric equilibrium exists, so the symmetry requirement is without loss of generality, at least if one restricts attention to equilibria in which the pricing strategy and beliefs held by a seller are polynomial functions of the royalties it observes.

[7]The outcome would be the same under the standard strong form of passive beliefs (corresponding to situations in which consumers do not change their equilibrium beliefs when observing out-of-equilibrium behavior). However, it would be harder to interpret some situations. For example, upon observing $p_0 > p_0^*$, a consumer who kept her beliefs about f^* and w^* should conclude that the sellers are accepting a contract that yields negative profits, for consumer demand is smaller than it should be in equilibrium (since we shall show later on that consumer

is that consumers always expect the interaction of sellers in the product market to be unaffected by the choice of p_0.

Because a seller anticipates such unsophisticated behavior by consumers when $p_0 \neq p_0^*$, it believes that $p_0 \neq p_0^*$ conveys no information about contract offers. Thus, sellers therefore form passive beliefs with respect to deviations in p_0. However, seller $i \in \{1,2\}$ is assumed to form "wary beliefs" (McAfee and Schwartz, 1994; Rey and Vergé, 2004) when it observes an unexpected contract offer. In such cases, it believes that the platform provider must have made an offer to $j \in \{1,2\}$ ($j \neq i$) that maximizes the platform's total profit given the price that it charges to consumers and the contract offered to seller $i \in \{1,2\}$. Of course, in equilibrium, a seller anticipates perfectly the offer made by the platform to the other seller, but the formation of wary beliefs by sellers implies that, if the platform deviates from equilibrium play, then sellers will correctly infer how it is deviating. We also assume that a seller that forms wary beliefs conjectures that the other seller also does, and also conjectures that the platform provider does not want to drive any seller out of the market.

The two-sidedness arises because, when deciding whether to accept the platform provider's offer, seller $i \in \{1,2\}$ cares about how many consumers will join the platform. Thus, there are indirect network effects between buyers and sellers. Moreover, consumers who contemplate purchasing the platform care not only about p_0 but also about the (foreseen) royalties charged by the platform provider to sellers. Thus, the platform's price structure has a non-trivial effect on membership decisions and the level of transactions.

5.1. **Contracts observable to sellers, but unobservable to consumers.** Before examining equilibrium play when the contract offer received by a seller is solely observed by such a seller, it is useful to examine an intermediate case in which sellers observe each other's contract, but consumers do not. As we shall see next, such unobservability gives rise to an endogenously formed adverse selection problem: consumers will (correctly) believe that the platform provider will induce sellers to charge high (collusive) prices. Sellers will earn more for each consumer that joins the platform, but the platform's value for consumers will be harmed by such beliefs. Both these forces induce the platform provider to lower access prices for buyers, thereby setting a negative markup on them.

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demand for the platform does not affect competition between sellers, which solely depends on royalty fees).

To see these issues formally, let us denote the contract offered to each seller in equilibrium by $(\widehat{f}, \widehat{w})$. Because consumers cannot observe deviations from this contract and form weakly passive beliefs when observing any p_0, their demand for the platform when observing price p_0 equals

$$x_0(p_0, \widehat{w}) = \frac{(1-\widehat{w})^2}{(1+\theta)(2-\theta)^2} - p_0.$$

Taking into account that the platform provider extracts all the surplus from the sellers, it follows that it chooses p_0, w_1 and w_2 to maximize

$$x_0\ (p_0, \widehat{w})[p_0 + p_1(w_1, w_2)q_1(w_1, w_2) + p_2(w_2, w_1)q_2(w_2, w_1)].$$

The first-order condition corresponding to w_i is as follows:

$$\frac{\theta(1-\theta)(2+\theta)^2 - (8-6\theta^2)w_i + 2\theta^3 w_j}{(1-\theta^2)(4-\theta^2)^2} = 0\ (i, j = 1, 2;\ i \neq j).$$

Rearranging this equation allows us to give it an interpretation that will be useful later on: when seller i receives an offer involving royalty fee w ($i \in \{1, 2\}$), it infers that the platform provider finds it optimal to charge seller $j \in \{1, 2\}$ ($j \neq i$) with a royalty fee equal to

$$\widehat{w}^*(w) = \frac{\theta\,(1-\theta)\,(2+\theta)^2 + 2\,\theta^3\,w}{2\,(4-3\,\theta^2)}. \tag{5}$$

As a result, $\widehat{w}^*(\cdot)$ can be interpreted as a seller's belief about the royalty fee offered to the other seller. Such a belief is correct both on and off the equilibrium path because the platform anticipates that sellers will have complete information when pricing, so there is no way to fool them. The function $\widehat{w}^*(\cdot)$ will serve as a useful benchmark when we further assume in the next subsection that sellers cannot observe each other's contract offers.

To fully solve the model, it is easy to show that in equilibrium it must hold that

$$\widehat{w} = \frac{\theta}{2}.$$

Thus, the royalty fee is positive if $\theta > 0$ and negative if $\theta < 0$. The first-order condition corresponding to p_0 can be written as

$$\frac{(2-\theta)^2}{4\,(1+\theta)\,(2-\theta)^2} - \frac{1}{2\,(1+\theta)} - 2p_0 = 0,$$

so

$$\widehat{p}_0 = -\frac{1}{8\,(1+\theta)} < 0.$$

In equilibrium, the platform induces seller i to charge price

$$\widehat{p}_i = \frac{1}{2} > 0$$

and gains

$$\widehat{\pi}_0 = \frac{9}{64\,(1+\theta)^2}.$$

To understand these results, we can proceed as in the previous sections. If the platform acts as if it was choosing price p_1 instead of royalty fee w_1, it would choose price p_1 according to the following first order condition:

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} + p_2 \frac{\partial q_2}{\partial p_1} \right) = 0.$$

Note that this first-order condition differs from that of the previous case because consumers do not observe changes in royalty fees, so their decision to buy the platform good depends only on their beliefs about the equilibrium royalty. In a symmetric equilibrium, it holds that

$$-\left(\frac{\partial q_1}{\partial p_1} + \frac{\partial q_2}{\partial p_1} \right) p_i = q_i.$$

Thus, the optimal implied price for sellers is positive. This contrasts with the result in the public contracts case, in which the optimal price was zero.

It is easy to see that the optimal price p_0 solves

$$p_0 = \frac{U(p_1, p_2) - p_1 q_1 - p_2 q_2}{2}.$$

This equation shows that the platform has incentives to lower p_0, in comparison with the public contracts case, for two reasons: to compensate the decrease in consumer surplus from consumption of seller goods ($U(p_1, p_2) < U(0,0)$), and because seller surplus per consumer increases ($p_1 q_1 + p_2 q_2 > 0$). In the case at hand, it turns out that the platform finds it optimal to set a negative access fee for consumers.

Finally, note that seller i chooses price p_i so that

$$x_0 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} - w_1 \frac{\partial q_1}{\partial p_1} \right) = 0.$$

Thus, the royalty needs to be positive if the cross-price effect $\partial q_2 / \partial p_1$ is positive, and negative if the cross-price effect is negative.

Summarizing, we find that when consumers do not observe royalty fees, they are less reactive to changes in the intensity of competition between sellers, since they cannot observe deviations from the royalty fees they expect in equilibrium.

As a consequence, the platform has incentives to behave opportunistically, and choose royalties to induce collusive pricing by sellers. Consumers correctly foresee an adverse selection problem when it comes to getting access to the platform, so their utility from having access to the platform decreases. The platform has incentives to lower access prices for consumers for two reasons: to compensate the lower demand for platform access, and because seller revenue per consumer increases.

5.2. **Contracts unobservable to sellers and consumers.** We now turn to the analysis of the cases in which the contract offer received by a seller is solely observed by such a seller, starting with the second stage.

At the beginning of the second stage, seller $i \in \{1, 2\}$ knows p_0, x_0, f_i and w_i, and has to choose a price for its product based on this information. Taking into account that seller i's overall demand product equals $Q_i(p_i, p_j) \equiv x_0 q_i(p_i, p_j)$, we can solve for the second-stage subgames. Recalling that we are examining symmetric equilibria, let $B(\widehat{w})$ denote the belief formed by seller $i \in \{1, 2\}$ about the royalty fee paid by seller $j \in \{1, 2\}$ $(j \neq i)$ to the platform provider.[8] We follow Rey and Vergé (2004), and restrict attention to equilibria in which seller i's belief about the royalty fee paid by the other seller does not depend on the fixed fee it observes. Not only is the pricing strategy of seller $i \in \{1, 2\}$ independent from the fixed fee it already paid, but it is also independent from p_0 (and hence from x_0). Such a price has no signaling role and it does not affect belief formation, which seems a reasonable assumption given that x_0 is simply a scaling factor in seller i's second-stage profit.[9]

In what follows, let $p_i(w_i)$ denote the strategy of seller $i \in \{1, 2\}$ in the second-stage subgame if it has observed an offer of (w_i, f_i) and price p_0. Having observed this, seller $i \in \{1, 2\}$ chooses p_i to maximize $(p_i - w_i)Q_i(p_i, p_j(B(w_i))) - f_i$ $(j \in \{1, 2\};\ j \neq i)$ with f_i already sunk, so its first-order condition is

$$1 - \theta + w_i - 2p_i(w_i) + \theta p_j(B(w_i)) = 0. \tag{6}$$

We now turn to analyzing the first stage of play. Regardless of the price p_0 that consumers observe, they believe that seller $i \in \{1, 2\}$ is charged a royalty fee of

[8]Because we are looking at symmetric equilibria, the belief function $B(\cdot)$ does not depend on the label of the seller receiving the unexpected offer. Note that, in general, $B(\cdot)$ is an unrestricted function except for the constraint that $B(w^*) = w^*$ (i.e., conjectured beliefs are fulfilled along the equilibrium path). In our case, we will restrict the function so that beliefs be wary.

[9]Therefore, it does not affect equilibrium pricing in the second-second if sellers believe that it does not convey some information, making it self-fulfilling that it is pointless for the platform provider to use it for signaling purposes.

w^*, so they expect a price

$$p_i^* = \frac{1-\theta+w^*}{2-\theta}$$

for each unit they purchase from seller $i \in \{1,2\}$ in the second stage. Given price p_0, the overall utility expected by consumer x equals

$$U_x(w^*, p_0) = \frac{(1-w^*)^2}{(1+\theta)\,(2-\theta)^2} - x - p_0,$$

so the demand for the platform good is

$$x(w^*, p_0) = \frac{(1-w^*)^2}{(1+\theta)\,(2-\theta)^2} - p_0.$$

The platform provider's total profit if it charges p_0 and makes a private offer of (w_1, f_1) and (w_2, f_2) to sellers 1 and 2, respectively, is as follows:

$$\begin{aligned}\pi_0(w_1, f_1, w_2, f_2, p_0) &= x(p_0)\,[p_0 + w_1\,q_1(p_1(w_1), p_2(w_2)) \\ &\quad + w_2\,q_2(p_1(w_1), p_2(w_2))] + f_1 + f_2,\end{aligned}$$

since the platform provider can perfectly forecast actual sales made by sellers 1 and 2. In order for seller 2 (say) to form wary beliefs, the inference made by such a seller about seller 1's contract upon observing a price of p_0 and an offer of (w_2, f_2) must be such that $B(w_2)$ maximizes $\pi_0(w, f, w_2, f_2, p_0)$ with respect to w and f subject to the constraint that $f \leq (p_1(w) - w)x(p_0)q_1(p_1(w), p_2(B(w)))$. Taking into account that the constraint must bind at the optimum and that

$$q_1(p_1(w), p_2(B(w))) = \frac{p_1(w) - w}{1-\theta^2}$$

by condition (6) yields that

$$B(w_2) \in \underset{w}{\operatorname{argmax}}\ \pi_0(w, w_2, f_2, p_0), \tag{7}$$

where

$$\begin{aligned}\pi_0(w, w_2, f_2, p_0) &= x(p_0)\,\Bigg\{p_0 + w\,q_1(p_1(w), p_2(w_2)) \\ &\quad + w_2\,q_2(p_2(w_2), p_1(w)) + \frac{[p_1(w) - w]^2}{1-\theta^2}\Bigg\} + f_2.\end{aligned}$$

Maximizing $\pi_0(w, w_2, f_2, p_0)$ with respect to w yields the following first-order condition:

$$
\begin{aligned}
0 \quad = \quad & q_1(p_1(w), p_2(w_2)) + \frac{2[p_1(w) - w]}{1-\theta^2}\left(\frac{dp_1(w)}{dw} - 1\right) \qquad (8) \\
& + \left[w\frac{\partial q_1(p_1(w), p_2(w_2))}{\partial p_1} + w_2\frac{\partial q_2(p_2(w_2), p_1(w))}{\partial p_1}\right]\frac{dp_1(w)}{dw}.
\end{aligned}
$$

Since our purpose at this stage is to build some intuition, let us assume for now that a unique solution to equation (8) exists for any w_2, denote it by $w_1^*(w_2)$, and note that it must coincide with $B(w_2)$ even if $w_2 \neq w^*$ because sellers form wary beliefs even when off the equilibrium path. Using the implicit function theorem, we obtain the following result:

$$
\frac{dB(w_2)}{dw_2} = \frac{dw_1^*(w_2)}{dw_2} = -\frac{\dfrac{\theta}{1-\theta^2}\left(\dfrac{dp_2(w_2)}{dw_2} + \dfrac{dp_1(w)}{dw}\right)}{\partial^2\pi_0(w, w_2, f_2, p_0)/\partial w^2}.
$$

If $\pi_0(w, w_2, f_2, p_0)$ is strictly concave with respect to w (as we shall later show), symmetry yields that

$$
\text{sign}\left(\frac{dB(w)}{dw}\right) = \text{sign}\left(\theta\frac{dp(w)}{dw}\right).
$$

Whenever it holds that $dp(w)/dw > 0$, which is an intuitive property that equilibrium prices should satisfy,[10] we have that $dB(w)/dw \gtreqless 0$ if and only if $\theta \gtreqless 0$, according well with what one may have expected. Sellers' prices are strategic complements if $\theta > 0$ and strategic substitutes otherwise (provided goods are not independent), and the platform provider aims at softening competition between sellers under strategic complementarity and at toughening such competition under strategic substitutability.

Having shed some light on some of the properties that the equilibrium satisfies, we proceed to showing existence and characterizing it. To this end, evaluating the first-order condition at $w = B(w_2)$ (recall condition (7)) and letting $p_i(w) = p(w)$ because of symmetry yields that the following equation must hold:

$$
\begin{aligned}
0 \quad = \quad & 1 - \theta - p(B(w_2)) + \theta p(w_2) + (\theta w_2 - B(w_2))\frac{dp(B(w_2))}{dw} \qquad (9) \\
& + 2[p(B(w_2)) - B(w_2)][\frac{dp(B(w_2))}{dw} - 1].
\end{aligned}
$$

[10] Note that we shall restrict attention to polynomial pricing strategies, and that in such cases there is no loss in further restricting them to be affine.

If one focuses on PBE such that $p(\cdot)$ and $B(\cdot)$ be polynomial functions, then Rey and Vergé (2004) show that there is no loss of generality in restricting attention to affine functions, so one can readily solve the system of differential equations given by (9) and (6) (after dropping subscripts) to obtain the following result.

Proposition 3. *The unique symmetric PBE in which $p(w)$ and $B(w)$ are polynomial functions is such that $p(w) = \Theta_\theta + \Sigma_\theta w$ and $B(w) = \Gamma_\theta + \Phi_\theta w$ for some constants $\Theta_\theta \in [0, 1]$, $\Sigma_\theta \in [\frac{1}{2}, 1] > 0$, $\Gamma_\theta \in [0, 1]$, and $\Phi_\theta \in [-1, 1]$. In such an equilibrium, it always holds that*

$$p_i^* = \Theta_\theta + \frac{\Sigma_\theta \Gamma_\theta}{1 - \Phi_\theta} \geq 0 \ (i = 1, 2)$$

and $w^* \geq 0$ for any $\theta \in (-1, 1)$, with $w^* = 0$ if and only if $\theta = 0$. Also, platform profits are

$$\pi_0^* = \left(\frac{1 - (p_i^*)^2}{2(1 + \theta)} \right)^2,$$

and consumer surplus is

$$cs^* = \frac{1}{2} \left(\frac{1 - (p_i^*)^2}{2(1 + \theta)} \right)^2.$$

Proof. See Appendix A. ■

Contrary to the case in which sellers can observe each other's royalties (subsection 5.1), royalty fees are never negative under private contracting, regardless of whether price competition between sellers displays strategic complementarity ($\theta > 0$) or strategic substitutability ($\theta < 0$). When sellers can observe each other's royalties, $\theta > 0$ implies that $d\widehat{w}^*(w)/dw > 0$ (see expression (5)), so an increase in the royalty fee a seller observed would (correctly) make it believe that the other seller's royalty offer must have increased, since the platform aims at softening competition, and hence in equilibrium $\widehat{w} = \theta/2 > 0$; the converse happens if $\theta < 0$ (so that $d\widehat{w}^*(w)/dw < 0$), with $\widehat{w} = \theta/2 < 0$ in these cases because the platform wishes to toughen competition. When sellers cannot observe each other's offers, their beliefs become more sensitive to observed royalties. This overreaction to changes in the royalty fee observed is a straightforward effect of the wary beliefs formed by sellers in face of opportunistic contracting by the platform. Figure 1 plots $d\widehat{w}^*(w)/dw$ (see solid curve) relative to $dB(w)/dw$ (see dashed curve) as parameter θ varies.

The determinants of how the equilibrium royalty fee relates to θ are different when sellers can observe each other's royalty offers and when they cannot. When they can observe them as in subsection 5.1, the platform's incentives to deviate

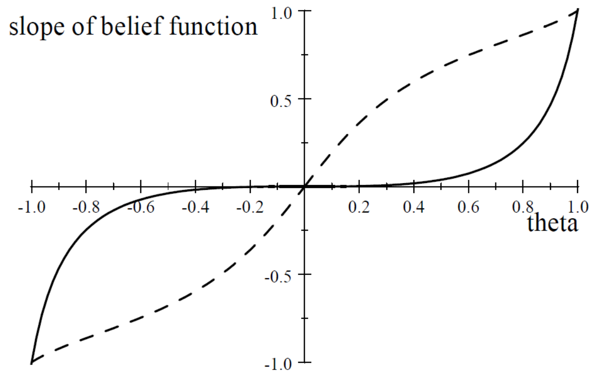

FIGURE 1. Comparison of beliefs

have to do with making competition between sellers softer (if $\theta > 0$) or tougher (if $\theta < 0$), as we just mentioned. When sellers cannot observe each other's royalty offers, the platform's incentives to deviate greatly depend on how a seller that receives an unexpected offer believes it is being treated. In particular, such a seller (correctly) infers that the platform must be simultaneously deviating with the other seller in a way that the opportunistic platform does not care about seller 2's profitability. Indeed, taking into account that the platform extracts all the surplus that seller i expects to make when observing royalty fee w_i, it holds that the payoff to the platform if it chooses w_1, w_2 and p_0 equals

$$\begin{aligned}
\widehat{\pi}_0(w_1, w_2, p_0) &= x(p_0)\Big\{p_0 + w_1\, q_1(p_1(w_1), p_2(w_2)) + w_2\, q_2(p_2(w_2), p_1(w_1)) \\
&\quad + [p_1(w_1) - w_1]\, q_1(p_1(w_1), p_2(B(w_1))) \\
&\quad + [p_2(w_2) - w_2]\, q_2(p_2(w_2), p_1(B(w_2)))\Big\} \\
&= x(p_0)\Big\{p_0 + w_1\, q_1(p_1(w_1), p_2(w_2)) + w_2\, q_2(p_2(w_2), p_1(w_1)) \\
&\quad + [p_1(w_1) - w_1]\, q_1(p_1(w_1), p_2(w_2)) \\
&\quad + [p_1(w_1) - w_1]\, \big[q_1(p_1(w_1), p_2(B(w_1))) - q_1(p_1(w_1), p_2(w_2))\big] \\
&\quad + [p_2(w_2) - w_2]\, q_2(p_2(w_2), p_1(B(w_2)))\Big\}.
\end{aligned}$$

Clearly, maximizing this payoff with respect to w_1 is equivalent to maximizing

$$\begin{aligned}
\widehat{\pi}_0'(w_1, w_2) &= [p_1(w_1) - w_1]\, q_1(p_1(w_1), p_2(w_2)) \\
&\quad + w_1\, q_1(p_1(w_1), p_2(w_2)) + w_2\, q_2(p_2(w_2), p_1(w_1)) \\
&\quad + [p_1(w_1) - w_1]\, \big[q_1(p_1(w_1), p_2(B(w_1))) - q_1(p_1(w_1), p_2(w_2))\big],
\end{aligned}$$

so the platform cares about seller 1's actual profit, the actual royalty revenue generated by each seller and the change in seller 1's profit because of the formation of wary beliefs. By the envelope theorem, seller 1's actual profit when w_1 varies a bit is equal to $-q_1(p_1(w_1), p_2(w_2))$, so $w^* = B(w^*)$ implies that

$$\left.\frac{\partial \widehat{\pi}_0'(w_1, w_2)}{\partial w_1}\right|_{w_1=w_2=w^*} = 0$$

is equivalent to

$$\left\{[p(w^*) - w^*]\,\theta\,\frac{dB(w^*)}{dw} - (1-\theta)\,w^*\right\}\frac{dp(w^*)}{dw} = 0.$$

The fact that $p(w^*) > w^*$ then implies that

$$w^* = \frac{\theta}{1-\theta}\,\frac{dB(w^*)}{dw}\,[p(w^*) - w^*]$$

must be nonnegative because we showed earlier that $\theta(dB(w^*)/dw) \geq 0$.

As we have shown, the sign of w^* depends on how the second argument of $q_1(p_1(w_1), p_2(B(w_1)))$ varies with w_1, that is, on whether an increase in w_1 will stimulate seller 1's sales via the conjectured price change performed by seller 2. Because seller 1 always believes that this is indeed the case, w^* is always nonnegative. When sellers can observe each other's offer, we showed in subsection 5.1 that the equilibrium royalty fee is positive if and only if competition between sellers displays strategic complementarity. Figure 2 compares royalty fees in the two models (the dashed curve corresponds to the case of private contracting).

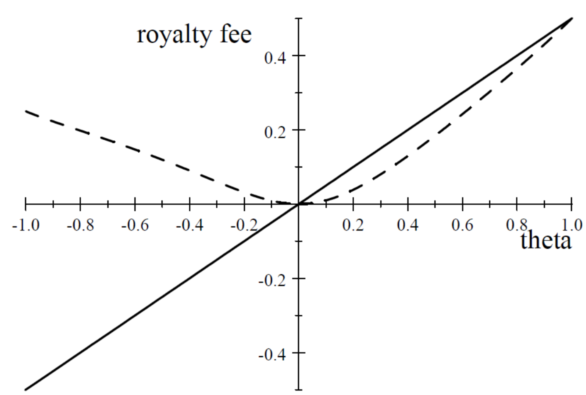

FIGURE 2. Comparison of royalty fees

Because $w^* < \widehat{w}$ if and only if $\theta > 0$, it should come as no surprise that the comparison of sellers' prices in both situations is as illustrated by Figure 3 (the dashed curve represents the situation when seller cannot observe each other's offer).

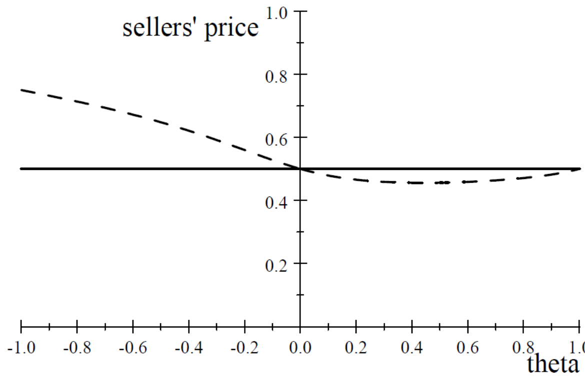

FIGURE 3. Comparison of seller prices

Relative to when sellers can observe each other's contract, it holds when they cannot that the platform provider loses part of its market power vis-à-vis sellers because of its opportunistic behavior when dealing with each on a one-on-one basis (as in Rey and Vergé, 2004). This smaller market power implies that the platform provider cannot sufficiently raise sellers' prices through the royalty fees when goods are substitutes; when goods are complements, the smaller market power of the platform provider implies that it cannot sufficiently lower prices charged by sellers so as to mitigate the double marginalization problem first pointed out by Cournot (1838) for the case of perfect complements.

The difference in pricing by sellers illustrated by the previous figure has key implications for platform pricing, since one of the two determinants of platform demand is how much utility consumers expect to attain given the anticipated pricing by sellers. When $\theta < 0$, consumers correctly anticipate that sellers will charge higher prices when they cannot observe each other's offer than when they can, so the platform provider has an incentive to lower the platform's price relative to when sellers can observe each other's offer. When $\theta > 0$, the sellers charge lower prices when they cannot observe each other's offer than when they can, so the platform provider has an incentive to raise the platform's price relative to when sellers can observe each other's offer.

The other determinant of platform pricing is how much overall profit is generated per consumer through the two sellers. Figure 4 shows how total profit generated by sellers per customer varies with θ (the dashed curve represents the situation when seller cannot observe each other's offer).

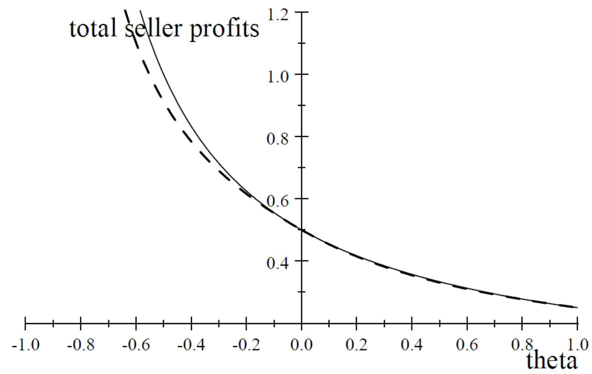

FIGURE 4. Comparison of seller profits per customer

Because sellers are induced to price collusively when they can observe each other's offer, it holds that per-consumer profitability is at least as large as when they cannot observe each other's offer. This implies that, regardless of the value of θ, the platform provider has an incentive to set a higher price for the platform when sellers cannot observe each other's offer than when they can. Interestingly, note that the incentive is very small when $\theta > 0$: in such cases, the platform provider's opportunistic behavior is hardly costly in terms of generating sellers' profits. The effect highlighted by Rey and Vergé (2004) is present, but it is not very strong.

Overall, we find that pricing by the platform is driven by the anticipated effect of sellers' prices on consumer utility. On the one hand, when $\theta > 0$, the platform provider prices higher when sellers cannot observe each other's offer than when they can: the effect on consumer demand of having lower prices dominates the effect of appropriating less profit through sellers. On the other hand, when $\theta < 0$, the effect of having lower consumer utility when sellers cannot observe each other's offer always dominates the lower per-consumer profitability that arises when sellers cannot observe each other's offer. This is illustrated by Figure 5 (the dashed curve represents the situation when seller cannot observe each other's offer).

It should then not be very surprising that platform profits are greater when sellers cannot observe each other's offer than when they can if and only if $\theta > 0$, as the Figure 6 shows (the dashed curve represents the situation when seller cannot observe each other's offer).

A similar result holds for consumer and total welfare, since they are proportional to platform profits both when sellers cannot observe each other's offer and when they can.

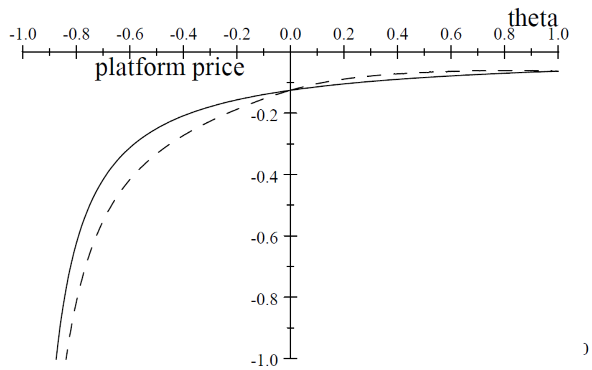

FIGURE 5. Comparison of consumer access prices

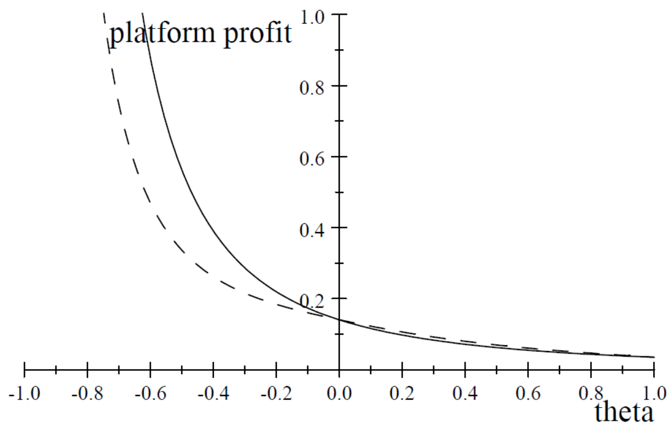

FIGURE 6. Comparison of platform profit

We now turn to our main result. In particular, the following proposition shows the effects of private contracts in a two-sided market by comparing the equilibrium of this subsection with the equilibria of the previous two sections.

Proposition 4. *Equilibrium royalties can be positive or negative in a one-sided market with public contracts, are negative in a two-sided market with public contracts, and are positive in a two-sided market with private contracts. The price of the platform good for consumers is positive in a two-sided market with public contracts, and is negative in a two-sided market with private contracts. Comparing two-sided market models, private contracts lead to lower profit, consumer surplus, and welfare.*

The first two claims in the proposition follow from comparing the equilibria of the models in Sections 3, 4, and 5. The proof for the last claim is included in the proof of Proposition 3.

6. Policy implications

6.1. **Most-favored nation and resale price maintenance clauses.**

Contracts between platform and sellers in two-sided markets often use Most-Favored Nation (MFN) or Resale Price Maintenance (RPM) clauses.[11] We next study the welfare implications of each instrument.

Suppose first that it is commonly known that the platform provider includes MFN clauses in their contracts with sellers. This is a way in which the platform provider commits not to price discriminate between sellers, so the situation corresponds to McAfee and Schwartz's (1994) "symmetry beliefs," that is, $B(w) = w$. It can be shown in these cases that results are identical to Section 5.1, so the introduction of MFN clauses makes the platform and consumers worse off when goods are substitutes, but it makes all of them better off when goods are complements.

Proposition 5. *Relative to when MFN clauses are not used, the introduction of such clauses increases total welfare if and only if* $\theta < 0$.

Suppose now that the platform is known to include RPM clauses in its contract with sellers. The best the platform can do is to replicate the results in Section 4 in order to make the adverse selection problem disappear and thus alleviate consumers' fears, so it is dominant for the platform to include price ceilings at the level of 0. Doing so makes not only the platform better off, but also consumers, which leads to our next result.

Proposition 6. *Relative to when RPM clauses are not used, the introduction of such clauses always increases total welfare.*

Our results suggest that RPM (in the form of imposing price ceilings) is advantageous to the platform and to consumers as well because it commits to low prices by sellers and hence stimulates platform adoption.

6.2. **Integration between platform and sellers.**

A natural alternative to contracting with sellers is to acquire one or both. Clearly, integration with both

[11] See http://www.theverge.com/2015/5/19/8621581/sony-music-spotify-contract for details of the secret contract signed between music streaming platform Spotify and record company Sony. Such contract contained MFN clauses.

sellers allows the platform to commit to the prices charged by sellers and is therefore a Pareto improvement that results in a situation such as the one in Section 4. However, the incentive to integrate need not be monotonic in the number of sellers acquired by the platform provider. To this end, consider what happens if the platform owns seller 1. In the second stage, the platform will choose p_1 to maximize $p_1 q_1(p_1, p_2) + w_2 q_2(p_2, p_1)$, whereas seller 2 will choose p_2 to maximize $(p_2 - w_2) q_2(p_2, p_1)$. It readily follows that prices as a function of w_2 are

$$p_1(w_2) = \frac{(\theta+2)(1-\theta) + 3\theta w_2}{(2-\theta)(2+\theta)}$$

and

$$p_2(w_2) = \frac{(\theta+2)(1-\theta) + (2+\theta^2) w_2}{(2-\theta)(2+\theta)}.$$

If consumers expect that seller 2 pays a royalty fee of w_2^I, then their demand equals

$$x_0(w_2^I, p_0) = \frac{2(2+\theta)(2+\theta) - 2(1+\theta)(2+\theta)^2 w_2^I + (1+\theta)(4+5\theta^2)(w_2^I)^2}{2(1+\theta)(2-\theta)^2(2+\theta)^2} - p_0.$$

Seller 2 anticipates earning $(p_2^*(w_2) - w_2) x_0(w_2^I, p_0) q_2(p_2^*(w_2), p_1^*(w_2)) - f_2$, so the platform chooses w_2 and p_0 to maximize

$$x_0(w_2^I, p_0)[p_0 + p_1^*(w_2) q_1(p_1^*(w_2), p_2^*(w_2)) + p_2^*(w_2) q_2(p_2^*(w_2), p_1^*(w_2))].$$

It is easy to show that

$$w_2^I = \frac{\theta(2+\theta)^2}{2(4+5\theta^2)},$$

which is positive if and only if $\theta > 0$. Also,

$$p_0^I = -\frac{8 - 4\theta + 13\theta^2 + \theta^3}{16(1+\theta)(4+5\theta^2)} < 0$$

and

$$x_0^I = \frac{24 + 4\theta + 23\theta^2 + 3\theta^3}{16(1+\theta)(4+5\theta^2)} > 0,$$

so

$$\pi_0^I = \frac{(24 + 4\theta + 23\theta^2 + 3\theta^3)^2}{256(1+\theta)^2(4+5\theta^2)^2}.$$

Comparing this with π_0^*, we have that the platform would have an incentive to integrate with seller 1 (say) if and only if $\theta < 0$. When $\theta > 0$, the consumers' fear of being taken advantage of is more mitigated by private contracting than by the platform's control of one of the sellers. In this sense, the loss in the platform's market power vis-à-vis sellers is preferred by consumers over a stricter (but not full) control of sellers' pricing behavior.

The following proposition summarizes all our results regarding integration.

Proposition 7. *Integration with both sellers makes the platform and consumers better off, so it is always socially desirable. Integration with just one seller makes the platform and consumers better off if $\theta < 0$ and worse off otherwise, so it is socially desirable if and only if $\theta < 0$.*

6.3. **The agency model of sales.** Even though we have abstracted away from any costs of (full) integration, such a move may be rather costly, and there is a contractual alternative that achieves the same outcome. Such an alternative involves the platform provider contracting upon the prices to be charged by sellers and committing to them when consumers have to decide whether to acquire the platform. This contractual approach would roughly correspond to what is known as the "agency model" of sales (used for example by Apple).[12] In the current case, the platform would find it dominant to force sellers to sell at a price of 0 and thus obtain the same outcome as in Section 4. Referring to the approach followed by the platform in Section 4 as the wholesale model of sales, we therefore have the following result.

Proposition 8. *Platform profits and consumer welfare are both higher when the platform controls sellers' prices directly than when it simply controls their royalty fees, so the agency model of sales is socially preferred over the wholesale model of sales.*

7. Concluding remarks

When contracts between the platform and sellers are private rather than public, we have shown that the pricing structure is basically driven by consumers' fear of being taken advantage of when purchasing the platform. Transparency is beneficial because it allows the platform to commit not to trick consumers into purchasing a platform that will have expensive goods sold by sellers. Another way to make adverse selection disappear is to contract on sellers' prices (as in iTunes). Both the platform and consumers would benefit from such price-forcing contracts. Also, we have shown that integration of the platform with one of the

[12]The agency model of sales is a business model through which sellers act as sale agents of the platform (Johnson, 2014). It is not to be confused with a Principal-Agent problem. In particular, note that in this section we assume that information is symmetric.

sellers is harmful when sellers sell substitutes, but going all the way and integrating with both sellers would allow the platform to get back to the public contracting outcome and do better without harming consumers.

Our results show that giving more market power to a dominant platform (in the form of making private contracts public or allowing for forcing contracts or RPM clauses) may make everybody better off because it removes informational frictions: everyone benefits from consumers not being so wary about the value delivered by the platform when deciding whether to get access to it. This insight does not only apply when consumers purchase the platform without observing the costs associated to using it, but rather it is more general. It also holds in cases in which consumers do not observe the quality of the goods sold by sellers before acquiring the platform, or when they do not observe the full variety of goods that will be offered through the platform. This may explain why quality assurance by platforms is common (as is the case for Nintendo). We believe that these topics present an interesting direction for further research.

Appendix A: Proofs

Proof of Proposition 3. If $p(w) = \Theta + \Sigma w$ and $B(w) = \Gamma + \Phi w$ for some parameters Θ, Σ, Γ and Φ to be determined, conditions (6) and (9) can be rewritten as

$$(1-\theta)(1-\Theta)-2\Sigma\Gamma+(\Theta+\Sigma\Gamma-\Gamma)2(\Sigma-1)+[2\Sigma(\theta-\Phi)+\Phi(\Sigma-1)2(\Sigma-1)]w_2 = 0$$

and

$$1-\theta+\theta\Sigma\Gamma-(2-\theta)\Theta+(1-2\Sigma+\theta\Sigma\Phi)w_2 = 0.$$

Since these two conditions should be satisfied for all w_2, we must have

$$(1-\theta)(1-\Theta)-2\Sigma\Gamma+(\Theta+\Sigma\Gamma-\Gamma)2(\Sigma-1) = 0, \tag{10}$$

$$2\Sigma(\theta-\Phi)+\Phi(\Sigma-1)2(\Sigma-1) = 0, \tag{11}$$

$$1-\theta+\theta\Sigma\Gamma-(2-\theta)\Theta = 0 \tag{12}$$

and

$$1-2\Sigma+\theta\Sigma\Phi = 0. \tag{13}$$

Rey and Vergé (2004) have already shown that there exists a unique tuple (Θ, Σ, Γ, Φ) that solves these equations and the required second-order conditions for the platform's maximization program, but we will give closed-form solutions that will prove useful later on.

When $\theta = 0$, it is easy to see that there is a unique solution to equations (10)-(13), given by $\Theta = 1/2$, $\Sigma = 1/2$, $\Gamma = 0$ and $\Phi = 0$. From (13), one obtains

$$\Phi = \frac{2\Sigma-1}{\theta\Sigma},$$

since it can be shown that there can be no solution with $\Sigma = 0$. Plugging this value for Φ in (11) allows us to rewrite it as the following cubic equation:

$$\Sigma^3 - \left(\frac{7-\theta^2}{2}\right)\Sigma^2 + \frac{5}{2}\Sigma - \frac{1}{2} = 0. \tag{14}$$

Letting

$$a \equiv -\frac{7-\theta^2}{2},$$

$$b \equiv \frac{5}{2},$$

$$c \equiv -\frac{1}{2},$$

$$K \equiv \frac{3b-a^2}{9}$$

and

$$L \equiv \frac{9ab - 27c - 2a^3}{54},$$

the solutions to the cubic equation are the following:

$$\Sigma_k = 2\sqrt{-K} \cos\left(\frac{1}{3} \arccos\left(\frac{L}{\sqrt{-K^3}}\right) + \frac{2\pi k}{3}\right) - \frac{a}{3} \ (k = 0, 1, 2).$$

The three roots are real, given that the discriminant $K^3 + L^2$ is negative for all $\theta \in (-1, 1)$. Plotting the three roots for all values of θ, it is easy to see that the only one which is equal to $1/2$ when $\theta = 0$ is Σ_2. Given that the solution must be continuous in θ, we know that $\Sigma = \Sigma_2$, that is,

$$\Sigma = \frac{7-\theta^2}{6} - \frac{(19 - 14\theta^2 + \theta^4)^{1/2}}{3} \sin\left(\frac{\pi}{6} - \frac{1}{3} \arccos\left(\frac{(1-\theta^2)(82 - 20\theta^2 + \theta^4)}{(19 - 14\theta^2 + \theta^4)^{3/2}}\right)\right).$$

From equation (12), we obtain

$$\Gamma = \frac{(2-\theta)\Theta - (1-\theta)}{\theta\Sigma},$$

so plugging it into (10) and rearranging yields that

$$\Theta = \frac{(1-\theta)[(6+\theta)\Sigma - 2(1+\Sigma^2)]}{4(3-\Sigma)\Sigma + 2\theta - (3\theta + \theta^2)\Sigma - 4}.$$

It therefore follows from (10) that

$$\Gamma = \frac{(1-\theta)(2\Sigma - 1)}{4(3-\Sigma)\Sigma + 2\theta - (3\theta + \theta^2)\Sigma - 4}.$$

Making it explicit that Θ, Σ, Γ and Φ depend on θ by writing Θ_θ, Σ_θ, Γ_θ and Φ_θ, it is easy to plot them and see that $0 \le \Theta_\theta \le 1$, $1/2 \le \Sigma_\theta \le 1$, $0 \le \Gamma_\theta \le 1$ and $-1 \le \Phi_\theta \le 1$ for all $\theta \in (-1, 1)$. Note that beliefs must be fulfilled in equilibrium, so $w^* = B(w^*)$ implies that

$$w^* = \frac{\Gamma_\theta}{1 - \Phi_\theta} \ge 0.$$

Also, the platform should find it optimal to choose $p_0 = p_0^*$ and $w_1 = w_2 = w^*$, so $(w^*, w^*, p_0^*) \in \underset{w_1, w_2, p_0}{\operatorname{argmax}}\ \widehat{\pi}_0(w_1, w_2, p_0)$, where

$$\begin{aligned} \widehat{\pi}_0(w_1, w_2, p_0) \quad = \quad & x(p_0) \left\{p_0 + w_1\, q_1(p_1(w_1), p_2(w_2)) + w_2\, q_2(p_2(w_2), p_1(w_1)) \right. \\ & + [p_1(w_1) - w_1]\, q_1(p_1(w_1), p_2(B(w_1))) \\ & \left. + [p_2(w_2) - w_2]\, q_2(p_2(w_2), p_1(B(w_2)))\right\}. \end{aligned}$$

Note that the optimal choices of w_1 and w_2 do not depend on the choice of p_0, so the platform provider can maximize with respect to w_1 and w_2 ignoring the value of p_0; the analysis above leading to expression (9) shows that private offers are chosen optimally, since second-order conditions are satisfied. To see this, note that (8) and the fact that

$$\frac{dq_1(p_1(w_1), p_2(B(w_1)))}{dw_1} = \frac{1}{1-\theta^2}\left(\frac{dp_1(w_1)}{dw_1} - 1\right),$$

imply that

$$\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1^2} = \frac{2(\Sigma_\theta^2 - 3\Sigma_\theta + 1)}{1-\theta^2}$$

and

$$\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1 \partial w_2} = \frac{2\theta\Sigma_\theta}{1-\theta^2}.$$

Thus, it follows from the fact that $\Sigma_\theta \geq 1/2$ that

$$\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1^2} \leq 0.$$

Also, it holds that

$$\left(\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1^2}\right)^2 - \left(\frac{\partial^2 \pi_0(w_1, f_1, w_2, f_2, p_0)}{\partial w_1 \partial w_2}\right)^2$$
$$= \frac{\Sigma_\theta(\Sigma_\theta^2 - 3\Sigma_\theta + 1)(\Sigma_\theta - 1) - (2\Sigma_\theta - 1)(\Sigma_\theta^2 - 3\Sigma_\theta + 1) - \theta^2\Sigma_\theta^2}{\left(\frac{1-\theta^2}{2}\right)^2},$$

which is nonnegative because $1/2 \leq \Sigma_\theta \leq 1$ and $(2\Sigma_\theta - 1)(\Sigma_\theta^2 - 3\Sigma_\theta + 1) + \theta^2\Sigma_\theta^2 = 0$ by (14). Thus, second-order conditions hold.

As for the optimal choice of p_0 given that seller $i \in \{1, 2\}$ receives an offer equal to (w^*, f^*), we need that

$$x(p_0) + [p_0 + 2p_1(w^*)q_1(p_1(w^*), p_2(w^*))]\frac{dx(p_0)}{dp_0} = 0,$$

so

$$p_0^* = \frac{(1-w^*)^2}{2(1+\theta)(2-\theta)^2} - \frac{2(\Theta_\theta + \Sigma_\theta w^*)(1 - \Theta_\theta - \Sigma_\theta w^*)}{2(1+\theta)},$$

which is negative for all $\theta \in (-1, 1)$. Finally, note that

$$p_i^* = \Theta_\theta + \frac{\Sigma_\theta \Gamma_\theta}{1 - \Phi_\theta} \ (i = 1, 2),$$

so $0 \leq p_i^* \leq 1$. It readily follows that

$$q_i^* = \frac{1 - p_i^*}{1+\theta} > 0,$$

$$\pi_0^* = \left(\frac{1 - (p_i^*)^2}{2(1+\theta)}\right)^2,$$

and

$$cs^* = \frac{1}{2}\left(\frac{1-(p_i^*)^2}{2(1+\theta)}\right)^2.$$

■

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